

# DAILY DEAL WEBSITES IN MARKETS WITH ASYMMETRIC INFORMATION

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## **Abstract**

We examine the role daily deal websites may serve in a two sided market with asymmetric information. Daily deal websites act as advertising intermediaries (platforms) by selling vendors' deals to consumers. We investigate whether these websites can reduce uncertainty by segmenting each side of the market. Such segmentation ensures an improved match between vendors and consumers in terms of the quality provided by the former and the willingness to pay for quality of the latter. Specifically, while one intermediary can potentially match relatively high quality vendors with high willingness to pay consumers, the other can match the opposite profiles of vendors and consumers. We derive two different patterns of the segmenting equilibrium, the characterization of which depends on the size of the segment of consumers who use the daily deal websites for the purpose of acquiring information. However, there is a long range of parameter values for which segmentation fails altogether. In this case, the intermediaries are completely undifferentiated and they randomly match vendors with consumers. Such random matching leads to fierce Bertrand competition between the intermediaries and zero profits. We also investigate the welfare effect of segmentation in comparison to random matching and show that segmentation does not necessarily improve social welfare.

**Keywords:** Two Sided Markets; Platforms; Asymmetric Information; Market Segmentation; Daily Deal Websites; Vertical Differentiation

## 1. INTRODUCTION

Daily deal websites, such as Groupon, LivingSocial, Google Offers, Amazon Local, and Double Take Deals, are websites that sell deals on services or products. Consumers who visit these websites and purchase such deals are given gift certificates or gift vouchers to later redeem with a specific vendor who has entered into an agreement with the daily deal website. Daily deal websites are platforms seeking to attract both vendors and consumers in a two-sided market. Consumers visit these platforms because they provide a vehicle to learn more about a specific product or service without having to pay full price for it. Vendors sell deals through these platforms because the platforms have a vast reach due to large consumer subscription bases. Often times these vendors are local and/or independent mom and pop shops who utilize daily deal websites in order to attract consumers to their business at a reduced price. Their hope is that upon having a good experience with the service, these consumers will return in the future to purchase the service again at full price.

As is the case with most platforms looking to lure many consumers on one side of the market in an effort to create network benefits for producers on the other side of the market, daily deal websites offer their services to consumers with no usage or subscription fees in order to build a consumer base. Having built consumer bases to which vendors want access, daily deal websites are able to keep a fraction of the revenue made from each deal sold on their website. It has been noted that the fraction of revenue that daily deal websites keep for themselves can reach upwards of 50%.<sup>1</sup> These platforms operate at the national level and consider themselves as advertising intermediaries that help expose unknown vendors to consumers.

However, selling through daily deal websites also introduces risks to the vendors. One such risk is the fact that deals may attract consumers who have no intention of making a repeat purchase at full price. A second risk is related to the heterogeneity in the vendor and consumer populations. With vendors offering products of different quality and consumers having different willingness to pay for higher quality, a mismatch between vendors and consumers types may limit the ability of vendors to extract surplus from consumers. In this paper we explore the possibility that daily deal websites may reduce the second type of risk by segmenting both the vendor and consumer markets. At such a segmenting equilibrium the platforms can increase the likelihood of correct matches between vendors and consumers.

In our model there are two daily deal websites (platforms). The vendor population is differentiated by quality and the consumer population is differentiated by willingness to pay for higher quality. Consumers cannot distinguish between high and low quality vendors, and similarly, vendors cannot distinguish between high and low willingness to pay consumers. We explore the possible existence

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<sup>1</sup> See <http://www.forbes.com/sites/chunkamui/2011/06/03/groupon-hysteria/>

of a segmenting equilibrium whereby each platform specializes in matching different segments of vendors and consumers. Specifically, while one platform matches high quality vendors with high willingness to pay consumers (high quality “matchmaker”), the competing platform matches the opposite profiles of vendors and consumers (low quality “matchmaker”.) This possible matching of different segments of the populations results in vertically differentiated platforms. However, in contrast to traditional models of vertical product differentiation, at a segmenting equilibrium platforms are differentiated not because they actively choose to offer different qualities of service. Rather, when different segments of the vendor population self-select to be represented by different platforms, part of consumers’ uncertainty regarding quality is alleviated, and quality differentiation between platforms endogenously arises. Put differently, it is the selection of platforms by the different segments of the vendor and consumer populations and not the active choice of quality by the platforms themselves that generates the vertical differentiation between the platforms.

We find that market conditions characterizing the rewards for high quality that vendors receive determine whether or not segmentation of the vendor and consumer populations can be supported. When the reward high quality vendors expect from repeat purchases by consumers is significant, the high quality platform offers a more generous deal to consumers than the low quality platform, and when this reward is low, it is the low quality platform that offers the more generous deal. However, unless there are either significantly high or low rewards to quality, a segmenting equilibrium fails to exist. In the absence of segmentation, platforms are not differentiated and random matching of vendors and consumers arises. Random matching leads, however, to fierce (undifferentiated Bertrand) competition between platforms, and therefore, to zero profits. We find that such random matching may arise, indeed, for a large range of values of the parameters of the model. The possible non-existence of a segmenting equilibrium and the undifferentiated competition between the platforms, can explain, to some extent, the recent poor performance of the daily deal websites.<sup>2</sup>

The difficulty in obtaining a segmenting equilibrium by using the deal price as the only instrument can explain why platforms may have to resort to additional instruments in order to facilitate differentiation. Groupon, for instance, sometimes requires a certain number of customers for a deal to go through. LivingSocial has no such tipping point rule.<sup>3</sup> LivingSocial is also known to pay back merchants quicker than Groupon.<sup>4</sup> The recent attempt by some daily deal websites such as FabFind.ca and Vipdeals to establish themselves as leaders in niche markets that match luxury merchants with consumers who

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<sup>2</sup> For instance, Groupon and Living Social, the two largest daily deal websites, lost \$256.7 million and \$558 million, respectively in 2011. In addition, in just the last half of 2011, 798 deal sites (including consolidations) closed. (<http://www.fastcompany.com/1822518/do-groupon-and-livingsocial-do-more-harm-good>)

<sup>3</sup> See <http://blog.laptopmag.com/face-off-groupon-vs-living-social>

<sup>4</sup> See <http://www.csmonitor.com/Business/The-Reformed-Broker/2011/0711/Tech-stocks-the-difference-between-Groupon-and-Living-Social>

have a penchant for luxury<sup>5</sup> is also consistent with our prediction that platforms have incentives to serve different segments of the populations of merchants and consumers in order to avoid undifferentiated price competition.

Our study contributes to four streams of research. The first is the literature on platform competition in two-sided markets. Much of this literature has investigated competition between horizontally differentiated platforms (Gabszewics et al. 2002, Armstrong 2006, Hagiu 2009, Gal-Or et al. 2012). One exception is a study by Brown and Morgan (2009) who investigate whether two platforms can coexist in equilibrium when one offers superior service than the other. In our paper, however, vertical differentiation between the platforms arises through self-selection by heterogeneous populations of consumers and vendors. The quality of the platform is not under its control. Rather, it is the segments of the consumer and vendor populations who choose to transact with the platform that determine its quality.

Similar to our study, in Hagiu and Spulber (2012) and Caillaud and Juilen (2003) as well platforms differ because of users' expectations about how the other side of the market behaves. In our model the consumer and vendor populations are heterogeneous. While one intermediary matches relatively high quality vendors with high willingness to pay consumers, the other matches the opposite profiles of vendors and consumers. In contrast, in Hagiu and Spulber (2012) and Caillaud and Juilen (2003) the populations on both sides of the market are homogenous. Platforms differ because these homogenous users expect one platform to provide higher user participation on the opposite side of the market than the other platform.

Our study also contributes to the literature on market segmentation that is implied by quality differentiation (Mussa and Rosen 1978, Katz 1984, Moorthy 1984, Schmid-Mohr and Villas-Boas 2008). In this literature firms have full control over the qualities of the products they offer and consumers self-select among the vertically differentiated products offered by these firms. In order to support segmentation the firms need to satisfy only one incentive compatibility constraint related to self-selection by consumers. However, for platforms that seek to match vendors with consumers, there is an additional incentive compatibility constraint related to self-selection by vendors. We show that the additional vendor incentive compatibility constraint makes it harder for these platforms to implement an equilibrium in which both sides of the market are segmented.

In our paper, daily deal websites act as a means for vendors to offer introductory prices to consumers in order to entice trial (Bagwell 1987). If segmentation exists, these introductory prices can differ across platforms and provide information about the quality of the vendors offering their products through the respective platforms. Methods for inducing trials have also been discussed in the couponing

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<sup>5</sup> See <http://www.newswire.ca/en/story/1020431/teambuy-ca-nabs-luxury-deal-site-fabfind-com>; [http://www.vipdeals.ae/site/how\\_it\\_work](http://www.vipdeals.ae/site/how_it_work)

literature (Ward and Davis 1978, Levedahl 1983). However, much of the literature on couponing focuses on the price discrimination benefits that coupons can support (Narasimhan 1984, Shaffer and Zhang 1995, Lu and Moorthy 2007). In our study, the focus is on the informational role of the daily deals. Specifically, mom and pop shops use daily deals to induce trials and inform consumers about the quality of their products through their platform choices.

So far only a limited number of studies have investigated daily deal websites. Some of these are empirical (Byers et al. 2011 and Dhloika 2011). Two analytical studies that have examined daily deal websites are Edelman et al. (2011) and Kumar and Rajan (2012). However, both are from the perspective of vendors. Unlike our paper, these studies have no strategic intermediary or intermediary competition.

The goal of this paper is two-fold. First, we seek to characterize conditions under which platforms can segment the market, and second, we investigate how such segmentation impacts the welfare of vendors, consumers, and platforms.

The rest of the paper is organized as follows. In the next section we develop the model and conduct the analysis. In Section 3 we simplify the model in order to obtain a sharper description of our results. Section 4 concludes the paper and offers suggestions for extending our model. The Appendix contains the proofs of all the propositions.

## 2. MODEL

We model an environment where mom and pop stores use daily deal websites to promote their services.<sup>6</sup> The objective of the websites is to encourage consumers to purchase the services at reduced prices in order to acquire information about the vendors. Vendors hope that based upon their early experience, consumers will return to purchase their services at full price. Hence, vendors use the daily deal websites as platforms to increase consumer awareness of their service offerings. We model, therefore, a two sided market in which platforms act as intermediaries in an attempt to attract and match consumers and vendors.

Even for a specific geographic region, there is often a multitude of vendors who offer a certain service for sale (e.g. roofing, Italian dinner, massage, etc.). Given their large number, it is many times the case that consumers are not aware of all vendors and have difficulty in distinguishing among them in terms of the quality of service they provide. We model this environment by assuming that there exists a continuum of vendors within a specific service category that are uniformly distributed based upon quality along the interval  $[\underline{q}, \bar{q}]$ . Vendor  $q$  sells a service of quality  $q$ ; however, the quality of each vendor is

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<sup>6</sup> For expositional convenience, we will limit the firm side of the market to service providers, which we will refer to as vendors throughout this paper.

private information available only to the vendor himself.<sup>7</sup> Because there is a continuum of vendors, each vendor is a price-taker and no single vendor has the power to set a price. This assumption is realistic in that mom and pop stores that utilize daily deal websites have little market power and find themselves in competition amongst many other vendors selling a similar service. Therefore, we assume that the full market price of service of quality  $q$  is exogenously determined by the function  $p(q)$ , where  $p(q)$  is increasing in  $q$ ; namely if the quality of the vendor is known, the higher his quality the higher the price he can charge. The price schedule  $p(q)$  may be determined, for instance, as unit cost plus a certain profit margin that is commonly acceptable in the vendor's industry. As the unit cost of higher quality service is normally higher, the schedule  $p(q)$  is an increasing function of  $q$ . There are, however, many vendors of very similar quality to that offered by the vendor of quality  $q$ , implying that each vendor is too small to have the market power to set prices.

There are two daily deal websites that act as intermediary platforms between the consumers and vendors; these platforms compete for both the vendors' and the consumers' business. Each platform sells deals to the consumers at price<sup>8</sup>  $R_i$  for  $i \in \{H, L\}$ . We assume that each platform has full control over its own deal price  $R_i$  and that it is as uninformed about vendor quality as consumers, implying that the same deal price is offered for all the platform's vendors.<sup>9</sup> Our rationale for assuming that platforms have complete price setting power is justified for several reasons. First, platforms operate on a national level, but the vendors with whom they contract are often specific to a geographic area. Second, these platforms have large consumer subscription bases that can only be reached using the platform. Finally, daily deal websites such as Groupon feature a single deal of the day in a specific product category; hence, businesses compete for these select features and Groupon has the final power in selecting the deal that they wish to offer.<sup>10</sup> We assume that each platform randomly selects only one vendor for which it features a deal.<sup>11</sup> For its services, each platform takes a share of the revenue; this share is denoted by  $\alpha$

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<sup>7</sup> That is, all vendors sell the same service with the exception of a distinction in quality.

<sup>8</sup> Deals are usually offered in the form of discounts or gift certificates. Our formulation of setting the actual price of the deal can be interpreted as offering a gift certificate that guarantees the final discounted price of the service.

<sup>9</sup> If platforms had more information about the quality of the vendors, they could potentially offer different deal prices to different vendors. We later conjecture about the consequences of relaxing the assumption of platforms' lack of information.

<sup>10</sup> It is possible that deal-of-the-day websites will also contract with larger companies in an effort to advertise their own services and grow their user base. However, we seek to model the more common case of these daily deal websites featuring deals from mom and pop shops.

<sup>11</sup> It is possible that vendors can submit bids for the right to sell through the intermediaries. However, we assume that the increased costs associated with running auctions outweigh the benefits. Indeed, we do not observe vendor bidding in major daily deal websites such as Groupon and LivingSocial.

and we will assume it to be the same for each vendor and platform.<sup>12</sup> For simplicity, we assume that neither the vendor nor the platform incurs any cost.

There is a continuum of consumers who are uniformly distributed based upon their willingness-to-pay for quality along the interval  $[\underline{\theta}, \bar{\theta}]$ , where  $\underline{\theta} > 0$ . The parameter  $\theta$  of the consumer determines her willingness to pay for quality. Specifically, the gross utility gain that a consumer of type  $\theta$  derives from the service provided by vendor  $q$  is  $a + \theta q$ . Her net gain depends on whether she buys directly from the vendor at the full price  $p(q)$  or from the platform at the reduced prices  $R_H$  or  $R_L$ . Specifically, in equations (1) and (2) we express the consumer's net payoff when buying a deal via platform  $i$  or directly from the vendor, respectively:

$$u_1(\theta) = a + \theta q - R_i, \tag{1}$$

$$u_2(\theta) = a + \theta q - p(q). \tag{2}$$

The parameter  $a$  measures the basic willingness to pay for the service that is unrelated to its quality. We assume it to be big enough to ensure that the net payoff of the consumer is positive even when paying full price for the service and irrespective of the quality provided by the vendor. For instance, if the service is a meal at a restaurant, even meals of low quality can curb hunger and provide some net utility from consumption. Consumers can purchase at most one deal for a given service from either one of the platforms.

In addition to the heterogeneity of the consumers in terms of their willingness to pay for higher quality, consumers differ also in terms of their interest in acquiring information about vendors for the purpose of guiding their future purchasing decisions. In this regard, we assume two different segments. The first segment consists of consumers who seek to learn about the service offered by a given vendor in order to determine whether they will continue to buy the service from him in the future. In contrast to the information seeking segment, the second segment consists of one-time shoppers who buy the service only once by using the daily deal websites with no intention of ever buying the service again for full price. We assume that a fraction  $\beta$  of all consumers consists of information seekers and a fraction  $(1 - \beta)$  consists of one-time shoppers.<sup>13</sup> If information seeking consumers have a positive experience with the service, they may return to buy the service from the same vendor at full price. We assume that the probability of return  $\varphi(q)$  is an increasing function of  $q$ . Hence, an information seeking consumer of type  $\theta$  receives the following expected utility when obtaining a deal from platform  $i$ , denoted by  $EU_i^I(\theta)$ :

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<sup>12</sup> For simplicity, we consider  $\alpha$  to be an exogenous parameter that is common to both platforms. In reality, platforms may differ somewhat in their sharing rules. It may be interesting to extend our analysis, therefore, so that the sharing rule chosen by each platform is a decision variable. We later conjecture on how such an extension is likely to affect our results.

<sup>13</sup> We may think of these one-time shoppers as individuals who purchase a deal while on a trip or have no need to ever make a future purchase from a given product category. In addition,  $\beta$  may vary across product categories. For instance, a deal for a concert may be less likely to attract information-seekers than a deal for a restaurant.

$$EU_i^I(\theta) = a + \theta * E[q|\text{consumer } \theta \text{ visits platform } i] - R_i + E[\varphi(q)[a + \theta q - p(q)]|\text{consumer } \theta \text{ visits platform } i]. \quad (3)$$

Note that with segmentation of the vendor population consumers can improve their estimate of the average quality offered via each platform. The conditional expected quality terms in (3) capture the updating of the information that is facilitated at a segmenting equilibrium. While the first term of (3) measures the expected net benefit of the consumer from the initial purchase via the platform, the second term measures the expected net benefit when she returns to buy the product directly from the vendor.

One-time shoppers visit daily deal websites with a sole interest of obtaining a service at a reduced deal price. These consumers have no intention of making a repeat purchase at full price; therefore, the probability that one-time shoppers return is  $\varphi(q) = 0$  for all  $q$ . A one-time shopper of type  $\theta$  receives the following expected utility when visiting platform  $i$ , denoted  $EU_i^D(\theta)$ :

$$EU_i^D(\theta) = a + \theta * E[q|\text{consumer } \theta \text{ visits platform } i] - R_i. \quad (4)$$

Because of our focus on small mom and pop vendors we assume that before consumers choose the platform they are barely aware of the vendor's existence, and therefore, do not have access to any information regarding the vendor's full price or quality. This is the reason that in (3) and (4) the consumer has to calculate the expected price and quality she is likely to encounter by choosing a given platform. In addition, Groupon's or Living Social's deals include very vague information about the full prices charged by featured vendors. An ad may state, for instance, that the full value of an Italian meal offered is \$30. Such a statement does not specify the full prices of different items on the menu, and as a result, it is of limited informational value to consumers in learning about the specific full price of the items she likes. In our modeling approach, we implicitly assume, therefore, that consumers cannot infer information regarding quality from the deal's advertised full value (by inverting the function  $p(q)$ .) With such a formulation we avoid the added complication that the daily deal website may potentially state the full value of the deal in order to signal the underlying quality of the vendor.

The timing of the game is as follows. First, each platform simultaneously sets a deal price,  $R_i$ . Second, given the knowledge of  $R_i$ , each vendor decides whether to sell their "trial" service through platform  $H$  or platform  $L$ . Simultaneously, consumers choose whether to buy a deal from platform  $H$  or platform  $L$ . After trying the service and learning of the service's quality, the information-seeking consumers may or may not return to the vendor and make a subsequent purchase from the vendor at full price.

## 2.1 Analysis

We seek to characterize an equilibrium in which high  $q$ -type vendors and high  $\theta$ -type consumers self-select to transact with platform  $H$  and low  $q$ -type vendors and low  $\theta$ -type consumers self-select to



transact with platform  $L$ . We will refer to such an equilibrium that segments the markets as a segmenting equilibrium. Figure 1 depicts a segmenting equilibrium as defined above.<sup>14</sup> At the segmenting equilibrium vendors of type  $q > q^*$  and consumers of type  $\theta > \theta^*$  choose platform  $H$  and vendors of type  $q < q^*$  and consumers of type  $\theta < \theta^*$  choose platform  $L$ .

[INSERT FIGURE 1 HERE]

As can be seen in Figure 1, we seek equilibria where two segments exist on each side of the market and platforms are exclusively assigned to one segment on each side. If we were to assume that platforms have information about either side of the market (i.e., if the platforms were to track consumer behavior or learn about vendors through repeated interactions) platforms could further segment their populations of consumers and vendors by offering distinct deal prices to different groups.

In order to simplify the analysis we use specific functional forms for the full price,  $p(q)$ , and the return probability schedules,  $\varphi(q)$ , as follows:  $p(q) = bq$  and  $\varphi(q) = cq$ , where  $b, c > 0$ . Hence, the parameters  $b$  and  $c$  determine the steepness of these schedules. The linear specification of the functions implies that the price and probability schedules increase at constant rates as quality increases. If we assumed a nonlinear specification so that the rate of increase of the functions was diminishing (increasing), instead, the reward to higher quality would be more moderate (more significant, respectively) than in our model. In conducting the analysis, we use the slopes  $b$  and  $c$  as measures of the reward to high quality. With a nonlinear specification, additional variables related to the rate of change in these slopes would also have to be considered.

We start by analyzing the choice of the consumers between the two platforms. At a segmenting equilibrium, consumers know that platform  $H$  offers deals for vendors of relatively high quality ( $q > q^*$ ) and platform  $L$  offers deals for vendors of relatively low quality ( $q < q^*$ ). They use this information to update their expected net utility when buying from each platform. For the information seeking consumers the expected net utilities are:

$$EU_H^I(\theta) = a + \theta * E[q|q > q^*] - R_H + E[cq[a + \theta q - bq]|q > q^*], \quad (5)$$

$$EU_L^I(\theta) = a + \theta * E[q|q < q^*] - R_L + E[cq[a + \theta q - bq]|q < q^*]. \quad (6)$$

As pointed out earlier, information seekers use the platforms to obtain information about vendors. Because of the segmentation of the vendor population they know that platform  $H$  represents, on average, higher quality vendors and platform  $L$  represents lower quality vendors. They use this information in calculating the average quality of vendors serviced by each platform.

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<sup>14</sup> The designation of the platforms is made without any loss of generality. We could also solve for the opposite arrangement in which high  $q$ -type vendors and high  $\theta$ -type consumers interact with platform  $L$  and low  $q$ -type vendors and low  $\theta$ -type consumers interact with platform  $H$ .

From (5) and (6) we can calculate the utility gain that an individual of type  $\theta$  receives from visiting platform  $H$  as opposed to visiting platform  $L$ :

$$\Delta^I(\theta) \equiv EU_H^I(\theta) - EU_L^I(\theta) = \frac{(\bar{q}-q)}{2}\theta + c \left[ \frac{a(\bar{q}-q)}{2} + \frac{(\theta-b)(\bar{q}-q)(\bar{q}+q+q^*)}{3} \right] - (R_H - R_L). \quad (7)$$

From (7), it is clear that the utility gain of buying from  $H$  rather than  $L$  is increasing in  $\theta$ . Hence, segmentation of the information seeking consumers might be possible if there exists a  $\theta^{I^*}$ -type inside the support  $[\underline{\theta}, \bar{\theta}]$  such that  $\Delta^I(\theta^{I^*}) = 0$ . For types  $\theta < \theta^{I^*}$   $\Delta^I(\theta) < 0$  and for  $\theta > \theta^{I^*}$   $\Delta^I(\theta) > 0$ , implying that the information seeking consumers self-select the platforms as predicted at a segmenting equilibrium. Solving the equation  $\Delta^I(\theta) = 0$  for  $\theta$  in (7) yields:

$$\theta^{I^*} = \frac{c[2b(\bar{q}+q+q^*)-3a]}{[3+2c(\bar{q}+q+q^*)]} + \frac{6(R_H-R_L)}{(\bar{q}-q)[3+2c(\bar{q}+q+q^*)]}. \quad (8)$$

One-time shoppers have no intention of a repeat purchase. Therefore, they seek to maximize their net expected utility when buying the service only once via one of the platforms as follows:

$$EU_H^D(\theta) = a + \theta * E(q|q > q^*) - R_H, \quad (9)$$

$$EU_L^D(\theta) = a + \theta * E(q|q < q^*) - R_L. \quad (10)$$

Constructing the difference in the net utility from visiting platform  $H$  as opposed to platform  $L$ , we obtain:

$$\Delta^D(\theta) \equiv EU_H^D(\theta) - EU_L^D(\theta) = \frac{(\bar{q}-q)}{2}\theta - (R_H - R_L). \quad (11)$$

Segmentation of the one-time shoppers is feasible if there exists  $\theta^{D^*}$  such that  $\Delta^D(\theta^{D^*}) = 0$ . Because the function  $\Delta^D(\theta)$  is increasing in  $\theta$ , consumers of type  $\theta < \theta^{D^*}$  will choose platform  $L$  and those of type  $\theta > \theta^{D^*}$  will choose  $H$ . Solving the equation  $\Delta^D(\theta) = 0$  yields:

$$\theta^{D^*} = \frac{2(R_H-R_L)}{(\bar{q}-q)}. \quad (12)$$

Comparing the expressions derived for  $\theta^{I^*}$  and  $\theta^{D^*}$  in (8) and (12) we notice that whereas  $\theta^{I^*}$  depends on the values of the parameters  $b$  and  $c$ ,  $\theta^{D^*}$  does not. The parameters  $b$  and  $c$  are both related to the consequences of repeat purchase by consumers. The parameter  $b$  measures the steepness of the full price schedule and the parameter  $c$  is related to the probability of repeat purchase by the consumer. Because one-time shoppers do not intend to ever buy the service again for full price, the values of these parameters do not affect their behavior. The values of both  $\theta^{I^*}$  and  $\theta^{D^*}$  increase when the gap  $(R_H - R_L)$  increases, as more consumers opt to purchase the relatively cheaper deal from  $L$ . Note also that the value of  $\theta^{I^*}$  increases when  $b$  increases and  $a$  declines. Changes in the parameter  $c$  have an ambiguous effect on the values of  $\theta^{I^*}$ . When  $b$  is relatively big in comparison to  $a$ , purchasing high quality service is much more expensive than low quality service when the consumer returns to the same vendor and pays

full price for the service. Factoring this higher price differential implies that more consumers will choose to experiment with the deal offered by the low quality platform and  $\theta^{I*}$  increases.

For simplicity, define  $x$  as the fraction of information-seeking consumers who visit platform  $L$ , namely  $x = \frac{\theta^{I*} - \theta}{\theta - \underline{\theta}}$ . Therefore,  $1 - x$  is the fraction of information-seeking buyers who visit platform  $H$ . Similarly, define  $y$  as the fraction of one-time shoppers who visit platform  $L$ , namely  $y = \frac{\theta^{D*} - \theta}{\theta - \underline{\theta}}$ . Hence,  $1 - y$  is the fraction of one-time shoppers who visit platform  $H$ . Finally, define  $z$  as the fraction of vendors that sell a trial through platform  $L$  and  $1 - z$  as the fraction of vendors that sell a trial through platform  $H$ , thus  $z = \frac{q^* - q}{q - \underline{q}}$ , where  $q^*$  is the quality of the vendor who is indifferent between selling deals through platform  $L$  and selling deals through platform  $H$ .

With knowledge of consumer strategies, vendors seek to maximize their own profits by choosing whether to sell a product through platform  $H$  or platform  $L$ . A vendor of quality  $q$  receives the expected profit of  $E\pi_H(q)$  and  $E\pi_L(q)$  by selling through platform  $H$  and platform  $L$ , respectively, expressed by the following equations.<sup>15</sup>

$$E\pi_H(q) = \beta \frac{(1-x)}{(1-z)} [(1-\alpha)R_H + bcq^2] + (1-\beta) \frac{(1-y)}{(1-z)} (1-\alpha)R_H, \quad (13)$$

$$E\pi_L(q) = \beta \frac{x}{z} [(1-\alpha)R_L + bcq^2] + (1-\beta) \frac{y}{z} (1-\alpha)R_L. \quad (14)$$

The first term in both (13) and (14) is profits from information seekers and the second term is profits from one-time shoppers. While the profits from information seekers accrue both from the vendor's share of the deal ( $(1-\alpha)R_i$ ) and the expected revenues from repeat purchases ( $bcq^2$ ), the profits from one-time shoppers accrue only from the vendor's share of the deal. Note also that the expected volume of customers that are likely to patronize a certain vendor via a given platform depends on the density of consumers per vendor at this platform. In the population of information seekers, this density is  $\frac{(1-x)}{(1-z)}$  for the high quality platform and  $\frac{x}{z}$  for the low quality platform. For the population of one-time shoppers, the densities are  $\frac{(1-y)}{(1-z)}$  and  $\frac{y}{z}$  for  $H$  and  $L$ , respectively. Given that one vendor is selected randomly to be featured, it is not the absolute size of the consumer population who visit the platform, but their relative size in comparison to the size of the population of the vendors that choose this platform that determines the volume of customers that a given vendor can expect (for platform  $H$ , for instance, this relative size is given by  $\frac{(1-x)}{(1-z)}$  and  $\frac{(1-y)}{(1-z)}$  for the two segments of consumers). In order to focus on the informational benefits of daily deal websites, we have assumed that no consumers in the

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<sup>15</sup> Recall that each vendor is assumed to know his own quality level, but cannot observe either the quality of other vendors or willingness to pay for quality of the consumers.

population are directly familiar with the vendors unless they use the services of the platforms. This explains why there are no terms in (13) and (14) that capture profits that accrue from informed consumers who buy directly from the vendors without the assistance of the platforms<sup>16</sup>.

As with the consumer population, in order to ensure the existence of segmentation of the vendor population, there should be a vendor of quality  $q^*$  in the support of the vendor population  $[q, \bar{q}]$  such that this vendor is indifferent between the two platforms. All vendors of quality  $q < q^*$  should prefer platform  $L$  and those of quality  $q > q^*$  should prefer platform  $H$ . Designating by  $\Delta\pi(q)$  the added profits of a vendor of type  $q$  when transacting with  $H$  rather than  $L$ , we obtain from (13) and (14) that:

$$\begin{aligned} \Delta\pi(q) \equiv E\pi_H(q) - E\pi_L(q) &= \beta bcq^2 \left[ \frac{1-x}{1-z} - \frac{x}{z} \right] \\ &+ (1-\alpha) \left[ R_H \frac{\beta(1-x)+(1-\beta)(1-y)}{1-z} - R_L \frac{\beta x+(1-\beta)y}{z} \right]. \end{aligned} \quad (15)$$

The value of  $q^*$  satisfies the equation  $\Delta\pi(q^*) = 0$ . Note that the vendor of quality  $q$  takes the values  $\theta^{I^*}$ ,  $\theta^{D^*}$ , and  $q^*$  as given when calculating the added benefit he derives from platform  $H$  and  $L$ . Hence, it is only the first term of (15) that depends upon the vendor's own quality level. In particular, the sign of the coefficient of  $q$  in this term  $\left[ \frac{1-x}{1-z} - \frac{x}{z} \right]$  determines whether the function  $\Delta\pi(q)$  is increasing or decreasing in  $q$ . To ensure the existence of a segmenting equilibrium where high  $q$ -types transact with  $H$ , the function should increase in  $q$ , implying that:

$$\frac{1-x}{1-z} > \frac{x}{z}. \quad (16)$$

Hence, the density of customers per vendor in platform  $H$  should exceed the density of customers per vendor in platform  $L$  in order to support the segmentation of vendors. Put differently, in order to support segmentation, platform  $H$  should foster a more exclusive environment for vendors than platform  $L$ , in the sense that relative to its customer base, platform  $H$  represents fewer vendors than platform  $L$ . Inequality (16) implies that  $z > x$  at the segmenting equilibrium, indicating that platform  $L$  attracts more vendors than customers.<sup>17</sup>

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<sup>16</sup> If we allowed for a specific proportion of consumers to be informed and buy directly from merchants, our results would not be drastically affected as long as this proportion is sufficiently small as compared to the proportion of consumers using daily deal websites to buy merchants' products. During deal periods, this is likely to be the case for mom and pop stores who see an influx of consumers redeeming deals. See [http://www.cbsnews.com/8301-505143\\_162-41841181/should-your-company-use-groupon-to-increase-sales/](http://www.cbsnews.com/8301-505143_162-41841181/should-your-company-use-groupon-to-increase-sales/)

<sup>17</sup> We can interpret the condition  $z > x$  as the "Single Crossing Property" necessary to separate the segment of high quality vendors from the segment of low quality vendors. This property is usually imposed to guarantee the existence of separating equilibria in environments with asymmetric information.

Finally, we characterize the platforms' maximization problems given the strategies of the vendors and the consumers. Platform  $H$  seeks to maximize its expected payoff,  $EV_H$ , by choosing an optimal  $R_H$  and platform  $L$  seeks to maximize its expected payoff,  $EV_L$ , by choosing an optimal  $R_L$ :

$$\max_{R_H} EV_H = [\beta(1-x) + (1-\beta)(1-y)]\alpha R_H, \quad (17)$$

$$\max_{R_L} EV_L = [\beta x + (1-\beta)y]\alpha R_L. \quad (18)$$

From (17) and (18), we observe that a change in the deal price of a given platform has counteracting effects on its profits. On the positive side, it raises its markup, but on the negative side, it reduces the demand by shifting the indifferent consumer in (8) and (12) in favor of the competing platform, thus affecting the values of  $x$  and  $y$ . Note that even though platforms do not charge consumers any subscription fees their profits depend on their market shares among consumers ( $1-x$  and  $1-y$  for  $H$  and  $x$  and  $y$  for  $L$ .) When consumers sign up with one of the platforms they end up choosing to purchase a "trial" product from one of the vendors represented by this platform. Such purchases constitute the source of revenues of the platform.

In Proposition 1 we provide some insight to the characteristics of a segmenting equilibrium if it exists.

**Proposition 1**

- (i) *For a sufficiently large  $\beta$  close to one, if a segmenting equilibrium exists, the high quality platform offers a more generous deal to consumers than the low quality platform, so that  $R_L > R_H$ . At such an equilibrium all one-time shoppers purchase the deal from platform  $H$ , namely  $y = 0$ .*
- (ii) *A segmenting equilibrium does not exist if all consumers are one-time shoppers, namely if  $\beta = 0$ . For sufficiently small but positive  $\beta$  values close to zero, if a segmenting equilibrium exists, the low quality platform offers the more generous deal to consumers so that  $R_L < R_H$ .*
- (iii) *At any segmenting equilibrium the density of information seeking consumers per vendor is higher at  $H$  than at  $L$ , namely  $z > x$  (i.e.,  $\frac{1-x}{1-z} > \frac{x}{z}$ ).*

According to Proposition 1 when most of the consumer population consists of information seekers the high quality platform offers a better deal to consumers than the low quality platform at the segmenting equilibrium ( $R_H < R_L$ ). This result might appear counterintuitive, as high quality services are usually sold for higher prices. However, as high quality vendors seek to distinguish themselves from low quality vendors, they agree to offering consumers a more generous deal for the purpose of experimentation, with the knowledge that most of these consumers will return to purchase the service again. Consumers will then pay the full higher price that higher quality services can command. Low quality vendors do not have an incentive to mimic the choice of platform selected by high quality vendors

in this case because the lower revenue they can expect from repeat purchases does not compensate for the difference in deal prices ( $R_L - R_H$ ) that such vendors experience when switching from platform  $L$  to platform  $H$ . In our setting, the lower deal price offered by platform  $H$  facilitates separating high quality from low quality vendors. Note that all one-time shoppers choose platform  $H$  in this case, namely  $y = 0$ . They pay a lower price and, on average, anticipate higher quality services when buying the deal from  $H$ .

Part (ii) of Proposition 1 reports that in order to support segmentation it is essential that some consumers are information seekers who truly wish to learn about the service, so that the experience can guide them in future purchasing decisions. If all consumers are one-time shoppers, there is no incentive for high quality vendors to distinguish themselves from low quality vendors, as no consumer intends to ever buy the service for full price. Note that when  $\beta = 0$ ,  $\Delta\pi(q)$  in (15) is a constant independent of  $q$ , thus all vendors will have an identical preference for one of the platforms, and segmentation of vendors is not feasible. Part (ii) states also that for small positive values of  $\beta$  close to zero, if segmentation arises,  $R_L < R_H$ . When most of the population consists of one-time shoppers who never intend to pay full price, high quality vendors have no incentive to agree to a more generous deal offered to consumers by the platform, as they can expect only very few repeat purchases of their service. Instead, they insist on obtaining a higher price from the platform right away, consistent with the higher quality they provide.<sup>18</sup> Part (iii) of the proposition guarantees that platform  $H$  offers a less competitive environment for vendors than platform  $L$  because relative to the size of its customer base, platform  $H$  signs up fewer vendors. This assumption is necessary to ensure that the added benefit vendors derive from selling their deal via  $H$  rather than  $L$  is an increasing function of  $q$ . Specifically, the function  $\Delta\pi(q)$  defined in (15) is strictly increasing.

The results we report in Proposition 1 are rather weak. We are unable to describe the conditions necessary for existence, and we provide characterization of the segmenting equilibrium only in the neighborhoods of  $\beta = 1$  and  $\beta = 0$ . In order to obtain a more complete characterization, in the next section we further simplify our model by assuming that the probability of repeat purchase of an information seeking consumer is a constant, independent of  $q$ ; specifically,  $\varphi(q) = c$ .<sup>19</sup>

### 3. CONSTANT PROBABILITY OF REPEAT PURCHASE

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<sup>18</sup> Note that when  $\beta \rightarrow 0^+$  and  $R_H < R_L$ ,  $y = 0$  and from (15)  $\lim_{\beta \rightarrow 0^+} \Delta\pi(q) = \frac{(1-\alpha)R_H}{(1-z)} > 0$  for all  $q$ . Hence, all vendors will choose platform  $H$  and segmentation is not feasible. This implies that when  $\beta \rightarrow 0^+$ , it cannot be the case that  $R_H < R_L$ .

<sup>19</sup> We could also think of  $c$  as the probability that an information seeker will have available funds to purchase the service at full price. Such funds could be available if, for example, she does not need to spend much on other products/services in the second period.

In order to facilitate a more complete characterization, in this section we assume that the probability of repeat purchase by an information seeking consumer is constant and only the full market price is an increasing function of quality. This simplification implies that the reward to a higher quality vendor from repeat purchase stems only from the higher full market price,  $p(q) = bq$ , and not from the higher probability of purchase. Alternatively, we could have assumed that the market price is a constant independent of quality while the probability of repeat purchase is an increasing function of quality.

When the probability of repeat purchase is a constant equal to  $c$ , the expected utility of an information seeking consumer when buying a deal from platform  $i$  is:

$$EU_i^I(\theta) = a + \theta * E[q|\text{consumer } \theta \text{ visits platform } i] - R_i + cE[a + \theta q - bq|\text{consumer } \theta \text{ visits platform } i]. \quad (19)$$

We will continue to assume that there exists a segment of one-time shoppers of size  $(1 - \beta)$  who have no intention of returning to the service provider. Therefore,  $EU_i^D(\theta)$  remains the same as in (4) for such consumers.

As in (7), we can calculate the function  $\Delta^I(\theta)$  to measure the added benefit an information seeking consumer of type  $\theta$  derives when buying a deal from  $H$  rather than  $L$ . By solving the equation  $\Delta^I(\theta) = 0$ , we obtain the consumer who is indifferent between platforms  $H$  and  $L$  as follows:

$$\theta^{I*} = \frac{2(R_H - R_L)}{(1+c)(\bar{q}-q)} + \frac{cb}{(1+c)}. \quad (20)$$

Note that in contrast to the expression we obtained in (8), the solution for  $\theta^{I*}$  in (20) is independent of  $q^*$ . The simplification we use for the repeat probability purchase function leads to the simplification in the expression for  $\theta^{I*}$ . Note also that the expression for  $\theta^{D*}$  remains as in (12), given that one-time shoppers do not purchase the service a second time.

In Proposition 1 we reported that at the segmenting equilibrium the high quality platform may offer the more generous deal (part (i)) or the less generous deal (part (ii)) to consumers. In Proposition 2 and 3 we derive conditions to support each type of equilibrium for the simplified case we consider in the present section.

**Proposition 2** *When the probability of repeat purchase is constant, a segmenting equilibrium with  $R_H < R_L$  exists:*

(i) *If the market price schedule is sufficiently steep and the segment of information-seeking consumers is sufficiently big. Specifically,*<sup>20</sup>

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<sup>20</sup> To ensure that the ‘‘Single Crossing Property’’ for vendors holds (i.e.  $z > x$ ), an additional upper bound on  $b$  is necessary. Specifically,  $b < \frac{1+c}{c} \left[ \frac{(\bar{\theta}-\underline{\theta})(2+3\beta+3\sqrt{4\beta+\beta^2-4})}{4\beta} + \underline{\theta} \right]$ . This upper bound ensures also that  $x \in \left(\frac{1}{2}, 1\right)$ . It is bigger than  $LB_{(H<L)}$  for all values of  $c$  and  $\beta > 2\sqrt{2} - 2$ .

$$b > LB_{(H<L)} > \frac{1+c}{c} \left[ \frac{\bar{\theta}-\underline{\theta}}{2\beta} + \underline{\theta} \right], \text{ and} \quad (21)$$

$$\beta > 2\sqrt{2} - 2.$$

(ii) At such an equilibrium all the one-time shoppers purchase the service from the high quality platform ( $y = 0$ ), but the low quality platform obtains a larger share of both information seeking consumers and vendors ( $z > x > \frac{1}{2}$ ). Nevertheless, the density of information seekers per vendor is higher at the high quality platform ( $\frac{1-x}{1-z} > \frac{x}{z}$ ).

(iii) The gap between the deal prices of the two platforms is given as:

$$R_L - R_H = \frac{(\bar{q}-\underline{q})}{6} \left[ 2bc - (1+c) \left[ \frac{\bar{\theta}-\underline{\theta}}{\beta} + 2\underline{\theta} \right] \right].$$

This gap increases with  $b, c$ , and  $\beta$ .

According to part (i) of Proposition 2, a segmenting equilibrium with  $R_H < R_L$  exists if the reward to high quality vendors from repeat purchases by information seekers is sufficiently high. This reward is more significant when the size of the segment of information seeking consumers is big ( $\beta > 2\sqrt{2} - 2$ ) and when the market price schedule is very steep ( $b$  is relatively large). When the reward from future purchases is large, high quality vendors distinguish themselves from low quality vendors by agreeing to offer customers a better introductory deal via the platform that represents them. Low quality vendors have no incentive to mimic this behavior because they do not expect a very high reward from information seekers who return to purchase their service again, given the significant steepness of the market price schedule. Notice from (21) that the slope of the market price schedule has to significantly exceed the average valuation for quality in the population ( $\frac{\bar{\theta}+\underline{\theta}}{2}$ ) in order to support this type of segmenting equilibrium. Moreover, the smaller the parameters  $\beta$  and  $c$  are the bigger the required steepness of the price schedule,  $b$ , that can support this equilibrium. In addition, if we were to allow the information seekers to defect and buy a randomly selected vendor via the platforms a second time after experiencing the product, it would become even more difficult to obtain an equilibrium, as there will be less incentive for the high quality vendors to accept the lower deal price in the first period.<sup>21</sup>

According to part (ii), in spite of agreeing to offer customers a better introductory deal, the high quality platform actually attracts a smaller fraction of the information seeking consumers than the low quality platform ( $x > \frac{1}{2}$ ). Given that this type of equilibrium is supported only when  $b$  is relatively large

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<sup>21</sup> Whether or not a consumer would return to the platform would be a function of the match between the consumer's  $\theta$  and the vendor's  $q$ . In any case for deals for service categories that are run infrequently a customer may not have the option to return to the platform and purchase another deal from a different vendor in the same category.



information seekers anticipate paying a much higher price for higher quality when purchasing it again for full market price. More of them choose, therefore, to experiment with the platform that is known to represent low quality vendors. Even though this platform charges them a high initial deal price, if they have a positive experience with the selected vendor they can expect to pay a much lower full price when they purchase the service a second time<sup>22</sup>. In contrast, all one-time shoppers buy the deal from the high quality platform because they have no concern about future payments. Given that the high quality platform offers a better deal, and on average, represents higher quality vendors, all one-time shoppers choose to transact with  $H$ . A bigger fraction of the vendor population chooses the low quality platform ( $z > \frac{1}{2}$ ) because this platform attracts also a bigger portion of the information seekers. However, in order to ensure that segmentation of the vendor population is feasible we need that the density of customers per vendor is higher at  $H$  than at  $L$ .

Part (iii) of the Proposition shows that the gap in the deal prices of the two platforms is more significant when the parameters  $b$ ,  $c$ , and  $\beta$  are bigger. In fact, when  $c$  and  $\beta$  are bigger,  $LB_{(H<L)}$  in (21) is smaller, thus relaxing the lower bound on the slope of the price schedule that is necessary to support the equilibrium. Hence, a segmenting equilibrium characterized by  $R_H < R_L$  is more likely as the probability of repeat purchase, the size of the segment of information seekers, and the steepness of the quality contingent market price are all bigger.

**Proposition 3** *When the probability of the return purchase is constant, a segmenting equilibrium with  $R_H > R_L$  exists:*

(i) *If the market price schedule is sufficiently flat,<sup>23</sup> and the segment of information seekers is sufficiently small. Specifically,*

$$b < UB_{(H>L)} < \frac{(\bar{\theta} + \underline{\theta})(1+c)}{[3(1+c) - \beta c]}, \text{ and} \quad (22)$$

$$\beta < \hat{\beta} < 1.$$

(ii) *At such an equilibrium the high quality platform captures more than 50% of the information seeking consumers and vendors ( $x < z < \frac{1}{2}$ ). The density of information seeking consumers per vendor is higher*

<sup>22</sup> Note that we assume that consumers choose to return to the same vendor from which they bought the “trial” service if their experience was positive (with probability  $c$ ). In particular, they do not switch to another vendor when buying the service a second time. The lack of incentive to switch may be related to good rapport established with the current vendor, absence of information about competing vendors, or high switching costs.

<sup>23</sup> To ensure that  $x > 0$  and  $y < 1$ , two lower bounds on  $b$  have to be imposed as well:  $LB_1 = \frac{(1+c)[\underline{\theta}(2+3c(1-\beta)) - \bar{\theta}]}{c[3(1+c) - \beta(2+3c)]}$  for  $x > 0$  and  $LB_2 = \frac{[(1+c)\underline{\theta} - \bar{\theta}(2(1+c) - 3c\beta)]}{2c\beta}$  for  $y < 1$ . There exist values of the parameters  $c$  and  $\beta$  in the feasible region that ensure these lower bounds to be smaller than  $UB_{(H>L)}$ .

and the density of one-time shoppers per vendor is lower at the high quality platform than at the low quality platform  $\left(\frac{1-x}{1-z} > \frac{x}{z} \text{ and } \frac{1-y}{1-z} < \frac{y}{z}\right)$ .

(iii) The gap between the prices charged by the two platforms is given as:

$$R_H - R_L = \frac{(\bar{q}-q)[(1+c)(\bar{\theta}+\underline{\theta})-2bc\beta]}{6(1+c-\beta c)}.$$

This gap increases with  $c$  and  $\beta$  and declines with  $b$ .

According to Proposition 3 a segmenting equilibrium with  $R_H > R_L$  exists only if the reward from repeat purchases that higher quality vendors can expect is not too large ( $b$  and  $\beta$  are relatively small). At such an equilibrium the high quality platform commands a larger fraction of information seeking consumers and vendors  $\left(x < \frac{1}{2} \text{ and } z < \frac{1}{2}\right)$ . In order to ensure segmentation of the vendor population (Single Crossing Property), we need that  $x < z < y$ . When information seekers face a relatively flat quality contingent price schedule, they are encouraged to experiment using the deal offered by the high quality platform in spite of the higher price deal that this platform charges. Because they can expect to pay only marginally more for higher quality service if they decide to purchase the service again, they have a stronger incentive to experiment via the platform that represents the higher quality vendors. In contrast to the results reported in Proposition 2, segmentation in Proposition 3 is achieved with the deal price more correctly representing the quality of the set of vendors that self-select to transact with the platforms; namely the vendors that offer, on average, higher quality command also a higher price for the deal they offer to consumers via the platform.

Note that in both Propositions 2 and 3 segmentation is facilitated only when the rewards to quality are significantly high or low. The required large  $b$  values necessary to support the equilibrium in Proposition 2 and the small  $b$  values necessary to support the equilibrium in Proposition 3 are needed to ensure either high rewards or low rewards. Proposition 4 shows us that under moderate rewards to quality, segmentation cannot exist.

**Proposition 4** For intermediate values of the slope of the market price schedule  $b$  in the interval  $(UB_{(H>L)}, LB_{(H<L)})$  no segmenting equilibrium exists.

An inspection of the bounds imposed on  $b$  in (21) and (22) yields that  $UB_{(H>L)} < LB_{(H<L)}$ , implying that there is a nonempty interval of  $b$  values that cannot support segmentation. See Figure 2 for a visual representation of Proposition 4. Intermediate values of  $b \in (UB_{(H>L)}, LB_{(H<L)})$  do not generate sufficiently different deal prices offered by the platforms, thus precluding the segmentation of the two-sided market. In absence of segmentation the platforms are not differentiated and the matching of

consumers to vendors is completely random. In addition, in the absence of differentiation, platforms compete fiercely on deal prices, and marginal cost pricing implies that  $R_H = R_L = 0$ . Hence, platforms and vendors can definitely benefit from segmentation. In the absence of segmentation undifferentiated competition forces the platforms (or the vendors themselves) to offer “trial” service to new uninformed consumers for free. With segmentation the “trial” services can be offered at positive prices. Segmentation may have, however, an ambiguous effect on the welfare of consumers. On the positive side, segmentation guarantees an improved match between the quality of the vendor and the willingness to pay for quality of the customer. On the negative side, while with random matching consumers can experiment with the new service for free, with segmentation they have to pay a positive price when using the service for the first time.

[INSERT FIGURE 2 HERE]

In spite of the fact that the platforms can benefit from segmentation, Proposition 4 illustrates that it can be quite difficult for them to implement such an outcome. Only very extreme values of the parameters of the model can actually support the segmenting equilibrium. In particular, the slope of the quality-contingent price schedule,  $b$ , has to be either very steep or very flat, and at any rate much different from the average valuation of quality in the population  $\left(\frac{\bar{\theta} + \theta}{2}\right)$  to support segmentation. The mean valuation of quality in the population should normally determine the steepness of the price schedule. With segmentation, the platforms are vertically differentiated. However, this vertical differentiation is not the result of platforms actually having control over the quality of the service they provide. Instead, the differentiation is the result of different segments of the vendor and consumer populations choosing to interact with different platforms. In traditional models of vertical product differentiation, producers have full control over the qualities of the products they offer. In order to support segmentation they need to satisfy only one incentive compatibility constraint related to self-selection of the differentiated products by consumers. For intermediaries that seek to match vendors with consumers, there are, in fact, two separate incentive compatibility conditions that constrain the ability of the platforms to implement segmentation. It is not only the choice of consumers but that of the vendors as well that has to be incorporated in ensuring the segmentation of each side of the market. The additional self-selection constraint of the vendors makes it more difficult to implement equilibrium with vertically differentiated platforms.

[INSERT TABLES 1 & 2 HERE]

The numerical calculations we conduct in Tables 1 and 2 illustrate the difficulty in obtaining the segmentation of the vendor population (the condition  $z - x > 0$ ). In the example used in the Tables the mean willingness to pay for quality is  $\frac{(\bar{\theta} + \underline{\theta})}{2} = 1.55$ . To obtain the segmenting equilibrium with  $R_H < R_L$  (Table 1), large values of  $\beta$  and  $b$  are necessary. For  $\beta = 0.95$  and  $c = 0.95$  for instance, the condition  $z - x > 0$  holds if  $b > 3.9$ . Hence, the steepness of the price schedule has to be more than twice the mean willingness to pay for quality in the consumer population. For smaller values of  $\beta$  and  $c$  the price schedule should be even steeper to support segmentation (when  $\beta = c = 0.85$ ,  $b_{min} = 6.9$ .) Similarly, to support segmentation with  $R_H > R_L$  (Table 2),  $\beta$  and  $b$  should be relatively small. For instance, when  $\beta = 0.1$  and  $c = 0.95$  segmentation is feasible when  $b < 0.16$ . For smaller values of  $c$  and larger values of  $\beta$ , the price schedule should be even flatter (when  $\beta = 0.2$  and  $c = 0.95$ ,  $b_{max} = 0.11$ .) Hence, the steepness of the market price schedule should be much smaller than the average willingness to pay for quality in the population.

In order to illustrate the ambiguous welfare effects of the segmenting equilibrium, next we restrict attention to the case that  $\beta = 1$ , namely the case that all consumers are information seekers. In Corollary 1 we first characterize the equilibrium in this case.

**Corollary 1** *When the entire population of consumers are information seekers ( $\beta = 1$ ), the only feasible equilibrium is when  $R_H < R_L$ . This equilibrium exists if:<sup>24</sup>*

$b > \frac{(1+c)(\bar{\theta} + \underline{\theta})}{c} \frac{(\bar{q} - \underline{q})}{2}$ . At this equilibrium  $y = 0$  and  $z > x > \frac{1}{2}$ . The prices set by the platforms are:

$$R_L = \frac{(\bar{q} - \underline{q})}{6} [(1 + c)(\bar{\theta} - 2\underline{\theta}) + cb],$$

$$R_H = \frac{(\bar{q} - \underline{q})}{6} [(1 + c)(2\bar{\theta} - \underline{\theta}) - cb].$$

In Proposition 5 we evaluate the welfare effects of segmentation versus random matching of consumers and vendors for the case that  $\beta = 1$ .

**Proposition 5** *In comparison to random matching, the welfare effects of the segmenting equilibrium are ambiguous. Specifically, when  $\beta = 1$  platforms and vendors are unambiguously better off, but consumers may be worse off as a result of segmentation. The overall change in social welfare may be positive or negative. The change in total welfare can be expressed as follows:*

$$\Delta W = \frac{(1+c)(\bar{\theta} - \underline{\theta})(\bar{q} - \underline{q})x(1-x)}{4} - \frac{(\bar{q} - \underline{q})(z-x)}{2} \left[ cb - (1 + c) \frac{(\bar{\theta} + \underline{\theta})}{2} \right]. \quad (23)$$

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<sup>24</sup> An additional upper bound on  $b$  is required in order to guarantee that  $x < 1$ , namely  $b < \frac{(1+c)(2\bar{\theta} - \underline{\theta})}{c}$ .

The first term of  $\Delta W$  in (23) measures the positive effect of improved matching that segmentation facilitates. As the heterogeneity on both sides of the markets, as measured by  $(\bar{\theta} - \underline{\theta})$  and  $(\bar{q} - \underline{q})$ , increases the benefit from improved matching becomes more significant. Note that this positive benefit is largest when  $x = \frac{1}{2}$ , namely if the platforms serve identical sizes of the populations of consumers and smallest when one platform dominates the market completely (i.e.,  $x = 0$  or  $x = 1$ ). Random matching is equivalent to this kind of market domination. The second term is negative because of the required steepness  $\left(b > \frac{(1+c)(\bar{\theta}+\underline{\theta})}{c \cdot 2}\right)$  of the quality-price schedule combined with the “Single Crossing Property” necessary to support segmentation ( $z > x$ ). This negative effect on welfare is bigger the steeper the price schedule and the bigger the gap ( $z - x$ ) are. A big ( $z - x$ ) gap implies that there is significant imbalance in the sizes of the consumer and vendor populations that are matched by each platform, thus leading to reduced welfare. Platforms are unambiguously better off with segmentation because they can charge positive deal prices, whereas with random matching, undifferentiated Bertrand competition leads to zero prices. Vendors are also better off because they can obtain positive prices (their share of the deal prices) even before consumers have any experience with their service. With random matching, vendors have to offer their service for free to entice consumers to try them out. The effect of segmentation on consumer welfare is ambiguous. The higher prices they have to pay at a segmenting equilibrium may or may not offset the benefit they derive from the improved matching that segmentation facilitates.

#### 4. CONCLUDING REMARKS

As is clear from recent reports in the trade press, daily deals are not necessarily profitable for intermediaries that offer them. In this paper, we offer one reason for that. Our results show that for a large range of reasonable parameter values, platforms fail to implement an equilibrium in which they offer differentiated products/services to consumers. In the absence of differentiation, random matching of vendors and consumers arises, leading to intense competition between the platforms and zero profits. In our model, vertical differentiation is not the result of platforms having control over the quality of the services they provide. It is the result of vendor self-selection: higher (lower) quality vendors choosing a platform because they expect that it will attract customers with higher (lower) willingness to pay. That is, in contrast to firms competing in a one-sided market, for platforms competing in a two-sided market vertical differentiation is possible when not only consumers but also vendors self-select. We show that this requirement is difficult to satisfy. Such a segmenting equilibrium exists only when market conditions lead to significantly high (low) rewards for quality. In our model consumers cannot use social media sites such as Yelp to obtain direct information about vendors’ quality once they learn about their existence via

advertised deals. If we incorporated this possibility in our model it would be even more difficult for platforms to implement segmentation, because their role as information providers would diminish even further.

The difficulty in obtaining the segmenting equilibrium by using the deal price as the only instrument can explain why platforms may have to resort to additional instruments in order to facilitate differentiation. Groupon, for instance, offers less favorable payment terms to vendors than Living Social.<sup>25</sup> One possible explanation for these different terms might be the stronger bargaining position of Groupon due to its larger subscription base. However, another possible explanation may be related to the difficulty we demonstrate in our model of segmenting the vendor population when the only instrument available to the websites is the deal price offered to consumers. When this single instrument fails to yield segmentation, the websites may seek additional instruments in order to avoid random matching. One such instrument may be different payment terms offered to the vendors. When one platform offers less favorable terms to vendors than the other, high quality vendors may find it easier to separate themselves from low quality vendors by choosing to sell deals via the platform that offers less favorable payment terms. Higher quality vendors are more confident that consumers who try their services once by purchasing deals from the platform are more likely to purchase their services, once again, by buying directly from them for full price. Moreover, they anticipate that the full price will be higher when their higher quality is recognized. As a result, high quality vendors are willing to accept the less favorable terms in anticipation that they will be rewarded in the future with much higher prices. Low quality vendors do not mimic this choice because they can get better terms from the other platform and do not anticipate very high prices from repeat purchases by consumers. Hence, the differentiation in payment terms offered by these platforms might be an attempt on their part to facilitate segmentation of the vendor population and by doing so, avoid undifferentiated competition between themselves.

We make several simplifying assumptions in our model for the sake of tractability. Relaxing some of these assumptions is unlikely to affect our qualitative results. For instance, assuming non-uniform distributions and/or a nonlinear market price schedule is unlikely to change our main conclusion that the extra incentive compatibility constraint for vendors significantly complicates the ability of platforms to support segmentation. Enriching the set of instruments that the platforms can use, however, is likely to make segmentation easier. For instance, making the sharing rule  $\alpha$  a decision variable and allowing the platform to differentiate in the type and level of service offered to vendors and consumers,

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<sup>25</sup> While Groupon offers payment in 60 days to merchants and charges them a fee for customers using credit cards to buy the deals, LivingSocial offers payment in 15 days and does not charge a credit card transaction fee (<http://online.wsj.com/article/SB10001424052970204358004577027992169046500.html>, [http://money.cnn.com/2011/04/26/technology/groupon\\_vs\\_livingsocial/index.htm](http://money.cnn.com/2011/04/26/technology/groupon_vs_livingsocial/index.htm)).

all improve the likelihood that a segmenting equilibrium exists.<sup>26</sup> In addition, in our model platforms do not have exogenous information about vendor quality and consumers' willingness to pay. However, some daily deal websites are known to collect information about both consumers and vendors.<sup>27</sup> Using such information can help platforms divide the market into finer segments and provide better matching of the vendor and consumer populations. In fact, niche daily deal websites FabFind.ca and Vipdeals use data mining techniques to help match "high net worth individuals" with "luxury products and services."<sup>28</sup> We leave it to future research to investigate how platforms can leverage information to provide better targeting of consumers.

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<sup>26</sup> Other methods of differentiation include ad copy (Groupon uses clever ad copy as a selling point; <http://business.time.com/2011/05/31/how-humor-sells-silly-daily-deals-no-one-needs/>), vendor service (LivingSocial offers face-to-face meetings with vendors prior to running a deal; [http://money.cnn.com/2011/04/26/technology/groupon\\_vs\\_livingsocial/index.htm](http://money.cnn.com/2011/04/26/technology/groupon_vs_livingsocial/index.htm)), and consumer rewards programs (Groupon Rewards works with vendors to offer consumers customer loyalty programs; <http://www.fastcompany.com/1837024/can-groupon-rewards-transform-local-commerce>)

<sup>27</sup> See <http://articles.latimes.com/2011/jul/12/business/la-fi-groupon-20110712>

<sup>28</sup> See <http://www.newswire.ca/en/story/1020431/teambuy-ca-nabs-luxury-deal-site-fabfind-com>; [http://www.vipdeals.ae/site/how\\_it\\_work](http://www.vipdeals.ae/site/how_it_work).

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## APPENDIX

**Proof of Proposition 1:** We start by proving part (iii) of Proposition 1 because we will subsequently use it in our proofs for parts (i) and (ii) of the proposition.

(iii) In a segmenting equilibrium in which high  $q$ -type vendors deal with platform  $H$  and low  $q$ -type vendors deal with platform  $L$ , we need separation on the vendors' side of the market such that vendors with quality level  $q < q^*$  visit platform  $L$  and vendors with quality level  $q > q^*$  visit platform  $H$ . This requires (15) to be a monotonically increasing function of  $q$ . Therefore,  $\frac{\partial(E\pi_H - E\pi_L)}{\partial q} > 0$  if:

$$\frac{1-x}{1-z} - \frac{x}{z} > 0. \quad (A1)$$

(A1) can be simplified to  $z > x$ .

(i) When  $\beta = 1$ , (15) becomes:

$$E\pi_H(q) - E\pi_L(q) = bcq^2 \left[ \frac{1-x}{1-z} - \frac{x}{z} \right] + (1-\alpha) \left[ R_H \frac{(1-x)}{1-z} - R_L \frac{x}{z} \right]. \quad (A2)$$

Inequality (A1) guarantees that the first term of (A2) is positive in equilibrium. For a segmenting equilibrium to exist in which  $q^* \in (\underline{q}, \bar{q})$ , it must be the case that  $E\pi_H(q^*) - E\pi_L(q^*) = 0$ . Therefore, the second term of (A2) must be negative when  $q = q^*$ . The second term of (A2) can only be negative when  $R_H \frac{(1-x)}{1-z} < R_L \frac{x}{z}$ . However, because  $\frac{1-x}{1-z} > \frac{x}{z}$  the second term of (A2) can be negative only if  $R_H < R_L$ .

Finally, because  $E\pi_H(q) - E\pi_L(q)$  is a continuous function, as  $\beta \rightarrow 1$  it must be the case that this equilibrium also holds for some small  $\varepsilon$  such that  $\beta = 1 - \varepsilon$ . Or as we claim in Proposition 1, for sufficiently large values of  $\beta$ .

When  $R_H < R_L$ ,  $\theta^{D^*}$  in (12) is less than 0. Because  $\underline{\theta} > 0$  this implies that  $y = 0$ . Because all one-time shoppers are better off in expectation by visiting platform  $H$ , none chooses to buy from  $L$ .

(ii) When  $\beta = 0$ , all consumers are one-time shoppers and (15) becomes:

$$E\pi_H(q) - E\pi_L(q) = (1-\alpha) \left[ R_H \frac{(1-y)}{1-z} - R_L \frac{y}{z} \right]. \quad (A3)$$

However, the right hand side (RHS) of (A3) is no longer a function of  $q$ . Therefore, if the RHS is greater than 0 all vendors will visit platform  $H$ , and if the RHS is less than 0 all vendors will visit platform  $L$ . Therefore, no segmenting equilibrium (as we define in the text) exists when  $\beta = 0$ .

To show that an equilibrium where  $R_H < R_L$  cannot exist for sufficiently small  $\beta$ , we take the limit of (15) when  $\beta \rightarrow 0^+$ . This limit is equivalent to (A3). Note that when  $R_H < R_L$ , as mentioned in the proof of part (i) of Proposition 1, it must be the case that  $y = 0$ . However, when  $y = 0$ , it is never the case that  $(1 - \alpha) \left[ R_H \frac{(1-y)}{1-z} - R_L \frac{y}{z} \right] < 0$ , a condition that is necessary to guarantee separation on the vendors' side of the market.

**Proof of Proposition 2:**

(i) The platforms maximize their profits by choosing an optimal deal price that satisfies (17) and (18). Using (20) and the fact that  $y = 0$  when  $R_H < R_L$  as shown in the proof of Proposition 1, we simultaneously solve (17) and (18) to get:

$$R_H = \frac{(\bar{q}-q)}{6} \left[ 2\bar{\theta}(1+c) - bc - \underline{\theta}(1+c) \right] + \frac{2(1-\beta)(\bar{q}-q)(1+c)(\bar{\theta}-\underline{\theta})}{6\beta}, \quad (\text{A4})$$

$$R_L = \frac{(\bar{q}-q)}{6} \left[ bc + \bar{\theta}(1+c) - 2\underline{\theta}(1+c) \right] + \frac{(1-\beta)(\bar{q}-q)(1+c)(\bar{\theta}-\underline{\theta})}{6\beta}. \quad (\text{A5})$$

Note that  $R_H < R_L$  when:

$$b > \frac{(1+c)}{c} \left[ \frac{(\bar{\theta}-\underline{\theta})}{2\beta} + \underline{\theta} \right]. \quad (\text{A6})$$

For segmentation among the information-seeking consumers to exist, it must be the case that  $\underline{\theta} < \theta^{I^*} < \bar{\theta}$ , or put differently,  $0 < x < 1$ . Using (A4), (A5), and (20), this can be expanded to:

$$\underline{\theta} < \frac{bc}{3(1+c)} + \frac{\bar{\theta}+\underline{\theta}}{3} + \frac{(1-\beta)(\bar{\theta}-\underline{\theta})}{3\beta} < \bar{\theta}. \quad (\text{A7})$$

(A7) holds when:

$$b > \frac{(1+c)}{c} \left[ \underline{\theta} - \frac{(\bar{\theta}-\underline{\theta})}{\beta} \right], \quad (\text{A8})$$

$$b < \frac{(1+c)}{c} \left[ \frac{(2\beta-1)}{\beta} (\bar{\theta} - \underline{\theta}) + \bar{\theta} \right]. \quad (\text{A9})$$

A vendor of quality  $q$  seeks to maximize profit by choosing whether to sell deals through platform  $H$  or platform  $L$ . Recalling (15), the following equation can be used to determine if a firm of quality  $q$  finds it in its best interest to visit platform  $H$  or platform  $L$ :

$$E\pi_H(q) - E\pi_L(q) = \beta bcq \left[ \frac{1-x}{1-z} - \frac{x}{z} \right] + (1-\alpha) \left[ R_H \frac{\beta(1-x)+(1-\beta)(1-y)}{1-z} - R_L \frac{\beta x+(1-\beta)y}{z} \right]. \quad (\text{A10})$$

If (A10) is positive a vendor of type  $q$  will visit platform  $H$ , and if (A10) is negative a vendor of type  $q$  will visit platform  $L$ . For segmentation on the vendor side of the market to exist  $E\pi_H(q^*) - E\pi_L(q^*) = 0$ . We can use algebraic manipulations to express  $q^*$  in the last equation in terms of  $z = \frac{q^*-q}{\bar{q}-q}$ , and once again make use of the fact that  $y = 0$ :

$$\left(\bar{q} - \underline{q}\right) \beta b c z \left(\frac{1-x}{1-z} - \frac{x}{z}\right) + \beta b c \underline{q} \left(\frac{1-x}{1-z} - \frac{x}{z}\right) + (1 - \alpha) \left[ R_H \left[ \frac{\beta(1-x) + (1-\beta)}{1-z} \right] - R_L \left[ \frac{\beta x}{z} \right] \right] = 0.$$

Using the platforms' maximization problems, we can also express (A4) and (A5) as:

$$R_H = \frac{[\beta(1-x) + (1-\beta)](\bar{\theta} - \underline{\theta})(\bar{q} - \underline{q})(1+c)}{2\beta},$$

$$R_L = \frac{\beta x (\bar{\theta} - \underline{\theta})(\bar{q} - \underline{q})(1+c)}{2\beta},$$

respectively.

Solving for  $z$ , we obtain:

$$z = \frac{L + \sqrt{L^2 + 4bc\beta \left[ \frac{bc\beta q x}{(\bar{q} - \underline{q})} + \beta^2 x^2 R \right]}}{2bc\beta}, \quad (\text{A11})$$

where

$$L \equiv bc\beta x - \frac{bc\beta \underline{q}}{(\bar{q} - \underline{q})} - [[\beta(1-x) + (1-\beta)]^2 + \beta^2 x^2] R, \text{ and}$$

$$R \equiv \frac{(1-\alpha)(\bar{\theta} - \underline{\theta})(1+c)}{2\beta}.$$

From (A10) we know that  $z > x$  is necessary to ensure that the added benefit of selling a deal via H rather than L is an increasing function of  $q$ . From the solution for  $z$  in (A11) this is possible when:

$$2\beta^2 x^2 - \beta x(2 + \beta) + 1 < 0. \quad (\text{A12})$$

We define  $\omega \equiv \beta x$  and calculate the roots of  $\omega$  as follows:

$$\frac{2 + \beta \pm \sqrt{4\beta + \beta^2 - 4}}{4}.$$

For inequality (A12) to hold it must be that  $4\beta + \beta^2 - 4 > 0$ , implying that:

$$\beta > 2\sqrt{2} - 2. \quad (\text{A13})$$

Provided that (A13) holds, inequality (A12) is valid if we restrict  $x$  to the following interval:

$$\frac{2 + \beta - \sqrt{4\beta + \beta^2 - 4}}{4\beta} < x < \frac{2 + \beta + \sqrt{4\beta + \beta^2 - 4}}{4\beta}. \quad (\text{A14})$$

Since  $\frac{2 + \beta + \sqrt{4\beta + \beta^2 - 4}}{4\beta} < 1$  and  $\frac{2 + \beta - \sqrt{4\beta + \beta^2 - 4}}{4\beta} > 0$  for all values of  $\beta$ , the condition  $z > x$  imposes more

binding constraints on the value of  $x$  than the requirement that  $0 < x < 1$ , as given in (A8) and (A9).

From the expressions derived for  $R_H$  and  $R_L$  in (A4) and (A5) and the expression for  $\theta^{L*}$  in (20), (A14) holds if:

$$b < \frac{(1+c)}{c} \left[ \frac{(\bar{\theta} - \underline{\theta})(2 + 3\beta + 3\sqrt{4\beta + \beta^2 - 4})}{4\beta} + \underline{\theta} \right], \quad (\text{A15})$$

$$b > \frac{(1+c)}{c} \left[ \underline{\theta} + \frac{(\bar{\theta} - \underline{\theta})(2 + 3\beta - 3\sqrt{4\beta + \beta^2 - 4})}{4\beta} \right] \equiv LB_{H < L} \quad (\text{A16})$$

Notice that the lower bound on  $b$  from (A6) is stricter than that from (A8), and the lower bound from (A16) is stricter than that from (A6). The lower bound on the value of  $b$  given in part (i) of the proposition is defined in (A16). The upper bound given in the footnote of the proposition is derived from (A15).

(ii) When  $R_H < R_L$ ,  $y = 0$  as we established in the proof of Proposition 1. Note that the lower bound on  $x$  in (A14) is a decreasing function of  $\beta$  for  $\beta \geq 2\sqrt{2} - 2$ . Hence this lower bound is the least demanding when  $\beta = 1$ . Evaluating it at  $\beta = 1$ , yields that the minimum value of the this lower bound is  $\frac{1}{2}$ . Hence,  $x > \frac{1}{2}$  for all values of  $\beta$ . The condition  $z > x$  is necessary so that the RHS of (A10) is an increasing function of  $q$ .

(iii) By subtracting (A4) from (A5), we obtain the expression for  $R_L - R_H$  given in part (iii) of the Proposition. Using this expression, it is straightforward to show that:

$$\frac{\partial(R_L - R_H)}{\partial b} > 0, \quad \frac{\partial(R_L - R_H)}{\partial \beta} > 0,$$

and also that:

$$\frac{\partial(R_L - R_H)}{\partial c} = \frac{(\bar{q} - \underline{q})}{6} \left[ 2b - \frac{(\bar{\theta} - \underline{\theta})}{\beta} - 2\underline{\theta} \right] > 0,$$

for  $b$  values that satisfy the conditions given in the proposition.

### Proof of Proposition 3:

(i), (ii) To ensure that the RHS of (A10) is an increasing function of  $q$ , we still require  $z > x$ .

In an equilibrium where  $R_H > R_L$ , it is no longer the case that  $y = 0$ . We solve for the equilibrium where  $0 < y < 1$ .

The platforms maximize their profits by choosing an optimal deal price that satisfies (17) and (18).

Recalling that  $x = \frac{\theta^{I*} - \underline{\theta}}{\bar{\theta} - \underline{\theta}}$  and  $y = \frac{\theta^{D*} - \underline{\theta}}{\bar{\theta} - \underline{\theta}}$ , where  $\theta^{I*}$  and  $\theta^{D*}$  are given by (20) and (12), respectively, we

simultaneously solve the platforms' maximization problems to find the optimal values of  $R_H$  and  $R_L$ :

$$R_H = \frac{(\bar{q} - \underline{q})[(2\bar{\theta} - \underline{\theta})(1+c) - bc\beta]}{6(1+c-c\beta)}, \quad (\text{A17})$$

$$R_L = \frac{(\bar{q} - \underline{q})[(\bar{\theta} - 2\underline{\theta})(1+c) + bc\beta]}{6(1+c-c\beta)}. \quad (\text{A18})$$

Note that  $R_H > R_L$  when:

$$b < \frac{(\bar{\theta} + \underline{\theta})(1+c)}{2c\beta}. \quad (\text{A19})$$

For segmentation among the information-seeking consumers to exist, it must be the case that  $\underline{\theta} < \theta^{I*} < \bar{\theta}$ , which implies from (A17), (A18), and (20) that:

$$b > \frac{(1+c)}{c} \frac{[2+3c(1-\beta)]\underline{\theta}-\bar{\theta}}{[3(1+c)-\beta(3c+2)]}, \text{ and} \quad (\text{A20})$$

$$b < \frac{(1+c)}{c} \frac{[2+3c(1-\beta)]\bar{\theta}-\underline{\theta}}{[3(1+c)-\beta(3c+2)]}. \quad (\text{A21})$$

For segmentation among the one-time shoppers to exist, it must be the case that  $\underline{\theta} < \theta^{D^*} < \bar{\theta}$ , which implies from (A17), (A18), and (12):

$$b > \frac{(1+c)\underline{\theta}-[2(1+c)-3\beta c]\bar{\theta}}{2\beta c}, \text{ and} \quad (\text{A22})$$

$$b < \frac{(1+c)\bar{\theta}-[2(1+c)-3\beta c]\underline{\theta}}{2\beta c}. \quad (\text{A23})$$

For segmentation on the vendor side of the market to exist  $E\pi_H(q^*) - E\pi_L(q^*) = 0$ . As we did in the proof of Proposition 2, this equation can be written in terms of  $z$  instead of  $q^*$ . Solving the revised equation for  $z$  yields:

$$z = \frac{L + \sqrt{L^2 + 4bc\beta \left[ \frac{bc\beta q x}{(\bar{q}-q)} + [\beta x + (1-\beta)y]^2 R \right]}}{2bc\beta}, \quad (\text{A24})$$

where

$$L \equiv bc\beta x - \frac{bc\beta q}{(\bar{q}-q)} - [[\beta(1-x) + (1-\beta)(1-y)]^2 + [\beta x + (1-\beta)y]^2]R, \text{ and}$$

$$R \equiv \frac{(1-\alpha)(\bar{\theta}-\underline{\theta})(1+c)}{2(1+c-\beta c)}.$$

Next we will demonstrate that in order to satisfy the Incentive Compatibility constraint of the vendor it is necessary that  $y > z$ . Note that the platforms' optimal choices of  $R_H$  and  $R_L$  can be written in terms of  $x$  and  $y$  as follows:

$$R_H = \frac{[\beta(1-x) + (1-\beta)(1-y)]}{\beta \frac{\partial x}{\partial R_H} + (1-\beta) \frac{\partial y}{\partial R_H}}, \quad (\text{A25})$$

$$R_L = - \frac{[\beta x + (1-\beta)y]}{\beta \frac{\partial x}{\partial R_L} + (1-\beta) \frac{\partial y}{\partial R_L}}. \quad (\text{A26})$$

Noting that  $\frac{\partial x}{\partial R_H} = -\frac{\partial x}{\partial R_L} > 0$  and  $\frac{\partial y}{\partial R_H} = -\frac{\partial y}{\partial R_L} > 0$  and substituting (A25) and (A26) into (A10) yields:

$$E\pi_H(q) - E\pi_L(q) = \beta bcq \left[ \frac{1-x}{1-z} - \frac{x}{z} \right] + \frac{(1-\alpha)}{\beta \frac{\partial x}{\partial R_H} + (1-\beta) \frac{\partial y}{\partial R_H}} \left[ \frac{[1-(\beta x + (1-\beta)y)]^2}{1-z} - \frac{[\beta x + (1-\beta)y]^2}{z} \right]. \quad (\text{A27})$$

Because  $\frac{(1-\alpha)}{\beta \frac{\partial x}{\partial R_H} + (1-\beta) \frac{\partial y}{\partial R_H}} > 0$ , in order to satisfy  $E\pi_H(q^*) - E\pi_L(q^*) = 0$  it is necessary that:

$$\frac{[1-(\beta x + (1-\beta)y)]^2}{1-z} - \frac{[\beta x + (1-\beta)y]^2}{z} < 0. \quad (\text{A28})$$

We can rearrange (A28) to get the following inequality:

$$[1 - 2[\beta x + (1-\beta)y]]z + [\beta x + (1-\beta)y]^2(2z - 1) < 0. \quad (\text{A29})$$

To ensure (A29), it must be that:

$$\frac{1-2[\beta x+(1-\beta)y]}{[\beta x+(1-\beta)y]^2} < \frac{1-2z}{z}. \quad (\text{A30})$$

However, because  $R_H > R_L$  it follows from (A25) and (A26) that  $\beta x + (1 - \beta)y < \frac{1}{2}$  and therefore the LHS of (A30) is positive. The only way that it can be less than the RHS of (A30) is that the RHS is positive as well, or  $z < \frac{1}{2}$ . Because  $x < z$  to ensure the ‘‘Single Crossing Property’’ it follows that  $x < z < \frac{1}{2}$ . Note also that the LHS of (A30) is bigger than  $\frac{1-2[\beta x+(1-\beta)y]}{[\beta x+(1-\beta)y]}$  because  $\beta x + (1 - \beta)y$  is a fraction. The function  $f(t) \equiv \frac{1-2t}{t}$  is a declining function of  $t$ . Thus when (A30) is satisfied, it must be the case that:

$$z < \beta x + (1 - \beta)y. \quad (\text{A31})$$

Because  $x < z$  the only way that (A31) can be valid is that  $y > z$  which also implies that  $y > x$ .

Substituting the values of  $R_H$  and  $R_L$  from (A17) and (A18) in to the expression for  $\theta^{I*}$  in (20) we obtain:

$$\theta^{I*} = \frac{bc}{1+c} + \frac{(1+c)(\bar{\theta}+\underline{\theta})-2b\beta c}{3(1+c-\beta c)(1+c)}. \quad (\text{A32})$$

As well, from (12) and (20):

$$\theta^{D*} = \theta^{I*}(1 + c) - bc, \quad (\text{A33})$$

Because  $x = \frac{\theta^{I*}-\underline{\theta}}{\bar{\theta}-\underline{\theta}}$  and  $y = \frac{\theta^{D*}-\underline{\theta}}{\bar{\theta}-\underline{\theta}}$  it follows from (A33) that:

$$y = (1 + c)x - \frac{c(b-\underline{\theta})}{(\bar{\theta}-\underline{\theta})}. \quad (\text{A34})$$

For  $y > x$  to hold we must have:

$$x > \frac{b-\underline{\theta}}{\bar{\theta}-\underline{\theta}}. \quad (\text{A35})$$

Substituting the expression for  $\theta^{I*}$  from (A32) into  $x$  we can obtain an upper bound on  $b$  that is necessary in order to satisfy (A35) as follows:

$$b < \frac{(\bar{\theta}+\underline{\theta})(1+c)}{[3(1+c)-\beta c]}. \quad (\text{A36})$$

Note that this upper bound on  $b$  is more demanding than the upper bounds in (A19), (A21), and (A23).

Next we will demonstrate that the upper bound on  $b$  is even tighter than that specified in (A36). The tighter bound will be derived from the ‘‘Single Crossing Property’’ condition  $z > x$ . From the expression derived for  $z$  in (A24) this condition holds if:

$$\begin{aligned} & [\beta x + (1 - \beta)y]^2(1 - x) > [1 - (\beta x + (1 - \beta)y)]^2x, \text{ which can be rewritten as:} \\ & [y - \beta(y - x)]^2(1 - 2x) - 2\beta x(y - x) + 2xy - x > 0. \end{aligned} \quad (\text{A37})$$

Define the function  $H(\beta) \equiv (\text{LHS of (A37)})$  when holding  $x$  and  $y$  fixed. Specifically, the expressions for  $x$  and  $y$  derived from (A32) and (A33) depend on additional variables beyond  $\beta$  (including  $b, c, \bar{\theta}$ , and

$\theta$ ). The function  $H(\beta)$  is derived from (A37) by accompanying the changes in  $\beta$  with changes in the parameters  $b$ ,  $c$ ,  $\bar{\theta}$ , and  $\underline{\theta}$  to ensure that the values of  $x$  and  $y$  remain fixed. Given that  $x$  and  $y$  depend upon four parameters in addition to  $\beta$  such concurrent changes are indeed feasible. Hence,

$$H(\beta) \equiv [y - \beta(y - x)]^2(1 - 2x) - 2\beta x(y - x) + 2xy - x \text{ for fixed } x \text{ and } y. \quad (\text{A38})$$

To ensure that  $z > x$  there should be regions of  $\beta$  values for which  $H(\beta) > 0$ . The expression for  $H(\beta)$  can be rewritten as:

$$H(\beta) \equiv \beta^2(y - x)^2(1 - 2x) - 2\beta(y - x)[y(1 - 2x) + x] + [y^2(1 - 2x) + 2xy - x]. \quad (\text{A39})$$

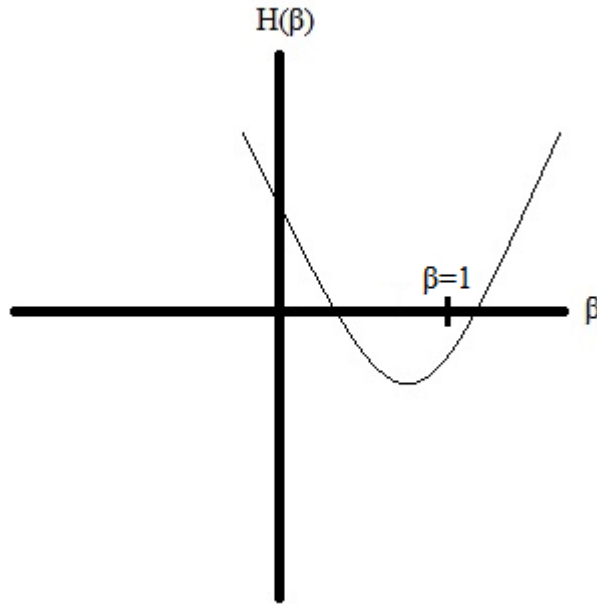
Note that  $H(\beta)$  is a convex function because the coefficient of  $\beta^2$  in (A39) is positive given that  $x < \frac{1}{2}$ .

As well,

$$H(1) = -x(1 - x)(1 - 2x) < 0. \quad (\text{A40})$$

For fixed  $x$  and  $y$ , the function  $H(\beta)$  can be drawn, therefore, as:

**FIGURE A1**



From Figure A1 the only way that there is a region of  $\beta$  values for which  $H(\beta) > 0$  is that  $H(0) > 0$ .

Substituting  $\beta = 0$  in (A39) yields that  $y^2(1 - 2x) + 2xy - x > 0$ . Solving the last inequality for  $y$  in terms of  $x$  yields:

$$y > \frac{\sqrt{x(1-x)} - x}{(1-2x)}. \quad (\text{A41})$$

Notice that the RHS of inequality (A41) is always bigger than  $x$  because  $x$  is a fraction smaller than  $\frac{1}{2}$ . Hence, inequality (A41) is more demanding than the condition  $y > x$ , which led to the upper bound on  $b$

given in (A36). The fact that  $H(I) < 0$  implies also that to support the equilibrium the parameter  $\beta$  cannot be too close to 1. In the Proposition we designate the biggest value that  $\beta$  can assume by  $\hat{\beta}$ .

Recalling from (A34) that  $y = (1 + c)x - \frac{c(b-\underline{\theta})}{(\underline{\theta}-\underline{\theta})}$ , inequality (A41) implies that:

$$c \left[ x - \frac{b-\underline{\theta}}{\underline{\theta}-\underline{\theta}} \right] > \left[ \frac{\sqrt{x(1-x)} - 2x(1-x)}{1-2x} \right], \quad (\text{A42})$$

where the RHS of (A42) is positive. Plugging the expression for  $x$  from (A32) into the LHS of (A42), we observe that the LHS of (A42) is a decreasing function of  $b$ . The condition  $y > x$  led to the requirement that  $\left[ x - \frac{b-\underline{\theta}}{\underline{\theta}-\underline{\theta}} \right] > 0$  and the upper bound on  $b$  in (A36). However, in (A42)  $\left[ x - \frac{b-\underline{\theta}}{\underline{\theta}-\underline{\theta}} \right] > k$  where  $k$  is a positive number. Because  $\left[ x - \frac{b-\underline{\theta}}{\underline{\theta}-\underline{\theta}} \right]$  is a decreasing function of  $b$ , a tighter upper bound on  $b$  is implied by (A42) than that specified in (A36). We designate this tighter upper bound as  $UB_{H>L}$  in the Proposition, where  $UB_{H>L} < \frac{(\bar{\theta}+\underline{\theta})(1+c)}{[3(1+c)-\beta c]}$ . The footnote included in the Proposition specifies the lower bound on  $b$  that is implied by (A20) and (A22).

(iii) The gap in the deal prices can be derived by subtracting (A18) from (A17).

It is straightforward to show that:

$$\frac{\partial(R_H-R_L)}{\partial c} > 0, \quad \frac{\partial(R_H-R_L)}{\partial \beta} > 0, \quad \frac{\partial(R_H-R_L)}{\partial b} < 0. \quad (\text{A43})$$

**Proof of Proposition 4:** Using the bounds on  $b$  derived in (21) and (22) of Propositions 2 and 3, it is straightforward to show that

$$\frac{(1+c)}{c} \left[ \frac{(\bar{\theta}-\underline{\theta})}{2\beta} + \underline{\theta} \right] - \frac{(\bar{\theta}+\underline{\theta})(1+c)}{[3(1+c)-\beta c]} > 0, \quad (\text{A44})$$

when  $0 \leq \beta \leq 1$ ,  $0 \leq c \leq 1$ , and  $0 < \underline{\theta} < \bar{\theta}$ . Hence  $LB_{H<L} > UB_{H>L}$  and the interval of possible values of  $b$  for the  $R_H < R_L$  equilibrium never overlaps with those for the  $R_H > R_L$  equilibrium. As a result, for moderate values of  $b$  in the interval  $(UB_{H>L}, LB_{H<L})$  no segmenting equilibrium exists.

**Proof of Corollary 1:** In our proof of Proposition 2 we show that for  $\beta = 1$  any segmenting equilibrium must be of the kind  $R_H < R_L$ . Inserting  $\beta = 1$  into (A15) and (A16), the following constraints must hold in order for equilibrium to exist:

$$b > \frac{(1+c)(\bar{\theta}+\underline{\theta})}{2c},$$

$$b < \frac{(1+c)(2\bar{\theta}-\underline{\theta})}{c}.$$

As shown in the proof of part (ii) of Proposition 2:

$$\frac{1}{2} < x < 1.$$



Finally, substituting  $\beta = 1$  in (A4) and (A5), we obtain the following deal prices:

$$R_H = \frac{(\bar{q}-\underline{q})}{6} [2\bar{\theta}(1+c) - bc - \underline{\theta}(1+c)],$$

$$R_L = \frac{(\bar{q}-\underline{q})}{6} [bc + \bar{\theta}(1+c) - 2\underline{\theta}(1+c)].$$

**Proof of Proposition 5:** Without segmentation consumers can obtain the deal at a price of zero because all producers are perceived to have the same average quality and platforms are undifferentiated.

Undifferentiated Bertrand competition leads to marginal cost pricing, which in our case implies zero prices. The expected consumer surplus in this case can be derived as:

$$ECS = (1+c) \left[ a + \frac{(\bar{\theta}+\underline{\theta})(\bar{q}+\underline{q})}{4} \right] - cb \frac{(\bar{q}+\underline{q})}{2}.$$

Vendors' expected profits accrue only from revenues that stem from the repeat purchases of consumers as follows:

$$E\pi = cb \frac{(\bar{q}+\underline{q})}{2}.$$

And the platforms make zero profits due to undifferentiated Bertrand competition. Summing up the consumer and vendor surpluses yields:

$$W = (1+c) \left[ a + \frac{(\bar{\theta}+\underline{\theta})(\bar{q}+\underline{q})}{4} \right].$$

Next, we seek to derive social welfare in an equilibrium with segmentation when  $\beta = 1$ . We utilize the constraints and optimal deal prices derived in Corollary 1.

The expected consumer surplus is derived by calculating the net surplus of the two different segments of consumers, those who purchase a deal from L and those that purchase a deal from H, and weighing each segment by its relative size. We obtain:

$$ECS = a(1+c) - \frac{(\bar{\theta} - \underline{\theta})(\bar{q} - \underline{q})(1+c)[(1-x)^2 + x^2]}{2} - cb \frac{(\bar{q} + \underline{q})}{2} \\ - \frac{(\bar{q}-\underline{q})}{2} (z-x) \left[ cb - (1+c) \frac{(\bar{\theta}+\underline{\theta})}{2} \right] + (1+c) \left[ \left( \frac{(\bar{\theta}+\underline{\theta})}{2} \right) \left( \frac{\bar{q}+\underline{q}}{2} \right) + \frac{(1-x)x(\bar{\theta}-\underline{\theta})(\bar{q}-\underline{q})}{4} \right].$$

The vendors' expected profit is the sum of the vendors' share of the expected deal prices and revenues from repeat purchases. Once again, the vendors' surplus is the sum of those selling via L and those selling via H. We obtain:

$$E\pi = \frac{(1-\alpha)(1+c)(\bar{\theta}-\underline{\theta})(\bar{q}-\underline{q})}{2} [(1-x)^2 + x^2] + \frac{cb(\bar{q}+\underline{q})}{2}.$$

The platforms' expected profits accrue only from their share of the deal revenues as follows:

$$EV = \frac{[(1-x)^2 + x^2]}{2} (1+c)(\bar{\theta} - \underline{\theta})(\bar{q} - \underline{q}) \alpha.$$

The change in platforms' expected profits is:

$$\Delta EV = \frac{[(1-x)^2+x^2]}{2} (1+c)(\bar{\theta} - \underline{\theta}) (\bar{q} - \underline{q}) \alpha > 0.$$

The change in vendors' expected profits is:

$$\Delta E\pi = \frac{(1-\alpha)(1+c)(\bar{\theta}-\underline{\theta})(\bar{q}-\underline{q})}{2} [(1-x)^2 + x^2] > 0.$$

The change in expected consumer surplus is:

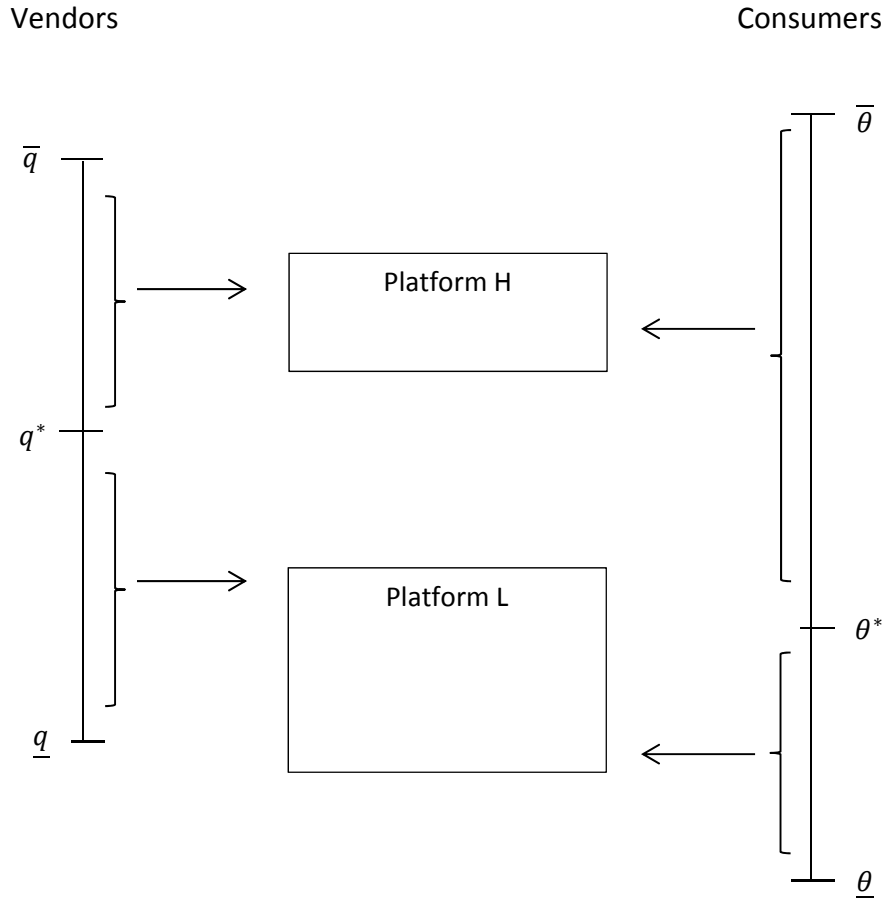
$$\Delta ECS = \frac{-(\bar{\theta}-\underline{\theta})(\bar{q}-\underline{q})(1+c)[(1-x)^2+x^2]}{2} - \frac{(\bar{q}-\underline{q})}{2} (z-x) \left[ cb - (1+c) \frac{(\bar{\theta}+\underline{\theta})}{2} \right] + (1+c) \frac{(\bar{\theta}-\underline{\theta})(\bar{q}-\underline{q})x(1-x)}{4},$$

which can be positive or negative.

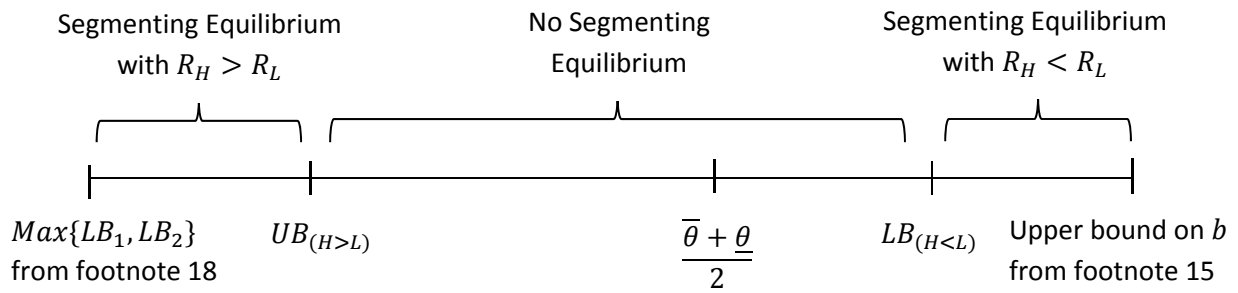
The overall change in total welfare is:

$$\Delta W = \frac{(1+c)(\bar{\theta}-\underline{\theta})(\bar{q}-\underline{q})x(1-x)}{4} - \frac{(\bar{q}-\underline{q})}{2} (z-x) \left[ cb - (1+c) \frac{(\bar{\theta}+\underline{\theta})}{2} \right].$$

The two terms of  $\Delta W$  have opposite signs. Hence, the change in social welfare may be positive or negative. The second term of  $\Delta W$  is negative because  $z-x > 0$  and  $b > \frac{(1+c)}{c} \frac{(\bar{\theta}+\underline{\theta})}{2}$  to support segmentation when  $\beta = 1$ .



**Figure 1- Segmenting Equilibrium**



**Figure 2 – Values of  $b$  Supporting Segmentation**

**Table 1: Investigating Equilibria Where  $R_H < R_L$  With  $\alpha = 0.5$ ,  $\bar{\theta} = 3$ ,  $\underline{\theta} = 0.1$ ,  $\bar{q} = 3$ ,  $\underline{q} = 0.1$**

		$\beta = 0.85$				$\beta = 0.95$			
		$c = 0.85$		$c = 0.95$		$c = 0.85$		$c = 0.95$	
		$R_L - R_H$	$z - x$	$R_L - R_H$	$z - x$	$R_L - R_H$	$z - x$	$R_L - R_H$	$z - x$
$b =$	3.6	-0.27152	-0.0349	-0.09809	-0.0303	0.04961	-0.0071	0.24040	-0.0038
	3.9	-0.02502	-0.0287	0.17741	-0.0244	0.29611	-0.0026	<b>0.51590</b>	<b>0.0005</b>
	4.2	0.22148	-0.0233	0.45291	-0.0193	<b>0.54261</b>	<b>0.0012</b>	<b>0.79140</b>	<b>0.0040</b>
	4.5	0.46798	-0.0187	0.72841	-0.0150	<b>0.78911</b>	<b>0.0045</b>	<b>1.06689</b>	<b>0.0069</b>
	4.8	0.71448	-0.0147	1.00391	-0.0113	<b>1.03561</b>	<b>0.0071</b>	<b>1.34239</b>	<b>0.0093</b>
	5.1	0.96098	-0.0112	1.27941	-0.0082	<b>1.28211</b>	<b>0.0094</b>	<b>1.61789</b>	<b>0.0112</b>
	5.4	1.20748	-0.0083	1.55491	-0.0056	<b>1.52861</b>	<b>0.0112</b>	<b>1.89339</b>	<b>0.0127</b>
	5.7	1.45398	-0.0058	1.83041	-0.0034	<b>1.77511</b>	<b>0.0126</b>	<b>2.16889</b>	<b>0.0139</b>
	6.0	1.70048	-0.0037	2.10591	-0.0016	<b>2.02161</b>	<b>0.0138</b>	<b>2.44439</b>	<b>0.0148</b>
	6.3	1.94698	-0.0019	2.38141	-0.0001	<b>2.26811</b>	<b>0.0146</b>	<b>2.71989</b>	<b>0.0153</b>
	6.6	2.19348	-0.0005	<b>2.65691</b>	<b>0.0010</b>	<b>2.51461</b>	<b>0.0152</b>	<b>2.99539</b>	<b>0.0157</b>
	6.9	<b>2.43998</b>	<b>0.0007</b>	<b>2.93241</b>	<b>0.0019</b>	<b>2.76111</b>	<b>0.0156</b>	<b>3.27089</b>	<b>0.0158</b>
7.2	<b>2.68648</b>	<b>0.0016</b>	<b>3.20791</b>	<b>0.0025</b>	<b>3.00761</b>	<b>0.0158</b>	<b>3.54639</b>	<b>0.0157</b>	

Bold values represent parameters that lead to an equilibrium

**Table 2: Investigating Equilibria Where  $R_L < R_H$  With  $\alpha = 0.5$ ,  $\bar{\theta} = 3$ ,  $\underline{\theta} = 0.1$ ,  $\bar{q} = 3$ ,  $\underline{q} = 0.1$**

		$\beta = 0.10$				$\beta = 0.20$			
		$c = 0.85$		$c = 0.95$		$c = 0.85$		$c = 0.95$	
		$R_H - R_L$	$z - x$	$R_H - R_L$	$z - x$	$R_H - R_L$	$z - x$	$R_H - R_L$	$z - x$
$b =$	0.1	<b>1.56584</b>	<b>0.0019</b>	<b>1.57012</b>	<b>0.0107</b>	1.64017	-0.0069	<b>1.64965</b>	<b>0.0018</b>
	0.11	<b>1.56537</b>	<b>0.0005</b>	<b>1.56962</b>	<b>0.0092</b>	1.63919	-0.0082	<b>1.64861</b>	<b>0.0005</b>
	0.12	1.56490	-0.0010	<b>1.56913</b>	<b>0.0077</b>	1.63821	-0.0095	1.64756	-0.0009
	0.13	1.56444	-0.0024	<b>1.56863</b>	<b>0.0061</b>	1.63723	-0.0107	1.64652	-0.0023
	0.14	1.56397	-0.0039	<b>1.56814</b>	<b>0.0046</b>	1.63626	-0.0120	1.64548	-0.0037
	0.15	1.56351	-0.0053	<b>1.56764</b>	<b>0.0031</b>	1.63528	-0.0133	1.64443	-0.0051
	0.16	1.56304	-0.0068	<b>1.56715</b>	<b>0.0015</b>	1.63430	-0.0146	1.64339	-0.0064
	0.17	1.56258	-0.0082	1.56665	$-3.7 \times 10^{-6}$	1.63332	-0.0159	1.64234	-0.0078
	0.18	1.56211	-0.0096	1.56616	-0.0015	1.63234	-0.0172	1.64130	-0.0092
	0.19	1.56165	-0.0111	1.56566	-0.0031	1.63137	-0.0184	1.64026	-0.0105

Bold values represent parameters that lead to an equilibrium