Game of Platforms:
Strategic Expansion into Rival (Online) Territory*

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Abstract

Online platforms, such as Google, Facebook, or Amazon, are constantly expanding their activities, while increasing the overlap in their service offering. In this paper, we study the scope and overlap of online platforms’ activities, when they are endogenously determined. We model an expansion game between two online platforms offering two different services to users for free, while selling user clicks to advertisers. At the outset, each platform offers one service, and users may subscribe to one platform or both (multihoming). In the second stage, each platform decides whether to expand by adding the service already offered by its rival. Platforms’ expansion decisions affect users’ mobility, and thus the partition of users in the market, which, in turn, affects platform prices and profits. We demonstrate that, in equilibrium, platforms may decide not to expand, even though expansion is costless, and improves exclusive users’ click probability for ads displayed by the platforms. Such strategic "no expansion" decisions are due to quantity and price effects of changes in user mobility, brought on by expansion. Both symmetric expansion and symmetric no-expansion equilibria may arise, as well as asymmetric expansion equilibria, even for initially symmetric platforms.

Keywords: Media economics, entry, online platforms, two-sided markets.

JEL classification: D43, L10, L41

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1 Introduction

Google Plus, Google’s social networking service, was introduced in 2011, in the wake of Buzz, Google’s previous and quite unsuccessful attempt to expand into social networking. With 25 million users in one month, industry analysts initially dubbed Google Plus the "Facebook Killer", expecting Google Plus to be the next Facebook or Twitter. This narrative was quick to change. Most Google Plus users were not active, and in 2012 analysts and bloggers largely referred to Google Plus as a "Ghost Town", compared to its very active and lively counterpart, Facebook. Today, Google Plus is defined as a "social layer" on top of Google. Its active users are predominantly from the tech community, and use it mainly for aggregating, sharing and discussing news items related to their common interests. Still, Google Plus is far behind Facebook, with 28 million unique monthly visitors spending around 7 minutes on average on site, compared to Facebook’s 142 million uniques, averaging almost 7 hours spent on the social network.1 In the online world, where traffic and time spent equal money, Google Plus is hardly a success story.2

Google Plus is just one example of expansion by an online platform into a territory already occupied by a rival platform. Google, Facebook, Amazon, and Apple - the "big four" of the tech industry - are constantly expanding their activities, while increasing their overlap. Google and Facebook now compete both in social networking, where Facebook was the incumbent, and - since the recent introduction of Facebook Graph Search - also in search, Google’s stronghold. Apple and Amazon compete in selling digital media and devices. More overlap between these platforms is found in cloud services, operating systems, smartphones, e-commerce, and the list goes on. With new services and products added each month, the overlap in these giants’ activities will continue to increase, as they strive to provide a one-stop shop for their users.3

And to what end? A major driver of these platforms’ expansion is cultivating exclusive and intimate relationships with users that translate into large advertising revenues45 (e.g., through improved ad targeting; see Kang 2012). Indeed, both Facebook and Google’s revenue comes predominantly from advertising, and just last year Amazon launched its ad exchange, which is estimated to generate $835 million this year, by allowing retargeting of shoppers after they leave Amazon.com (Edwards 2012, Griffith 2012, and Taube 2013).

Motivated by this, we study the expansion behavior of online platforms that capitalize on their users by selling ad clicks to advertisers. Our focus is on expansion into services already

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1 These figures from Nielsen are for desktop visitors in the US, in March 2013 (Wasserman 2013).
3 Also see The Economist, December 2012.
4 Evans 2009 surveys the evolution of online advertising methods, provides some industry numbers, and discusses privacy concerns.
5 Google’s ad revenue is now larger than that of the entire US print media, according to Edwards 2013.
offered by rival platforms. In our setting, platforms’ market power is derived from exclusive users, and thus decreases in the degree of user multihoming. We demonstrate that, in such an environment, platforms may choose not to expand, even when this is costless, due to expansion effects on the partition of users and the degree of multihoming.

We model a market with two platforms, through which advertisers may reach potential buyers, also referred to as platform users. The platforms provide free services to users, generating revenues by selling user clicks to advertisers.\(^6\) At the outset, each platform offers one service type, and has an exogenously given set of users, some of which multihome by subscribing to both platforms, while the rest are exclusive subscribers of the platform. This initial, possibly asymmetric, partition of users, represents platforms’ installed base, which is the result of past events outside the scope of the model (e.g. past interaction between the platforms, or different dates at which the platforms began operating in the market).

Platforms play an entry or expansion game, where each platform strategically chooses whether or not to add the type of service already offered by the rival platform. Such expansion is assumed to be costless. The game proceeds in two stages. In the first stage, platforms make their expansion decisions, determining the set of services offered in the market, and affecting users’ final partition, or choice of platform(s). In the second stage, platforms engage in a pricing game, in which they set prices per user click charged to advertisers. Advertisers then observe platform prices and users’ final partition, and choose where to place their ads, which, in turn, determines platforms’ expected profits.

The platform expansion game is solved by backward induction. For each pair of expansion decisions, the resulting user partition is derived, and platform pricing and profits follow. Platforms’ optimal expansion decisions are then determined.

In equilibrium, prices per click decrease with the size of the multihoming group. This is because prices are optimally set such that advertisers place ads on both platforms, but do not pay double for reaching multihomers twice. The multihoming group thus determines platforms’ market power. Specifically, a low degree of multihoming implies a high degree of market power, with high per-click prices, and a large degree of multihoming lowers platforms’ market power and the prices per clicked charged.

Platforms’ expected profits depend on the price per click charged, and on the expected number of clicks sold, where the latter depends on the click-through rate (henceforth, \(CTR\)) for users on the platform. The CTR is the probability that a user clicks on an ad viewed on the platform, and is higher with exposure to ads on two services rather than one. Therefore,

\(^6\)The advertisers’ side of the market fully subsidizes the buyers’ side. This price structure is commonly assumed in the media platforms literature (e.g. Anderson and Coate 2005, Anderson et al. 2012, Reisinger 2012, Ambrus et al. 2013).
the CTR for exclusive users increases with platform expansion from the one-service CTR to the two-service CTR, representing improved ad targeting following expansion.

As noted, platform expansion affects not only CTRs, but also the partition of users. Platform expansion decisions affect the number of services offered in the market, which, in turn, affects the level of user mobility, i.e., the fraction of non-captive users, who may switch between service providers. After expansion decisions are made, the non-captive users choose which platform(s) to join, creating the final user partition in the market.

Platform expansion thus has two main effects in the model. The first is the CTR effect. Expansion increases the CTR for exclusive platform users - the CTR effect is therefore positive, driving towards expansion. The second effect created by expansion decisions is the mobility effect, as the level of user mobility changes with the number of services available. This implies that expansion decisions affect the final user partition, which in turn affects platforms' prices and profits. The mobility effect is both direct and indirect. The direct mobility effect is the quantity effect, i.e. the change in the number of exclusive and multihoming clicks sold by each platform, resulting directly from the change in the final user partition. The indirect mobility effect is the price effect created by changes in the degree of multihoming, where increased multihoming lowers platform prices.

These effects imply that one or both platforms may not expand in equilibrium, even though expansion is costless and improves the CTR for exclusive users. Specifically, whenever expansion increases the degree of multihoming, the price effect of expansion is negative, while the quantity effect is positive, and stronger for the smaller platform. Expansion decisions depend on the level of the CTR and on the degree of multihoming, which affect the relative sizes of the price, quantity, and CTR effects. We show that, in this case, no-expansion is optimal for high-level CTRs, and for mid-level CTRs, whenever the opponent expands. As a result, asymmetric expansion may be an equilibrium, even for initially symmetric platforms, and both symmetric and asymmetric expansion equilibria may arise for asymmetric platforms.

Alternatively, when expansion decreases buyer multihoming, then the CTR and price effects are both positive, and the quantity effect is negative and stronger for the smaller platform, due to decreased user loss by its rival. Expansion decisions, in this case, depend on the magnitude of change in mobility, and on the degree of multihoming. No-expansion is optimal for both platforms when the change in mobility is very large, and for the smaller platform, when the degree of multihoming is large. Thus, for symmetric platforms, both symmetric expansion and no-expansion equilibria are possible, whereas, for asymmetric platforms, an equilibrium where only the larger platform expands is another possible outcome.

The paper is organized as follows. Section 2 discusses the related literature. Section 3 sets up the model, which is then analyzed in section 4. Section 5 presents our main results,
characterizing expansion equilibria for both increasing and decreasing mobility. Section 6 concludes, discussing the effect of different assumptions regarding migrating users' choice rule.

2 Related Literature

The paper relates to the two-sided markets literature, and specifically to the literature on media platforms.

The literature on two-sided markets has largely focused on platforms' pricing strategies, under varying assumptions regarding market characteristics (see, for example, Caillaud and Jullien 2001 and 2003, Roche and Tirole 2002, 2003, and 2006, Parker and Van Alstyne 2005, Armstrong 2006, Hagiu 2006 and 2009b, Jullien 2008, Weyl 2010, Cabral 2011, Halaburda and Yehezkel 2013a and 2013b). More recently, the platforms literature has been evolving to consider strategies other than pricing. These include decisions on openness and developer property rights (Parker and Van Alstyne 2008, Eisenmann et al. 2008), compatibility with rival platforms (Casadesus-Masanell and Ruiz-Aliseda 2009), the choice of exclusive contracts versus multihoming (Hagiu and Lee 2011), tying (Choi 2010, Amelio and Jullien 2012), exclusion of some user types (Hagiu 2011), and platform investment in proprietary content (Hagiu and Spulber 2013). Eisenmann et al. 2006, Boudreau and Hagiu 2008, and Hagiu 2009a analyze case studies in their discussions of different strategies in platform markets.

Studying advertising-financed platforms, we relate to the literature on media platforms. This literature has examined equilibrium ad prices, levels of advertising, content differentiation, and platform entry (e.g., Anderson and Coate 2005, Crampes et al. 2009, Anderson et al. 2011 and 2012, Reisinger 2012, and Ambrus et al. 2013). Compared to these papers, we employ simplifying assumptions regarding consumers and advertisers, to allow for tractability in solving the platform expansion game.

While simplifying, the main features of our model are consistent with previous work in the media platforms literature. Advertisers in our model may multihome, as is commonly assumed in the literature (an exception is Reisinger 2012). We further allow for user multihoming, as in Anderson et al. 2011 and 2012, Ambrus et al. 2013, and Athey et al. 2013. This gives rise to pricing which follows the "principle of incremental pricing" defined in Anderson et al. 2011, as pricing in our model is determined by the incremental benefit of placing ads on an additional platform. Our assumption that CTR increases with ad exposure, implies only

\[ \text{CTR} \propto \text{ad exposure} \]

We simplify by abstracting away from assumptions on consumers’ preferences for platforms, instead assuming installed based effects and use of a choice rule. We further assume that advertisers are homogeneous, characterized by a constant value per user click. Lastly, we assume no network effect is exerted by advertisers on users.
partial redundancy in reaching multihoming users, and thus the degree of multihoming exerts a negative effect on prices per click charged. In this sense we depart from Anderson et al. 2011, where the principle of incremental pricing implies that the degree of multihoming does not directly affect ad pricing (as only exclusive users count).

User mobility is a central feature in both our framework and in Athey et al. 2013, who consider competition between online news outlets, examining the effects of consumer switching and platforms’ tracking technologies on their profits. In both papers, higher user mobility (or switching, using Athey et al.’s terminology) implies increased multihoming, and user multihoming leads to some redundancy in advertising, whenever the advertisers themselves multihome. Another common feature is that advertisers’ choice of multihoming, i.e. placing ads on both platforms, is determined endogenously in both papers. However, in Athey et al. 2013 ad prices and platform profits may or may not decrease in the degree of user multihoming, due to advertisers’ heterogeneity.

Within the media platforms literature, several papers have focused on platforms’ endogenous differentiation (e.g., Gabszewicz et al. 2002, Dukes and Gal-Or 2003, Behringer and Filistrucchi 2009). Platforms in our model choose whether or not to expand by adding a second service already offered by their rival - our setting endogenizes both the number of services offered and their overlap. We thus analyze a different notion of endogenous differentiation, characteristic of online platforms’ expansion behavior.

Closely related, Eisenmann et al. 2011 study platform "envelopment", defined as "entry by one platform provider into another’s market by bundling its own platform’s functionality with that of the target’s". Envelopment is thus essentially the same as the expansion behavior we study, yet the specific research questions and approach in the two papers are quite different. Namely, Eisenmann et al. build a typology of envelopment attacks based on the level of complementarity between the attacker and target platforms, deriving conditions for success of envelopment attacks. We, on the other hand, model a strategic expansion game between two platforms, so there is no "attacker" and "target" - both platforms may or may not expand, and we derive conditions for different types of expansion equilibria. Furthermore, while Eisenmann et al. take a very broad perspective, we focus on advertising-financed online platforms, which allows for a tractable model.

3 The Model

We model a market with two platforms, offering online services to buyers or platform users, in order to attract advertisers. Platforms are ad-financed, and advertising revenues are collected on a per click basis.
The focus of the model is platforms’ strategic expansion behavior. We analyze an entry or expansion game between the two platforms, where each platform may or may not expand by adding the service initially offered by the rival platform. Following platform expansion decisions, some, but not all, users choose which platform(s) to join, and the number of services available affects the level of user mobility. Platform expansion decisions thus determine the final buyer partition in the market. Platforms then set prices per user click charged to advertisers. Advertisers observe the partition of buyers, and platforms’ prices, and choose their advertising strategy - placing ads on both platforms, on one of the platforms, or not advertising at all.

3.1 Platforms - Basic Assumptions and Notation

There are two platforms in the market and let $i \in \{1, 2\}$ denote the platform index. Platforms provide free services to their users, generating revenues by charging advertisers a price $p_i \in [0, \infty)$ for each user click on an ad displayed by the platform. At the outset, each platform is the provider of one type of service, different from the type offered by the rival platform.

A strategy for platform $i$ is a couple $(e_i, p_i)$ where $e_i \in \{E, \tilde{E}\}$ represents the platform’s expansion decision - either "expansion" denoted $E$ or "no expansion" denoted $\tilde{E}$; and $p_i \in [0, \infty)$ is the price per user click charged to advertisers on the platform. We assume that each platform may expand only by adding the service already offered by its rival, and that expansion is costless.

Platform profits are derived from user clicks sold to advertisers. We thus turn to introduce assumptions regarding buyers and advertisers in the market.

3.2 Buyers

There is a unit mass of buyers in the market. The initial partition of buyers is denoted $B = \{b_1, b_2, b_{12}\}$, where $b_i$ is the group of platform $i$’s exclusive subscribers and its mass, and $b_{12}$ is the initial group of multihomers, and its mass. Multihomers are buyers that subscribe to both platforms, using each platform’s core service. We assume that the market is covered at the outset, i.e. $b_1 + b_2 + b_{12} = 1$.\footnote{This assumption is made for simplicity, and is not necessary for obtaining our main results.}

The initial partition of buyers is exogenously given, representing each platform’s installed base. This implies that there may be asymmetry in platform size at the outset, where platform size is defined as the number of exclusive platform subscribers. WLOG let $b_1 \geq b_2$, and let $\delta \equiv b_1 - b_2 \geq 0$ denote the difference in platforms’ initial size. We will focus our analysis on the effects of the degree of multihoming, and on the effects of the initial asymmetry in the
market. We thus write group sizes as a function of $b_{12}$ and $\delta$:

$$b_1 = 0.5 (1 - b_{12} + \delta)$$  \hspace{1cm} (1)$$

$$b_2 = 0.5 (1 - b_{12} - \delta)$$  \hspace{1cm} (2)$$

Following platforms’ expansion decisions, some, but not all, buyers may switch platforms or become multihomers. The remaining buyers are captive users of the platform(s) they initially subscribed to - the initial partition, $B$, is thus "sticky". We refer to this property as imperfect mobility in the market, and to the group of non-captive, migrating, users as movers.

We first discuss movers’ simple choice rule. We then consider the property of imperfect mobility, allowing the level of mobility to change with the number of services offered in the market.

**Movers’ choice rule.** Movers’ are homogeneous, and employ the same choice rule. We assume that movers prefer subscription to both service types over just one. This implies that movers choose either multihoming or an expanded platform (if one or both platforms expand). We conduct the analysis under the assumption that migrating users choose to multihome. This could result from a higher quality for the core services, coupled with high compatibility between the two platforms.\(^9\)\(^10\) In section 5, we discuss the alternative assumption that non-captive users prefer an expanded platform over multihoming, due to low compatibility or a strong platform-related network effect.\(^11\)

**Imperfect endogenous mobility.** For each pair of expansion decisions $(e_1, e_2)$, movers constitute a fraction $\beta^{e_1e_2} \in (0, 1)$ of each user group $(b_1, b_2, \text{and } b_{12})$. We refer to $\beta^{e_1e_2}$ as the level of mobility in the market.

The level of mobility depends on the number of services offered, and is thus endogenously determined by platforms’ expansion decisions. Denote by $\beta^0 \in (0, 1)$ the level of mobility for a market with $n$ new services, where $n \in \{0, I, II\}$. This implies that $\beta^{EE} = \beta^0$, $\beta^{EE} = \beta^{EE} = \beta^I$, and $\beta^{EE} = \beta^{II}$. We further assume, for simplicity, that the change in mobility resulting from the introduction of one service is constant; this is denoted by $\Delta \beta \equiv \beta^{II} - \beta^I = \beta^I - \beta^0$.

The level of mobility is monotonic in $n$. We distinguish between two possible cases:

\(^9\)To formalize, consider mover utility to be the sum of qualities of the services used, minus a compatibility cost incurred only when multihoming. If core services are of higher quality than newly added services, then movers choose to multihome whenever the compatibility cost is sufficiently low.

\(^10\)Movers’ choice of multihoming may also result from a positive same-side network effect for services based on the initial partition, such that services with more subscribers at the outset are preferred. This network effect (trivially) implies that migrating users subscribe to the two original services, since newly added services had no subscribers in the initial partition.

\(^11\)We consider migrating users who apply the same choice rule to keep the analysis tractable. We could alternatively consider the case where some movers prefer to multihome and others prefer singlehoming on an expanded platform. Our main results depend on the endogeneity of the level of mobility, and will thus continue to hold.
1. Mobility increases in $n$: $\Delta \beta > 0$. This represents markets where users are more likely to sample and change service providers, when their choice set increases.

2. Mobility decreases in $n$: $\Delta \beta < 0$. This represents the case where expansion is perceived as decreasing specialization in core services. Since movers choose to multihome, expansion may decrease mobility (and the tendency to multihome).\textsuperscript{12}

Changes in mobility are bounded, $\Delta \beta \in (-0.5\beta^0, 0.5 (1 - \beta^0))$, to ensure that $\beta^{II} \in (0, 1)$.

**Buyers’ final partition.** Buyers’ final partition is derived from their initial partition, together with movers’ choice rule, taking into account the level of mobility, as determined by $(e_1, e_2)$. The final partition of buyers is denoted $\tilde{B} = \{\tilde{b}_1, \tilde{b}_2, \tilde{b}_{12}\}$, where $\tilde{b}_i$ denotes both the group of platform $i$’s subscribers and its mass; $\tilde{b}_i$ is given by:

$$\tilde{b}_i = (1 - \beta^{e_1e_2}) b_i + \Delta b_i$$

(3)

where $\Delta b_i$ depends on movers’ choice rule. $\tilde{b}_{12}$ is similarly defined.

Since movers choose to multihome, we write $\Delta b_i = 0$, and the mass of multihomers in the final partition is given by $\tilde{b}_{12} = (1 - \beta^{e_1e_2}) b_{12} + \beta^{e_1e_2}$.

To illustrate the effect of platform expansion on the partition of buyers, we turn our attention to figure 1. The figure depicts the case of increasing mobility, and thus the degree of multihoming increases following platform expansion. New services added by the platforms (when $e_i = E$) are enjoyed by their remaining exclusive users.

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\textbf{Buyers’ click-through rate.} The expected number of clicks generated by each group of buyers depends on its mass and click-through rate (henceforth, CTR). CTR is the probability that a user clicks on an ad, and thus increases in the number of exposures to the ad. Ad exposure occurs via platforms’ services. This implies that exclusive users are exposed to ads on one service when a platform has not expanded, and on two services when it has (given

\textsuperscript{12}Alternatively, one may think of users that become confused by the introduction of new services, such that they are less likely to sample and change providers when their choice set increases.

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Figure 1: Expansion effects on the partition of buyers - the case of increasing mobility.
that advertisers place ads on said platform). Similarly, multihomers are exposed to ads on one service if advertisers place ads on one of the platforms, and on two services if advertisers opt to advertise on both platforms.

Let \( \rho \in (0, 1) \) denote the one-service CTR, i.e. the probability a user clicks on an ad he sees on one service, and let \( \tilde{\rho} \) denote the two-service CTR, i.e. the probability he clicks on an ad following exposure on two services. We assume that the one-service CTR is the same for both service types.\(^{13}\) The two-service CTR is given by \( \tilde{\rho} = 2\rho - \rho^2 \), which is the probability that a user clicks at least once in two exposures. This definition of \( \tilde{\rho} \) represents an underlying assumption that users do not click twice on the same ad. Specifically, advertising on two services reduces the probability of a "no click" event, but the two-service CTR is less than double the one-service CTR: \( \rho < \tilde{\rho} < 2\rho \).

### 3.3 Advertisers

There is a unit mass of homogeneous advertisers in the market. Advertisers’ value for each user click on their ads is constant, and normalized to 1.

Advertisers’ strategy is a choice of platform or platforms on which to place ads, denoted \( \alpha \in A \equiv \{ \{1\}, \{2\}, \{1, 2\}, \emptyset \} \).

The expected value of advertising on \( \alpha \) is simply the difference between the expected benefit and cost of choice \( \alpha \). The expected benefit of advertising is the product of advertisers’ per click value, and the expected number of clicks on ads; the expected cost is the product of price per click paid to the platform(s), and the expected number of clicks on ads.

The expected value of \( \alpha \) thus depends on buyers’ final partition, the CTR for buyers reached through \( \alpha \), and the price per click charged. It is denoted \( V^\alpha \equiv V\left( \alpha \mid (e_i, p_i)_{i=1,2}, \tilde{B} \right) \), and given by:

\[
V^\alpha = \begin{cases} 
(1 - p_i) \left[ \rho_i \tilde{b}_i + \rho \tilde{b}_{12} \right] & \text{for } \alpha = \{i\} \\
\rho_1 \tilde{b}_1 + \rho_2 \tilde{b}_2 + \rho \tilde{b}_{12} - p_1 \left[ \rho_1 \tilde{b}_1 + \rho \tilde{b}_{12} \right] - p_2 \left[ \rho_2 \tilde{b}_2 + \rho \tilde{b}_{12} \right] & \text{for } \alpha = \{1, 2\} \\
0 & \text{for } \alpha = \emptyset 
\end{cases}
\]

(4)

Where \( \rho_i \) is the CTR for exclusive subscribers of platform \( i \), and thus depends on its expansion decision:

\[
\rho_i = \begin{cases} 
\rho & e_i = \bar{E} \\
\tilde{\rho} & e_i = \bar{E} \end{cases}
\]

(5)

\(^{13}\)We can generalize by assuming that the one-service CTR is different for each service type. Our results will continue to (qualitatively) hold.
When advertising on a single platform, \( \alpha = \{ i \} \), the expected number of clicks generated is \( \rho_i \bar{b}_i + \rho \bar{b}_{12} \), where \( \rho_i \bar{b}_i \) are the expected clicks from \( i \)'s exclusive subscribers, and \( \rho \bar{b}_{12} \) are the expected clicks of multihomers, who subscribe to one service offered by \( i \). Therefore, advertising on platform \( i \) is always at an expected cost of \( p_i [\rho_i \bar{b}_i + \rho \bar{b}_{12}] \). Since the expected per click value is 1, \( V^i = (1 - p_i) [\rho_i \bar{b}_i + \rho \bar{b}_{12}] \).

When advertising on both platforms, multihomers are reached twice, and advertisers pay both platforms for their clicks. While the expected number of clicks generated by multihomers is derived according to the two-service CTR, the expected payment to each platform for these clicks is derived according to the one-service CTR. Specifically, when \( \alpha = \{ 1, 2 \} \) multihomers' generate \( \bar{b}_{12} \) expected clicks, at the expected cost of \( (p_1 + p_2) \cdot \rho \bar{b}_{12} \), and recall that \( \bar{p} = 2\rho - \rho^2 \). Therefore, \( V^{12} < V^1 + V^2 \).

### 3.4 Platform Profits

Each pair of platform expansion decisions \((e_1, e_2)\) defines an expansion subgame, and determines \( \bar{B} \) in the subgame. Platform \( i \)'s expected profit in an expansion subgame \((e_1, e_2)\), for price \( p_i \), given the rival’s price \( p_j \), are denoted \( \pi^{e_1e_2}_i (p_i \mid p_j) \), and given by:

\[
\pi^{e_1e_2}_i(p_i \mid p_j) = \begin{cases} 
  p_i \cdot [\rho_i \bar{b}_i + \rho \bar{b}_{12}] & \text{if } i \in \alpha^* \\
  0 & \text{if } i \notin \alpha^*
\end{cases}
\]  

(6)

Where the expected number of clicks, \( \rho_i \bar{b}_i + \rho \bar{b}_{12} \), is comprised of \( \rho_i \bar{b}_i \) clicks by exclusive subscribers, and \( \rho \bar{b}_{12} \) clicks by multihomers, who use only the platform’s core service, regardless of \( e_i \).

### 3.5 Timeline

The timeline of the model is as follows:

1. Platforms make expansion decisions \( e_1 \) and \( e_2 \).
2. Final buyer partition \( \bar{B} \) is determined, given buyers’ choice rule and the level of mobility, \( \beta^{e_1e_2} \).
3. Platforms set prices per user click \( p_1 \) and \( p_2 \).
4. Advertisers choose the platform(s) on which they place their ads, \( \alpha \).
5. Platform profits are determined.
This is summarized in the following figure 2.

Figure 2: Timeline of the model.

### 3.6 Market Equilibrium

We define market equilibrium for the simultaneous move expansion game, which represents an environment where platforms’ development efforts are kept secret until new services are launched.

**Definition 1** Market equilibrium is a couple \( (\alpha^*, (e_i^*, p_i^*))_{i=1,2} \) such that, given the initial buyer partition \( B \), and movers’ choice rule:

1. Advertisers’ platform choice is optimal, given platforms’ expansion and pricing decisions, and the resulting buyer partition: \( \alpha^* = \arg\max_{\alpha \in A} \{ V(\alpha | (e_i, p_i)_{i=1,2}, B) \} \).

2. Platform pricing is Nash equilibrium, given platforms’ expansion decisions, the resulting buyer partition, and advertisers’ choice rule: \( p_i^* = \arg\max_{p_i} \pi_i^{e_1 e_2}(p_i | p_j^*) \). Equilibrium profits for an expansion subgame \((e_1, e_2)\), are denoted \( \Pi_i(e_1, e_2) \equiv \pi_i^{e_1 e_2}(p_i^* | p_j^*) \).

3. Expansion decisions are Nash equilibrium in the expansion game: \( e_i^* = \arg\max \Pi_i(e_i, e_j^*) \).

We consider the sequential move expansion game as a means of equilibrium selection, whenever multiple equilibria arise in the simultaneous move game. The simultaneous move game represents the case where platforms’ development strategies are kept secret until the new service is launched, whereas the sequential version represents a market where platforms’ development efforts are known.

The definition of market equilibrium for the sequential game will be similar, differing only in the equilibrium concept for optimal expansion decisions (item 3), which will be SGPE. We consider both the case where the larger platform is the first mover, as well as the case where it is the follower.

### 4 Analysis

Platform expansion increases the CTR for exclusive users; we thus begin by discussing the CTR effect of expansion. Expansion further affects the partition of users through its effect...
on mobility. We therefore proceed to derive buyers’ final partition and solve the platform pricing game, for each pair of expansion decisions, while discussing the quantity and price effects resulting from changes in mobility. Platform profits for each expansion subgame are then derived, and used to obtain optimal choice rules for expansion.

**The CTR effect of expansion.** The CTR effect refers to the increase in CTR for exclusive platform users brought on by expansion. Specifically, expansion increases CTR for these users from the one-service CTR, \( \rho \), to the two-service CTR, \((2\rho - \rho^2)\). The CTR effect is thus a positive quantity effect, which drives platforms towards expansion. It immediately follows that, absent mobility effects, when \( \Delta \beta = 0 \), both platforms will always expand and the equilibrium is \((E, E)\).

It is important to note that the magnitude of the CTR effect is non-monotonic in \( \rho \). The magnitude of this effect, represented by the change in CTR following expansion, \( \tilde{\rho} - \rho \), increases in \( \rho \) for \( \rho < 0.5 \), and decreases in \( \rho \) for \( \rho > 0.5 \).

**Buyer partition and the quantity effect of mobility.** Given a pair of expansion decisions \((e_1, e_2)\), the final partition of buyers, \( \bar{B} \), is such that:

\[
\bar{b}_i = (1 - \beta^{e_1 e_2}) b_i \quad \text{(7)}
\]

\[
\bar{b}_{12} = (1 - \beta^{e_1 e_2}) b_{12} + \beta^{e_1 e_2} \quad \text{(8)}
\]

The quantity effect of mobility is the change in the partition of users, directly resulting from the change in mobility. Following expansion decisions, a mass \( \beta^{e_1 e_2} b_i \) of each platform’s exclusive subscribers become multihomers. When mobility increases in the number of services, this mass increases, and expansion implies a positive quantity effect. The effect is positive due to the increase in multihoming clicks sold originating from exclusive users lost by the rival platform. This effect is stronger for the smaller platform, which benefits more from its opponent’s loss than the larger platform. Note that there is no quantity effect due to own users who become multihomers, as they are characterized by the same, one-service, CTR before and after expansion.

On the other hand, when mobility decreases in the number of services, the mass \( \beta^{e_1 e_2} (b_1 + b_2) \) of new multihomers decreases. Expansion thus entails a negative quantity effect due to reduced user loss by the rival, along with a positive quantity effect due to reduced own user loss, where these exclusive users are now characterized by a higher CTR. The overall effect will depend on platforms’ relative size and on the size of \( \rho \).

The quantity effect is the direct effect of the change in \( B \) following expansion. There further exists an indirect effect - the price effect of mobility, which we now discuss.
The pricing game and the price effect of mobility. We turn to platforms’ optimal pricing in a given expansion subgame. In equilibrium, platforms set prices per click such that advertisers choose to place ads on both platforms, and thus reach multihomers through both platforms. This implies that equilibrium prices are constrained by the degree of multihoming, since advertisers are not willing to pay double for multihomers’ clicks.

Equilibrium prices thus decrease in the mass of multihomers, $b_{12}$, and increase in the mass of exclusive users, $b_i$. Therefore, quite intuitively, market power in the model stems from the degree of exclusivity, and decreases in the degree of multihoming.

The price effect of mobility refers to the effect of changes in mobility on prices, via the change in buyer partition. Specifically, increased mobility leads to increased multihoming and lower exclusivity, which result in lower prices. The price effect of mobility is thus negative, whenever the change in mobility increases multihoming (and, at the same time, decreases exclusivity).

The following proposition 1 provides a characterization of the pricing equilibrium, and the resulting platform profits, given each pair of expansion decisions.

Proposition 1:

1. Given $(e_1, e_2)$ and the resulting $B$, platforms set prices at $p_i^* = \hat{p}_i^{e_1 e_2}$, for $i = 1, 2$. where:

$$\hat{p}_i^{e_1 e_2} = 1 - \frac{\rho^2 \tilde{b}_{12}}{\tilde{b}_i + \rho \tilde{b}_{12}}$$

and platform $i$’s profits are given by:

$$\pi_i^{e_1 e_2} = \rho \tilde{b}_i + \rho (1 - \rho) \tilde{b}_{12}$$

2. Equilibrium prices decrease in the degree of multihoming, $\tilde{b}_{12}$, and increase in the degree of exclusivity, $\tilde{b}_i$.

Proof:

1. Given $(e_1, e_2)$ and $B$, advertisers place ads on both platforms whenever $V^{12} \geq V^1, V^2, 0$, and choose a single platform $i$ whenever $V^i > V^{12}, V^j, 0$. Solving $V^{12} \geq V^j$ we find that $\alpha^* = \{1, 2\}$ whenever $p_i \leq \hat{p}_i^{e_1 e_2}$, for $i = 1, 2$. Furthermore, note that $V^i \geq V^j$ if and only if $p_i \leq \hat{p}_i^{e_1 e_2} (p_j)$, where $\hat{p}_i^{e_1 e_2} (p_j) \equiv \frac{\rho \tilde{b}_i - p_i \tilde{b}_j}{\rho \tilde{b}_i + \rho \tilde{b}_{12}} + p_j \frac{\tilde{b}_j + \rho \tilde{b}_{12}}{\rho \tilde{b}_i + \rho \tilde{b}_{12}}$. It is easily verified that $\hat{p}_i^{e_1 e_2} = \hat{p}_i^{e_1 e_2} (\hat{p}_j^{e_1 e_2})$, thus $V^1 = V^2 = V^{12} = \rho^2 \tilde{b}_{12}$ for $p_i = \hat{p}_i^{e_1 e_2}, i = 1, 2$.

We show that pricing at $\hat{p}_i^{e_1 e_2}$ is profit maximizing. First note that $i \in \alpha^*$ for all $p_i \leq \hat{p}_i^{e_1 e_2}$, and the profit maximizing price in this region is clearly $p_i = \hat{p}_i^{e_1 e_2}$. We now
consider \( p_i > \hat{p}^{e_2}_i \). Pricing at \( p_i \geq 1 \) leads to \( i \notin \alpha^* \) and zero profits, and is not profit maximizing. Therefore assume \( p_i \in (\hat{p}^{e_2}_i, 1) \): if \( p_j \in (\hat{p}^{e_2}_j, 1) \) then only one platform is chosen by advertisers - assume that \( i \) is chosen, i.e. \( V^i \geq V^j \). This implies zero profits for \( j \), and a profitable deviation to \( \hat{p}^{e_2}_j \). Alternatively, if \( p_i \in (\hat{p}^{e_2}_i, 1) \) and \( p_j = \hat{p}^{e_2}_j \) then \( V^j > V^i \) and thus \( i \notin \alpha^* \), and \( i \) has a profitable deviation to \( p_i = \hat{p}^{e_2}_i \). We have thus shown that for any price \( p_i \neq \hat{p}^{e_2}_i \) there exists a profitable deviation to \( p_i = \hat{p}^{e_2}_i \).

Nash equilibrium prices in a given subgame are thus \( p^*_i = \hat{p}^{e_2}_i \) for \( i = 1, 2 \). Substituting for \( \hat{p}^{e_2}_i \) in (6) yields the expression in (10) for platforms’ profits in subgame \((e_1, e_2)\).

2. Follows immediately from examining the first order derivatives of \( \hat{p}^{e_2}_i \) with respect to \( \tilde{b}_{12} \) and \( \tilde{b}_i \):

\[
\frac{\partial \hat{p}^{e_2}_i}{\partial \tilde{b}_{12}} = -\frac{\rho^2 \rho_i \tilde{b}_i}{(\rho_i \tilde{b}_i + \rho \tilde{b}_{12})^2} < 0
\]

\[
\frac{\partial \hat{p}^{e_2}_i}{\partial \tilde{b}_i} = \frac{\rho_i \rho^2 \tilde{b}_{12}}{[\rho_i \tilde{b}_i + \rho \tilde{b}_{12}]^2} > 0
\]

\[
\tilde{b}_{12} \text{ and } \tilde{b}_i.
\]

**Optimal expansion rules.** We proceed to derive conditions for platforms’ expansion decisions. Expansion rules will clearly take into account the CTR effect, and both the price and quantity effects of mobility, and will depend on the relative magnitudes of these effects.

Given platform \( j \)'s expansion decision, \( e_j, i \) will expand whenever \( \Pi_i (E, e_j) \geq \Pi_i (\tilde{E}, e_j) \).

(We assume that indifference is resolved by favoring expansion.) Solving yields, for \( \rho < \tilde{\rho}_{e_j} \):

\[
e_i | e_j = \begin{cases} E & \text{for } b_{12} \leq \tilde{b}_i^{e_j} \\ \tilde{E} & \text{for } b_{12} > \tilde{b}_i^{e_j} \end{cases}
\]

Whereas, for \( \rho > \tilde{\rho}_{e_j} \):

\[
e_i | e_j = \begin{cases} \tilde{E} & \text{for } b_{12} < \tilde{b}_i^{e_j} \\ E & \text{for } b_{12} \geq \tilde{b}_i^{e_j} \end{cases}
\]

Where \( \tilde{\rho}_{e_j} \) is given by:

\[
\tilde{\rho}_{e_j} = \frac{1 - \beta^{Ee_j} + \Delta \beta}{1 - \beta^{Ee_j} + 2 \Delta \beta}
\]

The threshold \( \tilde{b}_i^{e_j} \) depends on \( i, j \), such that:

\[
\tilde{b}_i^{e_2} = 1 + \delta \frac{(1 - \rho) (1 - \beta^{Ee_2}) - \Delta \beta}{(1 - \rho) (1 - \beta^{Ee_2}) - \Delta \beta + 2 (1 - \rho) \Delta \beta}
\]

\[
\tilde{b}_i^{e_1} = 1 - \delta \frac{(1 - \rho) (1 - \beta^{Ee_1}) - \Delta \beta}{(1 - \rho) (1 - \beta^{Ee_1}) - \Delta \beta + 2 (1 - \rho) \Delta \beta}
\]
The intuition behind the above thresholds is as follows. When the CTR is relatively small, \( \rho < \hat{\rho}_{e_j} \), the CTR effect is strong enough, such that the total positive quantity effect of CTR and mobility overcome the negative price effect, when the latter is not too large. This implies that the platforms will expand for relatively low levels of multihoming, \( b_{12} \leq \hat{b}_{e_j} \), and will not expand when the mass of multihomers is large, and the price effect of mobility overcomes the total quantity effect.

Alternatively, when the CTR is relatively large, \( \rho > \hat{\rho}_{e_j} \), then the CTR effect is small in magnitude, and the negative price effect dominates even for low degrees of multihoming. When this is the case, no-expansion is the likely outcome. However, note that the price effect increases in magnitude at a decreasing rate as multihoming increases, while the positive quantity effect of the rival’s user loss remains constant. As a result, no-expansion is optimal for low levels of \( b_{12} \), and expansion may be optimal for high levels of \( b_{12} \).\(^{14}\)

Using (13) and (14), and the thresholds defined in (15)-(17), equilibrium expansion decisions are derived.

5 Platform Expansion Equilibrium

Our main results regarding platforms’ equilibrium expansion decisions are hereby presented, first for the case of increasing mobility and then for the case of decreasing mobility.

5.1 Case 1: Mobility Increases with Expansion

When expansion increases mobility, the mass of multihomers increases with expansion. The price effect is clearly negative. The quantity effect of expansion is positive, and stronger for the smaller platform. This is, of course, in addition to the always-positive CTR effect.

We first examine equilibrium expansion decisions for initially symmetric platforms. The equilibrium is symmetric expansion for low levels of \( \rho \) due to dominance of the CTR and quantity effects, and symmetric no-expansion for high levels of \( \rho \) due to dominance of the price effect (both in dominant strategies). For mid-level CTRs, asymmetric equilibria arise, as expansion is optimal only given that the rival has not expanded. This is because the positive quantity and CTR effects dominate only for a small increase in multihoming.

For mid-level CTRs, we consider sequential expansion to determine which of the asymmetric equilibria arise. Clearly, while both platforms are characterized by the same level of mobility, the expanding platform enjoys the favorable CTR effect, not enjoyed by its rival. It follows that in the sequential move game, the first mover will expand and enjoy higher profits, and the second mover will not expand.

\(^{14}\)This domain exists only when \( \hat{b}^*_{e_j} < 1 \).
Proposition 2 provides a characterization of the equilibrium for the case of mobility that increases with expansion, and initially symmetric platforms. Figure 3 illustrates these results.

**Proposition 2: Increasing Mobility, Symmetric Platforms.** When $\Delta \beta > 0$ and $\delta = 0$, there exist $\tilde{\rho}_E, \tilde{\rho}_E$, such that $0 < \tilde{\rho}_E < \tilde{\rho}_E < 1$:

1. For $\rho \in (0, \tilde{\rho}_E)$: The equilibrium is $(E, E)$ - symmetric expansion.

2. For $\rho \in (\tilde{\rho}_E, \tilde{\rho}_E)$: The equilibria are $(E, \tilde{E})$ and $(\tilde{E}, E)$ - asymmetric expansion. When expansion is sequential, only the first mover expands.

3. For $\rho \in (\tilde{\rho}_E, 1)$: The equilibrium is $(\tilde{E}, \tilde{E})$ - symmetric no-expansion.

**Proof:** see appendix.

![Equilibrium expansion decisions, as a function of $\rho$, for $\Delta \beta > 0$ and $\delta = 0$.](image)

We turn to the case of asymmetric platforms. In this case, the smaller platform enjoys a stronger quantity effect as it gains more from its rival’s loss of users, and thus has a stronger incentive to increase mobility by expansion.

When the initial degree of multihoming is not very high, the resulting equilibria mirror those of the symmetric case, and the intuitions presented above carry over. As multihoming increases, the negative price effect is weakened. This, together with the stronger quantity effect enjoyed by the smaller platform, lead to its expansion. Thus, for large degrees of multihoming and CTRs in all but the lowest range, the equilibrium is asymmetric expansion where only the smaller platform expands.

The equilibrium characterization is presented in proposition 3 and in the figure that follows. Note that in figure 4 we use $\tilde{b} \equiv \min \left\{ \tilde{b}_1^1, \tilde{b}_2^2 \right\}$ for ease of notation. Corollary 4 highlights the main conclusions that follow immediately from propositions 2 and 3.

**Proposition 3: Increasing Mobility, Asymmetric Platforms.** When $\Delta \beta > 0$ and $\delta > 0$, there exist $\tilde{\rho}_E, \tilde{\rho}_E, \tilde{\rho}_E$, such that $0 < \tilde{\rho}_E < \tilde{\rho}_E < \tilde{\rho}_E < 1$:

1. For $\rho \in (0, \tilde{\rho}_E)$: The equilibrium is $(E, E)$ - symmetric expansion.

2. For $\rho \in (\tilde{\rho}_E, \tilde{\rho}_E)$ there exists $\tilde{b}_1^1 \in (0, 1 - \delta)$: The equilibrium is $(E, E)$ for $b_{12} \leq \tilde{b}_1^1$ and $(\tilde{E}, E)$ otherwise.
3. For \( \rho \in (\tilde{\rho}_E, \tilde{\rho}_E) \) there exist \( \hat{b}_E^1, \hat{b}_E^2 \in (0, 1 - \delta) \): The equilibrium is \((\bar{E}, E)\) for \( b_{12} > \min \{\hat{b}_E^1, \hat{b}_E^2\} \), and otherwise \((\bar{E}, \bar{E})\) and \((\bar{E}, E)\) are both equilibria. For \( b_{12} \leq \min \{\hat{b}_E^1, \hat{b}_E^2\} \)
and the case of sequential expansion, only the first mover expands.

4. For \( \rho \in (\tilde{\rho}_E, 1) \) there exists \( \bar{b}_E^2 \in (0, 1 - \delta) \): The equilibrium is \((\bar{E}, \bar{E})\) for \( b_{12} \leq \bar{b}_E^2 \) and \((\bar{E}, E)\) otherwise.

**Proof:** see appendix.

| \((\bar{E}, E)\) | \( b_{12} \leq \hat{b}_E^1 \) \((\bar{E}, E)\) | \( b_{12} \leq \hat{b}_E^1 \) \((\bar{E}, E)\) and \((\bar{E}, \bar{E})\) | \( b_{12} \leq \hat{b}_E^2 \) \((\bar{E}, \bar{E})\) | \( b_{12} > \hat{b}_E^2 \) \((\bar{E}, \bar{E})\) |
|---|---|---|---|
| 0 | \( \bar{E} \) | \( \bar{E} \) | \( \bar{E} \) | 1 | \( \rho \) |

Figure 4: Equilibrium expansion decisions as a function of \( \rho \), for \( \Delta \beta > 0 \) and \( \delta > 0 \).

**Corollary 4: Increasing Mobility.** When mobility increases with expansion:

(a) Asymmetric expansion may be an equilibrium, even for initially symmetric platforms.

(b) Both asymmetric and symmetric expansion equilibria may arise, for initially asymmetric platforms.

(c) For high-level CTRs, no-expansion is optimal for both platforms, and for mid-level CTRs, no-expansion is optimal whenever the opponent expands. This holds for symmetric platforms regardless of the degree of multihoming, and for asymmetric platforms when multihoming is not too high.

(d) For asymmetric platforms, when multihoming is high and the CTR is not too low, no-expansion is optimal only for the larger platform.

We conclude this subsection by considering the effect of changes in the CTR and in the initial degree of multihoming on platforms’ expansion decisions. An increase in the one-service CTR may result from improved targeting or better ad locations on the platform. The initial degree of multihoming, on the other hand, may increase when compatibility between the two platforms is improved.

Increases in the CTR weaken its effect, thus leading one or both platforms to no-expansion, as the price effect becomes more pronounced. Increases in the initial degree of multihoming may lead to expansion by the smaller platform, as its stronger quantity effect may now overcome the weakened price effect.

**Corollary 5: Increasing Mobility, Increases in CTR.** A small increase in \( \rho \) may change the expansion equilibrium:
(a) When $b_{12}$ is small, $b_{12} < \min \{ \tilde{b}_{1E}, \tilde{b}_{2E} \}$: From symmetric expansion to asymmetric expansion, or from asymmetric expansion to symmetric no-expansion.

(b) When $b_{12}$ is large, $b_{12} > \tilde{b}_{1E}$: From symmetric expansion to asymmetric expansion where only the smaller platform expands.

Corollary 6: Increasing Mobility, Increases in the degree of multihoming. An increase in $b_{12}$ may change the expansion equilibrium, leading to an equilibrium where only the smaller platform expands.

5.2 Case 2: Mobility Decreases with Expansion

When expansion decreases mobility, the mass of multihomers decreases with expansion. Hence, the price effect of expansion is positive, moving in the same direction as the CTR effect. Expansion further implies dual quantity effects: (1) A positive quantity effect due to increased exclusivity, with a higher CTR for retained users; along with (2) A negative quantity effect due to the rival’s reduced user loss.

We first consider the case of initially symmetric platforms. When the change in mobility is small, expansion is a dominant strategy for both platforms, due to the above-mentioned positive effects. However, when the change in mobility is very large, the negative quantity effect resulting from the opponent’s decreased loss of subscribers becomes stronger, and no-expansion equilibria may arise. Note that in this case the equilibrium is symmetric, either both platforms expand or both do not. This is because only a very large increase in multihoming (when $e_1 = e_2 = \bar{E}$) can sufficiently increase the total number of clicks sold, such that the price effect is offset by the quantity effect. Therefore, no expansion is optimal only given that the opponent chooses no expansion as well. Further note that when the change in mobility is very large, profits under symmetric no-expansion are higher than under symmetric expansion, and thus $(\bar{E}, \bar{E})$ is the SGPE of the sequential move game.

Proposition 7 provides a characterization of the expansion equilibria, and figure 5 illustrates the result for $\beta^0 > \frac{2}{5}$, in which case the equilibrium depends on $|\Delta \beta|$.

Proposition 7: Decreasing Mobility, Symmetric Platforms. When $-0.5 \beta^0 < \Delta \beta < 0$, and $\delta = 0$, the expansion equilibrium depends on $|\Delta \beta|$ and $\beta^0$:

1. For $\beta^0 \leq \frac{2}{3}$, or $\beta^0 > \frac{2}{3}$ and $|\Delta \beta| \leq 1 - \beta^0$: The equilibrium is $(E, E)$.

\(^{15}\)For $|\Delta \beta|$ to be large enough, we must also impose $\beta^0 > \frac{2}{3}$, otherwise the change in mobility will lead to a negative level of mobility (undefined in our setting).
2. For \( \beta^0 > \frac{2}{3} \) and \( |\Delta\beta| > 1 - \beta^0 \): Both \((E, E)\) and \((E, E)\) are equilibria. When expansion is sequential, the SGPE is \((E, E)\).

**Proof:** see appendix.

<table>
<thead>
<tr>
<th>(-0.5\beta^0)</th>
<th>(-(1 - \beta^0))</th>
<th>0</th>
<th>(\Delta\beta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>((E, E)) and ((E, E))</td>
<td>((E, E))</td>
<td>((E, E))</td>
<td>((E, E))</td>
</tr>
</tbody>
</table>

Figure 5: Equilibrium expansion decisions as a function of \(\Delta\beta\), when \(\Delta\beta < 0\), for \(\delta = 0\), \(\beta^0 > \frac{2}{3}\) (when \(\beta^0 \leq \frac{2}{3}\) the left region, where \(\Delta\beta < -(1 - \beta^0)\), does not exist).

When the platforms are initially asymmetric, asymmetric expansion equilibria may arise. As before, the smaller platform benefits more from increasing mobility, as it results in increased multihoming by rival’s users. Therefore, the smaller platform has a stronger incentive for no expansion. It will, indeed, not expand \((e_2 = \bar{E})\) when the degree of multihoming is large, and thus the incremental price effect of expansion is relatively small.

Therefore, for asymmetric platforms, the equilibrium outcome depends on the initial degree of multihoming, as well as on the mobility parameters, \(|\Delta\beta|\) and \(\beta^0\). Equilibrium characterization is provided in proposition 8, and the following figure 6 illustrates the result for \(\beta^0 > \frac{2}{3}\).

**Proposition 8: Decreasing Mobility, Asymmetric Platforms.** When \(-0.5\beta^0 < \Delta\beta < 0\), and \(\delta > 0\), there exist \(\tilde{b}_1^1, \tilde{b}_2^2 \in (0, 1 - \delta)\), such that \(\tilde{b}_1^1 < \tilde{b}_2^2\):

1. For \(\beta^0 \leq \frac{2}{3}\), or \(\beta^0 > \frac{2}{3}\) and \(|\Delta\beta| \leq 1 - \beta^0\): The equilibrium is \((E, E)\) whenever \(b_{12} \leq \tilde{b}_2^2\), and \((E, \bar{E})\) otherwise.

2. For \(\beta^0 > \frac{2}{3}\) and \(|\Delta\beta| > 1 - \beta^0\): The equilibria are \((E, E)\) and \((\bar{E}, \bar{E})\) for \(b_{12} \leq \tilde{b}_1^1\); \((E, E)\) for \(b_{12} \in (\tilde{b}_1^1, \tilde{b}_2^2)\), and \((E, \bar{E})\) otherwise. For \(b_{12} \leq \tilde{b}_1^1\), the SGPE is either \((E, E)\) or \((\bar{E}, \bar{E})\), depending on the levels of \(b_{12}, \rho, \beta^0, \Delta\beta\).

**Proof:** see appendix.

<table>
<thead>
<tr>
<th>(-0.5\beta^0)</th>
<th>(-0.5\beta^0)</th>
<th>0</th>
<th>(\Delta\beta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>((E, E)) and ((\bar{E}, \bar{E}))</td>
<td>((E, E))</td>
<td>((E, E))</td>
<td>((E, E))</td>
</tr>
</tbody>
</table>

Figure 6: Equilibrium expansion decisions as a function of \(\Delta\beta\) and \(b_{12}\), for \(\delta > 0\), and \(\beta^0 > \frac{2}{3}\) (when \(\beta^0 \leq \frac{2}{3}\), the left region, where \(\Delta\beta < -(1 - \beta^0)\), does not exist)
The following corollary summarizes the possible cases of optimal no-expansion, which follow directly from propositions 7 and 8.

**Corollary 9: No-expansion under decreasing mobility.** When mobility decreases with expansion:

(a) Symmetric no-expansion is an equilibrium when the change in mobility is large enough:

1. For initially symmetric platforms.
2. For initially asymmetric platforms, only when the initial degree of multihoming is small.

(b) Asymmetric expansion, where the smaller platform does not expand, is an equilibrium for initially asymmetric platforms, when the initial degree of multihoming is large.

We conclude by considering the effect of changes in $|\Delta \beta|$ and $b_{12}$ on the expansion equilibrium. A larger change in mobility (higher $|\Delta \beta|$), implying a decreased tendency to multihome, may result from platforms’ brand strengthening activities, and increased user loyalty. As previously noted, a higher degree of multihoming can be the result of higher compatibility. As expected, larger changes in mobility strengthen the quantity effect of exclusive users lost by the rival platform, and lead to no-expansion equilibria, for both symmetric and asymmetric platforms. Changes in $b_{12}$, on the other hand, will affect the expansion equilibrium only when platforms are asymmetric at the outset.

The effects of changes in $|\Delta \beta|$, and in $b_{12}$ are presented in corollaries 10 and 11, which directly follow from propositions 7 and 8.

**Corollary 10: Increases in $|\Delta \beta|$.** A small increase in $|\Delta \beta|$ may change the expansion equilibrium from symmetric expansion to symmetric no-expansion.

**Corollary 11: Increases in $b_{12}$.** For asymmetric platforms, an increase in $b_{12}$ may change the expansion equilibrium:

(a) From symmetric no-expansion to symmetric expansion (only when $b_{12}$ is small and $|\Delta \beta|$ is large); or -

(b) From symmetric expansion to asymmetric expansion where only the larger platform expands.
6 Concluding Remarks

We have presented a game theoretic framework for analysis of online platforms’ expansion decisions, focusing on expansion into services already offered in the market. The analysis demonstrates that platforms may not expand in equilibrium, even though expansion is assumed to be costless, and increases the CTR for exclusive users. This is because platform expansion affects the partition of users in the market, creating quantity and price effects.

Expansion effects on user partition operate through endogenous user mobility in the model. The analysis was conducted under the assumption that migrating users become multihomers, which represents a higher quality for platforms’ core services relative to new services added, or a positive network effect for these services. An alternative assumption is that migrating users prefer to singlehome, i.e. subscribing to a single platform, whenever at least one of the platforms expands.

Under this alternative assumption, movers continue to prefer subscription to two services over one, but multihome only when no platform expands. When one platform expands, it attracts all the movers in the market, and when both platforms expand, migrating buyers will subscribe to the platform which was larger at the outset. This is interpreted as a platform-related positive network effect.

In this case, when expansion raises the level of mobility, it decreases the degree of multihoming, and results in increased migration to the larger platform, whenever it expands. Both price and quantity effects are thus positive for the larger platform, which will always expand in equilibrium. Given that its opponent expands, the smaller platform may not expand in equilibrium, as expansion increases loss of own users (exclusive users and multihomers), and thus entails negative price and quantity effects.

As in the case of multihoming movers, the equilibrium will depend on the relative sizes of the CTR, price, and quantity effects. The smaller platform will not expand when the CTR is relatively large, when the change in mobility caused by expansion is large, or when the degree of multihoming is small. Therefore, possible market equilibria are symmetric expansion, and asymmetric expansion where the larger platform expands and the smaller platform does not.

Considering the above alternative assumption regarding buyers’ choice rule highlights the importance of endogenous user mobility in our modeling framework. When the level of mobility changes with the introduction of new services, expansion by one platform exerts strategic effects on its rival, directly through the change in user partition, and indirectly through prices. This feature leads to the various types of market equilibria that arise in the model.

Is it reasonable to assume that user mobility changes with the number of services in the market? While extremely difficult to measure, it seems that mobility in online markets does change over time, and with new service introduction. Mobility may increase with the number
of alternatives available to users in the market, especially when platform compatibility is high, and may also decrease over time, if platforms introduce exclusive new services, incompatible with rival platforms.

Appendix

Proof of proposition 2: For \( \delta = 0 \) the thresholds \( \tilde{b}_{ij} = 1 \) for \( i = 1, 2 \). Furthermore, \( \Delta \beta > 0 \) implies that \( 0 < \tilde{\rho}_E < \tilde{\rho}_E < 1 \). We now specify decision rules and the resulting equilibrium for each domain of \( \rho \):

1. For \( \rho \in (0, \tilde{\rho}_E) \): \( e_i|_{e_j} = E \) whenever \( b_{12} \leq 1 \). \( e_i = E \) is a dominant strategy for \( i = 1, 2 \), and the equilibrium is \( (E, E) \).

2. For \( \rho \in (\tilde{\rho}_E, \tilde{\rho}_E) \): \( e_i|_{\tilde{E}} = E \) whenever \( b_{12} \leq 1 \), and \( e_i|_E = \tilde{E} \) whenever \( b_{12} \leq 1 \). Both \( (E, \tilde{E}) \) and \( (\tilde{E}, E) \) are thus equilibria. Comparing profits for the two asymmetric equilibria yields \( \pi_{E1} > \pi_{E1}^E \) and \( \pi_{E2} > \pi_{E2}^E \). This implies that profits are higher for the expanding platform. Therefore, in a sequential move expansion game the first mover will expand and the follower will not.

3. For \( \rho \in (\tilde{\rho}_E, 1) \): \( e_i|_{\tilde{E}} = \tilde{E} \) whenever \( b_{12} \leq 1 \). \( e_i = \tilde{E} \) is a dominant strategy for \( i = 1, 2 \), and the equilibrium is \( (\tilde{E}, \tilde{E}) \).

Proof of proposition 3: For \( \delta > 0 \), the initial degree of multihoming is \( b_{12} < 1 - \delta \). We first derive conditions for \( \tilde{b}_{ij} < 1 - \delta \) for \( i = 1, 2 \).

\[ \tilde{b}_{e2}^1 < 1 - \delta \text{ whenever:} \]

\[ 1 + \delta \frac{(1 - \rho)(1 - \rho E_{e2}) - \Delta \beta}{(1 - \rho)(1 - \rho E_{e2}) - \Delta \beta + 2(1 - \rho) \Delta \beta} < 1 - \delta \]  \( (18) \)

This implies that for \( \rho < \tilde{\rho}_{e2} \): \( \tilde{b}_{e2}^1 < 1 - \delta \iff \rho > \tilde{\rho}_{e2}^1 \equiv \frac{1 - \beta E_{e2}}{1 - \beta E_{e2} + \Delta \beta} \), while for \( \rho > \tilde{\rho}_{e2} \):

\[ \tilde{b}_{e2}^1 < 1 - \delta \iff \rho < \tilde{\rho}_{e2}^1 \equiv \frac{1 - \beta E_{e2}}{1 - \beta E_{e2} + \Delta \beta} \]

\[ \tilde{b}_{e1}^2 < 1 - \delta \text{ whenever:} \]

\[ 1 - \delta \frac{(1 - \rho)(1 - \rho E_{e1}) - \Delta \beta}{(1 - \rho)(1 - \rho E_{e1}) - \Delta \beta + 2(1 - \rho) \Delta \beta} < 1 - \delta \]  \( (19) \)

This implies that for \( \rho < \tilde{\rho}_{e1} \): \( \tilde{b}_{e1}^2 < 1 - \delta \iff \Delta \beta < 0 \), while for \( \rho > \tilde{\rho}_{e1} \): \( \tilde{b}_{e1}^2 < 1 - \delta \iff \Delta \beta > 0 \).
Since $\Delta \beta > 0$: $\hat{b}_{i_1}^2 > 1 - \delta$ for $\rho < \tilde{\rho}_e$, and $\hat{b}_{i_1}^2 < 1 - \delta$ for $\rho > \tilde{\rho}_e$.

Furthermore, $\Delta \beta > 0$ implies that $0 < \tilde{ \rho}_i < \tilde{ \rho}_E < \tilde{ \rho}_E < 1$. We now specify decision rules and the resulting equilibrium for each domain of $\rho$:

1. For $\rho \in (0, \tilde{ \rho}_E)$: $e_i|_{e_j} = E$ whenever $b_{12} \leq \hat{b}_{i_1}^1$, and $\hat{b}_{i_1}^1 > 1 - \delta$ for $i = 1, 2$. $e_i = E$ is a dominant strategy for $i = 1, 2$, and the equilibrium is $(E, E)$.

2. For $\rho \in (\tilde{ \rho}_E, \tilde{ \rho}_E)$: $e_i|_{e_j} = E$ whenever $b_{12} \leq \hat{b}_{i_1}^1$. Since $\hat{b}_{i_1}^1 < 1 - \delta$, $e_2 = E$ is a dominant strategy. $\hat{b}_{i_1}^1 < 1 - \delta$, such that platform 1’s strategy depends on $b_{12}$. The equilibrium is $(E, E)$ for $b_{12} \leq \hat{b}_{i_1}^1$ and $(\tilde{E}, E)$ for $b_{12} > \hat{b}_{i_1}^1$.

3. For $\rho \in (\tilde{ \rho}_E, 1)$: $e_i|_{e_j} = E$ whenever $b_{12} \leq \hat{b}_{i_1}^1$, while $e_i|_{E} = E$ whenever $b_{12} > \hat{b}_{i_1}^1$. Since $\hat{b}_{i_1}^1 < 1 - \delta$, $\hat{b}_{i_1}^1 > 1 - \delta$, $\hat{b}_{i_1}^2 > 1 - \delta$, and $\hat{b}_{i_1}^2 < 1 - \delta$ - $e_1|_{e_j} = E$, and $e_2|_{E} = E$, while $e_1|_{E}$ and $e_2|_{E}$ depend on $b_{12}$. For $b_{12} > \min \{\hat{b}_{i_1}^1, \hat{b}_{i_1}^2\}$ the equilibrium is $(\tilde{E}, E)$, and otherwise $(\tilde{E}, E)$ and $(E, \tilde{E})$ are both equilibria. For $b_{12} \leq \min \{\hat{b}_{i_1}^1, \hat{b}_{i_1}^2\}$, comparing profits for the two asymmetric equilibria yields $\pi_{i}^{EE} > \pi_{i}^{E}E$ and $\pi_{2}^{EE} > \pi_{2}^{E}E$. This implies that profits are higher for the expanding platform. Therefore, in a sequential move expansion game the first mover will expand and the follower will not.

4. For $\rho \in (\tilde{ \rho}_E, 1)$: $e_i|_{e_j} = E$ whenever $b_{12} > \hat{b}_{i_1}^1$. Since $\hat{b}_{i_2}^1 > 1 - \delta$, $e_1 = \tilde{E}$ is a dominant strategy. $\hat{b}_{i_2}^1 < 1 - \delta$, such that platform 2’s strategy depends on $b_{12}$. The equilibrium is $(\tilde{E}, \tilde{E})$ for $b_{12} \leq \hat{b}_{i_2}^1$ and $(\tilde{E}, E)$ for $b_{12} > \hat{b}_{i_2}^1$.

**Proof of proposition 7:** For $\delta = 0$ the thresholds $\hat{b}_{i_1}^1 = 1$ for $i = 1, 2$. For $-0.5\beta^0 < \Delta \beta < 0$, there are two possible cases, depending on $\beta^0$ and $|\Delta \beta|$:

1. For $\beta^0 \leq \frac{2}{3}$: $-0.5\beta^0 > - (1 - \beta^0)$, thus for all $-0.5\beta^0 < \Delta \beta < 0$ and $\tilde{ \rho}_E, \tilde{ \rho}_E > 1$. This implies $\rho < \tilde{ \rho}_E, \tilde{ \rho}_E \forall \rho \Rightarrow e_i|_{e_j} = E$ for $b_{12} \leq 1$. $e_i = E$ is a dominant strategy for $i = 1, 2$, and the equilibrium is $(E, E)$.

2. For $\beta^0 > \frac{2}{3}$: (a) When $|\Delta \beta| \leq 1 - \beta^0$: the analysis is the same as in 1; (b) When $1 - \beta^0 < |\Delta \beta| \leq 0.5\beta^0$, the thresholds for $\rho$ are $\tilde{ \rho}_E < 0, \tilde{ \rho}_E > 1$. This implies $\tilde{ \rho}_E < \rho < \tilde{ \rho}_E \forall \rho \Rightarrow e_i|_{E} = E$ for $b_{12} \leq 1$, and $e_i|_{\tilde{E}} = \tilde{E}$ for $b_{12} \leq 1$. The equilibria are $(E, E)$ and $(\tilde{E}, \tilde{E})$. Comparing profits for the two symmetric equilibria yields $\pi_{i}^{EE} > \pi_{i}^{E}E$. Therefore, in a sequential move expansion game the SGPE is $(\tilde{E}, \tilde{E})$.

**Proof of proposition 8:** For $\delta > 0$ and $-0.5\beta^0 < \Delta \beta < 0$:
1. For $\beta^0 \leq \frac{2}{3}$, or $\beta^0 > \frac{2}{3}$ and $|\Delta \beta| \leq 1 - \beta^0$: the thresholds for $\rho$ are $\tilde{\rho}_E, \tilde{\rho}_E > 1$. This implies that $\rho < \tilde{\rho}_E, \tilde{\rho}_E \forall \rho \Rightarrow e_i | E = E$ for $b_{12} \leq \hat{b}_1$, where the thresholds for $b_{12}$ are $\hat{b}_{E1} > 1 - \delta, \hat{b}_{E2} < \hat{b}_E < 1 - \delta$. $e_1 = E$ is a dominant strategy, and the equilibrium is $(E, E)$ for $b_{12} \leq \hat{b}_E$, and $(E, \tilde{E})$ otherwise.

2. For $\beta^0 > \frac{2}{3}$ and $1 - \beta^0 < |\Delta \beta| \leq 0.5\beta^0$: the thresholds for $\rho$ are $\tilde{\rho}_E < 0, \tilde{\rho}_E > 1$. This implies that $\tilde{\rho}_E < \rho < \tilde{\rho}_E \forall \rho \Rightarrow e_i | E = E$ for $b_{12} \leq \hat{b}_E$, and $e_i | E = \tilde{E}$ for $b_{12} \leq \hat{b}_E$, where the thresholds for $b_{12}$ are $\hat{b}_{E1}, \hat{b}_{E2} < 1 - \delta$, and $\hat{b}_{E1}, \hat{b}_{E2} > 1 - \delta$. For $b_{12} \leq \hat{b}_E$ both $(E, E)$ and $(\tilde{E}, \tilde{E})$ are equilibria; for $b_{12} \in (\hat{b}_{E1}, \hat{b}_{E2})$ the equilibrium is $(E, E)$, and for $b_{12} \in (\hat{b}_{E2}, 1 - \delta)$ the equilibrium is $(E, \tilde{E})$. Lastly, for $b_{12} \leq \hat{b}_E$, and sequential expansion decisions: both $\pi_{iE} > \pi_{EE}$ and $\pi_{iE} < \pi_{EE}$ are possible, and the direction of the equilibrium depends on $b_{12}, \rho, \beta^0, \Delta \beta$.

References


