

Dynamic Platform Competition: Optimal Pricing and Piggybacking under Network Effects

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A repeated challenge of the two-sided market literature is the “chicken-and-egg” problem. In a single-period setting, subsidizing one side of the market to jumpstart the platform adoption process has been suggested as the solution. However, it is not known whether such subsidizing strategies remain optimal under dynamic platform competition. This paper develops a multi-period framework to study dynamic platform competition under cross-side network effects. First, we solve for the platforms’ optimal dynamic pricing strategies. Benchmarked with the single-period case, we identify regions when each competing platform should decrease prices, and more interestingly, when they should increase prices. Second, we study piggybacking as a new solution for the “chicken-and-egg” problem. We extend our baseline model to consider piggybacking, by introducing an initial installed base on the consumer side. We examine the asymmetric scenario when one piggybacking platform competes with another platform who does not piggyback. We then study optimal piggybacking strategies and examine the interaction effects among optimal pricing, optimal piggybacking, and network effects. Third, we extend our baseline model to consider platforms with asymmetric discount factors for their future profits. Practical implications are also discussed.

Key words: Platform, Competition, Pricing, Piggybacking, Subsidizing, Network Effects, Analytical Modeling, Economics of IS

1. Introduction

As more and more businesses (both physical and digital) search for their multi-sided platform business models, a primary challenge is how to grow the user bases with the interdependency issue among different user groups – known as the “chicken-and-egg” problem (Caillaud and Jullien 2003). The key solution proposed by the extant literature is to subsidize one or more user groups (e.g., Rochet and Tirole 2003, Parker and Van Alstyne 2005, Eisenmann et al. 2006) in order to jump-

start the platform adoption process. Such subsidizing strategies are indeed frequently employed by platforms. Paypal, for example, during its launching period, gave each new customer \$20 to speed the adoption of its payment platform (BlakeMasters 2012). More examples can be found in Parker et al. (2016).

The academic literature, surprisingly, has been largely focused on the single-period or static setting. Therefore, it is silent on optimal pricing strategies under dynamic platform competition despite the pressing needs from business practice. Taking the recent taxi-hailing market in China for example, two taxi-hailing startups, Didi and Kuaidi, competed head-to-head in this emerging market. To attract drivers and passengers to their respective platforms, both platforms have employed subsidizing strategies. The subsidy war is illustrated by Figure 1, where we highlight a couple of interesting observations: (1) both platforms initially subsidized both sides of the market simultaneously; (2) both reduced their subsidies over time; (3) both reduced subsidies on the passenger side faster than the driver side. Similar observations have been made in the US market where Uber competes with Lyft (Huet 2014, Lazzaro 2016).

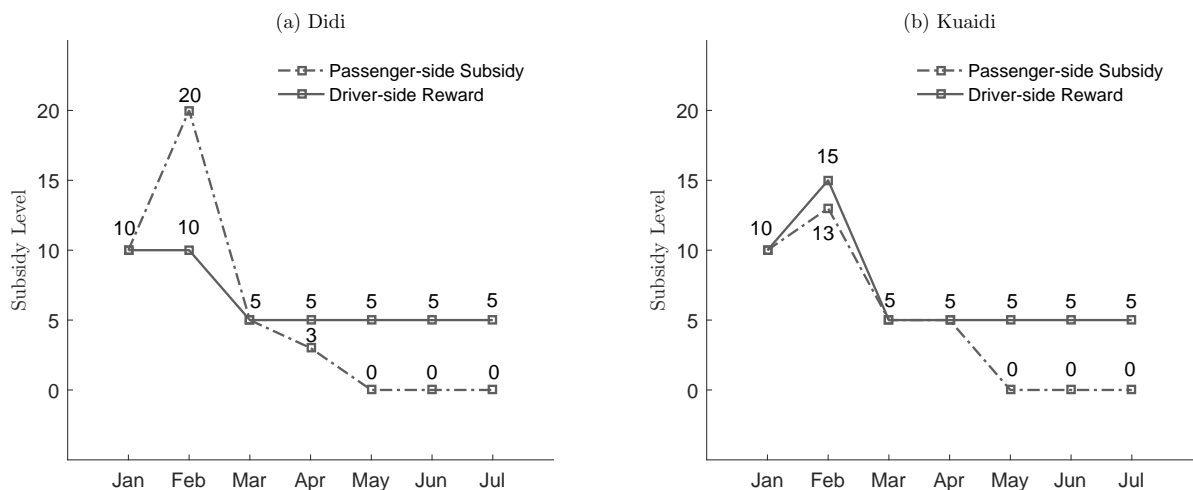


Figure 1 China's Taxi-hailing Subsidy War between Didi and Kuaidi in 2014 (amount in CNY)¹

In addition to subsidizing, practical platforms often use non-price strategies to address the above “chicken-and-egg” problem. One such popular strategy, summarized in Table 1, is piggybacking

¹ Data in Figure 1 are consolidated in Exhibit 3 in Su et al. (2016). 1 USD \approx 6.22 CNY in 2014.

where platforms are able to “connect with an existing user base from a different platform and stage the creation of value unit in order to recruit those users to participate” (Parker et al. 2016). Formal analysis of the piggyback strategy is rare, if not missing, in the academic literature despite its popularity in platform practices. As can be seen in Table 1, central to the piggyback strategy is the ability of the startup platform to bring in an initial installed base of users from a different established platform to its focal market. We formally model the piggyback strategy as an extension.

Approach	Definition	Examples
direct importing	offer direct gateways to users from external networks	user log-in with Facebook or Google account
business affiliation	acquire new users by serving as subsidiaries of external networks	embedded Google search on Yahoo! Youtube video tools on MySpace Zygna games on Facebook Paypal payment services on eBay
strategic poaching	convert users from other networks without upfront agreements	Airbnb posted house-rental listings on Craigslist to attract tenants
business model transformation	extend or switch to new business model with existing users	OpenTable expanded from restaurant management software to booking

Table 1 Summary of Different Approaches for Piggybacking in Platform Practice

This paper aims to take the first step towards filling this gap in the academic literature by developing a multi-period framework that enables us to study dynamic platform competition. We also extend our baseline model to consider a non-price strategy, piggybacking, used by practical platforms to address the “chicken-and-egg” problem. To the best of our knowledge, our paper is the first to formally study optimal piggybacking strategies in the context of dynamic platform competition under network effects. Our key research questions are: *Under dynamic platform competition and network effects: What are the platforms’ optimal pricing strategies over time? What are the platforms’ optimal piggybacking strategies?*

Our key findings include the following. First and foremost, we solve for the platforms' optimal dynamic pricing strategies. Benchmarked with the single-period case, we identify regions when each competing platform should decrease their prices, and more interestingly, when they should increase prices. Second, we extend our baseline model to consider piggybacking, which is captured in our model by introducing an initial installed base on the consumer side. We are particularly interested in the asymmetric scenario when one platform is able to implement piggybacking but its rival is unable to do so. One finding here, somewhat counter intuitive, is that it might not be necessary for the non-piggybacking platform to subsidize consumers when competing with a piggybacking platform, so long as the network effects are not too strong. Third, we study the optimal piggybacking strategy and show several insightful implications via sensitivity analysis. Third, we extend our baseline model to consider platforms with asymmetric discount factors for their future profits.

The rest of this paper is organized as follows: Section 2 reviews related literature and Section 3 introduces our model setups. In Section 4, we first introduce the benchmark single-period results from the literature, and then extend to a two-period duopoly setting. We consider the role of piggyback strategies in Section 5 and endogenize the piggybacking decision in Section 5.2. As model extensions, we consider in Section 6 a three-period model, as well as asymmetric competitions with different discount factors. Section 7 outlines the managerial implications and concludes.

2. Literature Review

Our research draws from the rich two-sided market literature pioneered by Rochet and Tirole (2003) and Parker and Van Alstyne (2005). A repeated challenge in this literature is the “chicken and egg” problem (Caillaud and Jullien 2003) discussed in the Introduction. Subsidizing one side of the market has been suggested as a solution. For example, Parker and Van Alstyne (2005) recommend giving away free access/products to either providers or consumers, depending on the cross-market elasticities. Rochet and Tirole (2006) introduce the “seesaw principle” under which a profit-maximizing platform should charge a high price on one side and a low price on the other

side. While both papers assumed user single-homing, the seesaw principle, is also optimal when users multi-home. Armstrong (2006) shows that when only one side multi-homes and other side single-homes (called “competitive bottleneck”), the platform is able to charge a higher price on the multi-homing side due to its monopoly power in providing access to the single-homing side for the multi-homing side. A similar setup (only one side multi-homes) has been used by Hagiu and Hałaburda (2014) and Economides and Tåg (2012). Hagiu and Hałaburda (2014) consider different types of user expectations when they join the platform. They find that platforms might be better off when users are less informed and form their expectations passively. As shown later, we extend the single-period model in Hagiu and Hałaburda (2014) to a two-period setting, using their findings as a benchmark.

It remains unknown whether the optimality of the one-side subsidizing strategy (or the seesaw principle), obtained in a single-period setting can be extended to a multi-period setting like ours. As shown in Figure 1, platforms often employ dynamic pricing strategies to compete. Only a few studies have considered platform dynamics. For example, Zhu and Zhou (2012) consider an entry game between an incumbent and an entrant platform. They assume that prices are fixed in order to focus on non-pricing factors such as quality, network effects, and consumer expectations. Cabral (2011) considers a discrete, traffic-based model with direct network effects, where a new group of users arrive at the beginning of each period, and a fixed proportion of all existing users leave the market at the end of each period. His key finding is that the optimal price in each period is monotonic in network size. Another line of research employs the differential pricing game approach. Sun and Tse (2007) find that “winner takes all” might not occur at the steady state of platform competition, depending on whether users are single-homing or multi-homing. Chen and Tse (2008) discover that, in addition to the multi-homing tendency, market segmentation is another driving factor for the “winner takes all” outcome to occur at the steady state.

The piggyback strategy, broadly speaking, falls into the literature on non-price platform controls (e.g., Boudreau 2010, Anderson Jr et al. 2013). In a single-period setting, this literature has examined several platform controls, such as adding initial developers in the software platform (Boudreau 2012), attracting early users with single-side functionalities (Hagiu and Eisenmann 2007), and

integrating the user base with a complementary platform (Li and Agarwal 2016). The value of piggybacking in dynamic platform competition remains unknown. Our paper contributes to this literature by characterizing the interaction between pricing and piggybacking in an asymmetric setting where one platform piggybacks while the other does not. We uncover a host of interesting new insights.

As an extension, we consider a platform whose objective is to maximize short-term profit. The entrepreneurship literature has suggested that startup firms often pursue operational goals which are different from those of the established firms (e.g., Sandberg and Hofer 1988). We extend our baseline model to an asymmetric duopoly where platforms differ in their operational objectives (e.g., Swinney et al. 2011). In contrast to the case of symmetric duopoly, we show that the short-term profit focused platform should refrain from competing (with the long-term profit focused platform) on the “money” side but monetize from the other side of the market.

3. Model Setup

Our baseline model extends the single-period duopoly setting in the literature (e.g., Hagiu and Hałaburda 2014) to a two-period setting. Platform A competes against platform B in a two-side market connecting consumers (denoted by c) and providers (denoted by d). We assume the lifecycle of the platform lasts for two periods. At the beginning of each period, each platform $k \in \{A, B\}$ simultaneously decides the access fee p_{ki}^m in period $i \in \{1, 2\}$ for all users on side $m \in \{c, d\}$.

Following the literature (e.g., Armstrong 2006, Hagiu and Hałaburda 2014), we assume that consumers single-home while providers multi-home. There are a group of new consumers arriving at the platform market place at the beginning of each period. New consumer arrivals are distributed along a Hotelling segment $[0, 1]$ with density ρ . Without loss of generality, we normalize the total consumer population size over two periods to 1 such that $\rho = \frac{1}{2}$. Since consumers are single homing, they choose between platform A and B upon arrival. Denote Q_{ki}^c (q_{ki}^c) as the accumulated (new) number of consumers for platform $k \in \{A, B\}$ in period $i \in \{1, 2\}$, respectively. In period 2, we assume a retention rate of δ ($\delta \in [0, 1]$) for all period 1 consumers. We have $Q_{k1}^c = q_{k1}^c$ and $Q_{k2}^c = \delta Q_{k1}^c + q_{k2}^c$.

Extending the consumer side demand function in Hagiu and Hałaburda (2014) to our two-period setting, we have

$$q_{Ai}^c = \rho \left[\frac{1}{2} + \frac{\beta(Q_{Ai}^d - Q_{Bi}^d) - p_{Ai}^c + p_{Bi}^c}{2t} \right], \quad q_{Bi}^c = \rho - q_{Ai}^c, \quad (1)$$

where $\beta > 0$ is the strength of cross-side network effects on consumer side, which represents the surplus derived by a consumer from the participation of each provider. t is the unit transportation cost which measures the degree of horizontal differentiation between two platforms.

Following the literature (e.g., Hagiu and Hałaburda 2014), the provider side demand function in period $i \in \{1, 2\}$ for platform $k \in \{A, B\}$ is given by

$$q_{ki}^d = \alpha Q_{ki}^c - p_{ki}^d, \quad (2)$$

where $\alpha > 0$ is the strength of cross-side network effects on the provider side, which represents the profit made by a provider on every participating consumer. Equation (2) suggests that each provider's adoption decision of one platform is not affected by the other platform, which captures the fact that providers are open to (but not necessarily) multi-homing. Note that the system of equations (1) and (2) characterizes the flow of consumer arrivals in each period, and incorporates cross-side network effects between consumers and providers.

Denote π_{ki} as the single-period profit for platform $k \in \{A, B\}$ in period $i \in \{1, 2\}$, and Π_{ki} as the discount future profit for platform $k \in \{A, B\}$ at the beginning of period $i \in \{1, 2\}$. We solve for the multi-period platform competition problem using backward induction. First, given Q_{k1}^c , each platform k solves for the period 2 problem:

$$\max_{p_{k2}^c, p_{k2}^d, k \in \{A, B\}} \Pi_{k2} = \pi_{k2} = p_{k2}^c Q_{k2}^c + p_{k2}^d Q_{k2}^d = p_{k2}^c (\delta Q_{k1}^c + q_{k2}^c) + p_{k2}^d Q_{k2}^d. \quad (3)$$

Then each platform k solves for the period 1 problem by discounting the future profit by λ .

$$\max_{p_{k1}^c, p_{k1}^d, k \in \{A, B\}} \Pi_{k1} = \pi_{k1} + \lambda \Pi_{k2}^* = p_{k1}^c Q_{k1}^c + p_{k1}^d Q_{k1}^d + \lambda \Pi_{k2}^*, \quad (4)$$

where Π_{k2}^* is a function of Q_{k1}^c , and Q_{k1}^c is determined by the equilibrium prices in period 1. Later we will extend the model to N periods in Section 6.1 where $N > 2$. To ensure that the profit

functions are well-behaved, following the literature (e.g., Armstrong 2006, Hagiu and Halaburda 2014), we further assume

$$4t > (\alpha + \beta)^2. \quad (5)$$

We summarize our key notation in Table 2.

k	Indicator for the platform, $k \in \{A, B\}$;
m	Side indicator, consumer side (c) or provider (d) side, $m \in \{c, d\}$;
i	Index of periods, $i \in \{1, 2\}$ in Section 3 to 5, and $i \in \{1, 2, 3\}$ in Section 6.1;
$\beta(\alpha)$	Strength of the consumer (provider) side network effects, $\alpha, \beta \geq 0$;
p_k^m	Equilibrium price of platform k on side m in the single-period duopoly;
p_{ki}^m	Equilibrium price of platform k on side m in the symmetric multi-period duopoly;
\tilde{p}_{ki}^m	Equilibrium price of platform k on side m in the asymmetric multi-period duopoly;
Δ_{ki}^m	Coefficient of Q_0 in the equilibrium prices in the asymmetric duopoly, i.e., $\Delta_{ki}^m = \frac{\partial \tilde{p}_{ki}^m}{\partial Q_0}$;
$q_{ki}^c (Q_{ki}^c)$	New (accumulated) consumers in period $i \in \{1, 2\}$ for platform k ;
π_{ki}	Platform k 's single-period profit in period i ;
Π_{ki}	Platform k 's discounted future profit at the beginning of period i ;
δ	Retention rate for period 1 consumers, $\delta \in [0, 1]$;
λ	Discount factor, $\lambda \in [0, 1]$;
t	Transportation cost, assume $4t > (\alpha + \beta)^2$;
Q_0	Imported consumer base of platform A ;
b	Cost coefficient for building Q_0 , $b \geq 0$;
ρ	Density for new consumer arrivals, $\rho = \frac{1}{2}(\frac{1}{3})$ for two-period (three-period) duopoly.

Table 2 Summary of Key Notation

4. Symmetric Duopoly

Consider a symmetric duopoly. Our baseline model includes the single-period model from the literature (e.g., Hagiu and Halaburda 2014) as a special case by solving the period 1 problem in Equation (4) with $\lambda = 0$. Denote p_k^c (p_k^d) as the platform k 's single-period pricing strategy for consumers (providers). The following Lemma 1 replicates the single-period symmetric duopoly results in the literature.

Lemma 1. *(Hagiu and Halaburda 2014) In a single-period symmetric duopoly, at equilibrium, the optimal pricing strategies for platform $k \in \{A, B\}$ are given by*

$$(p_k^c)^* = t - \frac{\alpha(\alpha + 3\beta)}{8}, \quad (p_k^d)^* = \frac{\alpha - \beta}{8}. \quad (6)$$

Given Lemma 1, Corollary 1 follows immediately.

Corollary 1. *In a single-period symmetric duopoly, at equilibrium, platforms decrease (increase) provider side price when the consumer (provider) side network effects are strengthened (i.e., $\frac{\partial(p_k^d)^*}{\partial\beta} < 0$, $\frac{\partial(p_k^c)^*}{\partial\alpha} > 0$). Platforms decrease consumer side price when the network effects are strengthened on either side (i.e., $\frac{\partial(p_k^c)^*}{\partial\alpha} < 0$, $\frac{\partial(p_k^d)^*}{\partial\beta} < 0$).*

The findings in Lemma 1 are consistent with the literature where only one side of the market multi-homes (e.g., Proposition 3 in Economides and Tåg 2012). We will use the single-period equilibrium price in Equation (6) repeatedly in the rest of this paper. Benchmarked by $(p_k^m)^*$, we are able to show how equilibrium prices under the dynamic competition (i.e., p_{ki}^m) would deviate from the single-period equilibrium. Proposition 1 summarizes our results in a two-period symmetric duopoly setting.

Proposition 1. *In a two-period symmetric duopoly, at equilibrium, the optimal pricing strategies for platform $k \in \{A, B\}$ are given by Table 3.*

Note that our two-period model is more general, as it contains the single-period model in the literature as a special case by setting $\delta = 0$. In this case, Proposition 1 is identical to Lemma 1. Our

Period	Market	$(p_{ki}^m)^*$	$(p_{ki}^m)^* - (p_k^m)^*$
1	Consumer	$\left(t - \frac{\alpha(\alpha+3\beta)}{8}\right) - \frac{t\lambda\delta(1+\delta)[16t - (\alpha^2+6\alpha\beta+\beta^2)]}{2[12t - (\alpha^2+4\alpha\beta+\beta^2)]}$	$-\frac{t\lambda\delta(1+\delta)[16t - (\alpha^2+6\alpha\beta+\beta^2)]}{2[12t - (\alpha^2+4\alpha\beta+\beta^2)]} \leq 0$
	Provider	$\frac{\alpha-\beta}{8}$	0
2	Consumer	$(1+\delta)\left(t - \frac{\alpha(\alpha+3\beta)}{8}\right)$	$\delta\left(t - \frac{\alpha(\alpha+3\beta)}{8}\right) \geq 0$
	Provider	$\frac{(1+\delta)(\alpha-\beta)}{8}$	$\frac{\delta(\alpha-\beta)}{8}$

Table 3 Equilibrium Prices in the Symmetric Two-period Duopoly

key interest of this paper lies in the case when $\delta > 0$. We are curious to know what new insights a dynamic setting can uncover, compared to a single-period duopoly. To show this, we compute the price gap $(p_{ki}^c)^* - (p_k^c)^*$ in the last column in Table 3. At equilibrium, we notice following interesting findings.

First, we show the relationship among the equilibrium prices (p_{ki}^m) , the discount factor (λ), and the retention rate (δ). On the consumer side, platforms offer a smaller price in period 1 (i.e., $(p_{k1}^c)^* - (p_k^c)^* \leq 0$). The magnitude of this price gap is amplified by a larger discount factor (i.e., $\partial|(p_{k1}^c)^* - (p_k^c)^*|/\partial\lambda \geq 0$) or a greater retention rate (i.e., $\partial|(p_{k1}^c)^* - (p_k^c)^*|/\partial\delta \geq 0$). However, it is suboptimal for the platforms to offer a smaller price to consumers in period 2. On the contrary, platforms should increase their prices (i.e., $(p_{k2}^c)^* - (p_k^c)^* \geq 0$).

On the provider side, platforms offer the single-period equilibrium price in period 1. In period 2, platforms may find it optimal to offer a smaller price (i.e., $(p_{ki}^d)^* - (p_k^d)^* < 0$) to providers if and only if $\beta > \alpha$ and this price gap is amplified by a larger retention rate (i.e., $\partial|(p_{ki}^d)^* - (p_k^d)^*|/\partial\delta \geq 0$ when $\beta \geq \alpha$).

Second, we further highlight the impact of the time horizon. We note that the optimal strategies on the consumer side are sensitive to the time horizon. Compared to the single-period setting, charging consumers a smaller price in the first period is optimal in a two-period setting because those consumers acquired in the early period not only attract providers but also contribute to the consumer side profit in the second period. In other words, consumers acquired in the early period

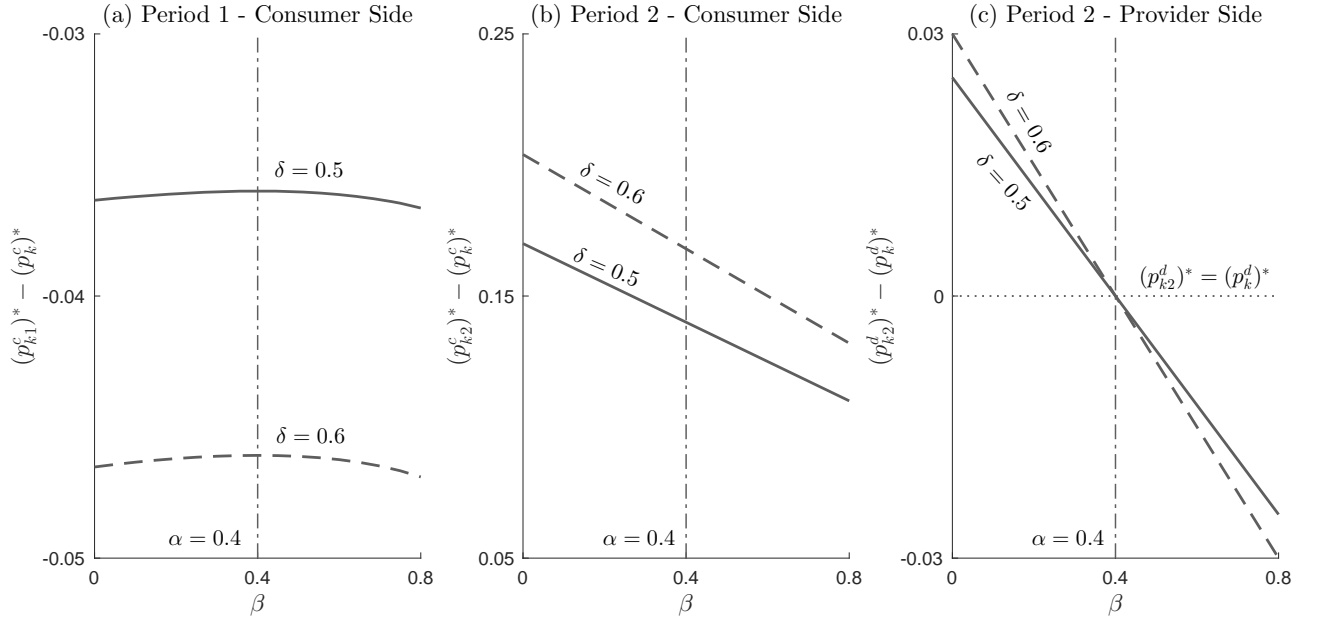


Figure 2 The Impact of δ on the Equilibrium Prices ($t = 4$, $\alpha = 1.5$)

become more valuable in a multi-period setting.

Interestingly, the optimal strategies on the provider side are not sensitive to the time horizon (i.e., $\frac{\partial (p_{k1}^d)^*}{\partial \lambda} = \frac{\partial (p_{k2}^d)^*}{\partial \lambda} = 0$). In period 2, the consumer side consists of both existing consumers and new consumers, resulting in a growing consumer installed base when δ is sufficiently large (i.e., when $\delta \geq 1 - \frac{q_{k2}^c}{Q_{k1}^c}$). In contrast, there is no similar installed base growth pattern on the provider side, as the provider side demand is jointly determined by the cross-side network effects and period 2 price (i.e., $Q_{k2}^d = \alpha Q_{k2}^c - p_{k2}^d$). Thus, optimal pricing on the provider side follows a similar logic as in the single-period setting, which is not sensitive to the time horizon.

Finally, we examine the role of cross-side network effects and their interactions with the consumer retention rate δ . As illustrated in Figure 2, when fixing $\alpha = 0.4$ and increasing β from 0 to 0.8, a smaller provider-side price is optimal in period 2 when $\beta < \alpha$ (decreasing curves in Figure 2c). On the consumer side, a greater β results in a smaller price in period 2 (decreasing curves in Figure 2b), but gives non-monotonic impacts in period 1. When β is smaller than α , indicating that consumer side network effects are relatively smaller, a greater β will push up consumer side price (increasing curves on the left side of Figure 2a). As β grows above α , a greater β will trigger a price war on the consumer side because both platforms find it optimal to subsidize consumers (decreasing curves on

the right side of Figure 2a). Furthermore, a greater retention rate δ will drive the price war more viciously when $\beta > \alpha$, because period 1 adopters becomes more valuable for platforms' period 2 profit (the curve with $\delta = 0.6$ is lower than that with $\delta = 0.5$ in Figure 2a).

5. The Piggyback Strategy

In this section, we extend our baseline model to consider the scenario when one startup, say platform A (but not platform B) employs the piggyback strategy. This is modeled formally by endowing platform A with an initial installed base of Q_0 on the consumer side. Such a formal treatment captures the essence of piggybacking where startups can redirect consumers from external networks to join their focal platforms, as we illustrated in Table 1.

We provide our analysis in two subsections. Subsection 5.1 considers an exogenous Q_0 , while subsection 5.2 endogenizes Q_0 .

5.1. Exogenous Q_0

Given an existing user base Q_0 , we modify the system of demand functions in Equation (1) as follows.

$$Q_{A1}^c = Q_0 + q_{A1}^c = Q_0 + \rho \left[\frac{1}{2} + \frac{\beta(Q_{A1}^d - Q_{B1}^d) - \tilde{p}_{A1}^c + \tilde{p}_{B1}^c}{2t} \right], \quad q_{B1}^c = \rho - q_{A1}^c, \quad (7)$$

where \tilde{p} denotes respective prices under Q_0 . Demand functions in period 2 remain the same as in Equation (1) when $i = 2$. At equilibrium, it can be shown that the optimal pricing strategies can be re-written as the combination of the optimal prices under the symmetric duopoly (defined by Proposition 1), and an additional term that is linear in Q_0 , i.e., $(\tilde{p}_{ki}^m)^* = (p_{ki}^m)^* + \Delta_{ki}^m Q_0$ where $\Delta_{ki}^m = \frac{\partial (\tilde{p}_{ki}^m)^*}{\partial Q_0}$. Δ_{ki}^m (specified in the appendix) are only functions of $\{t, \alpha, \beta\}$ but not Q_0 . This allows us to examine the marginal effects of Q_0 on $(\tilde{p}_{ki}^m)^*$. The following Proposition 2 summarizes our results.

Proposition 2. *In a two-period asymmetric duopoly where platform A is endowed with a positive initial installed base of Q_0 , Table 4 characterizes the comparative statics of Q_0 in period 1 equilibrium prices. Regions R1 to R8 are depicted in Figure 3 and 4.*

Market	Platform	$\frac{\partial(p_{k1}^m)^*}{\partial Q_0} < 0$	$\frac{\partial(p_{k1}^m)^*}{\partial Q_0} > 0$	$\frac{\partial(p_{k1}^m)^*}{\partial Q_0} = 0$
Consumer (c)	A	$\Delta_{A1}^c < 0$ (R1)	$\Delta_{A1}^c > 0$ (R2)	$\Delta_{A1}^c = 0$
	B	$\Delta_{B1}^c < 0$ (R5)	$\Delta_{B1}^c > 0$ (R6)	$\Delta_{B1}^c = 0$
Provider (d)	A	$\alpha < \beta$ (R3)	$\alpha > \beta$ (R4)	$\alpha = \beta$
	B	$\Delta_{B1}^d < 0$ (R7)	$\Delta_{B1}^d > 0$ (R8)	$\Delta_{B1}^d = 0$

Table 4 Comparative Statics of Q_0 in period 1. Period 2 equilibrium prices follow a similar pattern. Δ_{ki}^m are defined in the appendix. Regions R1 to R8 are visualized in Figure 3 and 4.

The impact of Q_0 is similar in each period. For brevity, we focus on period 1. For platform A , Proposition 2 says that \tilde{p}_{Ai}^c goes up as Q_0 increases, suggesting that acquiring Q_0 and subsidizing consumers with a smaller price substitute each other in most cases. On the provider side, when $\alpha < \beta$, platform A decreases price on the provider side in order to secure an even larger consumer base in the early period, suggesting complementarity between acquiring Q_0 and subsidizing providers; otherwise when $\alpha > \beta$, acquiring Q_0 and subsidizing consumers substitute each other.

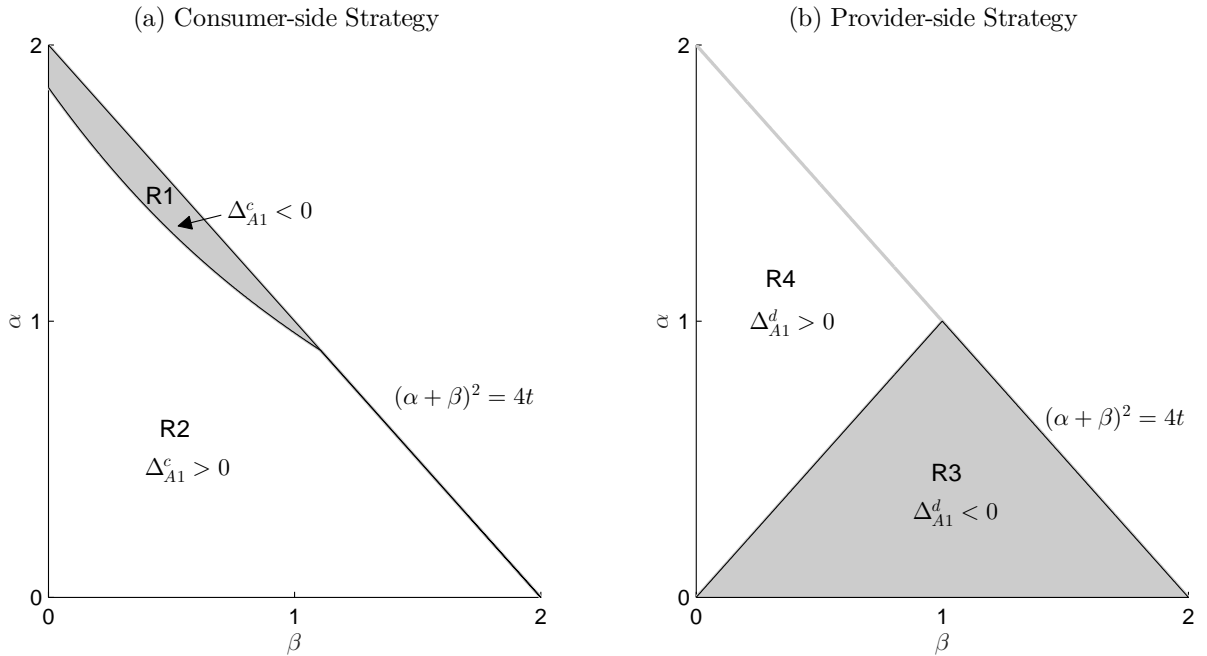


Figure 3 Platform A 's Period 1 Pricing Strategy ($t = 1$). Similar Pattern for Period 2.

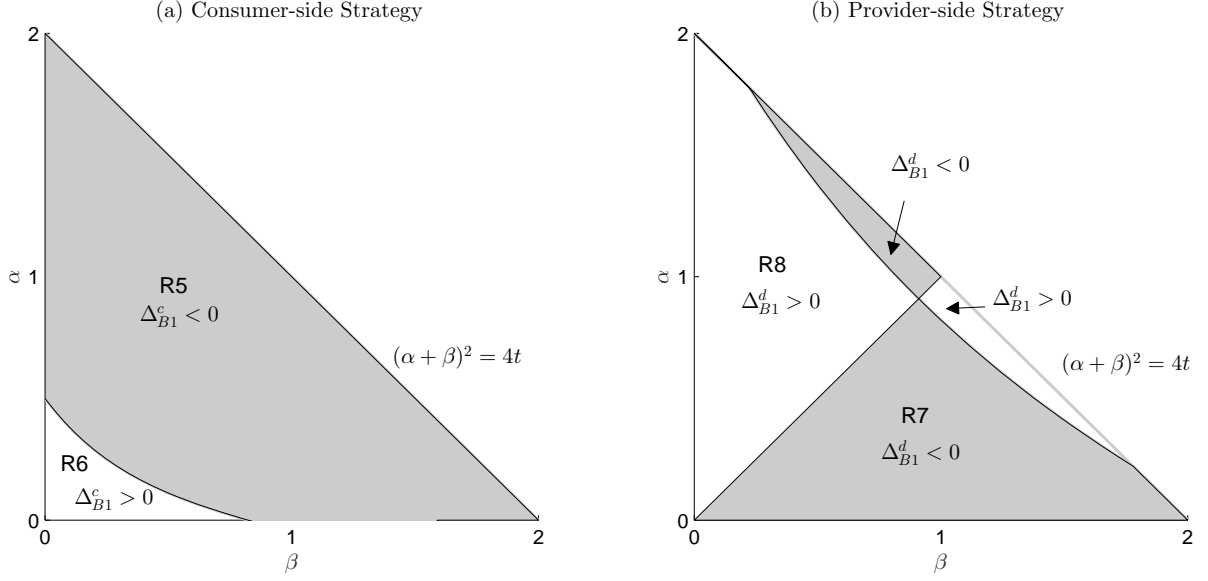


Figure 4 Platform B 's Period 1 Pricing Strategy ($t = 1$). Similar Pattern for Period 2.

To compete, the non-piggybacking platform's optimal pricing strategies are non-trivial. On the consumer side, given that platform A intends to raise \tilde{p}_{A1}^c in most cases, platform B may find it optimal to follow the suite by increasing \tilde{p}_{B1}^c when network effects of both directions are small (i.e., in region R6 of Figure 4a). This is given by platform B 's best responsive function. On the other hand, when network effects are sufficiently strong (in region R5 of Figure 4a), platform B has to offer a smaller \tilde{p}_{B1}^c . This is because that platform A 's initial Q_0 generates a powerful virtuous cycle between providers and consumers. Platform B has to differentiate himself by substantially decreasing price on the consumer side in order to maintain his consumer base.

On the provider side, platform B raises \tilde{p}_{B1}^d in two white regions of Figure 4b. (1) When β is small enough (see Region R8 of Figure 4b), the provider side is relatively more profitable (i.e., $\alpha > \beta$), thus both platforms raise prices on the provider side. (2) In the other white region (far right) of Figure 4b, because platform B has to aggressively subsidize consumers (see the respective region in R5 of Figure 4a), reducing prices \tilde{p}_{B1}^d to the providers at the same time would result in too much profit loss. Instead, platform B raises price in this white region.

To further illustrate the impact of Q_0 , we compare Figure 3 and 4 to search for regions where both platforms decrease prices (i.e., a price war) or increase prices (i.e., a reverse price war). As

shown in Figure 5, on the consumer side, such a price war occurs when the strength of cross-side network effects on the provider side (i.e., α) is strong enough, where provider participation is largely driven by consumer adoption. In this case, platforms compete for more consumers, even if platform A has been endowed with Q_0 .

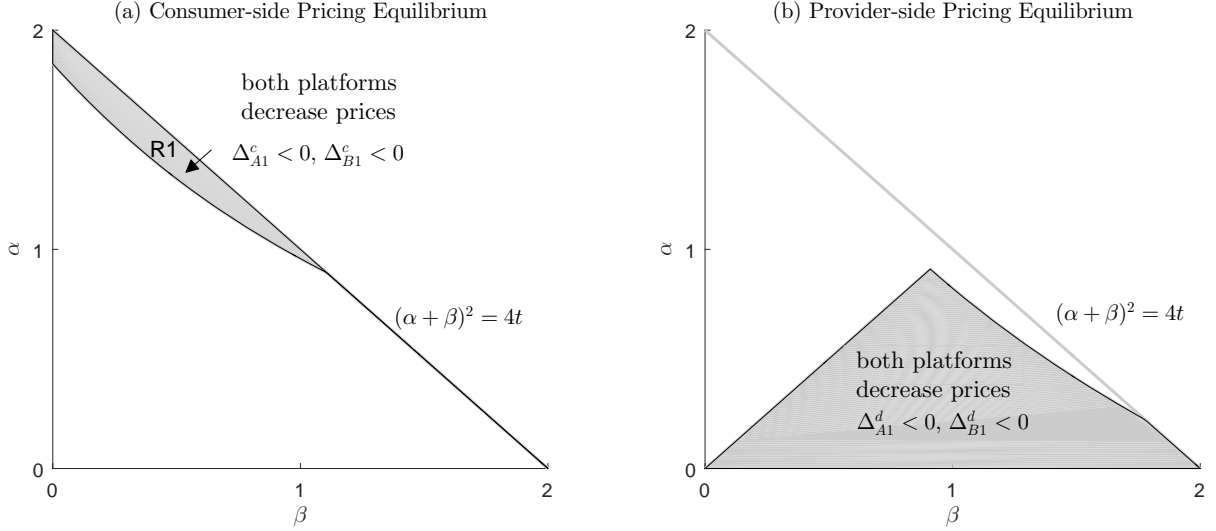


Figure 5 Regions Where Both Platforms Decrease Prices ($t = 1$)

Interestingly, there are regions where both platforms choose to increase prices (i.e., a reverse price war, see the shaded region in Figure 6). This implies that both platforms can benefit from the piggyback strategy of one of them. On the consumer side, this emerges when both α and β are small enough, such that the cross-side value of owning Q_0 is limited and platform A would like to monetize it early in period 1. On the provider side, both platforms raise prices if β is small enough, where provider participation is driven mostly by consumer adoption. In this case, Q_0 can help two platforms differentiate each other and soften the competition.

Finally, we examine the profit advantage of piggybacking over non-piggybacking.

Proposition 3. *If $\alpha = \beta$, platform A 's marginal profit advantage over platform B due to piggybacking is amplified by the strength of network effects, i.e., $\frac{\partial^2(\Pi_{A1}^* - \Pi_{B1}^*)}{\partial \alpha \partial Q_0} = \frac{\partial^2(\Pi_{A1}^* - \Pi_{B1}^*)}{\partial \beta \partial Q_0} \geq 0$.*

This result implies that, as the strength of network effects increases, the piggybacking platform

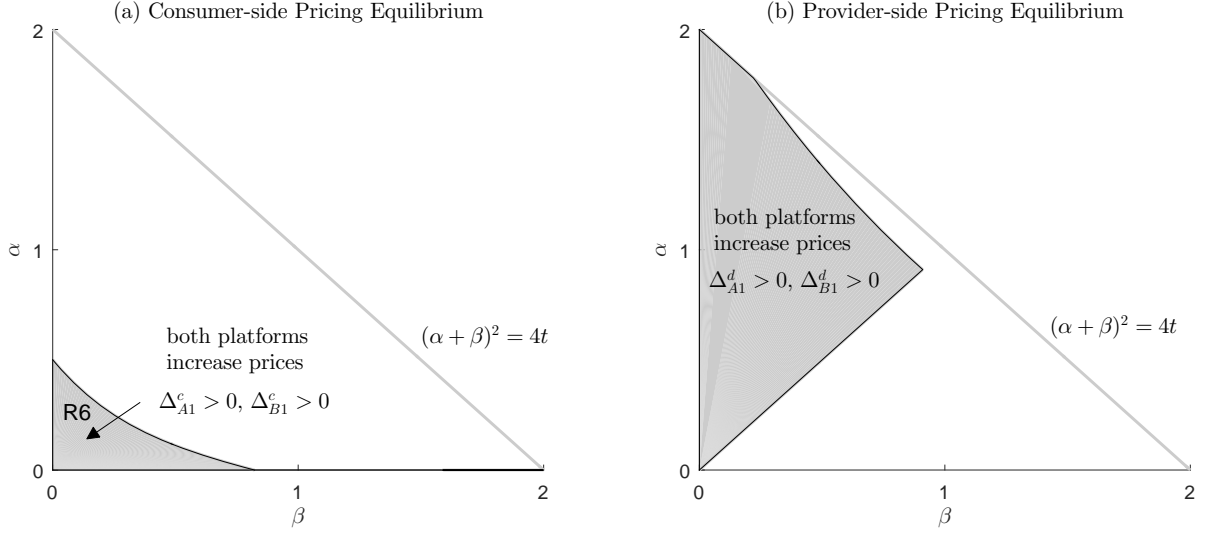


Figure 6 Regions Where Both Platforms increase prices ($t = 1$)

obtains an increasing profit advantage over her rival who does not piggyback. So far, we have assumed that piggybacking is costless for platform A. Next we endogenize Q_0 by considering the acquisition cost of Q_0 .

5.2. Endogenous Q_0

When acquiring Q_0 is costly, we modify the platform A's period 1 objective function in Equation (4) as follows

$$\max_{p_{A1}^c, p_{A1}^d, Q_0} \Pi_{A1} = \pi_{A1} + \lambda \Pi_{A2}^* = p_{A1}^c Q_{A1}^c + p_{A1}^d Q_{A1}^d - bQ_0^2 + \lambda \Pi_{A2}^*, \quad (8)$$

where bQ_0^2 represents the total cost of acquisition of Q_0 . Our assumption of a convex cost of acquiring Q_0 , captures the fact that it becomes increasingly difficult for the platform to import additional consumers. To avoid the trivial solution of $Q_0^* \rightarrow +\infty$ when b is too small, we assume that b is large enough such that $b > \hat{b}$ (\hat{b} is defined in the appendix). All other model setups remain the same as those in the baseline model. To align with Section 5.1, we assume again that platform B does not piggyback. For exposition simplicity, we further assume $\lambda = \delta = 1$.

Proposition 4 characterizes platform A's optimal strategies.

Proposition 4. *When platform A incurs a piggybacking cost of bQ_0^2 , at equilibrium, the following holds when b increases.*

1. Q_0^* decreases (i.e., $\frac{Q_0^*}{\partial b} < 0$);
2. On the consumer side, platform A increases period 1 price (i.e., $\frac{(\bar{p}_{A1}^c)^*}{\partial b} > 0$) only when $t < \hat{t}$ and $\frac{\beta}{\alpha} < \hat{u}$ (where \hat{t} and \hat{u} are shown to be unique in the appendix), otherwise platform A decreases price in period 1. Platform A always decreases period 2 price (i.e., $\frac{(\bar{p}_{A2}^c)^*}{\partial b} < 0$);
3. On the provider side, platform A decreases price in both periods (i.e., $\frac{\partial(\bar{p}_{Ai}^d)^*}{\partial b} < 0$) if and only if $\alpha > \beta$.

Proposition 4 uncovers additional insights. Benchmarked with Proposition 2, Proposition 4 suggests that, when Q_0 is endogenized, platforms need to balance the transportation cost t , optimal piggybacking Q_0^* , and optimal pricing in a non-trivial way, as illustrated in Figure 7.

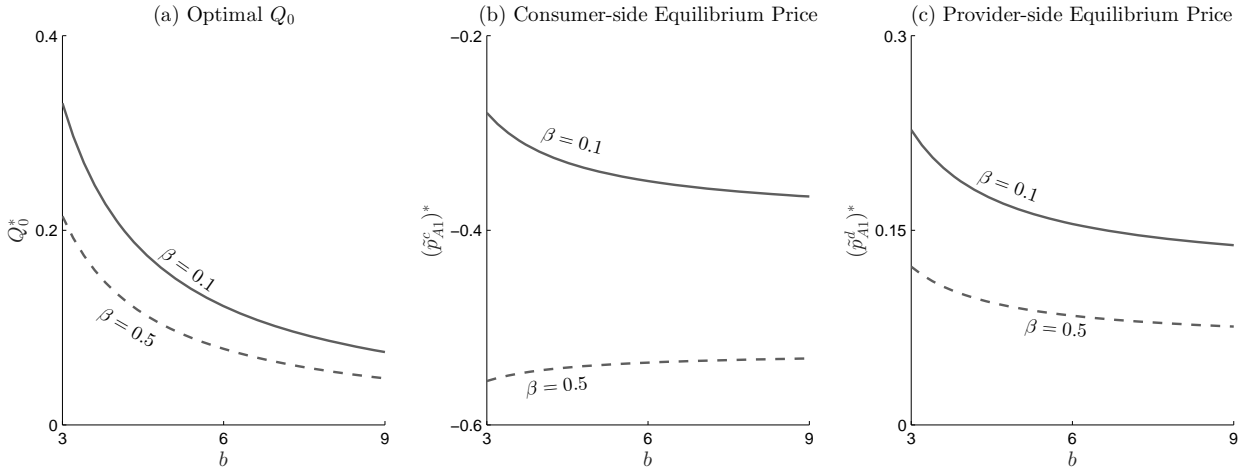


Figure 7 Comparative Statics of b ($\alpha = 1$, $t = 0.6$)

Consider the case of $\alpha > \beta$ when providers are more sensitive than consumers to the cross-side network effects. In this case, when b increases, Q_0^* decreases (see Figure 7a), which in turn, hurts the provider side more due to cross-side network effects. To counter, platform A decreases price on the provider side (see Figure 7c). Interestingly, when both t and $\frac{\beta}{\alpha}$ are small, the marginal impact of a decreasing Q_0 is less severe on the consumer side (versus a price increase), and charging a slightly higher price on the consumer side can overcome the loss due to a decreasing Q_0 (see the

dashed curve in Figure 7b where $\beta = 0.5$). Otherwise, when either t or β is large enough, platform A decreases consumer side price to attract more consumers. Furthermore, when β increases, meaning consumers are more sensitive to cross-network effects, all three curves in Figure 7 shift downward, in which case, reducing prices on both sides is more effective than increasing Q_0 .

6. Extensions

In this section, we extend our baseline model from Section 4. Subsection 6.1 address whether insights from our two-period setting can be generated in a three-period setting. Subsection 6.2 considers asymmetric discount factors of the platforms in an effort to be one step closer to real-world startups who are more sensitive than others to the short-term profit.

6.1. Three-period Duopoly

For analytical tractability, we use a three-period duopoly to highlight our model implications to an N -period competition where $N > 2$. W.o.l.g., we assume that the number of new consumer arrivals in each period equals $\frac{1}{3}$ (i.e., $\rho = \frac{1}{3}$) such that total consumer population remains at 1. Using backward induction, each platform first solves for its problem in period 3, followed by period 2 and period 1. Platform k 's problem in period 3 is

$$\max_{p_{k3}^c, p_{k3}^d, k \in \{A, B\}} \Pi_{k3} = \pi_{k3} = p_{k3}^c Q_{k3}^c + p_{k3}^d Q_{k3}^d.$$

Platform k 's problem in period $i \in \{1, 2\}$ is

$$\max_{p_{ki}^c, p_{ki}^d, i \in \{1, 2\}, k \in \{A, B\}} \Pi_{ki} = \pi_{ki} + \lambda \Pi_{ki+1}^* = p_{ki}^c Q_{ki}^c + p_{ki}^d Q_{ki}^d + \lambda \Pi_{ki+1}^*.$$

Solving, we have the following Proposition 5.

Proposition 5. *In a three-period symmetric duopoly, at equilibrium, the optimal pricing strategies for platform $k \in \{A, B\}$ are given by Table 5*

where

$$\Phi_1 = \frac{t\lambda\delta(1+\delta)[24t - (\alpha^2 + 6\alpha\beta + \beta^2)] \left[36t^2\lambda\delta^2 - (18t + \alpha^2 + 4\alpha\beta + \beta^2)^2 \right]}{72t^2\lambda\delta^2 [24t - (\alpha^2 + 6\alpha\beta + \beta^2)] - 2 [18t - (\alpha^2 + 4\alpha\beta + \beta^2)]^3} > 0;$$

Period	Market	$(p_{ki}^m)^*$	$(p_{ki}^m)^* - (p_k^m)^*$
1	Consumer	$\left(t - \frac{\alpha(\alpha+3\beta)}{12}\right) - \Phi_1$	$-\Phi_1$
	Provider	$\frac{\alpha-\beta}{12}$	0
2	Consumer	$\left(t - \frac{\alpha(\alpha+3\beta)}{12}\right) (1 + \delta) - \Phi_2$	$-\Phi_2 + \delta \left(t - \frac{\alpha(\alpha+3\beta)}{12}\right)$
	Provider	$\frac{(1+\delta)(\alpha-\beta)}{12}$	$\frac{\delta(\alpha-\beta)}{12}$
3	Consumer	$\left(t - \frac{\alpha(\alpha+3\beta)}{12}\right) (1 + \delta + \delta^2)$	$\delta(1 + \delta) \left(t - \frac{\alpha(\alpha+3\beta)}{12}\right)$
	Provider	$\frac{(1+\delta+\delta^2)(\alpha-\beta)}{12}$	$\frac{\delta(1+\delta)(\alpha-\beta)}{12}$

Table 5 Equilibrium Prices in Symmetric Three-Period Duopoly

$$\Phi_2 = \frac{t\lambda\delta(1 + \delta + \delta^2) [24t - (\alpha^2 + 6\alpha\beta + \beta^2)]}{2 [18t - (\alpha^2 + 4\alpha\beta + \beta^2)]} > 0.$$

Proposition 5 shows the following insights. First note that on the provider side, platforms' prices over three periods are not affected by λ . Second, on the consumer side, platforms' period 1 prices are decreasing and convex in the discount factor (i.e., $\frac{\partial(p_{k1}^c)^*}{\partial\lambda} < 0$, $\frac{\partial^2(p_{k1}^c)^*}{\partial\lambda^2} < 0$). Period 2 prices are decreasing and linear in the discount factor (i.e., $\frac{\partial(p_{k2}^c)^*}{\partial\lambda} < 0$, $\frac{\partial^2(p_{k2}^c)^*}{\partial\lambda^2} = 0$). Period 3 prices are not affected by λ .

We use an example to illustrate Proposition 5. As shown in Figure 8, as λ increases, prices drop more in period 1 than in period 2 (i.e., $\frac{\partial^2(p_{k1}^c)^*}{\partial\lambda^2} < 0$ and $\frac{\partial^2(p_{k2}^c)^*}{\partial\lambda^2} = 0$). In Figure 8a, when λ increases from 0.6 to 1, resulting in a greater price shift in period 1 (price decreases from -0.740 to -1.305) than in period 2 (price decreases from -0.600 to -1.000). There are no price changes in period 3 (0.717 in period 3 in Figure 8a) (see Figure 8b). On the other hand, when the retention rate δ increases from 0.6 to 1, as illustrated in Figure 8c, prices drop more in period 1 (from -0.574 to -1.305) than in period 2 (from -0.392 to -1.000). In period 3, platforms are able to harvest by charging high prices.

The provider side is another story. In period 1, they charge the same price as if it were a single

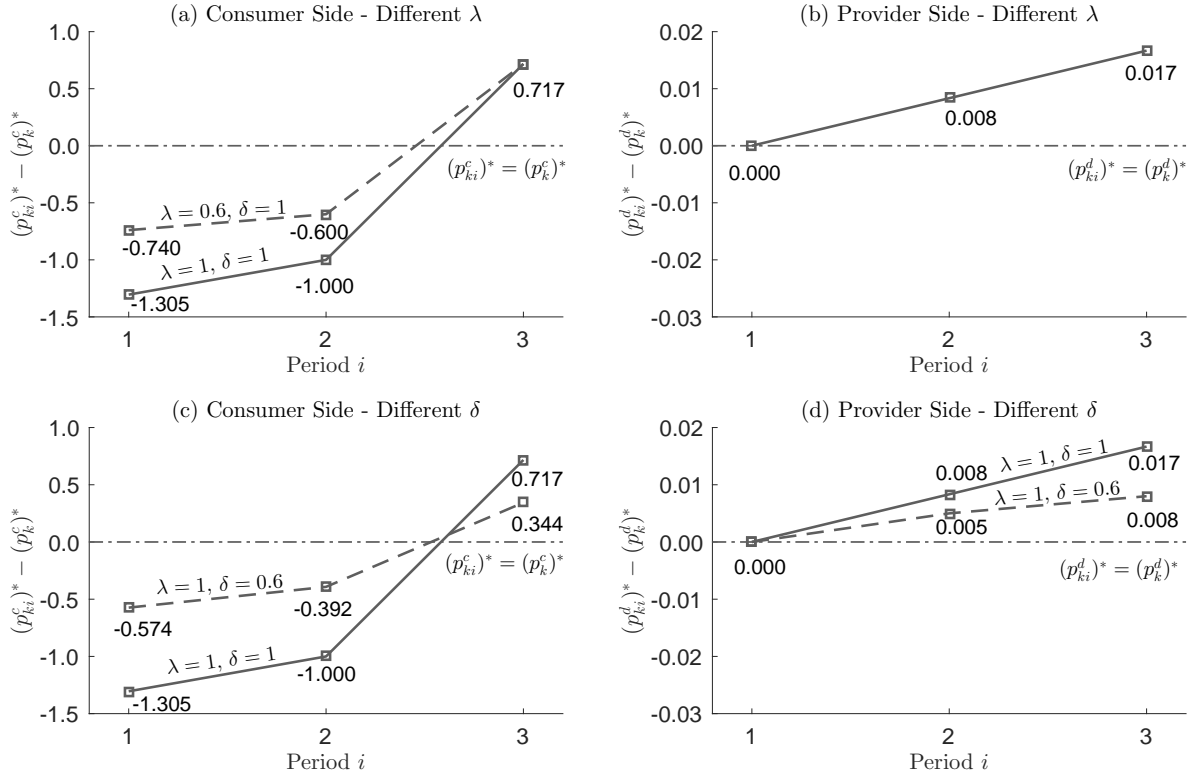


Figure 8 Impact of Discount Factor λ and Retention Rate δ in a Three-period Setting ($\alpha = 0.5, \beta = 0.4, t = 0.5$)

period. They raise prices over time from period 1 to period 2 to period 3. Such price increases are augmented by an increase in retention rate.

In sum, as the future profit is less discounted (a greater λ), platforms decrease prices more in the early periods to grow consumer base, but the provider side prices are not affected by λ . Proposition 5 shows that key insights from a two-period setting can be generalized to a three-period setting.

6.2. Asymmetric Discount Factors

Startup platforms have limited resources. In particular, they often have to deal with financial constraints and objectives in the short-run (e.g., Swinney et al. 2011). As a result, startup platforms may discount future profit more significantly, but focus on short-term profit (e.g., Zhu and Zhou 2012). To capture this scenario, we consider asymmetric discount factors λ_k ($\lambda_k \in [0, 1], k \in \{A, B\}$).

We modify Platform k 's period 1 objective function as follows.

$$\max_{p_{k1}^c, p_{k1}^d, k \in \{A, B\}} \Pi_{k1} = \pi_{k1} + \lambda \Pi_{k2}^* = p_{k1}^c Q_{k1}^c + p_{k1}^d Q_{k1}^d + \lambda_k \Pi_{k2}^*. \quad (9)$$

Without loss of generality, we assume that $1 \geq \lambda_A > \lambda_B \geq 0$. All other assumptions remain the same as in our baseline model except for exposition simplicity, we let $Q_0 = 0$ and $\delta = 1$. We have the following Proposition 6.

Proposition 6. *In a two-period asymmetric duopoly where platform k ($k \in \{A, B\}$) has a discount factor of λ_k ($\lambda_k \in [0, 1]$), at equilibrium, platforms' optimal prices are given in Table 6 where Ω_j ($j \in \{1, 2, 3, 4, 5\}$) is given by*

$$\begin{aligned}\Omega_1 &= -t(16t - \alpha^2 - 6\alpha\beta - \beta^2) \\ &\quad \times \frac{\left(\lambda_A \left(32t^2 \lambda_B (16t - \alpha^2 - 6\alpha\beta - \beta^2) - (\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^2 (16t - \alpha^2 - 5\alpha\beta - 2\beta^2) \right) \right.}{\left. - \lambda_B (\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^2 (8t - \alpha(\alpha + 3\beta)) \right)}{2(12t - \alpha^2 + 4\alpha\beta + \beta^2) (8t^2 (\lambda_A + \lambda_B) (-\alpha^2 - 6\alpha\beta - \beta^2 + 16t) + (\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^3)}; \\ \Omega_2 &= t(16t - \alpha^2 - 6\alpha\beta - \beta^2) \\ &\quad \times \frac{\left(\lambda_A \left(32t^2 \lambda_B (16t - \alpha^2 - 6\alpha\beta - \beta^2) - (\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^2 (8t - \alpha(\alpha + 3\beta)) \right) \right.}{\left. - \lambda_B (16t - \alpha^2 - 5\alpha\beta - 2\beta^2) (\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^2 \right)}{2(12t - \alpha^2 - 4\alpha\beta - \beta^2) (8t^2 (\lambda_A + \lambda_B) (-\alpha^2 - 6\alpha\beta - \beta^2 + 16t) + (\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^3)}; \\ \Omega_3 &= -\frac{4(\lambda_A - \lambda_B)t(16t - \alpha^2 - 6\alpha\beta - \beta^2)(-\alpha^2 - 4\alpha\beta - \beta^2 + 12t)}{8(\lambda_A + \lambda_B)t^2(16t - \alpha^2 - 6\alpha\beta - \beta^2) - (12t - \alpha^2 - 4\alpha\beta - \beta^2)^3} \geq 0; \\ \Omega_4 &= -\frac{2(\lambda_A - \lambda_B)t^2(16t - \alpha^2 - 6\alpha\beta - \beta^2)(8t - \alpha(\alpha + 3\beta))}{8(\lambda_A + \lambda_B)t^2(16t - \alpha^2 - 6\alpha\beta - \beta^2) - (12t - \alpha^2 - 4\alpha\beta - \beta^2)^3} \geq 0; \\ \Omega_5 &= -\frac{8(\lambda_A - \lambda_B)t^2(16t - \alpha^2 - 6\alpha\beta - \beta^2)}{8(\lambda_A + \lambda_B)t^2(16t - \alpha^2 - 6\alpha\beta - \beta^2) - (12t - \alpha^2 - 4\alpha\beta - \beta^2)^3} \geq 0.\end{aligned}$$

Corollary 2 summarizes the insights from Proposition 6.

Corollary 2. *In a two-period asymmetric duopoly where $\lambda_B < \lambda_A$, at equilibrium, compared to platform A, the followings hold.*

1. *On the consumer side, platform B charges a higher price in period 1 and a lower price in period 2;*
2. *On the provider side, platform B charges a lower price in both periods if and only if $\alpha > \beta$.*

When competing with platform A with greater λ , platform B has less freedom to subsidize consumers in period 1 (i.e., $(p_{A1}^c)^* < (p_{B1}^c)^*$). To recover from the loss of customer base in period

Period	Market	$(p_{Ai}^m)^*$	$(p_{Ai}^m)^* - (p_A^m)^*$	$(p_{Bi}^m)^*$	$(p_{Bi}^m)^* - (p_B^m)^*$
1	Consumer	$\left(t - \frac{\alpha(\alpha+3\beta)}{8}\right) - \Omega_1$	$-\Omega_1$	$\left(t - \frac{\alpha(\alpha+3\beta)}{8}\right) - \Omega_2$	$-\Omega_2$
	Provider	$\frac{(\alpha-\beta)(1+\Omega_3)}{8}$	$\frac{\Omega_3(\alpha-\beta)}{8}$	$\frac{(\alpha-\beta)(1-\Omega_3)}{8}$	$-\frac{\Omega_3(\alpha-\beta)}{8}$
2	Consumer	$2\left(t - \frac{\alpha(\alpha+3\beta)}{8}\right) + \Omega_4$	Ω_4	$2\left(t - \frac{\alpha(\alpha+3\beta)}{8}\right) - \Omega_4$	$-\Omega_4$
	Provider	$\frac{(\alpha-\beta)(1+\Omega_5)}{4}$	$\frac{\Omega_5(\alpha-\beta)}{4}$	$\frac{(\alpha-\beta)(1-\Omega_5)}{4}$	$-\frac{\Omega_5(\alpha-\beta)}{4}$

Table 6 Equilibrium Prices in the Asymmetric Duopoly with $\lambda_A > \lambda_B$ ($\delta = 1$)

1 due to a higher period 1 price, in period 2 platform B has to decrease price in order to attract new consumers, while his competitor, platform A , is ready to harvest by charging a higher price (i.e., $(p_{A2}^c)^* > (p_{B2}^c)^*$). We illustrate this in Figure 9a and 9c. It can be observed that this pattern exists either when $\alpha > \beta$ or $\alpha < \beta$.

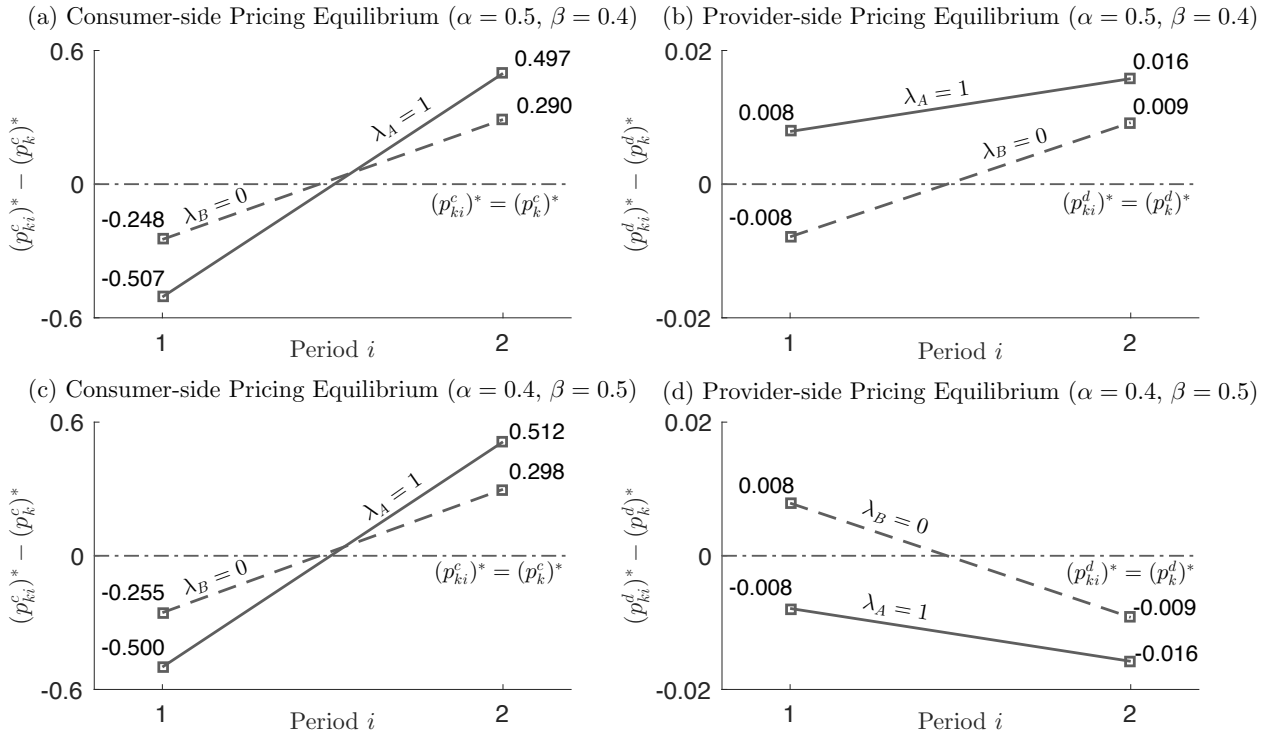


Figure 9 The Equilibrium Prices ($t = 0.5$)

On the provider side, platform B 's pricing strategies are more interesting. Corollary 2 suggests

that, counter-intuitively, when $\alpha > \beta$, platform B should offer a smaller price to providers, and it happens when the provider side is actually more profitable (i.e., see Figure 9b). Why? The motivation for maintaining a smaller price even under the short-term profit pressure is similar to the idea of price differentiation in Proposition 2. Specifically, significant provider side network effects give more incentives for platform A to harvest from the provider side with a higher price (platform A charges a higher price in period 1 in Figure 9b) while subsidizing consumers to induce more provider adoptions via cross-side network effects (platform A offers a lower price in period 1 in Figure 9a). In this case, it is optimal for platform B to differentiate himself from his competitor by offering a lower price to the providers. Otherwise, when $\alpha < \beta$ and provider side becomes less profitable, platform B starts to charge a higher provider price in both periods (see Figure 9d). In this case, the opportunity for platform B to charge a higher price to the provider side is, interestingly, when it is no-longer the “money” side for platform A .

7. Discussion and Conclusion

Single-period model		Two-period model ($\lambda = 1$)	
Single-homing side	Multi-homing side	Single-homing side	Multi-homing side
$p^c = 0.755$	$p^d = 0.000$	$p_1^c = -0.578$	$p_1^d = 0.000$
		$p_2^c = 1.510$	$p_2^d = 0.000$

Table 7 An Illustrative Example Comparing Single-Period vs. Two-Period Duopoly Strategies ($Q_0 = 0$, $\alpha = \beta = 0.7$, $t = 1$)

The rich literature on two-sided markets has established the optimality of the “seesaw principle” for platform pricing in the single-period setting. In this paper, we advance this literature by studying optimal pricing and piggybacking strategies in a dynamic setting. Our two-period model contains the single-period results as a special case (see Lemma 1), and we uncover a host of new insights not available in a single-period setting. To illustrate some of the new insights of our model,

we use an numerical example as summarized in Table 7. In this case, the single-homing side is identified as the “money side” where platforms charge a relatively higher price (0.755). The multi-homing side is subsidized (“free”). In the two-period setting, however, the single-homing side is strongly subsidized ($p_{k1}^c = -0.578$) in period 1, but charged a much higher price ($p_{k2}^c = 1.510$) in the second period. In contrast, the multi-homing side prices remain free, i.e., $p_1^d = p_2^d = 0$. Thus, platforms subsidize both sides in period 1, and there may not be a “money side” in each and every period. It can be observed that our dynamic model is richer than the single-period model and extends the single-period “seesaw principle” into a multi-period setting. We expand our discussion by considering a three-period model in Section 6.1. As visualized in Figure 8, our key insights under the two-period model can be generalized to a three-period setting, where platforms subsidize more aggressively on the single-homing side in the early period (Figure 8a). Collectively, our results provide a useful framework to examine practical platform competition, such as the case of Didi vs. Kuaidi competition we introduced in the Introduction section where both platforms start subsidizing on both sides of the markets in early periods, and consumer-side subsidy vanishes more quickly than the driver-side subsidy.

Platforms can solve the “chicken-and-egg” problem by piggybacking, i.e., importing users from external networks. However, the impact of piggybacking on pricing strategies remains unknown. Our findings suggest that pricing equilibrium is affected by piggybacking in a non-trivial way. The platform with the capability to import external users might still find it optimal to subsidize consumers (region R1 in Figure 3a) when the platform is not attractive enough for users (i.e., β is small). For instance, Zynga was a sub-platform of Facebook through which Zynga was able to recruit new users via viral acquisition. Nevertheless, it is reported that Zynga also made huge investments in paid marketing to harness its user adoption². Airbnb used to encourage cross-post housing information on Craigslist, which imported the user base from Craigslist to Airbnb³. More interestingly, as the user community constantly grew, Airbnb announced their free photography service for house owners to improve their houses’ online images, which eventually led to greater

² <http://www.ign.com/articles/2012/01/22/how-much-is-zynga-paying-for-new-gamers>

³ <http://hbswk.hbs.edu/item/how-uber-airbnb-and-etsy-attracted-their-first-1-000-customers>

profit margins from the traveler side⁴. This is consistent with our provider-side strategy illustrated in Figure 3: Airbnb used photography service to grow the attractiveness of the houses (i.e., a larger β). In this way, Airbnb could charge a greater price on the consumer side (Region R2), while still offering the free service to owners (Region R3). The interaction between piggybacking and subsidizing strategies is also reflected in Dropbox’s social media campaign in its early days. In contrast to the traditional approach using a “like us on Facebook” button, Dropbox offered 125MB free storage for users who connected to Dropbox with their Facebook accounts⁵.

When piggybacking becomes costly, platform owners face the challenge of balancing between subsidizing/pricing and importing new users. We address this issue in Section 5.2. As it becomes more difficult to acquire new external users, it is optimal for the platform to lower the price on the provider side, but not necessarily on the customer side (see Figure 7b). In 2010, when Zynga started to plan for splitting itself from Facebook due to weaker player acquisition⁶, Zynga also set up a new payment program (called “Platinum Purchase Program”) to focus on the high-margin customers by encouraging them to recharge with more than \$500 each time⁷. Our model suggests that this was an effort to remedy the loss due to weaker user acquisition from Facebook.

Finally, we consider the asymmetric competition between platforms who discount their future profits differently, as startup platforms maybe more sensitive to short-run profit goals. Our results suggest that engaging in a price war against established platforms on the “money” side is not optimal for startup platforms. Instead, it is optimal for them to monetize on the side where established firms might still subsidize. Our model is useful in understanding real-world platform competitions. Under attack from Microsoft when Windows Mediaplayer was bundled for free with Windows operating system, RealNetworks chose to leverage relationships with consumers and music companies to launch Rhapsody, a \$10-per-month music service. In this way, RealNetworks profits from consumers rather than subsidizing them (Eisenmann et al., 2006). Our model is also consistent

⁴ <https://www.airbnb.com/info/photography>

⁵ <https://blog.kissmetrics.com/dropbox-hacked-growth/>

⁶ <https://techcrunch.com/2010/05/07/zynga-gunning-up-and-lawyering-up-for-war-against-facebook-with-zynga-live/>

⁷ <http://gawker.com/5634379/the-secret-dealer-for-farmville-addicts>

with the ongoing competition in the video-streaming market where Youtube subsidizes content creators by sharing advertising revenue. Vimeo uses a business model by charging content creators for uploading⁸ contents. In this way, Vimeo has remained very popular in recent years as measured by its traffic record⁹. Our model provides a theoretical framework to analyze Vimeo's strategy. It will be interesting in the future to empirically test our model predictions using a large sample.

⁸ <https://vimeo.com/upgrade>

⁹ <http://www.alexa.com/siteinfo/vimeo.com>

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Appendix

Proofs

Proof of Lemma 1. Denote Q_k^m as the equilibrium demand on the m side of platform k in the single-period symmetric duopoly ($m \in \{c, d\}$, $k \in \{A, B\}$). Then Q_k^m is give by

$$Q_A^c = \rho \left[\frac{1}{2} + \frac{\beta(Q_A^d - Q_B^d) - p_A^c + p_B^c}{2t} \right], \quad Q_B^c = \rho - Q_A^c, \quad (\text{A.1})$$

$$Q_A^d = \alpha Q_A^c - p_A^d, \quad Q_B^d = \alpha Q_B^c - p_B^d.$$

Solving for the above system of demand functions with $\rho = \frac{1}{2}$, at equilibrium, we have

$$Q_A^c = \frac{2t - \alpha\beta - 2(p_A^c - p_B^c) - 2\beta(p_A^d - p_B^d)}{4(2t - \alpha\beta)},$$

$$Q_B^c = \frac{1}{2} - \frac{2t - \alpha\beta - 2(p_A^c - p_B^c) - 2\beta(p_A^d - p_B^d)}{4(2t - \alpha\beta)}, \quad (\text{A.2})$$

$$Q_A^d = \frac{\alpha [2t - \alpha\beta - 2(p_A^c - p_B^c) - 2\beta(p_A^d - p_B^d)]}{4(2t - \alpha\beta)} - p_A^d,$$

$$Q_B^d = \frac{\alpha}{2} - \frac{\alpha [2t - \alpha\beta - 2(p_A^c - p_B^c) - 2\beta(p_A^d - p_B^d)]}{4(2t - \alpha\beta)} - p_B^d.$$

Insert demand functions into the profit for Platform A .

$$\begin{aligned} \Pi_A &= p_A^c Q_A^c + p_A^d Q_A^d \\ &= \frac{-2(p_A^c)^2 + p_A^c [2t - \alpha\beta - 2(\alpha + \beta)p_A^d + 2(p_B^c + \beta p_B^d)] + p_A^d [(-8t + 2\alpha\beta)p_A^d + \alpha(2t - \alpha\beta + 2p_B^c + 2\beta p_B^d)]}{4(2t - \alpha\beta)}. \end{aligned}$$

The first-order derivatives are

$$\frac{\partial \Pi_A}{\partial p_A^c} = \frac{2t - \alpha\beta - 2[2p_A^c + (\alpha + \beta)p_A^d - p_B^c - \beta p_B^d]}{4(2t - \alpha\beta)}, \quad (\text{A.3})$$

$$\frac{\partial \Pi_A}{\partial p_A^d} = \frac{\alpha(2t - \alpha\beta + 2p_B^c + 2\beta p_B^d) - 2(\alpha + \beta)p_A^c - 4(4t - \alpha\beta)p_A^d}{4(2t - \alpha\beta)}.$$

Under the assumption of $4t > (\alpha + \beta)^2$, the second-order derivatives satisfy

$$\frac{\partial^2 \Pi_A}{\partial (p_A^c)^2} = -\frac{1}{2t - \alpha\beta} < 0, \quad \frac{\partial^2 \Pi_A}{\partial (p_A^d)^2} = -(1 + \frac{2t}{2t - \alpha\beta}) < 0,$$

which indicates that Π_A is concave in both p_A^c and p_A^d . The determinant of the Hessian Matrix is $\frac{8t - (\alpha + \beta)^2}{4(2t - \alpha\beta)} > 0$. Thus Π_A is jointly concave in $\{p_A^c, p_A^d\}$ and there exists a unique pair of $\{p_A^c, p_A^d\}$ that maximizes Π_A .

Similarly, we can show that there exists a unique pair of $\{p_B^c, p_B^d\}$ that maximizes Π_B . Given the symmetry of two platforms, we further impose $p_A^c = p_B^c$ and $p_A^d = p_B^d$ to solve the symmetric pricing duopoly. Lemma 1 follows by solving $\frac{\partial \Pi_A}{\partial p_A^c} = \frac{\partial \Pi_A}{\partial p_A^d} = \frac{\partial \Pi_B}{\partial p_B^c} = \frac{\partial \Pi_B}{\partial p_B^d} = 0$ simultaneously. \square

Proof of Corollary 1. Take the first-order derivatives to α and β in Equation (6). Following conditions always hold.

$$\begin{aligned}\frac{\partial (p_k^c)^*}{\partial \alpha} &= -\frac{2\alpha + 3\beta}{8} < 0, \\ \frac{\partial (p_k^c)^*}{\partial \beta} &= -\frac{3\alpha}{8} < 0, \\ \frac{\partial (p_k^d)^*}{\partial \beta} &= -\frac{1}{8} < 0.\end{aligned}$$

Corollary 1 follows immediately. \square

Proof of Proposition 1. First consider period 2.

At the beginning of period 2, given Q_{k1}^c and Q_{k1}^d ($k \in \{A, B\}$), we search for the equilibrium prices in period 2 (i.e., p_{k2}^c and p_{k2}^d). At equilibrium, period-2 demands satisfy the following conditions.

$$\begin{aligned}q_{A2}^c &= \frac{1}{2} \left[\frac{1}{2} + \frac{\beta(Q_{A2}^d - Q_{B2}^d) - p_{A2}^c + p_{B2}^c}{2t} \right], \quad q_{B2}^c = \frac{1}{2} - q_{A2}^c, \\ Q_{A2}^d &= \alpha(\delta Q_{A1}^d + q_{A2}^d) - p_{A2}^d; \quad Q_{B2}^d = \alpha(\delta Q_{B1}^d + q_{B2}^d) - p_{B2}^d.\end{aligned}$$

Solving for $q_{k2}^c(Q_{k1}^c, p_{k2}^c, p_{k2}^d)$ and $Q_{k2}^d(Q_{k1}^c, p_{k2}^c, p_{k2}^d)$, we have

$$\begin{aligned}q_{A2}^c &= \frac{2t - \alpha\beta - 2(p_{A2}^c - p_{B2}^c) - 2\beta(p_{A2}^d - p_{B2}^d) + 2\alpha\beta\delta(Q_{A1}^c - Q_{B1}^c)}{4(2t - \alpha\beta)}, \\ q_{B2}^c &= \frac{2t - \alpha\beta - 2(p_{B2}^c - p_{A2}^c) - 2\beta(p_{B2}^d - p_{A2}^d) + 2\alpha\beta\delta(Q_{B1}^c - Q_{A1}^c)}{4(2t - \alpha\beta)}, \\ Q_{A2}^d &= \frac{\alpha(2t - \alpha\beta) - 8tp_{A2}^d - 2\alpha[p_{A2}^c - p_{B2}^c - 4t\delta Q_{A1}^c - \beta(\alpha\delta(Q_{A1}^c + Q_{B1}^c) - p_{A2}^d - p_{B2}^d)]}{4(t - \alpha\beta)},\end{aligned}\tag{A.4}$$

$$Q_{B2}^d = \frac{\alpha(2t - \alpha\beta) - 8tp_{B2}^d - 2\alpha [p_{B2}^c - p_{A2}^c - 4t\delta Q_{B1}^c - \beta(\alpha\delta(Q_{A1}^c + Q_{B1}^c) - p_{A2}^d - p_{B2}^d)]}{4(2t - \alpha\beta)}.$$

Platform A 's period 2 profit is

$$\begin{aligned} \Pi_{A2} = \pi_{A2} &= p_{A2}^c Q_{A2}^c + p_{A2}^d Q_{A2}^d = p_{A2}^c (\delta Q_{A1}^c + q_{A1}^c) + p_{A2}^d Q_{A2}^d \\ &= \frac{1}{4(2t - \alpha\beta)} \left\{ \begin{array}{l} -2(p_{A2}^c)^2 \\ + p_{A2}^c [2t - \alpha\beta - 2(\alpha + \beta)p_{A2}^d + 2p_{B2}^c + 8t\delta Q_{A1}^c + 2\beta(p_{B2}^d - \alpha\delta(Q_{A1}^c + Q_{B1}^c))] \\ + p_{A2}^d [2p_{A2}^d(-4t + \alpha\beta) + \alpha(2t - \alpha\beta + 2p_{B2}^c + 8t\delta Q_{A1}^c + 2\beta(p_{B2}^d - \alpha\delta(Q_{A1}^c + Q_{B1}^c)))] \end{array} \right\}. \end{aligned}$$

Given platform B 's period 2 strategies $\{p_{B2}^c, p_{B2}^d\}$, we show that π_{A2} is jointly concave in $\{p_{A2}^c, p_{A2}^d\}$ such that there exists a unique pair of $\{p_{A2}^c, p_{A2}^d\}$ that maximizes platform A 's period 2 profit. The second-order derivatives satisfy that

$$\frac{\partial^2 \pi_{A2}}{\partial (p_{A2}^c)^2} = -\frac{1}{2t - \alpha\beta} < 0, \quad \frac{\partial^2 \pi_{A2}}{\partial (p_{A2}^d)^2} = -1 - \frac{2t}{2t - \alpha\beta} < 0, \quad \frac{\partial^2 \pi_{A2}}{\partial p_{A2}^d \partial p_{A2}^c} = -\frac{\alpha + \beta}{2(2t - \alpha\beta)}.$$

The determinant of the Hessian matrix is $\frac{16t^2 - (\alpha^2 + 6\alpha\beta + \beta^2)}{4(2t - \alpha\beta)^2} > 0$, thus the Hessian matrix is negative definite. Therefore π_{A2} is jointly concave in $\{p_{A2}^c, p_{A2}^d\}$. Platform A 's optimal responsive functions can be obtained by jointly solving $\frac{\partial \pi_{A2}}{\partial p_{A2}^c} = 0$ and $\frac{\partial \pi_{A2}}{\partial p_{A2}^d} = 0$.

Given that platform A and B are symmetric, we can follow a similar approach to obtain platform B 's optimal responsive functions by jointly solving $\frac{\partial \pi_{B2}}{\partial p_{B2}^c} = 0$ and $\frac{\partial \pi_{B2}}{\partial p_{B2}^d} = 0$. Solving the system of four equations $\frac{\partial \pi_{A2}}{\partial p_{A2}^c} = \frac{\partial \pi_{A2}}{\partial p_{A2}^d} = \frac{\partial \pi_{B2}}{\partial p_{B2}^c} = \frac{\partial \pi_{B2}}{\partial p_{B2}^d} = 0$, we have at equilibrium, period 2 optimal profit and optimal pricing strategies

$$\begin{aligned} (\Pi_{k2})^* &= (\pi_{k2})^* = \tag{A.5} \\ &= \frac{[16t^2 - (\alpha^2 + 6\alpha\beta + \beta^2)] [12t - (\alpha^2 + 4\alpha\beta + \beta^2) + 2(16t^2 - \alpha^2 - 4\alpha\beta - \beta^2)\delta Q_{k1}^c + 2(8t^2 - \alpha^2 - 4\alpha\beta - \beta^2)\delta Q_{-k1}^c]^2}{64 [12t - (\alpha^2 + 4\alpha\beta + \beta^2)]^2}, \\ (p_{k2}^c)^* &= \frac{(8t - \alpha(\alpha + 3\beta)) [12t - (\alpha^2 + 4\alpha\beta + \beta^2) + 2(16t^2 - \alpha^2 - 4\alpha\beta - \beta^2)\delta Q_{k1}^c + 2(8t^2 - \alpha^2 - 4\alpha\beta - \beta^2)\delta Q_{-k1}^c]}{8 [12t - (\alpha^2 + 4\alpha\beta + \beta^2)]}, \\ (p_{k2}^d)^* &= \frac{(\alpha - \beta) [12t - (\alpha^2 + 4\alpha\beta + \beta^2) + 2(16t^2 - \alpha^2 - 4\alpha\beta - \beta^2)\delta Q_{k1}^c + 2(8t^2 - \alpha^2 - 4\alpha\beta - \beta^2)\delta Q_{-k1}^c]}{8 [12t - (\alpha^2 + 4\alpha\beta + \beta^2)]}, \end{aligned}$$

where the subscript $-k$ represents the rival platform for platform k (i.e., $-k = B$ when $k = A$, and vice versa).

Next consider period 1. Similar to Equation (A.4), at equilibrium, period 1 demands are given by

$$\begin{aligned}
q_{A1}^c &= \frac{2t - \alpha\beta - 2(p_{A1}^c - p_{B1}^c) - 2\beta(p_{A1}^d - p_{B1}^d)}{4(2t - \alpha\beta)}, \\
q_{B1}^c &= \frac{2t - \alpha\beta - 2(p_{B1}^c - p_{A1}^c) - 2\beta(p_{B1}^d - p_{A1}^d)}{4(2t - \alpha\beta)}, \\
Q_{A1}^d &= \frac{\alpha(2t - \alpha\beta) - 8tp_{A1}^d - 2\alpha[p_{A1}^c - p_{B1}^c - \beta(p_{A1}^d + p_{B1}^d)]}{4(t - \alpha\beta)}, \\
Q_{B1}^d &= \frac{\alpha(2t - \alpha\beta) - 8tp_{B1}^d - 2\alpha[p_{B1}^c - p_{A1}^c - \beta(p_{A1}^d + p_{B1}^d)]}{4(2t - \alpha\beta)}.
\end{aligned} \tag{A.6}$$

Inserting π_{k2}^* , q_{k1}^c , and Q_{k1}^d into platform k 's period 1 objective function defined by Equation (4), we have

$$\begin{aligned}
\Pi_{A1} &= \frac{p_{A1}^c [2t - \alpha\beta - 2(p_{A1}^c - p_{B1}^c) - 2\beta(p_{A1}^d - p_{B1}^d)]}{4(2t - \alpha\beta)} \\
&+ \frac{p_{A1}^d [\alpha(2t - \alpha\beta) - 8tp_{A1}^d - 2\alpha(p_{A1}^d - p_{B1}^d) - \beta(p_{A1}^d + p_{B1}^d)]}{4(2t - \alpha\beta)} \\
&+ \frac{\lambda(16t - \alpha^2 - 6\alpha\beta - \beta^2) [8\delta t (p_{B1}^c + \beta p_{B1}^d - p_{A1}^c - \beta p_{A1}^d) + (\delta + 1)(2t - \alpha\beta)(12t - \alpha^2 - 4\alpha\beta - \beta^2)]^2}{64(2t - \alpha\beta)^2 (12t - \alpha^2 - 4\alpha\beta - \beta^2)^2}.
\end{aligned} \tag{A.7}$$

The second-order derivatives are

$$\begin{aligned}
\frac{\partial^2 \Pi_{A1}}{\partial (p_{A1}^c)^2} &= -\frac{1}{2t - \alpha\beta} + \frac{2\delta^2 \lambda t^2 (16t - \alpha^2 - 6\alpha\beta - \beta^2)}{(12t - \alpha^2 - 4\alpha\beta - \beta^2)^2 (2t - \alpha\beta)^2} < 0, \\
\frac{\partial^2 \Pi_{A1}}{\partial (p_{A1}^d)^2} &= \frac{2t \left(\alpha\beta + t \left(-\frac{\beta^2 \delta^2 \lambda (\alpha^2 + 6\alpha\beta + \beta^2 - 16t)}{(\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^2} - 2 \right) \right)}{(2t - \alpha\beta)^2} - 1 < 0, \\
\frac{\partial^2 \Pi_{A1}}{\partial p_{A1}^d \partial p_{A1}^c} &= -\frac{\alpha + \beta}{2(2t - \alpha\beta)} + \frac{2\beta \delta^2 \lambda t^2 (16t - \alpha^2 - 6\alpha\beta - \beta^2)}{(\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^2 (2t - \alpha\beta)^2}.
\end{aligned}$$

The determinant of the Hessian Matrix is $-\frac{(16t - \alpha^2 - 6\alpha\beta - \beta^2)(16\delta^2 \lambda t^2 - (\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^2)}{4(\alpha\beta - 2t)^2 (\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^2} > 0$, thus Π_{A1} is jointly concave in p_{A1}^c and p_{A1}^d . Then optimal reaction functions can be obtained by solving the first-order conditions (FOCs).

Following a similar approach used in the above solution for platform B 's period 1 problem, we obtain the period 1 equilibrium pricing strategies in Proposition 1. \square

Proof of Proposition 2. Following a similar approach as in the proof of Proposition 1, we can solve for the equilibrium prices in both periods (details available upon request). We focus on the

coefficients of Q_0 in all equilibrium prices to identify regions as illustrated in Figure 3 and 4. We obtained the following.

$$\begin{aligned} \Delta_{A1}^c &= \frac{\partial(\tilde{p}_{A1}^c)^*}{\partial Q_0} = 3t - \frac{\alpha^2 + 3\alpha\beta}{4} \\ &+ \frac{1}{4} \left(\frac{4t(\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^2(4t - \beta(\alpha + \beta))}{16\delta^2\lambda t^2(-\alpha^2 - 6\alpha\beta - \beta^2 + 16t) + (\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^3} - \frac{4\delta^2\lambda t(\alpha^2 + 6\alpha\beta + \beta^2 - 16t)}{\alpha^2 + 4\alpha\beta + \beta^2 - 12t} \right); \\ \Delta_{B1}^c &= \frac{\partial(\tilde{p}_{B1}^c)^*}{\partial Q_0} = t - \frac{1}{4}\alpha(\alpha + 3\beta) \\ &- \frac{t(4t - \beta(\alpha + \beta))(\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^2}{16\delta^2\lambda t^2(-\alpha^2 - 6\alpha\beta - \beta^2 + 16t) + (\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^3} - \frac{\delta^2\lambda t(\alpha^2 + 6\alpha\beta + \beta^2 - 16t)}{\alpha^2 + 4\alpha\beta + \beta^2 - 12t}; \\ \Delta_{A1}^d &= \frac{\partial(\tilde{p}_{A1}^d)^*}{\partial Q_0} = \frac{(\alpha - \beta) \left(16\delta^2\lambda t^2(-\alpha^2 - 6\alpha\beta - \beta^2 + 16t) + (\alpha^2 + 4\alpha\beta + \beta^2 - 16t)(\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^2 \right)}{4 \left(16\delta^2\lambda t^2(-\alpha^2 - 6\alpha\beta - \beta^2 + 16t) + (\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^3 \right)}; \\ \Delta_{B1}^d &= \frac{\partial(\tilde{p}_{B1}^d)^*}{\partial Q_0} = \frac{(\alpha - \beta) \left(16\delta^2\lambda t^2(-\alpha^2 - 6\alpha\beta - \beta^2 + 16t) + (\alpha^2 + 4\alpha\beta + \beta^2 - 8t)(\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^2 \right)}{4 \left(16\delta^2\lambda t^2(-\alpha^2 - 6\alpha\beta - \beta^2 + 16t) + (\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^3 \right)}; \\ \Delta_{A2}^c &= \frac{\partial(\tilde{p}_{A2}^c)^*}{\partial Q_0} = (8t - \alpha(\alpha + 3\beta)) \\ &\times \frac{(\alpha^2 + 4\alpha\beta + \beta^2)^3 + 128t^3(2\delta^2\lambda - 15) + 16t^2(28(\alpha^2 + 4\alpha\beta + \beta^2) - \delta^2\lambda(\alpha^2 + 6\alpha\beta + \beta^2)) - 36t(\alpha^2 + 4\alpha\beta + \beta^2)^2}{4 \left(16\delta^2\lambda t^2(-\alpha^2 - 6\alpha\beta - \beta^2 + 16t) + (\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^3 \right)}; \\ \Delta_{B2}^c &= \frac{\partial(\tilde{p}_{B2}^c)^*}{\partial Q_0} = (8t - \alpha(\alpha + 3\beta)) \\ &\times \frac{(\alpha^2 + 4\alpha\beta + \beta^2)^3 + 256t^3(\delta^2\lambda - 6) + 16t^2(26(\alpha^2 + 4\alpha\beta + \beta^2) - \delta^2\lambda(\alpha^2 + 6\alpha\beta + \beta^2)) - 36t(\alpha^2 + 4\alpha\beta + \beta^2)^2}{4 \left(16\delta^2\lambda t^2(-\alpha^2 - 6\alpha\beta - \beta^2 + 16t) + (\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^3 \right)}; \\ \Delta_{A2}^d &= \frac{\partial(\tilde{p}_{A2}^d)^*}{\partial Q_0} = (\alpha - \beta) \\ &\times \frac{(\alpha^2 + 4\alpha\beta + \beta^2)^3 + 128t^3(2\delta^2\lambda - 15) + 16t^2(28(\alpha^2 + 4\alpha\beta + \beta^2) - \delta^2\lambda(\alpha^2 + 6\alpha\beta + \beta^2)) - 36t(\alpha^2 + 4\alpha\beta + \beta^2)^2}{4 \left(16\delta^2\lambda t^2(-\alpha^2 - 6\alpha\beta - \beta^2 + 16t) + (\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^3 \right)}; \\ \Delta_{B2}^d &= \frac{\partial(\tilde{p}_{B2}^d)^*}{\partial Q_0} = (\alpha - \beta) \\ &\times \frac{(\alpha^2 + 4\alpha\beta + \beta^2)^3 + 256t^3(\delta^2\lambda - 6) + 16t^2(26(\alpha^2 + 4\alpha\beta + \beta^2) - \delta^2\lambda(\alpha^2 + 6\alpha\beta + \beta^2)) - 36t(\alpha^2 + 4\alpha\beta + \beta^2)^2}{4 \left(16\delta^2\lambda t^2(-\alpha^2 - 6\alpha\beta - \beta^2 + 16t) + (\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^3 \right)}. \end{aligned}$$

We go through each of the above partial derivatives to identify conditions when it is positive or negative. Without loss of generality, we focus on period 1 equilibrium prices by setting $\delta = \lambda = 1$. We then go through each region in Figure 3 and 4 to compute the signs of Δ_{k1}^c and Δ_{k1}^d . The signs of Δ_{k2}^c and Δ_{k2}^d can be shown in a similar way (the results when $\delta, \lambda \in [0, 1)$ are available upon

request).

Region R1 and R2 in Figure 3a: This is equivalent to show that, there exists a unique $\hat{\alpha}$ such that $\Delta_{A1}^c = \frac{\partial(p_{A1}^c)^*}{\partial Q_0} < 0$ when $\alpha > \hat{\alpha}$. This is true because the following three conditions hold simultaneously: (1) $\frac{\partial(\Delta_{A1}^c)^*}{\partial \alpha} < 0$, (2) $(\Delta_{A1}^c)^*|_{\alpha \rightarrow 2\sqrt{t}-\beta} < 0$ and (3) $(\Delta_{A1}^c)^*|_{\alpha \rightarrow 0, \beta \rightarrow 0} > 0$.

Region R3 and R4 in Figure 3b: This holds because $\Delta_{Ai}^d > 0$ when $\alpha > \beta$.

Region R5 and R6 in Figure 4a: This is equivalent to show that, there exists a unique $\bar{\alpha}_i$ in period i such that $\Delta_{Bi}^c = \frac{\partial(p_{Bi}^c)^*}{\partial Q_0} < 0$ when $\alpha > \bar{\alpha}_i$. This is true because the following three conditions hold simultaneously: (1) $\frac{\partial(\Delta_{Bi}^c)^*}{\partial \alpha} < 0$, (2) $(\Delta_{Bi}^c)^*|_{t \rightarrow \frac{(\alpha+\beta)^2}{4}} < 0$ and (3) $(\Delta_{Bi}^c)^*|_{\alpha \rightarrow 0, \beta \rightarrow 0} > 0$.

Region R7 and R8 in Figure 4b: This is equivalent to show that, there exists a unique $\hat{\beta}_i$ in period i such that when $(\alpha - \beta)(\beta - \hat{\beta}_i) < 0$, then $\Delta_{Bi}^d > 0$ holds. We split the discussion into two cases: (1) when $\alpha > \beta$, the above argument is true because the following three conditions hold simultaneously when $\alpha > \beta$: (1.1) $\frac{\partial(\Delta_{Bi}^d)^*}{\partial \beta} < 0$, (1.2) $(\Delta_{Bi}^d)^*|_{t \rightarrow \frac{(\alpha+\beta)^2}{4}, \alpha \rightarrow \beta^+} < 0$, and (1.3) $(\Delta_{Bi}^d)^*|_{\beta \rightarrow 0} > 0$. (2) When $\alpha < \beta$, the above argument is still true because the following three conditions hold simultaneously when $\alpha < \beta$: (2.1) $\frac{\partial(\Delta_{Bi}^d)^*}{\partial \beta} > 0$, (2.2) and $(\Delta_{Bi}^d)^*|_{t \rightarrow \frac{(\alpha+\beta)^2}{4}, \alpha \rightarrow \beta^-} > 0$, (2.3) $(\Delta_{Bi}^d)^*|_{\alpha \rightarrow 0} < 0$. \square

Proof of Proposition 3. The profit gap when $\alpha = \beta$ is

$$(\Pi_{A1}^* - \Pi_{B1}^*)|_{\beta \rightarrow \alpha} = \frac{2Q_0(2Q_0 + 1)t \left(4\delta^2 \lambda t^2 - 9(\alpha^2 - 2t)^2 \right)}{16\delta^2 \lambda t^2 - 27(\alpha^2 - 2t)^2}.$$

The cross derivative is

$$\frac{\partial^2(\Pi_{A1}^* - \Pi_{B1}^*)|_{\beta \rightarrow \alpha}}{\partial Q_0 \partial \alpha} = \frac{288\alpha\delta^2\lambda(4Q_0 + 1)t^3(2t - \alpha^2)}{\left(27(\alpha^2 - 2t)^2 - 16\delta^2\lambda t^2 \right)^2},$$

which is always nonnegative. \square

Proof of Proposition 4. The period 2 problem is identical to that in the proof of Proposition 1. In period 1, we first treat bQ_0^2 as exogenous so that we can solve for the optimal responsive functions of

$(\tilde{p}_{A1}^c)^*$ and $(\tilde{p}_{A1}^d)^*$, given platform B 's pricing strategies, $(\tilde{p}_{B1}^c, \tilde{p}_{B1}^d)$. Inserting the optimal responsive functions back to Platform A 's profit function, and computing the second-order derivative with respect to Q_0 , we have

$$\begin{aligned} \frac{\partial^2 \Pi_{A1}((\tilde{p}_{A1}^c)^*, (\tilde{p}_{A1}^d)^*, Q_0 | \tilde{p}_{B1}^c, \tilde{p}_{B1}^d)}{\partial Q_0^2} &= \frac{1}{8} (-17\alpha^2 - 6\alpha\beta - \beta^2 - 16b) - \frac{2t(4t - \alpha\beta)}{\alpha^2 + 4\alpha\beta + \beta^2 - 8t} \\ &+ \left(t + \frac{32t^3(4\alpha + \beta)}{\alpha(\alpha^2 - 16t)(\alpha^2 + 4\alpha\beta + \beta^2 - 16t)} + \frac{2\alpha^6(\alpha + 6\beta) + 32t^3(14\alpha - \beta) + 8\alpha^2t^2(59\alpha - 32\beta) - 16\alpha^4t(4\alpha + 11\beta)}{\alpha(\alpha^2 - 16t)(\alpha^2 + 6\alpha\beta + \beta^2 - 16t)} \right). \end{aligned}$$

There is only one term above containing b . Therefore, for the second-order derivative to be also negative, it requires $b > \hat{b}$ where \hat{b} is defined as

$$\begin{aligned} \hat{b} &= \frac{1}{16} (-17\alpha^2 - 6\alpha\beta - \beta^2) - \frac{t(4t - \alpha\beta)}{\alpha^2 + 4\alpha\beta + \beta^2 - 8t} \\ &+ \frac{1}{2} \left(t + \frac{32t^3(4\alpha + \beta)}{\alpha(\alpha^2 - 16t)(\alpha^2 + 4\alpha\beta + \beta^2 - 16t)} + \frac{2\alpha^6(\alpha + 6\beta) + 32t^3(14\alpha - \beta) + 8\alpha^2t^2(59\alpha - 32\beta) - 16\alpha^4t(4\alpha + 11\beta)}{\alpha(\alpha^2 - 16t)(\alpha^2 + 6\alpha\beta + \beta^2 - 16t)} \right). \end{aligned}$$

It can be shown that the right-hand-side of above is always positive, which indicates that a positive \hat{b} always exists. Platform B 's period 1 optimal responsive functions are identical to those in the proof of Proposition 2. The equilibrium prices and Q_0 can be shown by solving for the system of optimal reaction functions simultaneously. Take partial derivatives of Q_0^* with respect to b .

$$\begin{aligned} \frac{\partial Q_0^*}{\partial b} &= -16(-\alpha^2 - 4\alpha\beta - \beta^2 + 12t) \left((\alpha^2 + 4\alpha\beta + \beta^2)^3 - 1472t^3 + 32t^2(13\alpha^2 + 51\alpha\beta + 13\beta^2) - 36t(\alpha^2 + 4\alpha\beta + \beta^2)^2 \right)^2 \quad (\text{A.8}) \\ &\times \frac{((\alpha^2 + 4\alpha\beta + \beta^2)(\alpha^2 + 8\alpha\beta + \beta^2) + 224t^2 - 8t(4\alpha^2 + 21\alpha\beta + 4\beta^2))}{\left(\begin{aligned} &(\alpha^2 + 4\alpha\beta + \beta^2)^4(\alpha^2 + 10\alpha\beta + \beta^2) - 606208t^5 - 4t(\alpha^2 + 4\alpha\beta + \beta^2)^3(19\alpha^2 + 150\alpha\beta + 19\beta^2) \\ &+ 1024t^4(221\alpha^2 + 996\alpha\beta + 221\beta^2) + 48t^2(\alpha^2 + 4\alpha\beta + \beta^2)(47\alpha^4 + 488\alpha^3\beta + 1290\alpha^2\beta^2 + 488\alpha\beta^3 + 47\beta^4) \\ &- 256t^3(127\alpha^4 + 1179\alpha^3\beta + 2930\alpha^2\beta^2 + 1179\alpha\beta^3 + 127\beta^4) \\ &+ 16b(-\alpha^2 - 4\alpha\beta - \beta^2 + 12t) \left(-(\alpha^2 + 4\alpha\beta + \beta^2)^3 + 1472t^3 - 32t^2(13\alpha^2 + 51\alpha\beta + 13\beta^2) + 36t(\alpha^2 + 4\alpha\beta + \beta^2)^2 \right) \end{aligned} \right)^2. \end{aligned}$$

The denominator is quadratic and positive. The nominator is negative. Then $\frac{\partial Q_0^*}{\partial b} < 0$. Now consider \tilde{p}_{A1}^c . Again it can be verified that the denominator is quadratic and positive. Then $\frac{\partial \tilde{p}_{A1}^c}{\partial b} > 0$ if the nominator is positive. The nominator can be rewritten as $f(t)$ which is a function of t with the greatest order at t^{10} . The coefficient for t^{10} is negative. Thus $f(t)$ goes to infinitely negative when t is large enough. Furthermore, it can be shown that $\frac{df(t)}{dt} < 0$ always holds when $f(t) < 0$,

which implies that, if $f(t)|_{t \rightarrow \frac{(\alpha+\beta)^2}{4}} > 0$, then there exists a unique \hat{t} such that for all $t > \hat{t}$, $f(t) < 0$ always holds.

Finally, we show that $f(t) > 0$ is possible as t is small enough. To do this, we evaluate $f(t)$ at the lower bound of $t = \frac{(\alpha+\beta)^2}{4}$.

$$\begin{aligned} f(t)|_{t=\frac{(\alpha+\beta)^2}{4}} = & \\ & 4(\alpha^2 + \beta^2)(5\alpha^4 + 10\alpha^3\beta + 14\alpha^2\beta^2 + 10\alpha\beta^3 + 5\beta^4)(7\alpha^4 + 10\alpha^3\beta + 18\alpha^2\beta^2 + 10\alpha\beta^3 + 7\beta^4) \\ & \times (3\alpha^{10} + 28\alpha^9\beta + 75\alpha^8\beta^2 + 142\alpha^7\beta^3 + 156\alpha^6\beta^4 + 110\alpha^5\beta^5 + 4\alpha^4\beta^6 - 62\alpha^3\beta^7 - 79\alpha^2\beta^8 - 42\alpha\beta^9 - 15\beta^{10}). \end{aligned}$$

Only the polynomial in the last bracket can be negative. Divide the polynomial in the last bracket by α^{10} and denote $u = \frac{\beta}{\alpha}$. Rewrite function $f(\cdot)$ as a function $g(u)$.

$$g(u) = -15u^{10} - 42u^9 - 79u^8 - 62u^7 + 4u^6 + 110u^5 + 156u^4 + 142u^3 + 75u^2 + 28u + 3.$$

We have $g(0) = 3 > 0$ and $g(u)|_{u \rightarrow +\infty} \rightarrow -\infty$. Note that $\frac{dg(u)}{du} < 0$ always holds when $g(u) < 0$, which implies that once $g(u)$ becomes negative as u increases, $g(u)$ will never be positive again. Therefore, there exists a unique \hat{u} such that for $u < \hat{u}$, $f(t)$ is positive and $\frac{\partial(\bar{p}_{A1}^c)^*}{\partial b} > 0$. The signs of $\frac{\partial(\bar{p}_{A1}^d)^*}{\partial b}$, $\frac{\partial(\bar{p}_{A2}^c)^*}{\partial b}$, and $\frac{\partial(\bar{p}_{A2}^d)^*}{\partial b}$ can be shown directly by taking first-order derivatives in the equilibrium prices. \square

Proof of Proposition 5. The proof is similar to the previous proof of Proposition 1 but the backward induction starts from period 3. We omit it here for brevity (but the proof is available from the authors upon request). \square

Proof of Proposition 6. The period 2 problem is identical to that in the proof of Proposition 1 (see Equation A.5 for the equilibrium profit in period 2). In period 1, Π_{k1} can be obtained by replacing λ with λ_k ($k \in \{A, B\}$) in Equation (A.7). Similar to the proof of Proposition 1, we can show that the profit function Π_{k1} is jointly concave in $\{p_{k1}^c, p_{k1}^d\}$ ($k \in \{A, B\}$). The equilibrium prices in period 1 can be obtained by jointly solving all FOCs.

$$(p_{A1}^c)^* = 4t(16t - \alpha^2 - 6\alpha\beta - \beta^2)$$

$$\begin{aligned} & \times \frac{\left(\lambda_B (-\alpha^2 - 4\alpha\beta - \beta^2 + 14t) (\alpha^2 + 4\alpha\beta + \beta^2 - 12t) (8t - \alpha(\alpha + 3\beta)) + 32t^2 \lambda_B (-\alpha^2 - 6\alpha\beta - \beta^2 + 16t) \right)}{8(\alpha^2 + 4\alpha\beta + \beta^2 - 12t) (8t^2 (\lambda_A + \lambda_B) (-\alpha^2 - 6\alpha\beta - \beta^2 + 16t) + (\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^3)} \\ & + \frac{(\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^4 (8t - \alpha(\alpha + 3\beta))}{8(\alpha^2 + 4\alpha\beta + \beta^2 - 12t) (8t^2 (\lambda_A + \lambda_B) (-\alpha^2 - 6\alpha\beta - \beta^2 + 16t) + (\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^3)}; \end{aligned}$$

which can be rewritten as $(t - \frac{\alpha(\alpha+3\beta)}{8}) - \Omega_1$ where Ω_1 is given in Proposition 6. Other equilibrium prices in period 1 can be obtained using a similar approach. Next, we compute the equilibrium consumer demands in period 1, i.e., Q_{A1}^c and Q_{B1}^c , by inserting equilibrium prices into Equation (A.6).

$$\begin{aligned} (Q_{A1}^c)^* &= \frac{(\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^3 - 4t(-\alpha^2 - 6\alpha\beta - \beta^2 + 16t) (\lambda_A (-\alpha^2 - 4\alpha\beta - \beta^2 + 10t) + \lambda_B (\alpha^2 + 4\alpha\beta + \beta^2 - 14t))}{4(8t^2 (\lambda_A + \lambda_B) (-\alpha^2 - 6\alpha\beta - \beta^2 + 16t) + (\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^3)}; \\ (Q_{B1}^c)^* &= \frac{(\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^3 + 4t(-\alpha^2 - 6\alpha\beta - \beta^2 + 16t) (\lambda_A (-\alpha^2 - 4\alpha\beta - \beta^2 + 14t) + \lambda_B (\alpha^2 + 4\alpha\beta + \beta^2 - 10t))}{4(8t^2 (\lambda_A + \lambda_B) (-\alpha^2 - 6\alpha\beta - \beta^2 + 16t) + (\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^3)}. \end{aligned}$$

Inserting $(Q_{A1}^c)^*$ and $(Q_{B1}^c)^*$ into Equation (A.5) gives equilibrium prices in period 2 (given by Table 6). \square

Proof of Corollary 2. Based on the equilibrium prices in Table 6, we can compute the price gaps, i.e., $(p_{Ai}^m)^* - (p_{Bi}^m)^*$ where $i \in \{1, 2\}$ and $m \in \{c, d\}$. When $\lambda_A > \lambda_B$, we have

$$\begin{aligned} (p_{A1}^c)^* - (p_{B1}^c)^* &= \frac{(\lambda_A - \lambda_B)t(16t - \alpha^2 - 6\alpha\beta - \beta^2)(12t - \alpha^2 - 4\alpha\beta - \beta^2)(4t - \beta(\alpha + \beta))}{8(\lambda_A + \lambda_B)t^2(16t - \alpha^2 - 6\alpha\beta - \beta^2) + (\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^3} < 0; \\ (p_{A2}^c)^* - (p_{B2}^c)^* &= -\frac{4(\lambda_A - \lambda_B)t^2(16t - \alpha^2 - 6\alpha\beta - \beta^2)(8t - \alpha(\alpha + 3\beta))}{8(\lambda_A + \lambda_B)t^2(16t - \alpha^2 - 6\alpha\beta - \beta^2) + (\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^3} > 0; \end{aligned}$$

which proves (1) of Corollary 2. Similarly, on the provider side, we have

$$\begin{aligned} (p_{A1}^d)^* - (p_{B1}^d)^* &= -\frac{(\lambda_A - \lambda_B)t(\alpha - \beta)(16t - \alpha^2 - 6\alpha\beta - \beta^2)(12t - \alpha^2 - 4\alpha\beta - \beta^2)}{8(\lambda_A + \lambda_B)t^2(16t - \alpha^2 - 6\alpha\beta - \beta^2) + (\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^3}; \\ (p_{A2}^d)^* - (p_{B2}^d)^* &= -\frac{4(\lambda_A - \lambda_B)t^2(\alpha - \beta)(16t - \alpha^2 - 6\alpha\beta - \beta^2)}{8(\lambda_A + \lambda_B)t^2(16t - \alpha^2 - 6\alpha\beta - \beta^2) + (\alpha^2 + 4\alpha\beta + \beta^2 - 12t)^3}, \end{aligned}$$

both of which are positive when $\alpha > \beta$ and $\lambda_A > \lambda_B$. This proves (2) of Corollary 2. \square