

Strategic Pricing and Forecast Communication on On-Demand Service Platforms

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Abstract

On-demand service platforms (e.g., Uber, Lyft) match consumers with independent workers nearby at short notice. To manage fluctuating supply and demand conditions across market locations (zones), many on-demand platforms provide market forecasts to workers and practice *surge pricing*, wherein the price in a particular zone is temporarily raised above the regular price. We jointly analyze the strategic role of surge pricing and forecast communication explicitly accounting for workers' incentives to move between zones and the platform's incentive to share forecasts truthfully. Conventional wisdom suggests that surge pricing is useful in zones where demand for workers exceeds their supply. However, we show that when the platform relies on independent workers to serve consumers across different zones, surge pricing is also useful in zones where supply exceeds demand. Because individual workers do not internalize the competitive externality that they impose on other workers, too few workers may move from a zone with excess supply to an adjacent zone requiring additional workers. Moreover, the platform may have an incentive to misreport market forecasts to exaggerate the need for workers to move. We show how and why distorting the price in a zone with excess supply through surge pricing can increase total platform profit across zones, by incentivizing more workers to move and by making forecast communication credible. Our analysis offers insights for effectively managing on-demand platforms through surge pricing and forecast sharing, and the resulting implications for consumers and workers.

Keywords: Asymmetric Information, Dynamic Pricing, Forecast Sharing, On-Demand Economy, Peer-to-peer Platforms, Signaling, Surge Pricing

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“When an area is surging (having a surge price), it just kills demand, and a large fraction of the drivers leave the area and drive elsewhere... [Surge pricing] is not incentivizing drivers the way [Uber] hoped it would.” - Christo Wilson, co-author of Uber pricing study Chen et al. (2015), as quoted in *The San Francisco Chronicle* (Said, 2015).

“Was smack in the middle of the zone [with a high surge price] and yet no pings (customer requests)... If it’s surging, I would expect to see fewer cars available and as a driver, I would expect instant pings” - Uber driver on online forum (Uber People, 2015).

1 Introduction

The on-demand economy can be defined as "the economic activity created by digital market places that fulfill consumer demand with immediate access to and convenient provisioning of goods and services" (OnDemandEconomy.org, 2016). The on-demand economy has witnessed explosive growth in recent years due, in large part, to the emergence of peer-to-peer online platforms that match consumers in real time with independent workers who are available nearby and can serve consumers at short notice (Economist, 2015; Fowler, 2015). Leading examples of such platforms include Uber and Lyft for cabs; GrubHub, Instacart and Postmates for delivery of food, groceries or other items; TaskRabbit and Handy for household services; and, Glamsquad and Zeel for health and beauty services. A recent study estimates the annual U.S. consumer spending on on-demand services to be \$5.6 billion for cabs, \$4.6 billion for delivery, and \$8.6 billion for all other services (Colby and Bell, 2016). On-demand platforms have also become popular in many countries across the world; some notable examples in the case of on-demand cab platforms being Didi Chuxing in China, EasyTaxi in Latin America, Ola in India, and Yandex in Russia. Given their growing economic significance, it is important to understand the business strategies of on-demand platforms and how these platforms can be managed effectively. In this paper, we examine the pricing and forecast communication strategies adopted by many on-demand platforms to tackle fluctuating market conditions.

On-demand marketplaces are often characterized by fluctuating supply and demand conditions that, if not properly managed, can significantly hinder an on-demand platform’s ability to ensure effective service and generate revenue. On-demand platforms typically operate on a commission basis, retaining a percentage of worker revenues generated from serving consumers through the platform. For example, Uber and Lyft receive a commission of 20-25% of their drivers’ revenues. However, consumers can be served at short notice only if enough workers are available close to the consumers’ locations, since it typically requires time for workers to move between market locations. A sharp and unexpected increase in demand at a market location, for example, cannot be met if

there are not enough workers nearby. Fluctuating market conditions, therefore, pose a fundamental challenge for on-demand platforms since they can lead to significant loss of platform revenue.

This challenge is further compounded by the fact that the platform does not directly control worker availability at each market location. In order to provide on-demand service at attractive price points, many on-demand platforms rely on what has come to be known as the “gig economy” - a variable workforce of freelancers who work at their convenience and can be hired on-demand for a single project or task (Nunberg, 2016; Torpey and Hogan, 2016). These independent workers are attracted to on-demand platforms by the prospect of earning extra income in their free time and are, hence, willing to work for considerably lower compensation than a full-time worker. For example, workers for on-demand cabs and delivery service can typically be "anyone with a car" - college students, people from other professions, or retirees - who are willing to drive or deliver in between their other daily activities or work. Similarly, workers for on-demand household services can choose to work only at times when they do not have prior jobs acquired outside the platform. Thus, the platform obtains access to a workforce that is willing to serve consumers at relatively low prices. At the same time, these independent workers participate to work voluntarily and do not have to commit ahead of time to a fixed schedule of working days or time. Consequently, on-demand platforms do not know and cannot plan their supply of workers in advance, and cannot control their availability at each market location.

To operate effectively under such market conditions, on-demand platforms often adopt a two-pronged approach. First, they invest considerable resources to forecast supply and demand patterns ahead of time, and share these forecasts with the workers. For example, Uber uses advanced algorithms to forecast the need for workers at each market location based on historical patterns, holidays, weather, current local events and traffic conditions (Lin et al., 2014; Chen et al., 2015; Rosenblat and Stark, 2015). Uber shares these forecasts with drivers through the mobile application that drivers use to receive ride requests from Uber customers.¹

Second, many on-demand platforms employ a form of dynamic pricing known as *surge pricing*. Under surge pricing, there is a set *regular price* for the entire market region. However, depending on the prevailing supply and demand conditions, the platform can increase the current price at a given market location to be higher than the regular price; this higher price is referred to as a *surge price*.² To implement surge pricing, the market is split into several smaller regions or “zones”

¹The mobile application displays a map with the locations that are likely to need additional workers. The locations are marked yellow, orange or red to indicate the likelihood and extent of the need (Rosenblat and Stark, 2015).

²The surge price may be displayed to customers in the form of a *surge multiplier* (e.g., 2.5x), where the current price is the regular price times the surge multiplier.

and the platform periodically updates the current price in each market zone. Consumers learn the current price in their market zone at the time that they connect to the platform to request service. Surge pricing also affects workers' remuneration as they are paid more if they serve consumers in a zone with a surge price. Workers typically have access to "surge maps" that display the surge prices (and market forecasts) in their market zone and in adjacent market zones (Chen et al., 2015; Rosenblat and Stark, 2015). Examples of platforms that employ surge pricing include Uber, Lyft, Postmates, Instacart and Handy.

The conventional rationale for these platform strategies is quite straightforward. Sharing market forecasts with workers should encourage workers to be available where they are needed the most. Further, researchers and industry experts agree that surge pricing is useful to balance supply and demand specifically in market zones where there is a shortage of workers relative to demand (Gurley, 2014; Chen et al., 2015; Chen and Sheldon, 2015; Hall et al., 2015). Surge pricing is expected to work in two ways. First, by pricing out consumers who have lower willingness to pay, a surge price acts as an efficient mechanism to allocate or "ration" the limited supply of workers to consumers who value them the most. Second, because a surge price increases workers' compensation in that zone, it should attract more workers, thus, reducing the extent of shortage in supply. For both reasons, surge pricing should improve the welfare of consumers, workers and the overall market.

Thus, surge pricing is expected to be used in market zones where the demand for workers exceeds their supply. Moreover, surge pricing and forecast information sharing are both expected to improve the functioning of the on-demand marketplace by directing workers to zones requiring additional workers. For example, using company data from Uber, Hall et al. (2015) analyze a substantial surge pricing event that occurred in a particular market zone on New Year's eve. They find evidence to suggest that the surge price resulted in rides being allocated to consumers who valued them the most, and led to an increase in the supply of available drivers in that zone. Also based on data from Uber, Diakopolous (2015) finds that a surge price mainly attracted drivers from neighboring zones rather than attracting a fresh supply of drivers.

However, the above conventional wisdom does not seem to fully capture what is observed in practice. For example, one might expect that if surge pricing is used in market zones where the demand for workers exceeds their supply, then workers in that zone should not have to wait for relatively long to receive customer requests. Yet, as illustrated by the opening comment, many Uber drivers note that they regularly do not receive customer requests even when they are in a zone where surge pricing is in effect. A recent empirical study by Chen et al. (2015) adds weight

to such reports. Chen et al. (2015) collected data about pricing, demand and supply from Uber’s platform in New York City and San Francisco markets over a seven day period. Contrary to the conventional wisdom that a surge price is used to balance demand with the available supply, they find that a surge price systematically caused drivers in certain surge price zones to become idle, in effect creating an imbalance between supply and demand. In fact, instead of attracting new drivers to the market zone, they found that surge pricing frequently caused drivers to leave the surge price zone. Overall, they do not find sufficient support for the notion that surge pricing in a market zone is linked to stronger demand for rides (or a shortage of drivers) in that zone. Consequently, as illustrated by the opening comment, the authors conclude that surge pricing is not working in a manner consistent with conventional wisdom (Said, 2015).

Other researchers question whether sharing market forecasts with workers has been effective (Lee et al., 2015; Rosenblat and Stark, 2015). Based on driver reports and surveys, they find that a significant fraction of drivers routinely ignore the forecasts provided by the platform; for example, by not driving to market zones that are forecasted to have a shortage of drivers. Some drivers also express a lack of trust regarding whether the forecasts represent true market conditions, or are being used to manipulate drivers to move to different zones (Rosenblat, 2015). The lack of transparency surrounding surge pricing has also led to concerns from consumers that surge pricing is being used to fleece consumers even if there is sufficient supply of workers (Dholakia 2015), leading to calls for banning this practice (e.g., Politico 2015; Financial Times 2016). Thus, there may be more to these platform strategies than might initially meet the eye, which calls for more careful investigation.

Our objective in this paper is to conduct a model-based examination of the platform’s strategic incentives for surge pricing and forecast communication. We analyze a platform facing uncertain demand and supply conditions in adjacent market zones over successive time periods. We explicitly account for the workers’ incentives to move between different zones and the platform’s incentive to share forecasts truthfully with workers. By jointly studying two key strategies that platforms employ to tackle fluctuating market conditions, we shed light on how they may be interlinked. Our findings are likely to be relevant for managers to understand the role of surge pricing and forecast communication for effectively managing on-demand platforms that rely on independent workers. Our results may also help clarify some of the controversy surrounding these practices.

Our analysis provides the following insights. Having an additional worker in a market zone lowers the expected profit of all other workers in that zone. However, individual workers do not internalize this competitive externality. Consequently, even if workers know the market conditions in

different zones, too few workers may leave a zone with an excess supply of workers to serve adjacent zones that require additional workers; thus, workers seemingly ignore the market information from the platform and do not move to zones requiring additional workers. Hence, when workers are not under the direct control of the platform, simply informing workers about market conditions may not be sufficient to improve their availability.

We show, however, that the platform can force more workers to move from the market zone with excess supply by raising the price and choking demand in that zone. Consequently, even though it may be optimal to charge the regular price in a market zone with excess supply when this zone is viewed in isolation, we show that distorting the price in this zone through surge pricing can increase platform profit across market zones by improving worker availability. We further show that the platform may have an incentive to misreport market forecasts to exaggerate the need for workers to move. Consequently, workers may not trust the information provided by the platform. Hence, the platform may use a surge price to credibly communicate a greater need for workers to move. Importantly, a surge price in the market zone that the workers should leave is a less costly means for credible communication (than a surge price in the zone that workers should move to).

Thus, surge pricing in a market zone with excess supply can serve two distinct strategic purposes: to force independent workers to move to zones requiring more workers, and to credibly communicate the market need for them to move. We also examine the implications for consumers and workers. Ironically, consumers in the market zone with a surge price may not be served despite there being enough idle workers in their zone who are willing to serve. Moreover, surge pricing can hurt consumer, worker and market welfare even though it increases the availability of workers in markets with shortage of workers. Taken together, our findings may help understand the counterintuitive market observations and inform the debate regarding the use and impact of surge pricing.

2 Literature Review

Our work adds to the growing ongoing research on on-demand service platforms. Bai et al. (2016) and Taylor (2016) examine the steady state equilibrium in a queuing model where the arrival rate of users depends on the price charged by the platform, and the number of participating workers depends on the wage offered by the platform. Bai et al. (2016) show that the platform charges a higher price and pays a higher wage during periods of high demand. Taylor (2016) shows that congestion-driven service delays lead to lower prices and higher wages if the platform knows the valuations of consumers and workers; but this may not be the case if the valuations are uncertain. Also, using a queuing model, Riquelme et al. (2015) examine threshold-based surge pricing, wherein

the price is raised when the mismatch in supply and demand exceeds a certain threshold. They show that, under certain large market approximations, such a dynamic pricing strategy does not yield higher revenue than a static pricing strategy if the market parameters are known to the platform; but this dynamic strategy is more robust if the demand parameters are uncertain. Gurvich et al. (2015) find that worker flexibility to decide their own schedules reduces the number of participating workers and increases price levels. Cachon et al. (2016) study surge pricing in a single market zone. They show that surge pricing with a revenue-sharing arrangement between the platform and workers not only outperforms static pricing but may also capture much of the platform profit that is possible if worker wages are set independently.

We contribute to this literature by examining surge pricing across multiple market zones explicitly accounting for workers' incentives to move between zones. We also examine the implications of asymmetric information between the platform and workers, and the role of surge pricing in facilitating credible communication. We show that the platform may counterintuitively use surge pricing even in market zones with excess supply. Our work also differs from research on product-sharing platforms that allow users to rent assets from owners (e.g., Benjaafar et al. 2015; Einav et al. 2015; Fraiberger and Sundararajan 2015; Jiang and Tian 2016; Horton and Zeckhauser 2016); on product-sharing platforms, customers need not be served at short notice, they face a trade-off between owning and renting, the platform does not set the price, and there is no surge pricing.

More broadly, our work is related to research on dynamic pricing, location-based pricing, price signaling, and credible sharing of demand information, which we briefly discuss in turn. Research on dynamic pricing has examined the use of “peak-load pricing”, the practice of charging higher prices during periods of high demand, especially for non-storable resources such as electricity, telephone systems, and toll roads (e.g., Steiner 1957; Williamson 1966; Chao 1983; see Crew et al. (1995) for a survey of this literature). Researchers have shown that time-variant prices that reflect the true cost of production would lower the overall cost of meeting demand, thereby improving system efficiency. In particular, if the timing of the peak price periods is known in advance, then consumers may delay or advance their consumption to other times, thereby smoothening the demand and resulting in lower capacity investments. Much of this literature assumes that a single firm controls capacity and pricing decisions and does not consider interactions across market regions.³ We add to this literature by examining the pricing implications due to the strategic interactions between

³Some notable exceptions are Bohn et al. (1984) and Hogan (1992), who examine peak-load pricing in electricity markets considering the spatial properties of electricity production and demand, and Grimm and Zoettl (2013) who examine capacity decisions in a single market by competing producers in the presence of peak-load pricing.

the platform and independent workers who can move between market zones. Our work also differs from the literature on dynamic pricing under limited inventory (see Bitran and Caldentey (2003) and Elmaghraby and Keskinocak (2003) for a review), wherein the role of dynamic pricing is to manage the sales of a fixed amount of inventory occurring over a period of time.

Researchers have studied the competitive implications when a firm uses location-based pricing in order to price discriminate across consumers based on their location relative to the firm and its competitors (e.g., Corts 1998; Chen et al. 2001; Shaffer and Zhang 2002). In contrast, we study location-based pricing for better matching of supply and demand given the variation in the supply and demand for workers across market locations. Recently, Chen et al. (2016) examine the competitive implications of location-based pricing in the context of mobile geo-targeting. They show that if consumers can move between locations to avail a lower price available at a particular location, then firms may limit the extent to which they price-discriminate across market locations to discourage consumers from moving between locations, thereby softening price competition. We show that location-based surge pricing may be used to encourage workers to move between locations.

The role of price in signaling product quality has been examined by several researchers (e.g., Milgrom and Roberts 1986; Moorthy and Srinivasan 1995; Simester 1995; Shin 2005; Subramanian and Rao 2016). Stock and Balachander (2005) show that a firm may price a product too low and create product scarcity in order to credibly signal higher quality to consumers. We show that an on-demand platform may use surge pricing to exacerbate the extent of oversupply at a market location to credibly signal supply conditions to workers. Researchers have also examined the credible sharing of demand information between a downstream firm and an upstream supplier. Lee et al. (1997) show that the downstream firm can have an incentive to exaggerate demand conditions to ensure sufficient supply. Cachon and Lariviere (2001) and Özer and Wei (2006) study supply chain contracts that can facilitate credible information sharing. We add to this research stream by examining credible information sharing through surge pricing in a multi-period multi-market setting.

3 Model

Our model consists of three sets of players: an on-demand platform, independent workers, and consumers. We analyze the interactions in two adjacent market zones A and B over two successive periods 1 and 2. Consumers request on-demand service through the platform, and only workers in the same market zone as consumers can serve them at short notice. Hence, the platform matches consumers with workers available in the same market zone. The platform sets the price that consumers must pay workers for on-demand service, and receives a commission from the revenue generated.

A market zone can experience a *demand surge* (increase in demand), and the workers available in that zone may not be sufficient to satisfy the demand. Workers can move between market zones by incurring some cost and time. We model the following decisions. The platform decides the price for on-demand service in each period in each market zone. Workers decide whether to serve the market zone that they are currently in or to move to the adjacent zone. Consumers decide whether to request on-demand service. Our main interest is to understand when, why and in which market zones the platform will use a surge price, and the implications.

We start by describing consumers. In each period, there is a continuum of consumers in each market zone who require one unit of on-demand service in that period. These consumers require service at short notice and cannot wait. Their reservation price for on-demand service from the platform (relative to some outside option) is uniformly distributed over $[1, 2]$. We model the variation in demand over the two periods as follows. The number (mass) of consumers requiring service is initially the same in both market zones (in period 1), and subsequently increases in one of the market zones (in period 2). Either market zone can experience a demand surge with equal probability. Let $a > 0$ be the number of consumers requiring service in period 1 in each market zone. Let $a_H > a$ denote the number of consumers requiring service in period 2 in the market zone that experiences a demand surge; the number of consumers requiring service in the other market zone remains a . Let p_{ij} denote the price set by the platform in zone $i \in \{A, B\}$ in period $j \in \{1, 2\}$. The number of consumers who request service in zone i in period j is, therefore,

$$D_{ij} = a_{ij} (2 - p_{ij}), \tag{1}$$

where $a_{ij} \in \{a, a_H\}$ is the number of consumers requiring service. Thus, the demand surge (a_{ij}) has a multiplicative effect on demand. As explained later below, the implication is that, consistent with conventional wisdom, the platform will not have an incentive to use surge pricing if there are enough workers in a zone. Later, in §7, we discuss the implications of alternate demand formulations. For analytical convenience, we assume that all service requests take one period to complete, and originate and end in the same market zone. Hence, a worker who serves a consumer is busy for one period and remains in the same market zone at the end of that period.

The number of independent workers who are willing to work in each market zone on a given day is uncertain. As noted earlier, independent workers work at their convenience when they are not occupied with other work or activities. Hence, their availability is uncertain, which we model as follows. There is a registered pool of workers in each market zone. On a given day, a random subset of registered workers are available to serve consumers. Available workers join the platform

in their registered “home” market zones, which could be, for example, the market zone in which they live or get off from other work. Let $N_i \in \{n_H, n_L\}$ denote the number (mass) of workers who join the platform in market zone i at the start of period 1, where $n_H > n_L$. We assume that either $(N_A, N_B) = (n_H, n_L)$ or $(N_A, N_B) = (n_L, n_H)$, both being equally likely. In other words, there are either more workers to begin with in zone A than in zone B or vice-versa. Later, in §7, we discuss the implications of allowing the initial supply in each market zone to be distributed independently. While a worker starts in her home zone, she can move to the adjacent zone at a cost $c \geq 0$. Moving between zones requires one period and the worker is not available to serve consumers in either zone while moving. Workers who stay in their current zone are available to serve consumers in that zone. Let \tilde{N}_{ij} denote the number of workers available to serve consumers in zone i in period j . We say that the “initial supply in market i is high” if $N_i = n_H$. We assume that the uncertainty in the initial supply of workers is independent of the uncertainty in consumer demand.

Each period, the platform matches consumers requesting service in each zone with workers available to serve consumers in the same zone. If more workers are available than consumers requesting service, then demand is rationed and not all workers receive work requests; the platform randomly chooses from the available workers to match with consumers. If there are fewer workers available than consumers requesting service, then supply is rationed and not all consumers obtain service; the platform randomly chooses the consumers who are matched with available workers. The revenue generated from serving a consumer in zone i in period j is p_{ij} . The worker who serves the consumer receives a portion $\lambda \in (0, 1)$ of the revenue; we refer to this portion as the worker’s revenue. The platform receives the remaining portion, which we refer to as platform profit.

The platform can set the price p_{ij} to be equal to the regular price \bar{p} or higher. A price $p_{ij} > \bar{p}$ represents a surge price. We note that if the platform is not constrained by the availability of workers to serve consumers, then the price that maximizes its profit in zone i in period j is

$$p_{ij} = \arg \max_{p_{ij} \geq 1} (1 - \lambda) p_{ij} D_{ij} = \arg \max_{p_{ij} \geq 1} (1 - \lambda) a_{ij} (2 - p_{ij}) p_{ij} = 1. \quad (2)$$

To keep the analysis straightforward, we take the regular price $\bar{p} = 1$. Our approach ensures that, consistent with the common wisdom about surge pricing, it is optimal for the platform to charge the regular price and not use a surge price if there are enough workers to serve consumers in a zone. This approach thus enables us to delineate the strategic incentives for surge pricing in a zone with excess supply in the most straightforward manner. We remark that in our setting the platform cannot improve its profit by setting a regular price different than 1. As we discuss in §7, the surge pricing distortions that we identify still arise if $\bar{p} \neq 1$ and even if it is not profitable for the platform

to serve all consumers.

The sequence of events is as follows. At the start of period 1, Nature decides the number of workers that join the platform in period 1 in each zone, and the zone that will experience a demand surge in period 2. Then, in each period j the following events occur. Consumers requiring on-demand service that period join the platform. Next, the platform sets the price in each zone. Then, workers decide whether to stay in their current zone or to move to the adjacent zone. Consumers decide whether to request service. Lastly, the platform matches consumers requesting service with workers in the same zone who are available to serve consumers. At the end of the period, the workers who were matched with consumers complete their service, and the workers who decided to move reach the adjacent market zone. We assume that all players are rational, and maximize their respective expected utility or profit, and do not discount their utility or profit over the two periods.

It is useful to consider the market information that is known to the platform and workers. The model setting and parameters are common knowledge. The platform knows the number of workers who join the platform in each market zone in period 1. Further, the platform can forecast future demand. Hence, the platform knows in period 1 as to which market zone will have a demand surge in period 2. Workers too may be knowledgeable about supply and demand conditions, for example, based on their prior experience. Moreover, the platform may share its information with workers. We, therefore, analyze two information scenarios. First, in §5, we examine the *symmetric information* scenario, in which both the platform and the workers know (at the start of period 1) the initial supply in each market zone as well as which market zone will have a demand surge in period 2. This scenario allows us to examine the strategic role of surge pricing separate from concerns about truthful communication of market forecasts. This scenario also approximates situations in which the workers are sufficiently knowledgeable about market conditions based on their past experience, or the platform shares its information truthfully, for example, due to reputational concerns. Next, in §6, we analyze the *asymmetric information* scenario, in which only the platform knows the demand and supply conditions. In this case, the platform can share its forecasts with workers at the start of period 1. But, the shared forecasts are not verifiable and the platform can misreport the information.

To keep the analysis interesting and straightforward, we assume: (i) $a_H > n_H > n_L > a$, such that if workers did not move between zones then there is a shortage of workers in period 2 in the zone where demand surges, (ii) $n_H + n_L > a_H + a$, such that there is sufficient number of workers overall to meet the total demand across both zones in each period, and (iii) $a_H \leq (3 + 2\sqrt{2}) a$, such

that the demand surge is not too large and we can restrict attention to “interior” market outcomes in equilibrium. We discuss the implications of relaxing these assumptions in §7.

For ease of discussion, and without loss of generality, we fix the identity of the market zone whose demand surges in period 2 to be zone A . The discussion for the case in which demand surges in zone B is identical. Given the identity of the market zone whose demand surges, the platform can be in one of two states (types) depending on the initial supply of workers. We say that the platform is of type L if the initial supply is high in the market zone whose demand surges (zone A , in our discussion), and of type H if the initial supply is high in the other zone (zone B , in our discussion). Thus, there is lower mismatch between demand and the initial supply if the platform is type L . We use the notation $x|_t$ to denote variable x for a type $t \in \{H, L\}$ platform, and simply use x to denote the variable for either platform type. Table 1 summarizes our model notation.

Table 1: Model Notation

i	: Market zone, $i \in \{A, B\}$	\bar{p}	: Regular price for on-demand service ($= 1$)
j	: Market period, $j \in \{1, 2\}$	p_{ij}	: Price in zone i in period j , $p_{ij} \geq \bar{p}$.
t	: Platform's type, $t \in \{H, L\}$	r_{ij}	: Expected revenue of worker serving zone i in period j
a	: Number of consumers in zone without demand surge	π_{ij}	: Platform profit in zone i in period j
a_H	: Number of consumers in zone with demand surge	μ_i	: Proportion of zone i workers that move to adjacent zone
c	: Workers' cost of moving to adjacent zone	θ	: Workers' posterior belief that platform is type H
λ	: Workers' share of revenue	\hat{r}_{ij}	: Expected r_{ij} given workers' belief θ
N_i	: Initial number of workers in zone i ; $(N_A, N_B) = (n_H, n_L)$ or (n_L, n_H) , where $n_H > n_L$		
\tilde{N}_{ij}	: Number of workers available to serve consumers in zone i in period j		

4 Benchmark Market Outcomes

In this section, we examine two benchmark settings for the movement of workers between zones: (i) the *independent market zones* benchmark, in which workers cannot move between market zones; and (ii) the *first-best revenue* benchmark, in which workers move such that total market revenue is maximized. In subsequent sections, we explicitly consider how independent workers will in fact move to maximize their respective expected profit.

4.1 Independent Market Zones Benchmark

If workers cannot move between zones, then all workers that joined zone i in period 1 are available to serve consumers in zone i in both periods. Hence, $\tilde{N}_{ij} = N_i$. We obtain the platform's optimal pricing policy in the following lemma. All proofs are deferred to the Appendix.

Lemma 1. *If workers cannot move between market zones, then $p_{i1} = p_{B2} = 1$ and $p_{A2} = 2 - \frac{N_A}{a_H} > 1$.*

Lemma 1 establishes that, if workers cannot move between market zones, then the platform's use of surge pricing follows conventional wisdom. A surge price is used only in the market zone and time period in which demand exceeds supply, namely, zone A in period 2. Furthermore, the surge price rations the available workers to consumers with the highest valuations, i.e., $p_{A2} = 2 - \frac{N_A}{a_H}$ such that only $D_{A2} = N_A$ consumers with the highest reservation price obtain service.

Clearly, there is scope for a better market outcome if workers move between market zones, since there are idle workers in zone B and consumers that are not served in zone A . In fact, the independent market zones benchmark leads to the least number of consumers being served, the least market revenue, and the most number of idle workers. We next examine the implications if workers moved to zone A to maximize market revenue.

4.2 First-Best Revenue Benchmark

Recall from equation (2) that, if there are enough workers in a zone to serve all consumers, setting the regular price maximizes market revenue in that zone. Now, all consumers will request service at the regular price (since $D_{ij}(\bar{p}) = a_{ij}$). Moreover, it is feasible to serve all consumers since the total supply of workers is sufficient to meet total consumer demand, i.e. $n_H + n_L > a_H + a$. In particular, if $a_H - N_A$ workers move from zone B to zone A in period 1 then the demand surge in zone A in period 2 can be fully met and the market revenue in zone A is maximized. At the same time, this movement of workers will not affect the platform's ability to serve consumers in zone B since $\tilde{N}_{Bj} = n_H + n_L - a_H > a$. Hence, the market revenue in zone B is unaffected. Essentially, for maximizing market revenue, the value of having workers stay in zone B is zero (because there is an excess supply of workers in that zone), while the value of having workers move to zone A is positive (because there is a shortage of workers in that zone). Thus, total market revenue is maximized if workers move such that all consumers are served at the regular price in both zones. The following lemma describes this outcome.

Lemma 2. *Given that $N_A + N_B > a_H + a$, if workers move to maximize market revenue, then workers move such that $\tilde{N}_{ij} \geq a_{ij}$ and $p_{ij} = 1$.*

Thus, because there is sufficient supply of workers overall (and sufficient opportunity for workers to redistribute themselves), supply and demand can be matched to maximize revenue without the need for surge pricing. Since platform profit is proportional to market revenue, the first-best benchmark also maximizes platform profit. Consumer welfare is maximized in this benchmark since all consumers are served at the regular price. Furthermore, the number of idle workers is minimized. Consequently, worker utilization and aggregate worker revenues in each zone are also maximized.

5 Role of Surge Pricing with Independent Workers

We now examine the strategic interactions under symmetric information, explicitly accounting for the incentives of independent workers to move between zones. We start by asking whether simply informing workers about market conditions is sufficient to ensure that enough of them move to zone A to meet the demand surge, such that the platform attains the same profit as in the first-best revenue benchmark. Note that since platform profit is proportional to market revenue, the first-best revenue benchmark also represents the outcome if the platform could directly control the movement of workers. However, independent workers will move to maximize their respective expected profit. We first examine whether supply and demand be balanced solely based on the individually rational behavior of workers, without further intervention by the platform. Then, we examine the role and scope for surge pricing to influence workers and optimize platform profit.

We solve for the subgame-perfect pure strategy equilibrium. In particular, workers follow a pure strategy in period 1 regarding whether to move to the adjacent zone. Let $\mu_i \in [0, 1]$ denote the proportion of workers in zone i that decide to move to the adjacent zone in period 1. In equilibrium, workers correctly anticipate the behavior of all other workers.⁴ Let $\pi_{ij} = (1 - \lambda) p_{ij} \min \{ \tilde{N}_{ij}, D_{ij} \}$ denote the platform profit in zone i in period j . Let r_{ij} denote the expected revenue of a worker who serves market i in period j , taking into account the probability that the worker finds work.

5.1 Will Enough Workers Move if They are Informed?

We say that “enough workers move” if the resulting number of workers in zone A is $\tilde{N}_{A2} \geq a_H$. Let $\bar{\mu} = \frac{a_H - N_A}{N_B}$ denote the minimum proportion of workers that must move from zone B to zone A such that the first-best revenue is attained. Note that the platform can achieve the same profit as in the first-best revenue benchmark if and only if enough workers move to zone A and there is no surge pricing in equilibrium; this is because platform profit is maximized only by serving all consumers at the regular price. Therefore, we solve for the subgame equilibrium in period 1 given $p_{i1} = 1$, and determine whether enough workers move in equilibrium such that the platform does not have to resort to surge pricing in period 2. Proposition 1 provides the conditions when this is so.

Proposition 1. *If workers are informed about market conditions, then enough workers move to the market zone with demand surge even without surge pricing iff the following condition (C1) holds:*

$$c < \lambda \text{ and } n_H + n_L \geq a_H + \frac{2\lambda}{\lambda - c} a.$$

One might expect that not enough workers will move to zone A if their cost to move is high.

⁴Alternatively, one could examine a symmetric mixed strategy equilibrium in which all workers in zone i move with probability μ_i . It can be shown that, in our setting with atomistic workers, the mixed strategy equilibrium yields the same analytical results as the pure strategy equilibrium because of the law of large numbers.

This is indeed the case. From Proposition 1, we require that workers' cost to move, c , is less than the revenue-sharing rate λ . In the absence of surge pricing, $p_{A2} = 1$. If not enough workers move and there is a shortage of workers in zone A , then a worker serving zone A will obtain work with probability 1. Hence, the expected revenue of a worker serving zone A in period 2 is $r_{A2} = \lambda$. A worker that moves to zone A will obtain an expected profit $r_{A2} - c$. If $c > \lambda$ then the worker cannot profit from the move, and hence will not move.

Conversely, one might expect that enough workers will move if c is low. For, after all, the expected worker profit from moving to zone A is strictly positive if $c < \lambda$. However, Proposition 1 shows that not enough workers may move even if $c < \lambda$. In fact, not enough workers may move even if $c = 0$. From the overall market and platform perspective, the value of having workers stay in zone B is zero since there are enough workers to serve all consumers and additional workers will only add to the number of workers that will be idle. The value of having workers stay in zone B is also zero from the perspective of all workers in zone B taken together. However, from the perspective of an individual worker who starts in zone B , the value of staying in zone B is strictly positive. Despite the excess supply in zone B , there is a positive probability that a worker who stays in zone B obtains work (in one or both periods). Therefore, $r_{Bj} > 0$ and a worker's expected profit from staying in zone B ($r_{B1} + r_{B2}$) is strictly positive and may exceed the expected profit from moving to zone A ($r_{A2} - c$). Consequently, moving to zone A can be unattractive from the perspective of an individual worker who starts in zone B , even if it is valuable from the combined perspective of all such workers taken together.

Intuitively, an individual worker imposes a negative competitive externality on all other workers serving the same market zone as her. By staying in zone B , she lowers the probability of other workers serving zone B receiving a service request. However, because she does not internalize this externality, her value of staying in zone B is positive, and moving to zone A can be unattractive. In contrast, from the combined perspective of all workers who start in zone B (which accounts for the negative externality), the value of her staying in zone B is zero, whereas the value of her moving to zone A is positive if c is sufficiently small.

Proposition 1 shows that enough workers move without surge pricing if $c < \lambda$ only if the total supply of workers is also considerably higher than the total demand in period 2 ($n_H + n_L \geq a_H + \frac{2\lambda}{\lambda - c}a$); in this case, the resulting oversupply in zone B lowers the probability of a worker in that zone getting a service request to such an extent that enough workers move even though they do not internalize the externality that they impose on other workers. Consequently, the platform can

balance demand and supply without resorting to surge pricing. Conversely, if $n_H + n_L < a_H + 2a$, then not enough workers will move on their own without surge pricing even if their cost to move is zero and even though market conditions are known to them. Corollary 1 highlights this finding.

Corollary 1. *Even if workers are informed about market conditions and their cost to move between market zones is zero, without surge pricing, not enough workers move to the market zone with the demand surge iff $n_H + n_L < a_H + 2a$.*

One of the ways in which on-demand platforms seek to create value is to improve the availability of workers by lowering the information and transaction costs of workers to learn market conditions. Corollary 1, however, highlights the fact that simply informing workers about market conditions may not be sufficient to optimize their availability. Not enough workers may leave a zone with excess supply to serve adjacent zones requiring additional workers. In particular, this is so even if their cost to move between market zones is zero, and even though moving to the adjacent zone maximizes overall worker utilization and profit. Thus, seemingly, workers intentionally ignore the market conditions. But it is in fact rational for them to do so. Workers do not move to a zone requiring additional workers because there is positive value in staying in a zone with excess supply. Thus, workers require additional incentive to move out of a market zone with excess supply. Consequently, further intervention by the platform may be necessary.

5.2 Can Workers Be Made to Move With a Surge Price?

Note that if condition C1 in Proposition 1 does not hold, then $\mu_B < \bar{\mu}$ in the subgame equilibrium if $p_{i1} = 1$. Consequently, not enough workers move to zone A and the platform will use surge pricing in period 2 (such that $p_{A2} > 1$). We now examine whether a surge price in period 1 can induce more workers to move to zone A . We solve for the the subgame equilibrium in period 1 given $p_{i1} \geq 1$. Throughout the analysis, we make the dependence of the subgame equilibrium outcomes on period 1 prices explicit only where necessary.

Consider a worker that joins the platform in zone i in period 1. Her expected profit from serving zone i is $r_{i1} + r_{i2}$. Her expected profit from moving to serve zone j is $r_{j2} - c$. Therefore, it is rational for workers in zone i to move to zone j iff $r_{i1} + r_{i2} \leq r_{j2} - c$. It follows that, in equilibrium, if it is rational for workers in zone i to move to zone j , then it is not rational for workers in zone j to move to zone i . Therefore, either $\mu_A > 0$ or $\mu_B > 0$, but not both. The following lemma shows that since zone A has a shortage of workers to begin with, it must be that $r_{A2} > r_{B2}$ in any (subgame) equilibrium and, therefore, $\mu_A = 0$ and $\tilde{N}_{Bj} > a$.

Lemma 3. *In the subgame equilibrium given period 1 prices, workers in zone A stay in zone A, and there is always sufficient number of workers in zone B.*

Since $\mu_A = 0$ and $\mu_B \in [0, 1)$, we have $\tilde{N}_{A1} = N_A$, $\tilde{N}_{A2} = N_A + \mu_B N_B$ and $\tilde{N}_{B1} = \tilde{N}_{B2} = N_B(1 - \mu_B)$. If $\mu_B = 0$ in equilibrium, then we require that $r_{B1} + r_{B2} \geq r_{A2} - c$ such that workers in zone B (weakly) prefer to stay in zone B. If $\mu_B \in (0, 1)$, then we require that $r_{B1} + r_{B2} = r_{A2} - c$ such that workers in zone B are indifferent between staying and moving. Otherwise, either the workers who move or the workers who stay in zone B will have an incentive to deviate from their strategy. Therefore, the equilibrium worker movement condition can be written as

$$r_{B1} + r_{B2} \geq r_{A2} - c, \quad (3)$$

where equality holds if $\mu_B > 0$.

We have $r_{ij} = \lambda p_{ij} \min \left\{ \frac{D_{ij}}{N_{ij}}, 1 \right\}$, where $\min \left\{ \frac{D_{ij}}{N_{ij}}, 1 \right\}$ is the probability that a worker serving zone i in period j obtains a service request. Specifically, in zone B, there is always sufficient number of workers in equilibrium. Therefore, the platform's optimal price in period 2 is $p_{B2} = 1$ and $D_{B2} = a$. In period 1, $D_{B1} = a(2 - p_{B1})$. In zone A, if $\mu_B \leq \bar{\mu}$, then the platform's optimal price in period 2 is $p_{A2} = 2 - \frac{\tilde{N}_{A2}}{a_H}$ and $D_{A2} = \tilde{N}_{A2}$; if $\mu_B > \bar{\mu}$, then $p_{A2} = 1$ and $D_{A2} = a_H$.

We observe that the expected worker revenues in condition (3) are affected only p_{B1} and not p_{A1} . Therefore, the equilibrium μ_B depends only on p_{B1} . It follows that using a surge price in zone A in period 1 will only reduce platform profit, without affecting the number of workers that move to zone A. Therefore, it is not useful for the platform to employ a surge price $p_{A1} > 1$. Proposition 2 describes how the number of workers that move is influenced by a surge price in zone B.

Proposition 2. *The number of workers moving out of a zone with excess supply of workers is (weakly) increasing in the surge price in that zone in that period, and is not influenced by a surge price in the adjacent zone in that period.*

Proposition 2 establishes that the platform can use a surge price in a market zone with excess supply to induce workers to leave to serve other zones. One might have thought that a surge price should make it more attractive for workers to stay in a market zone because they can earn a higher revenue from a service request. However, this intuition does not hold if there is excess supply of workers in that zone. In this case, a surge price shrinks consumer demand and reduces a worker's probability of finding work. Thus, the surge price chokes demand and exacerbates the extent of oversupply, thereby making it more attractive for workers to leave. We next examine whether it is profitable for the platform to distort its pricing in this manner.

5.3 Surge Pricing in a Market Zone with Excess Supply

As noted earlier, the platform must engage in surge pricing if condition (C1) in Proposition 1 does not hold. The platform's optimal surge pricing strategy then depends on the trade-off between serving consumers in zone B in period 1 and serving consumers in zone A in period 2. Broadly speaking, the platform faces two alternatives for surge pricing. The first alternative is to avoid surge pricing in period 1 such that all consumers are served in period 1. This alternative corresponds to the use of surge pricing only in a market zone with insufficient supply. In particular, the platform sets the regular price in zone B in both periods, which optimizes the platform's profit from this zone. In zone A , the platform sets a surge price in period 2 because not enough workers move. The second alternative is to use a surge price in zone B in period 1 in order to make more workers move to zone A . This alternative corresponds to the use of surge pricing in a market zone with excess supply. It improves the platform's profit from zone A , but at the expense of the profit from zone B because the price in this zone is distorted above the optimal regular price.

We find that, in our setting, the platform may use a surge price in a market zone with excess supply (zone B) only if the initial supply is low in the market zone that requires additional workers (zone A), i.e., only if the platform is type H . This is because of two reasons. First, there is a greater shortage of workers in zone A if the platform is type H (than if it is type L). Hence, the marginal benefit of having an additional worker move to zone A is higher. Second, the extent of oversupply in zone B is higher if the platform is type H . Therefore, the probability of a worker in zone B getting a service request is lower. As a result, a surge price is more effective in making workers leave. Consequently, the type L platform does not find it profitable to use a surge price in period 1, and uses a surge price only in period 2 in zone A . We derive the market conditions under which the type H platform uses a surge price in period 1 in the following proposition.

Proposition 3. *If workers are informed about market conditions, the platform uses a surge price in a market zone with excess supply of workers iff the following condition (C2) holds: $a_H > 4a$, $n_H > a + \sqrt{a^2 + 2aa_H}$, $n_L < a_H - \frac{aa_H}{n_H} - \frac{1}{2}n_H$ and $\lambda \left(\frac{n_H + n_L}{a_H} \right) < c < \lambda \left(2 - \frac{a}{n_H - \phi} - \frac{n_L + \phi}{a_H} \right)$, where $\phi = a_H - n_L - \sqrt{(a_H - n_L)^2 - aa_H}$.*

Proposition 3 describes how the supply and demand factors affect whether the platform will use a surge price in a market zone with excess supply. Intuitively, such a strategy is more attractive for the platform if there is greater need to have workers move to zone A , and if the surge price is more effective in inducing workers to move from zone B . Consequently, we find that the platform uses surge pricing in period 1 if the demand surge in period 2 is sufficiently high ($a_H > 4a$), and

if the extent of undersupply in zone A and the extent of oversupply in zone B are sufficiently high (n_H is sufficiently high, n_L is sufficiently low). Moreover, the cost for workers to move (c) should neither be too low nor too high. If c is low, then more workers move even without a surge price, and the need to use a surge price is lower. If c is high, then a surge price is not sufficiently effective in moving workers and, hence, not useful.

More importantly, Proposition 3 establishes that, contrary to conventional wisdom, the platform may use surge pricing even in a market zone with excess supply. Essentially, the platform willingly foregoes demand and underserves consumers in an initial period (zone B in period 1), in order to better serve consumers in a future period (zone A in period 2). Ironically, consumers in the zone with a surge price are underserved, despite there being sufficient number of workers available in that zone to serve all consumers and even though there are workers idle in this zone in equilibrium. It follows from Lemma 3 that even when the type H platform uses a surge price in period 1, enough workers are available in zone B to serve all consumers, i.e., $\tilde{N}_{Bj}|_H > a$. Yet, not all consumers are served because of the surge price. In fact, under the linear demand formulation, we find that if it is optimal for the type H platform to use a surge price in period 1, then it must maximize the number of workers moving to zone A by setting a surge price $p_{B1}|_H = 2$, such that $D_{B1}|_H = 0$ and $r_{B1}|_H = 0$. Consequently, no consumers are served in zone B in period 1 even though there are sufficient number of idle workers available in equilibrium.

It is important to note that while a surge price is still in response to a demand surge, its strategic purpose and implications are different than when it is used for the conventional purpose of rationing supply. In its conventional role, a surge price is used only in a market zone with insufficient supply of workers and is meant to allocate available workers to consumers with the highest reservation price, thereby maximizing platform profit in this zone. In this case, workers do not move out of the zone in which there is a surge price, and may even move in from an adjacent zone anticipating higher revenue. This is the case when a surge price is used in zone A in period 2. However, a surge price in a market zone with an excess supply is meant to exacerbate the extent of oversupply by deliberately choking demand. In this case, the surge price lowers the expected worker revenue and induces them to leave to serve adjacent zones. In equilibrium, there are idle workers in the zone with the surge price and, yet, not all consumers are served. Further, platform profit in the zone with the surge price is reduced because demand is foregone.

Our findings regarding the strategic use of a surge price in a market zone with excess supply may clarify the counterintuitive market observations regarding surge pricing. As illustrated by

the opening comments in §1, Uber drivers have complained about encountering weaker demand in a market zone with a surge price. Their observations are confirmed by Chen et al. (2015), who find that a surge price often has a negative impact on demand causing drivers to become idle. They further find that, instead of attracting more drivers, a surge price may even cause drivers to move out of a zone with a surge price, leaving it with fewer drivers. These market observations, while contrary to conventional wisdom, are consistent with what our model predicts regarding the platform’s strategic use of surge pricing in a zone with excess supply. Researchers also note that a significant fraction of drivers do not seem to respond to the market forecasts shared by the platform, and prefer to stay in their current zone instead of driving to zones requiring more drivers (Lee et al., 2015; Rosenblat and Stark, 2015). Our analysis shows that not enough workers may move to a zone requiring more workers because there is positive value to staying in a zone with excess supply.

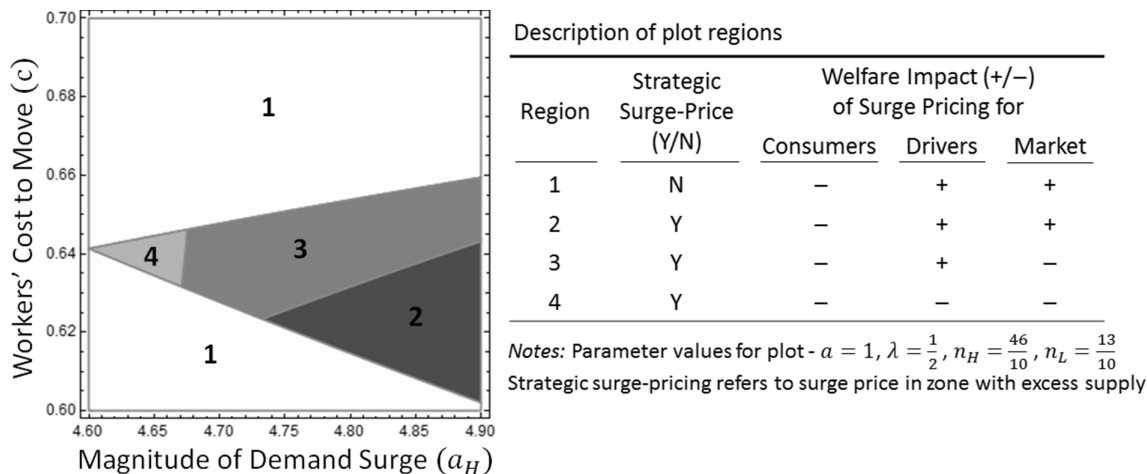
5.4 Welfare Implications

On-demand platforms have been accused of using surge pricing to fleece consumers even when there is sufficient supply of workers, leading to calls for banning this practice (e.g., Politico 2015; Financial Times 2016). Our analysis shows that the platform may indeed use surge pricing in market zones with excess supply. Even though the platform’s intent may not be to fleece consumers, nevertheless surge pricing prevents some consumers from obtaining service despite there being sufficient supply of idle workers in their vicinity to serve them. At the same time, this form of surge pricing allows for consumers in the market zone with a demand surge receive better service. Therefore, consumer complaints may be less of a concern if this form of surge pricing led to an improvement in market performance through better matching of supply and demand. To investigate this further, we compare the consumer, worker and market welfares with and without surge pricing, i.e., we compare the outcomes if the platform is able to use surge pricing with the outcomes in the counterfactual scenario in which surge pricing is not allowed.

We find that surge pricing may in fact have a negative impact on consumers, workers and the overall market. Specifically, it is useful to distinguish between instances of non-strategic surge pricing, where a surge price is used (in equilibrium) only in a zone with insufficient supply of workers, and strategic surge-pricing, where a surge price is also used in a zone with sufficient supply. Non-strategic surge pricing improves worker and market welfare, since it leads to higher compensation and utilization for workers and better matching of supply and demand for the market. However, non-strategic surge pricing does not necessarily benefit consumers; the higher price can offset the benefits of increased worker availability and allocation of workers to consumers with higher valuation.

More importantly, because strategic surge pricing chokes demand to induce workers to move, it can hurt even worker and overall market welfare. Figure 1 illustrates these outcomes as a function of the magnitude of the demand surge a_H and the workers' cost to move c for given values of the remaining parameters. Figure 1 shows the values of a_H and c for which strategic and non-strategic pricing is used in equilibrium, and the welfare impacts of surge pricing for these parameter values. Our findings thus support consumer complaints and regulatory concerns about surge pricing.

Figure 1: Welfare Impact of Surge Pricing



6 Surge Pricing and Strategic Forecast Communication

We now analyze the strategic interactions under asymmetric information. The platform shares forecasts with workers at the start of period 1. We first examine whether the platform can share forecasts truthfully without relying on period 1 prices to reveal or signal the information; that is to say, truthful communication can occur without an “accompanying” surge price. Then, we examine whether and how an accompanying surge price facilitates credible communication.

We solve for a perfect Bayesian equilibrium. We note at the outset that the platform does not have an incentive to misinform workers about the market zone that will experience a demand surge. For example, if the demand surge will occur in zone A , then the platform benefits if workers in zone A know that they must stay in that zone and workers in zone B know that they must move. Therefore, the platform will share this information truthfully. Consequently, it is sufficient to analyze the implications of asymmetric information about the initial supply condition, i.e., the platform's type. As before, and without loss of generality, we fix the identity of the market zone that will experience a demand surge to be zone A , and this is known to the workers (via the platform's communication). Hence, the platform is type H if the initial supply in zone A is low.

Let $s \in \{H, L\}$ denote the platform's communication about its type. At the start of period

1, workers have the (correct) prior belief that the probability of the platform being type H is $\frac{1}{2}$. Workers update their belief based on s and the period 1 price $\mathbf{p}_1 = (p_{A1}, p_{B1})$. Let $\theta \in [0, 1]$ denote their posterior belief that the platform is of type H . We show the dependence of outcomes on θ and \mathbf{p}_1 explicitly only where necessary. Given θ and \mathbf{p}_1 , as before, either none of the workers move or only some workers in zone B move. Let μ_B denote the proportion of workers in zone B who move to zone A , and $\tilde{N}_{ij}(\mu_B)$ denote the resulting number of workers available to serve zone i in period j . Therefore, $\tilde{N}_{A1}(\mu_B) = N_A$, $\tilde{N}_{A2}(\mu_B) = N_A + N_B\mu_B$ and $\tilde{N}_{Bj}(\mu_B) = N_B(1 - \mu_B)$.⁵ Then, in period 2, the platform's optimal price is $p_{i2}(\mu_B) = \max\left\{1, 2 - \frac{\tilde{N}_{i2}(\mu_B)}{a_{i2}}\right\}$.

Let $r_{ij}(\mu_B)|_t$ denote the expected worker revenue given the platform's type t . We have

$$\begin{aligned} r_{i1}(\mu_B)|_t &= \lambda p_{i1}|_t \min\left\{\frac{D_{i1}}{\tilde{N}_{i1}(\mu_B)|_t}, 1\right\}, \\ r_{i2}(\mu_B)|_t &= \lambda \max\left\{2 - \frac{\tilde{N}_{i2}(\mu_B)|_t}{a_{i2}}, 1\right\} \min\left\{\frac{a_{i2}}{\tilde{N}_{i2}(\mu_B)|_t}, 1\right\}. \end{aligned}$$

At the start of period 1, let $\hat{r}_{ij}(\theta)$ denote workers' expectation of their expected revenue in zone i in period j given their posterior belief θ , given by

$$\hat{r}_{ij}(\theta) = \theta r_{ij}(\mu_B)|_H + (1 - \theta) r_{ij}(\mu_B)|_L, \quad (4)$$

The worker equilibrium movement condition is then given by

$$\hat{r}_{B1}(\theta) + \hat{r}_{B2}(\theta) \geq \hat{r}_{A2}(\theta) - c, \quad (5)$$

where equality holds if $\mu_B > 0$. The following lemma establishes that there is a unique $\mu_B(\theta)$ that satisfies condition (5). Moreover, $\mu_B(\theta)$ is (weakly) increasing in θ .

Lemma 4. *Given period 1 prices and workers posterior belief, there is a unique $\mu_B(\theta)$ that satisfies the equilibrium worker movement condition. If $\mu_B(\theta) > 0$ for $\theta = \theta'$, then $\mu_B(\theta)$ is strictly increasing in θ for all $\theta \geq \theta'$.*

6.1 Can Platform Communicate Truthfully without Surge Pricing?

We now examine whether truthful communication is feasible in the absence of an accompanying surge price. That is, we analyze a separating equilibrium in which $\mathbf{p}_1 = (1, 1)$, the platform truthfully reports its type $s|_t = t$, and the workers trust the message by updating their belief to $\theta = 0$ if $s = L$ and $\theta = 1$ if $s = H$. For truthful communication to be feasible, the platform must not have an incentive to misreport its type given that workers trust the message.

⁵To determine the behavior of workers in a consistent and straightforward manner under all possible beliefs, including incorrect beliefs, we assume the following: given θ and \mathbf{p}_1 , in zone i , the proportion of workers that move in any randomly chosen subset of workers in that zone is the same and, hence, equal to μ_i . That is to say the behavior of workers in a zone is uniform and does not differ systematically across different groups of workers.

We find that the type L platform may misreport its type in order to induce more workers to move. From Lemma 4, $\mu_B(\theta)$ is weakly increasing in θ . Therefore, a larger proportion of workers may move if they believe that the platform is type H (than if it is type L), i.e., $\mu_B(\theta = 1) \geq \mu_B(\theta = 0)$. This is because, on a type H platform, there is greater undersupply of workers in zone A and greater oversupply in zone B to begin with, compared to a type L platform. Consequently, compared to a type L platform, workers face more competition for service requests in zone B and less competition in zone A . Hence, the incentive for workers in zone B to move to zone A is higher if the platform is type H . The type L platform may, therefore, have an incentive to misreport its type, since having more workers move to zone A can be profitable.

There are three situations in which the type L platform does not have an incentive to misreport its type. First, if condition (C1) from Proposition 1 holds, then enough workers will move given the true market information, and there is no need for the type L platform to lie about its type. Second, if workers do not move even if they believe that the platform is type H , then workers' belief about the platform's type does not affect their behavior. Specifically, if $\mu_B(\theta = 1) = 0$, then $\mu_B(\theta) = 0$ for all θ (from the contrapositive of the second part of Lemma 4). Consequently, the type L platform cannot manipulate workers by misreporting information. Lastly, misreporting its type can cause too many workers to move from zone B such that the type L platform faces a shortage of workers to serve zone B , i.e., $\mu_B(\theta = 1)$ is sufficiently high that $\tilde{N}_{Bj}(\mu_B(\theta = 1))\Big|_L = n_L(1 - \mu_B(\theta = 1)) < a$. In this case, the type L platform cannot serve all consumers in zone B and the ensuing loss in revenue in zone B can offset the gain from having more workers move to zone A .

In the remaining situations, the type L platform can profit from misreporting its type. Consequently, there is no equilibrium in which the platform sends a truthful message $s|_t = t$ without an accompanying surge price, and the workers trust the message. In Proposition 4, we provide sufficient conditions under which such truthful communication is not feasible.⁶

Proposition 4. *In the absence of an accompanying surge price, the platform cannot truthfully communicate market forecasts with workers if the following condition (C3) holds: $a_H > \frac{3}{2}a$, $n_H > \max\left\{\frac{a+a_H}{2}, \frac{a-a_H+\sqrt{a^2+6aa_H+a_H^2}}{2}\right\}$, $n_L < \min\left\{n_H, \frac{2a_H(n_H-a)}{n_H}\right\}$ and $\lambda \max\left\{0, \left(1 - \frac{2a}{n_H+n_L-a_H}\right), \left(2 + \frac{n_H a}{n_L a_H} - \frac{2n_L}{n_H} - \frac{n_H+n_L}{a_H}\right)\right\} < c < \lambda\left(2 - \frac{n_L}{a_H} - \frac{2a}{n_H}\right)$.*

Researchers have found that workers on on-demand platforms may be skeptical whether the forecasts shared by the platform represent true market conditions, or are being used to manipulate

⁶Condition (C3) in Proposition 4 is a sufficient condition because it ensures that there is no shortage of workers in zone B if the type L platform misreports, such that the type L platform strictly benefits from the additional workers that move to zone A . However, misreporting can be profitable even if there is a shortage of workers in zone B .

workers to move to different zones (Rosenblat, 2015). Our analysis shows why there may be such lack of trust. Because workers' are more reluctant to move than what is optimal for the platform, the platform has an incentive to exaggerate market conditions so as to induce more workers to move than they would if the true market conditions were known to them. Consequently, truthful communication between the platform and workers may not be feasible, causing workers to distrust the market forecasts even when there is truly a greater need to move.

6.2 Can Surge Pricing Facilitate Truthful Communication?

We now investigate whether the platform can credibly communicate its market information by complementing the forecast with surge pricing. We also study what form such surge pricing can take. Specifically, we analyze a separating equilibrium in which $s|_t = t$ and the type H platform signals its type by using a surge price in one or both market zones in period 1, i.e., $p_{i1}|_L = 1$ and $p_{i1}|_H \geq 1$, where $p_{i1}|_H > 1$ for some i . At the start of period 1, workers update their belief to $\theta = 1$ if $s = H$ and $\mathbf{p}_1 = \mathbf{p}_1|_H$ and to $\theta = 0$ otherwise. We solve for the least-cost separating equilibrium (e.g., Desai 2001; Stock and Balachander 2005); the least-cost separating equilibrium essentially minimizes the surge price that is used to signal the platform's type and is the most profitable separating equilibrium for the type H platform.

There are two possible scenarios depending on whether or not surge pricing in period 1 is optimal for the type H platform under symmetric information. We consider each in turn. If condition (C2) in Proposition 3 is met, then it is optimal for the type H platform to employ surge pricing in zone B in period 1 under symmetric information. In this case, the type H platform's pricing strategy is $(p_{A1}, p_{B1})|_H = (1, 2)$, and the type L platform's pricing strategy is $(p_{A1}, p_{B1})|_L = (1, 1)$. If the platform adopts the same strategy under asymmetric information, then workers can infer the platform's type depending on whether a surge price is used. The type L platform can potentially mimic the type H platform by setting a surge price $p_{B1} = 2$; doing so would make workers believe that the platform is type H and cause more workers to move to zone A . In Proposition 5, we show that the ensuing loss in revenue in zone B in period 1 due to the surge price makes this deviation unprofitable for the type L platform. Further, the type H platform does not benefit from using a lower surge price $p_{B1} < 2$, even though a lower surge price would suffice to credibly reveal its type. Instead, as under symmetric information, the platform benefits from setting $p_{B1} = 2$ to maximize the number of workers that move to zone A . Essentially, a surge price is costly for the type L platform, but is optimal for the type H platform. Consequently, this separating equilibrium is also the unique least-cost separating equilibrium. Moreover, the surge price occurs in the market

zone that workers should leave and reveals that there is greater need for workers to move to the adjacent zone. We thus obtain the same equilibrium outcome under asymmetric information as under symmetric information.

If condition (C2) in Proposition 3 is not met, then, conditional on workers being informed about the market condition, surge pricing in period 1 is not optimal for the type H platform. Therefore, a surge price can be useful only for its signaling role. For such a separating equilibrium to be feasible, the type H platform should benefit more from communicating the true market condition than the type L platform benefits from exaggerating the market condition. To determine whether this is the case, we first note that, in a separating equilibrium, a larger proportion of workers in zone B will move to zone A if they infer the platform's type to be type H than type L (based on period 1 price). This is because $\mu_B(\theta)$ is increasing in θ (as shown in Lemma 4) and in p_{B1} (following the same arguments as in Proposition 2). Let $\Delta\mu$ denote this increase in proportion of workers that move. For separation to be feasible, the type H platform should benefit more than the type L platform from this increase in proportion of workers that move.

In Proposition 5, we show that the type H platform indeed benefits more for two reasons. First, the type H platform faces a greater shortage of workers in zone A . Hence, the marginal benefit of having one more worker move to zone A is higher for the type H platform than for the type L platform. Second, the type H platform also profits more from a given increase in the proportion of workers that move from zone B because the actual number of workers that move is higher. Specifically, the number of additional workers that move if they infer the platform's type to be type H than type L is $n_t\Delta\mu$, which is higher for a type H platform since $n_H > n_L$. Thus, for both reasons, the type H platform benefits more from signaling its type. Hence, it is feasible for surge pricing to facilitate credible communication.

Moreover, the separating equilibrium in which the type H platform uses a surge price only in zone B is the unique least-cost separating equilibrium. A surge price in zone A shrinks demand in zone A , but does not otherwise influence workers' incentives to move to zone A (as in Proposition 2). As a result, as a signaling instrument, a surge price in zone A is equally costly for both platform types. In contrast, a surge price in zone B is relatively less costly for the type H platform. A surge price in zone B not only shrinks demand in zone B , but also increases the proportion of workers that move to zone A (as in Proposition 2). This increase in the proportion of workers moving to zone A is more valuable for the type H platform for the same two reasons as discussed above; namely, the type H platform profits more from having more workers move, and more workers move if the

platform is type H . Consequently, a surge price in zone B is a more efficient signaling instrument. Intuitively, a surge price in zone A is akin to signaling by simply “burning money”, since it results in the same profit loss for both platform types. In contrast, a surge price in zone B results in a lower profit loss if there is truly a greater need for workers to move, making it more efficient than burning money.

Proposition 5. *If the platform cannot truthfully communicate market forecasts without an accompanying surge price, then it can do so with an accompanying surge price. In the unique least-cost separating equilibrium, the surge price is used in the market zone that workers should move from and signals that there is greater need for them to move.*

Thus, a surge price can facilitate forecast communication by credibly signaling to workers that there is a greater need for them to move. Importantly, the surge price is used in the market zone that workers should leave, because this minimizes the platform’s cost to signal to supply conditions. This use of surge pricing is also consistent with the market observations discussed before. Specifically, workers leave the market zone in which a surge price is used, and the platform does not serve all consumers in this zone despite there being sufficient number of idle workers available to serve them in equilibrium. These results shed further light on the rationale for the counterintuitive use of surge pricing in a market zone with sufficient supply of workers.

The scope for the strategic surge pricing in a zone with excess supply is higher under asymmetric information. As under symmetric information, the platform will use strategic surge pricing if condition (C2) in Proposition 3 met. In this case, the surge price is primarily used to incentivize workers to move depending on the initial supply condition. Furthermore, the platform will use strategic surge pricing also when condition (C2) in Proposition 3 is not met and credible communication is not otherwise feasible, as is the case if condition (C3) in Proposition 4 holds. In this case, the surge price is used to facilitate credible communication of market forecasts. For example, in the case of Figure 1 in §5.4, the platform will use strategic surge pricing as under symmetric information in the regions marked 2, 3 and 4. In addition, credible communication is not feasible without an accompanying surge price in the lower region marked 1; hence, the platform will also use strategic surge pricing in this region. In the upper region marked 1, the type L platform does not have an incentive to misreport its type; because workers’ cost to move is high, no workers move even if the platform is type H and the type H platform does not find it profitable to make workers move using a surge price. Hence, a surge price is used only in zone A in period 2 as under symmetric information.

7 Concluding Remarks

A fundamental challenge faced by many on-demand platforms is to ensure that their workforce of independent workers is available at the right market locations at the right time. In this paper, we examine the role of surge pricing and forecast communication in addressing this challenge. We explicitly account for the platform’s and workers’ incentives to serve consumers in different market locations. Our analysis provides insights about when and why, and contrary to conventional wisdom, a platform may use a surge price even in a market zone with sufficient supply of workers, and how this affects the functioning of the marketplace.

Some of our findings are as follows. When workers are not under the direct control of the platform, informing workers about market conditions is not sufficient to obtain the optimal distribution of workers. Because individual workers do not internalize the competitive externality that they impose on other workers in their market zone, too few workers may leave a zone with an excess supply of workers to serve an adjacent zone that requires additional workers. The platform can make more workers move by distorting the price in the market zone with excess supply through surge pricing to deliberately choke demand. Doing so improves total platform profit across zones if the need for additional workers to move is sufficiently high.

Moreover, in sharing market forecasts with workers, information about which market zone the workers should move to can be shared credibly. However, the platform can have an incentive to exaggerate the need for workers to move. Consequently, workers may ignore the forecasts provided by the platform even if there is truly a greater need for them to move. A surge price accompanying the forecasts can, however, facilitate truthful communication by serving as a credible signal that more workers must move; importantly, it is more profitable for the platform to use a surge price in the market zone that the workers should leave. Thus, for on-demand platforms that rely on independent workers, surge pricing in a market zone with excess supply can serve two distinct strategic purposes: to incentivize workers to move to zones requiring more workers, and to credibly communicate the market need for them to move.

Our model-based analysis can inform the debate and controversy surrounding surge pricing. Researchers have found that, contrary to conventional wisdom, demand can be weaker in a market with a surge price, causing workers to be idle or even leave the market zone (Chen et al. 2015). Also, a significant fraction of workers seemingly ignore market forecasts provided by the platform and do not move to zones requiring additional workers (Lee et al. 2015; Rosenblat and Stark 2015). The counterintuitive market observations are, however, consistent with what our model predicts

regarding the workers' strategic behavior and platform's strategic use of surge pricing. Consumer advocates have been concerned that surge pricing may be used to fleece consumers even in zones with sufficient supply (Dholakia 2015). We show that surge pricing may indeed be used in such zones, not to fleece consumers, but to improve the availability of workers across market zones. Nevertheless, this strategic use of surge pricing can hurt consumers, workers and even overall market welfare. Ironically, consumers in a market zone with excess supply may not be served even though there are enough idle workers willing to serve them.

We briefly discuss some limitations of our study. To keep the analysis straightforward and interesting, we imposed certain parameter restrictions. We expect qualitatively similar results if these restrictions are relaxed. If $n_H + n_L < a_H + a$ such that there are fewer workers overall than is needed to meet total demand in period 2, then the first-best revenue benchmark will involve a shortage of workers and a surge price in both market zones in period 2. Nevertheless, it is still the case that workers do not internalize the competitive externality that they impose on other workers, resulting in too few workers moving to the market zone with a demand surge. If $a_H > (3 + 2\sqrt{2})a$, then under asymmetric information, a surge price in the market zone that workers must leave (zone B) alone may not be sufficient to credibly communicate a greater need for workers to move. Instead, the platform must use a surge price in both market zones. Nevertheless, it is profitable for the platform to use the maximum surge price in the market zone that workers should leave.

To develop our insights in a straightforward manner, we adopted a particular demand formulation. The linear demand assumption is mainly useful to show the existence of a strategic surge pricing equilibrium under symmetric information. We expect our main insights to extend to other concave demand functions. We assumed that consumer valuations are high enough so that it is profitable to serve all consumers. Consumers with low valuations will be priced out of the market even if there is sufficient supply of workers and, hence, including them would not significantly affect our analysis of surge pricing. We assumed that demand elasticity is not affected by the demand surge in period 2. Consequently, consistent with conventional wisdom, surge pricing would not arise in our model absent supply constraints. Alternatively, if the demand surge increases the demand intercept, making the demand less elastic, then surge pricing will occur (in zone A in period 2) even if there is sufficient supply. However, this surge price maximizes market revenue in this zone and, hence, will attract more workers instead of causing them to leave. We assumed $\bar{p} = 1$ such that it is optimal to set the regular price absent supply constraints. The equilibrium outcome of our model is not affected if $\bar{p} < 1$. Further, workers will move out of a zone with a surge price under the

same conditions as in our current model. However, $p_{ij} > \bar{p}$ always in equilibrium, and the platform trivially always charges a surge price. If $\bar{p} > 1$, then we obtain qualitatively similar results, though the platform's profit is lower than if $\bar{p} = 1$.

For analytical ease, we assumed only two states for the initial number of workers in each zone, causing the initial supply to be negatively correlated across zones. Allowing for more supply states and independent initial supply will not affect our insights under symmetric information. Under asymmetric information, as before, the platform will not always have an incentive to truthfully share information since a larger fraction of workers will move under some supply states than for others. Consequently, a surge price will be necessary to credibly communicate a greater need for workers to move. Moreover, separation through a surge price in zone B will be more profitable for the same reasons as in our current model - either the separating type has more workers moving for a given increase in the proportion of workers that move (higher initial supply in zone B), or has a larger need for workers in zone A (lower initial supply in zone A). However, the analysis of a separating equilibrium with several supply states is more involved.

We analyzed the interactions across two market zones. We expect our key insights to extend to a setting with multiple market zones. Specifically, under asymmetric information, the platform will not have an incentive to mislead workers about which zone(s) to leave and to move to. Moreover, we expect that, all else equal, a surge price in the market zone(s) that workers should leave will be a more efficient means to credibly signal the extent of the need for workers to leave those zone(s). Nevertheless, we leave it for future work to analyze such situations formally. Lastly, we assumed the platform's commission rate to be exogenous. We note that our insights hold qualitatively for any positive commission rate less than 100%. In practice, the commission rate is set by the platform and is likely to be decided by various factors such as workers' reservation wage and platform competition. Future research could study how the commission rate is determined in equilibrium.

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Appendix

Proof for Lemma 1: Consider period 2. In zone A , since $\tilde{N}_{A2} = N_A < a_H$, the optimal price is

$$p_{A2} = \arg \max_{p \geq 1} (1 - \lambda) p \min \{N_A, a_H (2 - p)\} = 2 - \frac{N_A}{a_H}. \quad (6)$$

In zone B , since $\tilde{N}_{B2} = N_B > a$, the optimal price is

$$p_{B2} = \arg \max_{p \geq 1} (1 - \lambda) p \min \{N_B, a (2 - p)\} = 1. \quad (7)$$

Similarly, in period 1, since $N_{i1} = N_i > a$, the optimal price in zone i is $p_{i1} = 1$.

Proof for Lemma 2: Follows from the discussion in the text.

Proof for Proposition 1: Given $p_{i1} = 1$, we derive the necessary and sufficient conditions for a subgame equilibrium in which $p_{i2} = 1$. We require $\tilde{N}_{i2} \geq a_{i2}$. Otherwise, it will be profitable for the platform to deviate to $p_{i2} = 2 - \frac{\tilde{N}_{i2}}{a_{i2}} > 1$. We derive the workers' best response in period 1, and verify when $\tilde{N}_{ij} \geq a_{ij}$.

For $N_{A2} \geq a_H$, we require $\mu_B > 0$. We first show $\tilde{N}_{Bj} \geq a$ if $p_{ij} = 1$. Suppose towards a contradiction $\tilde{N}_{Bj} < a$. Then, a worker in zone B will receive service requests with probability 1. Hence, $r_{Bj} = \lambda p_{Bj} = \lambda$. Also, $r_{A2} \leq \lambda p_{A2} = \lambda$. Therefore, it will not be profitable for a worker to move from zone B to zone A and $\tilde{N}_{Bj} > a$, which is the the desired contradiction. It follows that $\mu_B < 1$. We next show $\mu_A = 0$ if $\mu_B > 0$. In period 1, a worker will move from zone i to j iff $r_{i1} + r_{i2} \leq r_{j2} - c$. Therefore, if it is attractive for workers in zone B to move to zone A ($\mu_B > 0$) then it is not attractive for workers in zone A to move to zone B . Hence, $\mu_A = 0$.

Thus, $\tilde{N}_{A2} = N_A + N_B \mu_B$ and $\tilde{N}_{B1} = \tilde{N}_{B2} = N_B (1 - \mu_B)$. If $\tilde{N}_{i2} \geq a_{i2}$ such that there is no surge pricing in equilibrium, then $r_{A2} = \lambda p_{A2} \frac{a_H}{N_{A2}}$ and $r_{Bj} = \lambda p_{Bj} \frac{a}{N_{Bj}}$. We require $r_{B1} + r_{B2} = r_{A2}$ so that workers in zone B are indifferent between serving zone B and moving to zone A . Otherwise,

some workers will deviate from their current strategy. Therefore,

$$\lambda \frac{2a}{N_B(1-\mu_B)} = \lambda \frac{a_H}{N_A + N_B \mu_B} - c, \quad (8)$$

where $N_A + N_B \mu_B \geq a_H$. The above condition holds iff $\lambda \left(\frac{2a}{n_H + n_L - a_H} \right) \leq \lambda - c$. Conversely, if $\lambda \left(\frac{2a}{n_H + n_L - a_H} \right) > \lambda - c$, then $\tilde{N}_{A2} < a_H$ and the platform will use surge pricing in period 2 in zone A if $p_{i1} = 1$.

Proof for Lemma 3: Suppose, towards a contradiction, that $\mu_A > 0$ and $\mu_B = 0$. Then, in period 2, $\tilde{N}_{A2} < a_H$ and $\tilde{N}_{B2} > a$. Hence, the platform's optimal pricing strategy is to set $p_{A2} > 1$ and all workers in zone A obtain service requests, and set $p_{B2} = 1$ and not all workers in zone B obtain service requests. Therefore, $r_{A2} > \lambda$ and $r_{B2} < \lambda$. However, this implies that workers in zone A must strictly prefer to stay in zone A , which is a contradiction. Next, suppose towards a contradiction that $\tilde{N}_{Bj} \leq a$. Then, in period 2, $\tilde{N}_{A2} > a_H$ since $n_H + n_L > a_H + a$. Therefore, $p_{A2} = 1$ and $r_{A2} < \lambda$, and $p_{B2} > 1$ and $r_{B2} > \lambda$ in equilibrium. However, this implies that workers in zone B must strictly prefer to stay in zone B and $\tilde{N}_{Bj} > a$, which is a contradiction.

Proof for Proposition 2: We first show that there is a unique μ_B given p_{i1} . The expected revenue of workers serving zone B ($r_{B1} + r_{B2}$) is strictly increasing in μ_B , whereas the expected revenue of workers serving zone A (r_{A2}) is strictly decreasing in μ_B . This is because having fewer workers in zone B reduces worker competition in zone B and increases worker competition in zone A . Therefore, the LHS of equilibrium worker movement condition (3) is strictly increasing in μ_B and the RHS is strictly decreasing in μ_B . Also, condition (3) must hold as an inequality if $\mu_B = \mu' > 0$ such that $\tilde{N}_{Bj} = a$. As shown in the proof for Lemma 3, if $\tilde{N}_{Bj} = a$, then workers will strictly prefer to stay in zone B since their expected revenue in zone B will be strictly higher. Since the LHS of condition (3) is strictly increasing in μ_B and the RHS is strictly decreasing in μ_B , it follows that either condition (3) must hold as an equality for some $\mu_B < \mu'$ or as an inequality for $\mu_B = 0$. Therefore, an equilibrium always exists and is unique. Next, observe that r_{B1} , and hence the LHS of condition (3), is decreasing in p_{B1} . Also, as noted above, the LHS of condition (3) is strictly increasing in μ_B and the RHS is strictly decreasing in μ_B . It follows that μ_B is strictly increasing in p_{B1} if condition (3) holds as an equality, and μ_B is not affected by p_{B1} otherwise.

Proof for Proposition 3: It will not be profitable for the platform to induce more than $\bar{\mu}_B$ workers to move. Therefore, for the relevant range of surge prices $p_{B2} > 1$, we have $\tilde{N}_{A2} = N_A + \mu_B N_B < a_H$. Therefore, $p_{A2} = 2 - \frac{\tilde{N}_{A2}}{a_H}$ and $p_{B2} = 1$. The equilibrium worker movement condition (3) is then

$$\lambda \left[\frac{a + a(2 - p_{B1})p_{B1}}{N_B(1 - \mu_B)} \right] \geq \lambda \left[2 - \frac{N_A + \mu_B N_B}{a_H} \right] - c, \quad (9)$$

The platform's subgame equilibrium profit is given by

$$\Pi_P = (1 - \lambda) \left[(2 - p_{B1}) p_{B1} + \left(2 - \frac{N_A + \mu_B N_B}{a_H} \right) (N_A + \mu_B N_B) + 2a \right], \quad (10)$$

The platform can benefit from a surge price only if the surge price causes workers to move, i.e., $\mu_B > 0$. We first obtain the optimal surge price conditional on inducing workers to move. Then, we verify whether this surge price is profitable. If workers move ($\mu_B > 0$), then condition (9) holds as an equality. Therefore, Π_P can be rewritten as

$$\Pi_P = (1 - \lambda) \left[\left(2 - \frac{N_A + \mu_B N_B}{a_H} \right) (N_A + N_B) - \frac{c}{\lambda} N_B (1 - \mu_B) + 2a \right], \quad (11)$$

which is strictly increasing in μ_B iff $c > \frac{\lambda(N_A + N_B)}{a_H}$. Therefore, conditional on making workers move, p_{B1} will be set to maximize μ_B if $c > \frac{\lambda(N_A + N_B)}{a_H}$ and to minimize μ_B otherwise. Hence, it can be profitable for the platform to set $p_{B1} > 1$ only if $c > \frac{\lambda(N_A + N_B)}{a_H}$. We focus on this case in the remainder of the analysis.

We first show that $\mu_B < \bar{\mu}$ for $p_{B1} = 2$. Hence, $p_{B1} = 2$ is the candidate surge price. Substituting $p_{B1} = 2$ and $p_{B2} = 1$ in condition (9) and rearranging yields

$$\frac{N_A + N_B \mu_B}{a_H} + \frac{a}{N_B(1 - \mu_B)} + \frac{c}{\lambda} - 2 \geq 0 \quad (12)$$

The LHS above is strictly increasing in μ_B . Also, the LHS is positive for $\mu_B = \bar{\mu}$ since $c > \frac{\lambda(N_A + N_B)}{a_H} > 1$. Therefore, $\mu_B < \bar{\mu}$ for $p_{B1} = 2$. Note also μ_B is increasing in p_{B1} only if $p_{B1} \leq 2$; if $p_{B1} > 2$, then $r_{B1} = 0$ and μ_B is not affected by p_{B1} .

If $\mu_B > 0$ for $p_{B1} = 1$, then it will be profitable for the platform to set a surge price $p_{B1} = 2$. We have that $\mu_B > 0$ for $p_{B1} = 1$ iff condition (9) does not hold for $p_{B1} = 1$ and $\mu_B = 0$. Therefore,

$$\lambda \frac{2a}{N_B} < \lambda \left(2 - \frac{N_A}{a_H} \right) - c. \quad (13)$$

If the platform is type H , then $(N_A, N_B) = (n_L, n_H)$ and, from (13), we require $\frac{c}{\lambda} < 2 - \frac{2a}{n_H} - \frac{n_L}{a_H}$. We also require $\frac{c}{\lambda} > \frac{n_H + n_L}{a_H}$ for a surge price to be optimal. Combining these two conditions we have $\lambda \frac{n_H + n_L}{a_H} < c \leq \lambda \left(2 - \frac{2a}{n_H} - \frac{n_L}{a_H} \right)$, which can hold iff $n_L < a_H - \frac{aa_H}{n_H} - \frac{n_H}{2}$, and $n_H > a + \sqrt{a^2 + 2aa_H}$ (since $n_H + n_L > a_H + a$) and $a_H > 4a$ (since $a_H > n_H > n_L > a$). Next, if the platform is type L , then proceeding similarly, we require that $a_H > (5 + 2\sqrt{7})a$, which is not feasible since $a_H \leq (3 + 2\sqrt{2})a$.

Lastly, if $\mu_B = 0$ for $p_{B1} = 1$, then for a surge price to be profitable we require that $\mu_B > 0$ for $p_{B1} = 2$ and that the platform makes higher profit for $p_{B1} = 2$ than for $p_{B1} = 1$. The platform

profit if $p_{B1} = 1$ and $\mu_B = 0$ is

$$\Pi'_P = (1 - \lambda) \left[3a + \left(2 - \frac{N_A}{a_H} \right) N_A \right]. \quad (14)$$

If platform is type H, then substituting $(N_A, N_B) = (n_L, n_H)$ and $p_{B1} = 2$ in condition (9) yields

$$\frac{c}{\lambda} = 2 - \frac{n_L + n_H \mu_B}{a_H} - \frac{a}{n_H(1 - \mu_B)}. \quad (15)$$

From equations (11) and (14), we have $\Pi_P > \Pi'_P$ iff

$$(1 - \lambda) n_H \left[2 - \frac{2a}{n_H} - \frac{n_L}{a_H} - \left[\frac{c}{\lambda} (1 - \mu_B) + \left(\frac{n_H + n_L}{a_H} \right) \mu_B \right] \right] > 0 \quad (16)$$

Substituting for c in the above inequality from equation (15), we obtain that $\mu_B > \frac{\phi}{n_H}$, where $\phi = a_H - n_L - \sqrt{(a_H - n_L)^2 - aa_H}$. It follows from equation (15) that $\frac{c}{\lambda} < 2 - \frac{a}{n_H - \phi} - \frac{n_L + \phi}{a_H}$. Next, we note that condition (9) holds for $p_{B1} = 1$ and $\mu_B = 0$ iff $\frac{c}{\lambda} > 2 - \frac{2a}{n_H} - \frac{n_L}{a_H}$. We also require that $\frac{c}{\lambda} > \frac{n_H + n_L}{a_H}$. Therefore, for condition (16) to hold, we require that $2 - \frac{2a}{n_H} - \frac{n_L}{a_H} > \frac{n_H + n_L}{a_H}$, which holds iff $n_L < a_H - \frac{aa_H}{n_H} - \frac{1}{2}n_H$, $n_H > a + \sqrt{a^2 + 2aa_H}$, and $a_H > 4a$. If the platform is type L , proceeding similarly yields the condition that $a_H > (5 + 2\sqrt{7})a$, which is not feasible since $a_H \leq (3 + 2\sqrt{2})a$.

Proof for Lemma 4: To establish uniqueness, we show that $\hat{r}_{B1}(\theta) + \hat{r}_{B2}(\theta) - \hat{r}_{A2}(\theta)$ is strictly increasing in μ_B and is strictly positive if μ_B is sufficiently large. Since the equilibrium worker movement condition must hold as an equality for $\mu_B > 0$, it follows that there is a unique $\mu_B \geq 0$ that satisfies this condition. The expected revenue of workers serving zone B given that platform is type t ($r_{B1}(\mu_B)|_t + r_{B2}(\mu_B)|_t$) is strictly increasing in μ_B , because having fewer workers in zone B reduces worker competition in zone B . We have

$$\hat{r}_{B1}(\theta) + \hat{r}_{B2}(\theta) = \theta (r_{B1}(\mu_B)|_H + r_{B2}(\mu_B)|_H) + (1 - \theta) (r_{B1}(\mu_B)|_L + r_{B2}(\mu_B)|_L). \quad (17)$$

Thus, $\hat{r}_{B1}(\theta) + \hat{r}_{B2}(\theta)$ is strictly increasing in μ_B . Similarly, the expected revenue of workers serving zone A in period 2 given that platform is type t ($r_{A2}(\mu_B)|_t$) is strictly decreasing in μ_B , because having more workers in zone A increases worker competition in zone A . Hence, $\hat{r}_{A2}(\theta)$ is strictly decreasing in μ_B . Thus, $\hat{r}_{B1}(\theta) + \hat{r}_{B2}(\theta) - \hat{r}_{A2}(\theta)$ is strictly increasing in μ_B . Next, note that if μ_B is sufficiently large such that $n_H(1 - \mu_B) < a$, then $\tilde{N}_{Bj} < a$ and $\tilde{N}_{A2} > a_H$ and there is a shortage of workers serving zone B and an excess of workers serving zone A in period 2. Therefore, $r_{B2}|_t > \lambda$ and $r_{A2}|_t < 1$. Hence, $\hat{r}_{B1}(\theta) + \hat{r}_{B2}(\theta) - \hat{r}_{A2}(\theta) > 0$ for $\mu_B > 0$ sufficiently large.

We next show that $\hat{r}_{B1}(\theta) + \hat{r}_{B2}(\theta) - \hat{r}_{A2}(\theta)$ is also strictly decreasing in θ . Since $\hat{r}_{B1}(\theta) + \hat{r}_{B2}(\theta) - \hat{r}_{A2}(\theta)$ is strictly increasing in μ_B and condition (5) must hold as an equality for $\mu_B(\theta) >$

0, it follows that $\mu_B(\theta)$ must be strictly increasing in θ . Note that $\tilde{N}_{Bj}(\mu_B)|_H > \tilde{N}_{Bj}(\mu_B)|_L$ since $n_H > n_L$. Therefore, there is more worker competition in zone B in the case of a type H platform, and we have: (i) $r_{B1}(\mu_B)|_H \leq r_{B1}(\mu_B)|_L$, where the inequality is strict iff $\tilde{N}_{B1}(\mu_B)|_H > a(2 - p_{B1})$, and (ii) $r_{B2}(\mu_B)|_H < r_{B2}(\mu_B)|_L$. Hence, from equation (17), $\hat{r}_{B1}(\theta) + \hat{r}_{B2}(\theta)$ is strictly decreasing in θ . Similarly, there is less worker competition in zone A in the case of a type H platform ($\tilde{N}_{A2}|_H < \tilde{N}_{A2}|_L$), and we have $r_{A2}(\mu_B)|_H > r_{A2}(\mu_B)|_L$. Therefore, $\hat{r}_{A2}(\theta)$ is strictly increasing in θ . Thus, $\hat{r}_{B1}(\theta) + \hat{r}_{B2}(\theta) - \hat{r}_{A2}(\theta)$ is strictly decreasing in θ .

Proof for Proposition 4: For the platform not to share market information truthfully, we require that condition (C1) in Proposition 1 does not hold. Hence,

$$\frac{c}{\lambda} > 1 - \frac{2a}{n_H + n_L - a_H}. \quad (18)$$

We also require that $\mu_B(\theta = 1) > 0$ such that $\mu_B(\theta = 1) > \mu_B(\theta = 0)$ and more workers move if they believe that the platform is type H (than type L). Therefore, the equilibrium worker movement condition (5) must not hold for $\mu_B = 0$ given $\theta = 1$, which yields

$$\begin{aligned} r_{B1}(\mu_B = 0)|_H + r_{B2}(\mu_B = 0)|_H &< r_{A2}(\mu_B = 0)|_H - c, \\ \implies \lambda \frac{2a}{n_H} &< \lambda \left[2 - \frac{n_L}{a_H} \right] - c, \\ \implies \frac{c}{\lambda} &< 2 - \frac{n_L}{a_H} - \frac{2a}{n_H}. \end{aligned} \quad (19)$$

Lastly, a sufficient condition for the type L platform to strictly benefit from misreporting its type is that there are enough workers to serve consumers in zone B even if it misreports its type: $\tilde{N}_{B1}|_L = n_L(1 - \mu_B(\theta = 1)) \geq a$, which yields $\mu_B(\theta = 1) \leq \mu_1 = 1 - \frac{a}{n_L}$. Therefore, the equilibrium worker movement condition (5) must hold as an inequality for $\mu_B = \mu_1$ given $\theta = 1$, which yields

$$\begin{aligned} r_{B1}(\mu_B = \mu_1)|_H + r_{B2}(\mu_B = \mu_1)|_H &> r_{A2}(\mu_B = \mu_1)|_H - c, \\ \implies \lambda \frac{2n_L}{n_H} &> \lambda \left[2 + \frac{n_H a}{n_L a_H} - \frac{n_H + n_L}{a_H} \right] - c, \\ \implies \frac{c}{\lambda} &> 2 + \frac{n_H a}{n_L a_H} - \frac{n_H + n_L}{a_H} - \frac{2n_L}{n_H}. \end{aligned} \quad (20)$$

We have two lower bounds and one upper bound for $\frac{c}{\lambda}$. Given the parameter restrictions in §3, we have $2 - \frac{n_L}{a_H} - \frac{2a}{n_H} > 2 + \frac{n_H a}{n_L a_H} - \frac{n_H + n_L}{a_H} - \frac{2n_L}{n_H}$ and $2 - \frac{n_L}{a_H} - \frac{2a}{n_H} > 1 - \frac{2a}{n_H + n_L - a_H}$. For $2 - \frac{n_L}{a_H} - \frac{2a}{n_H} > 0$, we require $a_H > \frac{3}{2}a$, $n_H > \max \left\{ \frac{a + a_H}{2}, \frac{a - a_H + \sqrt{a^2 + 6aa_H + a_H^2}}{2} \right\}$ and $n_L < \min \left\{ n_H, \frac{2a_H(n_H - a)}{n_H} \right\}$. We thus obtain condition (C3).

Proof for Proposition 5: Given that truthful communication is not feasible without an accompanying surge price, there are two cases depending on whether or not condition (C2) in Proposition

3 holds. For each case, we obtain the unique least-cost separating equilibrium by construction.

Consider a separating equilibrium in which $s|_t = t$, $\mathbf{p}_1|_L = (1, 1)$ and $\mathbf{p}_1|_H = (1, \hat{p})$, where $\hat{p} \in (1, 2]$. Workers update their belief to $\theta = 1$ if $\mathbf{p}_1 = \mathbf{p}_1|_H$ and $\theta = 0$ otherwise. Therefore, $\hat{r}_{ij} = r_{ij}|_t$ in equilibrium on a type t platform. Let $\mu_B^*|_L = \mu_B(\mathbf{p}_1|_L, \theta = 0)$ and $\mu_B^*|_H = \mu_B(\mathbf{p}_1|_H, \theta = 1)$ denote the equilibrium proportion of workers that move in a separating equilibrium if the platform is type L and type H , respectively. Let $\Pi_P^*|_L = \Pi_P(\mathbf{p}_1|_L, \theta = 0)|_L$ and $\Pi_P^*|_H = \Pi_P(\mathbf{p}_1|_H, \theta = 1)|_H$ denote the corresponding equilibrium platform profits for the type L and type H platform, respectively. We say $\mathbf{p}'_1 < \mathbf{p}_1|_H$ if $\mathbf{p}'_1 = (1, p')$ and $p' < \hat{p}$. Let $\Pi'_P|_L = \Pi_P(\mathbf{p}_1|_H, \theta = 1)|_L$ denote the type L platform profit if it mimics the type H platform. Let $\Pi'_P|_H = \Pi_P(\mathbf{p}'_1, \theta = 0)|_H$ denote the type H platform profit if it deviates to $\mathbf{p}'_1 < \mathbf{p}_1|_H$. We use $\mu_B(\mathbf{p}_1, \theta)$ to make the dependence of $\mu_B(\theta)$ on \mathbf{p}_1 explicit.

We first establish that the type L platform will not mimic the type H platform if \hat{p} is sufficiently high. For clarity, we breakdown the analysis into several lemmas. Lemma 5 shows that $\mu_B(\mathbf{p}_1, \theta)$ is increasing in \mathbf{p}_1 . Lemma 6 shows that there is always enough workers to serve zone B in equilibrium. Lemma 7 then shows the desired result.

Lemma 5. $\mu_B(\mathbf{p}_1, \theta)$ is unaffected by p_{A1} , and is strictly increasing in p_{B1} if $\mu_B(\mathbf{p}_1, \theta) > 0$.

Proof. We characterize $\mu_B(\mathbf{p}_1, \theta)$ by considering how \mathbf{p}_1 affects the terms in equilibrium condition (5) for a given value of μ_B . Since p_{A1} does not affect any of the terms, $\mu_B(\mathbf{p}_1, \theta)$ is not affected by p_{A1} . Only $\hat{r}_{B1}(\theta)$ is strictly decreasing in p_{B1} , while the other terms are unaffected. Specifically, for a given μ_B , $r_{B1}(\mu_B)|_t$ is strictly decreasing in p_{B1} . Since equilibrium condition (5) holds as an equality if $\mu_B(\mathbf{p}_1, \theta) > 0$ and $\hat{r}_{B1}(\theta) + \hat{r}_{B2}(\theta) - \hat{r}_{A2}(\theta)$ is strictly increasing in μ_B (see proof of Lemma 4), it follows that $\mu_B(\mathbf{p}_1, \theta)$ is strictly increasing in p_{B1} if $\mu_B(\mathbf{p}_1, \theta) > 0$. \square

Lemma 6. $\mu_B^*|_L < \bar{\mu}|_L$, $\mu_B^*|_H \in \left(\mu_B^*|_L, 1 - \frac{a}{n_H}\right)$.

Proof. If $\mu_B^*|_t \geq 1 - \frac{a}{n_t}$, then $\tilde{N}_{A2} > a_H$ and $\tilde{N}_{Bj} < a$ in equilibrium. Therefore, $p_{A2} = 1$ and $p_{B2} > 1$. But then $\hat{r}_{B2} = r_{B2}|_t > \lambda$ and $\hat{r}_{A2} = r_{A2}|_t < \lambda$, and the equilibrium worker movement condition (5) cannot hold as an equality. Therefore, $\mu_B^*|_t < 1 - \frac{a}{n_t}$. Also, for the type L platform to have an incentive to misreport its type in the absence of surge pricing, we require: $\mu_B^*|_L < \bar{\mu}|_L$ such that not enough workers move if the platform is type L ; and, $\mu_B(\mathbf{p}_1|_L, \theta = 1) > \mu_B^*|_L \geq 0$ such that more workers from zone B move if they believe the platform is type H . Also, $\mu_B^*|_H > \mu_B(\mathbf{p}_1|_L, \theta = 1)$ since $\mu_B(\mathbf{p}_1|_L, \theta = 1) > 0$ and $\hat{p} > 1$ (Lemma 5). Hence $\mu_B^*|_H > \mu_B^*|_L$. \square

Lemma 7. $\Pi'_P|_L < \Pi_P^*|_L$ if \hat{p} is sufficiently close to 2.

Proof. The type L platform's equilibrium profit is

$$\Pi_P^*|_L = (1 - \lambda) \left[3a + \left(2 - \frac{n_H + n_L \mu_B^*|_L}{a_H} \right) (n_H + n_L \mu_B^*|_L) \right]. \quad (21)$$

Specifically, the platform realizes profit of $\pi_{ij} = (1 - \lambda) a$ except in zone A in period 2, and $\pi_{A2} = (1 - \lambda) \tilde{N}_{i2} \left(2 - \frac{\tilde{N}_{i2}}{a_{i2}} \right)$ since $\mu_B^*|_L < \bar{\mu}|_L$ (Lemma 6). Since the RHS above is strictly increasing in $\mu_B^*|_L$, we obtain the following lower bound for $\Pi_P^*|_L$ by setting $\mu_B^*|_L = 0$

$$\Pi_P^*|_L \geq (1 - \lambda) \left[3a + \left(2 - \frac{n_H}{a_H} \right) n_H \right] \quad (22)$$

Suppose $\mathbf{p}_1|_H = \bar{\mathbf{p}}_1 = (1, 2)$. Consider the type L platform's profit from deviating to $\bar{\mathbf{p}}_1$. Its profit in zone B in period 1 is zero since no consumers will request service. To obtain an upper bound on its profit in period 2, note that at best all consumers are served in both zones in period 2, resulting in profit of $\pi_{A2} = (1 - \lambda) a_H$ and $\pi_{B2} = (1 - \lambda) a$. Also, its profit in period 1 in zone A is $\pi_{A1} = (1 - \lambda) a$. Therefore, its deviation profit

$$\Pi'_P|_L \leq (1 - \lambda) [2a + a_H]. \quad (23)$$

From (22) and (23), the profit gain from the deviation is bounded by

$$\Pi_P^*|_L - \Pi'_P|_L \geq (1 - \lambda) \left[a_H - a - \left(2 - \frac{n_H}{a_H} \right) n_H \right]. \quad (24)$$

The RHS above is strictly positive for $a_H \leq (3 + 2\sqrt{2}) a$. By continuity, $\Pi_P^*|_L > \Pi'_P|_L$ for \hat{p} sufficiently close to 2. \square

We now show by construction that separation is feasible if condition (C2) in Proposition 3 holds and obtain the unique least-cost separating equilibrium.

Lemma 8. *If condition (C2) in Proposition 3 holds, then the unique least-cost separating equilibrium is one in which $\mathbf{p}_1|_H = \bar{\mathbf{p}}_1 = (1, 2)$.*

Proof. As shown in Proposition 3, $\mathbf{p}_1|_H = \bar{\mathbf{p}}_1$ maximizes the type H platform's profit conditional on separation if condition (C2) holds. Therefore, the separating equilibrium in which $\mathbf{p}_1|_H = \bar{\mathbf{p}}_1$ is the unique least-cost separating equilibrium if separation is feasible. To see that separation is feasible, note that $\Pi_P^*|_L > \Pi'_P|_L$ (Lemma 7). Further, the type H platform profit is strictly increasing in the surge price conditional on separation. Therefore, for any price $\mathbf{p}'_1 < \bar{\mathbf{p}}_1$, we have $\Pi_P^*|_H > \Pi_P(\mathbf{p}'_1, \theta = 1)|_H$. Also, $\Pi_P(\mathbf{p}_1, \theta = 1)|_H \geq \Pi'_P|_H = \Pi_P(\mathbf{p}_1, \theta = 0)|_H$ because (weakly) more workers move to zone A if they believe that the platform is type H (Lemma 4), and there is a shortage of workers in zone A (Lemma 6). Hence, $\Pi_P^*|_H > \Pi'_P|_H$ for $\mathbf{p}'_1 < \bar{\mathbf{p}}_1$. \square

We next obtain the unique least-cost separating equilibrium if condition (C2) in Proposition 3 does not hold. The type L platform will not mimic if \hat{p} is sufficiently high (Lemma 7). Also, the type L platform will mimic if \hat{p} is sufficiently close to 1; since, the type L platform will misreport its type without an accompanying surge price. By continuity, there exists $\hat{p} \in (1, 2)$ for which $\Pi'_P|_L = \Pi^*_P|_L$. Let $\tilde{p} = \min \left\{ \hat{p} : \Pi'_P|_L = \Pi^*_P|_L \right\}$ be the minimum such \hat{p} , and $\tilde{\mathbf{p}}_1 = (1, \tilde{p})$. We first establish that separation is feasible for $\mathbf{p}_1|_H = \tilde{\mathbf{p}}_1$ by showing that the type H platform will not deviate to a lower price. Lemma 9 establishes bounds for the proportion of workers that move for off-equilibrium prices. Lemma 10 then shows the desired result.

Lemma 9. For $\mathbf{p}'_1 \leq \tilde{\mathbf{p}}_1$, $\mu_B(\mathbf{p}'_1, \theta = 0) < \bar{\mu}|_L$ and $\mu_B(\mathbf{p}'_1, \theta = 1) < \bar{\mu}|_H$.

Proof. The type L platform does not have an incentive to use a surge price under full information. Therefore, $\Pi^*_P|_L > \Pi_P(\mathbf{p}'_1, \theta = 0)|_L$ for any $\mathbf{p}'_1 > \mathbf{p}_1|_L$ (i.e., $\mathbf{p}'_1 = (1, p')$ such that $p' > 1$). Since $\Pi'_P|_L = \Pi^*_P|_L$ for $\mathbf{p}_1|_H = \tilde{\mathbf{p}}_1$, we have $\Pi'_P|_L \geq \Pi_P(\mathbf{p}'_1, \theta = 0)|_L$ for any $\mathbf{p}'_1 \leq \tilde{\mathbf{p}}_1$, where equality holds iff $\mathbf{p}'_1 = \mathbf{p}_1|_L$. But this is possible iff there is a shortage of workers in zone A for $\mathbf{p}'_1 \leq \tilde{\mathbf{p}}_1$ and $\theta = 0$, i.e., $\mu_B(\mathbf{p}'_1, \theta = 0) < \bar{\mu}|_L$. Then, equilibrium worker movement condition (5) must not hold for $\theta = 0$, $\mathbf{p}_1 = \mathbf{p}'_1$ and $\mu_B = \bar{\mu}|_L$, which yields

$$\lambda \left[\frac{a + ap'(2 - p')}{n_H + n_L - a_H} \right] > \lambda - c. \quad (25)$$

Note that $\tilde{N}_{ij}(\mu_B = \bar{\mu}|_H)|_H = a_H = \tilde{N}_{ij}(\mu_B = \bar{\mu}|_L)|_L$. Therefore, given \mathbf{p}_1 , the terms in the equilibrium condition (5) are identical for $\theta = 0$ and $\mu_B = \bar{\mu}|_L$ and $\theta = 1$ and $\mu_B = \bar{\mu}|_H$. Consequently, equilibrium condition (5) will not also hold for $\theta = 1$, $\mathbf{p}_1 = \mathbf{p}'_1$ and $\mu_B = \bar{\mu}|_H$, and we have $\mu_B(\mathbf{p}'_1, \theta = 1) < \bar{\mu}|_H$. \square

Lemma 10. For $\mathbf{p}_1|_H = \tilde{\mathbf{p}}_1$ and $\mathbf{p}'_1 < \tilde{\mathbf{p}}_1$, $\Pi^*_P|_H - \Pi'_P|_H > 0$.

Proof. We show that $\Pi^*_P|_H - \Pi'_P|_H > \Pi'_P|_L - \Pi_P(\mathbf{p}'_1, \theta = 0)|_L$. Since $\Pi'_P|_L = \Pi^*_P|_L > \Pi_P(\mathbf{p}'_1, \theta = 0)|_L$ (see proof of Lemma 9), we obtain the desired result. Consider a change in period 1 price from \mathbf{p}'_1 to $\tilde{\mathbf{p}}_1$. For such a change in price, let $\Delta\mu = \mu^*_B|_H - \mu_B(\mathbf{p}'_1, \theta = 0) > 0$ denote the increase in proportion of workers in zone B that move. Let $\Delta\pi_{ij}|_t = \pi_{ij}(\tilde{\mathbf{p}}_1, \theta = 1)|_t - \pi_{ij}(\mathbf{p}'_1, \theta = 0)|_t$ denote the corresponding difference in profit in zone i in period j for a type t platform. Note that $\Pi^*_P|_H - \Pi'_P|_H = \sum_{ij} \Delta\pi_{ij}|_H$ and $\Pi'_P|_L - \Pi_P(\mathbf{p}'_1, \theta = 0)|_L = \sum_{ij} \Delta\pi_{ij}|_L$. We need to show $\sum_{ij} \Delta\pi_{ij}|_H > \sum_{ij} \Delta\pi_{ij}|_L$. We show that $\Delta\pi_{ij}|_H \geq \Delta\pi_{ij}|_L$ for all i, j and the inequality is strict for zone A in period 2.

Consider zone A . In period 1, all consumers are served at both prices for either platform type. Therefore, $\Delta\pi_{A1}|_H = \Delta\pi_{A1}|_L = 0$. In period 2, if the platform sets period 1 price \mathbf{p}'_1 , then $\tilde{N}_{A2} < a_H$ since $\mu_B(\mathbf{p}'_1, \theta = 0) < \bar{\mu}|_L$ (Lemma 9). The type H platform has fewer available workers in zone A than the type L platform. Specifically, $\tilde{N}_{A2}|_t = n_H + n_L - n_t \left(1 - \mu_B(\mathbf{p}'_1, \theta = 0)\right)$ and $\tilde{N}_{A2}|_H < \tilde{N}_{A2}|_L$. Now, platform profit $\pi_{A2} = (1 - \lambda)\tilde{N}_{A2} \left(2 - \frac{\tilde{N}_{A2}}{a_H}\right)$ is strictly increasing and strictly concave in \tilde{N}_{A2} . Therefore, since $\tilde{N}_{A2}|_H < \tilde{N}_{A2}|_L$, the incremental profit from having a given number of additional workers in zone A is higher for a type H platform. Moreover, if the platform shifts to period 1 price $\tilde{\mathbf{p}}_1$, then the number of additional workers that move to zone A is $n_t\Delta\mu$, which is also higher for the type H platform. Therefore, $\Delta\pi_{A2}|_H > \Delta\pi_{A2}|_L$.

Next consider zone B . In period 1, all consumers requesting service will be served if the price is \mathbf{p}'_1 for either platform type since $\mu_B(\mathbf{p}'_1, \theta = 0) < \bar{\mu}|_L$, and the platform profit is the same for both types ($= (1 - \lambda)ap' \left(2 - p'\right)$). At the price $\tilde{\mathbf{p}}_1$, all consumers requesting service will be served if the platform is type H since $\mu_B^*|_H < \bar{\mu}|_H$, and either the same number or fewer consumers will be served if the platform is type L depending on whether $\tilde{N}_{B1}|_L \geq a \left(2 - p'\right)$; the platform profit is the same for both types if all consumers are served, and is higher for the type H platform otherwise. Therefore, $\Delta\pi_{B1}|_H \geq \Delta\pi_{B1}|_L$. In period 2, we have $\pi_{B2} = (1 - \lambda)a$ if $\tilde{N}_{B2} \geq a$ and $\pi_{B2} = (1 - \lambda)\tilde{N}_{B2} \left(2 - \frac{\tilde{N}_{B2}}{a}\right)$ otherwise. For either period 1 price, $\tilde{N}_{B2}|_H \geq a$ since $\mu_B(\mathbf{p}'_1, \theta = 0) < \mu_B^*|_H < \bar{\mu}|_H$. Therefore, $\Delta\pi_{B2}|_H = 0$. For a type L platform, $\tilde{N}_{B2}|_L > a$ if the period 1 price is \mathbf{p}'_1 since $\mu_B(\mathbf{p}'_1, \theta = 0) < \bar{\mu}|_L$. If $\tilde{N}_{B2}|_L \geq a$ for period 1 price $\tilde{\mathbf{p}}_1$, then $\Delta\pi_{B2}|_L = 0$. Otherwise, $\Delta\pi_{B2}|_L < 0$ since $\pi_{B2}|_L$ is strictly increasing in \tilde{N}_{B2} if $\tilde{N}_{B2}|_L < a$. Therefore, $\Delta\pi_{B2}|_H \geq \Delta\pi_{B2}|_L$. \square

We now establish that the candidate separating equilibrium in which $\mathbf{p}_1|_H = \tilde{\mathbf{p}}_1$ is the unique least-cost separating equilibrium if condition (C2) in Proposition 3 does not hold. If condition (C2) does not hold, then the type H platform profit conditional on separation is decreasing in the surge price. Also separation is not feasible for $\mathbf{p}_1|_H < \tilde{\mathbf{p}}_1$ since the type L platform will mimic. Hence, there is no separating equilibrium in which $p_{A1}|_H = 1$ that yields higher profit for the type H platform than the candidate equilibrium. Suppose, towards a contradiction, there is an alternate least-cost separating equilibrium in which $p_{A1}|_H > 1$. Let $\mathbf{p}_1|_H = \mathbf{p}_1^A = (\hat{p}_A, \hat{p}_B)$ in this separating equilibrium, where $\hat{p}_A > 1$ and $\hat{p}_B \geq 1$. Since condition (C2) does not hold, $\Pi_P(\mathbf{p}_1^A, \theta = 1)|_H$ is strictly decreasing in \hat{p}_A and \hat{p}_B . Therefore, for the alternate separating equilibrium to be least-cost, we require that $\hat{p}_B < \tilde{p}$; otherwise, $\Pi_P(\mathbf{p}_1^A, \theta = 1)|_H < \Pi_P(\tilde{\mathbf{p}}_1, \theta = 1)|_H$, which is a contradiction. Let $\mathbf{p}'_1 = (1, \hat{p}_B)$. Therefore, $\mathbf{p}'_1 < \tilde{\mathbf{p}}_1$. Also, $\mu_B(\mathbf{p}_1^A, \theta) = \mu_B(\mathbf{p}'_1, \theta)$ from Lemma 5. Note that,

platform profit in zone A in period 1 is determined only by p_{A1} ($\pi_{A1} = (1 - \lambda) a p_{A1} (2 - p_{A1})$) and platform profit in the remaining zones and periods is determined only by p_{B1} and μ_B . Therefore, $\Pi_P(\mathbf{p}_1^A, \theta)|_t$ and $\Pi_P(\mathbf{p}'_1, \theta)|_t$ differ only in the profit in zone A in period 1, and this difference is independent of the platform type t .

$$\Pi_P(\mathbf{p}_1^A, \theta = 1)|_H - \Pi_P(\mathbf{p}'_1, \theta = 1)|_H = \Pi_P(\mathbf{p}_1^A, \theta = 1)|_L - \Pi_P(\mathbf{p}'_1, \theta = 1)|_L. \quad (26)$$

For separation to be feasible in the alternate equilibrium, we require $\Pi_P(\mathbf{p}_1^A, \theta = 1)|_L \leq \Pi_P^*|_L$. By construction, $\Pi_P(\tilde{\mathbf{p}}_1, \theta = 1)|_L = \Pi_P^*|_L$. Hence, from equation (26)

$$\Pi_P(\mathbf{p}_1^A, \theta = 1)|_H - \Pi_P(\mathbf{p}'_1, \theta = 1)|_H \leq \Pi_P(\tilde{\mathbf{p}}_1, \theta = 1)|_L - \Pi_P(\mathbf{p}'_1, \theta = 1)|_L. \quad (27)$$

We will show that

$$\Pi_P(\tilde{\mathbf{p}}_1, \theta = 1)|_H - \Pi_P(\mathbf{p}'_1, \theta = 1)|_H > \Pi_P(\tilde{\mathbf{p}}_1, \theta = 1)|_L - \Pi_P(\mathbf{p}'_1, \theta = 1)|_L, \quad (28)$$

and, therefore, $\Pi_P(\tilde{\mathbf{p}}_1, \theta = 1)|_H > \Pi_P(\mathbf{p}_1^A, \theta = 1)|_H$ from condition (27), which is the desired contradiction. Rearranging the above inequality, we need to show

$$\Pi_P(\tilde{\mathbf{p}}_1, \theta = 1)|_H - \Pi_P(\tilde{\mathbf{p}}_1, \theta = 1)|_L > \Pi_P(\mathbf{p}'_1, \theta = 1)|_H - \Pi_P(\mathbf{p}'_1, \theta = 1)|_L.$$

The following lemma shows that $\Pi_P(\mathbf{p}'_1, \theta = 1)|_H - \Pi_P(\mathbf{p}'_1, \theta = 1)|_L$ is strictly increasing in \hat{p}_B .

Lemma 11. *For $\mathbf{p}'_1 = (1, \hat{p}_B) \leq \tilde{\mathbf{p}}_1$, $\Pi_P(\mathbf{p}'_1, \theta = 1)|_H - \Pi_P(\mathbf{p}'_1, \theta = 1)|_L$ is strictly increasing in \hat{p}_B .*

Proof. Let $\mu = \mu_B(\mathbf{p}'_1, \theta = 1)$. From Lemma 9, $\mu < \bar{\mu}|_H$. Hence, $\tilde{N}_{A2}|_H < a_H$, $\tilde{N}_{Bj}|_H > a$ and

$$\Pi_P(\mathbf{p}'_1, \theta = 1)|_H = (1 - \lambda) \left[2a + a(2 - \hat{p}_B)\hat{p}_B + \left(2 - \frac{n_L + n_H\mu}{a_H} \right) (n_L + n_H\mu) \right], \quad (29)$$

where the terms within the square brackets correspond to $\pi_{A1}|_H + \pi_{B2}|_H$, $\pi_{B1}|_H$ and $\pi_{A2}|_H$, respectively. There are two cases depending on whether $\tilde{N}_{B1}|_L = n_L(1 - \mu) < a(2 - \hat{p}_B)$. If $\tilde{N}_{B1}|_L < a(2 - \hat{p}_B)$, then $\tilde{N}_{A2}|_L > a_H$ and

$$\Pi_P(\mathbf{p}'_1, \theta = 1)|_L = (1 - \lambda) \left[a + a n_L(1 - \mu)\hat{p}_B + \left(2 - \frac{n_L(1 - \mu)}{a} \right) n_L(1 - \mu) + a_H \right], \quad (30)$$

where the terms within the square brackets correspond to $\pi_{A1}|_L$, $\pi_{B1}|_L$, $\pi_{B2}|_L$ and $\pi_{A2}|_L$, respectively. We have

$$\begin{aligned} \frac{d}{d\hat{p}_B} \left(\Pi_P(\mathbf{p}'_1, \theta = 1)|_H - \Pi_P(\mathbf{p}'_1, \theta = 1)|_L \right) = \\ - 2a(\hat{p}_B - 1) - n_L(1 - \mu) + \frac{2aa_H(n_H + n_L) + aa_H n_L \hat{p}_B - 2an_H(n_L + n_H\mu) - 2a_H n_L^2(1 - \mu)}{aa_H} \left(\frac{d\mu}{d\hat{p}_B} \right). \end{aligned} \quad (31)$$

We wish to show that the RHS is positive. The equilibrium worker movement condition (5) is

$$\frac{a+a(2-\hat{p}_B)\hat{p}_B}{n_H(1-\mu)} = 2 - \frac{n_L+n_H\mu}{a_H} - \frac{c}{\lambda}. \quad (32)$$

Using implicit differentiation, we obtain

$$\frac{d\mu}{d\hat{p}_B} = \frac{n_H[a_H(2-\frac{c}{\lambda})+n_H(2\mu-1)-n_L]}{2aa_H(\hat{p}_B-1)} = \frac{aa_H(1+(2-\hat{p}_B)\hat{p}_B)+n_H^2(1-\mu)^2}{2aa_H(\hat{p}_B-1)(1-\mu)}, \quad (33)$$

where we have substituted for c from equation (32). Substituting for $\frac{d\mu}{d\hat{p}_B}$ in equation (31) and rearranging we obtain a cubic polynomial in \hat{p}_B that we need to show is positive:

$$\begin{aligned} & -2a(\hat{p}_B-1)^2(1-\mu) - n_L(1-\mu)^2(\hat{p}_B-1) + \\ & [2aa_H(n_H+n_L) + aa_Hn_L\hat{p}_B - 2an_H(n_L+n_H\mu) - 2a_Hn_L^2(1-\mu)] \\ & [aa_H(1+(2-\hat{p}_B)\hat{p}_B) + n_H^2(1-\mu)^2], \quad (34) \end{aligned}$$

where \hat{p}_B and μ are such that $n_L(1-\mu) < a(2-\hat{p}_B)$. Therefore, $\mu \in [0, \bar{\mu}_H]$ and $\hat{p}_B \in [1, 2 - \frac{n_L(1-\mu)}{a}]$. It is straightforward though tedious algebra to show that the expression (34) is concave in \hat{p}_B (the second derivative is linear in \hat{p}_B , and negative for $\hat{p}_B = 1$ and $\hat{p}_B = 2 - \frac{n_L(1-\mu)}{a}$) and increasing for $\hat{p}_B = 1$. Therefore, expression (34) is minimum either at $\hat{p}_B = 1$ or $\hat{p}_B = 2 - \frac{n_L(1-\mu)}{a}$. It can be shown that expression (34) is positive for $\hat{p}_B = 1$ and $\hat{p}_B = 2 - \frac{n_L(1-\mu)}{a}$. Hence, $\Pi_P(\mathbf{p}'_1, \theta = 1)|_H - \Pi_P(\mathbf{p}'_1, \theta = 1)|_L$ is increasing in \hat{p}_B for $\tilde{N}_{B1}|_L < a(2-\hat{p}_B)$.

Next, consider $\tilde{N}_{B1}|_L > a(2-\hat{p}_B)$. Let $\Delta'_{ij} = \pi_{ij}(\mathbf{p}'_1, \theta = 1)|_H - \pi_{ij}(\mathbf{p}'_1, \theta = 1)|_L$ denote the zone and period-wise profit difference. We show that either $\Delta'_{A1} = \Delta'_{B1} = 0$, Δ'_{A2} is strictly increasing in \hat{p}_B , and either $\Delta'_{B2} = 0$ or Δ'_{B2} is strictly increasing in \hat{p}_B . Therefore, $\sum_{ij} \Delta'_{ij}$ is strictly increasing in \hat{p}_B , which is the desired result. Because $\tilde{N}_{A1}|_t > a$, $\pi_{A1}(\mathbf{p}'_1, \theta = 1)|_t = (1-\lambda)a$ and $\Delta'_{A1} = 0$. Similarly, $\tilde{N}_{B1}|_t > a(2-\hat{p}_B)$. Hence, $\pi_{B1}(\mathbf{p}'_1, \theta = 1)|_t = (1-\lambda)a\hat{p}_B(2-\hat{p}_B)$ and $\Delta'_{B1} = 0$. Since $\mu < \bar{\mu}_H$, $\tilde{N}_{A2}|_H < a_H$ and

$$\pi_{A2}(\mathbf{p}'_1, \theta = 1)|_H = (1-\lambda)\left(2 - \frac{n_L+n_H\mu}{a_H}\right)(n_L+n_H\mu).$$

If $\tilde{N}_{A2}|_L > a_H$, then $\pi_{A2}(\mathbf{p}'_1, \theta = 1)|_L = (1-\lambda)a_H$. Hence,

$$\Delta'_{A2} = (1-\lambda)\left[\left(2 - \frac{n_L+n_H\mu}{a_H}\right)(n_L+n_H\mu) - a_H\right],$$

which is strictly increasing in μ since $n_L+n_H\mu < a_H$. Since μ is strictly increasing in \hat{p}_B (Lemma 5), Δ'_{A2} is strictly increasing in \hat{p}_B . If $\tilde{N}_{A2}|_L < a_H$, then

$$\pi_{A2}(\mathbf{p}'_1, \theta = 1)|_L = (1-\lambda)\left(2 - \frac{n_H+n_L\mu}{a_H}\right)(n_H+n_L\mu).$$

Then,

$$\Delta'_{A2} = \frac{1-\lambda}{a_H} (n_H - n_L) (1 - \mu) [2a_H - (n_H + n_L) (1 + \mu)], \quad (35)$$

which is strictly increasing in μ and, hence, in \hat{p}_B . Since $\tilde{N}_{B2}|_H > a$, $\pi_{B2}(\mathbf{p}'_1, \theta = 1)|_H = (1 - \lambda) a$. If $\tilde{N}_{B2}|_L > a$, then $\pi_{B2}(\mathbf{p}'_1, \theta = 1)|_L = (1 - \lambda) a$ and $\Delta'_{B2} = 0$. If $\tilde{N}_{B2}|_L < a$, then

$$\pi_{B2}(\mathbf{p}'_1, \theta = 1)|_L = (1 - \lambda) \left(2 - \frac{n_L(1-\mu)}{a}\right) n_L (1 - \mu),$$

which is strictly decreasing in μ , and hence decreasing in \hat{p}_B . Hence, Δ'_{B2} is increasing in \hat{p}_B . \square