

Market Power and Mergers in Multi-Sided Markets

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Abstract

This paper generalizes market power measures for evaluating multi-sided markets. In addition, existing estimation techniques for determining the effects of multi-product sellers and mergers are generalized to multi-sided markets. Unlike traditional one-sided products, the integration of platform network externalities by a multi-platform seller affects platform pricing. For example, the integration of Facebook and Instagram profiles by Facebook Inc. might improve the network externalities between social media users and advertisers, which affects pricing and competition with other social media platforms such as Snapchat. An application of the theoretical findings to the sixth generation video game market demonstrates the importance of using these generalized techniques instead of traditional one-sided approaches, when considering platform market power and platform mergers.

Keywords: Market power, multi-sided markets, platform mergers, price-cost markups, two-sided markets, video games.

JEL Classifications: L11, L14, L40.

1 Introduction

A firm’s ability to earn profits through superior price-cost markups is a constant concern for managers, investors, economists, and policy makers. Greater market power by a firm relative to its competitors corresponds to better price-cost markups resulting in greater profitability. Similarly, greater market power in an industry suggests a market environment that is less competitive. While many the academic community across many fields has analyzed the sources and effects of market power in traditional one-sided markets, there is little research on market power in multi-sided markets.

In particular, a one-sided price-cost markup measure likely produces a biased measure of market power in multi-sided markets since platforms have more than a single price and equilibrium prices are often distorted by network externalities. In fact, a price can be below its marginal cost in multi-sided markets even with few platform sellers: Rochet and Tirole (2003), Parker and Van Alstyne (2005), Armstrong (2006), Hagiu (2006), and Weyl (2010).¹ In the market for video games, gaming consoles are often sold near marginal cost even though there are only three main competitors.² Similarly, media platforms like Facebook, Snapchat, Youtube, and Google are free to consumers but generate revenues from advertisers. Considering the price-cost margin of a single side of the market results in a mis-specified measure of market power. Thus, a measure of price-cost markups that is robust to these distortions is required for considering market power across all types of markets.

The measure of market power for a firm is also important in assessing a potential merger between platforms. For example, can two platforms with little market power merge and increase their market power to become the market leader? Similarly, how does market power change when two leading platforms merge? To analyze the effects of mergers on prices, shares,

¹In addition, behavioral externalities or search costs can result in some products being sold near marginal cost (home printers) while other complementary products (ink cartridges) are sold with greater markups (add-on pricing): Ellison (2005), Gabaix and Laibson (2006), Ellison and Ellison (2009) and Grubb (2009).

²See Lee (2013) for the sixth generation; “Report: PS3 to sell for \$399, cost \$494 to make,” by Gamespot for the seventh generation; and “Microsoft Xbox One Hardware Cost Comes in Below Retail Price,” by IHS for the eighth generation.

and market power, I develop a general model of differentiated platform competition where platforms compete in prices and participation on each side of the market. This framework generalizes existing structural techniques for determining unobserved marginal costs and for characterizing post-merger prices of hypothetical mergers. In addition, I find that merger simulations that do not consider the multi-sidedness of platforms produce biased estimates of post-merger prices and market shares.

An important aspect of mergers in multi-sided markets that differs from mergers in traditional one-sided markets is the ability to integrate network externalities in a platform merger. For example, Facebook Inc. has integrated user features across its social media platforms (e.g., users can integrate their profiles so that an Instagram post automatically posts to the user’s Facebook page).³ This benefits social media users but also might benefit advertisers from improved advertiser match due to better user data that is shared between the two platforms. Similarly, the Nintendo 64 Transfer Pak accessory allowed gamers that owned a Nintendo 64 console to purchase Nintendo Gameboy Color games to play on the Nintendo 64. This Transfer Pak is a partial integration between Nintendo’s console platform and portable gaming platform. I find that the magnitude of the change in prices due to a merger increases when two platforms merge with integration.

To illustrate the main theoretical results, an application to the sixth generation video game market is considered. The analysis of this market shows how the consideration of a multi-sided platform as a traditional one-sided product results in biased estimates of marginal costs, market power, and post-merger prices for hypothetical mergers. This bias shows the importance of using the multi-sided market marginal cost estimation techniques, market power, and merger simulation techniques that account for the multi-sidedness of platforms when analyzing platform market power, profitability, and mergers in multi-sided markets.

The literature on market power and mergers for multi-sided markets largely derives from an antitrust perspective using data on media markets (e.g. newspapers, magazines, or news

³Li and Agarwal (2016) find considerable evidence regarding the benefits users experience from integration between the Instagram and Facebook platforms.

websites).⁴ For example, merger simulations to determine price-cost markups is considered by Filistrucchi et al. (2012), Filistrucchi and Klein (2013), and Song (2015).⁵ In addition, many standard antitrust measures are extended to two-sided markets: Emch and Thompson (2006) and Filistrucchi et al. (2014) for market definition, Evans and Noel (2008) for Critical Loss Analysis, Filistrucchi (2008) for the SSNIP test, Affeldt et al. (2013) for upward pricing pressure measurements, and Behringer and Filistrucchi (2015) for the Areeda–Turner rule. However, product multi-sided markets like video game platforms, smartphones, eReaders, and tablets differ from media markets since the platform itself is a product that must be purchased by consumers along with additional content.⁶

The papers that are most closely related to this study are Song (2015) and the collection of Filistrucchi et al. (2012), its predecessor working paper (Filistrucchi et al. (2010)), and Filistrucchi and Klein (2013). Filistrucchi and Klein (2013) consider hypothetical newspaper mergers and they back out the two marginal costs for a newspaper: the marginal cost of a consumer that buys a newspaper and the marginal cost of an advertiser that buys an ad slot. Song (2015) does the same for magazines with consumers and advertisers. In each paper, the demand structure for the platform produces infinite chain rules when taking certain derivatives. These infinite chain rules are due to feedback loops that are left open in the prescribed demand functions. To handle this, they use partial derivatives when determining platform first-order conditions, instead of full derivatives that account for the feedback loops effects on demand. The use of these (mis-specified) first-order conditions ensures that all unobserved marginal costs are determined.⁷ However, by accounting for the full effect that prices have on demands, I find that only a single unobserved marginal cost can be identified.

⁴Evans and Schmalensee (2013) provide an extensive overview.

⁵Chandra and Collard-Wexler (2009) also consider mergers in the two-sided market for newspapers but do not measure costs or use cost data. Instead they find that newspaper pricing below marginal cost is a necessary condition to lower circulation prices post-merger; however, they find no relationship between newspaper concentration measures and platform prices.

⁶This analysis also applies to add-on products like printers with ink cartridges, component purchasing where a bundle of goods is required, DVD and Blu-ray players, and traditional cameras with film. More generally, Zëgners and Kretschmer (2017) refer to these as aftermarket.

⁷The Filistrucchi et al. (2012) paper uses the same demand structure as Song (2015) so the process of the merger simulations is the same.

Fortunately, it is often the case that one of the two marginal costs in a two-sided market can be obtained from contract agreements that exist in many platform industries. I take advantage of this in the video games market and I determine unobserved game console marginal cost for each of the three platforms in the sixth generation video game market.

In addition to the literature using media market data, there is a considerable amount of empirical literature across fields relating to other multi-sided markets. However, this literature largely avoids platform merger issues or the determination of the multiple unobserved marginal costs. For literature on platform mergers, the empirical analysis is one-sided (Fan (2013) for newspapers or Jeziorski (2014) for radio stations). Nevertheless, the empirical literature on two-sided markets using dynamic estimation techniques is growing: Rysman (2004) for yellow pages; Nair (2007), Dubé et al. (2010) and Lee (2013) for video game consoles; Gowrisankaran and Rysman (2012) for camcorders; and Gowrisankaran et al. (2014) for DVDs. In addition, the video game market has received considerable attention regarding exclusive vs. multi-homing game titles and game pricing: Shankar and Bayus (2003), Binken and Stremersch (2009), Corts and Lederman (2009), Liu (2010), Landsman and Stremersch (2011), Lee (2013), and Zhou (2016). This paper contributes to the empirical literature by characterizing the necessary information that is required to determine unobserved marginal costs of a platform seller and by determining market power and hypothetical post-merger equilibria in multi-sided markets with specific results for the sixth generation video game market.

The rest of the paper is organized as follows. Section 2 summarizes the traditional one-sided market with the traditional market power measure and the standard techniques for determining unobserved marginal costs. The model of the multi-sided market as well as equilibrium pricing, elasticities, and unobserved marginal cost determination is provided in Section 3. A generalized multi-sided market power measure is also developed in this section. In Section 4, sellers of multiple platforms and the effect of platform mergers are considered. To illustrate the main findings in this paper, an application to the sixth generation video

game market is presented in Section 5. Finally, Section 6 concludes, followed by the appendix that contains the proofs of all formal findings.

2 A Traditional One-Sided Market

For comparison purposes, consider first a traditional one-sided market. Suppose that there are N sellers of differentiated products that compete in prices. Firm X has demand:

$$q^X = q^X(p^X, \mathbf{p}^{-X}), \quad (1)$$

where p^X is the price set by firm X , \mathbf{p}^{-X} is the vector of prices of the other firms, and $q^X(p^X, \mathbf{p}^{-X})$ is the demand function for firm X 's product where $\frac{dq^X}{dp^X} < 0$ and $\frac{dq^X}{dp^Y} > 0$ for $Y \neq X$. Firm X has profits given by:

$$\Pi^X = (p^X - c^X) \cdot q^X(p^X, \mathbf{p}^{-X}), \quad (2)$$

where c^X is the constant marginal cost for firm X .

Standard measures of market power often depend on either price-cost markups or the distribution of market shares. The standard measure using price-cost markups is the Lerner Index, developed by Lerner (1934), which considers the price-cost markup normalized by the price.⁸ That is, the Lerner index of market power for firm X is given by:

$$L^X = \frac{p^X - c^X}{p^X}, \quad (3)$$

at the equilibrium price p^X . Thus, L^X is the ratio of the equilibrium price-cost markup and the price.

While this measure provides information on a firm's ability to price above marginal cost, the cost structure of a firm is often unobserved by a practitioner. For example, with

⁸See Elzinga and Mills (2011) for an overview of the Lerner index.

administrative cost data provided by a firm, determining the marginal cost for a specific product is likely impossible for firms that are large conglomerates. As a result, a large portion of the empirical industrial organization literature is dedicated to using variation in product characteristics within a market to estimate demand elasticities.⁹ With demand elasticities, unobserved marginal costs can be determined. More specifically, using the first-order condition of profit maximization by each firm provides the following result:

$$L^X = \frac{p^X - c^X}{p^X} = -\frac{1}{\eta^X} \quad \text{or} \quad p^X = c^X \cdot \left(1 + \frac{1}{\eta^X}\right)^{-1}, \quad (4)$$

where η^X is the own-price elasticity of demand for firm X . Thus, by estimating each firm's own-price elasticity of demand, the unobserved cost is determined by:¹⁰

$$c^X = p^X \cdot \left(1 + \frac{1}{\eta^X}\right). \quad (5)$$

This is an important result as it implies that the estimated elasticities allow for the determination of unobserved marginal costs. Thus, information on prices and elasticities allows a practitioner to determine the market power in traditional one-sided markets.

3 A Multi-Sided Market

Now consider a multi-sided market where there are N independent platform sellers that connect distinct groups who interact through the seller's platform. For example, Snapchat connects consumers with other consumers and it connects consumer with advertisers; however, its main competitor, Facebook, also connects consumers with games and allows consumers to "login with Facebook" to interact with online markets and websites more easily. Thus, competing platforms can differ in the number of interaction types that they facilitate, and

⁹Originating with seminal work by Berry (1994), Berry et al. (1995), and Nevo (2000).

¹⁰If firms sell more than one differentiated product, then the full elasticity matrix of own- and cross-price elasticities is required, as discussed in Berry (1994), Berry et al. (1995), and Nevo (2000).

each platform charges a price for each type of interaction that is facilitated by the platform.¹¹ The platforms are differentiated and compete in prices and interaction participation.

3.1 Demand for Interaction

Suppose that the N competing platforms facilitate at most K types of interactions and let the amount of type i interaction on Platform X be denoted by I_i^X . For example, consider the market for video games where there exists four types of interactions: the interaction between consumers and the platform in terms of console purchases (I_1), games purchased by consumers from game developers (I_2), console software kits purchased by game developers (I_3), and the online subscriptions (e.g. Xbox live subscriptions) purchased by consumers (I_4).

For each type of interaction I_i^X , the platform seller charges a single price or fee, p_i^X . For example, each video game platform charges prices for each type of interaction: the direct console price, the fee paid to the platform for each game sale, the price of a software development kit, and the online subscription price that allows gamers to interact with other gamers.¹² For interactions between the platform and one of its sides, the agents pay the price to the platform. Whereas, for interactions between different sides, it is assumed that the distribution of who pays the fee to the platform does not affect interaction demand.¹³ Thus, each type of interaction has a demand that depends on the competing platform prices — consistent with differentiated price competition — and that depends on the amount of each type of interaction that the platforms provide — consistent with multi-sided markets.

¹¹The term “interaction” is intentionally vague since a platform price is incurred differently depending on the platform market. For a media platform, an “interaction” between a consumer and advertiser is a click on an advertisement or a click-through purchase from an ad. Alternatively, an “interaction” between a video game consumer and a game developer is the consumer purchase of the game. Where the platform charges a price determines where an interaction, with interaction demand, exists.

¹²Similarly, Facebook and other social media platforms allow users, businesses, agencies, and advertisers to interact and potentially charge different prices for these different types of interaction.

¹³This assumption is largely consistent with results in Rochet and Tirole (2003) on transaction pricing in multi-sided markets where the total price (the sum of the fees set by the platform to each side for an interaction), is what effects the total demand for transactions.

The demand for interaction i on Platform X is given by:

$$I_i^X = I_i^X(p_i^X, \mathbf{p}_i^{-X}; \mathbf{I}_{-i}^1, \mathbf{I}_{-i}^2, \dots, \mathbf{I}_{-i}^N), \quad (6)$$

where \mathbf{p}_i^{-X} denotes the vector of prices by other platforms for the interaction type i , and the \mathbf{I}_{-i}^Y denote the vector of all other interaction types besides i on Platform Y for $Y = 1, \dots, N$ (including $Y = X$). Thus, the demand for interaction on a platform depends directly on the prices for that interaction and on the amount of other interactions across platforms. Consistent with differentiated price competition and markets with network effects, suppose that $\frac{\partial I_i^X}{\partial p_i^X} < 0$ and $\frac{\partial I_i^X}{\partial p_i^Y} \geq 0$ for $Y \neq X$. That is, interaction on a platform is decreasing in a platform's own price and increasing in the price of a competitor. In addition, suppose that $\frac{\partial I_i^X}{\partial I_{-i}^X} \geq 0$. This relates to the presence of network externalities where $\frac{\partial I_i^X}{\partial I_{-i}^X} > 0$ implies a positive network externality; for example, the case with video games has a positive network externality between console sales and game sales ($\frac{\partial I_1^X}{\partial I_2^X} > 0$). Alternatively, $\frac{\partial I_i^X}{\partial I_{-i}^X} < 0$ implies a negative network externality which can occur on media platforms where greater consumer advertisements diminishes consumer usage of the media platform.

To ensure that the second-order conditions hold so that a unique equilibrium exists, suppose that $\frac{\partial (I_i^X)^2}{\partial^2 p_i^X} \leq 0$ with $\frac{\partial I_i^X}{\partial p_i^X} \rightarrow 0$ as $p_i^X \rightarrow 0$. This ensures that network effects are not “too large” so that demand increases from price reductions do not continuously increase profits. In addition, suppose that $\frac{\partial (I_i^X)^2}{\partial^2 I_{-i}^X} < 0$, along with $\frac{\partial I_i^X}{\partial I_{-i}^X} \rightarrow 0$ as $I_{-i}^X \rightarrow 0$ when $\frac{\partial I_i^X}{\partial I_{-i}^X} > 0$. This ensures that network effects are not “too large” so that demand increases from greater interaction elsewhere on the platform does not persistently increase profits. Combined, these second-order assumptions ensure that the network externalities are not too strong so that a platform cannot keep increasing its size continuously with a slight price reduction.

In addition, to ensure a solution to the platforms' profit maximization problem, it is important to note that the current demand for interaction given by Equation (6) contains infinite feedback loop. This is due to the fact that I_i^X is a function of p_i^X and \mathbf{I}_{-i}^X , but \mathbf{I}_{-i}^X is

a function of I_i^X which is again a function of p_i^X and \mathbf{I}_{-i}^X , and so on. This is why the derivative assumptions above are partial derivatives. The full (or total) derivatives do not exist. However, a platform considers both the direct partial effect and the indirect effect through feedback loops when maximizing profit. Thus, to solve the platform's maximization problem, assume that for each interaction i and Platform X , the function $I_i^X(p_i^X, \mathbf{p}_i^{-X}; \mathbf{I}_{-i}^1, \mathbf{I}_{-i}^2, \dots, \mathbf{I}_{-i}^N)$ has a solution such that

$$I_i^X = \tilde{I}_i^X(\mathbf{p}^1, \mathbf{p}^2, \dots, \mathbf{p}^N), \quad (7)$$

where \mathbf{p}^Y for $Y = 1, \dots, N$ is the vector of all interaction prices for Platform Y .¹⁴ Thus, the function $\tilde{I}_i^X(\cdot)$ provides the interaction demands with respect to each of the platform prices. This implies that the feedback loops in the demand for interactions from Equation (6) are closed when:

$$I_i^X = I_i^X(p_i^X, \mathbf{p}_i^{-X}; \tilde{\mathbf{I}}_{-i}^1, \tilde{\mathbf{I}}_{-i}^2, \dots, \tilde{\mathbf{I}}_{-i}^N). \quad (8)$$

Thus, an interaction directly depends on all platform prices for that interaction and also on the other interaction sizes on other platforms with closed feedback loops.¹⁵

Platforms generate revenues (which can be positive or subsidies) from each type of interaction. Thus, Platform X has total profits given by:

$$\Pi^X = \sum_{i=0}^M \sum_{j=0}^M \{(p_i^X - c_i^X) \cdot I_i^X\} - F^X, \quad (9)$$

where c_i^X is the marginal cost incurred by Platform X for an interaction of type i and F^X is the fixed cost for Platform X .

¹⁴The first-order conditions with respect to the $I_i^X(\cdot)$ function imply that $\frac{d\tilde{I}_i^X}{dp_i^X} < 0$ and $\frac{d\tilde{I}_i^X}{dp_i^Y} \geq 0$ for $Y \neq X$; in addition, positive network externalities ($\frac{\partial I_i^X}{\partial I_{-i}^X} > 0$) imply that $\frac{d\tilde{I}_i^X}{dp_{-i}^X} < 0$ while negative externalities ($\frac{\partial I_i^X}{\partial I_{-i}^X} < 0$), imply that $\frac{d\tilde{I}_i^X}{dp_{-i}^X} > 0$.

¹⁵Interaction demands with closed feedback loops will be important for developing the techniques for determining unobserved marginal costs.

3.2 The Multi-Sided Market Equilibrium

The equilibrium concept is the standard Nash equilibrium. That is, platform strategies are their collection of prices $(\mathbf{p}^1, \mathbf{p}^2, \dots, \mathbf{p}^N)$, which constitute an equilibrium if $\Pi^X(\mathbf{p}^X, \mathbf{p}^{-X}) \geq \Pi^X(\hat{\mathbf{p}}^X, \mathbf{p}^{-X})$ for all $\hat{\mathbf{p}}^X$. The interaction demand structure provides equilibrium pricing that is standard in multi-sided markets:

Theorem 1 (Equilibrium Platform Prices). *There exist unique prices $(\mathbf{p}^1, \mathbf{p}^2, \dots, \mathbf{p}^N)$ in equilibrium such that Platform X 's price for the interaction of type i is given by:*

$$p_i^X = \underbrace{c_i^X}_{MC} + \underbrace{\frac{-I_i^X}{dI_i^X/dp_i^X}}_{Markup} - \underbrace{\sum_{k \neq i} \left\{ \frac{\partial I_k^X}{\partial I_i^X} \cdot (p_k^X - c_k^X) \right\}}_{\text{Marginal Profit Elsewhere}}. \quad (10)$$

All proofs are in the appendix.

The equilibrium platform price for an interaction provides a similar expression to the standard pricing results found in the multi-sided market literature. Namely, price equals marginal cost, plus a markup from a platform's market power, minus the gains from the other sides (or in this case, the marginal profits from the other sides). That is, the last term in Equation (10) is the marginal change in profits for interaction I_k when interaction I_i changes, summed across all $k \neq i$ interaction types.¹⁶

To better illustrate the platform's pricing strategy, consider the simple video game example where there are only two types of interactions: consumers purchasing consoles (I_1), and consumers purchasing games (I_2). In this case, Theorem 1 implies that a platform's console price is the marginal cost of a console, plus a market power markup term, minus the marginal profit to the platform from game sales fees. In particular, the marginal profit for game sales is the platform fee from a game sale minus the platform's cost to supply the game $(p_2 - c_2)$, times the change in game sales from a change in console sales: $\frac{\partial I_2^X}{\partial I_1^X}$. Note that the

¹⁶In the platform's complete solution, solving the system of first-order pricing conditions implies that prices are functions of all marginal costs, interaction elasticities, and interaction derivatives. The proof of Theorem 1 provides greater detail.

interaction derivative term is often observed in the data. For example, the average number of game purchases for a console owner serves as a simple measure for $\frac{\partial I_2^X}{\partial I_1^X}$ since one fewer console owner likely implies a decrease in game purchases equal to the average number of game sales for that console. This provides a tractable equilibrium pricing function without specific knowledge on the externality structure.¹⁷

This pricing result is especially useful, not only because it conforms to the multi-sided market literature, but also because the markup can be rewritten by using elasticities. This allows for comparison with the traditional one-sided relationship between a product's price, marginal cost, and demand elasticity. The elasticity of demand for interaction of type i on Platform X is $\eta_i^X = \frac{d\bar{I}_i^X}{dp_i^X} \cdot \frac{p_i^X}{\bar{I}_i^X} = \frac{d\bar{I}_i^X}{dp_i^X} \cdot \frac{p_i^X}{I_i^X}$. In equilibrium, the following relationship holds:

Theorem 2 (Equilibrium Elasticity). *In multi-sided markets, the equilibrium demand elasticity is given by:*

$$-\frac{1}{\eta_i^X} = \underbrace{\frac{p_i^X - c_i^X}{p_i^X}}_{\text{Lerner}} + \frac{\sum_{k \neq i} \frac{\partial I_k^X}{\partial I_i^X} \cdot (p_k^X - c_k^X)}{p_i^X} = \frac{\sum_{k=1}^K \frac{\partial I_k^X}{\partial I_i^X} \cdot (p_k^X - c_k^X)}{p_i^X}. \quad (11)$$

In a multi-sided market, the elasticity of interaction demand depends on the traditional Lerner index plus a normalization of the marginal profits elsewhere. Alternatively, the negative inverse of an elasticity of interaction type i is the sum of the marginal profits with respect to interaction i $\left(\sum_{k=1}^K \frac{\partial I_k^X}{\partial I_i^X} \cdot (p_k^X - c_k^X)\right)$, normalized by the price p_i^X . Thus, if a practitioner observes the equilibrium prices, marginal costs, and interaction derivatives, then the equilibrium interaction demand elasticities can be determined.

The results in Theorem 2 also illustrate potential issues for using the traditional Lerner

¹⁷In relating this pricing result to the transaction platform literature that stems from Rochet and Tirole (2003), a two-sided transaction platform in this framework has $\frac{\partial I_2^X}{\partial I_1^X}$ fixed and equal to 1 for all prices and interaction levels. That is, each transaction where a price p_1 is taken from side 1 and p_2 is taken from side 2 always results in a one-to-one relationship between the two sides when prices are incurred (e.g. a purchase with a credit card or debit card, a purchase on Amazon or eBay, or the click-through of an advertisement on a website are all examples where the interaction between the two sides is one-to-one with respect to platform prices and fees). Thus, this framework allows for the consideration of these transaction platform structures as well as platform products that have numerous additional interactions across sides.

index in multi-sided markets. For example, by using the traditional one-sided market relationship of the negative inverse of an elasticity equating the Lerner index results in a biased assessment. This implies that the traditional Lerner index is flawed on two accounts for multi-sided markets. First, the potential for prices below marginal costs when a platform seller has market power results in a Lerner index that is negative which provides a biased estimate of too little market power. Second, the relationship between the traditional Lerner and a platform's own-price elasticity does not hold. For these reasons, an alternative index is required to accurately account for market power in multi-sided markets.

3.3 A General Index of Market Power

Market power is often defined as a seller's ability to price above marginal cost. This ability generates profits but creates surplus distortions. To this end, both the distribution of a platform's price-cost margins and the distribution of a platform's interactions are important for evaluating market power in multi-sided markets. For example, suppose that there are two platforms that compete in a two-sided market where one side is subsidized by both platforms while the other side faces a markup. If shares by each platform are the same on each side but one platform charges higher prices on each side, then the platform with higher prices has greater market power. Alternatively, if prices are the same but one platform is the share leader on the profitable side of the market while the other platform is the market share leader on the subsidized side of the market, then the platform that dominates the profitable side clearly has greater market power. Thus, both the price-cost margins and the distribution of interactions should determine market power in multi-sided markets.

Definition 1 (The Multi-Sided Market Lerner Index). The market power of Platform X in a multi-sided market is given by:

$$\tilde{L}^X = \frac{\sum_i \{(p_i^X - c_i^X) \cdot I_i^X\}}{\sum_i \{p_i^X \cdot I_i^X\}}. \quad (12)$$

Thus, the generalized Lerner index weights each price-cost margin in the numerator and each price in the denominator by the corresponding amount of interaction. This is indeed a generalization since the traditional one-sided product market implies that $\tilde{L}^X = \frac{(p^X - c^X) \cdot I^X}{p^X \cdot I^X} = \frac{p^X - c^X}{p^X} = L^X$. The generalized Lerner index might also be easily computable using administrative data. That is, the generalized Lerner index is the ratio of platform variable profits and total revenues:

$$\tilde{L}^X = \frac{\sum_i \{(p_i^X - c_i^X) \cdot I_i^X\}}{\sum_i \{p_i^X \cdot I_i^X\}} = \frac{\Pi^X + F^X}{\text{TR}^X}, \quad (13)$$

where $\text{TR}^X = \sum_i \{p_i^X \cdot I_i^X\}$ is the total revenue generated by Platform X .

Determining the variable profits and total revenues for a platform that is owned by a larger conglomerate might be difficult since costs might be shared across other products (e.g. the Google search platform under the Google corporation, Bing or Xbox under the Microsoft corporation, the Playstation console under the Sony corporation, etc.). This issue is common in the literature and is not specific to platforms. However, the relationship between variable profit and total revenues may prove valuable for smaller independent platforms like Snapchat, Pandora, or Hulu where company data is largely with respect to a single platform.

3.4 Determining Unobserved Costs

To determine market power in multi-sided markets, each interaction marginal cost is necessary. However, certain marginal costs might be difficult to observe. Fortunately, a standard result in the literature is that unobserved costs are determinable in traditional one-sided markets where differentiated firms compete in prices. That is, given prices and market shares as well as cross- and own-price elasticity estimates, the unobserved marginal costs can be computed. Naturally, one would expect a similar result to exist for multi-sided markets where platform seller prices for each interaction, market shares of interactions, interaction derivatives, and all elasticities are sufficient to determine all the interaction unobserved marginal

costs. Unfortunately, this is not the case.

Theorem 3 (Indeterminable Unobserved Marginal Costs). *If a multi-sided platform has more than one type of interaction, then even with all platform interaction prices, elasticities, and interaction derivatives, the vector of unobserved interaction marginal costs is indeterminable.*

This result differs from the limited existing literature on determining unobserved marginal costs in multi-sided markets. In Song (2015) and the collection of Filistrucchi et al. (2012) and Filistrucchi and Klein (2013), each finds that both marginal costs can be determined for a two-sided platform. However, in each paper, the demand structure for the platform produces infinite chain rules when taking certain derivatives. These infinite chain rules are due to feedback loops that are left open in the prescribed demand structure. To handle this, they use partial derivatives when determining platform first-order conditions, instead of full derivatives that account for the effect that feedback loops have on demand. With the mis-specified first-order conditions that uses partial derivatives, the matrix that must be invertible to determine all unobserved costs is non-singular so that all marginal costs are determined. However, by accounting for the full effect that prices have on demands, I find that the corresponding matrix has rank equal to one. This implies that only a single unobserved marginal cost is determinable when the platform's solution includes the full price effect.

Corollary 1 (Determining a Single Unobserved Marginal Cost). *Determining a single unobserved interaction cost for platform X (c_i^X), requires platform X prices (\mathbf{p}^X), other marginal costs (\mathbf{c}_{-i}^X), all interaction derivatives with respect to that interaction ($\frac{\partial I_{-i}^X}{\partial I_i^X}$), and the elasticity of demand for that interaction (η_i^X). That is, $c_i^X = c_i^X \left(\eta_i^X; \mathbf{p}^X; \mathbf{c}_{-i}^X; \frac{\partial I_{-i}^X}{\partial I_i^X} \right)$.*

This result might appear weak for platforms that have many types of interactions; however, Corollary 1 is potentially very powerful for many two-sided markets where there are only two unobserved marginal costs and the inherent supply chain structure allows for a

clear assessment of one marginal cost. Typically, practitioners avoid using administrative and accounting cost data for a large corporation because it is often difficult to decipher what information is important for determining the marginal cost of a specific product within a corporation. However, for many two-sided platforms, there is often clear platform marginal cost information for at least one of the marginal costs of interaction.

For example, a consumer search on Google or a personal profile on Facebook, Snapchat, or Instagram are all nearly costless for the platform; similarly, the cost of a digital download of an app, song, video, picture, or ebook are all likely costless for digital platforms; lastly, for the video game console, DVD/Blu-Ray player, and CD/Record player platforms, the marginal cost of content (games, DVDs, or CDs) is often zero to the platform seller or is inferable from supply chain contract information. More specifically, consider the video game platform market. For games, the platform seller incurs no cost of supplying the games but takes a percent of the final price that the consumer pays for the game; alternatively, if the platform sets the price of content, then collection by different parts of the supply chain provides the cost information required.¹⁸ Exploiting these marginal cost features of the video game market allows for a rich development of the determination of unobserved console marginal costs, price-cost markups, market power, and a hypothetical merger analysis which are considered for the sixth generation video game market in Section 5.

4 General Multi-Sided Platform Ownership

To this point, the analysis has considered platform sellers as single product sellers. That is, each seller sells a single platform. However, many multi-sided markets have multi-platform sellers. For example, Facebook Inc. uses its multiple social media platforms (Facebook, Whatsapp, and Instagram), to compete with other social media platforms like Snapchat and Google's attempt with Google Plus. Similarly, in the market for video games, Nintendo

¹⁸For example, "Anatomy of a \$60 video game," by the LA Times in 2010 provides a breakdown of the marginal cost with respect to supply chain payments from a video game.

and Sony each produce two types of gaming platforms (console and handheld), to compete with Microsoft's console gaming platform and digital device gaming platforms like smartphones and tablets. Considering a more general environment with multi-platform sellers also naturally allows for the analysis of horizontal platform mergers.

4.1 Multi-Platform Sellers

Suppose that there are n platform sellers of the N available platforms and assume that $n < N$ so that there exist some multi-platform sellers.¹⁹ Suppose that a platform seller sells platforms X and Y . In this case, platform X 's prices are distorted from the single platform seller case determined in Theorem 1:

Theorem 4 (Multi-Platform Seller Prices). *If a platform seller owns platforms X and Y , then the equilibrium price for the interaction of type i on platform X is given by:*

$$p_i^X = c_i^X + \frac{-I_i^X}{d\bar{I}_i^X/dp_i^X} - \sum_{k \neq i} \frac{\partial I_k^X}{\partial I_i^X} \cdot (p_k^X - c_k^X) - \underbrace{D_i^{XY} \cdot (p_i^Y - c_i^Y)}_{\text{Direct Diversion}} - \underbrace{D_i^{XY} \cdot \sum_{k \neq i} \frac{\partial I_k^Y}{\partial I_i^Y} \cdot (p_k^Y - c_k^Y)}_{\text{Diversion Elsewhere}} \quad (14)$$

where $D_i^{XY} \equiv \frac{d\bar{I}_i^Y/dp_i^X}{d\bar{I}_i^X/dp_i^X} < 0$ is the diversion ratio between platforms X and Y of interaction type i .

With a multi-platform seller, the price of a platform for an interaction takes on the typical individual platform seller prices given by Theorem 1, plus two additional terms. The first additional term is the direct diversion to the other platform owned by the seller for the same type of interaction, while the second additional term is the feedback diversion to the other platform for all other types of interaction. In terms of relating this expression to a traditional market, the summed network externality term (Diversion Elsewhere) is a result of the multi-sidedness in platform markets. Thus, the consideration of a multi-sided market

¹⁹If $n = N$, then each platform is sold by independent sellers as in Section 3.

as a traditional one-sided market when there exist multi-platform sellers has compounded bias.²⁰

Also notice that if prices are greater than marginal costs so that no subsidies occur, then both additional terms result in additional price markups relative to the individual platform seller prices (since $-D_i^{XY} > 0$). For mergers, this implies that prices for each of the platforms will increase post-merger.²¹

In another light, Theorem 4 implies that if some interactions are subsidized by platforms, then a merger can result in price decreases as well as price increases. Such a possibility suggests that the post-merger outcome may be more efficient. Furthermore, a potential efficiency measure from some price increases when other prices decrease due to a platform merger is the generalized Lerner index which fully accounts for all prices relative to costs. If the collective price changes after the merge are such that the generalized Lerner index decreases for each platform, then the merger is likely welfare improving.

To illustrate the main pricing affects from platform mergers, hypothetical mergers in the sixth generation video game market are analyzed. However, before proceeding to the video game application, there is an additional aspect which is specific to platform mergers that must also be considered when analyzing post-merger pricing in multi-sided markets.

4.2 Platform Integration by Multi-Platform Sellers

Unlike traditional multi-product sellers, a multi-platform seller can potentially affect the value of each of its platforms by integrating certain interactions between its multiple platforms. For example, Facebook Inc. has integrated user features across its social media platforms (e.g. users can integrate their profiles so that an Instagram post automatically

²⁰This result is consistent in a more general platform ownership structure as well. The proof of Theorem 4 provides the solution of a general platform ownership matrix. Not surprisingly, there exists a direct diversion and diversion elsewhere terms for every other platform that is owned by the platform seller.

²¹This analysis is largely consistent with Affeldt et al. (2013) who extended the upward pricing pressure measure to two-sided markets. Their diversion ratio effects are consistent with Theorem 4; in addition, they allow for efficiency gains (reduced marginal costs) after the merger. They apply their two-sided market upward pricing pressure measure to a newspaper market and show that the two-sidedness is important for measuring upward pricing pressure.

posts to the user’s Facebook page). For social media advertisers, greater user information from improved user data due to the integration results in a greater willingness to use either platform as advertising outlets. Similarly, the Nintendo 64 Transfer Pak accessory allowed consumers that owned a Nintendo 64 to purchase Nintendo Gameboy Color games to play on the Nintendo 64. This Transfer Pak is a partial integration between Nintendo’s console and portable gaming platforms.

Consideration of this integration is especially important for a platform merger analysis. Does platform integration result in greater price markups or does platform integration taper markups? From Theorem 4, if there exist pre-merger subsidies, then some price effects are ambiguous from a merger. Thus, understanding the effects of platform integration is important for measuring post-merger platform market power and for considering platform integration policy for multi-platform sellers.

To consider platform integration more formally, suppose that platforms X and Y achieve full integration if each interaction on platform X depends on the interactions of other types summed across both platform X and platform Y : $I_i^X = \left(p_i^X, \mathbf{p}_i^{-X}; \tilde{\mathbf{I}}_{-i}^X + \tilde{\mathbf{I}}_{-i}^Y; \tilde{\mathbf{I}}_{-i}^1, \tilde{\mathbf{I}}_{-i}^2, \dots, \tilde{\mathbf{I}}_{-i}^N \right)$. That is, interactions are combined across the two platforms so that each side benefits from interaction with the other sides of both platforms. While this is an extreme case as partial platform integration is more likely to occur, it provides a base case for how platform integration by multi-platform sellers affects platform pricing strategies.

Theorem 5 (Platform Integration Prices). *If platforms X and Y merge and there do not exist price subsidies, then each interaction price for platforms X and Y increases and the increase is greater if the platforms integrate.*

Thus, when platforms do not subsidize an interaction, a platform merger with platform integration results in greater price markups. With subsidized interaction, the effect on prices is ambiguous. This provides a potential explanation for why multi-platform sellers like Facebook Inc. and Google Inc. have often integrated their many platforms with respect to user profiles. The greater price markups from integration implies greater revenues; however,

if integration is too costly, then multi-platform sellers may only provide limited integration between multiple platforms. Partial integration has largely occurred with Sony and Nintendo who provide some integrated profile features between their console and handheld gaming platforms.²²

From a policy perspective, this result suggests that the effect of multi-platform integration on price markups must be considered in platform merger cases. If post-merger platform integration is plausible, then price markups will likely be greater than the standard estimates.

However, even with greater price markups, the welfare effects from integration are less clear. Platform integration resulting in greater platform market power can lead to deadweight loss; however, users benefit from integration as well: improved advertising matching benefits consumers, gaming integration benefits consumers, and social media integration can save consumers time when posting on multiple media platforms. These benefits must also be considered when analyzing platform integration.

5 The 6th Generation Video Game Market

To illustrate the main results in this paper, the market for the sixth generation video games is considered. From October 2000 through 2005, three platform sellers competed in the video game market: Sony's Playstation 2 console (PS2), Microsoft's Xbox console, and Nintendo's Gamecube console(GC). Platform sellers generated revenues from two types of interactions: revenues from console sales directly to consumers and revenues from fees obtained through each game purchase by a consumer.²³ In this case, there are three platform sellers ($N = 3$), and there are two types of interactions ($K = 2$).

Let I_1 denote the console sales to consumers where p_1 is the console price, c_1 is the

²²This trend may change now with Nintendo's release of the fully integrated Switch platform that is both a console and a handheld gaming device. It remains unclear if Nintendo will discontinue either their pure console platforms or their pure handheld platforms.

²³While Microsoft's Xbox Live online subscription program released in November of 2002 to allow gamers to play collectively through the internet, data limitations prevent the incorporation of the Xbox Live subscription into the analysis.

marginal cost for a platform seller to produce a console, and η_1 is the consumer elasticity of console demand. Similarly, let I_2 denote the amount of game sales to consumers. In terms of game prices and costs with respect to the video game platforms, video games are often developed by third party publishers who sell the games and pay a royalty to the platform for each game sale. Thus, the royalty can be interpreted as a platform's marginal profit from a game sale, $(p_2 - c_2)$, so that data on video game prices and platform royalties provides the marginal cost to the platform for providing an individual game sale. That is, game marginal cost for the platform is taken as the game sales price minus the royalty.²⁴

Given marginal costs on the game side, the result from Theorem 3 for determining a single marginal cost implies that the marginal cost for video game consoles is determinable. This set up for the royalty structure implies that p_2 is the price of a game so that the platform changes the price of the game directly by changing the royalty. In addition, c_2 is the marginal cost to provide a game to a consumer and η_2 is the demand elasticity for a video game. Lastly, $\frac{\partial I_2^X}{\partial I_1^X}$ is the change in game sales from a change in console sales; or the average number of game purchases by a console owner, which is observable. While data on the sixth generation video game market is limited, data from existing literature provides sufficient information to determine unobserved console marginal costs and preform a hypothetical merger analysis.

5.1 Data

The majority of the data that is necessary to analyze the results in this paper is provided by Lee (2013). Lee estimates a dynamic model of consumer demand for consoles and video games and video game developer demand for consoles. Thus, consumer console demand

²⁴An alternative structure with the game royalties would be that p_2 is the royalty with c_2 as the marginal cost for the platform to provide the game which is likely zero for games developed by third party publishers. This implies that η_2 is the demand elasticity for a video game with respect to the royalty. However, this alternative approach results in η_2 values (computed using Theorem 2 and data given in the next subsection) that are unreasonable demand elasticities for video games: -0.515 for Playstation 2 games, -0.578 for Xbox games, and -0.676 for Gamecube games. Using the royalty as a platform's marginal profit of a game sale, $(p_2 - c_2)$, does provide reasonable imputed values of η_2 (-2.144 for Playstation 2 games, -2.063 for Xbox games, and -1.876 for Gamecube games), which are consistent with the game elasticities determined by Nair (2007).

estimates account for the inherent network externalities that exist in this two-sided market.²⁵ The estimates are used in simulations that consider the market without exclusive contracts between certain games and consoles. Lee provides information on console and game sales for each of the three platforms over the five year period. This provides total console sales, game sales, and average console price statistics for each of the three platforms as well as the average price of a game. In addition, Lee determines the own- and cross- price elasticity for consoles. This information is provided in Table 1.²⁶

Table 1: The Sixth Generation Video Game Market

	Playstation 2	Xbox	Gamecube
I_1 - Console Sales (mil.)	30.07	13.32	9.83
p_1 - Avg. Console Price (\$)	226.60	214.26	159.16
η_1 - Console Elasticity	-1.973	-2.004	-1.432
I_2 - Total Game Sales (mil.)	296.15	117.5	82
p_2 - Avg. Game Sales Price (\$)	25	25	25
$(p_2 - c_2)$ - Avg. Game Royalty (\$)	6	7	9
$\frac{\partial I_2}{\partial I_1}$ - Games per Console Sales	9.849	8.821	8.342

Platform royalty information is proprietary. However, some information on platform royalties for this generation has been revealed. Evans et al. (2006) find evidence that Sony charged between \$3 and \$9 royalties for Playstation 2 games (I use the \$6 average as provided in Table 1), while Microsoft averaged a \$7 royalty for Xbox games.²⁷ Royalty information for Nintendo is even more difficult to find. Coughlan (2001) finds that Nintendo had relatively high royalties for the previous generation: \$18 for Nintendo v.s. Sony’s \$9. However, royalties have been trending downward since this time period.²⁸ In addition, Nintendo announced that

²⁵Lee (2013) uses monthly transaction data for the sixth generation video game market that is provided by NPD Group, a leading consumer data provider.

²⁶Console sales and elasticities are provided in Tables 1 and 4 in Lee (2013); the average price is computed using Figures 1(a) and 1(c) where the original prices for the Xbox and PS2 were \$300 upon release while the Gamecube released at \$200. Then in May 2002 the retail price for the Xbox and PS2 were each cut by \$100 while the Gamecube price was cut by \$50. The total game sales are determined by using Table 2 and the fact that “The top 10 titles on the PS2, XB, and GC (listed in Table 2) accounted for 13%, 16%, and 20% of platform software sales;” and the average game sales price is computed using Figure 1(b).

²⁷Evans et al. (2006) pages 131 and 135.

²⁸The standard royalty rate has become 12% in the video game market or \$6-\$8 today. Sources: “The

it dropped its royalty rate during the Gamecube era.²⁹ Given the likely higher royalty for Nintendo, I assume the average royalty for the Gamecube is \$9.³⁰

Unlike today, price variation in games was considerable during this era of gaming; in addition, price changes for consoles also occurred over the five year period. Thus, average pricing is used to assess this market. This implies that the imputed console marginal costs are average marginal costs over the five year period. Constant hardware components for each of the consoles along with decreasing hardware costs suggests that the marginal cost of consoles decreased over time. Thus, an average marginal cost for consoles is suitable for measuring price-cost markups and market power in this market.

5.2 Determining Console Marginal Costs

Given the royalty data on the game side of the market, the equilibrium elasticity result in Theorem 2 implies that for each platform seller:

$$c_1^X = \underbrace{\left(1 + \frac{1}{\eta_1}\right) \cdot p_1^X}_{\text{Traditional}} + \underbrace{\frac{\partial I_2^X}{\partial I_1^X} (p_2^X - c_2^X)}_{\text{Bias}}, \quad (15)$$

where every term on the right-hand side is given in Table 1. This marginal cost formula shows that by using the traditional method given in Equation (5) to back out costs, a bias is introduced that distorts the cost estimates. Thus, accounting for the multi-sided features in platform markets is critical for accurately estimating unobserved marginal costs, price-cost markups, and market power.

Furthermore, notice how the platform royalty, $(p_2 - c_2)$, affects the console marginal cost estimates from Equation (15). That is, an increase in the royalty by \$1 results in an increase in the console marginal cost by $\frac{\partial I_2^X}{\partial I_1^X}$ which ranges between 8 and 10 across consoles. Thus,

Economics of Game Publishing,” IGN 2006; and “Anatomy of a \$60 video game,” LA Times 2010; “Video Games and Consoles,” Encyclopedia.com

²⁹Source: “Nintendo Drops Gamecube Royalty Rates,” Gamasutra 2003.

³⁰While the simulation results are largely consistent for other Nintendo royalty rates, a robustness check on platform royalties that is preformed in the next subsection suggests that royalties are reasonable.

with data on console marginal costs, the accuracy of the platform royalties could be assessed. The royalty also affects the bias in the console marginal cost estimates so that the bias from using the traditional marginal cost technique is amplified for greater royalties.

Using the data from Table 1 as inputs into Equations (5) and (15), the biased traditional marginal cost estimates and the corrected marginal cost estimates for consoles is determined. These marginal cost estimates are provided in Table 2. Using the imputed marginal costs, the average price-cost markups for the corrected and biased cost measures are also provided in Table 2.

Table 2: Average Marginal Costs and Price-Cost Markups of Game Consoles

	Playstation 2	Xbox	Gamecube
Biased Marginal Costs (\$)	111.75	107.35	48.01
Unbiased Marginal Costs (\$)	170.84	169.10	123.09
Biased Price-Cost Markups (\$)	114.85	106.92	111.14
Unbiased Price-Cost Markups (\$)	55.76	45.17	36.07

Notice that the marginal cost bias is substantial, especially for the Nintendo GameCube, and that the traditional estimates understate the average marginal cost for producing game consoles. However, the average price-cost margins for consoles are all positive. In addition, the biased estimates completely overstate the profitability of the console side of the market.

While marginal cost data is proprietary, existing literature and news media have speculated that the marginal costs of consoles in the video game industry are typically greater than console prices upon release. These speculated marginal costs are larger than both the biased and the corrected cost estimates provided in Table 2. For example, in Lee (2013): “most platforms subsidize hardware sales, selling consoles close to or below cost, while charging publishers and developers a royalty for every game sold.” Evans et al. (2006) suggest that the Playstation 2 costed over \$400 per unit while the Xbox costed \$360 at release. White and Weyl (2016) assume that console marginal costs are \$400 in a calibration of their model of

the sixth generation video game market.³¹ In addition, Clements and Ohashi (2005) suggest that the marginal production cost of a console being greater than or equal to the console price was true for older consoles as well.³² In comparing the literature with the imputed marginal costs found in Table 2, the biased estimates clearly contradict the existing literature and this further highlights the importance of considering multi-sidedness when imputing costs with elasticity measures.

While the imputed average marginal costs are less than average prices, this result is in fact consistent with much of the existing literature. For example, Lee (2013) points out how decreasing costs might result in increasing price markups over time: “Most platform providers initially sold hardware platforms close to or below cost, with margins increasing over time as production costs fell.” In addition, Evans et al. (2006) find evidence that Sony earned profits from Playstation 2 consoles sold later on in this generation of gaming.³³ Thus, by considering average prices and average marginal costs, the estimates provided in Table 2 are reasonable.³⁴

5.3 Market Power and Profitability of Video Game Platforms

Given that the platforms earn profits on both sides of the market, using the generalized Lerner index that is defined by Equation (12) provides a more accurate estimate of platform market power than using the traditional Lerner index. Table 3 provides the generalized Lerner index that accounts for market power on both sides of the market using the biased and unbiased marginal cost estimates. In addition, platform unbiased variable profits (profits not including fixed costs) are provided in Table 3.

³¹Liu (2010) documents existing academic literature and news reports which all suggest that initial production costs are at or near console prices across platforms and generations.

³²For the more recent Xbox One and Playstation 4 consoles, an IHS report concludes that the consoles were launched at prices just above costs. Source: “Microsoft Xbox One Hardware Cost Comes in Below Retail Price, IHS Teardown Reveals,” by IHS.

³³Clements and Ohashi (2005) make a similar claim for the older console generations.

³⁴As a robustness check, the average royalties required to reach console marginal costs of the console retail launch price (\$300 for the PS2 and Xbox and \$200 for the Gamecube) are \$19.11 for PS2, \$21.84 for Xbox, and \$18.22 for Gamecube which are all unreasonable when the average price of a game is \$25.

Table 3: Market Power and Variable Profit for Video Game Platforms

	Playstation 2	Xbox	Gamecube
Market Power with Biased Costs	0.368	0.388	0.506
Market Power with Unbiased Costs	0.243	0.246	0.302
Unbiased Variable Profit (\$ mil.)	3,453.62	1,424.15	1,092.53

These market power estimates suggest that all three platforms have pricing power across the two sides of the market. Not surprisingly, market power is measured greater when the downward biased estimate for marginal cost of consoles is used. Thus, correctly estimating unobserved marginal costs is important when assessing market power in multi-sided markets; otherwise, a practitioner will overstate platform market power. One important distinction is that these measures of market power are with respect to a platform’s ability to price above marginal costs; however, this does not account for the ability to capture greater market share. From Table 3 we see that the Playstation 2 has the least amount of market power but obtains the most variable profit. This stems from their greater market share. This also implies that the greater market power for Nintendo relative to Microsoft and Sony suggests that Nintendo has greater differentiation from Sony and Microsoft which has created a niche market that is willing to pay greater markups.³⁵ At the same time, Sony and Microsoft are more direct competitors which prevents either of them from obtaining Nintendo’s greater pricing power even though each obtains greater market share.

5.4 Calibrated Merger Analysis

To better analyze multi-platform sellers and platform mergers, calibrated merger simulations are examined for the sixth generation video game market. By considering three types of hypothetical mergers — Sony with Microsoft, Sony with Nintendo, and Microsoft with Nintendo — and comparing the hypothetical outcome with the actual market equilibrium

³⁵In the last decade, the trend of Nintendo differentiating from Sony and Microsoft has become even more pronounced. This suggests that such a pricing power relationship continues to exist for Nintendo.

allows for a rich analysis of pricing and market power effects from platform mergers. In addition, the biased one-sided approach for merger simulations is also conducted for the hypothetical mergers to show the importance of considering the multi-sidedness of platform markets when performing merger simulation techniques.

To consider hypothetical mergers between video game platforms, Theorem 4 implies that the diversion ratios for consoles (D_1^{XY} and D_1^{YX}), and games (D_2^{XY} and D_2^{YX}), from changes in the platform sellers' price of consoles and game fees are required. These diversion ratios for consoles are computable with the cross-elasticity measures provided by Lee (2013). However, Lee (2013) only provides the elasticities for the console side of the market. Thus, the merger simulations for the sixth generation video game market will only consider the effect of mergers on console prices where game royalties are held constant across platforms. This implies that the post-merger console prices derived here are an upper bound on the post merger console prices. Using the own- and cross-price elasticities found in Lee (2013), Table 4 provides the console diversion ratios.³⁶

Table 4: Console Diversion Ratios

	Diversion Ratio
$D_1^{PS2/Xbox}$	-0.00718
$D_1^{PS2/GC}$	-0.00828
$D_1^{Xbox/PS2}$	-0.16672
$D_1^{Xbox/GC}$	-0.02504
$D_1^{GC/PS2}$	-0.13031
$D_1^{GC/Xbox}$	-0.04542

A console diversion ratio (D_1^{XY}), can be interpreted as the ratio of the consoles diverted to platform Y relative to the lost consoles of platform X when platform X increases the price of its console. Note that relatively weak $D_1^{PS2/Xbox}$ and $D_1^{PS2/GC}$ compared to the $D_1^{Xbox/PS2}$ and $D_1^{GC/PS2}$ implies that Sony will lose greater console sales to its competitors from a price

³⁶The computations of the diversion ratios from the data provided in Lee (2013) is provided in the appendix.

increase than Microsoft and Nintendo would lose to Sony from their own price increases. The moderate magnitudes of $D_1^{Xbox/GC}$ and $D_1^{GC/Xbox}$ suggest that a price increase by either will have a similar diversion to the other platform. This suggests that a merger between Sony and Microsoft (Nintendo) would result in a price increase for the Xbox (Gamecube) while only resulting in a limited price increase for the Playstation 2. These diversion ratios are also consistent with the share of console sales by the three platforms; Sony has a nearly 60% market share and so it is not surprising that a price increase in the Playstation 2 would result in greater diversion to other consoles (Sony has more to lose).

To analyze the effect of mergers on console prices, the interaction demand for consoles and games must be specified. With the limited data, assuming linear demand allows for a complete analysis of price, market share, and market power effects due to hypothetical mergers. Thus, suppose interaction demand of type i for platform X is given by:

$$\tilde{I}_i^X(\mathbf{p}^1, \mathbf{p}^2, \mathbf{p}^3) = a_i^X + \sum_{Y=1}^3 b_1^{XY} \cdot p_1^Y + \sum_{Y=1}^3 b_2^{XY} \cdot p_2^Y, \quad (16)$$

where a_i^X is the intercept of console sales ($i = 1$) or game sales ($i = 2$) on platform X ³⁷ and $b_i^{XY} = \frac{d\tilde{I}_i^X}{dp_i^Y}$ is the slope of demand with respect to p_i^Y for $Y = 1, 2, 3$. Note that the interaction derivative assumptions imply that $b_i^{XY} < 0$ for $i = 1, 2$ when $Y = X$ and $b_i^{XY} > 0$ for $i = 1, 2$ when $Y \neq X$.

The information used to determine the diversion ratios in Table 4 also provide the b_1^{XY} . In addition, since the game side of the market is held constant, $A_i^X \equiv a_i^X + \sum_{Y=1}^3 b_2^{XY} \cdot p_2^Y$ is constant. Using the market equilibrium console prices and sales provided in Table 1 when each game console is owned independently, and solving Equation (16) for A_1^X for each $X = PS2, Xbox,$ and GC provides the necessary parameters to characterize the linear demand for each console. The complete computation of the necessary parameters as well as the calculations for the three hypothetical mergers (Sony merging with Microsoft, Sony

³⁷Which depends on platform/console specific characteristics that are assumed constant through a merger.

merging with Nintendo, and Microsoft merging with Nintendo) is provided in the appendix. The main findings on post-merger console prices and shares are presented Table 5.

Table 5: Post-Merger Console Prices and Sales

	PS2 Price	Xbox Price	GC Price	PS2 Sales	Xbox Sales	GC Sales
PS2/Xbox	\$227.41 (+0.81)	\$223.92 (+9.65)	\$159.34 (+0.18)	30.06 (-0.01)	12.12 (-1.20)	9.85 (+0.02)
PS2/GC	\$227.27 (+0.66)	\$214.39 (+0.13)	\$166.69 (+7.54)	29.99 (-0.08)	13.34 (+0.02)	9.17 (-0.66)
Xbox/GC	\$226.72 (+0.11)	\$215.73 (+1.46)	\$161.64 (+2.49)	30.10 (+0.03)	13.15 (-0.17)	9.61 (-0.22)

Terms in parenthesis provide relative changes with respect to the pre-merger equilibrium.

First notice that all console prices increase, even those for the console that is not a part of the merger. In addition, console sales only increase for the seller left out of the merger but the increase is relatively small. Now consider the mergers by either Microsoft or Nintendo with Sony. In these cases, the price of the Playstation 2 increases slightly while the price of the other console increases considerably (\$7-\$10). This result is consistent with the analysis of the diversion ratios where Sony is found to be the platform with the most console share to lose from a price increase, so in a Sony merger we expect Sony's console price to increase the least. In the third case where Microsoft and Nintendo merge, the price increases are moderate for each console (\$1.50-\$2.50), which is in line with the symmetric diversion ratios between the two platforms.

To show the importance of considering multi-sidedness when performing merger simulations on platforms, the biased one-sided merger simulation is provided. For a consistent comparison, the biased simulation uses the corrected marginal cost estimates so that marginal costs are the same across the two simulations. Thus, by comparing the post-merger results found in Table 5 with the results using the biased merger simulation approach, only the unconsidered multi-sidedness in the platform seller solution generates the bias. If biased costs and a biased simulation are used, then both biases are present which limits our un-

derstanding of the differences between the simulations.³⁸ The biased one-sided post-merger results are provided in Table 6.

Table 6: Biased One-Sided Post-Merger Console Prices and Sales

	PS2 Price	Xbox Price	GC Price	PS2 Sales	Xbox Sales	GC Sales
PS2/Xbox	\$258.35	\$253.34	\$197.78	22.51	8.67	6.61
	(+32.25)	(+39.08)	(+38.62)	(-7.19)	(-4.65)	(-3.22)
PS2/GC	\$257.48	\$215.20	\$202.73	22.51	13.44	6.05
	(+30.87)	(+0.94)	(+43.56)	(-7.56)	(+0.12)	(-3.78)
Xbox/GC	\$258.34	\$246.98	\$199.43	22.91	9.47	6.44
	(+31.73)	(+32.72)	(+40.27)	(-7.16)	(-3.85)	(-3.39)

Terms in parenthesis provide relative changes with respect to the pre-merger equilibrium.

By comparing the two post-merger results, the biased merger simulation clearly produces upward biased estimates of console prices, which in turn leads to greater lost console sales. By not accounting for the lost profits that a platform seller faces — due to lost game sales from each lost console sale — the biased merger simulation results in post-merger price estimates that are too large. Thus, accounting for every type of interaction that generates profits or losses for platform sellers is critical in determining post-merger prices and sales. For the case of video games, by not considering the effect on platform profits from game sales, a practitioner obtains post-merger console prices that are biased upwards considerably. Such a result might lead to an unnecessary injunction of a platform merger.

Now consider the effect of mergers on platform market power and profits for the three hypothetical mergers. When platforms merge, prices for all platforms in the market increase. Thus, we expect market power to increase for every platform. Table 7 provides the unbiased estimates of platform profits and market power using the generalized Lerner index measure for multi-sided markets.

³⁸This will occur if the practitioner estimates marginal cost without considering the second side of the market. However, in this case, the resulting prices are similar to those found in Table 5 (but price-cost markups differ). Teasing out the impact of each type of bias for a biased merger simulations with biased estimated marginal costs is left for future research.

Table 7: Post-Merger Platform Market Power and Profits

	PS2 M.P.	Xbox M.P.	GC M.P.	PS2 Profit	Xbox Profit	GC Profit
PS2/Xbox	0.404 (+0.161)	0.408 (+0.162)	0.475 (+0.173)	\$3476.88 mil. (+23.25)	\$1412.81 (-11.34)	\$1096.07 (+3.54)
PS2/GC	0.403 (+0.160)	0.388 (+0.142)	0.491 (+0.189)	\$3463.85 (+10.22)	\$1427.52 (+3.37)	\$1087.72 (-4.81)
Xbox/GC	0.402 (+0.159)	0.391 (+0.145)	0.480 (+0.178)	\$3460.41 (+6.79)	\$1424.99 (+0.84)	\$1092.53 (+0.00)

Terms in parenthesis provide relative changes with respect to the pre-merger equilibrium.

The profits that decrease are a result of profit shifting from one console to another. For example, if Sony and Microsoft merge, the joint company will increase the console price for both the Playstation 2 and the Xbox so that total profits between the two platforms are maximized. This occurs with high console prices so that the Xbox profits decrease but the Playstation 2 increase in profits is substantial. Also notice that market power increases for all platforms after a merger. The increase in market power is sizable, between a 57% and 67% increase relative to the individual platform seller equilibrium. This suggests that a merger in the 6th generation video game will result in a platform market that is less competitive.

6 Conclusion

In multi-sided markets with network externalities, platform sellers often obtain market power by tipping the market. However, platform competition often persists with several platforms that differ in terms of market shares and pricing. To more accurately assess platform market power in a competitive environment, a weighted price-cost measure is developed that considers the profit margins across all sides of a platform.

In addition, the traditional methods for considering multi-product sellers and mergers are extended to multi-sided markets. By comparing these results with a traditional one-side approach, it becomes clear that the consideration of multi-sidedness is critical in deter-

mining multi-platform seller pricing, platform merger incentives, and potential post-merger outcomes.

To illustrate the main findings, the techniques are applied to the sixth generation video game market. I show that not considering just one side of the market results in biased estimates of console marginal costs, platform market power, and post-merger equilibria for hypothetical mergers between gaming platforms. This highlights the importance of using the techniques developed in this paper to assess market power and mergers in multi-sided markets.

Appendix of Proofs

Proof of Theorem 1: Each Platform X for $X = 1, \dots, N$ maximizes profits with respect to each available price p_i^X for $i = 1, \dots, K$. From Equations (8) and (9), Platform X has profits given by:

$$\Pi^X = \sum_{i=0}^M \sum_{j=0}^M \left\{ (p_i^X - c_i^X) \cdot I_i^X \left(p_i^X, \mathbf{p}_i^{-X}; \tilde{\mathbf{I}}_{-i}^1, \tilde{\mathbf{I}}_{-i}^2, \dots, \tilde{\mathbf{I}}_{-i}^N \right) \right\} - F^X.$$

The first-order conditions of profit maximization imply that

$$0 = I_i^X + (p_i^X - c_i^X) \cdot \frac{\partial I_i^X}{\partial p_i^X} + (p_i^X - c_i^X) \cdot \sum_{k \neq i} \left\{ \frac{\partial I_i^X}{\partial \tilde{I}_k^X} \cdot \frac{d\tilde{I}_k^X}{dp_i^X} \right\} + \sum_{k \neq i} \left\{ (p_k^X - c_k^X) \frac{\partial I_k^X}{\partial \tilde{I}_i^X} \cdot \frac{d\tilde{I}_i^X}{dp_i^X} \right\}.$$

Since I_i^X is given by Equations (7) and (8) so that $I_i^X = \tilde{I}_i^X(\mathbf{p}^1, \mathbf{p}^2, \dots, \mathbf{p}^N)$ $= I_i^X(p_i^X, \mathbf{p}_i^{-X}; \tilde{\mathbf{I}}_{-i}^1, \tilde{\mathbf{I}}_{-i}^2, \dots, \tilde{\mathbf{I}}_{-i}^N)$, taking the derivative with respect to p_i^X on both sides implies that $\frac{d\tilde{I}_i^X}{dp_i^X} = \frac{\partial I_i^X}{\partial p_i^X} + \sum_{k \neq i} \left\{ \frac{\partial I_i^X}{\partial \tilde{I}_k^X} \cdot \frac{d\tilde{I}_k^X}{dp_i^X} \right\}$. This implies that the first-order condition reduces to:

$$0 = I_i^X + (p_i^X - c_i^X) \cdot \frac{d\tilde{I}_i^X}{dp_i^X} + \sum_{k \neq i} \left\{ (p_k^X - c_k^X) \frac{\partial I_k^X}{\partial \tilde{I}_i^X} \cdot \frac{d\tilde{I}_i^X}{dp_i^X} \right\}.$$

Lastly, noting that $\frac{\partial I_k^X}{\partial I_i^X}$ is the change in interaction of type k due to a change in interaction of type i so that $\frac{\partial I_k^X}{\partial I_i^X} = \frac{\partial I_k^X}{\partial I_i^X}$. Solving for p_i^X implies:

$$p_i^X = c_i^X + \frac{-I_i^X}{d\bar{I}_i^X/dp_i^X} - \sum_{k \neq i} \left\{ \frac{\partial I_k^X}{\partial I_i^X} \cdot (p_k^X - c_k^X) \right\}.$$

Note that the elasticity of demand for interaction i on Platform X is given by $\eta_i^X = \frac{d\bar{I}_i^X}{dp_i^X} \cdot \frac{p_i^X}{I_i^X} = \frac{d\bar{I}_i^X}{dp_i^X} \cdot \frac{p_i^X}{I_i^X}$. This implies that the first-order condition becomes:

$$\left(1 + \frac{1}{\eta_i^X}\right) \cdot p_i^X = c_i^X - \sum_{k \neq i} \left\{ \frac{\partial I_k^X}{\partial I_i^X} \cdot (p_k^X - c_k^X) \right\}.$$

Rearranging terms implies that for all i we have:

$$\left(1 + \frac{1}{\eta_i^X}\right) \cdot p_i^X + \sum_{k \neq i} \frac{\partial I_k^X}{\partial I_i^X} \cdot p_k^X = c_i^X + \sum_{k \neq i} \frac{\partial I_k^X}{\partial I_i^X} \cdot c_k^X.$$

The collection of the above rearranged K first-order conditions for each platform can be written in vector form:

$$\begin{bmatrix} \left(1 + \frac{1}{\eta_1^X}\right) & \frac{\partial I_2^X}{\partial I_1^X} & \frac{\partial I_3^X}{\partial I_1^X} & \cdots & \frac{\partial I_K^X}{\partial I_1^X} \\ \frac{\partial I_1^X}{\partial I_2^X} & \left(1 + \frac{1}{\eta_2^X}\right) & \frac{\partial I_3^X}{\partial I_2^X} & \cdots & \frac{\partial I_K^X}{\partial I_2^X} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial I_1^X}{\partial I_K^X} & \frac{\partial I_2^X}{\partial I_K^X} & \frac{\partial I_3^X}{\partial I_K^X} & \cdots & \left(1 + \frac{1}{\eta_K^X}\right) \end{bmatrix} \begin{bmatrix} p_1^X \\ p_2^X \\ \vdots \\ p_K^X \end{bmatrix} = \begin{bmatrix} 1 & \frac{\partial I_2^X}{\partial I_1^X} & \frac{\partial I_3^X}{\partial I_1^X} & \cdots & \frac{\partial I_K^X}{\partial I_1^X} \\ \frac{\partial I_1^X}{\partial I_2^X} & 1 & \frac{\partial I_3^X}{\partial I_2^X} & \cdots & \frac{\partial I_K^X}{\partial I_2^X} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial I_1^X}{\partial I_K^X} & \frac{\partial I_2^X}{\partial I_K^X} & \frac{\partial I_3^X}{\partial I_K^X} & \cdots & 1 \end{bmatrix} \begin{bmatrix} c_1^X \\ c_2^X \\ \vdots \\ c_K^X \end{bmatrix}$$

The above equation reduces to $\mathbf{A}^X \mathbf{p}^X = \mathbf{B}^X \mathbf{c}^X$ and solving for prices, \mathbf{p}^X gives the unique equilibrium prices: $\mathbf{p}^X = \mathbf{A}^{-1} \mathbf{B}^X \mathbf{c}^X$. Each platform X takes the prices of all other platforms as given. Thus, equilibrium prices are $p_i^X \left(\eta^X; \mathbf{c}^X; \frac{\partial I_k^X}{\partial I_i^X} \right)$.

The first derivative assumptions that $\frac{\partial I_i^X}{\partial p_i^X} < 0$ and $\frac{\partial I_i^X}{\partial p_i^Y} \geq 0$ for $Y \neq X$; along with the second derivative assumptions that $\frac{\partial (I_i^X)^2}{\partial^2 p_i^X} \leq 0$ and $\frac{\partial (I_i^X)^2}{\partial^2 I_{-i}^X} < 0$ imply that the second-order conditions hold (the second derivative Hessian matrix is negative definite). This implies that

the equilibrium prices result in a local maximum. The assumptions that $\frac{\partial I_i^X}{\partial p_i^X} \rightarrow 0$ and that $\frac{\partial I_i^X}{\partial I_{-i}^X} \rightarrow 0$ when $\frac{\partial I_i^X}{\partial I_{-i}^X} > 0$ imply that the local maximum is indeed a global max so that the platform equilibrium is unique. \square

Proof of Theorem 2: Given that the elasticity of demand for interaction of type i on Platform X is given by $\eta_i^X = \frac{d\tilde{I}_i^X}{dp_i^X} \cdot \frac{p_i^X}{\tilde{I}_i^X} = \frac{d\tilde{I}_i^X}{dp_i^X} \cdot \frac{p_i^X}{I_i^X}$, dividing both sides of Equation (10) by p_i^X implies:

$$\frac{p_i^X}{p_i^X} = \frac{c_i^X}{p_i^X} + \frac{-I_i^X/p_i^X}{d\tilde{I}_i^X/dp_i^X} - \frac{\sum_{k \neq i} \left\{ \frac{\partial I_k^X}{\partial I_i^X} \cdot (p_k^X - c_k^X) \right\}}{p_i^X}.$$

Moving the first and third terms of the right-hand side to the left-hand side implies:

$$\frac{p_i^X - c_i^X}{p_i^X} + \frac{\sum_{k \neq i} \left\{ \frac{\partial I_k^X}{\partial I_i^X} \cdot (p_k^X - c_k^X) \right\}}{p_i^X} = - \frac{I_i^X/p_i^X}{d\tilde{I}_i^X/dp_i^X} = - \frac{1}{\eta_i}.$$

\square

Proof of Theorem 3: Rearranging terms in the equilibrium price equation in Theorem 1, Equation (10), implies that for all k we have:

$$\left(1 + \frac{1}{\eta_i^X}\right) \cdot p_i^X + \sum_{k \neq i} \frac{\partial I_k^X}{\partial I_i^X} \cdot p_k^X = c_i^X + \sum_{k \neq i} \frac{\partial I_k^X}{\partial I_i^X} \cdot c_k^X.$$

where η_k^X is the demand elasticity for interaction of type k on platform X . This implies that there are K equations and K the unknown marginal costs (c_k^X), when prices, elasticities, and interaction derivatives are known. Thus,

$$\begin{bmatrix} \left(1 + \frac{1}{\eta_1^X}\right) & \frac{\partial I_2^X}{\partial I_1^X} & \frac{\partial I_3^X}{\partial I_1^X} & \cdots & \frac{\partial I_K^X}{\partial I_1^X} \\ \frac{\partial I_1^X}{\partial I_2^X} & \left(1 + \frac{1}{\eta_2^X}\right) & \frac{\partial I_3^X}{\partial I_2^X} & \cdots & \frac{\partial I_K^X}{\partial I_2^X} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial I_1^X}{\partial I_K^X} & \frac{\partial I_2^X}{\partial I_K^X} & \frac{\partial I_3^X}{\partial I_K^X} & \cdots & \left(1 + \frac{1}{\eta_K^X}\right) \end{bmatrix} \begin{bmatrix} p_1^X \\ p_2^X \\ \vdots \\ p_K^X \end{bmatrix} = \begin{bmatrix} 1 & \frac{\partial I_2^X}{\partial I_1^X} & \frac{\partial I_3^X}{\partial I_1^X} & \cdots & \frac{\partial I_K^X}{\partial I_1^X} \\ \frac{\partial I_1^X}{\partial I_2^X} & 1 & \frac{\partial I_3^X}{\partial I_2^X} & \cdots & \frac{\partial I_K^X}{\partial I_2^X} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial I_1^X}{\partial I_K^X} & \frac{\partial I_2^X}{\partial I_K^X} & \frac{\partial I_3^X}{\partial I_K^X} & \cdots & 1 \end{bmatrix} \begin{bmatrix} c_1^X \\ c_2^X \\ \vdots \\ c_K^X \end{bmatrix}$$

The above equation reduces to $\mathbf{A}^X \mathbf{p}^X = \mathbf{B}^X \mathbf{c}^X$ and solving for costs, \mathbf{c}^X , requires the inverse

matrix of \mathbf{B}^X . If $K > 1$, then \mathbf{B}^X is not invertible. Furthermore, the rank of \mathbf{B}^X is equal to 1 for any number of sides K . Thus, at most one unobserved cost, c_l , can be determined from the system of equations when all the other $K - 1$ costs are observed. In this case, only the variables in the first-order condition for interaction l are required to determine the unobserved c_l . Those variables are all prices (p_1, p_2, \dots, p_K) , all other marginal costs $(c_j$ for $j \neq l)$, all interaction derivatives with respect to that interaction $(\frac{\partial I_j}{\partial I_l}$ for all $j \neq l)$, and the elasticity of demand for that interaction (η_l) . That is, $c_i^X = c_i^X \left(\eta_l^X; \mathbf{p}^X; \mathbf{c}_{-i}^X; \frac{\partial \mathbf{I}_k^X}{\partial \mathbf{I}_i^X} \right)$ where \mathbf{p}^X denotes the vector of prices of platform X for all types of interactions, \mathbf{c}_{-i}^X denotes the vector of all other costs (not c_i^X) of platform X , and $\frac{\partial \mathbf{I}_k^X}{\partial \mathbf{I}_i^X}$ denotes the vector of interaction derivatives across all interactions with respect to interaction type i for platform X . \square

Proof of Theorem 4: Each platform seller maximizes their individual profits with respect to platform prices for the platforms that they own. This results in K first-order conditions for each of the N platforms. For the platform seller that sells platforms X and Y , the first-order condition with respect to price p_i^X implies:

$$0 = (p_i^X - c_i^X) \cdot \frac{d\tilde{I}_i^X}{dp_i^X} + I_i^X + \sum_{k \neq i} \left\{ \frac{\partial I_k^X}{\partial I_i^X} \cdot \frac{d\tilde{I}_i^X}{dp_i^X} \cdot (p_k^X - c_k^X) \right\} \\ + (p_i^Y - c_i^Y) \cdot \frac{d\tilde{I}_i^Y}{dp_i^X} + \sum_{k \neq i} \left\{ \frac{\partial I_k^Y}{\partial I_i^Y} \cdot \frac{d\tilde{I}_i^Y}{dp_i^X} \cdot (p_k^Y - c_k^Y) \right\}.$$

Solving for p_i^X implies:

$$p_i^X = c_i^X + \frac{-I_i^X}{d\tilde{I}_i^X/dp_i^X} - \sum_{k \neq i} \frac{\partial I_k^X}{\partial I_i^X} \cdot (p_k^X - c_k^X) - D_i^{XY} \cdot (p_i^Y - c_i^Y) - D_i^{XY} \cdot \sum_{k \neq i} \frac{\partial I_k^Y}{\partial I_i^Y} \cdot (p_k^Y - c_k^Y),$$

where $D_i^{XY} \equiv \frac{d\tilde{I}_i^Y/dp_i^X}{d\tilde{I}_i^X/dp_i^X} < 0$ is the diversion ratio between platforms X and Y of interaction type i .

Consider the first order conditions for platform X in vector form:

$$\begin{aligned}
& \begin{bmatrix} \left(1 + \frac{1}{\eta_1^X}\right) & \frac{\partial I_2^X}{\partial I_1^X} & \cdots & \frac{\partial I_K^X}{\partial I_1^X} \\ \frac{\partial I_1^X}{\partial I_2^X} & \left(1 + \frac{1}{\eta_2^X}\right) & \cdots & \frac{\partial I_K^X}{\partial I_2^X} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial I_1^X}{\partial I_K^X} & \frac{\partial I_2^X}{\partial I_K^X} & \cdots & \left(1 + \frac{1}{\eta_K^X}\right) \end{bmatrix} \begin{bmatrix} p_1^X \\ p_2^X \\ \vdots \\ p_K^X \end{bmatrix} + \begin{bmatrix} D_1^{XY} & D_1^{XY} \frac{\partial I_2^Y}{\partial I_1^Y} & \cdots & D_1^{XY} \frac{\partial I_K^Y}{\partial I_1^Y} \\ D_2^{XY} \frac{\partial I_1^Y}{\partial I_2^Y} & D_2^{XY} & \cdots & D_2^{XY} \frac{\partial I_K^Y}{\partial I_2^Y} \\ \vdots & \vdots & \ddots & \vdots \\ D_K^{XY} \frac{\partial I_1^Y}{\partial I_K^Y} & D_K^{XY} \frac{\partial I_2^Y}{\partial I_K^Y} & \cdots & D_K^{XY} \end{bmatrix} \begin{bmatrix} p_1^Y \\ p_2^Y \\ \vdots \\ p_K^Y \end{bmatrix} \\
& = \begin{bmatrix} 1 & \frac{\partial I_2^X}{\partial I_1^X} & \cdots & \frac{\partial I_K^X}{\partial I_1^X} \\ \frac{\partial I_1^X}{\partial I_2^X} & 1 & \cdots & \frac{\partial I_K^X}{\partial I_2^X} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial I_1^X}{\partial I_K^X} & \frac{\partial I_2^X}{\partial I_K^X} & \cdots & 1 \end{bmatrix} \begin{bmatrix} c_1^X \\ c_2^X \\ \vdots \\ c_K^X \end{bmatrix} + \begin{bmatrix} D_1^{XY} & D_1^{XY} \frac{\partial I_2^Y}{\partial I_1^Y} & \cdots & D_1^{XY} \frac{\partial I_K^Y}{\partial I_1^Y} \\ D_2^{XY} \frac{\partial I_1^Y}{\partial I_2^Y} & D_2^{XY} & \cdots & D_2^{XY} \frac{\partial I_K^Y}{\partial I_2^Y} \\ \vdots & \vdots & \ddots & \vdots \\ D_K^{XY} \frac{\partial I_1^Y}{\partial I_K^Y} & D_K^{XY} \frac{\partial I_2^Y}{\partial I_K^Y} & \cdots & D_K^{XY} \end{bmatrix} \begin{bmatrix} c_1^Y \\ c_2^Y \\ \vdots \\ c_K^Y \end{bmatrix}
\end{aligned}$$

The above equation reduces to $\mathbf{A}^X \mathbf{p}^X + \mathbf{D}^{XY} \mathbf{p}^Y = \mathbf{B}^X \mathbf{c}^X + \mathbf{D}^{XY} \mathbf{c}^Y$ and similarly for the first-order conditions for Platform Y there exists $\mathbf{A}^Y \mathbf{p}^Y + \mathbf{D}^{YX} \mathbf{p}^X = \mathbf{B}^Y \mathbf{c}^Y + \mathbf{D}^{YX} \mathbf{c}^X$. Solving for prices implies that equilibrium prices depend on owned interaction elasticities, diversion ratios, owned interaction derivatives, and owned costs: $p_i^X \left(\eta^X, \eta^Y; \mathbf{c}^X, \mathbf{c}^Y; \frac{\partial \mathbf{I}_k^X}{\partial I_i^X}, \frac{\partial \mathbf{I}_k^Y}{\partial I_i^Y}; \mathbf{D}^{XY}, \mathbf{D}^{YX} \right)$.

Equilibrium with a General Ownership Matrix:

More generally, consider the equilibrium with a general platform ownership structure. To solve the system of first-order conditions across all K prices for all N platforms, consider the following matrix notation. Recall that \mathbf{p}^X is the K dimensional vector of prices for platform X . Let \mathbf{c}^X be the K dimensional vector of marginal costs for platform X . Denote the $K \cdot N$ dimensional vector of all prices (marginal costs) by $\mathbf{p}(\mathbf{c})$. Similarly, let $\tilde{\mathbf{I}}^X$ be the K dimensional vector of interactions for platform X and denote the $K \cdot N$ dimensional vector of all interactions by $\tilde{\mathbf{I}}$. Now consider the derivative matrices. Let $\frac{d\tilde{\mathbf{I}}^X}{d\mathbf{p}^Y}$ denote the $K \times K$

matrix of derivatives of platform X 's interactions with respect to platform Y 's prices:

$$\frac{d\tilde{\mathbf{I}}^X}{d\mathbf{p}^Y} = \begin{bmatrix} \frac{d\tilde{I}_1^X}{dp_1^Y} & \frac{d\tilde{I}_1^X}{dp_2^Y} & \cdots & \frac{d\tilde{I}_1^X}{dp_K^Y} \\ \frac{d\tilde{I}_2^X}{dp_1^Y} & \frac{d\tilde{I}_2^X}{dp_2^Y} & \cdots & \frac{d\tilde{I}_2^X}{dp_K^Y} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{d\tilde{I}_K^X}{dp_1^Y} & \frac{d\tilde{I}_K^X}{dp_2^Y} & \cdots & \frac{d\tilde{I}_K^X}{dp_K^Y} \end{bmatrix}$$

Denote the $K \cdot N \times K \cdot N$ matrix of all derivatives by $\frac{d\tilde{\mathbf{I}}}{d\mathbf{p}}$ where

$$\frac{d\tilde{\mathbf{I}}}{d\mathbf{p}} = \begin{bmatrix} \frac{d\tilde{\mathbf{I}}^1}{d\mathbf{p}^1} & \frac{d\tilde{\mathbf{I}}^1}{d\mathbf{p}^2} & \cdots & \frac{d\tilde{\mathbf{I}}^1}{d\mathbf{p}^N} \\ \frac{d\tilde{\mathbf{I}}^2}{d\mathbf{p}^1} & \frac{d\tilde{\mathbf{I}}^2}{d\mathbf{p}^2} & \cdots & \frac{d\tilde{\mathbf{I}}^2}{d\mathbf{p}^N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{d\tilde{\mathbf{I}}^N}{d\mathbf{p}^1} & \frac{d\tilde{\mathbf{I}}^N}{d\mathbf{p}^2} & \cdots & \frac{d\tilde{\mathbf{I}}^N}{d\mathbf{p}^N} \end{bmatrix}$$

where each $\frac{d\tilde{\mathbf{I}}^X}{d\mathbf{p}^Y}$ is the $K \times K$ matrix given above.

Lastly, consider the platform ownership matrix. In the case where each platform is owned by an individual seller as in Section 3, the across platform derivatives (the ‘‘off-diagonal’’ matrices in $\frac{d\tilde{\mathbf{I}}}{d\mathbf{p}}$), do not directly affect platform pricing. To capture the cross platform effects that depend on platform ownership, the ownership matrix is required. Denote the $K \cdot N \times K \cdot N$ ownership matrix by \mathbf{O} :

$$\mathbf{O} = \begin{bmatrix} \mathbf{1} & \mathbf{1}_{12} & \mathbf{1}_{13} & \cdots & \mathbf{1}_{1N} \\ \mathbf{1}_{12} & \mathbf{1} & \mathbf{1}_{23} & \cdots & \mathbf{1}_{2N} \\ \mathbf{1}_{13} & \mathbf{1}_{23} & \mathbf{1} & \cdots & \mathbf{1}_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{1}_{1N} & \mathbf{1}_{2N} & \mathbf{1}_{3N} & \cdots & \mathbf{1} \end{bmatrix}$$

where $\mathbf{1}$ is the $K \times K$ matrix with every input being a 1 and $\mathbf{1}_{XY}$ is a $K \times K$ matrix with every input being a 1 if platforms X and Y are owned by the same platform seller, otherwise

$\mathbf{1}_{XY}$ is a $K \times K$ matrix with every input being a 0. Thus, the first-order conditions with a general platform ownership matrix are given by:

$$\mathbf{0} = (\mathbf{p} - \mathbf{c}) \frac{d\tilde{\mathbf{I}}}{d\mathbf{p}} \odot \mathbf{O} + \mathbf{I},$$

where $\mathbf{0}$ is the $K \cdot N$ dimensional zero vector and \odot denotes the input by input multiplication operator between matrices of the same dimension.

Thus, elasticities and diversion ratios with respect to all platforms owned by the same seller are need to determine equilibrium prices. \square

Proof of Theorem 5: From the proof of Theorem 4 we have the first-order conditions in matrix form so that $\mathbf{A}^X \mathbf{p}^X + \mathbf{D}^{XY} \mathbf{p}^Y = \mathbf{B}^X \mathbf{c}^X + \mathbf{D}^{XY} \mathbf{c}^Y$ and similarly for the first-order conditions for Platform Y there exists $\mathbf{A}^Y \mathbf{p}^Y + \mathbf{D}^{YX} \mathbf{p}^X = \mathbf{B}^Y \mathbf{c}^Y + \mathbf{D}^{YX} \mathbf{c}^X$. The second-order conditions imply that with integration, the $\frac{\partial I_k^X}{\partial I_i^X} > 0$ becomes smaller since $\frac{\partial (I_i^X)^2}{\partial^2 I_{-i}^X} < 0$, and the $\frac{d\tilde{I}_i^X}{dp_i^X} < 0$ becomes smaller in magnitude but remains negative since $\frac{\partial I_i^X}{\partial I_{-i}^X} > 0$.

This implies that with integration, the off diagonal inputs in matrices \mathbf{A}^X and \mathbf{B}^X decrease by the same amount at each input location; however, the diagonals in matrix \mathbf{A}^X also decrease since η_k^X becomes smaller in magnitude but remains negative so that $\left(1 - \frac{1}{\eta_k^X}\right)$ is smaller with integration. Lastly, each input in matrix \mathbf{D}^{XY} decreases with integration. In total, this implies that with integration, the matrix term \mathbf{A}^X is diminished the greatest so that in solving for prices the greater inverse of \mathbf{A}^X outweighs the other effects and prices increase with integration. \square

Computations of Table 4: To compute all six D_1^{XY} in Table 4, the nine $\frac{d\tilde{I}_1^X}{dp_1^Y}$ for $X, Y = PS2, Xbox,$ and GC must be computed. Fortunately, an elasticity η_1^{XY} is defined as $\eta_1^{XY} \equiv \frac{d\tilde{I}_1^X}{dp_1^Y} \cdot \frac{p_1^Y}{I_1^X}$ so that cross- and own-price elasticities for consoles provided by Lee (2013)³⁹ and

³⁹From Table 4 in Lee (2013), the cross elasticities of console demand are: $\eta^{PS2/PS2} = -1.973$, $\eta^{PS2/Xbox} = 0.032$, $\eta^{PS2/GC} = 0.050$, $\eta^{Xbox/PS2} = 0.148$, $\eta^{Xbox/Xbox} = -2.004$, $\eta^{Xbox/GC} = 0.068$, $\eta^{GC/PS2} = 0.061$, $\eta^{GC/Xbox} = 0.048$, and $\eta^{GC/GC} = -1.432$.

the data on p_1 and I_1 from Table 1 imply that all nine $\frac{d\tilde{I}_1^X}{dp_1^Y} = \eta_1^{XY} \cdot \frac{I_1^X}{p_1^Y}$ are determinable.

$$\begin{bmatrix} \frac{d\tilde{I}_1^1}{dp_1^1} & \frac{d\tilde{I}_1^1}{dp_1^2} & \frac{d\tilde{I}_1^1}{dp_1^3} \\ \frac{d\tilde{I}_1^2}{dp_1^1} & \frac{d\tilde{I}_1^2}{dp_1^2} & \frac{d\tilde{I}_1^2}{dp_1^3} \\ \frac{d\tilde{I}_1^3}{dp_1^1} & \frac{d\tilde{I}_1^3}{dp_1^2} & \frac{d\tilde{I}_1^3}{dp_1^3} \end{bmatrix} = \begin{bmatrix} -0.26181 & 0.02077 & 0.01153 \\ 0.00188 & -0.12458 & 0.00402 \\ 0.00217 & 0.00312 & -0.08845 \end{bmatrix}$$

where $1 = PS2$, $2 = Xbox$, and $3 = GC$; and the units are in millions of consoles.

Given all nine $\frac{d\tilde{I}_1^X}{dp_1^Y}$ for $X, Y = PS2, Xbox, \text{ and } GC$, the six diversion ratios are given by $D_1^{XY} = \frac{d\tilde{I}_1^Y/dp_1^X}{d\tilde{I}_1^X/dp_1^X}$ for $X \neq Y = PS2, Xbox, \text{ and } GC$. Computing provides the diversion ratios provide in Table 4. \square

Hypothetical Merger Simulations: First consider the parameter computations for console demands. Each $b_1^{XY} = \frac{d\tilde{I}_1^X}{dp_1^Y}$ which were used to Compute Table 4 and are provided in the above proof. Given the b_1 and the p_1 and I_1 from Table 1, the A_1^X for $X = 1, 2, 3$ are determined: $A_1^X = I_1^X - b_1^{XX} \cdot p_1^X - b_1^{XY} \cdot p_1^Y - b_1^{XZ} \cdot p_1^Z$. This implies that the linear console demand functions are approximately given by:

$$\begin{aligned} \tilde{I}_1^{PS2} &= 83.113 - 0.26181 \cdot p_1^{PS2} + 0.02077 \cdot p_1^{Xbox} + 0.01152 \cdot p_1^{GC}, \\ \tilde{I}_1^{Xbox} &= 38.948 + 0.00188 \cdot p_1^{PS2} - 0.12458 \cdot p_1^{Xbox} + 0.00402 \cdot p_1^{GC}, \\ \tilde{I}_1^{GC} &= 22.747 + 0.00217 \cdot p_1^{PS2} + 0.00312 \cdot p_1^{Xbox} - 0.08844 \cdot p_1^{GC}. \end{aligned}$$

With the game side of the market held constant, the first-order conditions with respect to the three console prices for a general ownership matrix reduces to:

$$(p_1^1 - c_1^1, p_1^2 - c_1^2, p_1^3 - c_1^3) \begin{bmatrix} \frac{d\tilde{I}_1^1}{dp_1^1} & \frac{d\tilde{I}_1^1}{dp_1^2} & \frac{d\tilde{I}_1^1}{dp_1^3} \\ \frac{d\tilde{I}_1^2}{dp_1^1} & \frac{d\tilde{I}_1^2}{dp_1^2} & \frac{d\tilde{I}_1^2}{dp_1^3} \\ \frac{d\tilde{I}_1^3}{dp_1^1} & \frac{d\tilde{I}_1^3}{dp_1^2} & \frac{d\tilde{I}_1^3}{dp_1^3} \end{bmatrix} \odot \mathbf{O} + (\tilde{I}_1^1, \tilde{I}_1^2, \tilde{I}_1^3)$$

$$+ (p_2^1 - c_2^1, p_2^2 - c_2^2, p_2^3 - c_2^3) \begin{bmatrix} \frac{\partial I_2^1}{\partial p_1^1} & \frac{\partial I_2^1}{\partial p_1^2} & \frac{\partial I_2^1}{\partial p_1^3} \\ \frac{\partial I_2^2}{\partial p_1^1} & \frac{\partial I_2^2}{\partial p_1^2} & \frac{\partial I_2^2}{\partial p_1^3} \\ \frac{\partial I_2^3}{\partial p_1^1} & \frac{\partial I_2^3}{\partial p_1^2} & \frac{\partial I_2^3}{\partial p_1^3} \end{bmatrix} \odot \mathbf{O} = (0, 0, 0).$$

where 1 = *PS2*, 2 = *Xbox*, and 3 = *GC*; \mathbf{O} is the 3×3 ownership matrix for the three consoles; and $\frac{\partial I_2^X}{\partial p_1^Y} = \frac{\partial I_2^X}{\partial \tilde{I}_1^X} \cdot \frac{d\tilde{I}_1^X}{dp_1^Y}$ with $X, Y = 1, 2, 3$.

Thus, there are three unknowns, the console prices (p_1^1, p_1^2, p_1^3) , and solving the three systems of equations for the each of the three different hypothetical mergers provides the merger simulation prices. To determine the biased price estimates, the second derivative matrix in the above simulation disappears (the game side is unaccounted for), and solving for the three prices without the second side terms provides the biased merger simulation prices. Consoles shares are determined by the linear demand equations $(\tilde{I}_1^1, \tilde{I}_1^2, \tilde{I}_1^3)$, provided above. □

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