Single-Dimensional Versus Multi-Dimensional Product Ratings in Online Marketplaces

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We examine the strategic implications of multi-dimensional and single-dimensional rating schemes in online review platforms when consumers have heterogeneous preferences regarding the product quality attributes. The multi-dimensional rating scheme provides a separate rating for each product attribute, whereas the single-dimensional scheme gives one overall rating for each product. The multi-dimensional rating scheme is superior to the single-dimensional rating scheme in reducing consumers' uncertainty about the value of a product. However, we show that, when sellers respond to product ratings by adjusting their prices, the multi-dimensional rating scheme does not always benefit consumers. Furthermore, the multi-dimensional rating scheme does not necessarily benefit the platform, the sellers or the society compared to the single-dimensional rating scheme. We find that factors such as whether firms specialize in an attribute ("*attribute specialization*"), whether competing firms offer differentiated products ("*firm differentiation*"), and whether the product is an *experience good* or a *credence good* ("*product type*") play significant roles in determining the impact of a rating scheme on the platform, consumers, and the society. The main driver of the results is that the multi-dimensional ratings amplify the consumers' perceptions of any underlying differentiation in the attributes of competing products, while simultaneously exposing the similarity between the products if they are homogeneous. In contrast, single-dimensional ratings mask the differentiation between attributes when the products are different but reflect the heterogeneous consumer preferences even when the products are similar. Therefore, the rating scheme alters the heterogeneity in consumer perceptions and utility, which, in turn, affects the upstream price competition. The results demonstrate that focusing on the information transfer aspect of rating schemes provides only a partial understanding of the true impact of rating schemes and that the platform's optimal choice of a rating scheme depends critically on the product markets served by the platform.

Key words: online product reviews; competition; retail platforms; analytical modeling; economics of IS

1. Introduction

Online retail platforms rely on product reviews to mitigate consumer uncertainty about the quality of products sold in these platforms (Hong and Pavlou 2014, Kwark et al. 2014). The popularity of third-party review sites such as Yelp.com also underscores the importance of product reviews in online marketplaces. Product reviews are especially valuable when consumers have heterogeneous preferences about product attributes and competing products differ in these attributes (Chen et al. 2017). In such cases, product reviews help consumers find products that offer better value to them.

Online platforms elicit and display reviews using a variety of formats, which vary in their structure and content. Many platforms use numerical and/or qualitative rating scales for structured reviews and free form text for unstructured reviews. For instance, in Amazon.com and Yelp.com, the reviewers provide an overall rating for a product on a 5-star scale and may also support their numerical rating with a text review. Some platforms such as TripAdvisor.com have expanded the idea of a single overall product rating and provide separate ratings for different product attributes. For instance, TripAdvisor.com rates restaurants on four attributes: food, service, value, and atmosphere. TripAdvisor's rating system is referred to as *multi-dimensional rating scheme* (MD hereafter) whereas Amazon's system of providing a single overall rating is referred to as *single-dimensional rating scheme* (SD hereafter). While each review system has its own merits and drawbacks, the purported intention common to all review systems is to efficiently transfer product information to prospective buyers so that they can make more informed purchase decisions.

Several studies have examined SD, MD, and text reviews from the information transfer perspective (Chen et al. 2017, Ge and Li 2015, Archak et al. 2011). Assessing a rating scheme on the basis of reduction in consumer uncertainty is useful; however, it provides only a partial characterization of the true impact of a rating scheme. The consumer uncertainty about product value plays a significant role in shaping the price competition between sellers, and therefore has impact on the platform, consumers, and the society. In particular, a platform's choice of the rating scheme will depend critically on how it affects the platform's profitability. Despite the potential strategic implications of rating schemes, the extant literature provides little guidance to platforms in understanding these impacts and choosing the appropriate rating scheme.

In this paper, we examine how SD and MD affect the competition between firms that sell on the platform, the platform's profitability, consumer surplus, and social welfare. We perform the analysis by developing a game theoretical model of a context in which a retail platform sells products from two competing firms. Each product is characterized by two attributes. Consumers have heterogeneous preferences for these attributes and the products may also be differentiated along these attributes.

The study reveals several interesting findings that have important implications. The impact of SD or MD on the platform, consumers, and the society depends critically on three factors: (i) *attribute specialization* which refers to a firm choosing to specialize in an attribute by offering a high quality for that attribute and a low quality for the other attribute, (ii) *firm differentiation* which refers to competing products differing in the quality level on at least one attribute, and (iii) *product type* which refers to whether the product is an *experience good* or a *credence good*. If neither firm specializes in an attribute, then MD offers a higher profit to the platform than SD if and only if the product is a credence good and the firms are not differentiated. On the other

hand, if at least one firm specializes in an attribute, then MD offers a higher profit to the platform than SD only when the firms are differentiated, or the product is a credence good and the ratings have a low precision. The impact of the rating scheme on consumers is qualitatively opposite to that on the platform. That is, if MD is superior to SD from the platform's perspective, then MD is inferior to SD from the consumer surplus perspective. Therefore, while consumers have more precise information in MD than SD, consumers are not necessarily better off under MD. Generally, there is no difference between MD and SD from the social welfare perspective except when at least one firm specializes in an attribute and the firms are differentiated in which case either SD or MD can result in a higher social welfare. These results imply that any benefit of MD to consumers (platform) generally comes at the platform's (consumers') expense and therefore, a shift to MD from SD or vice versa does not result in a win-win situation for the platform and consumers. Finally, an increase in the rating precision decreases the relative attractiveness of MD from the platform's perspective in all cases except when one firm has a clear quality advantage over the other in both attributes, implying that in such scenarios a platform is more likely to prefer deploying MD when the product is a credence good than when it is an experience good.

The main intuition for the above results relates to how the rating schemes influence consumers' perceptions of competing products. Clearly, a heterogeneous consumer population (in terms of their quality perceptions and utility) enables competing sellers to target different consumer segments and reduce the intensity of price competition between them, thereby helping sellers as well as the platform that receives commission from the sellers. MD highlights, and could possibly amplify, any underlying differentiation in quality levels of attributes of competing products by providing a rating for each attribute of each product. On the other hand, when the products are homogeneous in their quality levels, MD also exposes the similarity between the products. In contrast, SD masks the differentiation between attributes by forcing consumers to aggregate the ratings of attributes; however, even when the products are homogeneous, SD still reflects the heterogeneous consumer preference and therefore fails to expose the similarity between products. Consequently, the heterogeneity in ratings and hence in the consumers' perceptions of product quality, which are influenced by the rating scheme as well as market/product characteristics, is exploited by the sellers and the platform. Essentially, the price competition between the upstream firms could be fundamentally altered by the rating scheme. As a result, the impact of rating schemes goes well beyond consumer uncertainty reduction and the strategic implications on seller competition affect all parties in the marketplace including the platform, consumers, and the society.

1.1. Relationship to Prior Literature

Much of the literature on online product reviews has focused on the impact of single-dimensional product ratings on product sales and firm revenue (Godes and Mayzlin 2004, Liu 2006, Dellarocas

et al. 2007, Forman et al. 2008, Zhu and Zhang 2010, Moe and Trusov 2011). For instance, Luca (2016) found that a one star increase in a restaurant's Yelp rating leads to a 5-9% increase in its revenue. However, several other studies have yielded mixed findings on the question of whether online ratings have a significant impact on sales or revenue (Chevalier and Mayzlin 2006, Duan et al. 2008). In particular, Chevalier and Mayzlin (2006) found that online ratings influence book sales. On the other hand, Duan et al. (2008) concluded that online ratings have no notable impact on movies' box office revenue. Scholars have explained these mixed findings by pointing out the shortcoming of single dimensional ratings in ignoring the multi-dimensional aspects of product quality and heterogeneous consumer preferences for these dimensions (Archak et al. 2011, Godes and Silva 2012, Moe and Schweidel 2012). Furthermore, some studies cautioned that single dimensional ratings may not perfectly reveal the true quality of products (Li and Hitt 2008, Hu et al. 2009, Godes and Silva 2012, Ho et al. 2017, Hu et al. 2017). For instance, Ho et al. (2017) showed that an individual is more likely to leave a review when the magnitude of disconfirmation - the difference between expected and experienced quality of a product - she encounters is larger. As a result, the rating she gives may not necessarily reflect her post-purchase evaluation only.

The main premise of research on multi-dimensional ratings is that they are more effective in transferring consumer experience than single-dimensional ratings (Archak et al. 2011, Godes and Silva 2012, Moe and Schweidel 2012, Chen et al. 2017). To that end, Chen et al. (2017) showed that a multi-dimensional rating system helps consumers find products that better fit their preferences and increases the confidence in their choices. Ghose and Ipeirotis (2011) used text mining to uncover the important product attributes from review text and quantified how these attributes affect product sales. Moghaddam and Ester (2011) proposed a Linear Discriminant Analysis (LDA) model to estimate the dimensional ratings from review text. Ge and Li (2015) argued that consumers' ratings are influenced by extreme experience in some dimensions only. Using text mining of online reviews, they showed that a multi-dimensional rating system suffers less from this issue than a single-dimensional rating system.

Our work differs from the previous literature on product reviews in the sense that we focus on the strategic implications of a platform's rating scheme choice on the upstream competing firms that sell through the platform. Prior literature has mostly focused on the influence of a rating scheme on downstream consumers. Furthermore, we perform the analysis using a game theoretical model whereas previous studies have primarily employed empirical and data mining techniques to characterize the value of product ratings.

Our work is also related to the research on product information disclosure by firms. Several studies in this stream have focused on the strategic implications of information disclosure on multiple product attributes (Hotz and Xiao 2013, Sun 2011). For instance, Hotz and Xiao (2013) examined

the incentives of firms to disclose product quality information. They showed that quality disclosure could intensify price competition, and therefore firms could choose nondisclosure. Sun (2011) showed that a monopolist does not always reveal the horizontal location information when the quality of its product is publicly known. Our work differs from the product information disclosure literature in several ways. First, we consider disclosure of two quality attributes in two scenarios, namely disclosure using a single measure and disclosure using a separate measure for each attribute. On the other hand, the product information disclosure literature focused on disclosure of a single attribute, either quality or fit. Second, we consider the case when reviewers may not perfectly assess the quality. Moreover, even when consumers can perfectly assess the quality, consumers' heterogeneity in their preference for the quality attributes plays a significant role on how they rate a product using a single measure. Such heterogeneity in preferences and subsequent ratings has not been addressed in the product information disclosure literature.

Finally, our study relates to the literature on the impact of product reviews on firms' profits in different market conditions (Li et al. 2011, Kwark et al. 2017, Li 2017). Li et al. (2011) concluded that online product reviews influence firms' pricing decisions for products with repeated purchases. Kwark et al. (2017) showed that retailers can use the upstream pricing scheme as a strategic tool to benefit from online product reviews. Li (2017) identified conditions under which revealing average ratings is more profitable than not revealing them. Yet, none of these studies considers the impact of different rating schemes on upstream competition and therefore on the platform profit, consumer surplus as well as social welfare. We contribute to the literature on the impact of product reviews on firms' profits by focusing on online retail platform's optimal choice of rating system and by identifying the conditions under which having separate ratings on product attributes benefits or hurts the online retail platform, consumers and the society.

2. Model

We consider an online marketplace that has a retail platform (R), two competing firms (1 and 2), and a continuum of consumers with heterogeneous preferences. The firms sell their products on the platform which charges a commission equal to α fraction of the sale price for each sale. Without loss of generality, we assume that the fixed and marginal production costs are zero. The platform deploys an online review system that provides information about the attributes of both products. Consumers, each of whom has a unit demand, use this information to decide which product to purchase on the marketplace.

2.1. Product Attributes

Each product is characterized by two attributes, namely a and b. These are *vertical quality* attributes in economic sense that all consumers prefer more of each quality attribute to less, *ceteris*

paribus. We assume, for expositional clarity, that an attribute for a product takes a value of either *high* (H) or *low* (L). We denote the quality of attribute j, $j \in \{a, b\}$, of product i, $i \in \{1, 2\}$ as $q_i^j \in \{H, L\}$, and refer to it as quality j of product i hereafter.

2.2. Consumer Utility and Consumer Segments

Consumers differ in their preference regarding the *relative* importance they attach to each of the quality attributes. We model consumer preference (also referred to as consumer type) using the parameter θ, $\theta \in [0, 1]$. For a consumer with $\theta = 1$, only attribute a is important and attribute b provides zero additional utility, and for a consumer with $\theta = 0$, only attribute b is important and attribute a offers zero additional utility. Similar to the Hotelling model of consumer preference, we assume that there is a negative correlation between the importance levels for the two attributes. That is, consumers that attach the most importance to attribute a attach the least importance to attribute b, and vice versa. Accordingly, the net utility of product i to a consumer of type θ, $U_i(\theta)$, is given by:

$$U_i(\theta) = v + \theta q_i^a + (1 - \theta) q_i^b - p_i \tag{1}$$

where v is the base utility of each product. Figure 1 illustrates how the (gross) utility of a consumer varies with her type.
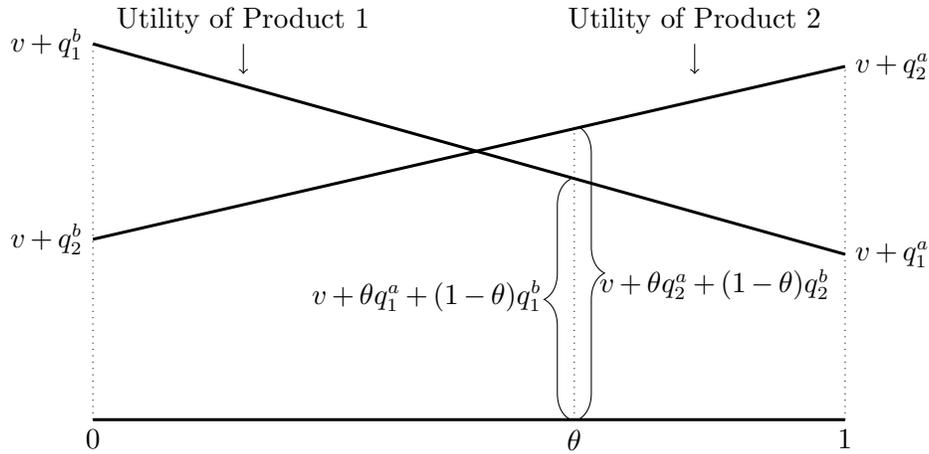

Figure 1   Consumers' gross utility for products 1 and 2 as a function of their type θ.

2.3. Consumer Uncertainty and Online Reviews

Consumers are uncertain about product qualities before purchase. Moreover, for some products, the uncertainty may not be fully resolved even after purchase and consumption. The products for which the consumers are unable to perfectly assess the product characteristics before consumption but are able to assess them perfectly after consumption are known in the literature as *experience*

goods, and those for which the consumers are unable to perfectly assess the characteristics even after consumption as *credence goods* (Nelson 1970, Darby and Karni 1973). Accordingly, we assume that consumers seek information about product quality prior to making a purchase and that online reviews on the platform provide the needed information to consumers.

Online review systems vary in terms of breadth and depth of reviews offered, the source of reviews (e.g., consumer versus experts), and the review format (e.g., structured rating scales versus unstructured texts). We abstract away the details and assume that the information content of the reviews regarding product quality on each attribute dimension can be summarized using a single numerical rating. If the platform implements MD, then the reviewers rate a product along each quality attribute and we denote the average review rating for attribute j of product i as $\bar{r}_i^j$. On the other hand, if the platform implements SD, then the reviewers provide one overall quality rating for product i and we denote the average overall rating as $\bar{r}_i$.

2.3.1. Multi-Dimensional Rating Scheme. In MD, each reviewer rates each quality attribute separately. Let there be N reviewers who provide a rating for each attribute of each product. Let reviewer k rate attribute j of product i as $r_{i,k}^j$. Consistent with the notion of experience and credence goods, we assume that reviewers may be imperfect in assessing the quality of a product. Therefore, we assume that review $r_{i,k}^j$ comes from an unbiased uniform distribution, i.e., $r_{i,k}^j \sim Uniform[q_i^j - \delta, q_i^j + \delta]$. Since the primary goal of this paper is to provide insights into how the rating scheme choice that allows reviewers to provide aggregated or disaggregated ratings affects the various players, we abstract away from potential shortcomings of rating schemes such as reviewer bias or strategic manipulation of reviews in our model. The parameter δ can be interpreted as the inverse of the rating precision in reflecting the true quality level of an attribute. Then, the average quality rating for attribute j of product i is given by $\bar{r}_i^j = \frac{\sum_k r_{i,k}^j}{N}$. Assuming N is sufficiently large, using the Central Limit Theorem, we conclude that $\bar{r}_i^j$ is distributed as:

$$\bar{r}_i^j \sim Normal\left(q_i^j, \frac{\delta^2}{3N}\right) \tag{2}$$

Furthermore, we define γ^j as the difference in the average rating of attribute j between products 1 and 2, i.e., $\gamma^j = \bar{r}_1^j - \bar{r}_2^j$. γ^j is then distributed as:

$$\gamma^j \sim Normal\left(q_1^j - q_2^j, \frac{2\delta^2}{3N}\right) \tag{3}$$

2.3.2. Single Dimensional Rating Scheme. In SD, each reviewer provides one overall rating for a product. However, reviewers differ in the relative importance they attach to each attribute, akin to consumers. Consequently, even with the same set of assessments at the individual attribute level, each reviewer could give a different overall rating for the product depending on her type.

We assume that reviewers come from the same population as consumers so that their types also follow the same uniform distribution as those of consumers. Thus, a reviewer k with type θ_k and attribute level assessments $r_{i,k}^a$ and $r_{i,k}^b$ for product i rates product i as $r_{i,k} = \theta_k r_{i,k}^a + (1 - \theta_k) r_{i,k}^b$. Then, the average rating for product i, $\bar{r}_i = \frac{\sum_k r_{i,k}}{N}$ is distributed as follows:

$$\bar{r}_i \sim Normal \left(\frac{q_i^a + q_i^b}{2}, \frac{3(q_i^a - q_i^b)^2 + 8\delta^2}{36N} \right) \tag{4}$$

Let the difference in the two product ratings under SD be denoted as $x = \bar{r}_1 - \bar{r}_2$. Then, it is easy to show that x follows a Normal distribution with mean $\mu = \frac{q_1^a + q_1^b}{2} - \frac{q_2^a + q_2^b}{2}$ and variance $\sigma^2 = \frac{(q_1^a - q_1^b)^2 + (q_2^a - q_2^b)^2}{12N}$.

We note that a mixed rating scheme which displays both single and multi-dimensional rating scores is also a potential option. For instance, AirBnB uses such a scheme. However, a mixed rating scheme does not convey any additional information beyond what is offered by a MD in our model because the SD rating for a product simply aggregates the individual MD ratings on the two quality dimension of the product. Therefore, the implications of a mixed rating scheme are the same as those of MD.

2.4. Timing of the Game

The game consists of three stages. In the first stage, the platform chooses the rating scheme and reviewers provide their ratings based on the chosen scheme. In the second stage, firms simultaneously set their prices p_i, after observing the reviewers' ratings. In the final stage, consumers purchase the product that offers the highest expected net utility, after observing prices and reviewers' ratings.

A consumer makes her decision based on her type, the product prices, and the rating information (one overall rating for each product in case of SD; a separate rating for each attribute of each product in case of MD). A consumer knows his/her type, but she does not know the preferences of other consumers or reviewers. Sellers do not know the individual consumer type but know the distribution of consumer types. All other model parameters are common knowledge. All players are risk neutral. We assume that v is high enough such that a consumer purchases either one of the products under both rating schemes. Figure 2 summarizes the timing of the game and Table 1 summarizes the notations used in this paper.

3. Analysis of Single-Dimensional and Multi-Dimensional Rating Schemes for Experience Goods

In this section, we consider experience goods for which the reviewers observe the quality levels perfectly before providing their ratings, which implies $\delta = 0$. We examine the case of credence goods for which $\delta > 0$ in Section 4. We first derive the subgame perfect equilibrium assuming that
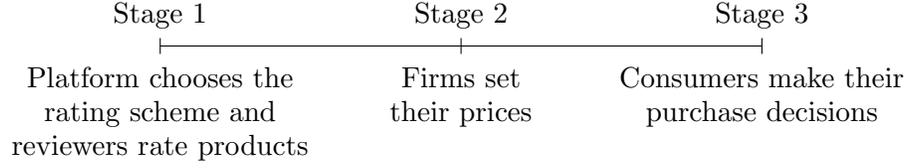
Stage 1	Stage 2	Stage 3
Platform chooses the rating scheme and reviewers rate products	Firms set their prices	Consumers make their purchase decisions

Figure 2   Timing of the game.

Notation	Definition
Parameters	
i	Index for products/firms
j	Index for product attributes
k	Index for reviewers
R	Online retail platform
U_i	A consumer's net utility derived from product i
v	Base utility of each product
θ	A consumer's preference for quality attributes (i.e., consumer type)
$r_{i,k}$	Single dimensional rating for product i of reviewer k
$\bar{r}_i$	Average single dimensional rating for product i
x	Difference in the average single dimensional ratings between products, $\bar{r}_1 - \bar{r}_2$
$r_{i,k}^j$	Multi-dimensional rating for quality attribute j of product i of reviewer k
$\bar{r}_i^j$	Average multi-dimensional rating for quality attribute j of product i
γ^j	Difference in the average ratings of quality attribute j between products, $\gamma^j = \bar{r}_1^j - \bar{r}_2^j$
ϵ	An arbitrarily small positive number
Decision Variables	
N	Number of reviewers who purchased and rated the products
α	Fraction of the price on each sale the platform charges the firms (i.e., commission rate)
δ	Inverse of the precision of consumer reviews in reflecting the true quality level
q_i^j	Quality level of attribute j of product i
p_i	Unit price of product i
Variables of Interest	
D_i	Demand of firm i
π_i^{SD}, π_i^{MD}	Profit of firm i in single-dimensional and multi-dimensional rating schemes
π_R^{SD}, π_R^{MD}	Profit of platform in single-dimensional and multi-dimensional rating schemes
S	Consumer surplus
W	Social welfare

Table 1   Summary of notations.

the platform chooses SD in stage 1 followed by the subgame perfect equilibrium if the platform implements MD in stage 1. We then compare the two equilibria with respect to platform profit, consumer surplus, and the social welfare.

3.1. Single-Dimensional Rating Scheme

In stage 3 of the game, a consumer purchases the product that offers a higher net expected utility based on the information she has about the products. In SD, consumers observe the overall ratings for the two products, $\bar{r}_1$ and $\bar{r}_2$, and prices, p_1 and p_2, before purchase. The pre-purchase expected net utility of product i for a consumer of type θ is $\mathbb{E}[U_i(\theta)] = v + \theta\,\mathbb{E}[q_i^a|\bar{r}_i] + (1-\theta)\,\mathbb{E}[q_i^b|\bar{r}_i] - p_i$. Since SD provides only one aggregate rating for each product, lacking any other information, a consumer's expected rating for each attribute is same and is identical to the overall product rating. That is, $\mathbb{E}[q_i^a|\bar{r}_i] = \bar{r}_i$ and $\mathbb{E}[q_i^b|\bar{r}_i] = \bar{r}_i$. Then, $\mathbb{E}[U_i(\theta)] = v + \theta\bar{r}_i + (1-\theta)\bar{r}_i - p_i = v + \bar{r}_i - p_i$. Note that, regardless of the true qualities for the two attributes of a product, since consumers cannot differentiate the qualities of the two attributes, consumers' own preference for an attribute does not affect their expected net utilities. If $\mathbb{E}[U_1(\theta)] > \mathbb{E}[U_2(\theta)]$, i.e., $\bar{r}_1 - p_1 > \bar{r}_2 - p_2$, consumers purchase product 1, and they purchase product 2, otherwise. Therefore, firm i's demand function in stage 3 is stated as the following.

$$D_i = \begin{cases} 1 & \text{if } p_{\bar{i}} - p_i > \bar{r}_{\bar{i}} - \bar{r}_i \\ \dfrac{1}{2} & \text{if } p_{\bar{i}} - p_i = \bar{r}_{\bar{i}} - \bar{r}_i \\ 0 & \text{otherwise} \end{cases}$$

where i represents the focal firm and $\bar{i}$ represents its competitor.

In stage 2 of the game, firms set their prices simultaneously after observing $\bar{r}_1$ and $\bar{r}_2$. The equilibrium prices are set as follows:

$$\{p_1^*, p_2^*\} = \begin{cases} \bar{r}_1 - \bar{r}_2 - \epsilon, 0 & \text{if } \bar{r}_1 > \bar{r}_2 \\ 0, \bar{r}_2 - \bar{r}_1 - \epsilon & \text{if } \bar{r}_2 > \bar{r}_1 \\ 0, 0 & \text{if } \bar{r}_1 = \bar{r}_2 \end{cases}$$

where ϵ is an arbitrarily small positive number. Substituting the equilibrium prices in the firms' demand functions, we obtain the equilibrium demands as follows:

$$\{D_1^*, D_2^*\} = \begin{cases} 1, 0 & \text{if } \bar{r}_1 > \bar{r}_2 \\ 0, 1 & \text{if } \bar{r}_2 > \bar{r}_1 \\ \dfrac{1}{2}, \dfrac{1}{2} & \text{if } \bar{r}_2 = \bar{r}_1 \end{cases}$$

In stage 1 of the game, the equilibrium expected prices, demands, and platform/firm profits depend on the probability distribution of $x = \bar{r}_1 - \bar{r}_2$. Lemma 1 summarizes the key equilibrium outcomes in SD in the limit when ϵ approaches zero.

LEMMA 1. *In equilibrium, the expected prices, demands, and firm and platform profits in the single-dimensional rating system for experience goods are as follows:*

(a) Prices:

$$\mathbb{E}(p_1^*) = \begin{cases} \dfrac{exp(-\dfrac{\mu^2}{2\sigma^2})\sigma}{\sqrt{2\pi}} + \dfrac{\mu}{2}(1 + Erf[\dfrac{\mu}{\sqrt{2}\sigma}]) & \text{if } \sigma \neq 0 \\ \mu & \text{if } \sigma = 0 \text{ and } \mu > 0 \\ 0 & \text{if } \sigma = 0 \text{ and } \mu \leq 0 \end{cases}$$

$$\mathbb{E}(p_2^*) = \begin{cases} \dfrac{exp(-\dfrac{\mu^2}{2\sigma^2})\sigma}{\sqrt{2\pi}} - \dfrac{\mu}{2}(1 - Erf[\dfrac{\mu}{\sqrt{2}\sigma}]) & \text{if } \sigma \neq 0 \\ 0 & \text{if } \sigma = 0 \text{ and } \mu > 0 \\ -\mu & \text{if } \sigma = 0 \text{ and } \mu \leq 0 \end{cases} \tag{5}$$

(b) Demands:

$$\mathbb{E}(D_1^*) = \begin{cases} \dfrac{1}{2}\left(1 + Erf[\dfrac{\mu}{\sqrt{2}\sigma}]\right) & \text{if } \sigma \neq 0 \\ 1 & \text{if } \sigma = 0 \text{ and } \mu > 0 \\ 1/2 & \text{if } \sigma = 0 \text{ and } \mu = 0 \\ 0 & \text{if } \sigma = 0 \text{ and } \mu < 0 \end{cases}$$

$$\mathbb{E}(D_2^*) = \begin{cases} \dfrac{1}{2}\left(1 - Erf[\dfrac{\mu}{\sqrt{2}\sigma}]\right) & \text{if } \sigma \neq 0 \\ 0 & \text{if } \sigma = 0 \text{ and } \mu > 0 \\ 1/2 & \text{if } \sigma = 0 \text{ and } \mu = 0 \\ 1 & \text{if } \sigma = 0 \text{ and } \mu < 0 \end{cases} \tag{6}$$

(c) Firm profits:

$$\mathbb{E}(\pi_1^{SD*}) = \begin{cases} (1-\alpha)\left(\dfrac{exp(-\dfrac{\mu^2}{2\sigma^2})\sigma}{\sqrt{2\pi}} + \dfrac{\mu}{2}(1 + Erf[\dfrac{\mu}{\sqrt{2}\sigma}])\right) & \text{if } \sigma \neq 0 \\ (1-\alpha)\mu & \text{if } \sigma = 0 \text{ and } \mu > 0 \\ 0 & \text{if } \sigma = 0 \text{ and } \mu \leq 0 \end{cases}$$

$$\mathbb{E}(\pi_2^{SD*}) = \begin{cases} (1-\alpha)\left(\dfrac{exp(-\dfrac{\mu^2}{2\sigma^2})\sigma}{\sqrt{2\pi}} - \dfrac{\mu}{2}(1 - Erf[\dfrac{\mu}{\sqrt{2}\sigma}])\right) & \text{if } \sigma \neq 0 \\ 0 & \text{if } \sigma = 0 \text{ and } \mu > 0 \\ -(1-\alpha)\mu & \text{if } \sigma = 0 \text{ and } \mu \leq 0 \end{cases} \tag{7}$$

(d) Platform profit:

$$\mathbb{E}(\pi_R^{SD*}) = \begin{cases} \alpha\left(exp(-\dfrac{\mu^2}{2\sigma^2})\sigma\sqrt{\dfrac{2}{\pi}} + \mu Erf[\dfrac{\mu}{\sqrt{2}\sigma}]\right) & \text{if } \sigma \neq 0 \\ \alpha\mu & \text{if } \sigma = 0 \text{ and } \mu > 0 \\ -\alpha\mu & \text{if } \sigma = 0 \text{ and } \mu \leq 0 \end{cases} \tag{8}$$

where $\mu = \dfrac{q_1^a - q_2^a + q_1^b - q_2^b}{2}, \sigma = \sqrt{\dfrac{(q_1^a - q_1^b)^2 + (q_2^a - q_2^b)^2}{12N}},$ *and Erf is the error function.*

Proof.   All proofs are in the appendix unless indicated otherwise.

Lemma 1 shows that the product with a higher average quality enjoys a higher expected demand, a higher expected price, and a higher expected profit than the one with a lower average quality, i.e., if $\mu > 0$, then product 1 has a higher expected demand, higher expected price, and higher expected profit than product 2. Furthermore, an increase in the quality difference increases the difference in demands, prices and profits between the products. The above observations are intuitive. We also find that, for a given pair of average quality levels for the two products, a larger difference in the qualities of the two attributes of a product increases the expected profit of both firms. That is, the expected profit of either firm is increasing in $|q_1^a - q_1^b|$ and $|q_2^a - q_2^b|$. This result can be attributed to the fact that a larger difference in the qualities of the two attributes of a product causes a larger variance in the rating difference. A larger rating difference benefits (hurts) the firm with a higher (lower) rating, and an increase in the variance of rating difference increases the likelihood of a large realized rating difference. Note that, not only the product with the higher average quality benefits from the increased variance, but also the product with the lower average quality. This is because a higher variance has a higher potential to (falsely) reverse the rating difference in favor of the product with the lower average quality. In essence, an increase in quality difference between the two attributes of a product causes the products to be perceived as more differentiated in consumers' view, which softens the competition between firms and allows both firms to earn higher profits.

3.2.   Multi-dimensional Rating Scheme

In MD, consumers observe average ratings of each quality attribute for each product, i.e., $\bar{r}_i^a, \bar{r}_i^b$ for $i \in \{1, 2\}$. Thus, for a consumer of type θ, the expected net utility of product i is $\mathbb{E}[U_i(\theta)] = v + \theta \bar{r}_i^a + (1 - \theta) \bar{r}_i^b - p_i$. Clearly, a consumer of type θ will buy product 1 if $\mathbb{E}[U_1(\theta)] > \mathbb{E}[U_2(\theta)]$ and will buy product 2, otherwise. Therefore, given the ratings, the purchase decision can vary depending on the consumer type in MD.

In order to find the indifferent consumer (i.e., consumer type who is indifferent between buying product 1 or product 2), we first note that $\gamma^a = \bar{r}_1^a - \bar{r}_2^a = q_1^a - q_2^a$ and $\gamma^b = \bar{r}_1^b - \bar{r}_2^b = q_1^b - q_2^b$. For illustrative purposes, let us assume that firm 1 has a higher rating in dimension a and firm 2 has a higher rating in dimension b, i.e., $\gamma^a > 0$ and $\gamma^b < 0$. Then, the preference of the indifferent consumer can be found as $\theta^* = \frac{p_1 - p_2 - \gamma^b}{\gamma^a - \gamma^b}$ after solving for θ in equality $\mathbb{E}[U_1] = \mathbb{E}[U_2]$. In this case, demand of firm 1 is $D_1 = 1 - \theta^*$ and demand of firm 2 is $D_2 = \theta^*$. We obtain equilibrium prices by simultaneously solving the following maximization problems.

$$\max_{p_1} \quad p_1 D_1 = p_1 \left(\frac{\gamma^a - p_1 + p_2}{\gamma^a - \gamma^b} \right)$$
$$\max_{p_2} \quad p_2 D_2 = p_2 \left(\frac{p_1 - p_2 - \gamma^b}{\gamma^a - \gamma^b} \right) \tag{9}$$

The following lemma summarizes the equilibrium in stage 2 for any given γ^a and γ^b.

LEMMA 2. *Conditional on ratings, in equilibrium, prices, demand, firm profits and platform profit in the multi-dimensional rating scheme for experience goods are shown as follows:*

(a) Prices:

$$\{p_1^*, p_2^*\} = \begin{cases} \dfrac{2\gamma^b - \gamma^a}{3}, \dfrac{\gamma^b - 2\gamma^a}{3}, & \text{if } \gamma^b \geq 2\gamma^a \text{ and } \gamma^b \geq \frac{\gamma^a}{2} \\ \dfrac{2\gamma^a - \gamma^b}{3}, \dfrac{\gamma^a - 2\gamma^b}{3} & \text{if } \frac{\gamma^a}{2} \geq \gamma^b \text{ and } 2\gamma^a \geq \gamma^b \\ \gamma^a, 0 & \text{if } 2\gamma^a \geq \gamma^b \geq \gamma^a \geq 0 \\ \gamma^b, 0 & \text{if } \gamma^a \geq \gamma^b \geq \frac{\gamma^a}{2} \geq 0 \\ 0, -\gamma^a & \text{if } 0 \geq \gamma^a \geq \gamma^b \geq 2\gamma^a \\ 0, -\gamma^b & \text{if } 0 \geq \frac{\gamma^a}{2} \geq \gamma^b \geq \gamma^a \end{cases} \qquad (10)$$

(b) Demand:

$$\{D_1^*, D_2^*\} = \begin{cases} \dfrac{2\gamma^b - \gamma^a}{3(\gamma^b - \gamma^a)}, \dfrac{\gamma^b - 2\gamma^a}{3(\gamma^b - \gamma^a)} & \text{if } \gamma^b \geq 2\gamma^a \text{ and } \gamma^b \geq \frac{\gamma^a}{2} \\ \dfrac{2\gamma^a - \gamma^b}{3(\gamma^a - \gamma^b)}, \dfrac{\gamma^a - 2\gamma^b}{3(\gamma^a - \gamma^b)} & \text{if } \frac{\gamma^a}{2} \geq \gamma^b \text{ and } 2\gamma^a \geq \gamma^b \\ 1, 0 & \text{if } 2\gamma^a \geq \gamma^b \geq \frac{\gamma^a}{2} \geq 0 \\ 0, 1 & \text{if } 0 \geq \frac{\gamma^a}{2} \geq \gamma^b \geq 2\gamma^a \end{cases} \qquad (11)$$

(c) Firm profits:

$$\{\pi_1^{MD*}, \pi_2^{MD*}\} = \begin{cases} (1-\alpha)\left(\dfrac{(2\gamma^b - \gamma^a)^2}{9(\gamma^b - \gamma^a)}\right), (1-\alpha)\left(\dfrac{(\gamma^b - 2\gamma^a)^2}{9(\gamma^b - \gamma^a)}\right) & \text{if } \gamma^b \geq 2\gamma^a \text{ and } \gamma^b \geq \frac{\gamma^a}{2} \\ (1-\alpha)\left(\dfrac{(2\gamma^a - \gamma^b)^2}{9(\gamma^a - \gamma^b)}\right), (1-\alpha)\left(\dfrac{(\gamma^a - 2\gamma^b)^2}{9(\gamma^a - \gamma^b)}\right) & \text{if } \frac{\gamma^a}{2} \geq \gamma^b \text{ and } 2\gamma^a \geq \gamma^b \\ (1-\alpha)\gamma^a, 0 & \text{if } 2\gamma^a \geq \gamma^b \geq \gamma^a \geq 0 \\ (1-\alpha)\gamma^b, 0 & \text{if } \gamma^a \geq \gamma^b \geq \frac{\gamma^a}{2} \geq 0 \\ 0, -(1-\alpha)\gamma^a & \text{if } 0 \geq \gamma^a \geq \gamma^b \geq 2\gamma^a \\ 0, -(1-\alpha)\gamma^b & \text{if } 0 \geq \frac{\gamma^a}{2} \geq \gamma^b \geq \gamma^a \end{cases} \qquad (12)$$

(d) Platform profit:

$$\pi_R^{MD*} = \begin{cases} \alpha\left(\dfrac{(2\gamma^b - \gamma^a)^2 + (\gamma^b - 2\gamma^a)^2}{9(\gamma^b - \gamma^a)}\right) & \text{if } \gamma^b \geq 2\gamma^a \text{ and } \gamma^b \geq \frac{\gamma^a}{2} \\ \alpha\left(\dfrac{(2\gamma^a - \gamma^b)^2 + (\gamma^a - 2\gamma^b)^2}{9(\gamma^a - \gamma^b)}\right) & \text{if } \frac{\gamma^a}{2} \geq \gamma^b \text{ and } 2\gamma^a \geq \gamma^b \\ \alpha\gamma^a & \text{if } 2\gamma^a \geq \gamma^b \geq \gamma^a \geq 0 \\ \alpha\gamma^b & \text{if } \gamma^a \geq \gamma^b \geq \frac{\gamma^a}{2} \geq 0 \\ -\alpha\gamma^a & \text{if } 0 \geq \gamma^a \geq \gamma^b \geq 2\gamma^a \\ -\alpha\gamma^b & \text{if } 0 \geq \frac{\gamma^a}{2} \geq \gamma^b \geq \gamma^a \end{cases} \qquad (13)$$

where $\gamma^a = \bar{r}_1^a - \bar{r}_2^a = q_1^a - q_2^a$ *and* $\gamma^b = \bar{r}_1^b - \bar{r}_2^b = q_1^b - q_2^b.$

Lemma 2 shows that when one product has a dominant quality advantage over the other in both attributes, then the product with the advantage captures the whole market. The last four conditions in Equation (10) characterize the dominant product scenario. When neither product has a dominant quality advantage on both attributes, both products enjoy some demand in the

market. The first two conditions in Equation (10) characterize this scenario. When neither product has a dominant quality advantage on both attributes, the product with a higher average quality has a higher price (but not necessarily a higher demand due to the lower price of its competitor's) and enjoys a higher profit than the other.

3.3. Comparison of Single-Dimensional and Multi-dimensional Rating Schemes

We now examine how the rating schemes affect the platform, firms, consumers, and the society by comparing the expected platform profits, expected firm profits, expected consumer surplus, and expected social welfare under SD and MD. The comparisons depend on the true quality levels of attributes within and across firms. Since $q_i^j \in \{H, L\}$ for $i \in \{1, 2\}$ and $j \in \{a, b\}$, there are 16 possible combinations of quality attributes. Taking advantage of the symmetry of consumers' preference distribution, we can reduce the 16 combinations to 6 equivalent cases; all combinations within each case yield the same quantitative and qualitative results. Furthermore, for expositional clarity and ease of interpretation, we group the 6 cases along dimensions related to attribute specialization and firm differentiation as discussed below.

Attribute Specialization: We say that a firm specializes in attribute j if and only if it chooses a high quality for attribute j and low quality for the other. When one or both firms in the market choose to specialize in an attribute, we refer to it as the *specialized firms* market.

Firm Differentiation: We say that the two firms in the market are differentiated if their quality levels differ for at least one attribute and refer to it as the *differentiated firms* market.

We provide the exhaustive list of 16 possible combinations of qualities and 6 equivalent scenarios in the appendix. Table 2 summarizes the classifications of these 6 scenarios along the attribute specialization and firm differentiation dimensions.

The following result provides insights into the platform's preference for a rating scheme.

PROPOSITION 1. *In the case of experience goods,*

(a) if neither firm specializes in an attribute (i.e., cases 1 and 5), then the platform profit is identical in MD and SD,

(b) if at least one firm specializes in an attribute, then the platform profit is higher under MD than SD if the firms are differentiated (i.e., cases 2, 3 and 4), and vice versa if the firms are not differentiated (i.e., case 6).

Proposition 1 reveals that attribute specialization is a prerequisite for the rating scheme to affect platform profitability for experience goods. The intuition for this finding is straightforward and it is the following. When neither firm specializes in an attribute, both firms have the same quality level (either H or L) for the two attributes of their product. A consequence of this is that even though consumers value each attribute differently, the overall utility from a product, which is a

Specialized Firms?

	Yes	No
Differentiated Firms? — Yes	*Case 2:* Att. A Att. B Firm 1 | H H Firm 2 | L H *Case 4:* Att. A Att. B Firm 1 | H L Firm 2 | L H *Case 3:* Att. A Att. B Firm 1 | H L Firm 2 | L L	*Case 1:* Att. A Att. B Firm 1 | H H Firm 2 | L L
Differentiated Firms? — No	*Case 6:* Att. A Att. B Firm 1 | H L Firm 2 | H L	*Case 5:* Att. A Att. B Firm 1 | H H Firm 2 | H H

Table 2   Different Market Scenarios.

convex combination of the quality levels of both attributes, is identical for all consumers regardless of their attribute preference. In other words, disaggregating a single-dimensional rating into its constituent ratings for the two attributes does not alter the utility and hence the purchase decision of any consumer. Consequently, the firms face identical demand functions, set same prices, and hence enjoy the same profit under both SD and MD.

On the other hand, if one or both firms specialize in an attribute, whether the platform realizes a higher profit in MD or SD depends exclusively on whether the firms are differentiated in at least one attribute. This finding is the result of a subtle impact of the rating scheme on consumers' perception of product differentiation. We explain the intuition behind this impact in the following paragraphs.

Consider SD. Consumers form their perceptions about the quality levels of (and hence the utility derived from) the two products using a single rating. Since reviewers differ in the relative value they assign to the two attributes, even though the reviewers know the exact quality of each attribute of a product, the aggregate rating from reviewers for each product follows a normal distribution with a positive variance. Recall that the difference in ratings between the two products follows $Normal(\frac{q_1^a - q_2^a + q_1^b - q_2^b}{2}, \frac{(q_1^a - q_1^b)^2 + (q_2^a - q_2^b)^2}{12N})$. We note that the variance in ratings exists whether or not the firms are truly differentiated; this variance is solely driven by the heterogeneous consumer preferences for the attributes and the fact that at least one firm specializes in an attribute. For instance, quality levels (H,L) and (H,L) for two products imply no differentiation between firms. Yet, the difference in ratings has a positive variance of $(H - L)^2/6N$. This positive variance in the rating difference implies that the two firms are not necessarily homogeneous from consumers' perspective. The firms exploit this perceived differentiation (whether the differentiation is real or

not) enabled by the single dimensional rating scheme and engage in less intense price competition on average to enjoy a positive expected profit.

Suppose the platform implements MD. Consumers learn the true quality of each attribute for both products from the reviews. In this case, if the firms are not truly differentiated, consumers will also realize the absence of differentiation between the two products. Therefore, the firms will engage in Bertrand-like price competition. If the firms are truly differentiated and at least one firm specializes in an attribute, then consumers will realize the true extent of differentiation under MD. Furthermore, the heterogeneity in attribute preferences will create two consumer segments - consumers that prefer firm 1 and those that prefer firm 2. Realizing the consumer segments, firms will be able to target their "core" consumer segment and charge higher prices.

Figures 3(a) and 3(b) illustrate how the rating scheme affects the consumers' perceived differentiation between the two products in (1) a specialized and non-differentiated market and (2) a specialized and differentiated market, respectively. Figure 3(a) shows that there is a probability mass of one for zero perceived differentiation in MD, but a positive probability for non-zero perceived differentiation in SD. SD enables the firms to take advantage of the perceived differentiation between the two products in this case. Figure 3(b) shows that the perceived differentiation is uniformly distributed, implying a large variance in MD, compared to a normal distribution in SD (albeit with the same mean as that of the distribution in MD). Firms are able to benefit more in MD than SD in this case.

(a) Perceived differentiation between two products in specialized and non-differentiated market (*Case 6*).

(b) Perceived differentiation between two products in specialized and differentiated market (*Cases 2 & 3*).
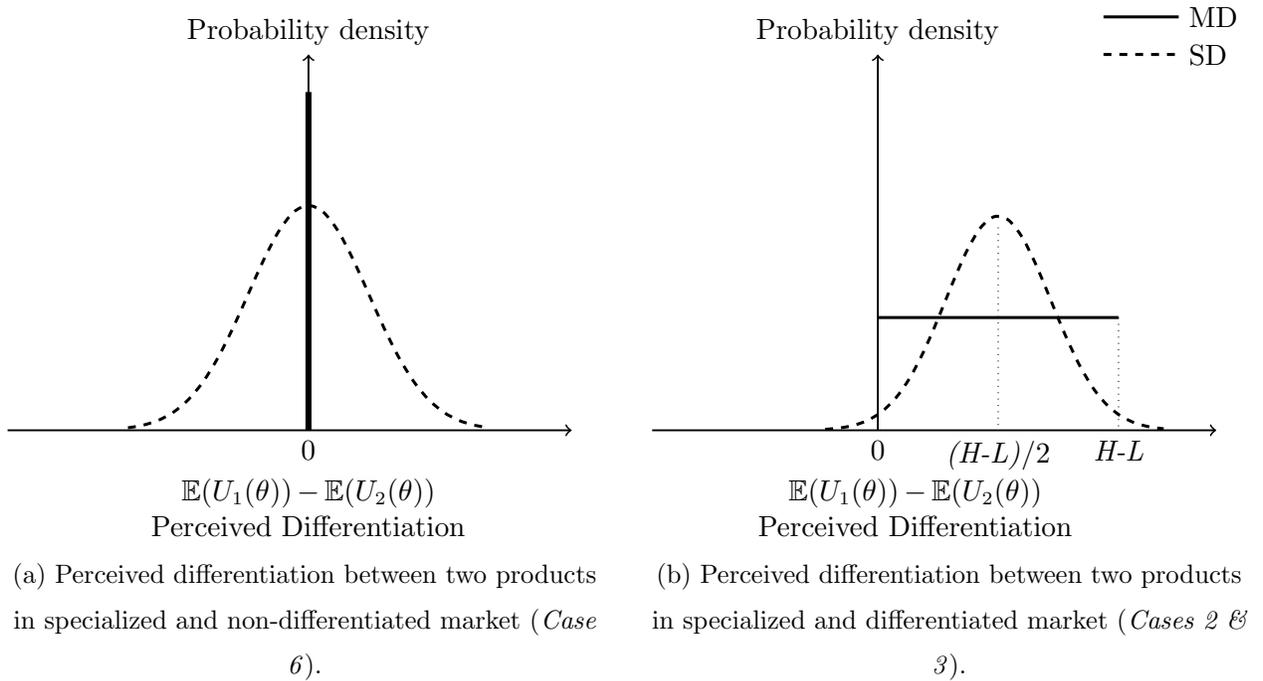
Figure 3   **Perceived differentiation between two products.**

The net effect of the impact of a rating scheme on consumers' perceived differentiation is that when the market has specialized firms, the differentiation falsely implied by SD benefits the platform and the sellers if the firms are not truly differentiated, but enabling the consumers to realize the true differentiation using MD benefits the platform and sellers when the firms are truly differentiated.

An important implication of Proposition 1 is that providing finer (i.e., less coarse) information to consumers does not always benefit the platform. Clearly, the finer information is never inferior to coarser information in resolving the consumer uncertainty about product quality. However, when sellers respond strategically to reviews by adjusting their prices, the platform itself may be hurt when it implements MD, which is more consumer-friendly (in terms of resolving consumer uncertainty) than SD.

We next examine how the rating schemes affect the consumers by comparing the aggregate consumer net utility (i.e., consumer surplus) under the two rating schemes.

PROPOSITION 2. *In the case of experience goods,*

(a) if neither firm specializes in an attribute (i.e., cases 1 and 5), then the consumer surplus is identical in MD and SD,

(b) if at least one firm specializes in an attribute, then the consumer surplus is higher under MD than under SD if and only if the firms are not differentiated (i.e., case 6), and vice versa, if the firms are differentiated (i.e., cases 2, 3, and 4).

Proposition 2(a) is intuitive given the discussion following Proposition 1, referring to the observation that the rating scheme does not alter either the prices or consumers' purchase decisions if neither firm specializes in an attribute. Hence, the effect of the two rating schemes on the consumer surplus and the platform profit are identical. On the other hand, Proposition 2(b) reveals that the comparison between the two ratings schemes along the consumer surplus dimension is qualitatively the opposite of that along the platform profit dimension. That is, consumers enjoy a higher consumer surplus under MD than SD only when the firms are not differentiated. The impact of a rating scheme on price competition between firms discussed following Proposition 1, i.e., in a non-differentiated firms market the price competition is more intense under MD than SD, partly explains this finding. However, we note that the consumer surplus is determined by misfit costs incurred by consumers also, in addition to prices. Clearly, MD can never be inferior to SD in enabling consumers to find the products that match their attribute preferences and identify the product that offers a better fit. Therefore, if MD induces a more intense price competition between firms, as in the case of a market with non-differentiated firms, then the consumers are better off in MD than in SD with respect to both price and misfit cost. Proposition 2(b) shows that in a

differentiated firms market the adverse impact of MD on consumers in the form of less intense price competition compared to SD offsets the benefit in terms of reduction in misfit costs such that consumers are worse off with superior information.

Proposition 2 is a key result with significant implications in light of the commonly held view that MD is a tool that benefits consumers who have heterogeneous preferences. However, our findings reveal that it is not necessarily the case. We show that if firms are able to differentiate themselves in terms of their offerings (i.e., specialize in different attributes) and adjust prices in response to reviews, then consumers are indeed hurt by the finer product ratings enabled by MD. Indeed, Archak et al. (2011) show that firms can mine the textual reviews to understand the relative importances assigned by consumers to various product attributes and derive significant pricing power. Thus, when firms are able to respond to reviews and adjust prices in this manner, the conclusion that MD is a tool that benefits consumers is questionable.

The following result compares the two rating schemes from a societal perspective in terms of welfare implication. We define social welfare as the sum of firm profits, platform profit, and consumer surplus.

PROPOSITION 3. *In the case of experience goods,*

(a) if neither firm specializes in an attribute and/or firms are not differentiated (i.e., cases 1, 5, and 6), then the social welfare is identical in MD and SD.

(b) If at least one firm specializes in an attribute and the firms are differentiated, then the social welfare is:

(b1) higher in MD only when each firm has a higher quality than the other in one of the dimensions (i.e., case 4)

(b2) higher in SD only when one firm has a higher quality than the other in one dimension and neither firm has a higher quality than the other in the other dimension (i.e., cases 2 and 3).

As seen from Propositions 1 and 2, when neither firm specializes in any attribute, the rating scheme does not alter either the profits of platform and firms or the consumer surplus. Therefore, social welfare is unaffected by the rating scheme when no firm specializes in an attribute. When at least one firm specializes but the firms are not differentiated, unlike platform profit and consumer surplus, the social welfare is again unaffected by the rating scheme; in essence, the decrease in firms' and the platform's profit in MD relative to SD is exactly offset by the increase in consumer surplus in MD relative to SD. In other words, the consumers' total misfit cost, which is the only factor that affects the social welfare comparison, is the same under both rating schemes when firms are not differentiated. This result has an intuitive explanation. When firms are not differentiated, in MD the prices are driven down to the marginal cost and each consumer buys the product that

offers her the highest utility because consumers deduce the product qualities from the reviews perfectly. However, under SD, the prices are not necessarily equal to the marginal cost because there is always a positive probability that ratings for the two firms are different. As a result, every consumer buys the same product that has a higher rating. Recognizing this, the firm that has a superior aggregate rating extracts the excess consumer surplus exactly by charging a price equal to the excess surplus.

Proposition 3(b) reveals an interesting result that the social welfare is higher in SD than in MD under some conditions; namely, when one firm has a higher quality than the other in one dimension and neither firm has a higher quality than the other in the other dimension. This result is counter-intuitive because it suggests that finer information does not necessarily reduce the overall consumer misfit costs. The explanation for this interesting result is the following. Suppose firm 1 has a high quality and firm 2 has a low quality for attribute a and both firms have a high quality for attribute b. Then, firm 1 is superior to firm 2 for all consumers regardless of their preference. Under MD, firm 1 targets consumers that attach a high importance to attribute a and leaves consumers that do not value attribute a highly to firm 2. Consequently, even though consumers that buy from firm 2 have a higher (gross) utility for the product from firm 1, they end up buying firm 2's product because it has a lower price than firm 1's product. On the other hand, SD masks the superiority of firm 1 because of aggregation and reduces the variability in the difference in observed ratings for the two products. Consequently, firm 1's targeting ability diminishes in SD compared to MD and it seeks to target a larger consumer base by decreasing the price. Therefore, more consumers end up buying firm 1's product, which results in a higher social welfare in SD than MD.

Propositions 1-3 provide significant insights into the conditions under which the platform, firms, consumers, and a social planner will prefer one rating scheme over the other. Naturally, another question of interest is how much one rating scheme is superior over the other from the perspectives of different players. We find that the magnitude of the difference in the payoff between the two rating schemes for each player depends on the difference in high and low quality levels (i.e., $H - L$). The following result provides insights into how the quality level difference ($H - L$) affects the extent to which one rating scheme is better than the other.

PROPOSITION 4. *In the case of experience goods, an increase in (H-L) affects the benefit of using MD compared to SD for the platform, consumers and society as summarized below, where an upward (downward) arrow indicates an increase (decrease) and a horizontal arrow indicates no change.*

Benefit from using MD compared to SD (MD-SD)

			Specialized Firms?	
			Yes	No
Platform		Yes	↑	⟷
Profit		No	↓	⟷
Consumer	Differentiated	Yes	↓	⟷
Surplus	Firms?	No	↑	⟷
Social		Yes	↑ *(Case 4: Perfectly Specialized)* ↓ *(Cases 2 and 3: Partially Specialized)*	⟷
Welfare		No	⟷	⟷

Based on Propositions 1-3, it is straightforward to verify that the value of $(H - L)$ does not affect the platform profit, consumer surplus, or the social welfare if neither firm specializes in an attribute. Proposition 4 shows that when at least one firm specializes in an attribute and therefore chooses a high quality for one of the attributes, for the platform, a larger difference between the high and low quality levels enhances the superiority of the rating scheme that offers it a higher profit. Recall from Proposition 1 that when at least one firm specializes in an attribute, the platform realizes a higher profit under MD than SD if and only if firms are differentiated. Thus, when the firms are differentiated, a larger $(H - L)$ increases the attractiveness of MD over SD. On the other hand, when the firms are not differentiated, a larger $(H - L)$ increases the attractiveness of SD over MD. In effect, if we denote $(H - L)$ as the extent of quality differentiation (i.e., between high and low quality levels), then quality differentiation complements firm differentiation regarding the relative impacts of the two rating schemes on the platform; a higher quality differentiation enhances the attractiveness of the rating scheme preferred by the platform. Thus, the platform has significant incentives to deploy MD in a market characterized by differentiated firms and a high quality differentiation.

The impact of quality differentiation on consumer surplus is opposite to that on the platform. When MD hurts (benefits) consumers relative to SD, an increase in quality differentiation exacerbates (enhances) the consumer surplus. Quality differentiation does not affect the social welfare unless firms are specialized as well as differentiated. When both firms specialize in different attributes (case 4 - perfectly specialized), an increase in quality differentiation enhances the attractiveness of MD over SD from the social welfare perspective, similar to the platform's perspective. On the other hand, only one firm specializes in an attribute (cases 2 and 3 - partially specialized), the attractiveness of SD over MD increases as quality differentiation increases from the social welfare perspective, similar to the consumers' perspective. These results are driven by a larger fraction of consumers purchasing a better fit product under MD in case of perfect specialization and under SD in case of partial specialization.

4. Analysis of Rating Schemes for Credence Goods

We analyze the case of credence goods for which $\delta > 0$ in this section. Since there are no closed form solutions for the quantities of interest - expected platform profit, expected consumer surplus and expected social welfare - when the products are credence goods, we resort to numerical integration to evaluate the impact of rating schemes. For the numerical analysis, we use the following parameters:[1]

High Quality (H) = Mean Quality $(M) + K$,

Low Quality (L) = Mean Quality (M) - K,

$M \in \{10, 20, 30, 40, 50\}$,

$K \in \{1, 2, 3, 4, 5\}$, and

$\delta \in \{0, 1, 2, 3, 4, 5\}$. [2]

We use parameter K as a measure of quality heterogeneity. We set the number of reviews (N) to be 100 and platform's commission rate (α) to be 0.1. By varying parameters δ, K, and M, we create 150 scenarios to characterize the impact δ and K.[3] We present our results only for the case $M = 10$ because the impacts of δ and K do not change qualitatively for other values of M.

4.1. Impact of Rating Schemes on Platform Profit

The expected prices, demands, and profits in stage 1 of the game depend on the probability distributions of γ^a and γ^b. We showed in equation (3) that $\gamma^a \sim \text{Normal}(q_1^a - q_2^a, \frac{2\delta^2}{3N})$, $\gamma^b \sim \text{Normal}(q_1^b - q_2^b, \frac{2\delta^2}{3N})$. Using Lemmas 1 and 2 and the probability distributions of γ^a and γ^b, we can compute the expected prices, demands, and profits in SD and MD. For example, we can calculate the expected platform profits as follows.

$\mathbb{E}(\pi_R^{SD*}) = \alpha\Big(exp(-\frac{\mu^2}{2\sigma^2})\sigma\sqrt{\frac{2}{\pi}} + \mu Erf[\frac{\mu}{\sqrt{2}\sigma}]\Big)$

$\mathbb{E}(\pi_R^{MD*}) = \alpha(\int_{-\infty}^{0}(\int_{-\infty}^{2\gamma^a}\frac{(2\gamma^a-\gamma^b)^2+(\gamma^a-2\gamma^b)^2}{9(\gamma^a-\gamma^b)}f(\gamma^b)\,\mathrm{d}\gamma^b - \int_{2\gamma^a}^{\gamma^a}\gamma^a f(\gamma^b)\,\mathrm{d}\gamma^b - \int_{\gamma^a}^{\frac{\gamma^a}{2}}\gamma^b f(\gamma^b)\,\mathrm{d}\gamma^b + \int_{\frac{\gamma^a}{2}}^{\infty}\frac{(2\gamma^b-\gamma^a)^2+(\gamma^b-2\gamma^a)^2}{9(\gamma^b-\gamma^a)}f(\gamma^b)\,\mathrm{d}\gamma^b)f(\gamma^a)\,\mathrm{d}\gamma^a + \int_{0}^{\infty}(\int_{-\infty}^{\frac{\gamma^a}{2}}\frac{(2\gamma^a-\gamma^b)^2+(\gamma^a-2\gamma^b)^2}{9(\gamma^a-\gamma^b)}f(\gamma^b)\,\mathrm{d}\gamma^b + \int_{\frac{\gamma^a}{2}}^{\gamma^a}\gamma^b f(\gamma^b)\,\mathrm{d}\gamma^b + \int_{\gamma^a}^{2\gamma^a}\gamma^a f(\gamma^b)\,\mathrm{d}\gamma^b + \int_{2\gamma^a}^{\infty}\frac{(2\gamma^b-\gamma^a)^2+(\gamma^b-2\gamma^a)^2}{9(\gamma^b-\gamma^a)}f(\gamma^b)\,\mathrm{d}\gamma^b)f(\gamma^a)\,\mathrm{d}\gamma^a)$

where $\mu = \frac{q_1^a-q_2^a+q_1^b-q_2^b}{2}, \sigma = \sqrt{\frac{3(q_1^a-q_1^b)^2+3(q_2^a-q_2^b)^2+16\delta^2}{36N}}, \gamma^a \sim \text{Normal}(q_1^a - q_2^a, \frac{2\delta^2}{3N}), \gamma^b \sim \text{Normal}(q_1^b - q_2^b, \frac{2\delta^2}{3N})$, and Erf is the error function.

Figure 4 plots the expected platform profit as a function of δ for the various cases of specialization and differentiation. The figure is shown for $K = 1$, but the conclusions derived from these figures

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[1] We conducted an extensive analysis by varying these parameters and found the same qualitative results.

[2] We keep δ small enough such that the rating associated with high quality, H, is higher than the rating associated with low quality, L. In other words, although the ratings in credence goods are noisy, they are informative and do not mislead consumers.

[3] We ran an extensive analysis on the impact of K, but we only present the results for the impact of δ in this section. In our analysis, we found the impact of K to be qualitatively the same as that shown in Proposition 4 for experience goods.

also hold for other values of K. Figure 4 reveals that as δ increases, the attractiveness of MD vis-à-vis SD for the platform increases in all cases except when one firm has a clear quality advantage over the other in both quality dimensions (case 1). Specifically, when firms do not specialize and are not differentiated (case 5), while the platform enjoys the same level of profit under both SD and MD for experience goods, it enjoys a higher profit under MD than SD for credence goods; when firms specialize and are also differentiated (case 2), the platform enjoys an even greater profit under MD compared to SD for credence goods than experience goods; when firms specialize but are not differentiated (case 6), the platform's profit becomes greater under MD compared to SD when the uncertainty for credence goods crosses a threshold value. In all these cases, the profit difference for the platform increases in favor of MD as review uncertainty increases (or, alternatively, as the product becomes "more" of a credence good).

On the other hand, when the firms do not specialize but are differentiated (case 1), the review uncertainty has qualitatively opposite impact on the relative attractiveness of MD for the platform; namely, MD is less attractive than SD for credence goods, but the platform enjoys the same profit under the two rating schemes for experience goods.

The impact of δ on the platform's profit can be explained using the following argument. An increase in δ increases the variability of ratings for each product under both SD and MD. However, the increase in the variance of ratings from the sellers' perspective is higher under MD than SD because the aggregation (or pooling) of the two ratings into a single rating by reviewers under SD mutes its variability; however, reviewers do not perform such aggregation under MD. A higher variance in ratings for the two products enables the sellers to better target different consumer segments under MD, which in turn benefits both sellers and the platform, except when one seller dominates the other on both quality dimensions (case 1). Therefore, in all cases except case 1, a higher δ benefits the platform more under MD than SD, as shown by Figures 4(b) through 4(d). On the contrary, in case 1, a higher variance in ratings increases the likelihood that the consumer utility for the two products are close to each other and hence a more intense competition and a smaller seller and platform profits. Therefore, MD, which results in a higher variance of consumer utility than SD, ends up hurting the platform more in case 1.

4.2. Impact of Rating Schemes on Consumer Surplus

We calculate the expected consumer surplus in SD and MD as:

$$E(CS^{SD*}) = \int_{-\infty}^{0} (\int_{0}^{1} (\theta q_2^a + (1-\theta)q_2^b + x)\,\mathrm{d}\theta) f(x)\,\mathrm{d}x + \int_{0}^{\infty} (\int_{0}^{1} (\theta q_1^a + (1-\theta)q_1^b - x)\,\mathrm{d}\theta) f(x)\,\mathrm{d}x$$

$$E(CS^{MD*}) = \int_{-\infty}^{0} (\int_{-\infty}^{2\gamma^a} (\int_{0}^{\theta^*} (\theta q_2^a + (1-\theta)q_2^b - p_2)\,\mathrm{d}\theta + \int_{\theta^*}^{1} (\theta q_1^a + (1-\theta)q_1^b - p_1)\,\mathrm{d}\theta) f(\gamma^b)\,\mathrm{d}\gamma^b +$$
$$\int_{2\gamma^a}^{\gamma^a} (\int_{0}^{1} (\theta q_2^a + (1-\theta)q_2^b + \gamma^a)\,\mathrm{d}\theta) f(\gamma^b)\,\mathrm{d}\gamma^b + \int_{\gamma^a}^{\frac{\gamma^a}{2}} (\int_{0}^{1} (\theta q_2^a + (1-\theta)q_2^b + \gamma^b)\,\mathrm{d}\theta) f(\gamma^b)\,\mathrm{d}\gamma^b + \int_{\frac{\gamma^a}{2}}^{\infty} (\int_{0}^{\theta^*} (\theta q_1^a +$$
$$(1-\theta)q_1^b + p_2)\,\mathrm{d}\theta + \int_{\theta^*}^{1} (\theta q_2^a + (1-\theta)q_2^b + p_1)\,\mathrm{d}\theta) f(\gamma^b)\,\mathrm{d}\gamma^b) f(\gamma^a)\,\mathrm{d}\gamma^a + \int_{0}^{\infty} (\int_{-\infty}^{\frac{\gamma^a}{2}} (\int_{0}^{\theta^*} (\theta q_2^a + (1-\theta)q_2^b + (1-\theta)q_2^a) \,\mathrm{d}\gamma^a + \int_{0}^{\infty} (\int_{-\infty}^{\frac{\gamma^a}{2}} (\int_{0}^{\theta^*} (\theta q_2^a + (1-\theta)q_2^a + (1-\theta)q_2^a) \,\mathrm{d}\gamma^a + \int_{0}^{\infty} (\int_{-\infty}^{\frac{\gamma^a}{2}} (\int_{0}^{\theta^*} (\theta q_2^a + (1-\theta)q_2^a) \,\mathrm{d}\gamma^a + \int_{0}^{\infty} (\int_{-\infty}^{\frac{\gamma^a}{2}} (\int_{0}^{\theta^*} (\theta q_2^a + (1-\theta)q_2^a) \,\mathrm{d}\gamma^a + \int_{0}^{\infty} (\int_{-\infty}^{\frac{\gamma^a}{2}} (\int_{0}^{\theta^*} (\theta q_2^a + (1-\theta)q_2^a) \,\mathrm{d}\gamma^a + \int_{0}^{\infty} (\int_{-\infty}^{\frac{\gamma^a}{2}} (\int_{0}^{\theta^*} (\theta q_2^a + (1-$$

(a) Case 1: Non-specialized, differentiated firms.

(b) Case 5: Non-specialized, non-differentiated firms.

(c) Case 2: Specialized, differentiated firms.

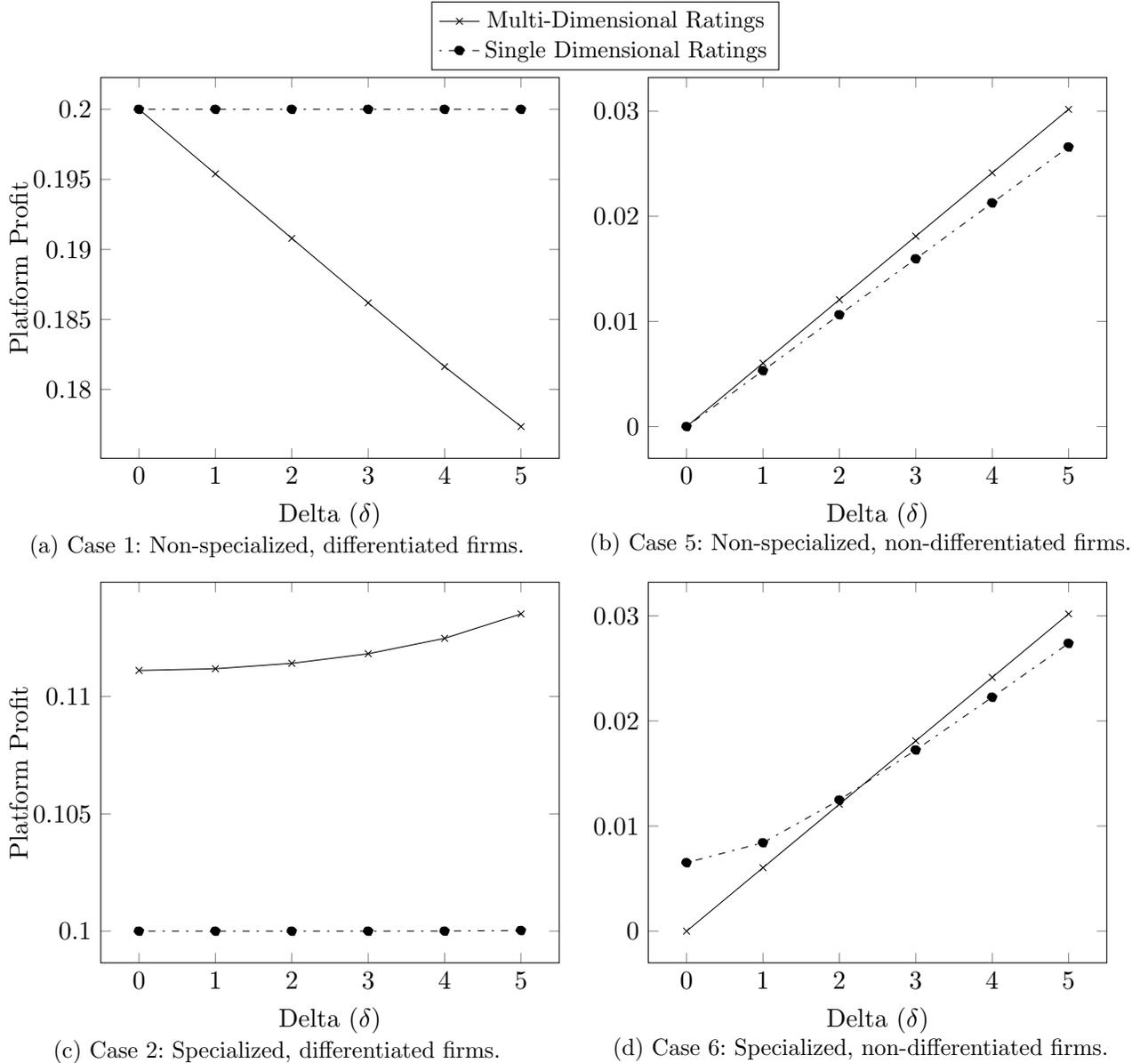(d) Case 6: Specialized, non-differentiated firms.

Figure 4   Comparison of platform profits under the two rating schemes for credence goods (M=10, $K=1$).

$\theta) q_2^b - p_2)\, \mathrm{d}\theta + \int_{\theta^*}^1 (\theta q_1^a + (1-\theta) q_1^b - p_1)\, \mathrm{d}\theta) f(\gamma^b)\, \mathrm{d}\gamma^b + \int_{\frac{\gamma^a}{2}}^{\gamma^a} (\int_0^1 (\theta q_1^a + (1-\theta) q_1^b - \gamma^b)\, \mathrm{d}\theta) f(\gamma^b)\, \mathrm{d}\gamma^b +$
$\int_{\gamma^a}^{2\gamma^a} (\int_0^1 (\theta q_1^a + (1-\theta) q_1^b - \gamma^a)\, \mathrm{d}\theta) f(\gamma^b)\, \mathrm{d}\gamma^b + \int_{2\gamma^a}^\infty (\int_0^{\theta^*} (\theta q_1^a + (1-\theta) q_1^b + p_2)\, \mathrm{d}\theta + \int_{\theta^*}^1 (\theta q_2^a + (1-\theta) q_2^b +$
$p_1)\, \mathrm{d}\theta) f(\gamma^b)\, \mathrm{d}\gamma^b) f(\gamma^a)\, \mathrm{d}\gamma^a$

where $x \sim \mathrm{Normal}(\frac{q_1^a + q_1^b - q_2^a - q_2^b}{2}, \frac{3(q_1^a - q_1^b)^2 + 3(q_2^a - q_2^b) + 16\delta^2}{36N})$, $\gamma^a \sim \mathrm{Normal}(q_1^a - q_2^a, \frac{2\delta^2}{3N})$, $\gamma^b \sim \mathrm{Normal}(q_1^b - q_2^b, \frac{2\delta^2}{3N})$, $\theta^* = \frac{\gamma^a - 2\gamma^b}{3(\gamma^a - \gamma^b)}$, $p_1 = \frac{2\gamma^a - \gamma^b}{3}$, $p_2 = \frac{\gamma^a - 2\gamma^b}{3}$.[4]

Figure 5 plots the expected consumer surplus as a function of δ for the various cases of specialization and differentiation. As in the case of platform profit, the figure is shown for $K = 1$, but the

[4] Since v does not have any qualitative impact on the comparison of consumer surplus under MD and SD, we let $v = 0$.

conclusions derived from these figures also hold for other values of K. A comparison of Figure 4 and Figure 5 shows that the impact of δ on consumer surplus is qualitatively the opposite to that on platform profit. In particular, except in case 1, an increase in δ decreases the consumer surplus under both rating schemes and the decrease is higher under MD than SD. In case 1, an increase in δ increases the consumer surplus more under MD than SD. The explanation for the opposite impacts on consumer surplus vis-à-vis platform profit is the same as that provided in Section 4.1 in that an increase in δ enables sellers to target consumers better under MD than under SD except in case 1.
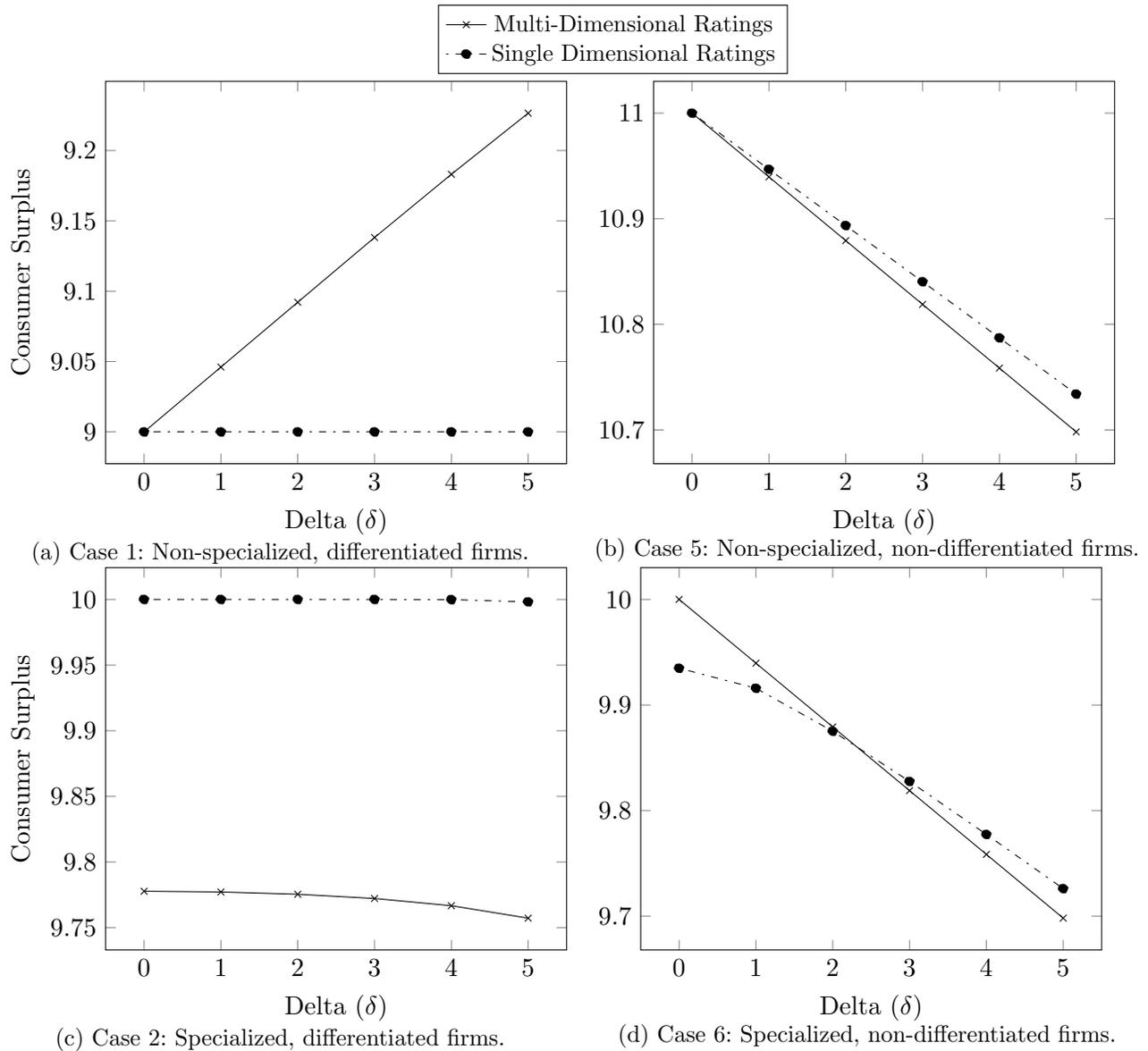

(a) Case 1: Non-specialized, differentiated firms.

(b) Case 5: Non-specialized, non-differentiated firms.

(c) Case 2: Specialized, differentiated firms.

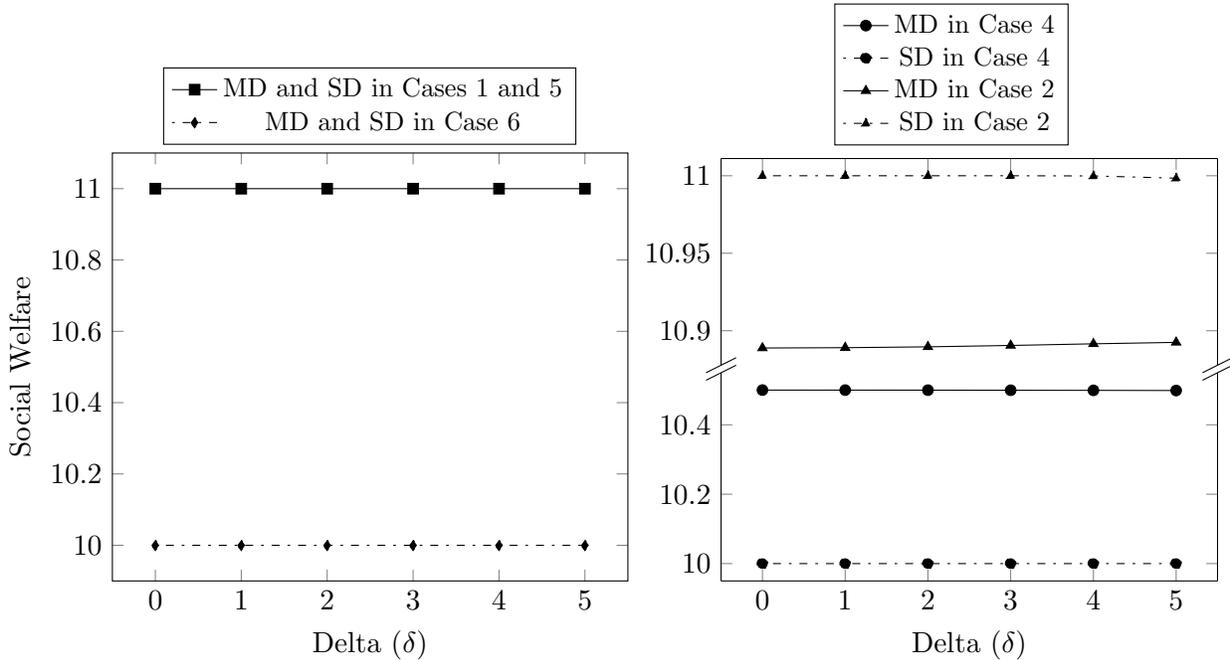(d) Case 6: Specialized, non-differentiated firms.

Figure 5   **Comparison of consumer surplus under the two rating schemes for credence goods (M=10, $K = 1$).**

4.3. Impact of Rating Schemes on Social Welfare

Figure 6 shows the impact of δ on social welfare for credence goods. We find that Figure 6 is consistent with Proposition 3 in the sense that the qualitative comparison between SD and MD with respect to social welfare is the same for experience and credence goods. More importantly, we also find that δ does not qualitatively impact the social welfare for credence goods except in case 2 (i.e. when one firm specializes in an attribute and the other firms does not specialize in any attribute). As given in Proposition 3(a), when neither firm specializes in an attribute (i.e., Cases 1 and 5) or when at least one of the firms specialize in an attribute but firms are not differentiated (i.e., Case 6), social welfare is identical in both rating schemes for experience goods. We find that as δ increases, this result still holds for credence goods, as shown in Figure 6(a). Furthermore, we find consistent results in differentiated firms market (i.e. Cases 2, 3 and 4). Proposition 3(b1) suggests that when firms are differentiated and perfectly specialized (i.e., Case 4), social welfare in MD is higher for experience goods. We observe the same result for credence goods, as shown in Figure 6(b). Proposition 3(b2) suggests that when firms are differentiated and partially specialized (i.e., Cases 2 and 3), social welfare in SD is higher for experience goods. We obtain the same result for credence goods as shown in Figure 6(b). Thus, the impact of δ on social welfare for credence goods is consistent with our findings for experience goods.

(a) Case 1: Non-specialized, differentiated firms,
Case 5: Non-specialized, non-differentiated firms,
Case 6: Specialized, non-differentiated firms.

(b) Case 4 (Perfectly specialized, differentiated firms) and Case 2 (Partially specialized, differentiated firms).

Figure 6   Comparison of social welfare under the rating schemes for credence goods (M=10, $K = 1$).

In summary, our numerical analysis for credence goods indicates that an increase in review uncertainty about product quality generally increases the attractiveness of MD relative to SD for the platform except in a differentiated firms market with no attribute specialization. However, the uncertainty has the opposite effect on consumers. An increase in uncertainty does not qualitatively alter the impacts of MD and SD on the society.

5. Model Extension: Skewed Consumer Preference

The baseline model assumes that consumer preference is uniform across the two attributes. However, it is possible that the consumer preference is skewed in the sense that more consumers place a larger weight on one of the attributes. For instance, food quality could be more important than ambiance for a majority of consumers when they choose a restaurant. In this section, we extend our model to allow skewness in consumer preference.

We assume without loss of generality that more consumers assign a larger weight on attribute b. We model this scenario using the declining linear probability density function for θ given by $f(\theta) = \bar{\theta} - 2\theta(\bar{\theta} - 1)$, as shown in Figure 7. The cumulative probability distribution function is given by $F(\theta) = \bar{\theta}\theta - \bar{\theta}\theta^2 + \theta^2$ for $\theta \in [0,1]$. Clearly, $\bar{\theta} \in [1,2]$ must hold for the density function to be linear. When $\bar{\theta} = 1$, it reduces to the uniform distribution considered in the baseline model. All other aspects remain the same as those in the baseline model. We focus on the platform profit for experience goods in this extension. Lemma 3 and Lemma 4 provided in the appendix summarize, respectively, the equilibria under SD and MD for the extension.
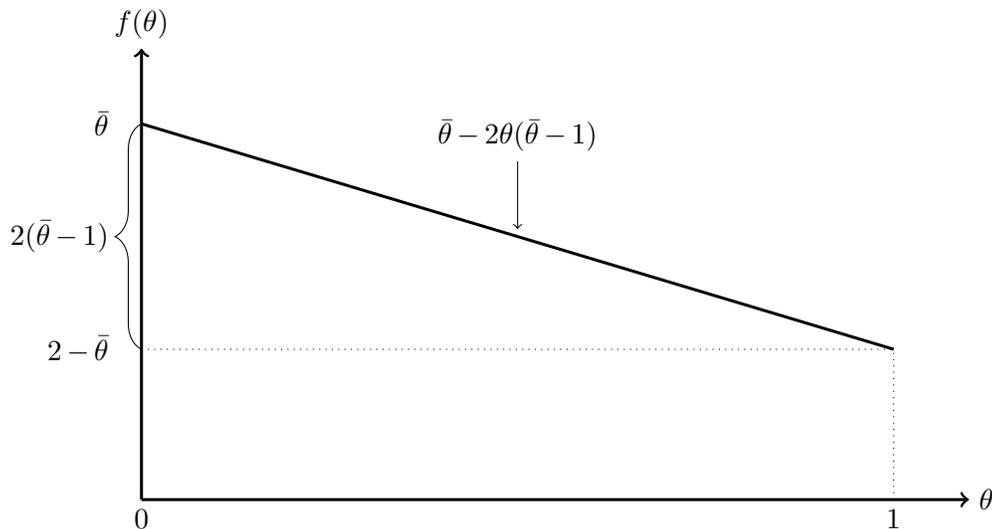

Figure 7   **A skewed distribution of consumer type** θ **with intercept** $\bar{\theta}$ **and slope** $2(\bar{\theta} - 1)$**.**

We distinguish two sets of sub cases of specialized, partially differentiated firms market (i.e., cases 2 and 3). Cases 2a and 3a refer to sub cases in which firms are differentiated on the attribute

preferred by fewer consumers. For example, in our model, sub cases 2a and 3a denote the scenario where firms are differentiated in attribute a. On the other hand, cases 2b and 3b refer to sub cases in which firms are differentiated in the attribute preferred by more consumers. That is, in our model, cases 2b and 3b denote the scenario where firms are differentiated in attribute b.

The following result compares SD and MD when consumer preference distribution is skewed.

PROPOSITION 5. *Assume that consumers have skewed preferences for product attributes. For experience goods,*

(a) If neither firm specializes in an attribute (i.e., cases 1 and 5), then the platform profit is identical in MD and SD.

(b) If at least one firm specializes in an attribute, then

(b1) the platform profit is higher under MD than under SD in the following scenarios: (i) when the firms are perfectly differentiated (i.e., case 4), (ii) when the firms are differentiated only in attribute a (i.e., the attribute preferred by fewer consumers - cases 2a and 3a), and (iii) when the firms are differentiated only in attribute b (i.e., the attribute preferred by more consumers - cases 2b and 3b) and $\bar{\theta} < \bar{\theta}^*$,[5] and

(b2) the platform profit is higher under SD than under MD in all other scenarios (i.e., case 6, cases 2b and 3b and $\bar{\theta} > \bar{\theta}^*$).

A comparison of Proposition 1 and Proposition 5 reveals that the qualitative result regarding the comparison of the two rating schemes on platform profit is the same under uniform distribution and skewed distribution of consumer preference, except in sub cases 2b and 3b. While MD provides a higher platform profit than SD under uniform consumer preference in these sub cases, this result holds only when $\bar{\theta}$ is smaller than a threshold under skewed linear consumer preference. This is an interesting result in the sense that when the firms are differentiated in the attribute preferred by more consumers, providing an aggregate single dimensional rating may reduce the price competition between the firms. At first, this finding may seem unintuitive, but the explanation is as follows. Consider MD, and assume that the firms are differentiated in attribute b and both have the same quality in attribute a. When consumer preferences are uniformly distributed, the firm with the low quality in attribute b does not reduce its price too much since it could still realize sizable demand from the consumers who prefer attribute a on which the firm does not suffer a quality disadvantage. However, when consumer preferences are tilted heavily towards attribute b, there is not enough demand from consumers who prefer attribute a. As a result, the firm with the lower quality reduces its price more, which in turn leads to a more intense competition. Thus, when a

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[5] $\bar{\theta}^*$ is the solution to the following equality: $148\bar{\theta}^{*9} + 570\bar{\theta}^{*8} - 5209\bar{\theta}^{*7} + 8881\bar{\theta}^{*6} + 7756\bar{\theta}^{*5} - 51228\bar{\theta}^{*4} + 85988\bar{\theta}^{*3} - 76780\bar{\theta}^{*2} + 39616\bar{\theta}^* - 9904 = 0$, which is approximately 1.93958.

large fraction of consumers favors one attribute over the other attribute, MD exposes the quality for each attribute and consequently exacerbates the competition between sellers, which eventually hurts the platform.

In summary, the findings of this section show that the results and implications regarding the impacts of the two rating schemes on the platform hold in a more general setting than the one examined using the baseline model, thereby confirming the robustness of our results.

6.   Conclusion and Implications

We summarize the key findings of our paper in Table 3. The table shows which rating scheme (MD or SD) offers the highest platform profit, consumer surplus, and social welfare. It is clear from the table that (i) attribute specialization, (ii) firm differentiation, and (iii) the product type play important roles in the rating scheme preferred by the platform, consumers, and the social planner.

			Experience Goods		Credence Goods	
			Specialized Firms?			
			Yes	No	Yes	No
Platform Profit		Yes	MD	Equal	MD	SD
		No	SD	Equal	SD (when $\delta < \delta^*$) MD (when $\delta > \delta^*$)	MD
Consumer Surplus	Differen-tiated Firms?	Yes	SD	Equal	SD	MD
		No	MD	Equal	MD (when $\delta < \delta^*$) SD (when $\delta > \delta^*$)	SD
Social Welfare		Yes	SD (Partially Spec.) MD (Perfectly Spec.)	Equal	SD (Partially Spec.) MD (Perfectly Spec.)	Equal
		No	Equal	Equal	Equal	Equal

Table 3   Summary of key findings. The rating scheme which offers the highest profit, surplus and welfare is presented in each cell. MD stands for multi-dimensional rating scheme and SD stands for single-dimensional rating scheme. Partially Spec. stands for partially specialized firms market (Cases 2 and 3) and Perfectly Spec. stands for perfectly specialized firms market (Case 4). δ^* represents a certain threshold.

The above findings have important implications for both academics and practitioners. From an academic's perspective, focusing on the information transfer aspect of a rating scheme, which relates solely to the consumer side, provides only a partial picture on the impacts of the rating scheme. More importantly, from the viewpoint of the platform which controls the choice of a rating scheme, the impact of the rating scheme on firms that sell through the platform is equally important. Our findings demonstrate the need to examine how a rating scheme affects both sides of the platform, i.e., consumers and sellers. The study provides several testable hypotheses about the effects of

rating schemes on platform and upstream sellers. In particular, it shows that the impacts vary depending on the rating scheme, market characteristics such as firm differentiation and product specialization, and product type. While the existing studies on the influence of reviews have largely focused on product sales, our study points to a need to account for platform profitability and the factors we have identified in this paper that have an impact on the platform profitability for a more complete understanding of the role played by online reviews.

From a practitioner perspective, the results suggest that there is no one rating scheme that is appropriate (i.e., offers the highest profit) for platforms across all product markets. Some product markets are highly amenable to specialization and differentiation such as those that sell products with a high variety (for example, TV, computers and other electronics). A multi-dimensional rating scheme could be more profitable to the platform in these markets. However, some others may not be easily susceptible to specialization and differentiation. For instance, competing hotels or restaurants located in a geographical proximity may not be able specialize or differentiate in location attribute. In such cases, a multi-dimensional rating scheme may not be attractive for the platform. Consequently, platforms will have carefully assess the market characteristics before choosing the rating scheme for their consumer review system.

Finally, from a consumer perspective, our study cautions against a commonly held view that a more refined rating scheme that provides individual ratings on various product features are always beneficial to consumers. Even though such a scheme enables consumers to obtain a clearer understanding about products, consumers are not necessarily better off with more granular product information. The consumers should realize that sellers also take advantage of this granular information and engage in less severe price competition to attract consumers.

Our study can be extended in several directions. Our model is particularly relevant to the product categories in which consumers make only a single purchase. A model that accounts for repeated purchases could be a potential extension to our research. Analyzing the impact of repeat purchases would require a dynamic model where the payoff function would aggregate consumer utility derived from a purchase made in the current period and purchases made in future periods using a discount factor. However, not every product category may demand multiple purchases. Therefore, a dynamic model may not necessarily be more appropriate for every product category. Nevertheless, analyzing the impact of repeated purchases on platform's rating scheme decision could be a potential extension to our study.

Another follow-up research may consider firms' strategic decisions regarding the quality levels fpr product attributes. However, when firms choose quality levels, several additional issues will have to addressed. First, a cost function, which captures the cost of quality for the two dimensions, will become part of firms' optimization problems, and the model will have to account for such cost

functions. Second, a new stage in in which the firms make decisions about the quality attributes, either sequentially or simultaneously, should be added to the game. The main research question in this paper relates to the platform's optimal choice between single and multi-dimensional rating schemes under different market conditions (i.e, exogenous quality choices by firms), and therefore, we did not endogenize the levels of product attributes in our model. Nevertheless, a valuable extension is to examine how competing firms' product design decisions could be affected by the review rating scheme deployed by the retail platform.

Appendix A: Multi-Dimensional Rating Scheme

Proof of Equation 2. Let $\theta \sim Uniform[0,1]$ and $\bar{r}_i^j = \frac{\sum_k r_{i,k}^j}{N}$. We know that $r_{i,k}^j \sim Uniform[q_i^j - \delta, q_i^j + \delta]$. We also know that sum of random variables are normally distributed with a mean μ and a variance σ^2 due to Central Limit Theorem. We can find the mean μ as $\mu = E[r_{i,k}^j] = q_i^j$. For variance, we need to solve $\sigma^2 = \frac{Var[r_{i,k}^j]}{N}$. Since $r_{i,k}^j$ follows a uniform distribution, variance of a uniform random variable can be calculated as follows: $Var[r_{i,k}^j] = \frac{(q_i^j + \delta - (q_i^j - \delta))^2}{12} = \frac{\delta^2}{3}$. Therefore $\sigma^2 = \frac{\delta^2}{3N}$. $\square$

Proof of Equation 3. Let $\gamma^j = \bar{r}_1^j - \bar{r}_2^j$. We know that $\bar{r}_1^j$ and $\bar{r}_2^j$ are normally distributed with means μ_1, μ_2 and variances σ_1^2, σ_2^2 respectively. Difference of two normal random variables is also a normal random variable, with mean $\mu_1 - \mu_2$ and variance $\sigma_1^2 + \sigma_2^2$. Then, $\gamma^j \sim Normal(q_1^j - q_2^j, \frac{2\delta^2}{3N})$. $\square$

Appendix B: Single-Dimensional Rating Scheme

Proof of Equation 4. We let $\theta \sim Uniform[0,1]$ and $r_{i,k} = \theta_k r_{i,k}^a + (1 - \theta_k) r_{i,k}^b$. We also know that $r_{i,k}^j \sim Uniform[q_i^j - \delta, q_i^j + \delta]$. We would like to find the distribution of $\bar{r}_i$ where $\bar{r}_i = \frac{\sum_k r_{i,k}}{N}$. We know that sum of random variables are normally distributed with a mean μ and a variance σ^2 due to Central Limit Theorem. Let us find μ and σ^2. We know that $\mu = \mathbb{E}[r_{i,k}] = \mathbb{E}[\theta_k r_{i,k}^a + (1 - \theta_k) r_{i,k}^b] = \mathbb{E}[\theta_k r_{i,k}^a] + \mathbb{E}[(1 - \theta_k) r_{i,k}^b] = \frac{q_i^a + q_i^b}{2}$. We also know that $\sigma^2 = \frac{Var[r_{i,k}]}{N}$. Then, we need to find $Var[r_{i,k}]$. $Var[r_{i,k}] = Var[\theta_k r_{i,k}^a + (1 - \theta_k) r_{i,k}^b] = Var[\theta_k r_{i,k}^a] + Var[(1 - \theta_k) r_{i,k}^b] + 2Cov[\theta_k r_{i,k}^a, (1 - \theta_k) r_{i,k}^b] = \mathbb{E}[\theta_k^2 (r_{i,k}^a)^2] - \mathbb{E}[\theta_k r_{i,k}^a]^2 + \mathbb{E}[(1 - \theta_k)^2 (r_{i,k}^b)^2] - \mathbb{E}[(1 - \theta_k) r_{i,k}^b]^2 + 2(\mathbb{E}[\theta_k r_{i,k}^a (1 - \theta_k) r_{i,k}^b] - \mathbb{E}[\theta_k r_{i,k}^a] \mathbb{E}[(1 - \theta_k) r_{i,k}^b]) = \mathbb{E}[\theta_k^2] \mathbb{E}[(r_{i,k}^a)^2] - (\mathbb{E}[\theta_k] \mathbb{E}[r_{i,k}^a])^2 + \mathbb{E}[(1 - \theta_k)^2] \mathbb{E}[(r_{i,k}^b)^2] - (\mathbb{E}[(1 - \theta_k)] \mathbb{E}[r_{i,k}^b])^2 + 2(\mathbb{E}[\theta_k (1 - \theta_k)] \mathbb{E}[r_{i,k}^a] \mathbb{E}[r_{i,k}^b] - \mathbb{E}[\theta_k] \mathbb{E}[r_{i,k}^a] \mathbb{E}[(1 - \theta_k)] \mathbb{E}[r_{i,k}^b]) = \frac{3(q_i^a)^2 + \delta^2}{9} - (\frac{q_i^a}{2})^2 + \frac{3(q_i^b)^2 + \delta^2}{9} - (\frac{q_i^b}{2})^2 + 2(\frac{q_i^a q_i^b}{6} - \frac{q_i^a q_i^b}{4}) = \frac{3(q_i^a - q_i^b)^2 + 8\delta^2}{36}$. Therefore, $\sigma^2 = \frac{3(q_i^a - q_i^b)^2 + 8\delta^2}{36N}$. $\square$

Appendix C: Proof of Lemma 1

Proof of Lemma 1. Let $x = \bar{r}_1 - \bar{r}_2$. We know that $\bar{r}_i$ follows a Normal distribution with mean $\frac{q_i^a + q_i^b}{2}$ and variance $\frac{3(q_i^a - q_i^b)^2 + 8\delta^2}{36}$. Difference of two normal random variables with means μ_1, μ_2 and variances σ_1^2, σ_2^2 is also a normal random variable, with mean $\mu_1 - \mu_2$ and variance $\sigma_1^2 + \sigma_2^2$. Therefore, x follows a Normal distribution with mean $\frac{q_1^a + q_1^b}{2} - \frac{q_2^a + q_2^b}{2}$ and variance $\frac{3(q_1^a - q_1^b)^2 + 3(q_2^a - q_2^b)^2 + 16\delta^2}{36}$. Since we focus on experience goods, $\delta = 0$. Using equilibrium prices and demands explained in §3.1, we can calculate the expected prices, demands, firm profits and platform profits as follows (assuming ϵ to be zero). $\mathbb{E}(p_1^*) = \int_0^\infty x f(x) \, dx = \frac{exp(-\frac{\mu^2}{2\sigma^2})\sigma}{\sqrt{2\pi}} + \frac{\mu}{2}(1 + Erf[\frac{\mu}{\sqrt{2}\sigma}])$. $\mathbb{E}(p_2^*) = \int_{-\infty}^0 -x f(x) \, dx = \frac{exp(-\frac{\mu^2}{2\sigma^2})\sigma}{\sqrt{2\pi}} - \frac{\mu}{2}(1 - Erf[\frac{\mu}{\sqrt{2}\sigma}])$. $\mathbb{E}(D_1^*) = \int_0^\infty f(x) \, dx = \frac{1}{2}\left(1 + Erf[\frac{\mu}{\sqrt{2}\sigma}]\right)$. $\mathbb{E}(D_2^*) = \int_{-\infty}^0 f(x) \, dx = \frac{1}{2}\left(1 - Erf[\frac{\mu}{\sqrt{2}\sigma}]\right)$. $\mathbb{E}(\pi_1^{SD*}) = \int_0^\infty (1 - \alpha) x f(x) \, dx = (1 - \alpha)\left(\frac{exp(-\frac{\mu^2}{2\sigma^2})\sigma}{\sqrt{2\pi}} + \frac{\mu}{2}(1 + Erf[\frac{\mu}{\sqrt{2}\sigma}])\right)$. $\mathbb{E}(\pi_2^{SD*}) = \int_{-\infty}^0 (1 - \alpha) - x f(x) \, dx = (1 - \alpha)\left(\frac{exp(-\frac{\mu^2}{2\sigma^2})\sigma}{\sqrt{2\pi}} - \frac{\mu}{2}(1 - Erf[\frac{\mu}{\sqrt{2}\sigma}])\right)$. $\mathbb{E}(\pi_R^{SD*}) =$

$\int_0^\infty \alpha x f(x)\,dx + \int_{-\infty}^0 \alpha(-x)f(x)\,dx = \alpha\left(exp(-\frac{\mu^2}{2\sigma^2})\sigma\sqrt{\frac{2}{\pi}} + \mu Erf(\frac{\mu}{\sqrt{2}\sigma})\right)$. In all the equations above, $\mu = \frac{q_1^a + q_1^b - q_2^a - q_2^b}{2}, \sigma = \sqrt{\frac{(q_1^a - q_1^b)^2 + (q_2^a - q_2^b)^2}{12N}}$. Also, note that Erf is the error function.

However, the above results are only applicable when there is variance in x. Let us find equilibrium outputs when there is no variance, $\sigma = 0$. We know that $\sigma^2 = \frac{3(q_1^a - q_1^b)^2 + 3(q_2^a - q_2^b)^2 + 16\delta^2}{36N}$. Since we focus on experience goods, we know that $\delta = 0$. Then, $\sigma^2 = \frac{(q_1^a - q_1^b)^2 + (q_2^a - q_2^b)^2}{12N}$. When $q_1^a - q_1^b = q_2^a - q_2^b = 0$, there is no variance in x, i.e. $\sigma = 0$. When there is no variance, $\mathbb{E}[p_1^*] = \mu$ if $\mu \geq 0$ and $\mathbb{E}[p_1^*] = 0$ if $\mu \leq 0$. Likewise, $\mathbb{E}[p_2^*] = 0$ if $\mu \geq 0$ and $\mathbb{E}[p_2^*] = -\mu$ if $\mu \leq 0$. Then, $\mathbb{E}[D_1^*] = 1, \mathbb{E}[D_2^*] = 0$ if $\mu > 0$, $\mathbb{E}[D_1^*] = 1/2, \mathbb{E}[D_2^*] = 1/2$ if $\mu = 0$, and $\mathbb{E}[D_1^*] = 0, \mathbb{E}[D_2^*] = 1$ if $\mu < 0$. Moreover, we can calculate $\mathbb{E}[\pi_1^*] = (1-\alpha)\mu$ if $\mu \geq 0$ and $\mathbb{E}[\pi_1^*] = 0$ if $\mu \leq 0$. Likewise, $\mathbb{E}[\pi_2^*] = 0$ if $\mu \geq 0$ and $\mathbb{E}[\pi_2^*] = -(1-\alpha)\mu$ if $\mu \leq 0$. Finally, $\mathbb{E}[\pi_R^{SD*}] = \alpha\mu$ if $\mu \geq 0$ and $\mathbb{E}[\pi_R^{SD*}] = -\alpha\mu$ if $\mu \leq 0$, where $\mu = \frac{q_1^a + q_1^b - q_2^a - q_2^b}{2}$, and as ϵ approaches to zero. $\square$

Appendix D: Proof of Lemma 2

Proof of Lemma 2. First, we need to find the indifferent consumer's type. We know that $\mathbb{E}[U_i(\theta)] = \theta\bar{r}_i^a + (1-\theta)\bar{r}_i^b - p_i$. Let $\gamma^a = \bar{r}_1^a - \bar{r}_2^a$ and $\gamma^b = \bar{r}_1^b - \bar{r}_2^b$. There are several different cases that changes the functional form of the indifferent consumer's type, θ. Let us define and analyze these cases.

Case 1: When $\gamma^a \geq 0$, $\gamma^b \leq 0$, type of indifferent consumer is $\theta^* = \frac{p_1 - p_2 - \gamma^b}{\gamma^a - \gamma^b}$. In this case, demand of firm 1 is $D_1 = 1 - \theta^*$ and demand of firm 2 is $D_2 = \theta^*$. We obtain equilibrium prices by simultaneously solving the following maximization problems: $\max\limits_{p_1} p_1 D_1 = p_1\left(\frac{\gamma^a - p_1 + p_2}{\gamma^a - \gamma^b}\right), \max\limits_{p_2} p_2 D_2 = p_2\left(\frac{p_1 - p_2 - \gamma^b}{\gamma^a - \gamma^b}\right)$. When we solve the maximization problems, we get $p_1^* = \frac{2\gamma^a - \gamma^b}{3}$, $p_2^* = \frac{\gamma^a - 2\gamma^b}{3}$. When we plug the equilibrium prices into the demand functions, we get $D_1^* = 1 - \theta^* = \frac{2\gamma^a - \gamma^b}{3(\gamma^a - \gamma^b)}$ and $D_2^* = \theta^* = \frac{\gamma^a - 2\gamma^b}{3(\gamma^a - \gamma^b)}$. In this case, $0 \leq \theta^* \leq 1$. Multiplying the demand and equilibrium prices, we calculate the equilibrium firm profits: $\pi_1^{MD*} = (1-\alpha)\frac{(2\gamma^a - \gamma^b)^2}{9(\gamma^a - \gamma^b)}$, $\pi_2^{MD*} = (1-\alpha)\frac{(\gamma^a - 2\gamma^b)^2}{9(\gamma^a - \gamma^b)}$. Finally, we can calculate the platform profit as follows, $\pi_R^{MD*} = \alpha\frac{(2\gamma^a - \gamma^b)^2 + (\gamma^a - 2\gamma^b)^2}{9(\gamma^a - \gamma^b)}$.

Case 2: When $\gamma^a \leq 0$, $\gamma^b \geq 0$, type of indifferent consumer is $\theta^* = \frac{p_1 - p_2 - \gamma^b}{\gamma^a - \gamma^b}$. In this case, demand of firm 1 is $D_1 = \theta^*$ and demand of firm 2 is $D_2 = 1 - \theta^*$. We obtain equilibrium prices by simultaneously solving the following maximization problems: $\max\limits_{p_1} p_1 D_1 = p_1\left(\frac{p_1 - p_2 - \gamma^b}{\gamma^a - \gamma^b}\right), \max\limits_{p_2} p_2 D_2 = p_2\left(\frac{\gamma^a - p_1 + p_2}{\gamma^a - \gamma^b}\right)$. When we solve the maximization problems, we get $p_1^* = \frac{2\gamma^b - \gamma^a}{3}$, $p_2^* = \frac{\gamma^b - 2\gamma^a}{3}$. When we plug the equilibrium prices into the demand functions, we get $D_1^* = \theta^* = \frac{2\gamma^b - \gamma^a}{3(\gamma^b - \gamma^a)}$ and $D_2^* = 1 - \theta^* = \frac{\gamma^b - 2\gamma^a}{3(\gamma^b - \gamma^a)}$. In this case, $0 \leq \theta^* \leq 1$. Multiplying the demand and equilibrium prices, we calculate the equilibrium firm profits: $\pi_1^{MD*} = (1-\alpha)\frac{(2\gamma^b - \gamma^a)^2}{9(\gamma^b - \gamma^a)}$, $\pi_2^{MD*} = (1-\alpha)\frac{(\gamma^b - 2\gamma^a)^2}{9(\gamma^b - \gamma^a)}$. Finally, we can calculate the platform profit as follows, $\pi_R^{MD*} = \alpha\frac{(2\gamma^b - \gamma^a)^2 + (\gamma^b - 2\gamma^a)^2}{9(\gamma^b - \gamma^a)}$.

Case 3: When $\gamma^b \geq \gamma^a \geq 0$, type of indifferent consumer is $\theta^* = \frac{p_1 - p_2 - \gamma^b}{\gamma^a - \gamma^b}$. In this case, demand of firm 1 is $D_1 = \theta^*$ and demand of firm 2 is $D_2 = 1 - \theta^*$. We obtain equilibrium prices by simultaneously solving the following maximization problems: $\max\limits_{p_1} p_1 D_1 = p_1\left(\frac{p_1 - p_2 - \gamma^b}{\gamma^a - \gamma^b}\right), \max\limits_{p_2} p_2 D_2 = p_2\left(\frac{\gamma^a - p_1 + p_2}{\gamma^a - \gamma^b}\right)$. When we solve the maximization problems, we get $p_1^* = \frac{2\gamma^b - \gamma^a}{3}$, $p_2^* = \frac{\gamma^b - 2\gamma^a}{3}$. When we plug the equilibrium prices into the demand functions, we get $D_1^* = \theta^* = \frac{2\gamma^b - \gamma^a}{3(\gamma^b - \gamma^a)}$ and $D_2^* = 1 - \theta^* = \frac{\gamma^b - 2\gamma^a}{3(\gamma^b - \gamma^a)}$. Since θ^* can be greater than 1, we need to find when $\theta^* \geq 1$. When we solve the following inequality, $\frac{2\gamma^b - \gamma^a}{3(\gamma^b - \gamma^a)} \geq 1$, we get $2\gamma^a \geq \gamma^b$. Therefore, when $2\gamma^a \geq \gamma^b \geq \gamma^a \geq 0$, $D_1^* = 1$ and $D_2^* = 0$. In that case, $p_1^* = \gamma^a$ and $p_2^* = 0$. Multiplying the demand and equilibrium prices, we calculate the equilibrium firm profits as $\pi_1^{MD*} = (1-\alpha)\gamma^a$, $\pi_2^{MD*} = 0$, and platform profit as $\pi_R^{MD*} = \alpha\gamma^a$. Otherwise, $D_1^* = \theta^*$ and $D_2^* = 1 - \theta^*$, where θ^* is solved above. Multiplying the

demand and equilibrium prices, we calculate the equilibrium firm profits: $\pi_1^{MD*} = (1-\alpha)\frac{(2\gamma^b-\gamma^a)^2}{9(\gamma^b-\gamma^a)}$, $\pi_2^{MD*} = (1-\alpha)\frac{(\gamma^b-2\gamma^a)^2}{9(\gamma^b-\gamma^a)}$. Finally, we can calculate the platform profit as follows, $\pi_R^{MD*} = \alpha\frac{(2\gamma^b-\gamma^a)^2+(\gamma^b-2\gamma^a)^2}{9(\gamma^b-\gamma^a)}$.

Case 4: When $\gamma^a \geq \gamma^b \geq 0$, type of indifferent consumer is $\theta^* = \frac{p_1-p_2-\gamma^b}{\gamma^a-\gamma^b}$. In this case, demand of firm 1 is $D_1 = 1-\theta^*$ and demand of firm 2 is $D_2 = \theta^*$. We obtain equilibrium prices by simultaneously solving the following maximization problems: $\max_{p_1} p_1 D_1 = p_1\left(\frac{\gamma^a-p_1+p_2}{\gamma^a-\gamma^b}\right), \max_{p_2} p_2 D_2 = p_2\left(\frac{p_1-p_2-\gamma^b}{\gamma^a-\gamma^b}\right)$. When we solve the maximization problems, we get $p_1^* = \frac{2\gamma^a-\gamma^b}{3}$, $p_2^* = \frac{\gamma^a-2\gamma^b}{3}$. When we plug the equilibrium prices into the demand functions, we get $D_1^* = 1-\theta^* = \frac{2\gamma^a-\gamma^b}{3(\gamma^a-\gamma^b)}$ and $D_2^* = \theta^* = \frac{\gamma^a-2\gamma^b}{3(\gamma^a-\gamma^b)}$. Since θ^* can be less than 0, we need to find when $\theta^* \leq 0$. When we solve the following inequality, $\frac{\gamma^a-2\gamma^b}{3(\gamma^a-\gamma^b)} \leq 0$, we get $\gamma^b \geq \frac{\gamma^a}{2}$. Therefore, when $\gamma^a \geq \gamma^b \geq \frac{\gamma^a}{2} \geq 0$, $D_1^* = 1$ and $D_2^* = 0$. In that case, $p_1^* = \gamma^b$ and $p_2^* = 0$. Multiplying the demand and equilibrium prices, we calculate the equilibrium firm profits as $\pi_1^{MD*} = (1-\alpha)\gamma^b$, $\pi_2^{MD*} = 0$, and platform profit as $\pi_R^{MD*} = \alpha\gamma^b$. Otherwise, $D_1^* = 1-\theta^*$ and $D_2^* = \theta^*$, where θ^* is solved above. Multiplying the demand and equilibrium prices, we calculate the equilibrium firm profits: $\pi_1^{MD*} = (1-\alpha)\frac{(2\gamma^a-\gamma^b)^2}{9(\gamma^a-\gamma^b)}$, $\pi_2^{MD*} = (1-\alpha)\frac{(\gamma^a-2\gamma^b)^2}{9(\gamma^a-\gamma^b)}$. Finally, we can calculate the platform profit as follows, $\pi_R^{MD*} = \alpha\frac{(2\gamma^a-\gamma^b)^2+(\gamma^a-2\gamma^b)^2}{9(\gamma^a-\gamma^b)}$.

Case 5: When $0 \geq \gamma^b \geq \gamma^a$, type of indifferent consumer is $\theta^* = \frac{p_1-p_2-\gamma^b}{\gamma^a-\gamma^b}$. In this case, demand of firm 1 is $D_1 = \theta^*$ and demand of firm 2 is $D_2 = 1-\theta^*$. We obtain equilibrium prices by simultaneously solving the following maximization problems: $\max_{p_1} p_1 D_1 = p_1\left(\frac{p_1-p_2-\gamma^b}{\gamma^a-\gamma^b}\right), \max_{p_2} p_2 D_2 = p_2\left(\frac{\gamma^a-p_1+p_2}{\gamma^a-\gamma^b}\right)$. When we solve the maximization problems, we get $p_1^* = \frac{2\gamma^b-\gamma^a}{3}$, $p_2^* = \frac{\gamma^b-2\gamma^a}{3}$. When we plug the equilibrium prices into the demand functions, we get $D_1^* = \theta^* = \frac{2\gamma^b-\gamma^a}{3(\gamma^b-\gamma^a)}$ and $D_2^* = 1-\theta^* = \frac{\gamma^b-2\gamma^a}{3(\gamma^b-\gamma^a)}$. Since θ^* can be less than 0, we need to find when $\theta^* \leq 0$. When we solve the following inequality, $\frac{2\gamma^b-\gamma^a}{3(\gamma^b-\gamma^a)} \leq 0$, we get $\frac{\gamma^a}{2} \geq \gamma^b$. Therefore, when $0 \geq \frac{\gamma^a}{2} \geq \gamma^b \geq \gamma^a$, $D_1^* = 0$ and $D_2^* = 1$. In that case, $p_1^* = 0$ and $p_2^* = -\gamma^b$. Multiplying the demand and equilibrium prices, we calculate the equilibrium firm profits as $\pi_1^{MD*} = 0$, $\pi_2^{MD*} = -(1-\alpha)\gamma^b$, and platform profit as $\pi_R^{MD*} = -\alpha\gamma^b$. Otherwise, $D_1^* = \theta^*$ and $D_2^* = 1-\theta^*$, where θ^* is solved above. Multiplying the demand and equilibrium prices, we calculate the equilibrium firm profits: $\pi_1^{MD*} = (1-\alpha)\frac{(2\gamma^b-\gamma^a)^2}{9(\gamma^b-\gamma^a)}$, $\pi_2^{MD*} = (1-\alpha)\frac{(\gamma^b-2\gamma^a)^2}{9(\gamma^b-\gamma^a)}$. Finally, we can calculate the platform profit as follows, $\pi_R^{MD*} = \alpha\frac{(2\gamma^b-\gamma^a)^2+(\gamma^b-2\gamma^a)^2}{9(\gamma^b-\gamma^a)}$.

Case 6: When $0 \geq \gamma^a \geq \gamma^b$, type of indifferent consumer is $\theta^* = \frac{p_1-p_2-\gamma^b}{\gamma^a-\gamma^b}$. In this case, demand of firm 1 is $D_1 = 1-\theta^*$ and demand of firm 2 is $D_2 = \theta^*$. We obtain equilibrium prices by simultaneously solving the following maximization problems: $\max_{p_1} p_1 D_1 = p_1\left(\frac{\gamma^a-p_1+p_2}{\gamma^a-\gamma^b}\right), \max_{p_2} p_2 D_2 = p_2\left(\frac{p_1-p_2-\gamma^b}{\gamma^a-\gamma^b}\right)$. When we solve the maximization problems, we get $p_1^* = \frac{2\gamma^a-\gamma^b}{3}$, $p_2^* = \frac{\gamma^a-2\gamma^b}{3}$. When we plug the equilibrium prices into the demand functions, we get $D_1^* = 1-\theta^* = \frac{2\gamma^a-\gamma^b}{3(\gamma^a-\gamma^b)}$ and $D_2^* = \theta^* = \frac{\gamma^a-2\gamma^b}{3(\gamma^a-\gamma^b)}$. Since θ^* can be greater than 1, we need to find when $\theta^* \geq 1$. When we solve the following inequality, $\frac{\gamma^a-2\gamma^b}{3(\gamma^a-\gamma^b)} \geq 1$, we get $\gamma^b \geq 2\gamma^a$. Therefore, when $0 \geq \gamma^a \geq \gamma^b \geq 2\gamma^a$, $D_1^* = 0$ and $D_2^* = 1$. In that case, $p_1^* = 0$ and $p_2^* = -\gamma^a$. Multiplying the demand and equilibrium prices, we calculate the equilibrium firm profits as $\pi_1^{MD*} = 0$, $\pi_2^{MD*} = -(1-\alpha)\gamma^a$, and platform profit as $\pi_R^{MD*} = -\alpha\gamma^a$. Otherwise, $D_1^* = 1-\theta^*$ and $D_2^* = \theta^*$, where θ^* is solved above. Multiplying the demand and equilibrium prices, we calculate the equilibrium firm profits: $\pi_1^{MD*} = (1-\alpha)\frac{(2\gamma^a-\gamma^b)^2}{9(\gamma^a-\gamma^b)}$, $\pi_2^{MD*} = (1-\alpha)\frac{(\gamma^a-2\gamma^b)^2}{9(\gamma^a-\gamma^b)}$. Finally, we can calculate the platform profit as follows, $\pi_R^{MD*} = \alpha\frac{(2\gamma^a-\gamma^b)^2+(\gamma^a-2\gamma^b)^2}{9(\gamma^a-\gamma^b)}$. $\square$

Appendix E: Comparison of Single-Dimensional and Multi-dimensional Rating Schemes

Case 1: A firm is better than the other firm in both dimensions. There are two possible scenarios in Case 1 which are presented below.

	Attribute A	Attribute B
Firm 1	H	H
Firm 2	L	L

	Attribute A	Attribute B
Firm 1	L	L
Firm 2	H	H

Case 2: A firm is better in one dimension and both firms are equal (H) in the other dimension. There are four possible scenarios in Case 2 which are presented below.

	Attribute A	Attribute B
Firm 1	H	H
Firm 2	L	H

	Attribute A	Attribute B
Firm 1	L	H
Firm 2	H	H

	Attribute A	Attribute B
Firm 1	H	H
Firm 2	H	L

	Attribute A	Attribute B
Firm 1	H	L
Firm 2	H	H

Case 3: A firm is better in one dimension and both firms are equal (L) in the other dimension. There are four possible scenarios in Case 3 which are presented below.

	Attribute A	Attribute B
Firm 1	H	L
Firm 2	L	L

	Attribute A	Attribute B
Firm 1	L	L
Firm 2	H	L

	Attribute A	Attribute B
Firm 1	L	H
Firm 2	L	L

	Attribute A	Attribute B
Firm 1	L	L
Firm 2	L	H

Case 4: A firm is better in one dimension, the other firm is better in the other dimension. There are two possible scenarios in Case 4 which are presented below.

	Attribute A	Attribute B
Firm 1	H	L
Firm 2	L	H

	Attribute A	Attribute B
Firm 1	L	H
Firm 2	H	L

Case 5: Both firms are either H or L in both dimensions. There are two possible scenarios in Case 5 which are presented below.

	Attribute A	Attribute B
Firm 1	H	H
Firm 2	H	H

	Attribute A	Attribute B
Firm 1	L	L
Firm 2	L	L

Case 6: Both firms are H in the same dimension and L in the other dimension. There are two possible scenarios in Case 6 which are presented below.

	Attribute A	Attribute B
Firm 1	H	L
Firm 2	H	L

	Attribute A	Attribute B
Firm 1	L	H
Firm 2	L	H

Appendix F: Proof of Proposition 1

Proof of Proposition 1. For case 1, let $q_1^a = q_1^b = H$ and $q_2^a = q_2^b = L$. Then, $\sigma = 0$ and $\gamma^a = \gamma^b = H - L$. We can calculate platform's expected profit as follows: $\mathbb{E}(\pi_R^{SD*}) = \alpha\mu = \alpha\left(\frac{\gamma^a + \gamma^b}{2}\right) = \alpha(H - L)$ and $\mathbb{E}(\pi_R^{MD*}) = \alpha\gamma^a = \alpha(H - L)$. In summary, $\mathbb{E}(\pi_R^{SD*}) = \mathbb{E}(\pi_R^{MD*})$ in case 1.

For case 2, let $q_1^a = q_1^b = H$ and $q_2^a = L, q_2^b = H$. Then, $\sigma \neq 0$ and $\gamma^a = H - L, \gamma^b = 0$. When we plug in $\mu = \frac{H-L}{2}$ and $\sigma = \frac{H-L}{\sqrt{12N}}$ to platform's expected profit formulation, we get $\mathbb{E}(\pi_R^{SD*}) = \alpha\big(exp(\frac{-3N}{2})\frac{H-L}{\sqrt{6N\pi}} + \frac{H-L}{2}Erf(\sqrt{\frac{3N}{2}})\big)$. Note that as N approaches to infinity, the expected profit for the platform converges to $\lim_{N \to \infty} \mathbb{E}(\pi_R^{SD*}) = \alpha\big(\frac{H-L}{2}\big)$. Platform's profit in multi-dimensional ratings can be calculated as follows: $\mathbb{E}(\pi_R^{MD*}) = \alpha\big(\frac{(2\gamma^a-\gamma^b)^2+(\gamma^a-2\gamma^b)^2}{9(\gamma^a-\gamma^b)}\big) = \alpha\big(\frac{5(H-L)}{9}\big)$. Therefore, $\mathbb{E}(\pi_R^{MD*}) > \mathbb{E}(\pi_R^{SD*})$ in case 2 for any values of H and L, when $N \geq 1$.

For case 3, let $q_1^a = H, q_1^b = L$ and $q_2^a = q_2^b = L$. Then, $\sigma \neq 0$ and $\gamma^a = H - L, \gamma^b = 0$. When we plug in $\mu = \frac{H-L}{2}$ and $\sigma = \frac{H-L}{\sqrt{12N}}$ to the formula in Lemma 3, we get $\mathbb{E}(\pi_R^{SD*}) = \alpha\big(exp(\frac{-3N}{2})\frac{H-L}{\sqrt{6N\pi}} + \frac{H-L}{2}Erf(\sqrt{\frac{3N}{2}})\big)$. Note that as N approaches to infinity, the expected profit for the platform converges to $\lim_{N \to \infty} \mathbb{E}(\pi_R^{SD*}) = \alpha\big(\frac{H-L}{2}\big)$. Platform's expected profit in multi-dimensional ratings becomes $\mathbb{E}(\pi_R^{MD*}) = \alpha\big(\frac{(2\gamma^a-\gamma^b)^2+(\gamma^a-2\gamma^b)^2}{9(\gamma^a-\gamma^b)}\big) = \alpha\big(\frac{5(H-L)}{9}\big)$. Therefore, $\mathbb{E}(\pi_R^{MD*}) > \mathbb{E}(\pi_R^{SD*})$ in case 3 for any values of H and L, when $N \geq 1$.

For case 4, let $q_1^a = H, q_1^b = L$ and $q_2^a = L, q_2^b = H$. Then, $\sigma \neq 0$ and $\gamma^a = H - L, \gamma^b = L - H$. When we plug in $\mu = 0$ and $\sigma = \frac{H-L}{\sqrt{6N}}$ to above formula, we get $\mathbb{E}(\pi_R^{SD*}) = \alpha\big(\frac{H-L}{\sqrt{3N\pi}}\big)$. To calculate platform's expected profit in multi-dimensional ratings, $\mathbb{E}(\pi_R^{MD*}) = \alpha\big(\frac{(2\gamma^a-\gamma^b)^2+(\gamma^a-2\gamma^b)^2}{9(\gamma^a-\gamma^b)}\big) = \alpha(H - L)$. Therefore, $\mathbb{E}(\pi_R^{MD*}) > \mathbb{E}(\pi_R^{SD*})$ in case 4 for any values of H and L, when $N \geq 1$.

For case 5, let $q_1^a = q_1^b = H$ and $q_2^a = q_2^b = H$. Then, $\sigma = 0$ and $\gamma^a = \gamma^b = 0$. $\mathbb{E}(\pi_R^{SD*}) = \alpha\mu = 0$. Platform's expected profit in multi-dimensional ratings is $\mathbb{E}(\pi_R^{MD*}) = \alpha\gamma^a = 0$. Therefore, $\mathbb{E}(\pi_R^{SD*}) = \mathbb{E}(\pi_R^{MD*})$ in case 5.

For case 6, let $q_1^a = H, q_1^b = L$ and $q_2^a = H, q_2^b = L$. Then, $\sigma \neq 0$ and $\gamma^a = \gamma^b = 0$. When we plug in $\mu = 0$ and $\sigma = \frac{H-L}{\sqrt{6N}}$, we get $\mathbb{E}(\pi_R^{SD*}) = \alpha\big(\frac{H-L}{\sqrt{3N\pi}}\big)$. Since $\gamma^a = \gamma^b = 0$, $\mathbb{E}(\pi_R^{MD*}) = \alpha\gamma^a = 0$. Therefore, $\mathbb{E}(\pi_R^{SD*}) > \mathbb{E}(\pi_R^{MD*})$ in case 6. $\square$

Appendix G: Proof of Proposition 2

Proof of Proposition 2. For case 1, let $q_1^a = q_1^b = H$ and $q_2^a = q_2^b = L$. Then, $\sigma = 0$ and $\mu > 0$. We can calculate the equilibrium prices in SD using Lemma 1 as $p_1^* = \mu = \bar{r}_1 - \bar{r}_2 = H - L, p_2^* = 0$. Likewise, demand functions are: $D_1^* = 1, D_2^* = 0$. Then, consumer surplus in SD becomes: $S_{SD} = \int_0^1 v + \theta H + (1-\theta)H - (H - L)\,\mathrm{d}\theta = v + L$. In MD, price and demand functions in equilibrium are as follows: $p_1^* = \gamma^a = H - L, p_2^* = 0$ and $D_1^* = 1, D_2^* = 0$. Finally, consumer surplus in multi-dimensional ratings is: $S_{MD} = \int_0^1 v + \theta H + (1-\theta)H - (H - L)\,\mathrm{d}\theta = v + L$. Therefore, consumer surplus are the same in both rating systems in case 1, i.e. $S_{MD} = S_{SD}$.

For case 2, we let $q_1^a = q_1^b = H$ and $q_2^a = L, q_2^b = H$. Then, $\sigma \neq 0$. Let x be the difference in the average ratings in SD, $x = \bar{r}_1 - \bar{r}_2$. When $x > 0$, consumers purchase product 1, and when $x < 0$ they purchase product 2. Firms set their prices in equilibrium as $p_1^* = min\{0, x\}$ and $p_2^* = min\{0, -x\}$. Consumer surplus in SD can be calculated as: $S_{SD} = \int_0^\infty \big(\int_0^1 v + \theta H + (1-\theta)H - x\,\mathrm{d}\theta\big)f(x)\,\mathrm{d}x + \int_{-\infty}^0 \big(\int_0^1 v + \theta L + (1-\theta)H - (-x)\,\mathrm{d}\theta\big)f(x)\,\mathrm{d}x$, where x $\sim Normal\big(\frac{H-L}{2}, \frac{(H-L)^2}{12N}\big)$. When we solve this equation we get: $S_{SD} = v + \frac{H+L}{2} - \frac{H-L}{exp(\frac{3N}{2})\sqrt{6N\pi}} - \frac{H-L}{4}\big(1 - Erf(\sqrt{\frac{3N}{2}})\big)$. Note that as N approaches to infinity, consumer surplus converges to $\lim_{N \to \infty} S_{SD} = v + \frac{H+L}{2}$. In MD, price and demand functions in equilibrium can be calculated as follows: $p_1^* = \frac{2\gamma^a-\gamma^b}{3} = \frac{2(H-L)}{3}, p_2^* = \frac{\gamma^a-2\gamma^b}{3} = \frac{H-L}{3}$ and $D_1^* = \frac{2\gamma^a-\gamma^b}{3(\gamma^a-\gamma^b)} = \frac{2}{3}, D_2^* = \frac{\gamma^a-2\gamma^b}{3(\gamma^a-\gamma^b)} = \frac{1}{3}$. Finally, consumer surplus in MD is: $S_{MD} = \int_0^{\frac{1}{3}} v + \theta L + (1-\theta)H - \frac{H-L}{3}\,\mathrm{d}\theta + \int_{\frac{1}{3}}^1 v + \theta H + (1-\theta)H - \frac{2(H-L)}{3}\,\mathrm{d}\theta = v + \frac{7H+11L}{18}$. Therefore, $S_{SD} > S_{MD}$ for any values of v, H and L, when $N \geq 1$.

For case 3, we let $q_1^a = H, q_1^b = L$ and $q_2^a = q_2^b = L$. Then, $\sigma \neq 0$. Consumer surplus in SD can be calculated as explained before: $S_{SD} = \int_0^\infty \left(\int_0^1 v + \theta H + (1 - \theta)L - x \, d\theta \right) f(x) \, dx + \int_{-\infty}^0 \left(\int_0^1 v + \theta L + (1 - \theta)L - (-x) \, d\theta \right) f(x) \, dx$, where x $\sim Normal\left(\frac{H-L}{2}, \frac{(H-L)^2}{12N} \right)$. Then we get $S_{SD} = v + L - \frac{H-L}{exp(\frac{3N}{2})\sqrt{6N\pi}} - \frac{H-L}{4}\left(1 - Erf(\sqrt{\frac{3N}{2}}) \right)$. Note that as N approaches to infinity, consumer surplus converges to $\lim_{N \to \infty} S_{SD} = v + L$. In MD, price and demand functions in equilibrium can be calculated as: $p_1^* = \frac{2\gamma^a - \gamma^b}{3} = \frac{2(H-L)}{3}, p_2^* = \frac{\gamma^a - 2\gamma^b}{3} = \frac{H-L}{3}$ and $D_1^* = \frac{2\gamma^a - \gamma^b}{3(\gamma^a - \gamma^b)} = \frac{2}{3}, D_2^* = \frac{\gamma^a - 2\gamma^b}{3(\gamma^a - \gamma^b)} = \frac{1}{3}$. Finally, consumer surplus in MD is: $S_{MD} = \int_0^{\frac{1}{3}} v + \theta L + (1 - \theta)L - \frac{H-L}{3} \, d\theta + \int_{\frac{1}{3}}^1 v + \theta H + (1 - \theta)L - \frac{2(H-L)}{3} \, d\theta = v + \frac{-H+10L}{9}$. Therefore, $S_{SD} > S_{MD}$ for any values of v, H and L, when $N \geq 1$.

For case 4, we let $q_1^a = H, q_1^b = L$ and $q_2^a = L, q_2^b = H$. Then, $\sigma \neq 0$. We can calculate consumer surplus in SD as: $S_{SD} = \int_0^\infty \left(\int_0^1 v + \theta H + (1 - \theta)L - x \, d\theta \right) f(x) \, dx + \int_{-\infty}^0 \left(\int_0^1 v + \theta L + (1 - \theta)H - (-x) \, d\theta \right) f(x) \, dx$, where x $\sim Normal\left(0, \frac{(H-L)^2}{6N} \right)$. When we solve this equation we get: $S_{SD} = v + \frac{H+L}{2} - \frac{H-L}{\sqrt{3N\pi}}$. Note that as N approaches to infinity, consumer surplus converges to $\lim_{N \to \infty} S_{SD} = v + \frac{H+L}{2}$. In MD, price and demand functions in equilibrium can be calculated as follows: $p_1^* = \frac{2\gamma^a - \gamma^b}{3} = H - L, p_2^* = \frac{\gamma^a - 2\gamma^b}{3} = H - L$ and $D_1^* = \frac{2\gamma^a - \gamma^b}{3(\gamma^a - \gamma^b)} = \frac{1}{2}, D_2^* = \frac{\gamma^a - 2\gamma^b}{3(\gamma^a - \gamma^b)} = \frac{1}{2}$. Finally, consumer surplus in multi-dimensional ratings is: $S_{MD} = \int_0^{\frac{1}{2}} v + \theta L + (1 - \theta)H - (H - L) \, d\theta + \int_{\frac{1}{2}}^1 v + \theta H + (1 - \theta)L - (H - L) \, d\theta = v + \frac{-H+5L}{4}$. Therefore, $S_{SD} > S_{MD}$ for any values of v, H and L, when $N \geq 1$.

For case 5, let $q_1^a = q_1^b = H$ and $q_2^a = q_2^b = H$. Then, $\sigma = 0$ and $\mu = 0$. We can calculate the equilibrium prices in SD using Lemma 1 as $p_1^* = 0, p_2^* = 0$. Likewise, demand functions are: $D_1^* = \frac{1}{2}, D_2^* = \frac{1}{2}$. Then, consumer surplus in SD becomes: $S_{SD} = \int_0^{\frac{1}{2}} v + \theta H + (1 - \theta)H \, d\theta + \int_{\frac{1}{2}}^1 v + \theta H + (1 - \theta)H \, d\theta = v + H$. In MD, price and demand functions in equilibrium are as follows: $p_1^* = \gamma^a = 0, p_2^* = \gamma^b = 0$ and $D_1^* = \frac{1}{2}, D_2^* = \frac{1}{2}$. Finally, consumer surplus in MD is: $S_{MD} = \int_0^{\frac{1}{2}} v + \theta H + (1 - \theta)H \, d\theta + \int_{\frac{1}{2}}^1 v + \theta H + (1 - \theta)H \, d\theta = v + H$. Therefore, $S_{MD} = S_{SD}$.

For case 6, we let $q_1^a = H, q_1^b = L$ and $q_2^a = H, q_2^b = L$. Then, $\sigma \neq 0$. We can calculate consumer surplus in SD as: $S_{SD} = \int_0^\infty \left(\int_0^1 v + \theta H + (1 - \theta)L - x \, d\theta \right) f(x) \, dx + \int_{-\infty}^0 \left(\int_0^1 v + \theta H + (1 - \theta)L - (-x) \, d\theta \right) f(x) \, dx$, where x $\sim Normal\left(0, \frac{(H-L)^2}{6N} \right)$. When we solve this equation we get: $S_{SD} = v + \frac{H+L}{2} - \frac{H-L}{\sqrt{3N\pi}}$. Note that as N approaches to infinity, consumer surplus converges to $\lim_{N \to \infty} S_{SD} = v + \frac{H+L}{2}$. In MD, price and demand functions in equilibrium are as follows: $p_1^* = \gamma^a = 0, p_2^* = \gamma^b = 0$ and $D_1^* = \frac{1}{2}, D_2^* = \frac{1}{2}$. Finally, consumer surplus in MD is: $S_{MD} = \int_0^{\frac{1}{2}} v + \theta H + (1 - \theta)L \, d\theta + \int_{\frac{1}{2}}^1 v + \theta H + (1 - \theta)L \, d\theta = v + \frac{H+L}{2}$. Therefore, $S_{MD} > S_{SD}$ for any values of v, H and L, when $N \geq 1$. $\square$

Appendix H: Proof of Proposition 3

Proof of Proposition 3.   Social welfare is calculated as the sum of expected profit of the platform and consumer surplus. For case 1, we can calculate the social welfare in both rating systems as: $W_{SD} = \mathbb{E}(\pi_R^{SD*}) + S_{SD} = v + H$, $W_{MD} = \mathbb{E}(\pi_R^{MD*}) + S_{MD} = v + H$. Therefore, $W_{SD} = W_{MD}$.

For case 2, social welfare can be calculated as: $W_{SD} = \mathbb{E}(\pi_R^{SD*}) + S_{SD} = v + \frac{3H+L}{4} + \frac{H-L}{4}Erf\left(\sqrt{\frac{3N}{2}} \right)$, $W_{MD} = \mathbb{E}(\pi_R^{MD*}) + S_{MD} = v + \frac{17H+L}{18}$. Therefore, $W_{SD} > W_{MD}$ for any values of v, H and L, when $N \geq 1$.

In case 3, social welfare can be calculated as: $W_{SD} = \mathbb{E}(\pi_R^{SD*}) + S_{SD} = v + \frac{H+3L}{4} + \frac{H-L}{4}Erf\left(\sqrt{\frac{3N}{2}} \right)$, $W_{MD} = \mathbb{E}(\pi_R^{MD*}) + S_{MD} = v + \frac{4H+5L}{9}$. Therefore, $W_{SD} > W_{MD}$ for any values of v, H and L, when $N \geq 1$.

In case 4, social welfare can be calculated as: $W_{SD} = \mathbb{E}(\pi_R^{SD*}) + S_{SD} = v + \frac{H+L}{2}$, $W_{MD} = \mathbb{E}(\pi_R^{MD*}) + S_{MD} = v + \frac{3H+L}{4}$. Therefore, $W_{MD} > W_{SD}$ for any values of v, H and L, when $N \geq 1$.

In case 5, social welfare can be calculated as: $W_{SD} = \mathbb{E}(\pi_R^{SD*}) + S_{SD} = v + H$, $W_{MD} = \mathbb{E}(\pi_R^{MD*}) + S_{MD} = v + H$. Therefore, $W_{SD} = W_{MD}$.

In case 6, social welfare can be calculated as: $W_{SD} = \mathbb{E}(\pi_R^{SD*}) + S_{SD} = v + \frac{H+L}{2}$, $W_{MD} = \mathbb{E}(\pi_R^{MD*}) + S_{MD} = v + \frac{H+L}{2}$. Therefore, $W_{SD} = W_{MD}$. $\quad\square$

Appendix I: Proof of Proposition 4

Proof of Proposition 4. *Cases 1 and 5*: The expected platform profit, consumer surplus and social welfare in SD is the same as MD for all values of v, H and L when $N \geq 1$. Therefore, the changes in $H - L$ does not impact the difference of expected platform profit, consumer surplus and social welfare between the two rating schemes.

Case 2: Expected platform profit: $\mathbb{E}(\pi_R^{MD*}) = \frac{5(H-L)}{9}$, $\lim_{N\to\infty} \mathbb{E}(\pi_R^{SD*}) = \frac{H-L}{2}$. Then, $\lim_{N\to\infty}(\mathbb{E}(\pi_R^{MD*}) - \mathbb{E}(\pi_R^{SD*})) = \frac{H-L}{18}$. As $H - L$ increases, the superiority of expected platform profit in multi-dimensional rating scheme always increases. Consumer surplus: $S_{MD} = v + \frac{7H+11L}{18}$, $\lim_{N\to\infty} S_{SD} = v + \frac{H+L}{2}$. Therefore, $\lim_{N\to\infty}(S_{SD} - S_{MD}) = \frac{H-L}{9}$. It is clear that when $H - L$ increases, the superiority of consumer surplus in single-dimensional ratings always increases. Social welfare: $\lim_{N\to\infty} W_{SD} = v + H$, $W_{MD} = v + \frac{17H+L}{18}$. Then, $\lim_{N\to\infty}(W_{SD} - W_{MD}) = \frac{H-L}{18}$. As $H - L$ increases, the difference of social welfare in single-dimensional rating scheme always increases.

Case 3: Expected platform profit: $\mathbb{E}(\pi_R^{MD*}) = \frac{5(H-L)}{9}$, $\lim_{N\to\infty} \mathbb{E}(\pi_R^{SD*}) = \frac{H-L}{2}$. Then, $\lim_{N\to\infty}(\mathbb{E}(\pi_R^{MD*}) - \mathbb{E}(\pi_R^{SD*})) = \frac{H-L}{18}$. As $H - L$ increases, the superiority of expected platform profit in multi-dimensional rating scheme always increases. Consumer surplus: $S_{MD} = v + \frac{-H+10L}{9}$, $\lim_{N\to\infty} S_{SD} = v + L$. Therefore, $\lim_{N\to\infty}(S_{SD} - S_{MD}) = \frac{H-L}{9}$. It is clear that when $H - L$ increases, the superiority of consumer surplus in single-dimensional ratings always increases. Social welfare: $\lim_{N\to\infty} W_{SD} = v + \frac{H-L}{2}$, $W_{MD} = v + \frac{4H+5L}{9}$. Then, $\lim_{N\to\infty}(W_{SD} - W_{MD}) = \frac{H-L}{18}$. As $H - L$ increases, the difference of social welfare in single-dimensional rating scheme always increases.

Case 4: Expected platform profit: $\mathbb{E}(\pi_R^{MD*}) = H - L$, $\lim_{N\to\infty} \mathbb{E}(\pi_R^{SD*}) = 0$. Then, $\lim_{N\to\infty}(\mathbb{E}(\pi_R^{MD*}) - \mathbb{E}(\pi_R^{SD*})) = H - L$. As $H - L$ increases, the superiority of expected platform profit in multi-dimensional rating scheme always increases. Consumer surplus: $S_{MD} = v + \frac{-H+5L}{4}$, $\lim_{N\to\infty} S_{SD} = v + \frac{H+L}{2}$. Therefore, $\lim_{N\to\infty}(S_{SD} - S_{MD}) = \frac{3(H-L)}{4}$. It is clear that when $H - L$ increases, the superiority of consumer surplus in single-dimensional ratings always increases. Social welfare: $\lim_{N\to\infty} W_{SD} = v + \frac{H+L}{2}$, $W_{MD} = v + \frac{3H+L}{4}$. Then, $\lim_{N\to\infty}(W_{MD} - W_{SD}) = \frac{H-L}{4}$. As $H - L$ increases, the difference of social welfare in multi-dimensional rating scheme always increases.

Case 6: Expected platform profit: $\mathbb{E}(\pi_R^{MD*}) = 0$, $\mathbb{E}(\pi_R^{SD*}) = \frac{H-L}{\sqrt{3N\pi}}$. Then, $\mathbb{E}(\pi_R^{SD*}) - \mathbb{E}(\pi_R^{MD*}) = \frac{H-L}{\sqrt{3N\pi}}$. As $H - L$ increases, the superiority of expected platform profit in single-dimensional rating scheme always increases. Consumer surplus: $S_{MD} = v + \frac{H+L}{2}$, $S_{SD} = v + \frac{H+L}{2} - \frac{H-L}{\sqrt{3N\pi}}$. Therefore, $S_{MD} - S_{SD} = v + \frac{H-L}{\sqrt{3N\pi}}$. It is clear that when $H - L$ increases, the superiority of consumer surplus in multi-dimensional ratings always increases. Social welfare: $W_{SD} = v + \frac{H+L}{2}$, $W_{MD} = v + \frac{H+L}{2}$. Then, $(W_{SD} - W_{MD}) = 0$. The changes in $H - L$ does not impact the difference of social welfare between the two rating schemes. $\quad\square$

Appendix J: Lemmas 3 and 4

LEMMA 3. *In equilibrium, the expected prices, demands, and firm and platform profits in the single-dimensional rating system are as follows:*

(a) Prices:

$$
\mathbb{E}(p_1^*) = \begin{cases} \dfrac{exp(-\dfrac{\mu^2}{2\sigma^2})\sigma}{\sqrt{2\pi}} + \dfrac{\mu}{2}(1 + Erf[\dfrac{\mu}{\sqrt{2}\sigma}]) & \text{if } \sigma \neq 0 \\ \mu & \text{if } \sigma = 0 \text{ and } \mu \geq 0 \\ 0 & \text{if } \sigma = 0 \text{ and } \mu \leq 0 \end{cases}
$$

$$
\mathbb{E}(p_2^*) = \begin{cases} \dfrac{exp(-\dfrac{\mu^2}{2\sigma^2})\sigma}{\sqrt{2\pi}} - \dfrac{\mu}{2}(1 - Erf[\dfrac{\mu}{\sqrt{2}\sigma}]) & \text{if } \sigma \neq 0 \\ 0 & \text{if } \sigma = 0 \text{ and } \mu \geq 0 \\ -\mu & \text{if } \sigma = 0 \text{ and } \mu \leq 0 \end{cases}
$$

(14)

(b) Demands:

$$
\mathbb{E}(D_1^*) = \begin{cases} \dfrac{1}{2}\left(1 + Erf[\dfrac{\mu}{\sqrt{2}\sigma}]\right) & \text{if } \sigma \neq 0 \\ 1 & \text{if } \sigma = 0 \text{ and } \mu > 0 \\ \dfrac{1}{2} & \text{if } \sigma = 0 \text{ and } \mu = 0 \\ 0 & \text{if } \sigma = 0 \text{ and } \mu < 0 \end{cases}
$$

$$
\mathbb{E}(D_2^*) = \begin{cases} \dfrac{1}{2}\left(1 - Erf[\dfrac{\mu}{\sqrt{2}\sigma}]\right) & \text{if } \sigma \neq 0 \\ 0 & \text{if } \sigma = 0 \text{ and } \mu > 0 \\ \dfrac{1}{2} & \text{if } \sigma = 0 \text{ and } \mu = 0 \\ 1 & \text{if } \sigma = 0 \text{ and } \mu < 0 \end{cases}
$$

(15)

(c) Firm profits:

$$
\mathbb{E}(\pi_1^{SD*}) = \begin{cases} (1-\alpha)\left(\dfrac{exp(-\dfrac{\mu^2}{2\sigma^2})\sigma}{\sqrt{2\pi}} + \dfrac{\mu}{2}(1 + Erf[\dfrac{\mu}{\sqrt{2}\sigma}])\right) & \text{if } \sigma \neq 0 \\ (1-\alpha)\mu & \text{if } \sigma = 0 \text{ and } \mu \geq 0 \\ 0 & \text{if } \sigma = 0 \text{ and } \mu \leq 0 \end{cases}
$$

$$
\mathbb{E}(\pi_2^{SD*}) = \begin{cases} (1-\alpha)\left(\dfrac{exp(-\dfrac{\mu^2}{2\sigma^2})\sigma}{\sqrt{2\pi}} - \dfrac{\mu}{2}(1 - Erf[\dfrac{\mu}{\sqrt{2}\sigma}])\right) & \text{if } \sigma \neq 0 \\ 0 & \text{if } \sigma = 0 \text{ and } \mu \geq 0 \\ -(1-\alpha)\mu & \text{if } \sigma = 0 \text{ and } \mu \leq 0 \end{cases}
$$

(16)

(d) Platform profit:

$$
\mathbb{E}(\pi_R^{SD*}) = \begin{cases} \alpha\left(exp(-\dfrac{\mu^2}{2\sigma^2})\sigma\sqrt{\dfrac{2}{\pi}} + \mu Erf(\dfrac{\mu}{\sqrt{2}\sigma})\right) & \text{if } \sigma \neq 0 \\ \alpha\mu & \text{if } \sigma = 0 \text{ and } \mu \geq 0 \\ -\alpha\mu & \text{if } \sigma = 0 \text{ and } \mu \leq 0 \end{cases}
$$

(17)

where $\mu = \dfrac{(4-\bar{\theta})(q_1^a - q_2^a) + (\bar{\theta}+2)(q_1^b - q_2^b)}{6}, \sigma = \sqrt{\dfrac{(3-(\bar{\theta}-1)^2)((q_1^a - q_1^b)^2 + (q_2^a - q_2^b)^2)}{36N}}$, *and Erf is the error function.*

LEMMA 4. *In equilibrium, prices, demand, firm profits and platform profit in multi-dimensional rating system are shown as follows:*

(a) Prices:

$$\{p_1^*, p_2^*\} = \begin{cases} p_1^*(\bar{\theta}, \gamma^a, \gamma^b), p_2^*(\bar{\theta}, \gamma^a, \gamma^b) & if \begin{array}{ll} (1) & \gamma^a \geq 3\gamma^b \geq 0 \\ (2) & 3\gamma^b \geq \gamma^a \geq 2\gamma^b \geq 0 \ and \ \frac{\gamma^a - \gamma^b}{\gamma^b} \geq \bar{\theta} \geq 1 \\ (3) & 0 \geq \frac{\gamma^b}{2} \geq \gamma^a \geq \gamma^b \ and \ 2 \geq \bar{\theta} \geq \frac{3\gamma^a - \gamma^b}{\gamma^a} \end{array} \\[2em] p_2^*(\bar{\theta}, -\gamma^a, -\gamma^b), p_1^*(\bar{\theta}, -\gamma^a, -\gamma^b) & if \begin{array}{ll} (4) & 0 \geq 3\gamma^b \geq \gamma^a \\ (5) & 0 \geq 2\gamma^b \geq \gamma^a \geq 3\gamma^b \ and \ \frac{\gamma^a - \gamma^b}{\gamma^b} \geq \bar{\theta} \geq 1 \\ (6) & \gamma^b \geq \gamma^a \geq \frac{\gamma^b}{2} \geq 0 \ and \ 2 \geq \bar{\theta} \geq \frac{3\gamma^a - \gamma^b}{\gamma^a} \end{array} \\[2em] \gamma^a, 0 & if \quad \gamma^b \geq \gamma^a \geq \frac{\gamma^b}{2} \ and \ \frac{3\gamma^a - \gamma^b}{\gamma^a} \geq \bar{\theta} \geq 1 \\[1em] \gamma^b, 0 & if \begin{array}{ll} (7) & 2\gamma^b \geq \gamma^a \geq \gamma^b \geq 0 \\ (8) & 3\gamma^b \geq \gamma^a \geq 2\gamma^b \geq 0 \ and \ 2 \geq \bar{\theta} \geq \frac{\gamma^a - \gamma^b}{\gamma^b} \end{array} \\[2em] 0, -\gamma^a & if \quad 0 \geq \frac{\gamma^b}{2} \geq \gamma^a \geq \gamma^b \ and \ \frac{3\gamma^a - \gamma^b}{\gamma^a} \geq \bar{\theta} \geq 1 \\[1em] 0, -\gamma^b & if \begin{array}{ll} (9) & 0 \geq \gamma^b \geq \gamma^a \geq 2\gamma^b \\ (10) & 0 \geq 2\gamma^b \geq \gamma^a \geq 3\gamma^b \ and \ 2 \geq \bar{\theta} \geq \frac{\gamma^a - \gamma^b}{\gamma^b} \end{array} \end{cases} \tag{18}$$

(b) Demand:

$$\{D_1^*, D_2^*\} = \begin{cases} D_1^*(\bar{\theta}, \gamma^a, \gamma^b, \Sigma), D_2^*(\bar{\theta}, \gamma^a, \gamma^b, \Sigma) & if \begin{array}{ll} (1) & \gamma^a \geq 3\gamma^b \geq 0 \\ (2) & 3\gamma^b \geq \gamma^a \geq 2\gamma^b \geq 0 \ and \ \frac{\gamma^a - \gamma^b}{\gamma^b} \geq \bar{\theta} \geq 1 \\ (3) & 0 \geq \frac{\gamma^b}{2} \geq \gamma^a \geq \gamma^b \ and \ 2 \geq \bar{\theta} \geq \frac{3\gamma^a - \gamma^b}{\gamma^a} \end{array} \\[2em] D_2^*(\bar{\theta}, \gamma^a, \gamma^b, -\Sigma), D_1^*(\bar{\theta}, \gamma^a, \gamma^b, -\Sigma) & if \begin{array}{ll} (4) & 0 \geq 3\gamma^b \geq \gamma^a \\ (5) & 0 \geq 2\gamma^b \geq \gamma^a \geq 3\gamma^b \ and \ \frac{\gamma^a - \gamma^b}{\gamma^b} \geq \bar{\theta} \geq 1 \\ (6) & \gamma^b \geq \gamma^a \geq \frac{\gamma^b}{2} \geq 0 \ and \ 2 \geq \bar{\theta} \geq \frac{3\gamma^a - \gamma^b}{\gamma^a} \end{array} \\[2em] 1, 0 & if \begin{array}{ll} (7) & \gamma^b \geq \gamma^a \geq \frac{\gamma^b}{2} \ and \ \frac{3\gamma^a - \gamma^b}{\gamma^a} \geq \bar{\theta} \geq 1 \\ (8) & 2\gamma^b \geq \gamma^a \geq \gamma^b \geq 0 \\ (9) & 3\gamma^b \geq \gamma^a \geq 2\gamma^b \geq 0 \ and \ 2 \geq \bar{\theta} \geq \frac{\gamma^a - \gamma^b}{\gamma^b} \end{array} \\[2em] 0, 1 & if \begin{array}{ll} (10) & 0 \geq \frac{\gamma^b}{2} \geq \gamma^a \geq \gamma^b \ and \ \frac{3\gamma^a - \gamma^b}{\gamma^a} \geq \bar{\theta} \geq 1 \\ (11) & 0 \geq \gamma^b \geq \gamma^a \geq 2\gamma^b \\ (12) & 0 \geq 2\gamma^b \geq \gamma^a \geq 3\gamma^b \ and \ 2 \geq \bar{\theta} \geq \frac{\gamma^a - \gamma^b}{\gamma^b} \end{array} \end{cases} \tag{19}$$

(c) Firm profits:

$$
\{\pi_1^{MD*}, \pi_2^{MD*}\} = \begin{cases}
\begin{aligned}
&(1-\alpha)\pi_1^*(\bar{\theta}, \gamma^a, \gamma^b), \\
&(1-\alpha)\pi_2^*(\bar{\theta}, \gamma^a, \gamma^b)
\end{aligned} & if \begin{array}{l} (1) \quad \gamma^a \geq 3\gamma^b \geq 0 \\ (2) \quad 3\gamma^b \geq \gamma^a \geq 2\gamma^b \geq 0 \ and \ \frac{\gamma^a - \gamma^b}{\gamma^b} \geq \bar{\theta} \geq 1 \\ (3) \quad 0 \geq \frac{\gamma^b}{2} \geq \gamma^a \geq \gamma^b \ and \ 2 \geq \bar{\theta} \geq \frac{3\gamma^a - \gamma^b}{\gamma^a} \end{array} \\[3em]
\begin{aligned}
&(1-\alpha)\pi_{-1}^*(\bar{\theta}, \gamma^a, \gamma^b), \\
&(1-\alpha)\pi_{-2}^*(\bar{\theta}, \gamma^a, \gamma^b)
\end{aligned} & if \begin{array}{l} (4) \quad 0 \geq 3\gamma^b \geq \gamma^a \\ (5) \quad 0 \geq 2\gamma^b \geq \gamma^a \geq 3\gamma^b \ and \ \frac{\gamma^a - \gamma^b}{\gamma^b} \geq \bar{\theta} \geq 1 \\ (6) \quad \gamma^b \geq \gamma^a \geq \frac{\gamma^b}{2} \geq 0 \ and \ 2 \geq \bar{\theta} \geq \frac{3\gamma^a - \gamma^b}{\gamma^a} \end{array} \\[3em]
(1-\alpha)\gamma^a, 0 & if \quad \gamma^b \geq \gamma^a \geq \frac{\gamma^b}{2} \ and \ \frac{3\gamma^a - \gamma^b}{\gamma^a} \geq \bar{\theta} \geq 1 \\[2em]
(1-\alpha)\gamma^b, 0 & if \begin{array}{l} (7) \quad 2\gamma^b \geq \gamma^a \geq \gamma^b \geq 0 \\ (8) \quad 3\gamma^b \geq \gamma^a \geq 2\gamma^b \geq 0 \ and \ 2 \geq \bar{\theta} \geq \frac{\gamma^a - \gamma^b}{\gamma^b} \end{array} \\[2em]
0, -(1-\alpha)\gamma^a & if \quad 0 \geq \frac{\gamma^b}{2} \geq \gamma^a \geq \gamma^b \ and \ \frac{3\gamma^a - \gamma^b}{\gamma^a} \geq \bar{\theta} \geq 1 \\[2em]
0, -(1-\alpha)\gamma^b & if \begin{array}{l} (9) \quad 0 \geq \gamma^b \geq \gamma^a \geq 2\gamma^b \\ (10) \quad 0 \geq 2\gamma^b \geq \gamma^a \geq 3\gamma^b \ and \ 2 \geq \bar{\theta} \geq \frac{\gamma^a - \gamma^b}{\gamma^b} \end{array}
\end{cases} \tag{20}
$$

(d) Platform profit:

$$
\pi_R^{MD*} = \begin{cases}
\alpha(\pi_1^*(\bar{\theta}, \gamma^a, \gamma^b) + \pi_2^*(\bar{\theta}, \gamma^a, \gamma^b)), & if \begin{array}{l} (1) \quad \gamma^a \geq 3\gamma^b \geq 0 \\ (2) \quad 3\gamma^b \geq \gamma^a \geq 2\gamma^b \geq 0 \ and \ \frac{\gamma^a - \gamma^b}{\gamma^b} \geq \bar{\theta} \geq 1 \\ (3) \quad 0 \geq \frac{\gamma^b}{2} \geq \gamma^a \geq \gamma^b \ and \ 2 \geq \bar{\theta} \geq \frac{3\gamma^a - \gamma^b}{\gamma^a} \end{array} \\[3em]
\alpha(\pi_{-1}^*(\bar{\theta}, \gamma^a, \gamma^b) + \pi_{-2}^*(\bar{\theta}, \gamma^a, \gamma^b)) & if \begin{array}{l} (4) \quad 0 \geq 3\gamma^b \geq \gamma^a \\ (5) \quad 0 \geq 2\gamma^b \geq \gamma^a \geq 3\gamma^b \ and \ \frac{\gamma^a - \gamma^b}{\gamma^b} \geq \bar{\theta} \geq 1 \\ (6) \quad \gamma^b \geq \gamma^a \geq \frac{\gamma^b}{2} \geq 0 \ and \ 2 \geq \bar{\theta} \geq \frac{3\gamma^a - \gamma^b}{\gamma^a} \end{array} \\[3em]
\alpha\gamma^a & if \quad \gamma^b \geq \gamma^a \geq \frac{\gamma^b}{2} \ and \ \frac{3\gamma^a - \gamma^b}{\gamma^a} \geq \bar{\theta} \geq 1 \\[2em]
\alpha\gamma^b & if \begin{array}{l} (7) \quad 2\gamma^b \geq \gamma^a \geq \gamma^b \geq 0 \\ (8) \quad 3\gamma^b \geq \gamma^a \geq 2\gamma^b \geq 0 \ and \ 2 \geq \bar{\theta} \geq \frac{\gamma^a - \gamma^b}{\gamma^b} \end{array} \\[2em]
-\alpha\gamma^a & if \quad 0 \geq \frac{\gamma^b}{2} \geq \gamma^a \geq \gamma^b \ and \ \frac{3\gamma^a - \gamma^b}{\gamma^a} \geq \bar{\theta} \geq 1 \\[2em]
-\alpha\gamma^b & if \begin{array}{l} (9) \quad 0 \geq \gamma^b \geq \gamma^a \geq 2\gamma^b \\ (10) \quad 0 \geq 2\gamma^b \geq \gamma^a \geq 3\gamma^b \ and \ 2 \geq \bar{\theta} \geq \frac{\gamma^a - \gamma^b}{\gamma^b} \end{array}
\end{cases} \tag{21}
$$

where $\Sigma = \sqrt{(\bar{\theta}(9\bar{\theta} - 16) + 16)(\gamma^a)^2 - 2(7\bar{\theta}(\bar{\theta} - 2)\bar{\theta} + 16)\gamma^a\gamma^b + (\bar{\theta}(9\bar{\theta} - 20) + 20)(\gamma^b)^2}$,

$p_1^*(\bar{\theta}, \gamma^a, \gamma^b) = \frac{(\bar{\theta}(3\bar{\theta} - 10) + 10)(\bar{\theta}(\gamma^a + \gamma^b) - 2\gamma^b) - ((\bar{\theta} - 6)\bar{\theta} + 6)\Sigma}{16(\bar{\theta} - 1)((\bar{\theta} - 2)\bar{\theta} + 2)}, p_2^*(\bar{\theta}, \gamma^a, \gamma^b) = \frac{(\bar{\theta}(3\bar{\theta} - 2) + 2)(2\gamma^b - \bar{\theta}(\gamma^a + \gamma^b)) + ((\bar{\theta} + 2)\bar{\theta} - 2)\Sigma}{16(\bar{\theta} - 1)((\bar{\theta} - 2)\bar{\theta} + 2)}$,

$D_1^*(\bar{\theta}, \gamma^a, \gamma^b, \Sigma) = \frac{(8\gamma^a - 3\bar{\theta}\gamma^a - 10\gamma^b + 5\bar{\theta}\gamma^b + \Sigma)(5\bar{\theta}\gamma^a - 8\gamma^a + 6\gamma^b - 3\bar{\theta}\gamma^b + \Sigma)}{64(\bar{\theta} - 1)(\gamma^a - \gamma^b)^2}$,

$D_2^*(\bar{\theta}, \gamma^a, \gamma^b, \Sigma) = \frac{(2\gamma^b - 5\bar{\theta}\gamma^a + 3\bar{\theta}\gamma^b - \Sigma)(5\bar{\theta}\gamma^b - 3\bar{\theta}\gamma^a - 2\gamma^b + \Sigma)}{64(\bar{\theta} - 1)(\gamma^a - \gamma^b)^2}$,

$\pi_1^*(\bar{\theta}, \gamma^a, \gamma^b) = \frac{((5\bar{\theta} - 8)\gamma^a - 3(\bar{\theta} - 2)\gamma^b + \Sigma)((8 - 3\bar{\theta})\gamma^a + 5(\bar{\theta} - 2)\gamma^b + \Sigma)(\bar{\theta}(3\bar{\theta} - 10) + 10)(\bar{\theta}(\gamma^a + \gamma^b) - 2\gamma^b) - ((\bar{\theta} - 6)\bar{\theta} + 6)\Sigma)}{1024(\bar{\theta} - 1)^2((\bar{\theta} - 2)\bar{\theta} + 2)(\gamma^a - \gamma^b)^2}$,

$$\pi_2^*(\bar{\theta}, \gamma^a, \gamma^b) = \frac{(3\bar{\theta}\gamma^b - 5\bar{\theta}\gamma^a + 2\gamma^b - \Sigma)(5\bar{\theta}\gamma^b - 3\bar{\theta}\gamma^a - 2\gamma^b + \Sigma)((\bar{\theta}(3\bar{\theta}-2)+2)(2\gamma^b - \bar{\theta}(\gamma^a + \gamma^b)) + (\bar{\theta}(\bar{\theta}+2)-2)\Sigma)}{1024(\bar{\theta}-1)^2((\bar{\theta}-2)\bar{\theta}+2)(\gamma^a - \gamma^b)^2},$$

$$\pi_{-1}^*(\bar{\theta}, \gamma^a, \gamma^b) = \frac{(3\bar{\theta}\gamma^b - 5\bar{\theta}\gamma^a + 2\gamma^b + \Sigma)(5\bar{\theta}\gamma^b - 3\bar{\theta}\gamma^a - 2\gamma^b - \Sigma)((\bar{\theta}(3\bar{\theta}-2)+2)(2\gamma^b - \bar{\theta}(\gamma^a + \gamma^b)) + (\bar{\theta}(\bar{\theta}+2)-2)\Sigma)}{1024(\bar{\theta}-1)^2((\bar{\theta}-2)\bar{\theta}+2)(\gamma^a - \gamma^b)^2},$$

$$\pi_{-2}^*(\bar{\theta}, \gamma^a, \gamma^b) = \frac{((5\bar{\theta}-8)\gamma^a - 3(\bar{\theta}-2)\gamma^b - \Sigma)((3\bar{\theta}-8)\gamma^a - 5(\bar{\theta}-2)\gamma^b + \Sigma)((\bar{\theta}(3\bar{\theta}-10)+10)(\bar{\theta}(\gamma^a + \gamma^b) - 2\gamma^b) + ((\bar{\theta}-6)\bar{\theta}+6)\Sigma)}{1024(\bar{\theta}-1)^2((\bar{\theta}-2)\bar{\theta}+2)(\gamma^a - \gamma^b)^2},$$

and $\gamma^a = q_1^a - q_2^a, \gamma^b = q_1^b - q_2^b$.

Appendix K: Proof of Proposition 5

We will work with the equilibrium outputs we present in Lemmas 3 and 4 throughout this section. Due to the complexity of calculations, we consider N to be sufficiently high.

For case 1, let $q_1^a = q_1^b = H$ and $q_2^a = q_2^b = L$. Then, $\sigma = 0$ and $\gamma^a = \gamma^b = H - L$. We can calculate platform's expected profit as follows: $\mathbb{E}(\pi_R^{SD*}) = \alpha\mu = \alpha(H-L)$ and $\mathbb{E}(\pi_R^{MD*}) = \alpha\gamma^a = \alpha(H-L)$. In summary, $\mathbb{E}(\pi_R^{SD*}) = \mathbb{E}(\pi_R^{MD*})$ in case 1.

For case 2a, let $q_1^a = q_1^b = H$ and $q_2^a = L, q_2^b = H$. Then, $\sigma \neq 0$ and $\gamma^a = H - L, \gamma^b = 0$. When we plug in $\mu = \frac{(4-\bar{\theta})(H-L)}{6}$ and $\sigma = \frac{(H-L)\sqrt{2+(2-\bar{\theta})\bar{\theta}}}{\sqrt{36N}}$ to platform's expected profit formulation, we get $\mathbb{E}(\pi_R^{SD*}) = \alpha\frac{(4-\bar{\theta})(H-L)}{6}$. Platform's profit in multi-dimensional ratings can be calculated as follows: $\mathbb{E}(\pi_R^{MD*}) = \alpha\frac{(\bar{\theta}(\bar{\theta}(\bar{\theta}(\bar{\theta}(-9\bar{\theta}+3\sqrt{\bar{\theta}(9\bar{\theta}-16)+16}+38)-10(\sqrt{\bar{\theta}(9\bar{\theta}-16)+16}+11))+50\sqrt{\bar{\theta}(9\bar{\theta}-16)+16}+144)-8(10\sqrt{\bar{\theta}(9\bar{\theta}-16)+16}+9)))(H-L)}{128(\bar{\theta}-1)^2((\bar{\theta}-2)\bar{\theta}+2)} + \alpha\frac{(H-L)40\sqrt{\bar{\theta}(9\bar{\theta}-16)+16}}{128(\bar{\theta}-1)^2((\bar{\theta}-2)\bar{\theta}+2)}$. Therefore, $\mathbb{E}(\pi_R^{MD*}) > \mathbb{E}(\pi_R^{SD*})$ in case 2a for any values of $\bar{\theta}$, H and L, when N is sufficiently high.

For case 2b, let $q_1^a = q_1^b = H$ and $q_2^a = H, q_2^b = L$. Then, $\sigma \neq 0$ and $\gamma^a = 0, \gamma^b = H - L$. When we plug in $\mu = \frac{(2+\bar{\theta})(H-L)}{6}$ and $\sigma = \frac{(H-L)\sqrt{2+(2-\bar{\theta})\bar{\theta}}}{\sqrt{36N}}$ to platform's expected profit formulation, we get $\mathbb{E}(\pi_R^{SD*}) = \alpha\frac{(2+\bar{\theta})(H-L)}{6}$. Platform's profit in multi-dimensional ratings can be calculated as follows: $\mathbb{E}(\pi_R^{MD*}) = \alpha\frac{(\bar{\theta}(\bar{\theta}(\bar{\theta}(\bar{\theta}(9\bar{\theta}+3\sqrt{\bar{\theta}(9\bar{\theta}-20)+20}-116)-14\sqrt{\bar{\theta}(9\bar{\theta}-20)+20}+446)+102\sqrt{\bar{\theta}(9\bar{\theta}-20)+20}-804)-176\sqrt{\bar{\theta}(9\bar{\theta}-20)+20}+760))(H-L)}{256(\bar{\theta}-1)^2((\bar{\theta}-2)\bar{\theta}+2)} + \alpha\frac{(H-L)88\sqrt{\bar{\theta}(9\bar{\theta}-20)+20}-304}{256(\bar{\theta}-1)^2((\bar{\theta}-2)\bar{\theta}+2)}$. Then, $\mathbb{E}(\pi_R^{MD*}) > \mathbb{E}(\pi_R^{SD*})$ in case 2b when $\bar{\theta} < T$ where T is a threshold and the solution of the following equality: $148\bar{\theta}^9 + 570\bar{\theta}^8 - 5209\bar{\theta}^7 + 8881\bar{\theta}^6 + 7756\bar{\theta}^5 - 51228\bar{\theta}^4 + 85988\bar{\theta}^3 - 76780\bar{\theta}^2 + 39616\bar{\theta} - 9904 = 0$, which is approximately 1.93958. Likewise, $\mathbb{E}(\pi_R^{SD*}) > \mathbb{E}(\pi_R^{MD*})$ in case 2b when $1.93958 < \bar{\theta}$. These results hold for any values of H and L, and when N is sufficiently high.

For case 3a, let $q_1^a = H, q_1^b = L$ and $q_2^a = q_2^b = L$. Then, $\sigma \neq 0$ and $\gamma^a = H - L, \gamma^b = 0$. The calculations are identical with case 2a. Likewise, for case 3b, let $q_1^a = L, q_1^b = H$ and $q_2^a = q_2^b = L$. Then, $\sigma \neq 0$ and $\gamma^a = 0, \gamma^b = H - L$. The calculations are identical with case 2b.

For case 4, let $q_1^a = H, q_1^b = L$ and $q_2^a = L, q_2^b = H$. Then, $\sigma \neq 0$ and $\gamma^a = H - L, \gamma^b = L - H$. When we plug in $\mu = \frac{(1-\bar{\theta})(H-L)}{3}$ and $\sigma = \frac{(H-L)\sqrt{4+(2-\bar{\theta})\bar{\theta}}}{\sqrt{36N}}$ to above formula, we get $\mathbb{E}(\pi_R^{SD*}) = \alpha\frac{(\bar{\theta}-1)(H-L)}{3}$. To calculate platform's expected profit in multi-dimensional ratings, $\mathbb{E}(\pi_R^{MD*}) = \alpha\frac{((\bar{\theta}-2)\bar{\theta}(2(\bar{\theta}-2)\bar{\theta}(\sqrt{8(\bar{\theta}-2)\bar{\theta}+17}-5)+41\sqrt{8(\bar{\theta}-2)\bar{\theta}+17}-71)+42\sqrt{8(\bar{\theta}-2)\bar{\theta}+17}-70)(H-L)}{64(\bar{\theta}-1)^2((\bar{\theta}-2)\bar{\theta}+2)}$. Therefore, $\mathbb{E}(\pi_R^{MD*}) > \mathbb{E}(\pi_R^{SD*})$ in case 4 for any values of $\bar{\theta}$, H and L, when N is sufficiently high.

For case 5, let $q_1^a = q_1^b = H$ and $q_2^a = q_2^b = H$. Then, $\sigma = 0$ and $\gamma^a = \gamma^b = 0$. $\mathbb{E}(\pi_R^{SD*}) = \alpha\mu = 0$. Platform's expected profit in multi-dimensional ratings is $\mathbb{E}(\pi_R^{MD*}) = \alpha\gamma^a = 0$. Therefore, $\mathbb{E}(\pi_R^{SD*}) = \mathbb{E}(\pi_R^{MD*})$ in case 5.

For case 6, let $q_1^a = H, q_1^b = L$ and $q_2^a = H, q_2^b = L$. Then, $\sigma \neq 0$ and $\gamma^a = \gamma^b = 0$. When we plug in $\mu = 0$ and $\sigma = \frac{(H-L)\sqrt{4+(2-\bar{\theta})\bar{\theta}}}{\sqrt{36N}}$, we get $\mathbb{E}(\pi_R^{SD*}) = \alpha\frac{\sqrt{\frac{2+(2-\bar{\theta})\bar{\theta}}{N}}(H-L)}{3\sqrt{\pi}}$. Since $\gamma^a = \gamma^b = 0$, $\mathbb{E}(\pi_R^{MD*}) = \alpha\gamma^a = 0$. Therefore, $\mathbb{E}(\pi_R^{SD*}) > \mathbb{E}(\pi_R^{MD*})$ in case 6. $\square$

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