

# A Model for Value Co-Creation through Cross-Producer Bundles

Hemant K. Bhargava<sup>1</sup>

hemantb@gmail.com

University of California Davis

## ABSTRACT

Many markets feature an economic structure in which value is co-created by multiple producers and their outputs are collected and sold as a common bundle by a producer-consortium or by a separate and independent firm, a retailer. Examples include technology goods and services, e.g., software platforms such as *Slack*, multi-sourced data platforms, patent pools, and in-home video entertainment. This paper develops an economic model to study demand, production, and revenue-sharing in such markets and examines market dynamics covering both the causes and effects of changes in industry structure. Producers in these markets are not rivalrous competitors in the usual zero-sum sense, because output of each casts an externality on production decisions of others and total market demand expands with total output, albeit with diminishing returns. This property allows multiple producers to flourish in equilibrium (vs. just one with the most favorable technological or cost structure), and more so when the market expands less quickly with total output. Equilibrium production quantities of competitors are strategic complements, yet competition between producers does manifest itself, e.g., if one acquires better production technology (i.e., makes value units at lower cost) then the equilibrium production levels of other producers are reduced. Insights are also derived for alternative market structures, e.g., producers have more output and earn higher profit when organized into a distribution consortium vs. relying on a separate retailer. Mergers between producers have similar effect. The formulation enables us to rigorously answer economic questions ranging from pricing, revenue sharing, and production levels in a static setting, to market dynamics covering both the causes and effects of changes in industry structure.

**Keywords:** value co-creation, bundling, revenue-sharing, platforms.

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# 1 Introduction

*What is common to in-home video entertainment (cable TV bundles), software platforms Slack, Trello, Intuit and Dropbox, multi-hospital patient data platforms such as PSCANNER, and season/ground passes for sports tournaments, auto shows, technology exhibitions, county fairs, carnivals, and arts and music shows?*

A common thread in these examples is that individual outputs from multiple ( $K$ ) independent producers are bundled into a “product” by a separate actor (e.g., retailer, platform, community organizer, or a consortium, see the left panel of Fig. 1) who sets the bundle price and shares bundle revenue with producers. Unlike markets where competing producers are rivals in vying for customers (e.g., automobile industry), here every producer “serves” all customers who purchase the bundle (though each buyer may actually consume only a subset of items in it), hence the output of one producer benefits the others, making them non-rivalrous collaborators in supplying value to consumers. Yet, rather than being just “team players,” producers do compete for a share of revenues and their production and revenue-sharing decisions are governed by selfish interest. This paper develops an economic model for analyzing this market setting, computes equilibrium outcomes and strategies under different market structures, and examines the drivers of changes in market structure.

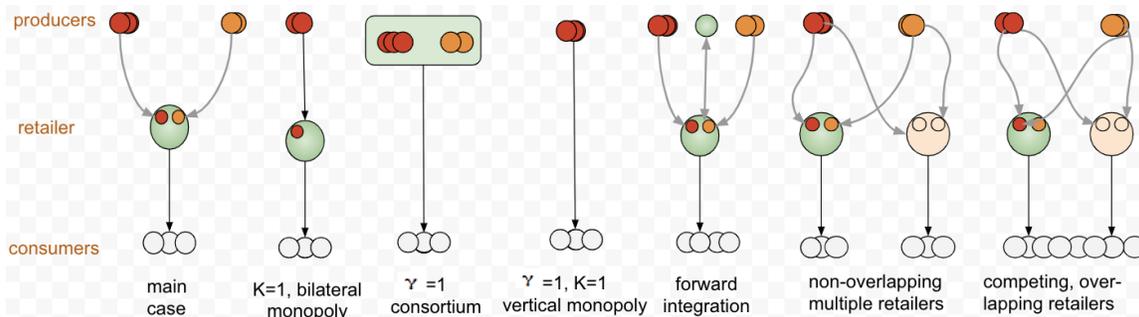


Figure 1: Some market structure variations for multi-producer bundle goods.  $K$  is the number of independent producers,  $\gamma$  is the total revenue share of producers, while  $1-\gamma$  that of the retailer.

This paper was inspired by the market for in-home video entertainment (movies and TV shows consumed on TVs and other personal devices) where cable TV providers (and more recently,

streaming providers such as Netflix) offer a bundle of entertainment content sourced from an oligopoly of multiple content providers such as studios and programming networks. Such value co-creation is also a defining characteristic of a category of platforms, a business architecture that is based on value co-creation (Ceccagnoli et al., 2012). For example, team productivity tools *Slack* and *Trello* both contain dozens of “integrations” or features (covering capabilities such as polling, task management, graphic communication, etc.) that are sourced from various software developers and made available to buyers under a collective single price. A variation is *forward integration* where the platform also produces output rather than act as a pure aggregator of third-party output (e.g., *Netflix*, *Slack*). A special case is *Adobe Creative Cloud*, where also multiple apps are offered for one bundle price, except that all apps are owned by the same company, *Adobe*.

Our goal in analyzing markets that feature cooperative production and bundling is to answer economic questions ranging from pricing, revenue sharing, and production levels in a static setting, to market dynamics covering both the causes and effects of changes in industry structure. One perspective which is fundamental to these questions is the nature of competition amongst producers, and between producers and the retailer. There is a vast literature that covers many forms of competition: quantity competition with homogeneous goods (Bertrand, Cournot), horizontal product differentiation (Hotelling line, Salop circle, Chen and Riordan’s Spokes), complementary and composite goods, and decisions by teams, co-operatives, and conglomerates; however, none of these reflects the economics of bundling in a meaningful way. We elaborate on this perspective in §2.1. A related, and second, perspective for studying these markets is value co-creation in platforms, which we discuss in §2.2. A third perspective is product bundling. While the literature on bundling covers both the mechanics of bundling and its optimal design, it primarily employs a micro-level analysis of bundling which does not scale up to a market or industry-level analysis (§2.3 elaborates on this point). This is because the derivation of bundle demand faces deep mathematical complexities from the need to convolve demand distributions for multiple bundle components, either with or without correlation, super- or sub-additivity in valuations across components, and asymmetric demand profiles of bundle components. Even the simplest bundle setting (two products, no cor-

relation and no super- or sub-additivity in valuations) is analytically intractable, and micro-level models stand little chance of addressing broader industry-level questions.

This paper develops and applies a method for analyzing markets that involve co-production of goods, i.e., where market demand is defined over a combination of multiple outputs from multiple producers (e.g., in software platforms, art festivals, etc.). What is a suitable model for such analysis? Developing such a model, specifically an industry-level model, is a key contribution of this paper. After discussing the relevant literature and challenges in modeling (§2), §3 develops a reduced-form specification for bundle demand which fits and respects the characteristics of bundling across a wide spectrum of bundling scenarios, is computationally tractable in terms of computing optimal bundle policies under different market structures, and produces useful insights regarding the market. §4 describes equilibrium market outcomes, including demand, production and revenue-sharing. Next, §5 examines the drivers and consequences of changes in market structures and how these affect market outcomes. The reduced-form demand model does not accommodate individual-level demand preferences for specific products or producers or for combinations of them, nor does it explore or advise regarding what specific outputs are made by each producer. However, it does capture higher-level requirements faithfully in a way that is analytically tractable and produces meaningful conclusions.

## **2 Perspectives from Related Literature**

We cover three relevant perspectives, models of competition in §2.1, then value co-creation in platforms in §2.2, and finally the literature on bundling in §2.3.

### **2.1 Competition**

Firms that make a homogeneous good (i.e., outputs are substitutes) compete directly by choosing quantity (*Cournot* competition) and/or price (*Bertrand* competition). Market price depends on total output, reduces as output increases, and the lower-cost firm gets higher output and profits

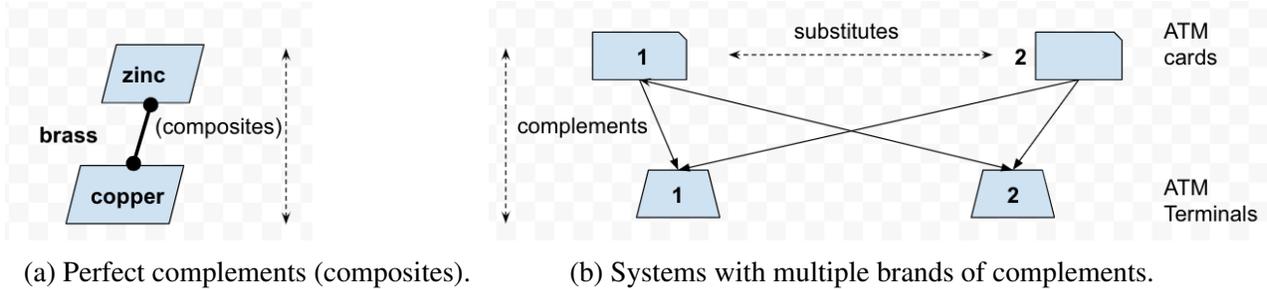


Figure 2: Production and competition with composite goods and systems.

(Varian, 1992, Ch. 16). Firms' price responses move in the same direction as competitor's price (i.e.,  $\frac{\partial P_j(P_i)}{\partial P_i} < 0$ ) whereas their production quantities move in opposite direction ( $\frac{\partial Q_j(Q_i)}{\partial Q_i} > 0$ ), i.e., they are *strategic substitutes*. This fundamental property holds under variations such as differentiated goods or partial substitutes. In contrast, we will see that the natural behavior under value co-creation is for output levels to be *strategic complements* ( $\frac{\partial Q_j(Q_i)}{\partial Q_i} > 0$ ). A similar contrast arises against firms that compete through vertical (i.e., "quality") or horizontal (i.e., "location") differentiation (see, e.g., Shaked and Sutton (1982), Shaked and Sutton (1987), Gabszwick and Thisse (1986), Ferreira and Thisse (1996), and Y. Chen and Riordan (2007)).

The form of competition more closely related to the present paper arises between producers of complements, with the extreme case being that of *composite goods* (Fig. 2a). Cournot (1929) provided the example of brass as a composite of zinc and copper, and showed that sourcing the components from different producers leads to higher prices. Unlike with substitutes, competing firms are co-producers of the composite good, combining outputs of different producers raises the market price, and price increase by one firm weakens demand and profits for the other. A generalization of this structure appears in "systems competition" (Fig. 2b) where components are complements but there are multiple brands of each component good (i.e., they compete directly), for instance ATM cards that require, and interoperate on, ATM machines (Katz and Shapiro, 1994). Economides and Salop (1991) examined price and quantity under alternative market structures (e.g, vertical integration) with two component types and two producers of each.

The economic form of interest in this paper blends cooperative production (a non-rivalrous

complementary or composite good effect) with competition (between multiple brands of a component), and with an underlying demand structure based on bundling. Component providers are co-producers (e.g., in a TV bundle, crime thrillers and live news act as composites in a multi-genre bundle), but some subsets of components are also imperfect and competing substitutes (e.g., crime thrillers from multiple producers). Unlike the abovementioned literature on systems and composite goods (where individual components are offered in the market), the composite good or bundle is the only one that is offered and priced. Lerner and Tirole (2004) examined the welfare implications of such collaboration in the context of patent pools and technology licensing. Bhargava (2012) examined pricing equilibria in this setting and showed that Cournot's over-pricing result holds even when the composite relationship is weak (i.e., not all components are necessary, although demand does increase with more components). The present paper goes beyond price formation to consider issues of provision, revenue-sharing, and the consequences and drivers of alternative market structures.

## **2.2 Platforms and Value Co-Creation**

Value co-creation has recently been discussed in the context of technology-enabled platforms, which facilitate multiple groups of entities (say, shoppers and merchants) to congregate, discover, and transact with each other (Choudary et al., 2016). Platforms focus on *enabling* value creation and exchange, rather than value production itself (e.g., *Facebook* users enjoying connecting with their friends; *OpenTable* diners get value when they book affiliated restaurants, and restaurants derive value from outreach to potential diners). A few platforms such as *Slack* and *pSCANNER* combine partner-producer contributions into a single bundle; in most other platforms, value co-creation occurs under a different economic framework in which product-related decision rights (e.g., on pricing) are held by producer partners (Haggiu, 2009; Nocke et al., 2007).

Ceccagnoli et al. (2012) empirically examine the effect on small producers' performance when they participate in a platform's value co-creation ecosystem. Foerderer et al. (2018) examine the

effect on complement-provision and innovation when platform owners also make complements. Demirezen et al. (2018) examine collaboration between two firms who are jointly responsible for some output, when one of them can lead and define a contract for the contribution of the other. Adner et al. (2016) build a “frenemies” model of competition between platforms that also make apps and decide whether to offer their apps on competing platforms. While these papers examine important issues in platforms and value co-creation, their goals and results are distinct from those of this paper. More generally, although there is a substantial and growing literature on platforms, existing papers have primarily considered micro-level decisions (e.g., business model design, level of openness, product line expansion, salesforce compensation). In contrast the present paper aims to jointly examine (for platforms such as *Slack* that bundle third-party apps with the platform) a wide spectrum of issues including platform pricing, producers’ output decisions, revenue-sharing with producers, and the effects of alternate market structures.

### **2.3 Bundling**

Product bundling, one of the simplest and widely practiced business strategies, improves seller profits with little extra effort especially when component goods have low marginal costs. There is a vast literature on bundling, across marketing, economics and information systems. The earliest papers noted that bundling increases profits by reducing dispersion in product valuations across consumers (Stigler, 1963; Adams and Yellen, 1976; Schmalensee, 1984; McAfee et al., 1989). Other advantages of bundling include supply-side economies of scope (Evans and Salinger, 2005; Surowiecki, 2010), lower consumer transaction costs or other demand-side conveniences and network effects (Lewbel, 1985; Prasad et al., 2010), and strategic leverage across products (Burstein, 1960; Carbajo et al., 1990; Eisenmann et al., 2011; Stremersch and Tellis, 2002). For a discussion of emerging issues and past literature, see Rao et al. (2018), Kobayashi (2005) and Venkatesh and Mahajan (2009).

Analysis of bundle choice and bundle pricing is challenging because derivation of bundle de-

mand from demand for individual components must deal with possible correlation between valuations of individual components, sub- or super-additivity of valuations, and asymmetry in demand profiles across components. Consider the simplest case of two component goods  $i=1, 2$  for which consumer valuations are independent, distributed uniformly in  $[0, b_1]$  and  $[0, b_2]$  respectively, and bundle valuation  $v_{iB}$  for consumer  $i$  is simply  $v_{i1}+v_{i2}$ . The bundle demand curve is obtained from the distribution of the  $v_{iB}$ 's which is the convolution of the two uniform distributions (see Fig. 3). Now, if bundle valuations sub-additive, one can write  $v_{iB} = v_{i1}+\phi_i v_{i2}$  (if  $v_{i1}>v_{i2}$ , or  $v_{iB} = v_{i2}+\phi_i v_{i1}$  otherwise), with each  $\phi_i \in [0, 1]$  (or  $>1$  for super-additive valuations). This heterogeneity in  $\phi$  alone makes it difficult to express the bundle demand function, and the problem goes out of bounds when considering correlation and  $k>2$  products. For two-item bundles with additive valuations, Y. Chen and Riordan (2013) use copula functions in the Frechét family (i.e., with uniform marginal distributions for component goods) to provide a rigorous treatment of correlated valuations.

While expressions for bundle demand are not available except for the simplest two-item settings, the literature does establish a few basic properties of bundle demand: that across-consumer valuations for the bundle exhibit less variation (relative to mean) than for individual components; that demand is “flatter in the middle”; and that these properties are, *ceteris paribus*, amplified as bundle size increases. This behavior is vividly described in Bakos and Brynjolfsson (2000, Fig. 1), reproduced as Fig. 3. The structure of bundle demand for more complicated settings can be understood by simulating and aggregating valuations for the component goods (see e.g., Olderog and Skiera (2000)). These properties will play a vital role in development of the market demand function in the next section (see §A.1 and §3.3).

This paper goes beyond existing bundling literature in two main ways. First, past literature considers only bundles where the component goods are sourced from a single firm, which is also the firm that forms the bundle (e.g., a *MS Office* bundle). Exceptions include a few recent papers starting with Bhargava (2012) which have examined multi-producer bundling, however these are limited to two-item bundles. Second, goals of past literature (including on multi-producer bun-

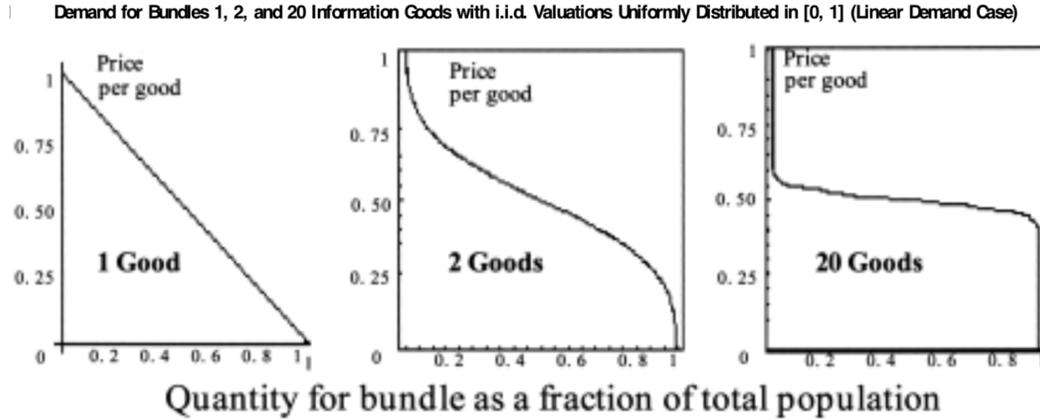


Figure 3: Bundle demand gets more “flatter in the middle” as bundle size increases. Reproduced from Bakos and Brynjolfsson (2000, Fig. 1).

dles) are either to examine the optimality of bundling (e.g., Armstrong, 2013) or to specify optimal prices or mix of bundling: e.g., Bhargava (2013) for two-item bundles; Bakos and Brynjolfsson (1999) and Ibragimov and Walden (2010) for bundles of enormous or infinite number of components with identical demand profiles; Hitt and P. Chen (2005) for customized bundling, where the firm specifies a price for  $n$  items and customers pick the items. Bundling literature has not examined the provision of bundle components, the effect of market structure on provision, producer participation in the bundle, or the inter-dependencies between producers. Doing so requires a closed-form expression of bundle demand and a richer consideration of bundle settings. The next section examines this task.

### 3 Modeling a Co-Created Bundle Economy

The complexities in specifying bundle demand and the nature of competition raises unique challenges in building an aggregate model of demand, supply, and revenue-sharing, for an economy where a retailer builds a bundle with outputs from multiple producers. The starting point in our framework is to define the bundle product and producer output in terms of canonical “value units” which represent a combination of quantity, variety, and quality. The bundle, measured by its mag-

nitude of  $Q$  value units, is offered to consumers under an unlimited-use price  $P$ . Bundle demand is  $D(P, Q)$ , and  $Q$  is an aggregation of outputs  $Q_i$  from multiple producers ( $i = 1 \dots \mathcal{I}$ ). The focus in specifying the framework is to ensure that all relevant concepts—aggregate demand, marginal demand, total supply, marginal supply, and revenue sharing in the industry—are consistent with this measure of value units.

For clarity, the discussion frequently employs a concrete setting which inspired this paper, that of “TV bundles” that are offered to consumers by communications firms (cable operators, telecom, satellite service providers) using content sourced from multiple studios and programming networks. Multi-producer bundling has been present during the entertainment industry’s evolution over the last 150 years, across multiple eras each characterized by new innovative technologies which influence the equilibrium market structure and its dynamics across the eras. Consumers evaluate the bundle based on quantity (more is better, though at diminishing rate), variety (e.g., for a TV bundle buyers want a mix of movies, TV shows, political thrillers, children-oriented content, comedy, and so on, that comprise many genres and appeal across many moods, age groups, tastes etc.), and quality (creative aspects, star talent, production quality etc.). The industry structure in the initial model setup represents the “cable era” of in-home entertainment where most market regions had a single dominant provider. The section on market structure variations and dynamics reflects the transition from the cable era to the present, streaming, era.

### **3.1 Bundle Production, Distribution, Pricing and Revenue-Sharing**

Producers make the bundle components that consumers value, but lacking direct reach into the consumer market must rely on a specialist firm, a retailer, to sell to consumers. The retailer sources bundle components of aggregate value  $Q$  from producers and uses its distribution infrastructure, built at fixed cost  $F$ , to market the bundle at a per-subscriber cost of  $w^R(Q)$ .  $F$  will play no role in the main optimization problem, however  $F$  and  $\frac{\partial D(P, Q)}{\partial Q}$  can explain why multi-producer outputs are served as a bundle (e.g., economies of scope in distribution, and consumer preferences

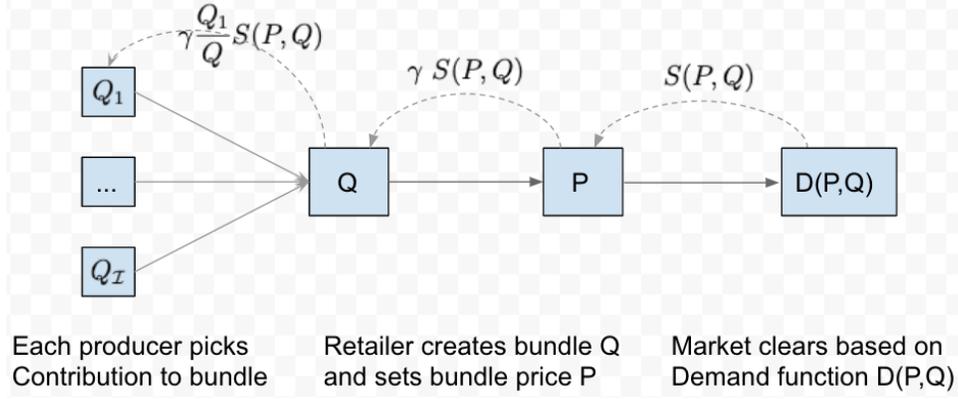


Figure 4: Sequence of decisions.

for size, variety, flexibility) rather than each producer’s goods separately (e.g., as in a supermarket). The variable costs  $w^R(Q)$  include, for instance, market research, price determination, digital transmission and account management costs. The retailer sets bundle price  $P$ , creating a bundle distribution surplus  $S(Q) = (P - w^R(Q))D(P, Q)$ , which is industry surplus without considering fixed costs  $F$  of the retailer’s distribution infrastructure and variable costs  $c_i(Q_i)$  of production by producers. For convenience in exposition we shall refer to  $S(Q)$  as industry surplus. We impose a basic regularity requirement on rate of growth in  $S(Q)$  in order to ensure that the problem does not become unbounded and vacuous.

**Requirement 1** (Bounded  $S(Q)$ ). *As  $Q$  increases,  $S(Q)$  should increase at a diminishing rate, i.e.,  $\frac{\partial^2 S(Q)}{\partial Q^2} < 0$  (with  $\frac{\partial S(Q)}{\partial Q} \geq 0$ ).*

Industry surplus  $S(Q)$  is shared between the retailer and producers according to their relative market power. For example, for in-home entertainment, producers have some power because ultimately consumer demand is for content (as expressed in the oft-stated maxim “content is king”), while the retailer’s power is driven by expertise and technology for delivering content (e.g., content delivery firms such as cable or satellite, who hold the conduit to deliver content into homes). The revenue-share between producers and the retailer will vary, e.g., based on the level of concentration within each layer. We start by assuming that the retailer is a monopolist. Let  $(1 - \gamma)$  denote the retailer’s market power, so that its profit is  $(1 - \gamma)S(Q)$ , while the remainder  $\gamma S(Q)$  (sourcing costs paid to producers) becomes the total revenue available to producers. Producers split their share pro-

portional to the value-units they provide, i.e., producer  $i$  receives  $\gamma \frac{Q_i}{Q} S(Q)$ . With this structure, the retailer sets bundle price  $P$  to maximize its profit,  $\Pi_R(Q) = \max_P (1 - \gamma) ((P - w^R(Q)) D(P, Q))$ . Fig. 4 depicts the production-distribution (bundling) relationship between industry players as well as the sequence of decisions made by them.

Producers vary in their production technology, captured by heterogeneity in production cost. For producers, the cost of creating output combines two competing effects: i) economies of scale and fixed costs for content production (e.g., studios, sets, equipment etc.) and ii) increasingly higher costs for achieving a unit increase in market demand. We assume that the net of these costs increases with  $Q$  at a faster rate than  $Q$ 's effect on demand, so that cost of adding value units exceeds its positive impact on demand beyond some level of output (otherwise the optimal production would be unbounded). Likewise, we ensure that the retailer's cost  $w^R(Q)$  rises faster with  $Q$  than the demand-side effect of  $Q$ .

**Requirement 2** (Supply-side Costs). *The costs of producing bundle components and of distributing the bundle increase with  $Q$  at a faster rate than the increase in value from higher  $Q$ .*

For ease of isolating and explicating the multi-producer mechanics in this setting, we adopt a linear production cost function ( $c_i Q_i$ ) for producers, shifting the burden of satisfying Requirement 2 to the bundle demand function (which we specify in §3.3). This enables us to characterize each producer with a single parameter,  $c_i$ . Producers choose output level simultaneously, with producer  $i$  picking  $Q_i$  to maximize its profit  $\Pi_i$  which is its share of the surplus less its own production costs  $c_i(Q)$ . Let  $Q_{-i}$  denote aggregate output of all producers other than  $i$ , then  $\pi_i(Q_i, Q_{-i}) = \gamma \frac{Q_i}{Q} S(Q) - c_i(Q)$ , where  $Q = Q_i + Q_{-i}$ . The individual rationality (IR) constraint for producers is that they make positive (or zero) profit, i.e., that  $\frac{\gamma S(Q)}{Q} \geq c_i$  for producer  $i$  (i.e., average revenue exceeds average cost). In equilibrium, producers with cost parameter higher than  $\frac{\gamma S(Q)}{Q}$  have no output, while the rest choose  $Q_i$  that maximizes own profit. This yields the system

of equations,

$$\text{active producers} \quad K = |\{i : c_i \leq \frac{\gamma S(Q^*)}{Q^*}\}| \quad (1a)$$

$$\text{optimality conditions} \quad \forall i \in 1 \dots K : \frac{\gamma S(Q^*)}{Q^*} - \frac{Q_i^*}{Q^*} \left( \frac{\gamma S(Q^*)}{Q^*} - \frac{\partial S(Q^*)}{\partial Q^*} \Big|_{Q^*} \right) = c_i. \quad (1b)$$

$$\text{adding them up over } i: \quad K \left( \frac{\gamma S(Q^*)}{Q^*} - \bar{c}_{(K)} \right) = \left( \frac{\gamma S(Q^*)}{Q^*} - \frac{\partial S(Q^*)}{\partial Q^*} \Big|_{Q^*} \right). \quad (1c)$$

where  $K$  is the number of producers with positive output (i.e.,  $i = 1 \dots K$ ) and  $\bar{c}_{(K)}$  is the average of cost parameters for those producers. Eq. 1b suggests the plausible result that output levels of producers are inversely related to their cost parameters. However, its assertion requires computation of the equilibrium value  $Q^*$  (and  $K$ ), and the model needs further precision in order to establish and identify a unique or globally optimal solution.

### 3.2 Example and Preliminary Insights

Before analyzing the general equilibrium solution, we set up the simplest two-producer example to derive some crucial preliminary insights regarding the outcomes in this economic structure.

**Example 1** (Two Producers). *Compared against a single-producer market (selling through a retailer with revenue-sharing parameter  $\gamma$ ), a market with two producers who have identical cost functions i) has higher total equilibrium output  $Q$ , ii) satisfies more demand at higher price, iii) yields lower collective profit for producers, and iv) produces higher profit for the retailer.*

Let  $Q^{(1)}$  be the single-producer equilibrium output level, when the producer maximizes profit  $\pi = \gamma S(Q) - cQ$  (with  $\gamma S(Q^{(1)}) > cQ^{(1)}$  for positive profit). Applying Eq. 1b we have  $\gamma \frac{\partial S(Q)}{\partial Q} \Big|_{Q^{(1)}} = c$ . For the two-producer setting with symmetric outputs  $(\frac{Q^{(2)}}{2}, \frac{Q^{(2)}}{2})$  and profit functions  $\pi_i = \left( \gamma \frac{Q_i}{Q} S(Q) - c_i Q_i \right)$ , Eq. 1b yields  $\frac{\gamma}{2} \frac{\partial S(Q)}{\partial Q} \Big|_{Q^{(2)}} + \frac{\gamma}{2} \frac{S(Q^{(2)})}{Q^{(2)}} = c$ . Now, if  $Q^{(2)} \leq Q^{(1)}$ , then  $\gamma \frac{\partial S(Q)}{\partial Q} \Big|_{Q^{(2)}} > c$  (from  $\gamma \frac{\partial S(Q)}{\partial Q} \Big|_{Q^{(1)}} = c$  and Requirement 1,  $\frac{\partial^2 S(Q)}{\partial Q^2} < 0$ ) and  $\gamma \frac{S(Q^{(2)})}{Q^{(2)}} > c$  (because  $\gamma S(Q^{(1)}) > cQ^{(1)}$ ), hence the LHS will always exceed, and never equal, the RHS, creating a contradiction. The reverse case  $Q^{(2)} > Q^{(1)}$  poses no such contradiction, establishing  $Q^{(2)} > Q^{(1)}$  (part i of the example),

which also generates the other results in the example. To summarize, Example 1 generates two important insights about this multi-producer bundle setting.

1. Aggregate output is higher with  $K > 1$  heterogeneous producers than under a single producer whose cost function is the average cost of the  $K$  producers. (Consequently, total producer profit is lower while the retailer has higher profit with  $K > 1$ .)
2. Multiple producers sustain positive production (i.e., not just the lowest-cost producer), unless there is a huge gap between production costs of producers.

These insights remain valid even when the two producers have asymmetric cost functions, and also when there are  $K > 2$  heterogeneous producers. We develop and prove these results formally in the later sections (Proposition 3), including what it means for a  $c_i$  to be so high that producer  $i$  is forced into zero output (Eq. 6 and Proposition 2).

### 3.3 Specific Demand and Cost Functions

Next we develop a specific form of the demand function in order to derive expressions and make the computations concrete. The choice of demand function is driven primarily by the nature of the bundled good, i.e., valuations for quantity, variety, quality, and preference-heterogeneity along multiple dimensions including levels of additivity and correlations among these valuations. §A.1 in the Appendix lists several alternative demand functions that were considered. The analysis of their properties suggests a specific demand model that best represents the multi-producer bundle setting,  $D(P, Q) = \sqrt{AQ^\theta - bP}$ , (with  $\theta \in [0, 2/3]$ ), where  $AQ^\theta = M(Q) = D(0, Q)^2$  represents (the square of) market saturation level for a bundle of  $Q$  value units. The parameter  $\theta$  measures how elastic  $M$  is to bundle size  $Q$  (i.e.,  $\theta = \frac{\partial M}{\partial Q} / (M/Q)$ ), and can be interpreted as market propensity for bigger bundles. The equation can be generalized to  $D(P, Q) = (AQ^\theta - bP)^\alpha$  (where  $\alpha \in [0, 1]$ ), however  $\alpha = \frac{1}{2}$  makes the exposition easier to follow. Further, while we pick this particular demand form as most suited to the bundle setting, the qualitative results would be unchanged if using, for instance, linear, quadratic, negative exponential or constant elasticity demand functions.

While the demand function  $D(P, Q)$  is defined over an abstract measure of value units,  $Q$  can be estimated via its effect on the maximum level of market demand, i.e.,  $Q^\theta = (D^{-1}(0))^2/A$ . Consistent with this expression and the requirements, a useful retailing cost function is  $c(Q) = cQ^\theta$ , implying both that the retailer enjoys economies of scale and also that as  $Q$  increases, cost increases more rapidly than demand. It can be confirmed that the linear production cost function  $c_i Q_i$  satisfies Requirement 1 in combination with the diminishing returns embedded in  $D(P, Q)$  as defined above. We will see that, despite the linear cost function, this formulation yields an interior solution for content production (i.e., even high-cost or low-value or niche producers have positive production) rather than a corner solution in which the most advantaged producers secure the entire market. We adopt the convention that the  $i$ 's are arranged in ascending order.

## 4 Equilibrium Analysis

The sequence of decisions in this multi-firm economy is that heterogeneous producers (with production costs  $c_i Q_i$  and collective market power  $\gamma$  which is encoded into a revenue-sharing parameter with the retailer) choose their  $Q_i$ 's, the retailer aggregates these outputs into a bundle  $Q = \sum_i Q_i$  in exchange for transfer prices  $F_i$ , and the retailer distributes the bundle (incurring additional cost  $cQ^\theta$ ) at market price  $P$ . We solve the problem in backward sequence, first identifying optimal  $P$  which maximizes the retailer's profit given  $Q$ , then determining  $Q_i$ 's while satisfying the aggregation constraint ( $Q = \sum_i Q_i$ ) and producer's participation constraints ( $\pi_i(P^*(Q), Q_i, Q_{-i}) \geq 0$ , where  $Q_{-i}$  is the vector of all  $Q_i$ 's except  $Q_i$ ). The worth of this modeling framework is in the results it produces: ease of generating them, what they cover, how meaningful they are, and their credibility. Lemma 1 starts by describing the industry equilibrium solution, which we develop and explain in the rest of this section.

**Lemma 1** (Equilibrium Solution). *With producers' costs  $c_i$  per value unit arranged in ascending order, the equilibrium numbers of producers  $i = 1 \dots K$  who make content, their magnitude of value*

units produced, and the market price set by the retailer are

$$K = \max\{i : c_i \leq \frac{c_1 + \dots + c_i}{i - (1 - 3\theta/2)}\} \quad (2a)$$

$$\forall i = 1 \dots K : Q_i = \left(2 - \frac{c_i(2 - 3\theta)}{\bar{c}}\right) \frac{Q}{(2 - 3\theta)} \quad (2b)$$

$$\text{with } Q = \sum_{i=1}^K Q_i = \left[\frac{\gamma}{b\bar{c}} \left(2 - \frac{(2 - 3\theta)}{K}\right)\right]^{2/(2-3\theta)} \left(\frac{A-bc}{3}\right)^{3/(2-3\theta)} \quad (2c)$$

$$P^* = \frac{2A+bc}{3b} Q^\theta. \quad (2d)$$

When Eq. 2a yields  $K=1$  (i.e.,  $c_2 > \frac{2c_1}{3\theta}$ ), then  $Q_1 = Q = \left(\frac{3\gamma\theta}{bc_1}\right)^{\frac{2}{2-3\theta}} \left(\frac{A-bc}{3}\right)^{\frac{3}{2-3\theta}}$ .

## 4.1 Pricing

Price determination is straightforward and done the usual way. Given the total available content  $Q$ , the retailer sets the optimal price to maximize profit, which is its bundle revenues less the cost of sourcing content from producers,

$$P^* = \arg \max_P \Pi_R(P, Q) :$$

$$\Pi_R(P, Q) = (1-\gamma)(P-cQ^\theta)D(P, Q) = (1-\gamma)(P-cQ^\theta)\sqrt{AQ^\theta - bP} \quad (3a)$$

$$\text{which yields } P^* = \frac{2A+bc}{3b} Q^\theta \quad (3b)$$

$$D^* = \sqrt{\frac{A-bc}{3}} Q^\theta \quad (3c)$$

$$\text{and } \Pi_R^* = \frac{2}{b}(1-\gamma) \left(\frac{A-bc}{3} Q^\theta\right)^{3/2} \quad (3d)$$

$$\text{with } S^*(Q) = \frac{2}{b} \left(\frac{A-bc}{3} Q^\theta\right)^{3/2}. \quad (3e)$$

The final term  $S^*(Q)$  is the overall industry surplus when a bundle of magnitude  $Q$  is offered to consumers at  $P^*$ . Notice that the surplus and profit terms increase with  $Q$  (unlike Cournot quantity competition where price and profit would fall as supply increased), and less than linearly (with

$\theta \in [0, \frac{2}{3}]$ ). This suggests that  $Q$  is better thought of as *quality* than *quantity*. The equilibrium level of demand also increases with  $Q$  but at a diminishing rate, while price-per-unit- $Q$  falls with  $Q$ . Hence the model satisfies the price equilibrium Requirement 5. The analysis would be the same if the retailer's profit function were set up (instead of Eq. 3a) as a constant fraction of net revenues (with producers getting the rest).

## 4.2 Production

Producers pick their output levels simultaneously. The equilibrium levels of output are such that no producer gains by unilaterally deviating from chosen output level, given the choices of other producers. Each producer's  $Q_i$  is chosen to maximize own profit  $\pi_i$  given  $Q_{-i}$ .

$$Q_i^* = \arg \max_{Q_i \geq 0} \pi_i : \quad \pi_i(P, Q_i, Q_{-i}) = \left[ \frac{2}{b} \left( Q^\theta \frac{A-bc}{3} \right)^{3/2} \right] \frac{Q_i}{Q} \gamma - c_i Q_i \quad (4a)$$

$$\text{set } \frac{\partial \pi_i}{\partial Q_i} = 0 : \quad c_i = \frac{2\gamma}{bQ^2} \left( Q^\theta \frac{A-bc}{3} \right)^{3/2} \left( Q - \frac{(2-3\theta)Q_i}{2} \right) \quad (4b)$$

$$\Leftrightarrow \quad Q_i = \left[ 2 - \frac{bc_i Q}{\gamma} \left( \frac{1}{Q^\theta} \frac{3}{A-bc} \right)^{3/2} \right] \frac{Q}{(2-3\theta)} \quad (4c)$$

$$\text{share of production} \quad \frac{Q_i}{Q} = \left[ 2 - \frac{bc_i Q}{\gamma} \left( \frac{3}{AQ^\theta - bc} \right)^{3/2} \right] \frac{1}{(2-3\theta)} \quad (4d)$$

$$\text{IR constraint : } \forall i \quad Q_i \geq 0 \Leftrightarrow c_i \leq \frac{2\gamma}{bQ^{\frac{2-3\theta}{2}}} \left( \frac{A-bc}{3} \right)^{3/2} \quad (4e)$$

$$\left( Q = \sum_i^{\hat{K}} Q_i \right) \quad \Leftrightarrow Q \leq \left( \frac{2\gamma}{bc_{\hat{K}}} \right)^{\frac{2}{2-3\theta}} \left( \frac{A-bc}{3} \right)^{\frac{3}{2-3\theta}} \quad (4f)$$

where  $\hat{K}$  is the highest  $i$  for which the RHS of Eq. 4e holds. For each  $i$ , Eq. 4c represents the optimal output level given the levels  $Q_{-i}$  of other “feasible” producers (i.e.,  $i=1\dots\hat{K}$ ). The collection of Eq. 4c for feasible producers defines the industry-level supply equilibrium, however it is an implicit condition stated in terms of  $Q = \sum_i Q_i$  (Eq. 4f). Next, to figure out the equilibrium output levels, repeat and add up Eq. 4c for  $i=1\dots\hat{K}$ , and let  $\bar{c}_{(\hat{K})} = \sum_i c_i$  denote the average cost

parameter for content production. This yields

$$Q = \frac{\hat{K}Q}{(2-3\theta)} \left[ 2 - \frac{b\bar{c}}{\gamma} Q^{\frac{2-3\theta}{2}} \left( \frac{3}{A-bc} \right)^{3/2} \right] \quad (5a)$$

$$\equiv Q \left( \frac{1}{Q^\theta} \frac{3}{A-bc} \right)^{3/2} = \frac{\gamma}{b\bar{c}_{(\hat{K})}} \left( 2 - \frac{2-3\theta}{\hat{K}} \right) \quad (5b)$$

$$\equiv Q = \left[ \frac{\gamma}{b\bar{c}_{(\hat{K})}} \left( 2 - \frac{(2-3\theta)}{\hat{K}} \right) \right]^{2/(2-3\theta)} \left( \frac{A-bc}{3} \right)^{3/(2-3\theta)} \quad (5c)$$

Combining Eq. 4e, for each  $i$ , with above equations (Eq. 5b is the most useful) yields that participation is limited to producers with the following cost parameters.

$$\text{feasible cost vector : } (c_1, \dots, c_{\hat{K}}) \text{ such that } c_{\hat{K}} \leq \frac{\bar{c} \cdot \hat{K}}{\hat{K} - (1 - 3\theta/2)}. \quad (6)$$

Among all the feasible vectors, the optimal one trivially is the highest  $\hat{K}$  for which the feasibility condition is satisfied, denoted as  $K$  in Lemma 1. Procedurally,  $K$  can be identified by testing the condition first with all  $\mathcal{I}$  producers; if it fails then the highest  $c_i$  is removed from the vector, successively, until the condition is satisfied. The first vector that achieves the condition is a feasible set of producers and each will then have non-negative  $Q_i$ . Once this is done, Eq. 5c describes the product bundle, given the various parameters of the problem, and combining this with Eq. 4c produces  $Q_i$  the value supplied by each producer. Given that producers with index higher than  $K$  have no production, henceforth  $K$  will represent the (equilibrium) number of producers in the market.

### 4.3 Equilibrium Properties

Plugging Eq. 5b into Eq. 4c yields the optimal  $Q_i$  in the form given in Lemma 1 Eq. 2b. Plugging Eq. 5b into Eq. 4d yields each producer's fractional share of total product value in equilibrium, specifically  $\frac{Q_i}{Q} = \left( 2 - \frac{c_i}{\bar{c}} \right) \left( \frac{2}{2-3\theta} - \frac{1}{K} \right)$ . The equilibrium expressions in Lemma 1 for

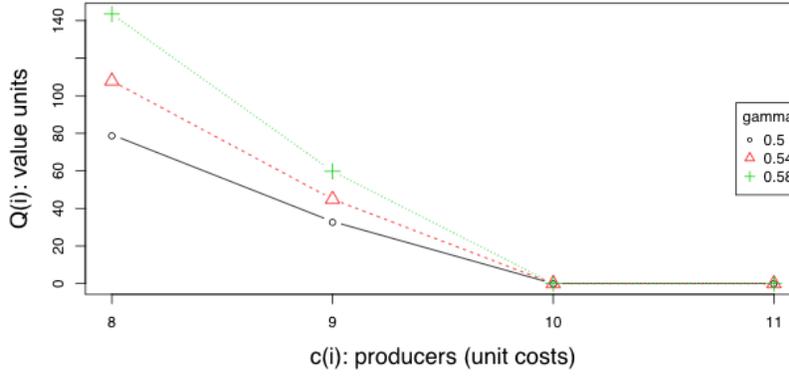


Figure 5: Impact of cost structure and revenue-sharing on number of producers and production levels. Other parameters are  $\theta=0.5$ , and  $A, b, c$  such that  $\frac{A-bc}{3}=10$ .

$Q_i, Q, P^*, D^*$  and the profits of retailer and producers can be derived similarly. Fig. 5 depicts simulation results that confirm intuitive properties: producers make more output as their share of revenue  $\gamma$  increases (each producer's share  $\frac{Q_i}{Q}$  is unchanged), producers with better production technology (i.e., lower cost per value unit) have a competitive advantage and produce more, and producers with sufficiently high enough cost cannot sustain positive production. Specifically, content producers who can produce value units at lower unit cost will supply a greater amount of content to the retailer (i.e.,  $\frac{\partial Q_i}{\partial c_i} < 0$ ).

**Proposition 1** (Production levels vs. costs). *Lower-cost (i.e., more efficient) producers make more content, i.e.,  $(c_i < c_j)$  implies  $Q_i > Q_j$ .*

The linear production cost in our model ( $c_i Q_i$ ) raises the possibility of a bang-bang equilibrium solution in which only the most efficient or lowest-cost producer has positive production level. This is because producers “draw from the same well” for revenue and marginal revenue is linked to total bundle size rather than how much each producer has made, giving the lowest-cost producer an advantage for every next unit of production (i.e., marginal revenue less cost is highest for producer 1), regardless of existing production levels. Our analysis reveals the opposite result.

**Proposition 2** (Multi-producer output). *Multiple producers have positive output (i.e.,  $K \geq 2$ ) so long as the cost gap between the top two most-efficient producers is not too high, i.e.,  $(c_2 \leq \frac{2c_1}{3\theta})$ .*

In general, producer  $i$  has positive output only if

$$c_i \leq \frac{\bar{c}_{(i-1)}(i-1)}{(i-1) - (1-3\theta/2)}, \quad (7)$$

where  $\bar{c}_{(i-1)}$  is the average cost of producers  $1 \dots i-1$ .

Why can higher-cost producers sustain positive production even though producer 1 has a persistent economic advantage at every level of  $Q$ , hence can push  $Q^*$  above the level where marginal gain equals the marginal cost of other producers? The reason is threefold. First, combating the most obvious counter-argument against the result (i.e., producer 1 chooses  $Q_1$  where marginal gain equals her marginal cost, while leaving others to ponder about the “next” unit of output at higher marginal cost) producers’ output decisions are made independently and concurrently, hence producer 1’s cost advantage does not translate into enforcing a sequential decision framework on other producers. Second, each producer  $i$  has a different marginal gain from incremental output because this revenue benefit is based on a production externality linked to output of others (i.e.,  $Q_{-i}$ ). This aspect of the logic is most vividly seen in Example 3.2) where the oversupply result is obtained despite both producers having identical costs. Third, the reticence of producer 1 to make even more output and drive others out of the market is constrained by the diminishing returns from higher  $Q$  ( $\frac{\partial^2 S(Q)}{\partial Q^2} < 0$  because of the exponent  $\theta$  in  $D(P, Q)$ ), leaving room for some higher-cost producers to sustain at lower levels of output (see Proposition 1). This last reason leads to an informative corollary.

**Corollary 1.** *The number of active producers in the market ( $K$ ) is inversely proportional to the intensity  $\theta$  with which demand increases against  $Q$ , i.e.,  $\frac{\partial K}{\partial \theta} < 0$ .*

Corollary 1 explicates the effect that the term  $Q^\theta$  has on demand and value creation. Think of  $\theta$  as representing, approximately, consumers’ greed or propensity for “bigger” bundles, i.e., the inverse of their budget constraint for consuming bundles (higher  $\theta$  implies a more elastic constraint). The strongest producers are more aggressive in value creation when  $\theta$  is high (i.e., greater gains from higher  $Q_i$ ), leaving little room for high-cost producers to have marginal gain that exceeds marginal cost. At the limiting value ( $\theta < \frac{2}{3}$ ), only producer 1 can survive, no matter how close  $c_2$  is

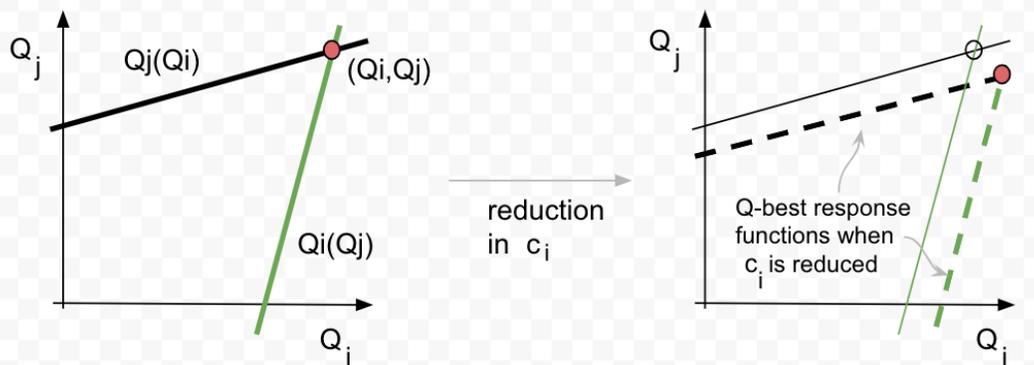


Figure 6: Content provision strategy, and effect of change in one producer's technology.

to  $c_1$ , because producer 1 has unbounded gain from greater output. Lower values of  $\theta$  allow additional producers to survive because lower-cost producers are less keen on producing more output (see Eq.1).

What is the nature of interdependence between outputs of competing producers? In traditional competition (e.g., Hotelling competition with covered market), market share is a zero-sum game so that higher output by producer  $i$  implies lower output by  $j$ , and production levels are *strategic substitutes* (recall from §2.1). With cross-producer bundling, in contrast, higher output by any producer has a “rising tide lifts all boats” effect: it improves market demand for the bundle (which is common to all producers), and this provides partial benefit to competing producers. This property can formally be derived from the behavior of the  $Q_j$  best-response against  $Q_i$ .

**Corollary 2** (Cross-producer harmony). *Increased production by producer  $i$  motivates, ceteris paribus, higher production by producer  $j$ .*

Corollary 2 illustrates and confirms why “competitors” are in fact “collaborators” in this market form, and that their outputs are *strategic complements* rather than substitutes. Notably, this result holds with other parameters being the same, and because  $Q_i$ 's are themselves endogenous the result is more usefully interpreted as describing the mechanism leading to the equilibrium (see the depiction of the best-response functions in Fig. 6). Consider now why the premise of the Corollary, i.e., why might an increase in  $Q_i$  occur? The most direct reason is that producer  $i$  is able to reduce its production cost  $c_i$ , e.g., by acquiring improved talent or technology. The previous result would

indicate that lower  $c_i$  has a positive effect on  $Q_j$ , but a formal consideration reveals the opposite and sheds further light on the nature of competition between producers.

**Corollary 3** (Cross-producer conflict). *Reduction (improvement) in one producer's cost forces others to reduce production and can cause some producers to exit. Formally,  $\frac{\partial Q_j}{\partial c_i} > 0$ .*

Suppose producer  $i$  is able to reduce  $c_i$ , then intuitively it wants to produce higher  $Q_i$  for any given choices of other producers. Because  $i$  produces more, other producers  $j$  observe a lower marginal revenue at the existing level  $Q_j$  (because of diminishing marginal gains from higher  $Q$ ) and must ramp production down to the point where marginal revenue equals marginal cost (this can be confirmed by computing  $\frac{\partial Q_j}{\partial c_i} > 0$ ). Hence, their best-response functions in response to the lower  $c_i$  shift towards lower  $Q_j$  (see the right panel in Fig. 6). In equilibrium, with lower  $c_i$ , producer  $i$  makes more and other producers make less. This result holds when  $K$  remains the same, i.e., no  $Q_j$  crosses the boundary  $Q_j \geq 0$  (at the boundary, if some producers are driven out of the market, then it is possible for other producers to have higher  $Q_j$  than before.) The result further illuminates the intuitive understanding that while producers are engaged in collaborative production in this setting, a competitive effect emerges because consumer spending is shared across them collectively.

## 5 Variations in Market Structure

The previous two sections have proposed a reduced-form model and equilibrium outcomes for markets with multiple producers and a separate single retailer (shown in the left panel of Fig. 1). Collectively, the results presented thus far confirm the distinctive aspects of this market structure and also validate the reduced-form demand structure employed in setting up the model. Alternative market structures can readily be analyzed through variations of this model. For instance, the main model considered multiple producers and a separate single retailer (shown in the left panel of Fig. 1). Setting both  $K=1$  and  $\gamma=1$  corresponds to a vertically integrated monopoly, and setting just  $\gamma=1$  (with  $K>1$ ) represents a production consortium where the consortium makes pricing decisions (rather than a separate retailer who shares revenues). Varying  $K$  affects the number of

producers in the market, with  $K=1$  representing a bilateral monopoly comprising a single producer and single retailer.

Consider how variation in  $K$  (number of producers) affects total output  $Q$  (Eq. 4c). Intuitively, since market demand  $D(P, Q)$  is responsive to  $Q$  rather than  $K$ , this would suggest higher output under a single producer than if production and profits were shared among multiple producers. More generally, the nature of competition implied in Corollary 3 also suggests that output would be higher under fewer producers. However, computing the expression  $\frac{\partial Q}{\partial K}$  using Eq. 5c, we find that higher  $K$  leads to greater supply of content.

**Proposition 3** (Oversupply with more producers). *Ceteris paribus, higher  $K$  leads to higher output (i.e.,  $\frac{\partial Q}{\partial K} > 0$ ), and greater market coverage for the bundle. Total output would be lower under a single producer (with unit cost same as  $\bar{c}$  of existing producers).*

This unusual result is obtained because cross-producer bundling and revenue-sharing has an effect analogous to the productivity- and production-enhancing effect of technology. Generally, a producer's output level is set to equate marginal cost of making more output (here, a constant  $c_i$ ) with its marginal revenue. However, under multi-producer bundling, producer  $i$ 's benefit from every dollar spent on production (cost  $c_i$ ) gets amplified. Producer  $i$  benefits from the higher market demand and price that arises due to the  $Q_j$ 's of other producers, but can enjoy these gains only proportional to its share of content. As all producers evaluate their output decisions this way, the result is an oversupply of output. Together, Corollary 3, and Proposition 3 explain the interplay of collaboration-competition in this market structure: one, producers do compete because higher production by one crowds out others, but conversely each producer's production also creates some gains for others. Notice that the result arises purely on account of co-production externality, rather than due to any dependence of  $K$  on either bundle price or the level of revenue-sharing with the retailer. Analysis of individual-level output decisions leads to the next result.

**Proposition 4** (Mergers between producers). *A merger between producers, such that the new entity has cost parameter equal to average of merged producers, causes all producers to make less output, but the merged entity earns higher profit.*

How do mergers or splits, or changes in  $K$ , affect profit-sharing between the retailer and producers? From Corollary 3, reduction in  $K$  causes lower  $Q$ , hence lowers the retailer's profit. Moreover, it can also shift the revenue-sharing parameter away from the retailer. With a high  $K$ , a retailer can afford losing a producer with whom it can't reach a profit sharing agreement, and this threat of being shut out forces producers to give a high share to the retailer. In contrast, a small  $K$  (and the extreme,  $K=1$ ) makes the producer(s) more consequential to the retailer's survival, and the retailer must surrender a higher share ( $\gamma$ ) of bundle revenues to the producer(s). Hence, mergers and acquisitions among producers have a doubly harmful effect on the retailer, who earns lower revenues on account of lower  $Q$  and higher  $\gamma$ . This result is another peculiarity of the bundling structure inherent in selling in-home entertainment content. It contrasts industries where producers compete for individual customers (through a retailer), and consolidation among producers generally leads to higher prices and higher margins for both producers and the retailer.

**Proposition 5** (Horizontal mergers among producers). *Mergers and acquisitions between producers, and other actions that reduce  $K$ , reduce the retailer's profits,  $\frac{\partial \Pi_R}{\partial K} > 0$ .*

Next, consider equilibrium outcomes when producers can organize into a bundling consortium and directly offer the bundle to the market (vs. revenue-sharing with a retailer). This structure occurs, for instance, in technology patent licensing (Lerner and Tirole, 2004) and it roughly describes Hulu's position in distribution of in-home entertainment (as a joint venture between The Walt Disney Company, AT&T Warner Media, and Comcast-NBC). To examine this ( $\gamma=1$  and  $K>1$ ), suppose that the consortium faces the same additional cost of distribution  $cQ^\theta$  as would a separate retailer. More generally, consider the effect of  $\gamma$  on equilibrium outcomes. Now, given  $Q$ , the consortium would set price exactly as the retailer would. because  $\gamma$  linearly affects the retailer's profit, it does not directly impact bundle price, hence  $P^*$  is as given in Lemma 1. However the higher  $\gamma$  motivates producers to create more output, increasing  $Q$  more than linearly in  $\gamma$ .

**Proposition 6** (Consortium vs. a Retailer). *Producers supply more content when selling content bundles as a consortium rather than through a separate retailer. Generally, producers make more content when they can get higher share of content subscription revenues, i.e.,  $\frac{\partial Q}{\partial \gamma} > 0$ .*

## 6 Conclusion

This paper has developed a model for analyzing markets in which a retailer firm offers a bundle of outputs sourced from multiple producers. While such markets have existed for long (e.g., art and music festivals, county fairs, sports tournaments, and other events with a ground or seasons pass), information technology has made them more prominent by facilitating the merging cross-producer outputs when the market values variety, rather than just quantity and quality. This is observed in digital platforms, where value co-creation is a defining characteristic and the platform becomes the base for distribution of outputs from multiple producers. Building on existing perspectives on competition, bundling and value co-creation, our goal is to model the entire economic system, explaining the retailer or platform's pricing as well as producers' output decisions, modeling revenue-sharing between them, and exploring the effects of alternate market structures. Given the well-known challenges in representation and analysis of bundle demand, we achieve these goals by first developing a reduced-form specification for bundle demand which fits and respects the characteristics of bundling across a wide spectrum of bundling scenarios, and then deriving market outcomes under alternative market structures as well as the drivers and consequences of changes in market structures.

Several useful insights are derived under the main model setting which features a single retailer and multiple producers. Crucially, we show that multiple producers can flourish and have positive output, rather than being vanquished by the dominant, i.e., lowest-cost, producer. This result is tightly related to market propensity for bigger bundles ( $\theta$ ). When this propensity is low (correspondingly, high), the dominant producer has less (respectively, more) incentive to expand output, but the lower (or higher) level of output encourages (or dissuades) higher-cost producers from being active. Whether a specific producer has feasible output depends on the cost structure of the focal producer relative to the average cost of its superior producers. For those that survive, outputs levels are strategic complements, but a cost improvement by one producers forces others to cut output in equilibrium. Total industry output with multiple  $K$  producers is higher (i.e., more

inefficient) compared with a single producer whose cost is the average of the  $K$  producers, and consequently high  $K$  leads to lower total producer profit and higher profit for the retailer, even with the same revenue-sharing parameter. Therefore, mergers (or acquisitions) between producers are beneficial at the producer layer and reduce profit at the distribution layer.

The modeling framework of this paper is limited in the sense that it does not accommodate individual-level demand preferences for specific products or producers, or for combinations of them, nor does it explore or advise regarding what specific outputs are made by each producer. However, it captures higher-level requirements in a way that is analytically tractable and leads to meaningful conclusions, and as a foundation for analysis of additional market structures. The main setting—that all outputs are combined into a universal single bundle offered to all buyers—works best when outputs of individual producers are of roughly comparable value, e.g., oligopoly with a few large and powerful producers. What if, for instance, there are hundreds of tiny or niche producers, in addition to the oligarchs? This variation could be approximated by treating  $Q$  as an aggregation only across the large  $i$  producers (i.e., undercounting  $Q$  a little) and assigning very high  $c_i$ 's to the niche producers, with the understanding that they have alternative motivations (e.g., advertising) rather than revenue-share from the retailer. Alternately, if some producers are powerful outliers with extremely high-value , then following bundling theory, these high-value items (e.g., HBO) can be separated out of the main bundle and offered as an “add-on” (see partial bundling, e.g., Bhargava (2013)). The model could also be applied to examine forward integration (the retailer doubling as a producer, e.g., *Netflix*) by treating the retailer as a producer with cost parameter  $c_R$ , while also recognizing the unique profit motivation when optimizing the retailer's decisions in this new structure. Another useful extension involves multiple retailers, either with distinct and non-overlapping market regions (which raises the possibility of mergers in the distribution layer, especially because each smaller retailer is less able to extract the benefits of cross-producer bundling) or with strong overlap in which case the analysis would also need to consider single- vs multi-homing behavior of consumers. All these are exciting prospects for research for which this paper can now provide a formal and relevant modeling and analytical framework.

# A Appendix

## A.1 Bundle Demand

Given the well-known computational challenges that arise in constructing bundle demand from a micro-level specification of demand for individual components by individual consumers (see §2.3), we instead consider the aggregate characteristics that arise when a bundle with several components is offered to a large number of consumers who are heterogeneous on multiple dimensions (including in level of additivity and correlation between their valuations for subsets of bundle components).

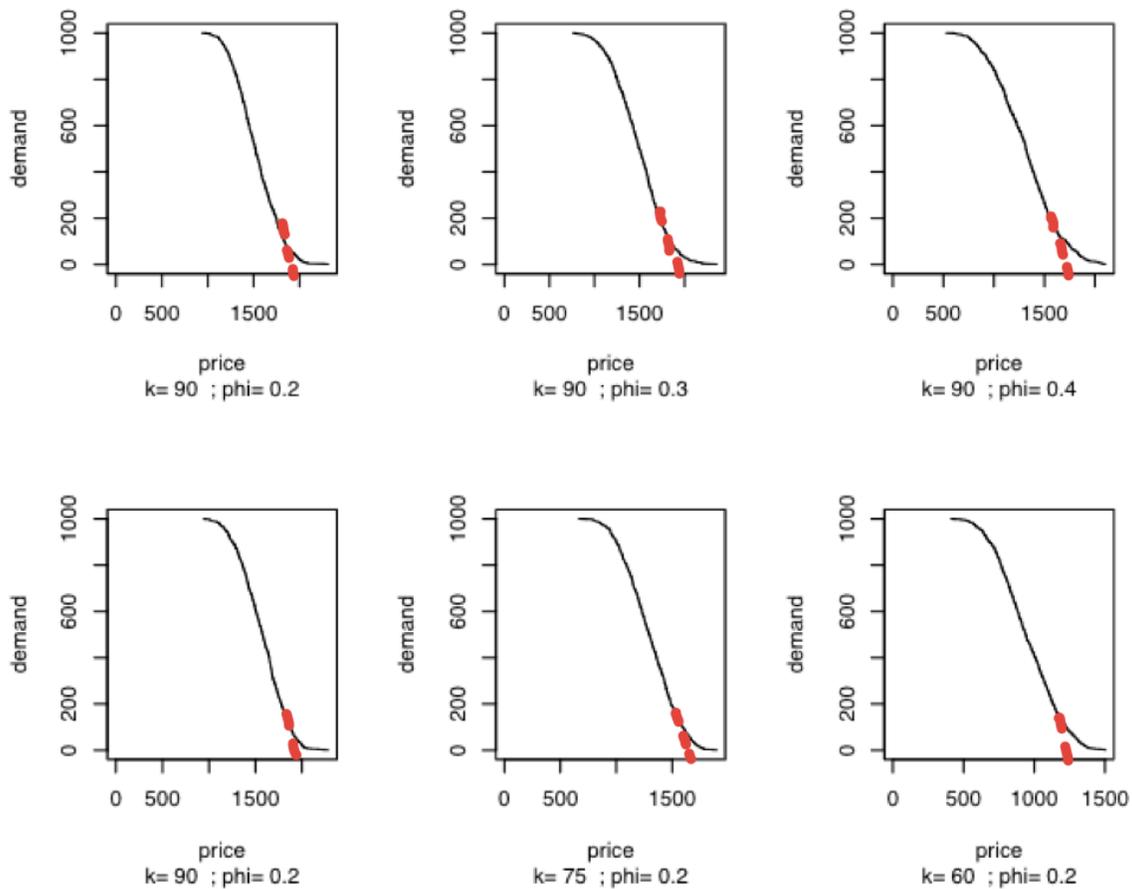


Figure 7: Simulations of bundle demand with  $k$  products, with varying levels of sub-additivity in valuations ( $\phi$ ) and asymmetry in demand profiles.

As discussed in §2.3 (Fig. 3), a core property of bundle demand is that it is “flatter in the middle” of the price region. Bakos and Brynjolfsson (2000) demonstrated this using goods with independent and additive valuations and identical demand profiles. With sufficient number of goods, we observe the same behavior when any or all of these conditions are relaxed. Fig. 7, based on simulation results, depicts this by displaying demand as a function of price (rather than the usual *inverse* demand curve), and comports with Figs 2-5 in Olderog and Skiera (2000). We encode this property in the following requirement.

**Requirement 3** (Demand response to price). *The rate at which bundle demand drops as price increases is higher at higher prices.*

$$\frac{\partial D(P, Q)}{\partial P} < 0, \frac{\partial^2 D(P, Q)}{\partial P^2} < 0. \quad (8)$$

Requirement 3 specifies that bundle demand is not only decreasing in price (which is standard for any downward-sloping demand function), but also that the rate of decline increases rapidly as price increases. To be precise, the  $\frac{\partial^2 D(P, Q)}{\partial P^2} < 0$  property fails when price is quite high (then demand drops more slowly as price increases, see Fig. 3 and Fig. 7); however this high-price low-demand region is irrelevant from an equilibrium perspective because demand smoothing causes a high-demand low-price optimal solution. Hence, outcomes would remain identical if we replaced this region with the dashed curve as shown in Fig. 7.

Next, consider how  $D(P, Q)$  varies with  $Q$  and particularly how  $D(0, Q)$  varies with  $Q$ , the total output across all producers. Using the saturation level of TV bundle demand (i.e.,  $\hat{D}(Q) = D(P=0, Q)$ ) to illustrate, demand increases with  $Q$ , although at slower rate when  $Q$  is already high (because of consumers’ finite time and other bio-costs of consuming content). More generally, this behavior also holds at non-zero prices, and is unlike traditional competition where a lower price is the effect of increase in  $Q$ . Moreover, the higher-demand effect of  $Q$  should be greater at higher levels of price than at lower levels where more of the market is already captured. These requirements are encompassed in our next requirement, consistent with Requirement 1.

**Requirement 4** (Effect of bundle magnitude on demand). *Bundle demand increases as  $Q$  in-*

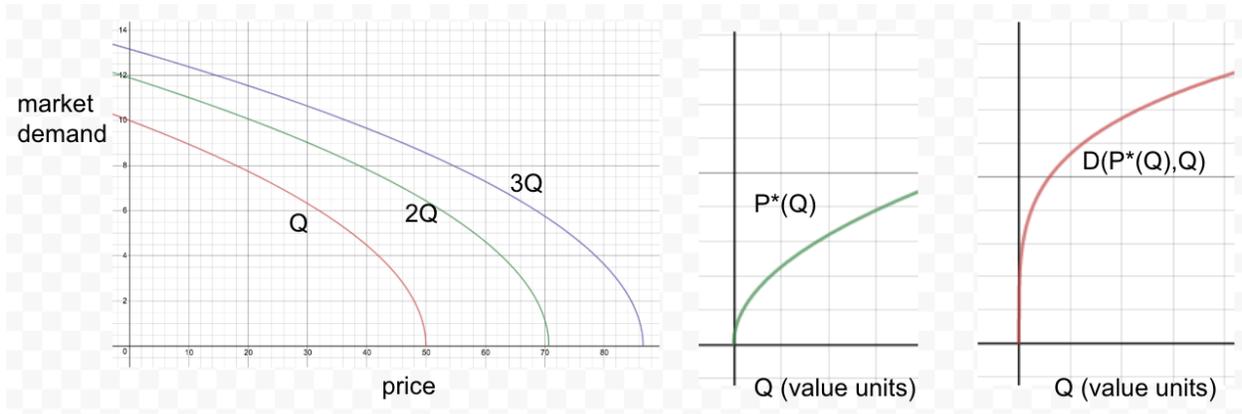


Figure 8: Ideal shape of demand curve for content bundle, and optimal price and demand level as a function of content value units.

creases, but at slower rates for higher  $Q$ , and more rapidly for higher prices.

$$\frac{\partial D}{\partial Q} > 0, \frac{\partial^2 D}{\partial Q^2} < 0, \frac{\partial \hat{D}(Q)}{\partial Q} > 0, \frac{\partial^2 \hat{D}(Q)}{\partial Q^2} < 0, \frac{\partial^2 D}{\partial P \partial Q} > 0. \quad (9)$$

The requirements laid out above are visualized with demand functions in Fig. 8 (left panel). Additional restrictions can be derived by considering equilibrium behavior of bundle demand. Insights from bundling literature are summarized in the right panel of Fig. 8: (i) the equilibrium demand level should increase with  $Q$  even though (ii) the equilibrium market price also increases in  $Q$ , and (iii) the *price per value unit* should decrease in  $Q$ . This is because repeated increases in  $Q$  are subject to diminishing gains in consumer value from variety and quality, hence provide successively lower monetization power. We combine these observations into the following requirement.

**Requirement 5** (Pricing Equilibrium). *Market demand  $D = D(P, Q)$  should exhibit the following equilibrium-price behavior.*

$$\frac{\partial D(P^*, Q)}{\partial Q} > 0, \frac{\partial (P^*/Q)}{\partial Q} < 0, \quad (\text{and by derivation}) \frac{\partial^2 S(Q)}{\partial Q^2} < 0. \quad (10)$$

Table 1 (in Appendix) lists multiple demand models that were examined, including multiple ways to incorporate  $Q$  into the demand function. As evident from the table, several of the standard demand formulations are not well great at capturing the bundle structure of demand, suggesting

$D(P, Q)$	unit cost	$P^*$	$D(P^*, Q)$	$\frac{\partial^2 D}{\partial P^2} > 0?$	$\frac{\partial(\frac{P^*}{Q})}{\partial Q} < 0$ at $c=0$	$\frac{\partial D(P^*, Q)}{\partial Q} > 0?$	$\frac{\partial^2 D}{\partial P \partial Q} > 0?$
$A - b \frac{1}{Q} P$	$c \ln Q$	$\frac{AQ}{2b} + \frac{c \ln Q}{2}$	$\frac{A}{2} - \frac{bc \ln Q}{2Q}$	<b>No</b>	<b>No</b>	<b>Yes</b>	<b>Yes</b>
$A - b \frac{1}{\ln Q} P$		$\frac{A \ln Q}{2b} + \frac{c \ln Q}{2}$	$\frac{A}{2} - \frac{bc}{2}$	<b>No</b>	<b>Yes</b>	<b>No</b>	<b>Yes</b>
$A Q^\theta - b P$	$c \ln Q$	$\frac{AQ^\theta}{2b} + \frac{c \ln Q}{2}$	$AQ^\theta - \frac{bc}{2} \ln Q$	<b>No</b>	<b>Yes</b>	<b>?</b>	<b>?</b>
$A \ln Q - b P$		$\frac{A \ln Q}{2b} + \frac{c \ln Q}{2}$	$(A - \frac{bc}{2}) \ln Q$	<b>No</b>	<b>Yes</b>	<b>Yes</b>	<b>?</b>
$A \ln Q - b P^2$		$\frac{A \ln Q}{2b} + \frac{c \ln Q}{2}$	$(A - \frac{bc}{2}) \ln Q$	<b>Yes</b>	<b>Yes</b>	<b>Yes</b>	<b>?</b>
$\sqrt{A \ln Q - b P}$	$c \ln Q$	$\frac{2A+bc}{3b} (\ln Q)$	$\sqrt{\frac{A-bc}{3}} (\ln Q)$	<b>Yes</b>	<b>Yes</b>	<b>Yes</b>	<b>Yes</b>
$\sqrt{A Q^\theta - b P}$	$c Q^\theta$	$\frac{2A+bc}{3b} Q^\theta$	$\sqrt{\frac{A-bc}{3}} Q^\theta$	<b>Yes</b>	<b>Yes</b>	<b>Yes</b>	<b>Yes</b>
$A Q^\theta e^{-\frac{P}{b}}$	$c \ln Q$	$b + c \ln Q$	$A Q^\theta e^{-\frac{b+c \ln Q}{b}}$	<b>No</b>	<b>No</b>	<b>Yes</b>	<b>No</b>
$A \ln Q e^{-\frac{P}{b}}$		$b + c \ln Q$	$A \ln Q e^{-\frac{b+c \ln Q}{b}}$	<b>No</b>	<b>No</b>	<b>Yes</b>	<b>No</b>
$A e^{-\frac{P}{b} Q^\theta}$	$c \ln Q$	$b Q^\theta + c \ln Q$	$A e^{-\frac{b Q^\theta + c \ln Q}{b}}$	<b>No</b>	<b>Yes</b>	<b>Yes</b>	<b>No</b>

Table 1: Alternate ways to model demand for content at bundle price  $P$ . The exponent  $\theta$  above is assumed to lie in  $(0, 1)$  and further restricted to  $(0, 2/3)$  in some cases. The “?” indicates that the property may hold or not depending on certain parameter values.

the chosen demand model,  $D(P, Q) = \sqrt{A Q^\theta - b P}$ , (with  $\theta \in [0, 2/3]$ ).

## A.2 Technical Details and Proofs

**Proof of Lemma 1.** The proof is divided into two parts representing the retailer’s price optimization problem, and the provision decisions  $Q_i^*$  of producers jointly with computation of  $Q^*$  and  $K$ , the number and set of active producers.

**Retailer’s optimal price  $P^*$ :** The pricing problem is of usual form, given  $Q$  as an input. The retailer maximizes its payoff function, which is a  $1-\gamma$  fraction of total surplus (Eq. 3a), and optimality conditions yield the price  $P^*$ . Equilibrium levels of demand and  $S(Q)$ , given  $Q$ , are obtained by substitution.

**Production decisions ( $Q_i$ ) and  $\hat{K}$ :** Each producer chooses  $Q_i$  to maximize  $\gamma \frac{Q_i}{Q} S(Q)$  where  $Q$  is the aggregation of  $Q_i$ ’s for active producers (i.e.,  $c_i \leq \frac{\gamma S(Q)}{Q}$ ). Optimality conditions yield a series of

equations across all active producers,  $Q_i = \left[ 2 - \frac{bc_i Q}{\gamma} \left( \frac{1}{Q^\theta} \frac{3}{A-bc} \right)^{3/2} \right] \frac{Q}{(2-3\theta)}$ . Let  $\hat{K}$  be the number of active producers. Then, (i) because of the ascending order on cost, the set of IR constraints reduces to  $c_{\hat{K}} \leq \frac{\gamma S(Q)}{Q}$ , and (ii) adding the series of equations (and using  $Q = \sum_{i=1}^{\hat{K}} Q_i$ ) yields Eq. 5b. Write  $Z = \left( \frac{1}{Q^\theta} \frac{3}{A-bc} \right)^{\frac{3}{2}}$ , which is the common term between  $c_{\hat{K}} \leq \frac{\gamma S(Q)}{Q}$  and Eq. 5b. Then, writing those two expressions in terms of  $Z$ , we have (i)  $\frac{QZb}{\gamma} \leq \frac{2}{c_{\hat{K}}}$  and (ii)  $\frac{QZb}{\gamma} = \frac{1}{\bar{c}_{(\hat{K})}}$ . Algebraic rearrangement produces the feasibility condition in the form  $c_{\hat{K}} \leq \frac{\bar{c}_{\hat{K}}}{\hat{K} - (1-3\theta/2)}$ . Now, because producers are arranged in ascending order of cost, the set of active producers is exactly the set  $\{1 \dots K\}$  where  $K$  is the highest  $\hat{K}$  which satisfies the feasibility condition. ■

**Proof of Proposition 1.** The result follows trivially because  $Q_i$  is inversely related to  $c_i$ , see Eq. 4c. ■

**Proof of Proposition 2.** Write  $c_1 + \dots + c_i = (c_1 + \dots + c_{i-1}) + c_i$ , which equals  $c_i + (i-1)\bar{c}_{i-1}$ . Plugging this into Eq. 2a, yields the result after algebraic simplification, and also generates the special case for  $i=2$ . ■

**Proof of Propositions 3 and 4.** Consider the equilibrium when the economy has  $K$  active producers. Suppose producers  $\{i, j\}$  (with  $j > i$  and both  $\leq K$ ) merge yielding a new entity whose cost parameter is the mean of  $c_i$  and  $c_j$ . Now, from Proposition 2, this merger has no effect on the participation decisions of producers with index higher than  $j$ , because average cost of their superior producers remains the same. Hence we only need to consider the change in  $Q_i$ 's of all active producers. This effect is obtained from Eq. 2b and Eq. 2c in Lemma 1, but these must be considered simultaneously because they yield interrelated outputs  $Q_i$ 's and  $Q$ . For convenience rewrite Eq. 2b as  $Q_i = \left( \frac{2}{2-3\theta} - \frac{c_i}{\bar{c}} \frac{1}{K} \right) Q$ . With this format—and, for the moment, ignore any change in  $Q$  itself—it is evident that for all active producers other than  $\{i, j\}$ , the only impact on output decision occurs on account of lower  $K$ , i.e., they have less output in equilibrium. For producers  $i$  and  $j$  the reduction effect is starker; this can be seen by comparing their new output with the sum of previous output of  $i$  and  $j$ , and recognizing that  $\frac{2}{2-3\theta}$  lies between 1 and infinity. There are two

effects to note while comparing. (i) The previous joint output of  $\{i, j\}$  has a higher term  $2\frac{2}{2-3\theta}$  vs. just  $\frac{2}{2-3\theta}$  of the merged entity; (ii) the merged entity has a greater term  $-\frac{c_i+c_j}{2}$  vs.  $c_i+c_j$  of the joint output. However, part (ii) is multiplied by  $\frac{1}{eK}$  hence has a smaller effect than part (i). Therefore, the only viable effect on  $Q$  is that the post-merger value is lower. This lower value (which is a multiple in Eq. 2b) ensures that every producer has lower output. If the merged producers have a better cost structure than the average of  $c_i$  and  $c_j$  then it is possible that the merged entity has higher output. However, the merger has positive impact on profit of the merged producers even with the lower output, because this lower output veers more towards the efficient level for them. ■

**Proof of Proposition 6.** Consider the general case first, with  $\gamma < 1$ . Because  $\gamma$  linearly affects the retailer's profit, it does not directly impact bundle price, and  $P^*$  is as given in Lemma 1. For production decisions, though, a higher  $\gamma$  gives producers higher marginal gain for each unit of cost, leading to higher  $Q_i$  and  $Q$ . It is evident from Eq. 2c that this effect is more than linear in  $Q$ , because  $(2-3\theta) < 2$ . For the consortium case, with  $\gamma=1$ , the profit function Eq. 3a is replaced by simply  $(P - cQ^\theta)D(P, Q)$  (since there is no retailer), and the rest of the apparatus can be executed to yield the results for this extreme case. ■

**Proof of Proposition 5.** From Proposition 3, post-merger total output is lower. From Eq. 3e, retailer's total profit in equilibrium (and, similarly,  $S^*(Q)$ ) is an increasing function of  $Q$  (assuming the same level of  $\gamma$ ), hence the merger reduces the retailer's profit. ■

## References

Adams, William James and Janet L. Yellen (1976). "Commodity Bundling and the Burden of Monopoly". *The Quarterly Journal of Economics* 90.3, pp. 475–498. URL: <http://ideas.repec.org/a/tpr/qjecon/v90y1976i3p475-98.html>.

- Adner, Ron, Jianqing Chen, and Feng Zhu (2016). “Frenemies in Platform Markets: The Case of Apple’s iPad vs. Amazon’s Kindle”. *Harvard Business School Technology & Operations Mgt. Unit Working Paper* 15-087.
- Armstrong, Mark (2013). “A more general theory of commodity bundling”. *Journal of Economic Theory* 148, pp. 448–472.
- Bakos, Yannis and Erik Brynjolfsson (1999). “Bundling information goods: Pricing, profits, and efficiency”. *Management Science* 45.12, pp. 1613–1630.
- (2000). “Bundling and Competition on the Internet”. *Marketing Science* 19.1, pp. 63–82. URL: <http://mktsci.journal.informs.org/cgi/content/abstract/19/1/63>.
- Bhargava, Hemant K. (2012). “Retailer-Driven Product Bundling in a Distribution Channel”. *Marketing Science* 31.6 (November-December), pp. 1014–1021.
- (2013). “Mixed Bundling of Two Independently-Valued Goods”. *Management Science*.
- Burstein, M. L. (1960). “The Economics of Tie-In Sales”. *The Review of Economics and Statistics* 42 (1), pp. 68–73.
- Carbajo, José, David de Meza, and Daniel J. Seidmann (1990). “A Strategic Motivation for Commodity Bundling”. *The Journal of Industrial Economics* 38.3.
- Ceccagnoli, Marco, Chris Forman, Peng Huang, and DJ Wu (2012). “Co-Creation of Value in a Platform Ecosystem: The Case of Enterprise Software”. *MIS Quarterly* 36, pp. 263–290. DOI: 10.2307/41410417.
- Chen, Yongmin and Michael H. Riordan (2007). “Price and Variety in the Spokes Model”. *The Economic Journal* 117.522, pp. 897–921. ISSN: 00130133, 14680297. URL: <http://www.jstor.org/stable/4625536>.
- (2013). “Profitability of Product Bundling”. *International Economic Review* 54.1, pp. 35–57. ISSN: 1468-2354. DOI: 10.1111/j.1468-2354.2012.00725.x. URL: <http://dx.doi.org/10.1111/j.1468-2354.2012.00725.x>.

- Choudary, Sangeet Paul, Marshall W. Van Alstyne, and Geoffrey G. Parker (2016). *Platform Revolution: How Networked Markets Are Transforming the Economy—And How to Make Them Work for You*. 1st. W. W. Norton & Company. ISBN: 0393249131, 9780393249132.
- Cournot, Antoine Augustin (1929). *Researches into the mathematical principles of the theory of wealth/ Augustin Cournot, 1838; translated by Nathaniel T. Bacon ; with an essay on Cournot and mathematical economics and a bibliography of mathematical economics, by Irving Fisher*. Macmillan Company, New York.
- Demirezen, Emre M, Subodha Kumar, and Bala Shetty (2018). *Two Is Better Than One: A Dynamic Analysis of Value Co-Creation*.
- Economides, Nicholas and Steven C Salop (1991). *Competition and integration among complements and network market structure*. New York University, Leonard N. Stern School of Business.
- Eisenmann, Thomas, Geoffrey Parker, and Marshall Van Alstyne (2011). “Platform envelopment”. *Strategic Management Journal* 32.12, pp. 1270–1285. DOI: 10.1002/smj.935.
- Evans, David S. and Michael A. Salinger (2005). “Why Do Firms Bundle and Tie? Evidence from Competitive Markets and Implications for Tying Law”. *Yale Journal on Regulation* 22.1.
- Ferreira, Rodolphe Dos Santos and Jacques-François Thisse (1996). “Horizontal and vertical differentiation: The Launhardt model”. *International Journal of Industrial Organization* 14.4, pp. 485–506.
- Foerderer, Jens, Thomas Kude, Sunil Mithas, and Armin Heinzl (2018). “Does platform owner’s entry crowd out innovation? Evidence from Google photos”. *Information Systems Research*.
- Gabszwick, J Jaskold and Jacques-François Thisse (1986). “On the nature of competition with differentiated products”. *The Economic Journal*, pp. 160–172.
- Hagiu, Andrei (2009). “Two-Sided Platforms: Product Variety and Pricing Structures”. *Journal of Economics & Management Strategy* 18.4, pp. 1011–1043.
- Hitt, Lorin M. and Pei-yu Chen (2005). “Bundling with Customer Self-Selection: A Simple Approach to Bundling Low-Marginal-Cost Goods”. *Management Science* 51.10, pp. 1481–1493.

- Ibragimov, Rustam and Johan Walden (2010). “Optimal Bundling Strategies Under Heavy-Tailed Valuations”. *Management Science* 56.11, pp. 1963–1976. DOI: 10.1287/mnsc.1100.1234. eprint: <http://mansci.journal.informs.org/cgi/reprint/56/11/1963.pdf>. URL: <http://mansci.journal.informs.org/cgi/content/abstract/56/11/1963>.
- Katz, Michael L and Carl Shapiro (1994). “Systems Competition and Network Effects”. *J. Econ. Perspect.* 8.2, pp. 93–115.
- Kobayashi, Bruce H. (2005). “Does Economics Provide a Reliable Guide to Regulating Commodity Bundling by Firms? a Survey of the Economic Literature”. *Journal of Competition Law and Economics* 1.4, pp. 707–746. DOI: 10.1093/joclec/nhi023. eprint: <http://jcle.oxfordjournals.org/content/1/4/707.full.pdf+html>. URL: <http://jcle.oxfordjournals.org/content/1/4/707.abstract>.
- Lerner, Josh and Jean Tirole (2004). “Efficient Patent Pools”. *Am. Econ. Rev.* 94.3, pp. 691–711.
- Lewbel, Arthur (1985). “Bundling of substitutes or complements”. *International Journal of Industrial Organization* 3.1, pp. 101–107.
- McAfee, R. Preston, John McMillan, and Michael D. Whinston (1989). “Multiproduct Monopoly, Commodity Bundling, and Correlation of Values”. *The Quarterly Journal of Economics* 104.2, pp. 371–83.
- Nocke, Volker, Martin Peitz, and Konrad Stahl (2007). “Platform ownership”. *Journal of the European Economic Association* 5.6, pp. 1130–1160.
- Olderog, Torsten and Bernd Skiera (2000). “The Benefits of Bundling Strategies”. *Schmalenbach Business Review* 1, pp. 137–160.
- Prasad, Ashutosh, R. Venkatesh, and Vijay Mahajan (2010). “Optimal Bundling of Technological Products with Network Externality”. *Management Science* 56.12, pp. 2224–2236.
- Rao, Vithala R, Gary J Russell, Hemant Bhargava, Alan Cooke, Tim Derdenger, Hwang Kim, Nanda Kumar, Irwin Levin, Yu Ma, Nitin Mehta, John Pracejus, and R Venkatesh (2018).

- “Emerging Trends in Product Bundling: Investigating Consumer Choice and Firm Behavior”. *Customer Needs and Solutions* 5.1, pp. 107–120.
- Schmalensee, Richard (1984). “Gaussian Demand and Commodity Bundling”. *The Journal of Business* 57.1, S211–230.
- Shaked, Avner and John Sutton (1982). “Relaxing price competition through product differentiation”. *The review of economic studies*, pp. 3–13.
- (1987). “Product differentiation and industrial structure”. *The Journal of Industrial Economics*, pp. 131–146.
- Stigler, George (1963). “United States v. Loew’s Inc: A Note on Block-Booking”. *The Supreme Court Review*, pp. 152–157.
- Stremersch, Stefan and Gerard J. Tellis (2002). “Strategic Bundling of Products and Prices: A New Synthesis for Marketing”. *Journal of Marketing* 66.1, pp. 55–72.
- Surowiecki, James (2010). “Bundles of Cable”. *The New Yorker*.
- Varian, H.R. (1992). *Microeconomic Analysis*. Norton International edition. Norton. ISBN: 9780393960266.  
URL: <https://books.google.com/books?id=m20iQAAACAAJ>.
- Venkatesh, R. and Vijay Mahajan (2009). “The Design and Pricing of Bundles: a Review of Normative Guidelines and Practical Approaches”. In: *Handbook of Pricing Research in Marketing*. Ed. by Vithala Rao. Northampton, MA: Edward Elgar Publishing, Inc.1.