

Optimal Platform Price Discrimination and Measuring the Value of Network Effects

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Abstract

Theory suggests that firms with market power should discriminate between users when setting prices or advertising levels. For platforms, this price discrimination should take into account the heterogeneous network effects provided by users on different sides of the market in addition to heterogeneity in elasticity of demand across users. We construct and illustrate a practical approach for estimating optimal platform pricing strategies, the social welfare implications of a change in regulatory policy or market structure, or changes in platform value or participation after a demand shock. We find that a profit maximizing platform should decrease fees or advertising for users who elastically demand the platform (the direct effect) and who create high amounts of network value for other profitable users who themselves demand the platform elastically (the network effect). We evaluate our model using data collected from a survey of over 40,000 US internet users on their demand for Facebook. Our non-parametric estimate, using only local information about marginal valuations, suggests that Facebook could increase profits by decreasing the amount of advertising on some market segments and increasing it for others. Our parametric estimates, which assumes demand for Facebook follows a logistic distribution, suggests that a large increase in social welfare is possible from Facebook reducing advertising.

1 Introduction

Much of the value of many digital platform businesses comes from what are known as “network effects”. A network effect is an externality that one participant in a market,

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digital platform, or similar system provides to others. But how exactly can one measure and exploit the value of network effects for any particular business or industry? In this paper we propose and implement a flexible strategy for the measurement and optimal harnessing of network effects.

Our paper begins by introducing a model of platform participation that allows for several dimensions of heterogeneity. Users vary in their opportunity cost for using the platform, the value they get from other types of users using the platform, and the disutility they receive from advertising. It is a model of an n-sided network in the sense that each individual or market segment can be thought of as a side of the network.¹ In this setting, the optimal price discrimination strategy is to decrease fees or advertising for users who elastically demand the platform (the direct effect) and who create high amounts of network value for other profitable users who themselves demand the platform elastically (the network effect).

A major contribution of our model is that it is designed to be implemented using real world data available to regulators and asset pricers who do not have inside information from the platform. After introducing and analyzing our model, we proceed to an empirical illustration. We collected information on US internet users' demand for Facebook across over 40,000 surveys conducted through Google Surveys. We categorize the surveyed into ten demographic groups by their age and gender. To collect information on Facebook demand and network effects, we use an experimental choice approach (Brynjolfsson et al., 2019).² We adapt this approach to our case by giving consumers the choice to give up access to a subset of their network in exchange for monetary compensation. Using this information about demand for Facebook, we estimate marginal elasticities (for the non-parametric analysis) and estimate the parameters of a logistic demand curve (for the parametric analysis).

In our non-parametric analysis, we estimate the marginal consequence of Facebook raising or lowering the quantity of advertisements (ad load) on users of different types. The disutility from advertising is relatively low, and groups demand Facebook relatively inelastically, implying that the direct effect of raising the level of advertising would be to increase revenues. However, taking network effects into account, our results suggest that Facebook should reduce the level advertising on most groups, and only raise them

¹When conceived of this way, any platform, including a one-sided platform, can be thought of as a n-sided platform once we account for the heterogeneity in users within a side. For example, a telephone network, which is the classic example of a one-sided network, can be thought of as consisting of multiple sides that can be distinguished based on various characteristics including business vs. personal use, demographics, regional location, heterogeneity in activity (frequent users or not) and type of activity (always callers, callers and receivers, always receivers).

²Brynjolfsson et al. (2019) measure the consumer surplus generated by digital goods by conducting discrete choice experiments where they offer consumers the choice to give up access to the good in exchange for monetary compensation.

for for males aged 25 to 34 and females aged 35 to 44.

In a parametric analysis, we use information from the distribution of surveys to fit a logistic model of demand for Facebook for each demographic group. Using this model we quantitatively estimate the impact on Facebook use and social welfare of Facebook eliminating advertisements on its platform (as it would if forced into perfect competition, with a marginal cost of zero). Our parametric results suggests a large increase in social welfare, on the order of \$90 billion a month, would result from such a change.

The paper concludes with a discussion of the strengths and weaknesses of this approach to modelling platform businesses and possible extensions.

2 Related Literature

A rich stream of theoretical literature studying network effects in the context of platform businesses has evolved over the past decade and a half. Following the seminal work of Parker and Van Alstyne (2005) and Rochet and Tirole (2003), platform researchers have extensively studied the impact of direct and indirect network effects on various strategic issues including pricing (Hagiu (2009)), launch (Evans and Schmalensee (2010)) and openness (Boudreau (2010)). The core insight of this research is that it can be optimal for a two-sided platform to subsidize one side and increase fees for the other side (Eisenmann et al. (2006)).

The above papers all focus on what are known as one or two-sided platforms. Examples of two-sided platforms are Uber (riders and drivers) and Ebay (sellers and buyers). In a two-sided platform, it can make sense to price discriminate based on side, because different types of users may provide different network externalities. For example, an additional Uber driver in a region provides a positive externality to riders (they will get a ride faster) but a negative externality to other drivers (they will have to wait longer in-between fares). However, a large literature suggests that even within a ‘side’ of a one or two-sided platform, users are heterogenous in the effect their actions have on the network. The empirical literature on network effects uses several techniques for their estimation, including studying exogenous shocks to the network (e.g. Tucker (2008)), using an instrumental variable approach (e.g. Aral and Nicolaides (2017)) and conducting field experiments (e.g. Aral and Walker (2012)).

There are several recent papers which model pricing in the presence of multi-dimensional network effects. For example, Bernstein and Winter (2012) determines a mechanism for optimally renting storefronts in a shopping mall where stores have heterogenous externalities on other stores. Candogan et al. (2012) and Fainmesser and Galeotti (2015) consider monopolistic pricing of a divisible network good, where

utility from the good is quadratic in the amount consumed and linear in the impact of neighbors’ consumption. In (Candogan et al., 2012), the platform firm has perfect knowledge about all individuals’ utility functions, but allows for individuals to vary in their utility from the platform good (although this utility must be quadratic). They show that the problem of determining profit maximizing prices is NP hard, but derive an algorithm guaranteeing 88% of the maximum. Fainmesser and Galeotti (2015) considers a similar model but assumes that all individuals have the same demand for the network good, while allowing for a random distribution of network connections. They find that allowing for the network to lower prices on ‘influencers’ must increase social welfare, but allowing firms to fully price discriminate might be harmful. The paper in this literature with a model most similar to ours is Weyl (2010). That paper, like ours, considers an indivisible platform good with network effects. It also, like this paper, allows for groups to vary in both their network effect on other groups and in their opportunity cost for using the platform. It finds that a wedge exists between the profit maximizing and social welfare maximizing pricing strategy.³

Our paper builds on these prior papers along several dimensions. First, our model features more realistic monetization, allowing for different types of users to face different levels of disutility from the firm increasing their level of advertising. This is in contrast to (Candogan et al., 2012) and (Fainmesser and Galeotti, 2015) which do not allow for such variation, and Weyl (2010) which features an unrealistic pricing scheme, where users are charged based on the level of participation of other users (i.e. an ‘insulating tariff’). Weyl (2010) use of insulating tariffs in pricing forces users to immediately jump to a desired equilibrium in response to a price change, which prevents a dynamic analysis of a pricing change. Second, unlike (Candogan et al., 2012) and (Fainmesser and Galeotti, 2015) our model has a realistic amount of uncertainty within a side of a model, meaning that first degree price discrimination that drives consumer surplus to zero is impossible.⁴ The most important contribution of our model is that it is the first one to allow for straightforward calibration. To the best of our knowledge, no previous paper has made quantitative model-based recommendations about multi-sided platform pricing, or quantitatively evaluated the welfare consequences of a platform regulation market structure change.

The illustration in our paper is of Facebook, a platform primarily monetized through advertising. Most platforms keep the quantity of ads (“ad load” to those in the in-

³The exact nature of this wedge – as a marginal, not an average distortion – was clarified in a published comment (Tan and Wright, 2018).

⁴The fact that platforms cannot fully first-degree price discriminate is testified to by papers that shows users benefit considerably on average from joining a platform. For example, Ceccagnoli et al. (2011) find that independent software publishers experience an increase in sales and a greater likelihood of issuing an IPO after joining a major platform ecosystem, and Brynjolfsson et al. (2019) find large consumer surplus from the use of digital platforms.

dustry) shown per user fixed while showing different ads to different users based on their characteristics and bid outcomes of ad auctions (e.g. Google (Hohnhold et al., 2015), Pandora (Huang et al., 2018a)). Platforms with a newsfeed, such as Facebook, WeChat and LinkedIn, understand the tradeoff between ad load and user engagement. Some of them show the same number of ads per person (see Huang et al. (2018b) for advertising on WeChat), while others fix the number of ads a user sees based on the expected revenue generated by the user in the long term (Yan et al. (2019) describe LinkedIn’s ad load strategy). While this optimization takes user engagement into account, network externalities generated by a user are not explicitly modeled and users generating different amounts of network externalities end up seeing the same number of ads.⁵

3 Analytic Model

The foundational element of a model of network effects is a stance on how agents connect to and gain welfare from the network. In our model, individuals with heterogeneous characteristics decide whether or not to participate in a network. Their desire to participate in the network is a function of their expectation of which other individuals will participate. For example, Jane Doe’s desire to use Instagram is a function of which of her friends are also using Instagram. The key term in the model is the externality that users gain from others. Unlike other models of platforms, we allow for individuals of different characteristics to gain different amounts of value from the participation of others on the network. These different market segments are the different sides of the platform.

We use the example of a social network, because our implementation section takes place in that setting. Therefore, in our baseline model, other incidental network characteristics mimic that of an internet social network. Once two users are using the network, there is no additional cost for them to form a connection. All connections where both users gain weakly positive value are immediately formed. We assume that the fee or subsidy faced by each participating network user is a binary function of their decision to participate on the network. This assumption is easy to modify for other contexts where fees are a function of the number or type of connections or interactions.⁶

The platform’s monetization is also modeled. Users face disutility depending on

⁵Based on informal conversation with researchers who have worked with Facebook, our understanding is that in constructing its newsfeed, Facebook gives every potential entry a score, based on the amount of engagement the entry is expected to create in the user who sees the ad, the amount of revenue that might be generated (if it is an advertisement) and a penalty for being similar to a recently displayed entry.

⁶An example of an online platform with network effects and a non-binary fee structure would be an online auction house like eBay. eBay’s main source of revenue is a progressive fee on the value of every transaction.

how intensely they are monetized by the platform. This may correspond to the unpleasantness of advertisements or the disutility of knowing one’s data will be harvested and resold. Alternatively, it may correspond to an explicit participation charge, such as WhatsApp’s original \$1 subscription cost.

This model is implementable in the sense that there is a clear strategy for measuring all the terms that appear in the model. It is scalable in the sense that these terms can be measured with as much precision and for as small a market segment as desired. As a first pass, a platform might distinguish between the network externalities and demand characteristics of broad user groups such as women and men. A more sophisticated platform with a larger research budget might estimate and incorporate into their optimization network externalities at the individual level. In the parametric calibration of the model, we will make additional assumptions about the functional form of user demand for the platform.

3.1 Consumers

A consumer i chooses whether to participate in the platform ($\mathbf{P}_i = 1$) or not ($\mathbf{P}_i = 0$).⁷ If the consumer i uses the platform ($\mathbf{P}_i = 1$), they expect to receive

$$E[U_i(\mathbf{P}_i = 1)] = \mu_i(P_1, \dots, P_I, -\phi_i) \tag{1}$$

where P_j is the probability individual j participates on the platform. ϕ_i is the revenue the platform raises from individual i . A firm which monetizes using advertising might raise \$1 in revenue by displaying additional ads which create \$.20 in additional disutility (i.e. $\frac{\partial \mu_i}{\partial \phi_i} = .2$). Local telephone calls and pre-2106 WhatsApp monetized by charging a flat fee for participation (i.e. $\frac{\partial \mu_i}{\partial \phi_i} = \1).⁸ Note that users do not directly care about what other users are charged, but it is indirectly important to them insofar as it causes other users to participate on the network.

$\frac{\partial \mu_i}{\partial P_j}$ is the marginal utility of j being on the network to i (if i participates). For convenience, we will sometimes write the marginal value of a user j to a user i as

$$U_i(j) = \frac{\partial \mu_i}{\partial P_j} \tag{2}$$

⁷Note that while demand functions are here defined at the individual level, as a practical matter firms may estimate them at the level of a demographic or social group. We consider an example with ten market segments in our calibration.

⁸In general, platforms monetize in many different ways. Some monetize by charging fees for transactions (Ebay, AirBnB, etc), some subsidize one side while charging others (Credit Cards), some by charging a flat fee for participation (Local telephone calls, pre-2106 WhatsApp), and some monetize by charging advertisers or selling advertisements (social networks). Our baseline model is best suited for evaluating the latter two approaches, but can be straightforwardly modified to handle other monetization methods.

and the marginal disutility of advertising as

$$a_i = \frac{\partial \mu_i}{\partial \phi_i} \tag{3}$$

In our non-parametric analysis, our only assumption is that μ_i be continuously differentiable. In our parametric analysis, which is designed to make inferences about equilibria far from the current one, we further assume that utility from the platform is linearly additive in the network effect from friends and disutility from ϕ . In other words, the parametric analysis assumes that $U_i(j)$ and a_i are constant.⁹

The value to a consumer of not using the platform, their ‘opportunity cost’, is an ex-ante unknown random variable.

$$U_i(\mathbf{P}_i = 0) = \epsilon_i \tag{4}$$

where ϵ_i are independent random variables (not necessarily symmetrical or mean 0). ϵ ’s distribution may vary by individual. This means that the probability of participating on a network, P , conditional on a given level of utility from the network good $U(\mathbf{P} = 1)$ is consumer specific.

The distribution of ϵ_i determines how elastic i will be to changes in the platforms’ attractiveness. Consider the case where ϵ_i is expected to be approximately equal to the utility of participation $U_i(\mathbf{P}_i = 1)$ – in other words, that it is likely that the user is ‘on the fence’ about using the platform. In this case, changes in ϕ_i or other consumers’ participation will be highly likely to change i ’s participation.

Each consumer gets to see the resolution of their private outside option ϵ_i before participating, but not the resolution of anyone else’s. Therefore, they base their decision to participate on the platform based on their beliefs in the likelihood of others participating. The *ex-post* consumer demand function is

$$\begin{cases} \mathbf{P}_i = 1 & \text{if } E[U_i(\mathbf{P}_i = 1)] > \epsilon_i \\ \mathbf{P}_i = 0 & \text{otherwise} \end{cases}$$

⁹The assumption that the value of platform connections are linearly additive is not a harmless one, despite being made in all of the most similar papers extant ((Candogan et al., 2012), (Fainmesser and Galeotti, 2015), and (Weyl, 2010) all make this assumption). It means, for example, the additional value that Jane Doe gets from James Smith joining Instagram isn’t a function of whether any third person is already on Instagram. This is a useful simplification in the context of social networks, but in the case of other networks it is likely unrealistic. Taking a food delivery platform as an example, it is likely the case that the 10th pizza delivery service joining the platform provides less platform value to the typical user than the 1st. A related simplification is the assumption that the value of a connection is only a function of the characteristics of the connected individuals. In general, the value of a connection to one individual may be a function of that individuals’ connections to other individuals. We abstract from these possibilities in the parametric model. The measurement of non-linearly additive network effects introduces large measurement challenges beyond the scope of this paper’s illustration.

Note that P_i 's are independent because ϵ_i 's are independent.

We can write the *ex-ante* demand function (i.e. expected demand before ϵ_i is known) as:

$$P_i = \text{Prob}[E[(U_i(\mathbf{P}_i = 1))] > \epsilon_i] = \Omega_i(\mu_i) \quad (5)$$

for more useful notation, define

$$U_i \equiv E[U_i(\mathbf{P}_i = 1)] = \mu_i \quad (6)$$

The network is in equilibrium when individuals' decisions to participate are optimal responses to their beliefs about every other individuals' decision to participate.

For the symmetric network (i.e. where all individuals have the same ϵ distribution, A , and network externality), where utility is linearly additive in the network effects and disutility from advertising, the equilibrium is stable so long as

$$1 > \frac{\partial \Omega}{\partial U} U(i)(I - 1) \quad (7)$$

where $U(i)$ is the value from any consumer participating in the network to any other consumer. The derivation of this equation is in appendix 40.

4 Profit Maximization

There are many questions you can ask about optimal platform strategy in this setting. Here we focus on profit maximization by a social network which can price discriminate among its users taking their demand functions (as well as the actions of competitors) as given. Platforms in this setting can price discriminate either by directly charging or subsidizing some users, or by giving some subset of users more or less advertisements.

Firms maximize expected total profits. After uncertainty is resolved, the firm's revenues are

$$\Phi = \sum_i^I \phi_i P_i - F \quad (8)$$

Where ϕ_i is the revenue collected from or distributed to consumer i if they participate in the network. It is a choice variable from the perspective of the firm. P_i is a binary indicator of whether the consumer participates. F is the fixed cost of the platform firms operation.¹⁰

¹⁰We assume the platform faces no marginal costs, but adding a marginal cost does not change the qualitative results.

P_i 's are independent random variables, so firms maximize

$$E[\Phi] = \sum_i^I \phi_i P_i - F \quad (9)$$

where

$$P_i = E[P_i] = \Omega_i(\mathbf{U}_i) \quad (10)$$

the probability of a consumer participating P_i is an individual specific function of \mathbf{U}_i . Ω_i is the effective individual specific demand function.

The firm seeks to maximize revenues

$$\max_{\phi_i} E[\Phi] = \sum_i^I \phi_i P_i \quad (11)$$

s.t.

$$P_i = \Omega_i(\mathbf{U}_i) \quad (12)$$

This yields the following first order condition

$$\frac{\partial \Phi}{\partial \phi_i} = P_i + \phi_i \frac{\partial P_i}{\partial \phi_i} + \sum_{j \neq i}^J \phi_j \frac{\partial P_j}{\partial \phi_i} \quad (13)$$

where

$$\frac{\partial P_i}{\partial \phi_i} = \frac{\partial \Omega_i}{\partial \mathbf{U}_i} \left(-\frac{\partial \mu_i}{\partial \phi_i} + \sum_j^J \left(\frac{\partial \mu_i}{\partial P_j} \frac{\partial P_j}{\partial \phi_i} \right) \right) \quad (14)$$

and,

$$\frac{\partial P_j}{\partial \phi_i} = \frac{\partial \Omega_j}{\partial \mathbf{U}_j} \left(\frac{\partial \mu_j}{\partial P_i} \frac{\partial P_i}{\partial \phi_i} + \sum_{k \neq i}^K \frac{\partial \mu_j}{\partial P_k} \frac{\partial P_k}{\partial \phi_i} \right) \quad (15)$$

This recursion is natural as P_i is a function of P_j , which is a function of P_i , etc. Equation (15) will converge to a finite value so long as each recursion of the network effect dampens out (which will occur so long as the equilibrium is stable).

4.1 Profit Maximization Problem Simplified

Equation 13 gives conditions for the optimal schedule of fees (or other revenue raising monetization strategies) and subsidies for the general case. Even if not enough is known about the entire curve of functions to find the optimum, knowing the first derivative of the goal with respect to the choice parameters is useful. *An experimenting firm can simply use these equations to inch towards a local maximum via gradient decent.*

To simplify the recursion of network effects, we retain only first term in brackets in

14 and 15. In other words, the following equations only take into account one cascade of network effects.¹¹ For clarity and parsimony, we also make the substitutions from equations 2 and 3

$$\frac{\partial P_i}{\partial \phi_i} = \frac{\partial \Omega_i}{\partial U_i} \left(-a_i + \sum_{j \neq i}^J \left(U_j(j) \frac{\partial P_j}{\partial \phi_i} \right) \right) \quad (16)$$

and,

$$\frac{\partial P_j}{\partial \phi_i} = \frac{\partial \Omega_j}{\partial U_j} \left(U_j(i) \frac{\partial P_i}{\partial \phi_i} + \sum_{k \neq i}^K U_j(k) \frac{\partial P_k}{\partial \phi_i} \right) \quad (17)$$

Then, substituting into 13, yields a new simplified first order condition

$$\frac{\partial \Phi}{\partial \phi_i} = \underbrace{P_i - \phi_i a_i \frac{\partial \Omega_i}{\partial U_i}}_{\text{Direct Effect}} - \underbrace{a_i \frac{\partial \Omega_i}{\partial U_i} \sum_{j \neq i}^J \phi_j \frac{\partial \Omega_j}{\partial U_j} U_j(i)}_{\text{Network Effect}} \quad (18)$$

The simplified first order condition consists of two sets of terms. The first two terms report the direct effect of raising the amount of advertising on individual i by one dollar. This will raise revenue, based on that individual's current likelihood of participation, and lose revenue based on how elastic that individual's participation is. The two direct effect terms are what normal firms have to consider when pricing their products (note that when $U_j(i) = 0 \forall i, j$, i.e. when no network effects are present, 18 reduces to this pair of terms).

The last term in equation 18 is the network effect of an advertising increase. The increase in advertising makes i less likely to participate (in this approximation, by an amount $a_i \frac{\partial \Omega_i}{\partial U_i}$) which leads others to stop participating (by an amount $\frac{\partial \Omega_j}{\partial U_j} U_j(i)$). When these third parties stop participating, the platform loses on the current revenues that they were paying ϕ_j .

In other words, the fee or level of disutilitous advertising should be increased on user i if the increased revenue (P_i) is greater than the decreased revenue from the person directly impacted possibly dropping out (second term) plus the decreased revenue from all the charged person's friends potentially dropping out (third term).

This simplified first order condition can be made more precise by taking into account additional cascades of the network effect. In other words, because user i 's fee increasing causes j to be less likely to participate, all those connected to j should be less likely to participate as well.

¹¹In the parametric section we will show that the first cascade of network effects is quantitatively much more important than subsequent cascades for a reasonable parameterization.

5 Empirical Illustration

The setting for our empirical illustration is Facebook. Facebook is an ad-supported social network. It was selected because it is used by a very large percentage of the US population, and previous research has demonstrated that many value it highly.

To illustrate how our method can be used by firms to price discriminate, we collected survey data to estimate our model. We conducted approximately 40,000 surveys on a representative sample of US internet population. Google Surveys provides information on a survey participants' gender and age group, so we distinguish market segments based on those characteristics. We divided Facebook users into ten market segments. These are a pair of genders and five age brackets. The list of surveys conducted is summarized in figure 1. Figure 2 gives examples of how the surveys appeared to respondents. Respondents answered these surveys either as part of Google Rewards or to access premium content on websites.

5.1 Modifying the Model for Facebook Market Segments

First, however, we must modify the model to allow for demographic groups of different size, and to allow for heterogenous rates of friendship between these groups. Let $P_{i,d}$ be the probability individual d on side i participates on the network. Then,

$$E[U_{i,d}(\mathbf{P}_{i,d} = 1)] = \mu_i(P_{1,1}, \dots, P_{1,D}, \dots, P_{I,1}, \dots, P_{I,D}, -\phi_i) \quad (19)$$

Assuming that individuals within a side i are identical except for their draw of an outside option, we can sum over the number of individuals on a side. So,

$$E[U_{i,d}(\mathbf{P}_{i,d} = 1)] = \mu_i(P_1 D_1, \dots, P_I D_I, -\phi_i) \quad (20)$$

where D_i is the potential number of platform users on side i .

Finally, we make a modification to match the nature of network effects on Facebook and how our network data is collected. We solicit the value of Facebook friends on the network. Individuals who are not friends on Facebook have minimal direct network effects on each other on average. Therefore, we assume that network value is only provided by the share of people within a demographic which are friends. Letting z_{ij} be the fraction of people of type j on the network who i is friends with yields

$$E[U_{i,d}(\mathbf{P}_{i,d} = 1)] = \mu_i(P_1 z_i(1) D_1, \dots, P_I z_i(I) D_I, -\phi_i) \quad (21)$$

For firm revenues, we also must adjust the expected revenue equation to account for the different amounts of people in each group.

$$E[\Phi] = \sum_i^I \phi_i P_i D_i \quad (22)$$

5.2 Non-Parametric Results

In our non-parametric analysis we are interested in the question of how a small change in a level of advertising ϕ_i would change Facebook revenues. In other words, we wish to estimate $\frac{\partial \Phi}{\partial \phi_i}$. Therefore, we re-derive equation 18 with the modifications described above. This produces equation 23

$$\frac{\partial \Phi}{\partial \phi_i} = P_i - a_i \phi_i \frac{\partial \Omega}{\partial U_i} - a_i \frac{\partial \Omega_i}{\partial U_i} \sum_{j \neq i}^J \phi_j \frac{\partial \Omega_j}{\partial U_j} U_j(i) z_j(i) D_j \quad (23)$$

the two additional terms are displayed in red and blue. $z_j(i)$ is the share of people of type i on platform who j 's are friends with. D_j is the number of potential users of type j . These two terms represent how many valuable friendships we would expect to exist between individuals in demographic group j to an individual in group i if every possible participant used the platform. Note that equation 23 gives the expected change in total monthly profit when increasing ϕ on a single individual in group i . To estimate the consequences of raising the fee on all individuals in group i , this estimate should be multiplied by the total population of i .¹²

To evaluate equation 23 we need information on the elasticity of FB use ($\frac{\partial \Omega_i}{\partial U_i}$), number of friends, the value of friends ($U_j(i)$), and the disutility of advertising by market segment (a_i). To collect this information we conducted additional surveys using Google Surveys.

Figures 3 report the fraction of users, by gender, who reject an offer of $\$x$ to give up Facebook for a month. These distributions can be thought of as a demand curve, as it reports the fraction of users by their consumer surplus from Facebook. The figures

¹²Note also that we can also evaluate additional iterations of network effects in this setting. This would be evaluated using the following equation

$$\frac{\partial \Phi}{\partial \phi_i} = P_i - \phi_i \frac{\partial P_i}{\partial \phi_i} - \sum_{j \neq i}^J \phi_j \frac{\partial P_j}{\partial \phi_i} \quad (24)$$

where

$$\frac{\partial P_j}{\partial \phi_i} = \frac{\partial \Omega_j}{\partial U_j} (-a_i \frac{\partial \Omega_i}{\partial U_i}) (U_j(i)) + \sum_{k \neq j}^K U_j(k) U_k(i) z_k(i) D_k \quad (25)$$

and

$$\frac{\partial P_i}{\partial \phi_i} = -a_i \frac{\partial \Omega_i}{\partial U_i} + \sum_{j \neq i}^J (U_i(j)) \frac{\partial P_j}{\partial \phi_i} z_j(i) D_j \quad (26)$$

reveal that there is significant heterogeneity between groups in the value they get from Facebook.

We find men aged 65+ are the least likely to use Facebook (41.4% don't use it) and females aged 55-64 are the most likely to use (22.8%). Males aged 55-64 are the most elastic users of Facebook (12.8% of users value it at less than 5 dollars a month) and males aged 25-34 are the least elastic Facebook users (only .020% of users value it at less than 5\$ a month).

We use the information from these demand curves to evaluate equation 23 in two ways. First, the share of users who value Facebook at 0-\$5 is used to estimate how elastic that group's use of Facebook is. A group with many users who just barely value Facebook more than 0 will be more likely to leave the platform after an increase in advertising or a decrease in positive network effects. Specifically, we estimate $\frac{\partial \Omega_i}{\partial U_i}$ as equal to the share of i who report a Facebook value of \$0 – \$5, less the share of the population in that group who do not use Facebook (estimated from the share responding "I don't use Facebook" in one of the other surveys) divided by 5. The division by 5 comes from assuming that the distribution of valuations is locally uniform.

Second, we use the total average valuation of Facebook by the group to rescale the total value that users get from each individual friend group on Facebook. In other words, we rescale the total value that users get from each group of friends so that they add to the average total Facebook valuation estimated with this question. The average surplus a group member gets from Facebook is calculated as the average consumer surplus from Facebook (calculated as the area under the linear curves in figure 3 divided by the share of the group which uses Facebook).

Figure 4 reports the average number of friends each user has of different types. To generate these estimates, we first took the average of the friend shares by demographic. We rescaled these so that the percentages added to 100%. We then multiplied these shares by the average total number of friends by demographic.

Using the average number of friends a person of a demographic has, as well as the average responses to their WTA an offer to block all friends of a given demographic, we can calculate the average marginal network effect of friends across demographic groups. Figure 5 reports these estimated average network effect values.

Figure 6 gives the total number of valuable friends someone on of a given market segment would have of another if everyone in the US used Facebook. To calculate this figure, we begin by assuming that individuals only have friends in the United States. We then divide the numbers from figure 4 by the share of people in a demographic in the US who currently use Facebook.

There are two terms in equation 23 that we still need sources for. ϕ , the current amount of revenue raised, is assumed to be uniform for all groups. It is taken from

the Facebook quarterly earnings report, which gives Facebook’s revenue per North American active user. Dividing to yield a monthly value gives $\phi = \$11.62$. a_i is calculated as the average reported disutility from advertisements (collected as one of the survey questions) divided by the previous number.

Table 1 gives the final results of estimates of (23). It shows how the significant heterogeneity between group utility functions leads to significant heterogeneity in how their level of advertising should change. The final results, given in the third column, are not primarily driven by the direct effect of an advertising increase on a particular user’s participation. Rather, taking the value an individual provides to others into account is critical.

Taking the results at face value, before considering network effects, Facebook should increase advertisements for all, especially females aged 35-44. They participate in Facebook at a high rate, and do so inelastically. However, not taking into account network effects would be folly, as two groups with an estimated similar direct response to an advertising increase – men aged 25-34 and women aged 55-64– have very different network values to other users.

Taking a single cascade of network effects and the disutility from advertising into account, Facebook should *decrease* advertisements on almost all groups. However, it should increase advertisements on females age 35-44 and males age 25-34.

5.3 Building the Parametric Model

In the above analysis, we estimate the impact on Facebook revenues of a small change in the level of advertising on a given group. However, in addition to understanding the impact of small changes, we are interested in the total effect of large changes in demand for Facebook or Facebook pricing. One important reason this is interesting is for the purpose of regulation. Some argue that monopolistic platforms, by taking into account network effects from users, are more likely to make social welfare optimizing pricing decisions than competitive firms. Others argue that monopoly power allows platforms to charge prices far above marginal cost, depressing social welfare.¹³

To perform this analysis we need estimates of

1. The market segment specific demand for Facebook function Ω_i and distribution of opportunity costs ϵ_i
2. The current revenue per user of a market segment collected by Facebook $\overline{\phi}_i$

¹³For example, Weyl (2010) derives the wedge between social optimal pricing and profit maximizing pricing in both the perfect competition and monopolistic case for a platform good with network effects. However, as explained in a published comment (Tan and Wright, 2018), the derived wedge is a marginal, not average one, meaning that it gives little guidance in general.

3. The market segment specific utility from Facebook as a function of the number of other potential users, their participation rates, and the user’s level of advertising μ_i
4. The total population of a market segment and the rate at which Facebook friendships are formed between individuals of a market segment D_i and $z_i(j)$

To estimate Ω_i we assume that it follows a logistic distribution. We estimate the parameters of Ω_i by running a logistic regression on responses to the question “Would you refuse \$X to stop using Facebook for a month?”. Figures 7 through 16 report responses to these questions, and the logistic line of best fit.¹⁴

Table 2 reports the estimates underlying these curves. We convert from estimates of the CDF logistic equation to the PDF of the distribution of ϵ_i ’s using the equations

$$\epsilon_i \sim \frac{e^{-\frac{x-\eta_i}{s_i}}}{s_i \left(1 + e^{-\frac{x-\eta_i}{s_i}}\right)^2} \quad (27)$$

where

$$s_i = (\text{Coef. on Cost}_i)^{-1} \quad (28)$$

and

$$\eta_i = (-\text{Intercept}_i)s_i \quad (29)$$

In the non-parametric analysis, we assumed that Facebook raised \$11.62 dollars a month in revenue from US users through displaying them advertisements.¹⁵ However, it is likely that they eyeballs of some US Facebook users are more valuable to Facebook than others. To calculate initial revenue per user $\bar{\phi}_i$ we take in data on the cost of advertising to users of different types from Facebook’s advertisement API. After selecting which demographic group you would like to target, Facebook tells you how many impressions you are estimated to receive per dollar of spending. We take the inverse of this measure to be the relative value of a demographic to Facebook’s ad revenue. By taking as given that the average value of a user per month is \$11.62, we can then calculate the revenue per user of a demographic using the following equations

¹⁴Although we also collected information on the fraction of users who would refuse \$1,000 to continue using Facebook, we drop these responses to surveys in the parametric analysis, as an unrealistically high share of individuals report high valuations on this question. In future work, we will include a parameterization using all of these questions’ data in a robustness analysis.

¹⁵This is derived from Facebook’s 2019 Q1 annual report, where they report \$ 34.86 in revenues per North American user per quarter.

$$\bar{\phi}_i = z \text{Relative Value}_i \quad (30)$$

and

$$11.62 = q \frac{\sum^I \text{Relative Value}_i \bar{P}_i D_i}{\sum^I \bar{P}_i D_i} \quad (31)$$

where q is a scaling term, \bar{P}_i is the estimate of the initial participation rate on Facebook by the demographic group (taken as $\Omega_i(0)$), and D_i is the total population of the group in the US.

We assume that μ_i , the utility a member of demographic group i gets from participating on the platform, takes the functional form

$$\mu_i = \sum^J U_i(j) P_j z_i(j) D_j - a_i \phi_i + C_i \quad (32)$$

We estimate $U_i(j)$ using the same survey questions and approach as in the non-parametric analysis, except that we do not rescale answers so that total average Facebook valuations are equal to the sum of the value from network effects. We estimate a_i , the linear disutility from a dollar raised in revenue from advertisements, as

$$a_i = \frac{A_i}{\bar{\phi}_i} \quad (33)$$

where A_i is the average disutility from current Facebook as reported in our surveys. C_i , the value or disutility from using Facebook if one had no Facebook connections is calculated so that μ_i takes the value 0 in the initial period.¹⁶ Finally, D_i and $z_i(j)$ are calculated exactly as in the non-parametric case.

We calculate the impact of a change in advertising strategy over the course of multiple cascades. We denote the period when platform changes its advertising level as $t = 1$. The participation rate on the platform for a demographic group after cascade t is

$$P_{i,t} = \Omega_i \left(\sum^J U_i(j) z_i(j) D_j P_{j,t-1} - a_i \phi_{i,t} + C_i \right) \quad (34)$$

where $P_{i,0} = \bar{P}_i$, the initial rate of platform participation for the market segment.

We calculate the perceived welfare to a user of demographic i from the existence of Facebook after cascade t as

$$\int_0^{P_{i,t}} ((\mu_i(\vec{P}_{j,t-1}, \phi_i) - e_i(\rho_i))) d\rho_i \quad (35)$$

¹⁶Of course, users of Facebook in the initial period will all have positive values from the decision to use Facebook, on net, because ϵ_i – the opportunity cost of using Facebook – takes on a negative value for them.

where e_i is the inverse of Ω_i , giving the implied opportunity cost of Facebook use for every percentile of the population, i.e.

$$e_i = -s_i \log\left(\frac{1-p_i}{p_i}\right) + \mu_i \quad (36)$$

the total welfare to a demographic group from the existence of Facebook is the above amount times the number of users of that demographic group.

The revenue to Facebook from user participation of a given demographic after t cascades is

$$\Phi_{i,t} = \phi_{i,t} D_i P_{i,t} \quad (37)$$

Assuming no cost to providing the digital service, total social welfare is the sum of private welfare and total revenues.

5.4 Parametric Results

We now can calculate the implications of different pricing strategies for total Facebook participation, revenue, and social welfare. While a wide variety of scenarios are possible, we focus on Facebook setting its level of advertising revenue per user equal to its marginal cost per user, assuming the marginal cost per user is zero. In other words, we consider the implications of Facebook eliminating advertisements. This could occur as a result of government regulations preventing Facebook from displaying advertisements, or if a decentralized system of Facebook alternatives – each of which allowing users on their platform to communicate to users on other platforms – were to emerge, with low fixed cost of entry.

Figures 17 through 21 display participation rates, change in welfare per user, change in Facebook revenues, and change in total social welfare initially and after N cascades of the network effect after advertisements on Facebook are eliminated.

We predict large and positive changes in long-run social welfare after advertisements on Facebook are eliminated. This is due to increased network surplus due to greatly increased participation on the Facebook network. After only one cascade of the network effect, that is, only taking into account the direct effect of eliminating advertisements, total social welfare actually decreases. This is because people’s disutility from advertisements are only about one fifth of the revenue raised by advertisements.

However, as a result of this decrease in advertisements, participation increases dramatically over several cascades. After 10 cascades of the network effect, participation increases by over 20 percentage points for some demographic groups. After 1000 cascades, participation rates are over 72 percent for all demographic groups. Increased participation dramatically increases welfare for those already on Facebook, by over 400 dollars a month for most demographic groups. In sum, our results suggest that

eliminating advertisements from Facebook would boost US social welfare by over 94 Billion dollars a month once all network effect cascades have kicked in.

6 Conclusion and Managerial Implications

Building on Parker and Van Alstyne (2005) and Weyl (2010) we construct and illustrate an approach for firms to incorporate network effects in their monetization strategies. The specific example we emphasize is a firm which can discriminate in its advertising to profit maximize. Taking the first order condition for profit maximization with respect to the advertising schedule yields a recursive equation that can be evaluated to the desired degree of precision. The managerial insight is that platform owners should increase advertising on market segments which inelastically demand the platform (the direct effect), don't have much disutility from advertisements, and don't create much network value for others. Platforms should decrease advertisements on those who elastically demand the platform and create high amounts of network value for other profitable users who demand Facebook elastically (the first recursion network effect).

We use this model to estimate the revenue and welfare consequences of different pricing strategies and market structures in a parametric and non-parametric manner. As far as we know this is the first paper to produce such quantitative yet theory informed predictions.

That being said, our approach is not without weaknesses. The main issue is trickiness in soliciting the necessary data to estimate the model. Consumers may not fully understand or reliably answer questions about their valuations for different friend groups. Poor memory may also be an obstacle. There may also be important differences between short and long-term elasticities of demand. Similarly, if individuals have very high variance or skewness in their platform valuations, network effects, or number of friends, the average of these values within a group may be a poor summary statistic – especially if these measures are correlated within a side of the market/demographic group. However, with a larger budget, incentive compatible experiments, smaller market segments or within-platform proprietary data, each of these concerns could be addressed. Finally, our model conceives of consumers as atomistic price takers. This ignores the possibility that highly valuable users with market power might bargain with the platform or that users might unionize to demand a better equilibrium. However, the implications of such a scenario could be estimated in an extension of the model. In any case an intriguing area for future investigation is to actually conduct experiments on platforms to see how well real world phenomena match our predictions.

In future work, we plan to generalize the model and use it to derive other implications. One possible use is for asset pricers and option traders. We can estimate the

implications to the platforms of demand shocks, such as “how would total Facebook revenues change if everyone under the age of 35 stopped using it”. Another important implication is to competition policy. Some believe that large internet platforms are monopolies that should be broken up, or otherwise restricted in their ability to price discriminate. Using our approach, practitioners can evaluate whether more consumer surplus is created by allowing or banning certain pricing strategies, or by reducing market concentration, for a particular type of social network service.

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7 Tables and Figures

<u>Question Text</u>	<u>Question Variations</u>	<u>Possible Responses</u>	<u>Number of Responses</u>
How many friends do you have on *Facebook*?	2 different response intervals	Survey 1: 0-100; 100-200; 200-300; 300-400; 400-500; More than 500; I do not use Facebook Survey 2: 0-50; 50-100; 100-500; 500-700; 700-900; More than 900; I do not use Facebook	2509
Would give up Facebook for *1 month* in exchange for \$[X]? Choose Yes if you do not use Facebook.	X = 5, 10, 20, 50, 100, 200, 500, 1000	Yes, I will give up Facebook; No, I would need more money	10033
"On Facebook, would you unfriend all your friends who are *[demo_group]* for 1 month in exchange for \$[X]? Choose Yes if you do not use Facebook."	demo_group = men between age 18 and 24; men between age 25 and 34; men between age 35 and 44; men between age 45 and 54; men between age 55 and 64; men aged 65 or over; women between age 18 and 24; women between age 25 and 34; women between age 35 and 44; women between age 45 and 54; women between age 55 and 64; women aged 65 or over X = 5, 10, 20, 50, 100, 200, 500, 1000	Yes, I will unfriend all these friends; No, I would need more money	13272
What *percentage* of your *friends on Facebook* are *[demo_group]*?	demo_group = under age 18; men between age 18 and 24; men between age 25 and 34; men between age 35 and 44; men between age 45 and 54; men between age 55 and 64; men aged 65 or over; women between age 18 and 24; women between age 25 and 34; women between age 35 and 44; women between age 45 and 54; women between age 55 and 64; women aged 65 or over	0-10%; 10%-20%; 20%-40%; 40%-60%; 60%-80%; 80%-100%; I do not use Facebook	13037
What is the *maximum* amount of money (in US \$) you would pay to personally *not see any advertisements on Facebook* for *1 month*? Select 0 if you do not use Facebook.	NA	0; \$1-\$5; \$5-\$10; \$10-\$15; \$15-\$20; \$20+	1001

Figure 1: List of surveys conducted

Please complete the following survey to access this premium content.

How many friends do you have on **Facebook**?

- 0-100
- 100-200
- 200-300
- 300-400
- 400-500
- More than 500
- I do not use Facebook

OR

[↻ Show me a different question](#)

[↶ Skip survey](#)

Google [INFO](#) [PRIVACY](#)

Figure 2: Google survey interface example. Note that each respondent only receives a single survey question, and that responses are limited to seven multiple choice options.

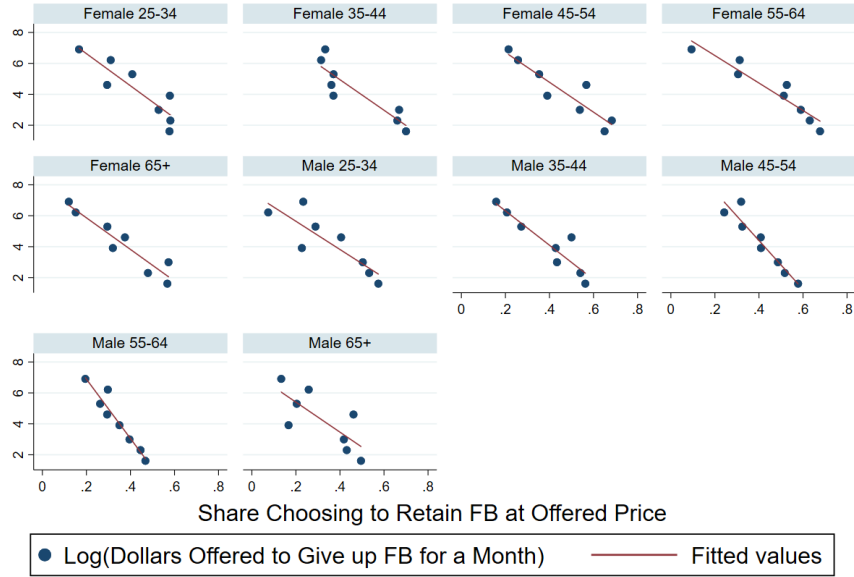


Figure 3: Fraction of users, by age and gender, who would reject an offered sum to retain access to Facebook for a month. This is $U_i - \epsilon_i$ in the model. $\frac{\partial \Omega_i}{\partial U_i}$, that is, how elastic demand is for a group i , is calculated as the share of individuals who use FB, but would stop using it for an offer of \$5 (our lowest offered valuation), divided by 5.

Gender	Age	Direct Effect	$\frac{\partial \Phi}{\partial \phi_i}$	$\frac{\partial \Phi}{\partial \phi_i}$
		$a = 1$	$a = 1$	$a = a_i$
Female	25-34	\$0.41	\$-3.27	\$-0.38
Female	35-44	\$0.62	\$-0.63	\$0.26
Female	45-54	\$0.50	\$-1.97	\$-0.05
Female	55-64	\$0.55	\$-1.43	\$-0.55
Female	65+	\$0.35	\$-3.28	\$-0.79
Male	25-34	\$0.55	\$-0.10	\$0.18
Male	35-44	\$0.51	\$-0.72	\$-0.15
Male	45-54	\$0.47	\$-1.46	\$-0.45
Male	55-64	\$0.30	\$-3.13	\$-0.17
Male	65+	\$0.38	\$-1.96	\$-0.16

Table 1: Non-Parametric Results: The estimated monthly profit consequences of increasing advertisements or fees on a random individual of type i using equation 23. Column one reports only the direct effect of the fee (i.e. the first two terms of 23) taking $a=1$ (i.e. raising a dollar of revenue creates 1 dollar of direct disutility). Column 2 reports the full estimate of equation 23, taking $a=1$. Column 3 also reports the full estimate, but with a taking a different value of a (the disutility caused by an additional dollar in revenue from advertising) for each demographic group (ranging from .14 to .66). The number should be multiplied by the US population in group i to get the estimate on total revenues of changing the advertising level for all individuals in that demographic.

GENDER	AGE	Female	Female	Female	Female	Female	Male	Male	Male	Male	Male
		25-34	35-44	45-54	55-64	65+	25-34	35-44	45-54	55-64	65+
Female	25-34	71.0	47.2	43.2	24.0	21.5	64.2	34.9	18.3	19.7	15.3
Female	35-44	50.8	52.6	41.1	25.0	19.2	25.0	31.3	18.6	15.1	10.6
Female	45-54	42.3	38.7	34.0	29.1	14.6	21.7	23.6	25.4	23.7	10.7
Female	55-64	22.8	29.3	34.4	37.2	21.5	12.2	17.4	20.6	14.9	11.9
Female	65+	10.5	18.9	20.8	20.1	16.7	8.3	13.0	11.1	16.6	11.0
Male	25-34	46.1	41.0	12.6	25.9	23.8	51.6	31.0	31.0	21.4	14.7
Male	35-44	46.3	46.2	33.3	39.1	9.6	37.4	39.0	31.4	22.9	19.3
Male	45-54	20.7	32.0	30.3	20.9	16.5	22.1	27.4	22.1	20.6	13.4
Male	55-64	21.3	20.4	27.1	22.8	19.9	18.5	20.0	27.2	19.5	17.2
Male	65+	12.7	14.1	17.8	20.6	12.3	10.4	7.0	20.2	17.7	14.1

Figure 4: Average number of friends someone in Y-axis market segment has of the type in the X-axis market segment.

GENDER	AGE	Female	Female	Female	Female	Female	Male	Male	Male	Male	Male
		25-34	35-44	45-54	55-64	65+	25-34	35-44	45-54	55-64	65+
Female	25-34	1.90	0.64	0.81	0.95	1.73	0.59	1.04	1.77	2.10	1.62
Female	35-44	2.48	0.95	0.99	1.20	1.44	1.87	1.35	2.11	2.56	3.90
Female	45-54	2.91	0.98	1.16	1.23	1.33	1.62	1.49	1.51	1.42	3.25
Female	55-64	1.66	1.60	1.16	0.97	0.96	2.74	1.60	1.75	2.48	2.91
Female	65+	1.43	2.98	1.68	1.47	1.66	3.37	1.92	2.58	1.56	2.83
Male	25-34	1.61	1.00	0.99	3.39	1.64	0.62	1.11	0.87	1.67	2.50
Male	35-44	3.67	0.91	0.93	1.29	1.10	1.02	0.99	1.13	1.86	2.22
Male	45-54	2.68	2.29	1.46	1.49	2.21	1.95	1.60	1.99	2.27	2.95
Male	55-64	2.17	1.96	1.90	1.60	1.69	2.32	1.73	1.42	1.97	2.33
Male	65+	3.41	2.95	2.76	1.77	1.32	3.53	5.25	1.74	2.26	2.48

Figure 5: Average value, in dollars, per month, of a friend of X-axis demographic to someone of Y-axis demographic. This is $U_i(j)$ in the model.

GENDER	AGE	Female	Female	Female	Female	Female	Male	Male	Male	Male	Male
		25-34	35-44	45-54	55-64	65+	25-34	35-44	45-54	55-64	65+
Female	25-34	100.7	62.2	56.3	31.1	29.2	108.1	58.0	27.7	32.9	26.1
Female	35-44	72.1	69.4	53.7	32.3	26.2	42.0	52.0	28.1	25.3	18.1
Female	45-54	60.0	51.0	44.4	37.7	19.8	36.5	39.1	38.6	39.7	18.2
Female	55-64	32.4	38.6	44.9	48.1	29.3	20.5	29.0	31.2	25.0	20.3
Female	65+	14.9	25.0	27.1	26.0	22.7	14.0	21.6	16.8	27.7	18.8
Male	25-34	65.4	54.1	16.4	33.6	32.4	86.8	51.4	47.0	35.8	25.1
Male	35-44	65.7	61.0	43.4	50.6	13.0	62.9	64.8	47.6	38.4	33.0
Male	45-54	29.3	42.2	39.6	27.1	22.5	37.2	45.5	33.6	34.6	22.8
Male	55-64	30.2	26.9	35.4	29.5	27.1	31.2	33.3	41.2	32.7	29.4
Male	65+	18.0	18.6	23.2	26.7	16.8	17.5	11.7	30.7	29.6	24.1

Figure 6: Total number of valuable friends someone on Y-axis would have of type on X-axis if everyone in the US used Facebook.

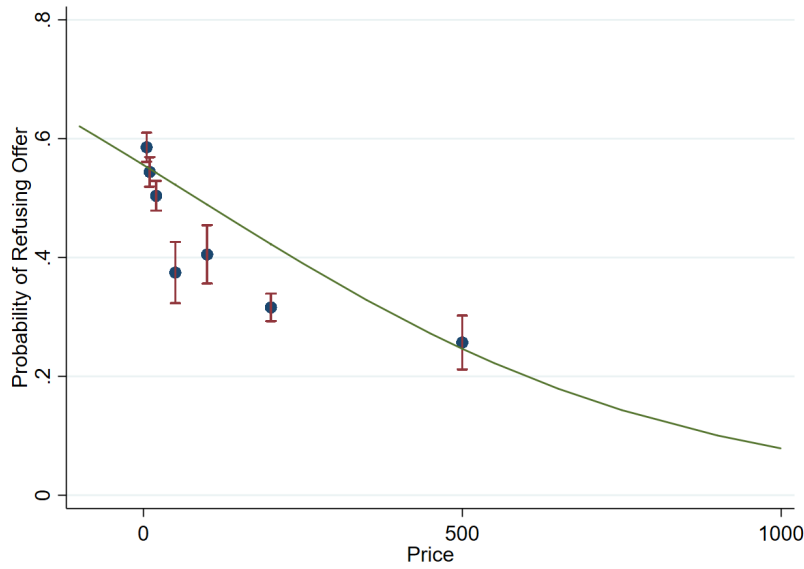


Figure 7: Underlying data and estimate of the the demand curve (Ω_i) for women age 25-34. The points are the mean response to the question “Would you give up Facebook for 1 month in exchange for \$X? Choose Yes if you do not use Facebook.” for individuals of the group. Confidence intervals are based on binomial statistics. The curve, in green, is the logistic line of best fit.

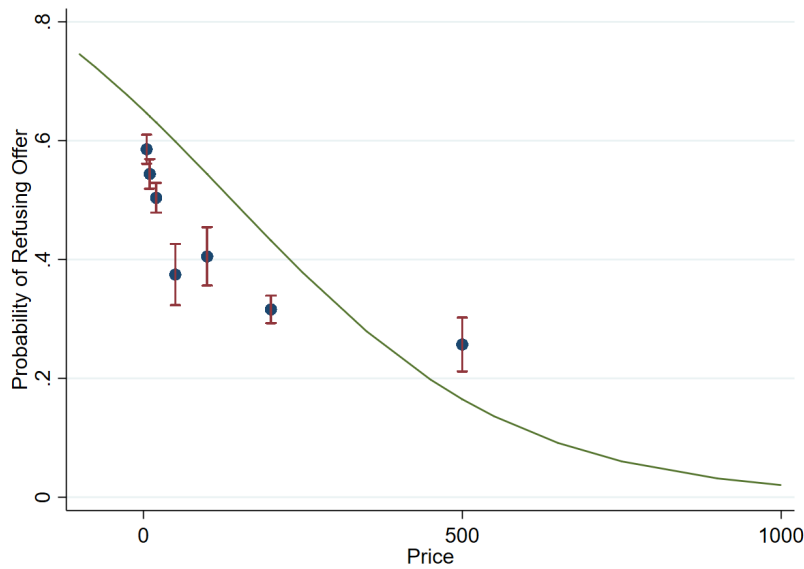


Figure 8: Underlying data and estimate of the the demand curve (Ω_i) for women age 35-44. The points are the mean response to the question “Would you give up Facebook for 1 month in exchange for \$X? Choose Yes if you do not use Facebook.” for individuals of the group. Confidence intervals are based on binomial statistics. The curve, in green, is the logistic line of best fit.

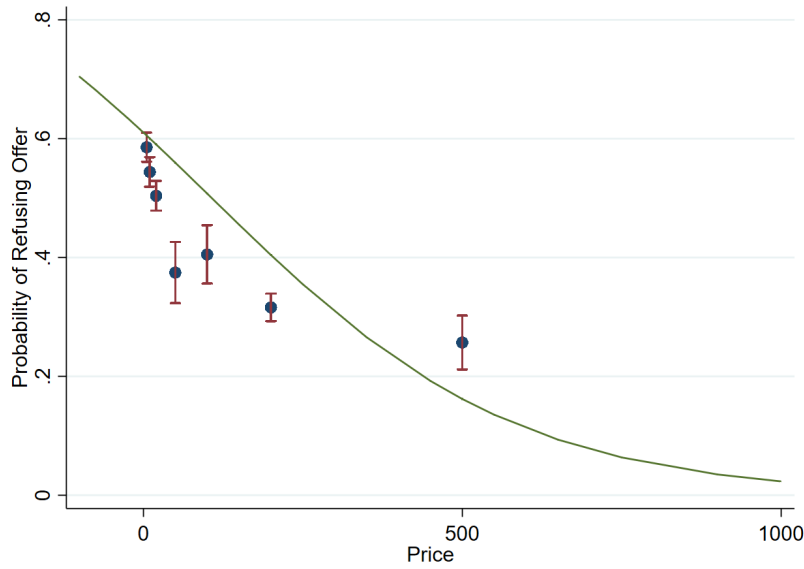


Figure 9: Underlying data and estimate of the the demand curve (Ω_i) for women age 45-54. The points are the mean response to the question “Would you give up Facebook for 1 month in exchange for \$X? Choose Yes if you do not use Facebook.” for individuals of the group. Confidence intervals are based on binomial statistics. The curve, in green, is the logistic line of best fit.

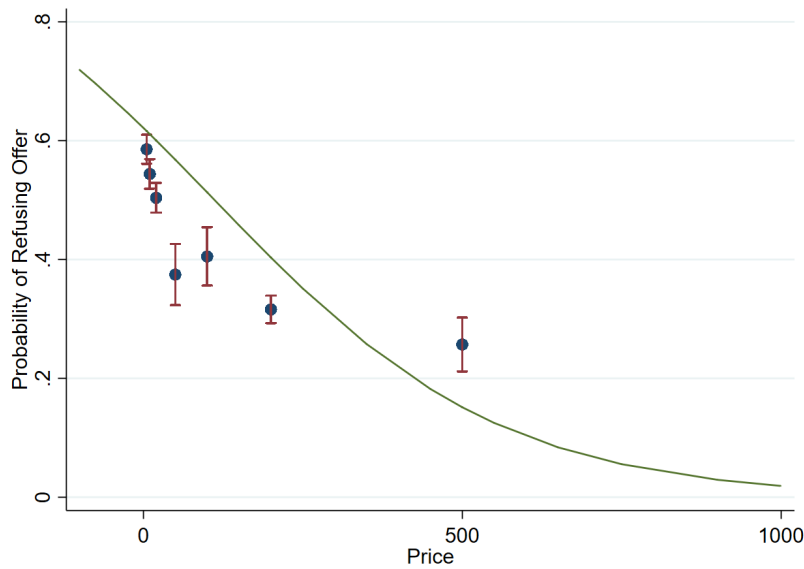


Figure 10: Underlying data and estimate of the the demand curve (Ω_i) for women age 55-64. The points are the mean response to the question “Would you give up Facebook for 1 month in exchange for \$X? Choose Yes if you do not use Facebook.” for individuals of the group. Confidence intervals are based on binomial statistics. The curve, in green, is the logistic line of best fit.

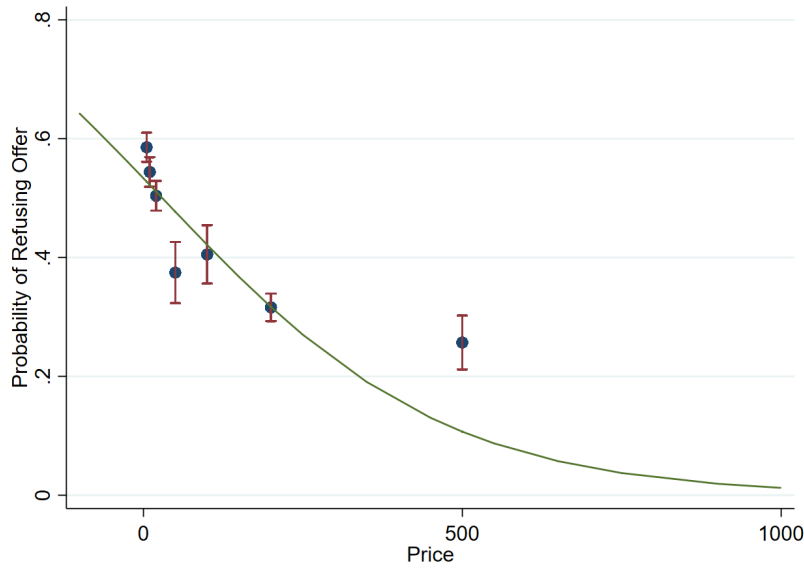


Figure 11: Underlying data and estimate of the the demand curve (Ω_i) for women age 65 or older. The points are the mean response to the question “Would you give up Facebook for 1 month in exchange for \$X? Choose Yes if you do not use Facebook.” for individuals of the group. Confidence intervals are based on binomial statistics. The curve, in green, is the logistic line of best fit.

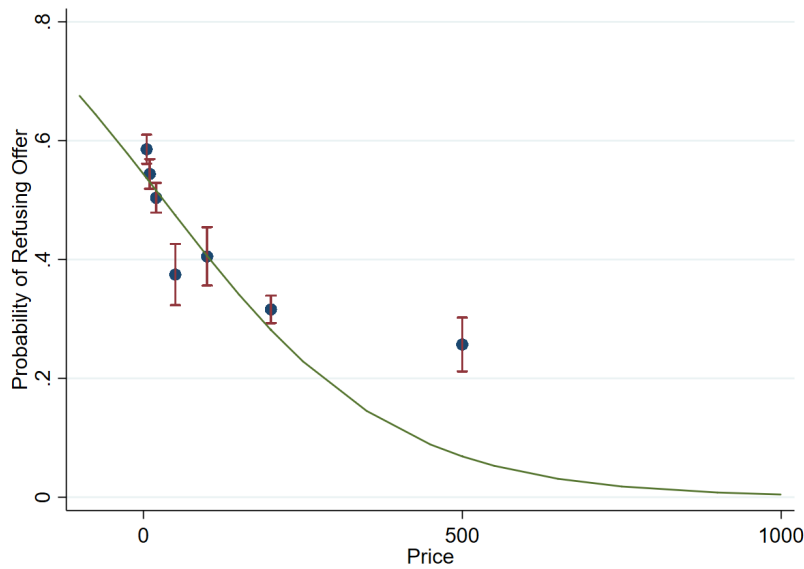


Figure 12: Underlying data and estimate of the the demand curve (Ω_i) for men age 25-34. The points are the mean response to the question “Would you give up Facebook for 1 month in exchange for \$X? Choose Yes if you do not use Facebook.” for individuals of the group. Confidence intervals are based on binomial statistics. The curve, in green, is the logistic line of best fit.

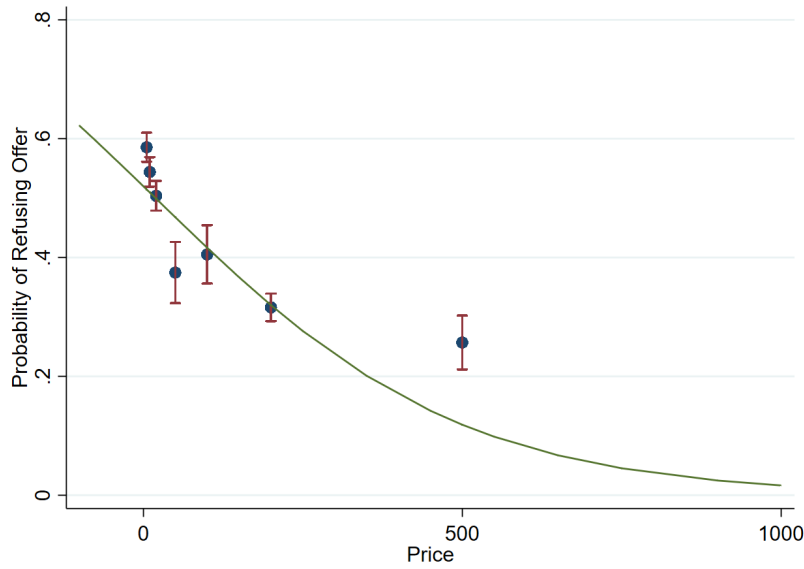


Figure 13: Underlying data and estimate of the the demand curve (Ω_i) for men age 35-44. The points are the mean response to the question “Would you give up Facebook for 1 month in exchange for \$X? Choose Yes if you do not use Facebook.” for individuals of the group. Confidence intervals are based on binomial statistics. The curve, in green, is the logistic line of best fit.

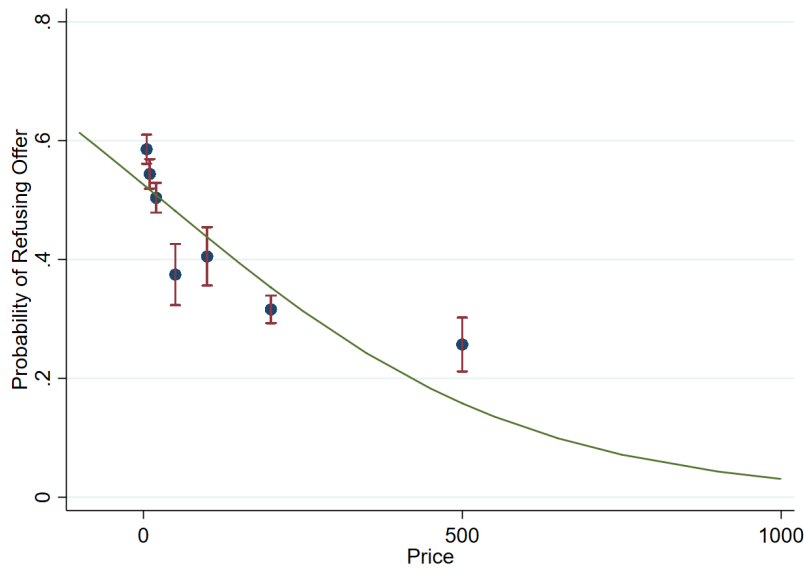


Figure 14: Underlying data and estimate of the the demand curve (Ω_i) for men age 45-54. The points are the mean response to the question “Would you give up Facebook for 1 month in exchange for \$X? Choose Yes if you do not use Facebook.” for individuals of the group. Confidence intervals are based on binomial statistics. The curve, in green, is the logistic line of best fit.

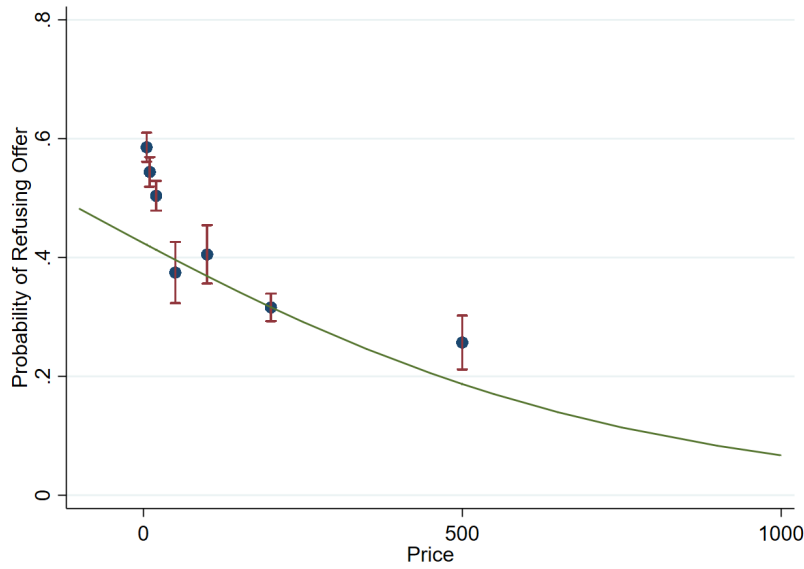


Figure 15: Underlying data and estimate of the the demand curve (Ω_i) for men age 55-64. The points are the mean response to the question “Would you give up Facebook for 1 month in exchange for \$X? Choose Yes if you do not use Facebook.” for individuals of the group. Confidence intervals are based on binomial statistics. The curve, in green, is the logistic line of best fit.

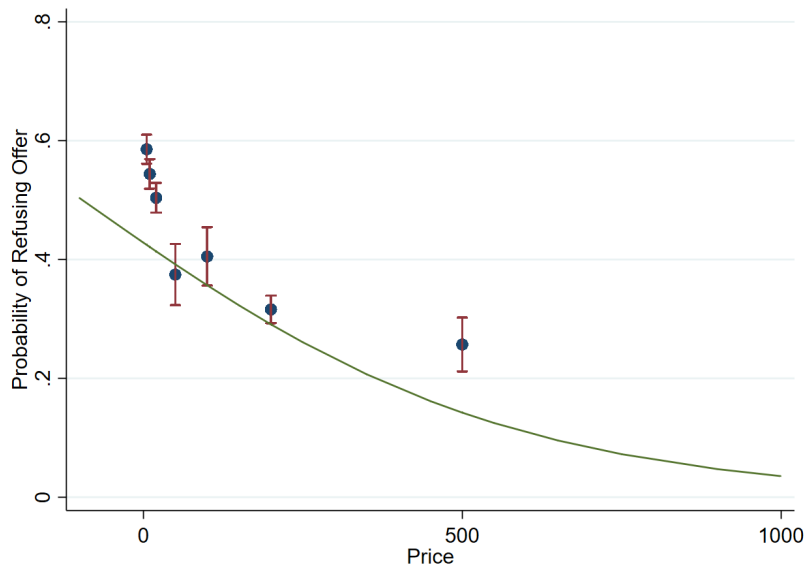


Figure 16: Underlying data and estimate of the the demand curve (Ω_i) for men age 65 or older. The points are the mean response to the question “Would you give up Facebook for 1 month in exchange for \$X? Choose Yes if you do not use Facebook.” for individuals of the group. Confidence intervals are based on binomial statistics. The curve, in green, is the logistic line of best fit.

Intercept	Coefficient on Cost	Demo Group
.2241054	.002684	Female 25-34
.6247908	.0044958	Female 35-44
.4487355	.0041855	Female 45-54
.4957726	.0044395	Female 55-64
.1337358	.0045129	Female 65+
.175812	.0055666	Male 25-34
.0794195	.0041705	Male 35-44
.1046415	.0035566	Male 45-54
-.3053136	.0023273	Male 55-64
-.2883383	.0030162	Male 65+

Table 2: Coefficient estimates from a logit regression of willingness to stop using Facebook on cost of Facebook proposed (equal to negative of the Price offered to stop using Facebook).

	Initial	Cascade 1	Cascade 2	Cascade 3	Cascade 10	Cascade 20	Cascade 1000
Women 25-34	0.556	0.557	0.561	0.566	0.691	0.806	0.807
Women 35-44	0.651	0.655	0.661	0.669	0.842	0.942	0.942
Women 45-54	0.610	0.613	0.619	0.628	0.816	0.931	0.931
Women 55-64	0.622	0.628	0.634	0.642	0.819	0.930	0.931
Women 65+	0.533	0.538	0.544	0.554	0.769	0.912	0.913
Men 25-34	0.544	0.553	0.561	0.572	0.812	0.944	0.944
Men 35-44	0.520	0.526	0.533	0.543	0.770	0.918	0.918
Men 45-54	0.526	0.531	0.537	0.544	0.726	0.870	0.870
Men 55-64	0.424	0.425	0.429	0.434	0.577	0.726	0.727
Men 65+	0.428	0.430	0.435	0.442	0.627	0.801	0.802

Figure 17: Initial rate of Facebook use by demographic, and use of Facebook after N cascades of network effects, after all advertisements on Facebook are eliminated.

	Initial	Cascade 1	Cascade 2	Cascade 3	Cascade 10	Cascade 20	Cascade 1000
Women 25-34	12.42M	12.46M	12.54M	12.66M	15.45M	18.02M	18.03M
Women 35-44	13.36M	13.43M	13.55M	13.72M	17.27M	19.31M	19.32M
Women 45-54	13.10M	13.16M	13.29M	13.48M	17.52M	19.98M	19.99M
Women 55-64	13.51M	13.66M	13.78M	13.96M	17.80M	20.22M	20.23M
Women 65+	15.09M	15.21M	15.40M	15.66M	21.75M	25.81M	25.83M
Men 25-34	12.50M	12.71M	12.89M	13.15M	18.68M	21.70M	21.71M
Men 35-44	10.59M	10.72M	10.87M	11.06M	15.69M	18.70M	18.71M
Men 45-54	11.00M	11.10M	11.22M	11.37M	15.18M	18.18M	18.19M
Men 55-64	8.59M	8.61M	8.69M	8.80M	11.69M	14.70M	14.72M
Men 65+	9.67M	9.71M	9.82M	9.98M	14.15M	18.08M	18.09M

Figure 18: Initial number of US Facebook users by demographic, and use of Facebook after N cascades of network effects, after all advertisements on Facebook are eliminated. Numbers reported in millions of users.

	Cascade 1	Cascade 2	Cascade 3	Cascade 10	Cascade 20	Cascade 1000
Women 25-34	-135.2M	-135.2M	-135.2M	-135.2M	-135.2M	-135.2M
Women 35-44	-147.5M	-147.5M	-147.5M	-147.5M	-147.5M	-147.5M
Women 45-54	-147.4M	-147.4M	-147.4M	-147.4M	-147.4M	-147.4M
Women 55-64	-151.5M	-151.5M	-151.5M	-151.5M	-151.5M	-151.5M
Women 65+	-136.0M	-136.0M	-136.0M	-136.0M	-136.0M	-136.0M
Men 25-34	-264.4M	-264.4M	-264.4M	-264.4M	-264.4M	-264.4M
Men 35-44	-119.1M	-119.1M	-119.1M	-119.1M	-119.1M	-119.1M
Men 45-54	-122.2M	-122.2M	-122.2M	-122.2M	-122.2M	-122.2M
Men 55-64	-86.5M	-86.5M	-86.5M	-86.5M	-86.5M	-86.5M
Men 65+	-82.7M	-82.7M	-82.7M	-82.7M	-82.7M	-82.7M
Total	-1392.5M	-1392.5M	-1392.5M	-1392.5M	-1392.5M	-1392.5M

Figure 19: Change in Facebook revenues, by demographic after N cascades of network effects, after all advertisements on Facebook are eliminated. Note that, because all advertisements are eliminated, the change in revenues does not vary with cascade. Reported in millions of dollars per month.

	Cascade 1	Cascade 2	Cascade 3	Cascade 10	Cascade 20	Cascade 1000
Women 25-34	1.38	6.97	14.60	201.95	395.46	396.56
Women 35-44	2.20	8.11	16.24	212.44	425.78	427.06
Women 45-54	1.58	7.90	16.61	224.70	448.59	449.93
Women 55-64	4.05	9.72	17.55	206.41	412.82	414.07
Women 65+	1.95	7.98	16.20	209.50	413.06	414.29
Men 25-34	3.63	9.32	17.25	201.82	403.10	404.35
Men 35-44	3.29	10.04	19.37	237.59	468.83	470.25
Men 45-54	2.93	9.12	17.57	219.68	427.70	428.93
Men 55-64	0.68	7.48	16.80	243.86	469.04	470.29
Men 65+	1.14	7.91	17.27	240.67	460.32	461.58

Figure 20: Change in average net-welfare from Facebook participation (i.e. $\int_{P_i}^{P_i} (\mu_i - \epsilon_i) - \int_0^{P_i} -\epsilon_i$), by demographic after N cascades of network effects, after all advertisements on Facebook are eliminated. Reported in dollars per month.

	Cascade 1	Cascade 2	Cascade 3	Cascade 10	Cascade 20	Cascade 1000
Women 25-34	0.03B	0.16B	0.33B	4.51B	8.84B	8.86B
Women 35-44	0.05B	0.17B	0.33B	4.36B	8.73B	8.76B
Women 45-54	0.03B	0.17B	0.36B	4.82B	9.63B	9.66B
Women 55-64	0.09B	0.21B	0.38B	4.49B	8.97B	9.00B
Women 65+	0.06B	0.23B	0.46B	5.93B	11.69B	11.72B
Men 25-34	0.08B	0.21B	0.40B	4.64B	9.27B	9.30B
Men 35-44	0.07B	0.20B	0.39B	4.84B	9.55B	9.58B
Men 45-54	0.06B	0.19B	0.37B	4.59B	8.94B	8.97B
Men 55-64	0.01B	0.15B	0.34B	4.94B	9.50B	9.53B
Men 65+	0.03B	0.18B	0.39B	5.43B	10.39B	10.42B
Profit Change	-1.39B	-1.39B	-1.39B	-1.39B	-1.39B	-1.39B
Total Social Welfare Change	-0.89B	0.48B	2.35B	47.16B	94.12B	94.40B

Figure 21: Change in social welfare for each demographic group, and total including decreased profit, after N cascades, due to advertisements being eliminated on Facebook. Reported in billions of dollars per month.

A Network Stability

A.1 Stability of Equilibria

An important first question is whether the network just described is stable. We define a network as stable at equilibrium \vec{P} if the derivative of a connected individual's best response function with response to these probabilities is less than 1.¹⁷ This is a version of a 'trembling hand' perfect equilibrium, meaning that the equilibrium is robust to small fluctuations in each individual's likelihood of participation.

For a symmetric network (i.e. every individuals' demand function Ω is identical), assuming that utility is linearly additive in the network effects and disutility from advertisements, the probability of participation for any individual is

$$P = \Omega\left(\sum^I U(i)P - a\phi\right) \quad (38)$$

where $U(i)$ is the value of any connection.¹⁸ Then the best response function is

$$\frac{\partial \Omega}{\partial P} = \frac{\partial \Omega}{\partial U} \left(\sum^I U(i) - a\phi \right) \quad (39)$$

And so a network equilibrium is stable so long as

$$1 > \frac{\partial \Omega}{\partial U} U(i)(I - 1) \quad (40)$$

In other words, a network equilibrium is stable so long as the average user doesn't have too many connections, is too elastic in their individual participation, or gains too much value from every additional connection. If the inequality is violated, small deviations from an equilibrium are liable to send participation to a boundary condition of 100% participation or zero participation.

A.2 Stability of Equilibrium to Demand Shock

Relatedly, we can also consider the resilience of a network equilibrium to a shock in preferences.

Theorem 1. *Consider a symmetric network where Ω is continuously differentiable and utility is linearly additive in network effects and the disutility from advertisement. Then for any stable equilibrium (as defined above) $\frac{P_i}{\phi_j}$ and $\frac{P_j}{\phi_i}$ are finite*

¹⁷This concept of equilibrium stability borrows from Jackson (2010) section (9.7.2). In that model, only some individuals are connected in the network, but in our model all are connected. In that model p corresponds to the percentage of neighbors who participate, but in our model it corresponds to the likelihood of anyone who participates.

¹⁸the value of a 'connection to oneself' is assumed to be 0

Proof. Rewriting equation 15 with the assumption all nodes are identical, before i gets hit with a fee, yields:

$$\frac{\partial P_j}{\partial \phi_i} = \frac{\partial \Omega}{\partial \mathbf{U}} \left(U(i) \frac{\partial P_i}{\partial \phi_i} + \sum_{k \neq i}^K U(i) \frac{\partial P_k}{\partial \phi_i} \right) \quad (41)$$

Substituting in 14 and summing yields

$$\frac{\partial P_j}{\partial \phi_i} = \frac{\partial \Omega}{\partial \mathbf{U}} \left((I-2) \frac{\partial P_j}{\partial \phi_i} U(i) + \frac{\partial \Omega}{\partial \mathbf{U}} U(i) \left(U(i)(I-1) \frac{\partial P_j}{\partial \phi_i} - \frac{\partial A}{\partial \phi_i} \right) \right) \quad (42)$$

Solving for $\frac{\partial P_j}{\partial \phi_i}$ yields

$$\frac{\partial P_j}{\partial \phi_i} = \frac{-\left(\frac{\partial \Omega}{\partial \mathbf{U}}\right)^2 U(i) \frac{\partial A}{\partial \phi_i}}{1 - \left(\frac{\partial \Omega}{\partial \mathbf{U}} U(i)(I-2) + \left(\frac{\partial \Omega}{\partial \mathbf{U}}\right)^2 (U(i))^2 (I-1)\right)} \quad (43)$$

The network will not unravel due to a welfare change so long as 43 is not infinite. This is equivalent to showing that the denominator is not equal to zero (as all other terms are finite).

However, the denominator never takes the value 0 when the network stability criteria is satisfied. Rearranging terms, the denominator can be written as

$$1 - \frac{\partial \Omega}{\partial \mathbf{U}} U(i) \left((I-2) + \left(\frac{\partial \Omega}{\partial \mathbf{U}}\right) (U(i))(I-1) \right) \quad (44)$$

From the assumption that the network is stable, we have

$$1 > \frac{\partial \Omega}{\partial \mathbf{U}} U(i)(I-1) \quad (45)$$

This implies

$$I-1 > (I-2) + \frac{\partial \Omega}{\partial \mathbf{U}} U(i)(I-1) \quad (46)$$

and applying 40 again implies

$$1 > \frac{\partial \Omega}{\partial \mathbf{U}} U(i) \left((I-2) + \left(\frac{\partial \Omega}{\partial \mathbf{U}}\right) (U(i))(I-1) \right) \quad (47)$$

And if $\frac{\partial P_j}{\partial \phi_i}$ is finite, clearly so to is $\frac{\partial P_i}{\partial \phi_i}$. So long as the network is stable in the normal sense, it is stable to welfare shocks. □

Lemma 2. *In a symmetric network, $\frac{\partial P_i}{\partial \phi_j} = 0$ if $\left(\frac{\partial \Omega}{\partial \mathbf{U}}\right)^2 \frac{\partial A}{\partial T} U(j) = 0$*

Proof. Directly from (43) □