Divide and Conquer in Two-Sided Markets: A Potential-Game Approach*

Lester T. Chan

Department of Economics

Boston University

lctlabc@bu.edu

July 2019

Abstract

Strong network effects typically lead to multiple equilibria in two-sided markets. In response to the methodological challenge in selecting a suitable equilibrium, this paper shows that many two-sided market models are weighted potential games, and thus potential-maximizer selection (Monderer and Shapley 1996) can always select a unique equilibrium in these models. As shown in the game theory literature, the selected equilibrium coincides with the unique equilibrium under global-game selection, p-dominance selection, perfect foresight dynamics, and log-linear dynamics; it is also robust to incomplete information and widely supported by experimental results. Under potential-maximizer selection, platforms often subsidize one side and charge the other side, i.e., divide and conquer. The fundamental determinant of which side to subsidize or monetize is the cross-side network effects only. This divide-and-conquer strategy implies that platforms are often designed to favor the money side much more than the subsidy side.

JEL Classifications: D42, D43, D85, L12, L13

Keywords: two-sided platforms, potential games, divide and conquer, equilibrium selection, network effects

*I am grateful to many people for useful comments and discussions, especially Roberto Burguet, Andrei Hagiu, Hsueh-Ling Huynh, Duozhe Li, Barton Lipman, Albert Ma, Henry Mak, Travis Ng, Fernando Payro, Marc Rysman, Tat-How Teh, Alexander White, Beixi Zhou, and seminar participants at the 2019 International Industrial Organization Conference, 2019 Asian Meeting of the Econometric Society, 2019 North American Summer Meeting of the Econometric Society, Boston University, and the Chinese University of Hong Kong.

1 Introduction

Two groups of agents often interact via platforms: men and women meet in a nightclub, buyers and sellers trade on a marketplace, consumers and merchants transact through a payment card, etc. These markets are known as two-sided markets.[1] Typically, positive cross-side network effects are present in these markets, creating strategic complementarities among the agents. For example, a man (woman) will join a heterosexual nightclub only if there are some women (men) joining the nightclub. Therefore, to attract men, the nightclub needs a lot of women, but to attract women, the nightclub needs a lot of men. This issue is known as the classic "chicken and egg" problem (Caillaud and Jullien 2003)—one of the most difficult challenges for many two-sided platforms, and a methodological challenge for researchers on two-sided markets.

Formally speaking, in a typical two-sided market model where platforms set prices in stage 1 and all agents simultaneously make their joining decisions in stage 2, agents often engage in a coordination game with multiple equilibria in stage 2. For example, if there is a monopoly platform and agents from the same side are identical, there can be two equilibria in stage 2: (i) all agents join the platform, and (ii) no one joins the platform. If there are competing platforms, all agents will coordinate on one of the platforms in the equilibrium when the network effects are sufficiently strong, but which platform will they coordinate on? In other words, how should we deal with the multiple equilibria issue?

In the two-sided market literature, researchers impose various equilibrium selection criteria to deal with multiple equilibria in stage 2. A popular selection criterion is *Pareto-dominance selection*, i.e., to select the Pareto-dominant equilibrium whenever there are multiple equilibria. However, this criterion often fails to select a unique equilibrium under platform competition because coordinating on one of the platforms does not necessarily Pareto-dominate the others. Under platform competition, a popular selection criterion is *focal-platform selection*, i.e., to assume that all agents always coordinate on a pre-specified platform whenever there are multiple equilibria (Caillaud and Jullien 2001; Hagiu 2006; Jul-

[1] See Rysman (2009) for an early survey of the two-sided market literature.

lien 2011).[2] The drawback of this criterion is that we have to impose this asymmetry among the competing platforms. Another selection criterion is to assume that the number of agents from each side who join the platform decreases with the platform's prices (Caillaud and Jullien 2003; Armstrong and Wright 2007).[3] Yet, this criterion is often too weak to select a unique equilibrium, even for the monopoly-platform markets.[4] More recently proposed selection criteria include the concept of coalitional rationalizability (Ambrus and Argenziano 2009) and insulating tariffs (Weyl 2010; White and Weyl 2016). Nevertheless, both criteria sometimes fail to select a unique equilibrium under platform competition.[5]

In addition to the above-mentioned limitations, different selection criteria often lead to different predictions and implications. Yet, there is no consensus in the two-sided market literature on which selection criterion we should or should not use. Hence, there is a methodological challenge in selecting a suitable equilibrium. In response to this challenge, this paper proposes using another approach—the potential-game approach—to resolve the multiple equilibria issue in two-sided markets. This approach is justified by many solid microfoundations in the game theory literature, widely supported by experimental results, and can select a unique equilibrium for many two-sided market models.

The concept of potential games was formalized by Monderer and Shapley (1996),[6] which I will explain in detail in Section 2.3. In short, a game is a potential game if it is strategically equivalent to an identical interest game, in which all players share the same utility function. In the corresponding identical interest game, (generically) there is a unique Pareto-dominant

[2] In the literature, this selection criterion is also called favorable/unfavorable (or good/bad) expectations (Caillaud and Jullien 2001; Hagiu 2006; Jullien 2011), optimistic/pessimistic beliefs (Halaburda and Yehezkel 2013), focality (Halaburda and Yehezkel 2019), or incumbency advantage (Biglaiser et al. 2019).

[3] This criterion imposes no restriction when the price increases on one side but decreases on the other.

[4] For instance, both Pareto-dominance selection and Pareto-dominated selection (i.e., to select the Pareto-dominated equilibrium whenever there are multiple equilibria) satisfy this criterion.

[5] Coalitional rationalizability fails when agents are sufficiently heterogeneous (Ambrus and Argenziano 2009, Abstract). Insulating tariffs fail when agents from the same side are homogeneous and the network effects are sufficiently strong (White and Weyl 2016, Proposition 4c).

[6] The use of potential games appeared in several earlier papers such as Rosenthal (1973) and Blume (1993).

equilibrium, called the *potential maximizer*. In the game theory literature, many selection criteria select the potential maximizer if a game is a potential game. For example, the unique equilibrium under global-game selection (Frankel et al. 2003, Theorem 4) and p-dominance selection (Morris and Ui 2005, Lemma 7) is the potential maximizer.[7] The potential maximizer is also the unique state that is absorbing and globally accessible under perfect foresight dynamics (Hofbauer and Sorger 1999; Oyama et al. 2008) and the unique stochastically stable state under log-linear dynamics (Blume 1993, Theorem 6.3, 6.5; Alos-Ferrer and Netzer 2010; Okada and Tercieux 2012).[8] Even without relying on other selection criteria, the potential maximizer itself is also robust to incomplete information (Ui 2001; Morris and Ui 2005). Given these solid microfoundations in the game theory literature, *potential-maximizer selection*, which was first proposed by Monderer and Shapley (1996, Section 5), is to select the potential maximizer whenever there are multiple equilibria.[9] Last but not least, potential-maximizer selection is supported by ample experimental evidence (Van Huyck et al. 1990; Goeree and Holt 2005; Chen and Chen 2011).[10]

[7] For games with strategic complements, these results hold for a more general class of potential games, called monotone potential games (Morris and Ui 2005, Section 6). For the concept of global games, see Carlsson and van Damme (1993) and Morris and Shin (2003). Jullien and Pavan (2018) study a two-sided market model under the global-game setting, but their paper is not about equilibrium selection (see p. 5 of their paper). By contrast, Sakovics and Steiner (2012) study a one-sided network model under global-game selection. For the concept of p-dominance, see Kajii and Morris (1997).

[8] For games with strategic complements, the results under perfect foresight dynamics and log-linear dynamics hold for monotone potential games and local potential games (the latter is a special case of monotone potential games; see Morris and Ui (2005, Section 6) for details) respectively. For the concept of perfect foresight dynamics, see Matsui and Matsuyama (1995).

[9] Clearly, we can also apply other selection criteria (e.g. global-game selection) to a potential game and obtain the potential maximizer as the unique equilibrium. Nevertheless, potential-maximizer selection is the most tractable selection criterion because we can directly work on a complete information game as demonstrated in this paper.

[10] Anderson et al. (2001) introduce the notion of stochastic potential by adding some noise to the standard potential (the former converges to the latter as the noise goes to zero; see p. 194 of their paper for details). Both Goeree and Holt (2005) and Chen and Chen (2011) find that subjects often end up at the maximizer of the stochastic potential.

Although potential-maximizer selection is applicable only to potential games, many two-sided market models are indeed potential games.[11] In particular, for the four most-cited papers on two-sided markets (namely, Rochet and Tirole 2003, 2006; Armstrong 2006; Caillaud and Jullien 2003), all of their main models with strategic complements are (weighted) potential games, where potential-maximizer selection is applicable. In fact, one contribution of this paper is to unveil the significant applications of potential games on two-sided markets.

The purpose of this paper is to demonstrate how potential-maximizer selection can resolve the multiple equilibria issue and to derive novel insights into two-sided markets. To achieve this goal, I study a few variants of Armstrong's (2006) models, which I outline below.

Sections 2 analyzes the baseline model, which is a special case of Armstrong's (2006, Section 3) monopoly-platform model where agents from the same side are identical. Under potential-maximizer selection, the platform has to leave enough surplus to the agents by setting sufficiently low prices in stage 1, so that all agents will join the platform in stage 2. It turns out that the platform's optimal pricing strategy is to fully subsidize one side and set the highest possible price on the other side, i.e., to *divide and conquer* (Caillaud and Jullien 2003). The only determinant of which side to monetize or subsidize is the relative size of the network effects, i.e., the platform monetizes the side that enjoys a larger *per-interaction benefit*. In other words, the *money/subsidy side* is independent of the total number of agents on each side and the costs of serving the agents. This divide-and-conquer strategy implies that the optimal design of the monopoly platform is to favor the money side only, which is socially suboptimal.

Two-sided platforms often divide and conquer in reality: women enjoy free admission on ladies' nights while men pay an admission fee; shoppers visit shopping malls for free while retailers pay the rent; consumers are paid to use credit cards while merchants pay for the service, etc. The existing literature shows that a monopoly platform would divide and conquer only when the elasticities of demand or the network effects of the two sides

[11] It means that every subgame in stage 2 of these two-sided market models is a potential game. Chan (2019, Section 3) shows that many one-sided network models are potential games.

are significantly different (Armstrong 2006; Rochet and Tirole 2003, 2006).[12] By contrast, under potential-maximizer selection, the monopoly platform's divide-and-conquer strategy is ubiquitous in the baseline model.

Section 3 extends the baseline model to allow for heterogeneous agents, in which they incur idiosyncratic personal costs from joining the platform. When agents are homogeneous on one side and heterogeneous on the other, the platform has an additional incentive to attract more agents from the latter side by lowering the price on this side. Nonetheless, under potential-maximizer selection, the platform monetizes the homogeneous side and subsidizes the heterogeneous side if and only if the per-interaction benefit of the former is larger than that of the latter—the same implication as in the baseline model—and this is irrespective of the details of the heterogeneity among the agents as long as some regularity conditions are satisfied. Moreover, the optimal design of the platform tends to favor one side (not necessarily the heterogeneous side) much more than the other side. Hence, all the key implications of the baseline model are naturally extended to this richer framework.

When agents are sufficiently heterogeneous on both sides as in Armstrong's (2006, Section 3) original model, the platform has a high incentive to attract more agents by lowering the prices on both sides; this in turn leaves a lot of surplus to the agents. Therefore, it is possible that the equilibrium outcome under potential-maximizer selection coincides with that under Pareto-dominance selection.

Section 4 analyzes a variant of Armstrong's (2006, Section 4) duopoly-platform model, in which the platforms are vertically (but not horizontally) differentiated. Under potential-maximizer selection, the platforms' price-setting stage is analogous to the standard Bertrand competition with vertical differentiation. The entire market tips to a platform in the equilibrium. The dominant platform always divides and conquers as in the baseline model. By contrast, the money/subsidy side depends on the relative size of the average per-interaction benefits across the competing platforms rather than its own per-interaction benefits of the

[12] Alternatively, Caillaud and Jullien (2003) and Jullien (2011) derive the divide-and-conquer strategy under platform competition together with focal-platform selection.

two sides. The optimal design of the platforms tends to favor both (one) sides when the platforms are very (not) competitive.

Section 5 discusses and compares potential-maximizer selection with focal-platform selection and discusses several extensions to show the generality of potential-maximizer selection. Section 6 concludes.

2 Monopoly Platform: Homogeneous Agents

Section 2.1 presents the baseline model, which is a special case of Armstrong's (2006, Section 3) monopoly-platform model where agents from the same side are identical. Section 2.2 analyzes the model under Pareto-dominance selection as a benchmark. For the baseline model, focal-platform selection (with the monopoly platform being "focal"), coalitional rationalizability, and insulating tariffs coincide with Pareto-dominance selection.[13] Section 2.3 introduces the potential-game approach and shows that the equilibrium outcome under potential-maximizer selection is very different from that under Pareto-dominance selection. This section studies the more realistic finite-agent model; the next section studies the more popular continuum-agent model.

2.1 The Baseline Model

A monopoly platform serves two groups of agents, namely, group 1 and group 2, and there are $N_1 \in \mathbb{N}$ group-1 agents and $N_2 \in \mathbb{N}$ group-2 agents. The game has two stages. In stage 1, the platform sets subscription fees $(p_1, p_2) \in \mathbb{R}^2$ to the two groups. In stage 2, all group-1 and group-2 agents observe the platform's prices and simultaneously decide whether to join the platform. Let $a_i^k \in \{0 \equiv \text{Not join}, 1 \equiv \text{Join}\}$ denote the action of agent $k \in \{1, \ldots, N_i\}$ from group $i = 1, 2$.

If the platform attracts $n_1 \equiv \sum_{k=1}^{N_1} a_1^k$ group-1 agents and $n_2 \equiv \sum_{k=1}^{N_2} a_2^k$ group-2 agents,

[13] Similarly, Armstrong (2006, p. 672) allows the platform to directly choose the agents' utility levels rather than setting prices, which is also equivalent to Pareto-dominance selection.

the payoff of a group-i agent from joining the platform is

$$u_i(n_j, p_i) = v_i n_j - p_i, \quad (j = 1, 2;\; j \neq i) \tag{1}$$

where $v_i \in \mathbb{R}_{++}$ is the *per-interaction benefit* of a group-i participant from interacting with each group-j participant. If an agent does not join the platform, his payoff is zero.

The platform's payoff is equal to its profit:

$$\pi(n_1, n_2, p_1, p_2) = (p_1 - c_1)\, n_1 + (p_2 - c_2)\, n_2, \tag{2}$$

where $c_i \in \mathbb{R}_+$ is the (sufficiently low) marginal cost of serving each group-i participant.

In what follows, I am interested in the pure strategy subgame perfect equilibria of this two-stage game.[14]

2.2 Pareto-Dominance Selection

I solve this game backwards, starting from stage 2. Multiple equilibria often arise in stage 2 due to the cross-side network effects between the two sides. In particular, there are two equilibria in stage 2 when $(p_1, p_2) \in [0, v_1 N_2] \times [0, v_2 N_1]$:[15]

1. all agents join the platform;

2. no one joins the platform.

Clearly, the first equilibrium Pareto-dominates the second equilibrium for all agents (as well as for the platform). As a benchmark, I apply *Pareto-dominance selection* to the

[14] There are some mixed strategy equilibria in the model. Nevertheless, these equilibria are unstable (i.e., they cannot be reached by any dynamic process) and never selected under Pareto-dominance selection or potential-maximizer selection (see footnote 18 for details). These equilibria play no role in the subsequent analysis, and I will not discuss them throughout this paper.

[15] If $p_i > v_i N_j$ ($p_i < 0$), there is a unique equilibrium with no (N_i) group-i participants. When $p_i = v_i N_j$ or $p_i = 0$, there are equilibria with only some (but not all) group-i agents joining the platform because they are indifferent between joining or not. Nevertheless, the platform can avoid these equilibria by lowering the price p_i a bit in stage 1. These equilibria play no role in the subsequent analysis, and I will not discuss them throughout this paper.

model, i.e., to assume that all agents always join the platform whenever there are multiple equilibria in stage 2. Under this selection criterion, the platform charges both groups the highest possible prices in stage 1 so that all agents will join the platform with zero surplus in stage 2, i.e.,

$$p_1^* = v_1 N_2, \quad p_2^* = v_2 N_1.$$

Hence, from (2), the platform's equilibrium profit is

$$\pi^* = (v_1 + v_2) N_1 N_2 - c_1 N_1 - c_2 N_2.$$

2.3 Potential-Maximizer Selection

I now analyze the model under potential-maximizer selection. Section 2.3.1 illustrates the potential-game approach with the simplest case where there are only one group-1 agent and one group-2 agent. Section 2.3.2 analyzes the general case with N_1 group-1 agents and N_2 group-2 agents. Section 2.3.3 discusses and compares the results under potential-maximizer selection with the benchmark results.

2.3.1 Simplest Case: $N_1 = N_2 = 1$

When there are only two agents, the subgame in stage 2 can be represented by the following payoff matrix:

	Join	Not join
Join	$v_1 - p_1, v_2 - p_2$	$-p_1, 0$
Not join	$0, -p_2$	$0, 0$

(3)

Consider a function P defined on the strategy space of the same game as below:

	Join	Not join
Join	$1 - \frac{p_1}{v_1} - \frac{p_2}{v_2}$	$-\frac{p_1}{v_1}$
Not join	$-\frac{p_2}{v_2}$	0

(4)

P is constructed in a way that the change in each agent's payoff from unilaterally switching actions in (3) is proportional to the corresponding change in P. To see this, the payoff difference between (Join, Join) and (Not join, Join) for the group-1 agent is $v_1 - p_1$, and the corresponding difference in P is $1 - \frac{p_1}{v_1} = \frac{1}{v_1}(v_1 - p_1)$. Similarly, his payoff difference between (Join, Not join) and (Not join, Not join) is $-p_1$, and the corresponding difference in P is $-\frac{p_1}{v_1} = \frac{1}{v_1}(-p_1)$. The same logic applies to the group-2 agent.

Hence, if we view (4) as an identical interest game in which the two agents share the same payoff function P, then this game is strategically equivalent to (3).[16] In particular, the best-response correspondence and the set of equilibria for these two games are identical. Therefore, we can view P as a sufficient statistic for the equilibrium analysis of (3): P is also called the *potential function* of (3). A game is a *weighted potential game* if such a potential function P exists, and thus (3) is a weighted potential game. The formal definition is given as follows. The mathematical definition is given in Appendix B.

Definition 1 *A game is a weighted potential game if there exists a function P defined on the strategy space of the game, such that the change in any player's payoff from unilaterally switching actions is (positively) proportional to the corresponding change in P. P is called the game's potential function.*

For a weighted potential game, the maximizer of the potential function (called the *potential maximizer*) always exists, and it is generically unique.[17] For example, the potential maximizer in (4) when $p_1, p_2 \geq 0$ is

$$\begin{array}{lll} \text{(Join, Join)} & \text{if} & \dfrac{p_1}{v_1} + \dfrac{p_2}{v_2} \leq 1, \\ \text{(Not join, Not join)} & \text{if} & \dfrac{p_1}{v_1} + \dfrac{p_2}{v_2} \geq 1. \end{array} \tag{5}$$

[16] In fact, these two games are von Neumann-Morgenstern equivalent as defined by Morris and Ui (2004).

[17] The potential function P is unique up to positive affine transformations, i.e., $P' \equiv C_1 P + C_2$ ($C_1 \in \mathbb{R}_{++}$; $C_2 \in \mathbb{R}$) is also a potential function of the game. Thus, the potential maximizer is invariant to the choice of the potential function.

Note that the potential maximizer is always a pure strategy Nash equilibrium of the game:[18] if someone deviates from the potential maximizer, the potential will decrease, and, by definition, the deviator will have a lower payoff.

Both (Join, Join) and (Not join, Not join) in (3) are equilibria when $(p_1, p_2) \in [0, v_1] \times [0, v_2]$. Nevertheless, (generically) only one of them is the potential maximizer as shown in (5). As stated in the Introduction (p. 4), many selection criteria in the game theory literature select the potential maximizer, and there is ample experimental evidence showing that subjects often end up at the potential maximizer. Therefore, the equilibrium selection criterion based on potential games is to select the potential maximizer. The formal definition is given as follows.

Definition 2 *Potential-maximizer selection is to select the potential-maximizing equilibrium of a weighted potential game.*

Under potential-maximizer selection, the (generically) unique equilibrium of the subgame for this simplest case is given by (5) when $p_1, p_2 \geq 0$. It implies that the two agents will join the platform only when the prices (p_1, p_2) set by the platform in stage 1 are sufficiently low. Otherwise, they will not join the platform. Note that (3) is a two-player two-action game where the potential-maximizing equilibrium coincides with the risk-dominant equilibrium.[19]

2.3.2 General Case

Section 2.3.1 shows that every subgame in stage 2 is a weighted potential game when there are one group-1 agent and one group-2 agent. I now prove the same result for the general case with N_1 group-1 agents and N_2 group-2 agents. Note that the potential function P of the subgame in stage 2 depends on the prices (p_1, p_2) set by the platform in stage 1. Given

[18] The potential of a mixed strategy Nash equilibrium is a convex combination of the potentials defined on the pure strategy action space. Therefore, (generically) a mixed strategy Nash equilibrium is not the potential maximizer.

[19] For the concept of risk dominance, see Harsanyi and Selten (1988).

that agents from the same side are identical, P is symmetric in the sense that it depends only on the number of participants n_1 and n_2.

Lemma 1 *Every subgame in stage 2 is a weighted potential game with the potential function*

$$P(n_1, n_2|p_1, p_2) = n_1 n_2 - \frac{p_1}{v_1} n_1 - \frac{p_2}{v_2} n_2. \tag{6}$$

Proof. The proof is to verify that the function P defined in (6) is indeed the potential function. For a group-1 agent, if there are n_1 (excluding himself) and n_2 participants, his payoff difference between joining the platform or not is

$$u_1(n_2, p_1) - 0 = v_1 n_2 - p_1. \quad \text{(by (1))}$$

The corresponding difference in P is

$$\begin{aligned} & P(n_1+1, n_2|p_1, p_2) - P(n_1, n_2|p_1, p_2) \\ = \; & \left((n_1+1)n_2 - \frac{p_1}{v_1}(n_1+1) - \frac{p_2}{v_2} n_2\right) - \left(n_1 n_2 - \frac{p_1}{v_1} n_1 - \frac{p_2}{v_2} n_2\right) \quad \text{(by (6))} \\ = \; & n_2 - \frac{p_1}{v_1}. \end{aligned}$$

Clearly, the change in the group-1 agent's payoff from unilaterally switching actions is proportional (with proportion v_1) to the change in P. The same logic applies to a group-2 agent (with proportion v_2 for him). Therefore, every subgame is a weighted potential game with the potential function given by (6). ■

After identifying the potential function, the next step is to identify the potential maximizer. If there is a unique equilibrium in the subgame (i.e., $(p_1, p_2) \notin [0, v_1 N_2] \times [0, v_2 N_1]$), the potential maximizer is the unique equilibrium. If there are two equilibria in the subgame (i.e., $(p_1, p_2) \in [0, v_1 N_2] \times [0, v_2 N_1]$), the potential maximizer is the equilibrium with a higher potential. By Lemma 1, the respective potentials of the two equilibria are

$$\begin{aligned} P(N_1, N_2|p_1, p_2) &= N_1 N_2 - \frac{p_1}{v_1} N_1 - \frac{p_2}{v_2} N_2, \\ P(0, 0|p_1, p_2) &= 0. \end{aligned}$$

Given the above analysis, the potential maximizer of the subgame, which is the selected equilibrium in stage 2 under potential-maximizer selection, is summarized as follows.[20]

Lemma 2 *When $p_1, p_2 \geq 0$, the unique equilibrium of the subgame in stage 2 under potential-maximizer selection is*

$$\begin{array}{ll} \textit{all agents join the platform} & \textit{if } \dfrac{p_1}{v_1 N_2} + \dfrac{p_2}{v_2 N_1} \leq 1, \\ \textit{no one joins the platform} & \textit{otherwise.}^{21} \end{array}$$

As shown in Lemma 2, the platform has to leave enough surplus to the participants by setting sufficiently low prices (p_1, p_2) in stage 1, so that all agents will join the platform in stage 2. Hence, from (2) and Lemma 2, the platform's profit-maximization problem in stage 1 becomes

$$\max_{p_1, p_2 \geq 0} (p_1 - c_1)N_1 + (p_2 - c_2)N_2 \quad \text{s.t.} \quad \frac{p_1}{v_1 N_2} + \frac{p_2}{v_2 N_1} \leq 1.$$

Generically and w.l.o.g., assume that group-1 agents enjoy less per-interaction benefit than group-2 agents do, i.e., $v_1 < v_2$. Solving the above optimization problem shows that the platform's optimal pricing strategy is to set zero price for group 1 and the highest possible price for group 2, i.e.,

$$p_1^* = 0, \quad p_2^* = v_2 N_1.$$

Hence, the platform's equilibrium profit is

$$\pi^* = v_2 N_1 N_2 - c_1 N_1 - c_2 N_2.$$

[20] Lemma 2 omits the cases where p_1 and/or p_2 are strictly negative because the platform will not set such prices in the equilibrium: joining the platform is the (weakly) dominant strategy for the agents whenever it is free to do so.

[21] Both equilibria are potential maximizers when $\frac{p_1}{v_1 N_2} + \frac{p_2}{v_2 N_1} = 1$. Nevertheless, the platform can lower the prices p_1 or p_2 a bit in stage 1 so that the former is the unique potential maximizer in stage 2. Therefore, we can assume for simplicity that the equilibrium in stage 2 is the former when $\frac{p_1}{v_1 N_2} + \frac{p_2}{v_2 N_1} = 1$.

2.3.3 Comparison and Discussion

Table 1 summarizes and compares the results with the benchmark results.

Pareto-dominance selection	Potential-maximizer selection
$p_1^* = v_1 N_2; \quad p_2^* = v_2 N_1$	$p_1^* = 0; \quad p_2^* = v_2 N_1$
$\pi^* = (v_1 + v_2) N_1 N_2 - c_1 N_1 - c_2 N_2$	$\pi^* = v_2 N_1 N_2 - c_1 N_1 - c_2 N_2$
Division of total surplus:	
Group 1: 0; Group 2: 0	Group 1: $v_1 N_1 N_2$; Group 2: 0
Platform: $(v_1 + v_2) N_1 N_2 - c_1 N_1 - c_2 N_2$	Platform: $v_2 N_1 N_2 - c_1 N_1 - c_2 N_2$

Table 1: Comparisons between the two selection criteria (with $v_1 < v_2$).

Under both selection criteria, the platform charges group-2 agents the same maximum price and fully extracts their surplus. By contrast, the platform provides free access for group-1 agents and leaves them a lot of surplus under potential-maximizer selection. Hence, the platform's equilibrium profit is much lower than that of the benchmark. In this case, we refer to group 1 as the *subsidy side* and group 2 as the *money side*.

There are three key implications in this model.

Divide-and-conquer strategy The platform's *divide-and-conquer* strategy that subsidizes one side and monetizes the other side is ubiquitous because the per-interaction benefits of the two sides are (generically) different. As mentioned in the Introduction (p. 5), two-sided platforms often divide and conquer in reality. To derive this divide-and-conquer strategy under the current framework (and without using potential-maximizer selection), one would need to rely on *Pareto-dominated selection*, i.e., to assume that all agents always coordinate on not joining the platform whenever there are multiple equilibria. Yet, this selection criterion is often regarded as even "less plausible" than Pareto-dominance selection. Surprisingly, the equilibrium outcome under Pareto-dominated selection actually coincides with that under potential-maximizer selection in the current model.[22] This equivalence no longer holds when I extend the model in the next two sections.

Money/subsidy side The *money/subsidy side* of the platform depends only on the relative size of the per-interaction benefits v_1 and v_2. In other words, the money/subsidy side is independent of the total number of agents N_1 and N_2 on each side, i.e., the platform does not necessarily monetize the group with more agents. For example, shopping malls have more shoppers than retailers, but only the latter are charged. The reason is that when there are more, say, group-1 agents, the platform can extract more surplus from group 1 by increasing p_1. Yet, having more group-1 agents also increases the group-2 agents' benefits from joining the platform. Hence, the platform can also extract more surplus from group 2 by increasing p_2. These two effects cancel out perfectly in this model, and thus the money/subsidy side is independent of the number of agents.

Similarly, the money/subsidy side is independent of the marginal costs c_1 and c_2 for serving the agents, i.e., the platform does not necessarily monetize the "more profitable" side. For example, for open-access academic journals, the marginal cost of an additional reader is zero and reviewing a paper is costly, but these journals only charge the authors. The reason is that, in the current model, all agents coordinate on either joining or not joining the platform in the equilibrium. Therefore, the total cost $c_1N_1 + c_2N_2$ incurred by the platform is equivalent to a fixed cost, which does not affect the decision on the money/subsidy side.

Optimal design Oftentimes, the agents' per-interaction benefits v_1 and v_2 are not exogenous, but rather the platform's endogenous choice. For example, shopping malls are often designed to maximize shoppers' travel distances by locating anchor stores far from each other

[22] Under Pareto-dominated selection, the platform needs to guarantee participation from one side by providing free access for that side. Then, the platform can charge the highest possible price on the other side. Clearly, the choice of the money/subsidy side under Pareto-dominated selection is the same as that under potential-maximizer selection.

and placing escalators at opposite ends; this benefits retailers but harms shoppers. Hence, the following discussion investigates the comparative statics of v_1 and v_2.

As shown in Table 1, the optimal design of the profit-maximizing platform under Pareto-dominance selection is to favor both sides, i.e., to increase both v_1 and v_2. This is not true under potential-maximizer selection. The platform's equilibrium profit is independent of v_1 as long as $v_1 < v_2$. Therefore, the platform only has the incentive to increase v_2: the optimal design of the platform is to favor the money side only. Under Pareto-dominance selection, social surplus is equal to the platform's equilibrium profit. Therefore, the optimal design of the platform maximizes social welfare. By contrast, the optimal design of the platform is socially suboptimal under potential-maximizer selection: the platform has no incentive to increase group-1 agents' surplus by increasing v_1.

Table 1 demonstrates how different selection criteria can lead to totally different predictions and implications: this is the methodological challenge in two-sided markets. Nevertheless, potential-maximizer selection, a selection criterion justified by many solid microfoundations and experimental evidence, yields more realistic predictions in this model. In fact, these predictions already capture many distinct features of two-sided markets. In other words, these distinct features do not rely on the heterogeneity of the agents within the same side nor a particular market structure; rather, they rely on a suitable equilibrium selection criterion. Sections 3 and 4 extend the baseline model to allow for heterogeneous agents and competing platforms respectively. All the above key implications are naturally extended to these richer frameworks.

3 Monopoly Platform: Heterogeneous Agents

Sections 3.1–3.2 extend the baseline model with heterogeneous agents on one side as in Armstrong and Wright (2007, Section 4), where agents from that side incur idiosyncratic personal costs from joining the platform.[23] Section 3.3 further extends the analysis to Arm-

[23] For example, for physical platforms such as shopping malls and trade fairs, transport cost is likely to be a major consideration for buyers but not for sellers. An alternative interpretation is that agents from

strong's (2006, Section 3) original model, in which agents are heterogeneous on both sides. Under these richer frameworks, every subgame in stage 2 remains a weighted potential game; potential-maximizer selection remains applicable. For convenience, this section presents the continuum-agent model; Appendix C analyzes the corresponding limiting case of the finite-agent model.

3.1 Model

The baseline model is modified as follows. There are now a continuum $[0, N_1]$ of identical group-1 agents and a continuum $[0, \overline{N}_2]$ of heterogeneous group-2 agents ($N_1, \overline{N}_2 \in \mathbb{R}_{++}$). If the platform attracts $n_1 \equiv \int_0^{N_1} a_1^k dk$ group-1 agents and $n_2 \equiv \int_0^{\overline{N}_2} a_2^k dk$ group-2 agents, the payoffs of a group-1 agent and agent $k \in [0, \overline{N}_2]$ (agent k always refers to a group-2 agent in this section) from joining the platform are

$$u_1(n_2, p_1) = v_1 n_2 - p_1, \quad u_2^k(n_1, p_2) = v_2 n_1 - p_2 - t(k), \tag{7}$$

where the function $t : [0, \overline{N}_2] \to \mathbb{R}_+$ specifies each group-2 agent's stand-alone cost from joining the platform. I assume that t is strictly increasing, convex, log-concave, twice-differentiable, $t(0) = 0$, and $t(\overline{N}_2) \to \infty$. Under these assumptions, group-2 agents are sufficiently heterogeneous in a smooth way.[24]

For simplicity, I assume away the marginal costs of serving the participants. Hence, the platform's profit is

$$\pi(n_1, n_2, p_1, p_2) = p_1 n_1 + p_2 n_2. \tag{8}$$

The rest of the model setup is the same as that of the baseline model.

one side have different reservation values. See p. 354 and 361 of Armstrong and Wright (2007) for more interpretations and examples. See also Hagiu and Spulber (2013) for a similar model.

[24]These assumptions merely guarantee that the platform's profit-maximization problem in stage 1 is well-behaved. Even if all these assumptions are violated, every subgame in stage 2 remains a weighted potential game; potential-maximizer selection remains applicable. Besides, the analysis can be easily extended to allow for negative values of t, i.e., group-2 agents can derive some stand-alone benefits from joining the platform.

3.2 Analysis

Compared to the baseline model, there is now an additional *demand expansion effect* on group 2. More precisely, for any group-2 price $p_2 \leq v_2 N_1$ set by the platform in stage 1,[25] not joining the platform is the strictly dominant strategy for agent $k \in (N_2, \overline{N}_2]$, where

$$0 \leq N_2 \equiv t^{-1}(v_2 N_1 - p_2) < \overline{N}_2. \tag{9}$$

These group-2 agents are "irrelevant", and I refer to the remaining group-1 and group-2 agents as *relevant agents*. In addition, if p_2 is negative,[26] joining the platform is the strictly dominant strategy for agent $k \in [0, \underline{N}_2)$, where

$$0 \leq \underline{N}_2 \equiv t^{-1}(-p_2) < N_2. \tag{10}$$

From the above discussions, it is clear that multiple equilibria often arise in stage 2. To be exact, there are two stable[27] equilibria and an unstable equilibrium in stage 2 if and only if

Case 1: $(p_1, p_2) \in [0, v_1 N_2] \times [0, v_2 N_1]$, or

Case 2: $(p_1, p_2) \in [v_1 \underline{N}_2, v_1 N_2] \times (-\infty, 0]$.

Denote $\mathbf{a}_i$ as group-i agents' action profile, $\mathbf{a}_i = \mathbf{0}$ and $\mathbf{a}_i = \mathbf{1}$ as no one and all agents from group i joining the platform respectively, and $\mathbf{1}_S$ as the indicator function where

$$\mathbf{1}_S(k) \equiv \begin{cases} 1 & \text{if } k \in S, \\ 0 & \text{if } k \notin S. \end{cases}$$

As shown in Figure 1 (left), the three equilibria in stage 2 for Case 1 are

[25] If $p_2 > v_2 N_1$, not joining the platform is the strictly dominant strategy for all group-2 agents. The platform never sets such a high group-2 price in the equilibrium, and thus I omit the discussion on these prices throughout this section.

[26] As pointed out by Armstrong (2006, footnote 5), "*It is often unrealistic to suppose negative prices are feasible*". For completeness, this section analyzes both cases, i.e., with or without the non-negative price constraint; see footnote 32 for equilibrium outcome under the non-negative price constraint. When I study platform competition in Section 4, I only analyze the model under the non-negative price constraint.

[27] An equilibrium is stable (unstable) if it can (cannot) be reached by some dynamic process.

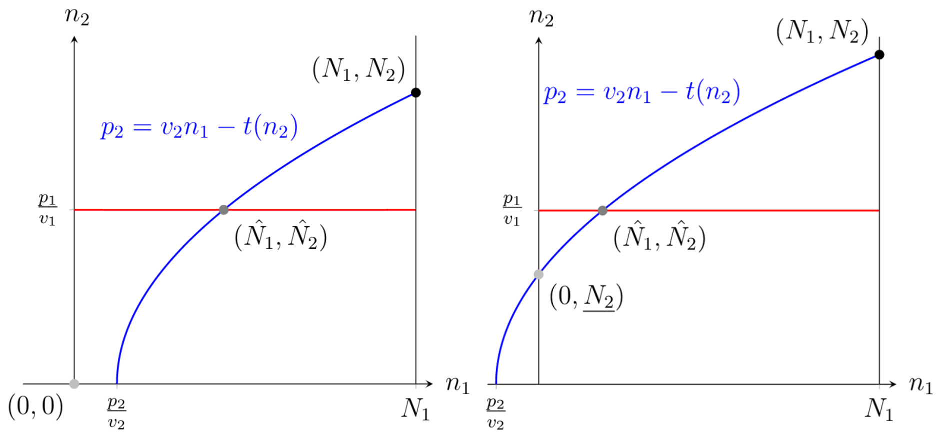

Figure 1: The equilibria of the subgame in stage 2 for Case 1 (left) and Case 2 (right).

1. Pareto-dominant equilibrium: $(\mathbf{a}_1^*, \mathbf{a}_2^*) = (\mathbf{1}, \mathbf{1}_{[0,N_2]})$;
2. unstable equilibrium: $(\mathbf{a}_1^*, \mathbf{a}_2^*) = (\mathbf{1}_{[0,\widehat{N}_1]}, \mathbf{1}_{[0,\widehat{N}_2]})$;[28]
3. Pareto-dominated equilibrium: $(\mathbf{a}_1^*, \mathbf{a}_2^*) = (\mathbf{0}, \mathbf{0})$.

As shown in Figure 1 (right), the three equilibria in stage 2 for Case 2 are

1. Pareto-dominant equilibrium: $(\mathbf{a}_1^*, \mathbf{a}_2^*) = (\mathbf{1}, \mathbf{1}_{[0,N_2]})$;
2. unstable equilibrium: $(\mathbf{a}_1^*, \mathbf{a}_2^*) = (\mathbf{1}_{[0,\widehat{N}_1]}, \mathbf{1}_{[0,\widehat{N}_2]})$;
3. Pareto-dominated equilibrium: $(\mathbf{a}_1^*, \mathbf{a}_2^*) = (\mathbf{0}, \mathbf{1}_{[0,\underline{N}_2]})$.

Clearly, both Pareto-dominance selection and Pareto-dominated selection are applicable to the current model. Appendix D analyzes these two benchmarks and shows that their equilibrium outcomes always differ from that under potential-maximizer selection.

I now analyze the model under potential-maximizer selection. First, I show that every subgame in stage 2 is a weighted potential game. Group-2 agents are heterogeneous in the

[28] Given that group-1 agents are identical, $\mathbf{a}_1^* = \mathbf{1}_{[0,\widehat{N}_1]}$ simply means that the mass of group-1 participants is $\widehat{N}_1$. Thus, strictly speaking, there is a continuum of unstable equilibria.

current framework, and thus the potential function P depends on group-2 agents' action profile $\mathbf{a}_2$ rather than the number of group-2 participants n_2.

Lemma 3 *Every subgame in stage 2 is a weighted potential game with the potential function*

$$P(n_1, \mathbf{a}_2 | p_1, p_2) = n_1 n_2 - \frac{p_1}{v_1} n_1 - \frac{p_2}{v_2} n_2 - \frac{1}{v_2} \int_0^{\overline{N}_2} t(k) a_2^k dk. \tag{11}$$

Proof. See Appendix C. ■

Compared to Lemma 1 in the baseline model, the extra term $\frac{1}{v_2} \int_0^{\overline{N}_2} t(k) a_2^k dk$ captures the total stand-alone cost incurred by group-2 participants.

After identifying the potential function, the next step is to identify the potential maximizer. It can be done so by following the same approach in Section 2.3.2 as shown below. Note that the unstable equilibrium is never selected under potential-maximizer selection.

Lemma 4 *Under potential-maximizer selection, the unique equilibrium of the subgame in stage 2 is*

1. *when* $0 \leq p_2 \leq v_2 N_1$*:*
$$\begin{aligned} (\mathbf{a}_1^*, \mathbf{a}_2^*) &= (\mathbf{1}, \mathbf{1}_{[0,N_2]}) \quad \textit{if } p_1 \leq \frac{v_1}{v_2 N_1} \int_0^{N_2} (t(N_2) - t(k)) dk, \\ (\mathbf{a}_1^*, \mathbf{a}_2^*) &= (\mathbf{0}, \mathbf{0}) \qquad \textit{otherwise.} \end{aligned}$$

2. *when* $p_2 \leq 0$*:*
$$\begin{aligned} (\mathbf{a}_1^*, \mathbf{a}_2^*) &= (\mathbf{1}, \mathbf{1}_{[0,N_2]}) \quad \textit{if } p_1 \leq v_1 \underline{N}_2 + \frac{v_1}{v_2 N_1} \int_{\underline{N}_2}^{N_2} (t(N_2) - t(k))\, dk, \\ (\mathbf{a}_1^*, \mathbf{a}_2^*) &= (\mathbf{0}, \mathbf{1}_{[0,\underline{N}_2]}) \quad \textit{otherwise.} \end{aligned}$$

Proof. See Appendix A.1. ■

As shown in Lemma 4, the platform has to leave enough surplus to the participants by setting sufficiently low prices (p_1, p_2) in stage 1, so that all relevant agents will join the platform in stage 2.[29] Therefore, for any group-2 price $p_2 \leq v_2 N_1$ set by the platform (which

[29] We can easily verify that the terms $\frac{v_1}{v_2 N_1} \int_0^{N_2} (t(N_2) - t(k)) dk$ and $v_1 \underline{N}_2 + \frac{v_1}{v_2 N_1} \int_{\underline{N}_2}^{N_2} (t(N_2) - t(k))\, dk$ in Lemma 4 are decreasing in p_2. Thus, lowering the group-2 price makes it easier for all relevant agents to coordinate on joining the platform.

determines the values of N_2 and $\underline{N}_2$ by (9) and (10) respectively), the platform optimally sets the highest possible group-1 price until the constraint in Lemma 4 binds, i.e.,

$$p_1^* = \begin{cases} \frac{v_1}{v_2 N_1} \int_0^{N_2} (t(N_2) - t(k)) dk & \text{if } 0 \le p_2 \le v_2 N_1, \\ v_1 \underline{N}_2 + \frac{v_1}{v_2 N_1} \int_{\underline{N}_2}^{N_2} (t(N_2) - t(k))\, dk & \text{if } p_2 \le 0. \end{cases} \tag{12}$$

It remains to derive the platform's optimal group-2 price p_2^*. From (8) and (12), the platform's profit-maximization problem in stage 1 becomes

$$\max_{p_2 \le v_2 N_1} \pi = \begin{cases} \frac{v_1}{v_2} \int_0^{N_2} (t(N_2) - t(k))\, dk + p_2 N_2 & \text{if } 0 \le p_2 \le v_2 N_1, \\ v_1 N_1 \underline{N}_2 + \frac{v_1}{v_2} \int_{\underline{N}_2}^{N_2} (t(N_2) - t(k))\, dk + p_2 N_2 & \text{if } p_2 \le 0. \end{cases} \tag{13}$$

Solving the above optimization problem gives us p_2^* (and thus N_2^* and $\underline{N}_2^*$). After that, we can derive p_1^* and π^* from (12) and (13) respectively. The equilibrium outcome of this game is characterized as follows.

Proposition 1 *Under potential-maximizer selection, there is a unique equilibrium in this model. When $v_1 \le v_2$, the platform's optimal group-2 price p_2^* and the equilibrium mass of group-2 participants N_2^* are implicitly given by*

$$p_2^* = v_2 N_1 - t(N_2^*) = \left(1 - \frac{v_1}{v_2}\right) N_2^* t'(N_2^*) \ge 0,\text{[30]} \tag{14}$$

and the platform's optimal group-1 price p_1^* and its equilibrium profit π^* are given by

$$\begin{aligned} p_1^* &= \frac{v_1}{v_2 N_1} \int_0^{N_2^*} (t(N_2^*) - t(k))\, dk, \\ \pi^* &= \frac{v_1}{v_2} \int_0^{N_2^*} (t(N_2^*) - t(k))\, dk + p_2^* N_2^*. \end{aligned}$$

When $v_1 \ge v_2$, p_2^*, N_2^*, and $\underline{N}_2^*$ are implicitly given by

$$p_2^* = -t(\underline{N}_2^*) = v_2 N_1 - t(N_2^*) = \left(N_2^* - \frac{v_1}{v_2}(N_2^* - \underline{N}_2^*)\right) t'(N_2^*) \le 0,\text{[31]} \tag{15}$$

[30] We can easily show that p_2^* and N_2^* exist. They are also unique because t is strictly increasing and convex. p_2^* is positive because $v_1 \le v_2$ and t is increasing.

[31] We can easily show that p_2^*, N_2^*, and $\underline{N}_2^*$ exist. They are also unique because t is strictly increasing, convex, and log-concave. We can also verify that p_2^* is negative. See Appendix A.2 for the formal proof.

and p_1^* *and* π^* *are given by*

$$
\begin{aligned}
p_1^* &= v_1 \underline{N}_2^* + \frac{v_1}{v_2 N_1} \int_{\underline{N}_2^*}^{N_2^*} (t(N_2^*) - t(k))\, dk, \\
\pi^* &= v_1 N_1 \underline{N}_2^* + \frac{v_1}{v_2} \int_{\underline{N}_2^*}^{N_2^*} (t(N_2^*) - t(k))\, dk + p_2^* N_2^*.
\end{aligned}
$$

As shown in Proposition 1, the platform's optimal pricing strategy depends crucially on the per-interaction benefits v_1 and v_2. When $v_1 \leq v_2$, the optimal group-2 price is equal to the standard monopoly markup $N_2^* t'(N_2^*)$, adjusted downward by the fraction $1 - \frac{v_1}{v_2}$ due to the positive network effects generated to group 1. When $v_1 > v_2$, the platform strictly subsidizes group 2.[32] The higher the v_1, the more the platform's incentive to subsidize group 2. Unlike the baseline model, the platform always charges group 1 due to the additional demand expansion effect on group 2. Nevertheless, the platform monetizes group 1 and subsidizes group 2 if and only if $v_1 \geq v_2$—the same feature as in the baseline model—and this is irrespective of the details of the stand-alone cost function t (as long as it satisfies the imposed assumptions in Section 3.1).

Table 2 summarizes the comparative statics results. Again, these results do not depend on the exact form of the stand-alone cost function t.

	N_2^*	p_1^*	p_2^*	$p_1^* N_1$	$p_2^* N_2^*$	π^*
N_1	+	+/−	+	+	+	+
v_1	+	+	−	+	−	+
v_2	+/−	+/−	+	+/−	+	+

	N_2^*	p_1^*	p_2^*	$p_1^* N_1$	$p_2^* N_2^*$	π^*
N_1	+	+	−	+	−	+
v_1	+	+	−	+	−	+
v_2	+	+/−	+	+/−	+	+

Table 2: Comparative statics when $v_1 \leq v_2$ (left) and $v_1 \geq v_2$ (right).

Proof. See Appendix A.3. ■

[32] If the platform is not allowed to strictly subsidize the agents (as mentioned in footnote 26), it will optimally set $p_2^* = 0$. The equilibrium mass of group-2 participants will be $N_2^* = t^{-1}(v_2 N_1)$ by (9), and p_1^* and π^* will be given by the same expressions in Proposition 1.

As shown in Table 2, when the mass of group-1 agents N_1 increases, the platform always generates more revenue from group 1. If additionally $v_1 \leq v_2$, the platform charges group-2 agents more because they now enjoy more benefits from joining the platform. On the other hand, the platform subsidizes group 2 when $v_1 \geq v_2$. When N_1 increases, it actually increases the subsidy (i.e., p_2^* decreases) to attract more group-2 participants, and then recovers the loss by charging group 1 even more. Similarly, when v_1 increases, the platform always lowers the group-2 price to attract more group-2 participants, so it can charge group 1 even more. In the literature, this is known as the "seesaw principle" (Rochet and Tirole 2006).[33] On the other hand, when v_2 increases, the platform always charges group 2 more. Yet, N_2^* might still increase because group-2 participants now enjoy a higher per-interaction benefit. Hence, depending on the parameter values, the platform may increase or decrease the group-1 price when v_2 increases.

I now discuss the optimal design of the platform, i.e., the comparative statics of v_1 and v_2 to the platform's equilibrium profit π^*. Unlike the baseline model, π^* is (strictly) increasing in both v_1 and v_2. Nevertheless, when v_1 (v_2) is relatively large, π^* actually increases more with a further increase in v_1 (v_2) than an increase in v_2 (v_1). Therefore, the optimal design of the two-sided platforms tends to favor one side much more than the other side. The following corollary formally states the result.

Corollary 1 $\frac{\partial \pi^*}{\partial v_1} \leq \frac{\partial \pi^*}{\partial v_2}$ *when* $2v_1 \leq v_2$*;* $\frac{\partial \pi^*}{\partial v_1} \geq \frac{\partial \pi^*}{\partial v_2}$ *when* $v_1 \geq v_2$*.*

Proof. See Appendix A.4. ■

Sections 3.1–3.2 demonstrate how potential-maximizer selection can be applied to Armstrong's model with heterogeneous agents on one side. Despite the additional demand expansion effect on the heterogeneous side, all the three key implications of the baseline model are naturally extended to the current model. In general, there is no closed-form solution for the current model. Appendix E provides two examples with closed-form solutions: the models

[33] The seesaw principle is defined by Rochet and Tirole (2006, p. 659), "*... a factor that is conducive to a high price on one side, to the extent that it raises the platform's margin on that side, tends also to call for a low price on the other side as attracting members on that other side becomes more profitable*".

with linear stand-alone cost and quadratic stand-alone cost. These examples verify all the above results (i.e., Proposition 1, Table 2, and Corollary 1) and derive further implications.

3.3 Armstrong's (2006, Section 3) Original Model

Given the analysis in Section 3.2, it is straightforward to extend the analysis to Armstrong's original model where agents are heterogeneous on both sides. The rest of this section discusses the main results. The formal analysis is given in Appendix F.

In Armstrong's original model, both group-1 and group-2 agents incur idiosyncratic personal costs from joining the platform; demand expansion effects are present on both sides. Similar to the previous models, multiple equilibria often arise in stage 2. When agents from each side are sufficiently heterogeneous in a smooth (and nice) way, there are only two stable equilibria in stage 2: (i) the Pareto-dominant equilibrium with high participation, and (ii) the Pareto-dominated equilibrium with low/no participation.

As explained in footnote 13, Armstrong (2006, Section 3) imposes Pareto-dominance selection and derives the platform's optimal prices (p_1^*, p_2^*) in stage 1. On the other hand, under potential-maximizer selection, the platform faces an additional constraint in stage 1: it has to set the prices (p_1, p_2) such that the Pareto-dominant equilibrium in stage 2 is also the potential maximizer. As shown in Sections 2.3.2 and 3.2, this additional constraint always binds in the equilibrium if agents are identical on at least one side. By contrast, when agents are sufficiently heterogeneous on both sides, sometimes this additional constraint does not bind in the equilibrium. The reason is that demand expansion effects are now present on both sides, and thus the platform has the incentive to attract more agents from both sides by lowering both p_1 and p_2.[34] Hence, the platform's optimal prices (p_1^*, p_2^*) under Pareto-dominance selection might be low enough that the Pareto-dominant equilibrium in stage 2 is also the potential maximizer. If this is the case, the equilibrium outcome under potential-maximizer selection coincides with that under Pareto-dominance selection; otherwise, their

[34] By contrast, if agents are identical on one side, the platform always fully extracts these agents' surplus under Pareto-dominance selection.

equilibrium outcomes differ.

Appendix F.3 shows that when the stand-alone cost functions on both sides are monomials[35] with the same degree $\alpha > 1$, the equilibrium outcome under potential-maximizer selection coincides with that under Pareto-dominance selection if and only if $\frac{v_1}{v_2} \leq \frac{1}{\alpha}$ or $\frac{v_1}{v_2} \geq \alpha$. In other words, their equilibrium outcomes are different whenever the per-interaction benefits of the two sides do not differ too much, i.e., $\frac{1}{\alpha} < \frac{v_1}{v_2} < \alpha$.

4 Platform Competition

This section demonstrates how potential-maximizer selection can resolve the multiple equilibria issue under platform competition and derives further insights into two-sided markets. The baseline model is naturally extended to a duopoly-platform model, which is a special case of Armstrong's (2006, Section 4) duopoly-platform model and almost equivalent to Caillaud and Jullien's (2003, Section 5) model.[36] Multiple equilibria naturally arise in both of their models, but they do not attempt to select an equilibrium.[37] By contrast, I apply potential-maximizer selection to the model and derive the unique equilibrium.

4.1 Model

The baseline model is modified as follows. There are now two competing platforms, indexed by A and B. The payoff of a group-i agent from joining platform $m \in \{A, B\}$ depends on

[35] Monomial stands for a polynomial which has only one term.

[36] The only difference to Caillaud and Jullien's model is that their agents can choose not to join any platforms, while mine have to join one of the platforms as in Armstrong's (2006, Section 4) model. My model is a special case of Armstrong's model with no transport cost (i.e., $t_1 = t_2 = 0$ in his model). Note that the platforms in my model can be vertically differentiated. In this sense, my model is more general than theirs.

[37] Armstrong (2006) actually does not analyze this special case; he only studies the cases where transport costs (i.e., t_1 and t_2 in his model) are sufficiently high so that there is a unique market-sharing equilibrium for the competing platforms. In fact, White and Weyl (2016, p. 3) call this special case the "previously intractable parameter values", and it remains intractable in their paper.

the number of group-j agents who join the same platform, i.e.,

$$u_i^m(n_j^m, p_i^m) = v_i^m n_j^m - p_i^m. \tag{16}$$

As shown in the above payoff function, the model allows group-i participants to enjoy different per-interaction benefits $v_i^m \in \mathbb{R}_{++}$ at different platforms.[38]

For simplicity, I assume away the marginal costs of serving the participants. Hence, platform m's profit is

$$\pi^m(n_1^m, n_2^m, p_1^m, p_2^m) = p_1^m n_1^m + p_2^m n_2^m. \tag{17}$$

Following Armstrong and Wright (2007), I assume that the subscription fees p_1^m and p_2^m set by the platforms are non-negative.[39] As they have argued (p. 356), this is a reasonable restriction for pure-subscription models because strictly subsidizing the participants will create obvious adverse selection and moral hazard problems. Armstrong (2006, footnote 5) also makes a similar argument.

The timing of the game is the same as before. In stage 1, the platforms simultaneously set prices $(p_1^m, p_2^m) \in \mathbb{R}_+^2$ to the two groups. In stage 2, all agents observe the platforms' prices and simultaneously decide which platform to join (they have to join one, and only one).

4.2 Analysis

Similar to the previous models, multiple equilibria often arise in stage 2. Denote the price differences between the two platforms as

$$\Delta p_1 \equiv p_1^A - p_1^B, \quad \Delta p_2 \equiv p_2^A - p_2^B.$$

[38] As explained in Section 2.3.3, different designs of the platforms can lead to different per-interaction benefits for the two sides. In reality, no two platforms share exactly the same design. Therefore, I allow the competing platforms to deliver different per-interaction benefits for each side.

[39] In Section 2, the platform will not set negative price(s) in the equilibrium as explained in footnote 20. Section 3 analyzes both cases (i.e., with or without the non-negative price constraint) for completeness as explained in footnote 26.

There are two equilibria in stage 2 when $\Delta p_i \in [-v_i^B N_j, v_i^A N_j]$ $(i,j = 1,2;\ j \neq i)$:[40]

1. all agents join platform A;

2. all agents join platform B.

Neither Pareto-dominance selection nor Pareto-dominated selection is applicable to the current model: coordinating on one platform does not necessarily Pareto-dominate the other. In fact, none of the popular selection criteria in the two-sided market literature is applicable except focal-platform selection; the latter also suffers from several drawbacks as I will discuss in Section 5 (p. 31–32). By contrast, potential-maximizer selection remains valid. First, I show that every subgame in stage 2 is a weighted potential game.

Lemma 5 *Every subgame in stage 2 is a weighted potential game with the potential function*

$$P(n_1^A, n_2^A | \Delta p_1, \Delta p_2) = n_1^A n_2^A - \frac{v_1^B N_2 + \Delta p_1}{v_1^A + v_1^B} n_1^A - \frac{v_2^B N_1 + \Delta p_2}{v_2^A + v_2^B} n_2^A. \tag{18}$$

Proof. See Appendix A.5. ■

After identifying the potential function, the next step is to identify the potential maximizer. When $\Delta p_i \in [-v_i^B N_j, v_i^A N_j]$ $(i,j = 1,2;\ j \neq i)$, by Lemma 5, the respective potentials of the two equilibria are

$$\begin{aligned} P(N_1, N_2 | \Delta p_1, \Delta p_2) &= \frac{v_1^A v_2^A - v_1^B v_2^B}{(v_1^A + v_1^B)(v_2^A + v_2^B)} N_1 N_2 - \frac{\Delta p_1}{v_1^A + v_1^B} N_1 - \frac{\Delta p_2}{v_2^A + v_2^B} N_2, \\ P(0, 0 | \Delta p_1, \Delta p_2) &= 0. \end{aligned}$$

Hence, the potential-maximizing equilibrium is given as follows.

Lemma 6 *When $\Delta p_i \in [-v_i^B N_j, v_i^A N_j]$ for all $i,j = 1,2;\ j \neq i$, the unique equilibrium of the subgame in stage 2 under potential-maximizer selection is*

$$\begin{cases} \textit{all agents join platform } A & \textit{if } v_1^A v_2^A - \left(\frac{v_2^A + v_2^B}{N_2} p_1^A + \frac{v_1^A + v_1^B}{N_1} p_2^A\right) \geq v_1^B v_2^B - \left(\frac{v_2^A + v_2^B}{N_2} p_1^B + \frac{v_1^A + v_1^B}{N_1} p_2^B\right), \\ \textit{all agents join platform } B & \textit{otherwise.} \end{cases}$$

[40] We can easily verify that (i) there is a unique equilibrium if the prices are out of this range, and (ii) the platforms' equilibrium prices (as characterized by Proposition 2) are indeed within this range.

As shown in Lemma 6, we can view all group-1 and group-2 agents as a "representative agent" who either joins A or B in stage 2: the "value" of platform $m \in \{A, B\}$ is $v_1^m v_2^m$, its "price" is $\frac{v_2^A+v_2^B}{N_2} p_1^m + \frac{v_1^A+v_1^B}{N_1} p_2^m$, and the representative agent joins the platform that offers the higher "net value". Hence, stage 1 is analogous to the standard Bertrand competition with vertical differentiation. Generically and w.l.o.g., assume that the "value" of A is higher, i.e., $v_1^A v_2^A > v_1^B v_2^B$. Standard analysis for Bertrand competition implies that B charges the minimum prices $p_1^{B*} = p_2^{B*} = 0$, and A slightly undercuts B to capture the entire market. From Lemma 6, this implies that

$$\frac{v_2^A + v_2^B}{N_2} p_1^{A*} + \frac{v_1^A + v_1^B}{N_1} p_2^{A*} = v_1^A v_2^A - v_1^B v_2^B. \tag{19}$$

Subject to the above constraint, A maximizes its profit by optimally allocating the prices to the two sides, i.e.,

$$\max_{p_1^A, p_2^A \geq 0} p_1^A N_1 + p_2^A N_2 \quad \text{s.t.} \quad \frac{v_2^A + v_2^B}{N_2} p_1^A + \frac{v_1^A + v_1^B}{N_1} p_2^A = v_1^A v_2^A - v_1^B v_2^B. \tag{20}$$

Generically and w.l.o.g., assume that the average per-interaction benefit across the competing platforms for group 1 is smaller than that of group 2, i.e., $v_1^A + v_1^B < v_2^A + v_2^B$. Solving (20) shows that A's optimal pricing strategy is to set zero group-1 price and a positive group-2 price such that (19) holds, i.e.,

$$p_1^{A*} = 0, \quad p_2^{A*} = \frac{v_1^A v_2^A - v_1^B v_2^B}{v_1^A + v_1^B} N_1.$$

Hence, A's equilibrium profit is

$$\pi^{A*} = \frac{v_1^A v_2^A - v_1^B v_2^B}{v_1^A + v_1^B} N_1 N_2.$$

The equilibrium outcome under potential-maximizer selection is summarized as follows.

Proposition 2 *Generically and w.l.o.g., suppose $v_1^A v_2^A > v_1^B v_2^B$ and $v_1^A + v_1^B < v_2^A + v_2^B$. Under potential-maximizer selection, there is a unique equilibrium in this model. Stage 1 is a Bertrand equilibrium with*

$$p_1^{A*} = 0, \quad p_2^{A*} = \frac{v_1^A v_2^A - v_1^B v_2^B}{v_1^A + v_1^B} N_1, \quad p_1^{B*} = 0, \quad p_2^{B*} = 0.$$

All agents join platform A in stage 2, and platform A's equilibrium profit is

$$\pi^{A*} = \frac{v_1^A v_2^A - v_1^B v_2^B}{v_1^A + v_1^B} N_1 N_2.$$

As shown in Proposition 2, the market tips to a dominant platform with the higher value of $v_1^m v_2^m$ irrespective of the total number of agents N_1 and N_2 on each side. Following the baseline model, I discuss the three key implications under the current framework.

Divide-and-conquer strategy Similar to the baseline model, the dominant platform (A) always extracts surplus from one side and provides free access for the other side. The weaker (in terms of v_1^B and v_2^B) the competitor, the more surplus the dominant platform can extract from the money side.

Money/subsidy side In contrast to the monopoly-platform models, the money/subsidy side of the dominant platform depends on the average per-interaction benefits $v_1^A + v_1^B$ and $v_2^A + v_2^B$ across the competing platforms rather than its own per-interaction benefits v_1^A and v_2^A. This implies that the decision on the money/subsidy side for the dominant platform is significantly affected by the per-interaction benefits delivered by other competing platforms, even if the competitors' market shares are negligible. Therefore, there can be a reversal of the money/subsidy side for the dominant platform under competition. To see this, consider the following example:

$$v_1^A = 3, \quad v_2^A = 2, \quad v_1^B = 1, \quad v_2^B = 5. \tag{21}$$

Platform A favors group 1 more than group 2, but platform B favors group 2 much more than group 1. Suppose that initially A is a monopolist. By Table 1 in Section 2.3.3 (with group 1 and group 2 interchanged in that table), A monetizes group 1 and subsidizes group 2; its optimal pricing strategy and equilibrium profit are

$$p_1^{A*} = 3N_2, \quad p_2^{A*} = 0, \quad \pi^{A*} = 3N_1 N_2. \tag{22}$$

Suppose now B enters the market. Under competition, A still dominates the market by Proposition 2, and its optimal pricing strategy and equilibrium profit are

$$p_1^{A*} = 0, \quad p_2^{A*} = \frac{1}{4}N_1, \quad \pi^{A*} = \frac{1}{4}N_1 N_2.$$

Now, A subsidizes group 1 and monetizes group 2: the money/subsidy side of the dominant platform is reversed. Besides, if B is the monopolist, by Table 1, its optimal pricing strategy and equilibrium profit are

$$p_1^{B*} = 0, \quad p_2^{B*} = 5N_1, \quad \pi^{B*} = 5N_1 N_2.$$

Compared to (22), B actually makes a higher profit if A and B are separate monopolists because B can extract more surplus from one side. This implies that the optimal design for a monopoly platform might not work well under platform competition; this leads us to the discussion on the optimal design of competing platforms.

Optimal design When the platforms are very competitive (say, $v_1^A v_2^A \approx v_1^B v_2^B$), the optimal design of the competing platforms tends to favor both sides (more precisely, to maximize $v_1^m v_2^m$) because the platform with a lower value of $v_1^m v_2^m$ has zero market share in the equilibrium. By contrast, when one of the platforms is inferior (say, $v_1^B \approx v_2^B \approx 0$), the optimal design of the superior platform tends to favor only the money side in order to extract more surplus from that side.[41]

When platform A dominates the market (i.e., $v_1^A v_2^A > v_1^B v_2^B$), social surplus is $(v_1^A + v_2^A)N_1 N_2$; it can be less than the social surplus $(v_1^B + v_2^B)N_1 N_2$ if all agents join platform B instead (see (21) as an example). Besides, the optimal design of the dominant platform (A) is likely to be socially suboptimal: when the platforms are very competitive, the optimal design of A tends to maximize $v_1^A v_2^A$ instead of $v_1^A + v_2^A$; when B is inferior, the optimal design of A tends to favor only the money side.

[41] In fact, when $v_1^B = v_2^B = 0$, platform A's optimal pricing strategy and equilibrium profit in Proposition 2 are equal to those of the monopoly platform in Table 1.

5 Discussion

This section discusses and compares potential-maximizer selection with focal-platform selection, the most popular selection criterion under platform competition. I also discuss several extensions. Some of them are analyzed in the appendix.

Focal-platform selection As mentioned in the Introduction (p. 2–3), focal-platform selection is asymmetric in the sense that all agents always coordinate on a pre-specified platform whenever there are multiple equilibria. By contrast, potential-maximizer selection treats every platform symmetrically, i.e., the identity of a platform does not matter. Focal-platform selection faces another challenge: without a specific context, we cannot determine which platform should be the "focal" platform. By contrast, potential-maximizer selection unambiguously identifies the dominant platform in the equilibrium. In the equilibrium, the dominant platform is indeed "focal" because all agents coordinate on the dominant platform. Yet, this is an equilibrium outcome rather than an assumption from the start.

When multiple equilibria exist, consumers' expectations are the key. But how these expectations are formed is even more important. As pointed out in the pioneering work on network economics by Katz and Shapiro (1985, p. 439), "*...the expectations formation process remains an important element of the market to model explicitly*". They repeated the same point subsequently (1994, p. 97), "*...the two equilibria are rather different, and one would like to have a theory that includes the factors that lead to one outcome or the other*". Yet, this issue is largely ignored in the literature: researchers almost always take for granted that consumers' expectations are exogenously given.

By contrast, potential-maximizer selection provides a microfoundation for the formation of consumers' expectations by endogenizing them. By doing so, consumers' equilibrium expectations depend crucially on the strengths of cross-side network effects of the competing platforms as shown in Proposition 2. I thereby address the longest debate in network economics: do network effects lead to inefficient lock-in? The answer is no. As shown in Proposition 2, an inferior platform that delivers lower per-interaction benefits for both sides is defeated in the equilibrium. In other words, quality largely explains the success of a dominant platform as repeatedly argued by Liebowitz and Margolis (1990, 1994, 1995, 1996, 1999, 2013). This argument is also supported by recent empirical evidence (Tellis et al. 2009a, 2009b; Gretz 2010) and experimental results (Hossain and Morgan 2009; Hossain et al. 2011). Now, it is further justified by the theoretical results in this paper.

Alternative pricing instruments This paper follows Armstrong's framework where agents are charged a subscription fee to join a platform. Yet, potential-maximizer selection is also applicable to models with alternative pricing instruments such as transaction fees and two-part tariffs. If a monopoly platform charges transaction fees instead of subscription fees, oftentimes there is a unique equilibrium in the model.[42] By contrast, multiple equilibria often arise even if the competing platforms use transaction fees. Hence, potential-maximizer selection can be applied to resolve the multiple equilibria issue.

Appendix G modifies the duopoly-platform model in Section 4 so that the competing platforms charge transaction fees instead of subscription fees. In this alternative framework, platforms can adjust the net per-interaction benefits of the two sides with transaction fees. Thus, the market tips to a dominant platform with a larger sum of per-interaction benefits $v_1^m + v_2^m$ rather than the product of them $v_1^m v_2^m$ as in Section 4. Both platforms often divide and conquer. The money/subsidy side depends on its own per-interaction benefits v_1^m and v_2^m rather than the average per-interaction benefits $v_1^A + v_1^B$ and $v_2^A + v_2^B$ across the competing platforms as in Section 4. In addition, the optimal design of the competing platforms always favor both sides (more precisely, to maximize $v_1^m + v_2^m$), which is socially optimal. Appendix G.3 demonstrates how the analysis can be extended to two-part tariffs.

Same-side network effects Sometimes same-side network effects are present on one or both sides of a platform; they can also be positive (e.g. peer/learning effect) or negative

[42] If the monopoly platform in Section 2 charges transaction fees instead, it is easy to see that there is a unique equilibrium in which the platform charges the highest possible transaction fees on both sides and captures all agents' surplus.

(e.g. competition/congestion effect). Appendix H.1 extends the baseline model with negative same-side network effects and shows that it is equivalent to the heterogeneous-agent model in Section 3 from the platform's point of view. In particular, the platform's profit-maximization problem in stage 1 is identical under these two frameworks. Thus, all the results and implications in Section 3 carry over to this alternative framework. Appendix H.2 extends the baseline model with positive same-side network effects and shows that all the three key implications in Section 2 carry over to this richer framework. In particular, the platform always divides and conquers, and the money/subsidy side is independent of the same-side network effects.

Heterogeneous per-interaction benefits Section 3 follows Armstrong's framework where agents are heterogeneous in their personal costs from joining the platform. Yet, potential-maximizer selection is applicable even if agents are also heterogeneous in their per-interaction benefits. In this case, *Spence distortions* become an important issue as emphasized by Weyl (2010), i.e., platforms internalize network effects to marginal rather than average users. Nevertheless, as I argue elsewhere (Chan 2019), Spence distortions should not arise under strong network effects with multiple tipping equilibria because platforms compete for the adoption of all users rather than competing for the marginal user. The main finding of that paper is that all popular selection criteria in the literature lead to Spence distortions under strong network effects while potential-maximizer selection does not. This provides another justification for using potential-maximizer selection in platform markets.

Generalizing potential-maximizer selection Not all two-sided market models are weighted potential games. For example, if agents can multihome, in general this is not a weighted potential game. Nevertheless, potential-maximizer selection is not restricted to weighted potential games; it is also applicable to other weaker forms of potential games (see footnotes 7 and 8 for details). Hence, extending potential-maximizer selection to two-sided market models that are other weaker forms of potential games is a fruitful direction for future research.

6 Conclusion

This paper demonstrates how potential-maximizer selection can resolve the multiple equilibria issue and derives novel insights into two-sided markets. As explained, potential-maximizer selection is justified by many solid microfoundations in the game theory literature and widely supported by experimental results. Moreover, this paper shows that many two-sided market models are weighted potential games, and thus potential-maximizer selection can be applied uniformly to these models. Furthermore, the predictions under potential-maximizer selection match the reality well. In particular, two-sided platforms often divide and conquer, and the fundamental determinant of the money/subsidy side is the cross-side network effects only. This divide-and-conquer strategy implies that platforms are often designed to favor the money side much more than the subsidy side, which is often socially suboptimal. Last but not least, potential-maximizer selection is very tractable as demonstrated in this paper. Given all the above-mentioned advantages of potential-maximizer selection, I thereby recommend using potential-maximizer selection to resolve the multiple equilibria issue in two-sided markets whenever possible.

References

[1] Alos-Ferrer, C., & Netzer, N. (2010). The logit-response dynamics. *Games and Economic Behavior*, *68*(2), 413–427.

[2] Ambrus, A., & Argenziano, R. (2009). Asymmetric networks in two-sided markets. *American Economic Journal: Microeconomics*, *1*(1), 17–52.

[3] Anderson, S. P., Goeree, J. K., & Holt, C. A. (2001). Minimum-effort coordination games: Stochastic potential and logit equilibrium. *Games and Economic Behavior*, *34*(2), 177–199.

[4] Armstrong, M. (2006). Competition in two-sided markets. *The RAND Journal of Economics*, *37*(3), 668–691.

[5] Armstrong, M., & Wright, J. (2007). Two-sided markets, competitive bottlenecks and exclusive contracts. *Economic Theory*, *32*(2), 353–380.

[6] Biglaiser, G., Calvano, E., & Cremer, J. (2019). Incumbency advantage and its value. *Journal of Economics & Management Strategy*, *28*, 41–48.

[7] Blume, L. E. (1993). The statistical mechanics of strategic interaction. *Games and Economic Behavior*, *5*(3), 387–424.

[8] Caillaud, B., & Jullien, B. (2001). Competing cybermediaries. *European Economic Review*, *45*(4–6), 797–808.

[9] Caillaud, B., & Jullien, B. (2003). Chicken & egg: Competition among intermediation service providers. *The RAND Journal of Economics*, 309–328.

[10] Chan, L. T. (2019). Strong network effects eliminate Spence distortions: a potential-game approach. *Working paper.*

[11] Chen, R., & Chen, Y. (2011). The potential of social identity for equilibrium selection. *American Economic Review*, *101*(6), 2562–89.

[12] Carlsson, H., & Van Damme, E. (1993). Global games and equilibrium selection. *Econometrica*, *61*(5), 989–1018.

[13] Frankel, D. M., Morris, S., & Pauzner, A. (2003). Equilibrium selection in global games with strategic complementarities. *Journal of Economic Theory*, *108*(1), 1–44.

[14] Goeree, J. K., & Holt, C. A. (2005). An experimental study of costly coordination. *Games and Economic Behavior*, *51*(2), 349–364.

[15] Gretz, R. T. (2010). Hardware quality vs. network size in the home video game industry. *Journal of Economic Behavior & Organization*, *76*(2), 168–183.

[16] Hagiu, A. (2006). Pricing and commitment by two-sided platforms. *The RAND Journal of Economics*, *37*(3), 720–737.

[17] Hagiu, A., & Spulber, D. (2013). First-party content and coordination in two-sided markets. *Management Science*, *59*(4), 933–949.

[18] Halaburda, H., & Yehezkel, Y. (2013). Platform competition under asymmetric information. *American Economic Journal: Microeconomics*, *5*(3), 22–68.

[19] Halaburda, H., & Yehezkel, Y. (2019). Focality advantage in platform competition. *Journal of Economics & Management Strategy*, *28*, 49–59.

[20] Harsanyi, J. C., & Selten, R. (1988). A general theory of equilibrium selection in games. *MIT Press Books*, *1*.

[21] Hofbauer, J., & Sorger, G. (1999). Perfect foresight and equilibrium selection in symmetric potential games. *Journal of Economic Theory*, *85*(1), 1–23.

[22] Hossain, T., Minor, D., & Morgan, J. (2011). Competing matchmakers: an experimental analysis. *Management Science*, *57*(11), 1913–1925.

[23] Hossain, T., & Morgan, J. (2009). The quest for QWERTY. *American Economic Review Papers and Proceedings*, *99*(2), 435–40.

[24] Jullien, B. (2011). Competition in multi-sided markets: Divide and conquer. *American Economic Journal: Microeconomics*, *3*(4), 186–220.

[25] Jullien, B., & Pavan, A. (2018). Information Management and Pricing in Platform Markets. *Review of Economic Studies*, forthcoming.

[26] Kajii, A., & Morris, S. (1997). The robustness of equilibria to incomplete information. *Econometrica*, *65*(6), 1283–1309.

[27] Katz, M. L., & Shapiro, C. (1985). Network externalities, competition, and compatibility. *American Economic Review*, *75*(3), 424–440.

[28] Katz, M. L., & Shapiro, C. (1994). Systems competition and network effects. *Journal of Economic Perspectives*, *8*(2), 93–115.

[29] Liebowitz, S. J., & Margolis, S. E. (1990). The fable of the keys. *The Journal of Law and Economics*, *33*(1), 1–25.

[30] Liebowitz, S. J., & Margolis, S. E. (1994). Network externality: An uncommon tragedy. *Journal of Economic Perspectives*, *8*(2), 133–150.

[31] Liebowitz, S. J., & Margolis, S. E. (1995). Path dependence, lock-in, and history. *Journal of Law, Economics, & Organization*, 205–226.

[32] Liebowitz, S. J., & Margolis, S. E. (1996). Should technology choice be a concern of antitrust policy. *Harvard Journal of Law and Technology.*, *9*, 284–317.

[33] Liebowitz, S. J., & Margolis, S. E. (1999). Winners, losers & Microsoft; Competition and antitrust in high technology. *Independent Institute*.

[34] Liebowitz, S. J., & Margolis, S. E. (2013). The troubled path of the lock-in movement. *Journal of Competition Law and Economics*, *9*(1), 125–152.

[35] Matsui, A., & Matsuyama, K. (1995). An approach to equilibrium selection. *Journal of Economic Theory*, *65*(2), 415–434.

[36] Monderer, D., & Shapley, L. S. (1996). Potential games. *Games and Economic Behavior*, *14*(1), 124–143.

[37] Morris, S., & Shin, H. S. (2003). Global Games: Theory and Applications. *Advances in Economics and Econometrics*, Eighth World Congress (M. Dewatripont, L. Hansen, and S. Turnovsky, eds.). Cambridge University Press.

[38] Morris, S., & Ui, T. (2004). Best response equivalence. *Games and Economic Behavior*, *49*(2), 260–287.

[39] Morris, S., & Ui, T. (2005). Generalized potentials and robust sets of equilibria. *Journal of Economic Theory*, *124*(1), 45–78.

[40] Okada, D., & Tercieux, O. (2012). Log-linear dynamics and local potential. *Journal of Economic Theory*, *147*(3), 1140–1164.

[41] Oyama, D., Takahashi, S., & Hofbauer, J. (2008). Monotone methods for equilibrium selection under perfect foresight dynamics. *Theoretical Economics*, *3*(2), 155–192.

[42] Rochet, J. C., & Tirole, J. (2003). Platform competition in two-sided markets. *Journal of the European Economic Association*, *1*(4), 990–1029.

[43] Rochet, J. C., & Tirole, J. (2006). Two-sided markets: a progress report. *The RAND Journal of Economics*, *37*(3), 645–667.

[44] Rosenthal, R. W. (1973). A class of games possessing pure-strategy Nash equilibria. *International Journal of Game Theory*, *2*(1), 65–67.

[45] Rysman, M. (2009). The economics of two-sided markets. *Journal of Economic Perspectives*, *23*(3), 125–43.

[46] Sakovics, J., & Steiner, J. (2012). Who matters in coordination problems?. *American Economic Review*, *102*(7), 3439–61.

[47] Sandholm, W. H. (2001). Potential games with continuous player sets. *Journal of Economic Theory*, *97*(1), 81–108.

[48] Tellis, G. J., Yin, E., & Niraj, R. (2009a). Does quality win? Network effects versus quality in high-tech markets. *Journal of Marketing Research*, *46*(2), 135–149.

[49] Tellis, G. J., Niraj, R., & Yin, E. (2009b). Reply to comments: Why and how quality wins over network effects and what it means. *Journal of Marketing Research*, *46*(2), 150–62.

[50] Ui, T. (2001). Robust equilibria of potential games. *Econometrica*, *69*(5), 1373–1380.

[51] Van Huyck, J. B., Battalio, R. C., & Beil, R. O. (1990). Tacit coordination games, strategic uncertainty, and coordination failure. *American Economic Review*, *80*(1), 234–248.

[52] Weyl, E. G. (2010). A price theory of multi-sided platforms. *American Economic Review*, *100*(4), 1642–72.

[53] White, A., & Weyl, E. G. (2016). Insulated platform competition. *Available at SSRN 1694317.*

Appendix

A Proofs

A.1 Lemma 4

First, I identify the potential maximizer for Case 1, and then for Case 2. For Case 1, by Lemma 3, the respective potentials of the three equilibria are

$$\begin{aligned}
P(N_1, \mathbf{1}_{[0,N_2]}|p_1,p_2) &= N_1N_2 - \frac{p_1}{v_1}N_1 - \frac{p_2}{v_2}N_2 - \frac{1}{v_2}\int_0^{N_2} t(k)dk \\
&= -\frac{p_1}{v_1}N_1 + \frac{1}{v_2}\int_0^{N_2}(t(N_2)-t(k))dk, \quad \text{(by (9))} \qquad (23) \\
P(\widehat{N}_1, \mathbf{1}_{[0,\widehat{N}_2]}|p_1,p_2) &= \widehat{N}_1\widehat{N}_2 - \frac{p_1}{v_1}\widehat{N}_1 - \frac{p_2}{v_2}\widehat{N}_2 - \frac{1}{v_2}\int_0^{\widehat{N}_2} t(k)dk \\
&= -\frac{p_2}{v_2}\widehat{N}_2 - \frac{1}{v_2}\int_0^{\widehat{N}_2} t(k)dk \ (\widehat{N}_2 = \frac{p_1}{v_1} \text{ as shown in Figure 1}) \qquad (24) \\
&\leq 0, \\
P(0,\mathbf{0}|p_1,p_2) &= 0.
\end{aligned}$$

Given that the potential maximizer is the equilibrium with the highest potential, the unique equilibrium in stage 2 under potential-maximizer selection for Case 1 is

$$\begin{aligned}
(\mathbf{a}_1^*, \mathbf{a}_2^*) &= (\mathbf{1}, \mathbf{1}_{[0,N_2]}) \quad \text{if } p_1 \leq \frac{v_1}{v_2N_1}\int_0^{N_2}(t(N_2)-t(k))dk, \\
(\mathbf{a}_1^*, \mathbf{a}_2^*) &= (\mathbf{0}, \mathbf{0}) \qquad \text{otherwise.}
\end{aligned}$$

I now turn to Case 2. The potentials of the Pareto-dominant equilibrium and the unstable equilibrium are the same as those in Case 1, i.e., given by (23) and (24) respectively. By Lemma 3, the potential of the Pareto-dominated equilibrium is

$$P(0, \mathbf{1}_{[0,\underline{N}_2]}|p_1,p_2) = -\frac{p_2}{v_2}\underline{N}_2 - \frac{1}{v_2}\int_0^{\underline{N}_2} t(k)dk \geq 0.$$

The above potential is positive because $-p_2 < t(k)$ for all $k \in (\underline{N}_2, \widehat{N}_2]$ by (10). Hence, the potential maximizer is

$$\begin{aligned}
(\mathbf{a}_1^*, \mathbf{a}_2^*) &= (\mathbf{1}, \mathbf{1}_{[0,N_2]}) \quad \text{if } p_1 \leq v_1\underline{N}_2 + \frac{v_1}{v_2N_1}\int_{\underline{N}_2}^{N_2}(t(N_2)-t(k))\,dk, \\
(\mathbf{a}_1^*, \mathbf{a}_2^*) &= (\mathbf{0}, \mathbf{1}_{[0,\underline{N}_2]}) \quad \text{otherwise.}
\end{aligned}$$

A.2 Proposition 1

First, I show that p_2^* and N_2^* in (15) are unique. After that, it is obvious that p_2^* is negative. To prove the first part, it suffices to show that the right-hand side of (15) decreases with p_2^*:

$$\begin{aligned}
&\frac{d\left(\left(N_2^* - \frac{v_1}{v_2}\left(N_2^* - \underline{N}_2^*\right)\right)t'(N_2^*)\right)}{dp_2^*} \\
=\ & \left(\left(\frac{v_1}{v_2} - 1\right)\frac{1}{t'(N_2^*)} - \frac{v_1}{v_2}\frac{1}{t'(\underline{N}_2^*)}\right)t'(N_2^*) - \left(N_2^* - \frac{v_1}{v_2}\left(N_2^* - \underline{N}_2^*\right)\right)\frac{t''(N_2^*)}{t'(N_2^*)} \\
=\ & -1 - \frac{v_1}{v_2}\left(\frac{t'(N_2^*)}{t'(\underline{N}_2^*)} - 1\right) + \frac{t(\underline{N}_2^*)t''(N_2^*)}{(t'(N_2^*))^2} \quad \text{(by (15))} \\
\leq\ & -1 + \frac{t(\underline{N}_2^*)t''(N_2^*)}{(t'(N_2^*))^2} \quad (t \text{ is increasing and convex}) \\
\leq\ & -1 + \frac{t(\underline{N}_2^*)}{t(N_2^*)} \quad (t \text{ is log-concave, i.e., } t(N_2^*)t''(N_2^*) \leq (t'(N_2^*))^2) \\
\leq\ & 0. \quad (t \text{ is increasing})
\end{aligned}$$

I now show that $p_2^* \leq 0$. When $p_2^* = 0$, the left-hand side of (15) is zero while the right-hand side is negative. Given that the left-hand side increases with p_2^* while the right-hand side decreases with p_2^*, p_2^* must be negative.

A.3 Table 2

I first compute the comparative statics for the left table (where $v_1 \leq v_2$), and then for the right table (where $v_1 \geq v_2$).

Left table Figures 2 and 3 in Appendix E show that the signs of $\frac{\partial N_2^*}{\partial v_2}$, $\frac{\partial p_1^*}{\partial v_2}$, and $\frac{\partial p_1^* N_1}{\partial v_2}$ are ambiguous. I now compute the comparative statics in the following order:

	N_2^*	p_1^*	p_2^*	$p_1^*N_1$	$p_2^*N_2^*$	π^*
N_1	[1]+	[9] +/−	[3]+	[11]+	[6]+	[13]+
v_1	[2]+	[10]+	[4]−	[12]+	[7]−	[14]+
v_2	+/−	+/−	[5]+	+/−	[8]+	[15]+

N_2^* is characterized by (14) in Proposition 1; its comparative statics is also based on (14):

[1] $\frac{\partial N_2^*}{\partial N_1} = \frac{v_2^2}{(2v_2-v_1)t'(N_2^*)+(v_2-v_1)N_2^*t''(N_2^*)} \geq 0.$

[2] $\frac{\partial N_2^*}{\partial v_1} = \frac{N_2^*t'(N_2^*)}{(2v_2-v_1)t'(N_2^*)+(v_2-v_1)N_2^*t''(N_2^*)} \geq 0.$

As shown in (9), p_2^* can be expressed as a function of N_2^*. Hence, its comparative statics is based on that of N_2^* in [1] and [2]:

[3] $\frac{\partial p_2^*}{\partial N_1} = v_2 - t'(N_2^*)\frac{\partial N_2^*}{\partial N_1} = \frac{v_2(v_2-v_1)(t'(N_2^*)+N_2^*t''(N_2^*))}{(2v_2-v_1)t'(N_2^*)+(v_2-v_1)N_2^*t''(N_2^*)} \geq 0.$

[4] $\frac{\partial p_2^*}{\partial v_1} = -t'(N_2^*)\frac{\partial N_2^*}{\partial v_1} \leq 0.$

[5] $\frac{\partial p_2^*}{\partial v_2} = N_1 - t'(N_2^*)\frac{\partial N_2^*}{\partial v_2} = \frac{(v_2-v_1)N_1(t'(N_2^*)+N_2^*t''(N_2^*))+\frac{v_1}{v_2}N_2^*(t'(N_2^*))^2}{(2v_2-v_1)t'(N_2^*)+(v_2-v_1)N_2^*t''(N_2^*)} \geq 0.$

The comparative statics of $p_2^*N_2^*$ is based on those of p_2^* and N_2^* in [1]–[5]:

[6] $\frac{\partial p_2^*N_2^*}{\partial N_1} = N_2^*\frac{\partial p_2^*}{\partial N_1} + p_2^*\frac{\partial N_2^*}{\partial N_1} \geq 0.$

[7] $\frac{\partial p_2^*N_2^*}{\partial v_1} = -\frac{v_1}{v_2}N_2^*t'(N_2^*)\frac{\partial N_2^*}{\partial v_1} \leq 0.$

[8] $\frac{\partial p_2^*N_2^*}{\partial v_2} = \frac{(v_2-v_1)N_1N_2^*(2t'(N_2^*)+N_2^*t''(N_2^*))+\left(\frac{v_1}{v_2}N_2^*t'(N_2^*)\right)^2}{(2v_2-v_1)t'(N_2^*)+(v_2-v_1)N_2^*t''(N_2^*)} \geq 0.$

As shown in Proposition 1, p_1^* is a function of N_2^*. Hence, its comparative statics is based on that of N_2^* in [1] and [2]:

[9] $$\begin{aligned}\frac{\partial p_1^*}{\partial N_1} &= \frac{v_1}{v_2N_1^2}\left(N_1N_2^*t'(N_2^*)\frac{\partial N_2^*}{\partial N_1} - \int_0^{N_2^*}(t(N_2^*)-t(k))\,dk\right)\\ &= \frac{v_1}{v_2N_1^2}\left(N_2^*t'(N_2^*)\frac{t(N_2^*)+\left(1-\frac{v_1}{v_2}\right)N_2^*t'(N_2^*)}{\left(2-\frac{v_1}{v_2}\right)t'(N_2^*)+\left(1-\frac{v_1}{v_2}\right)N_2^*t''(N_2^*)} - N_2^*t(N_2^*) + \int_0^{N_2^*}t(k)dk\right).\end{aligned}$$

$\frac{\partial p_1^*}{\partial N_1}$ can be positive as shown in (33) of Appendix E.1. I now show with an example that $\frac{\partial p_1^*}{\partial N_1}$ can also be negative. Suppose $N_1 = 2.6111$, $v_1 = 1$, $v_2 = 10$, and define

$$\widehat{t}(k) = \begin{cases} e^k & \text{if } 3 \leq k \leq \overline{N}_2, \\ e^3(x-2) & \text{if } 2 \leq k \leq 3, \\ 0 & \text{if } 0 \leq k \leq 2. \end{cases}$$

The stand-alone cost function $\widehat{t}$ does not satisfy all the imposed assumptions in Section 3.1 because it is not strictly increasing nor twice-differentiable. Yet, we can "perturb" $\widehat{t}$ a bit to another function t, such that t satisfies all the imposed assumptions in Section 3.1, $t(3) = t'(3) = t''(3) = e^3$, and $\int_0^3 t(k)dk$ is arbitrarily close to $\int_0^3 \widehat{t}(k)dk = \frac{e^3}{2}$. Given this stand-alone cost function t, by Proposition 1, the equilibrium mass of group-2 participants is $N_2^* = 3$. Hence, by [9], $\frac{\partial p_1^*}{\partial N_1} = -2.5618 \times 10^{-2} < 0$.

[10] $\frac{\partial p_1^*}{\partial v_1} = \frac{1}{v_2 N_1}\left(\int_0^{N_2^*} (t(N_2^*) - t(k))\, dk + v_1 N_2^* t'(N_2^*)\frac{\partial N_2^*}{\partial v_1}\right) \geq 0.$

The comparative statics of $p_1^* N_1$ is based on that of p_1^* in [9] and [10]:

[11] $\frac{\partial p_1^* N_1}{\partial N_1} = \frac{v_1}{v_2} N_2^* t'(N_2^*)\frac{\partial N_2^*}{\partial N_1} \geq 0.$

[12] $\frac{\partial p_1^* N_1}{\partial v_1} = N_1 \frac{\partial p_1^*}{\partial v_1} \geq 0.$

π^* is given by Proposition 1, and its comparative statics can be obtained by applying the envelope theorem:

[13] $\frac{\partial \pi^*}{\partial N_1} = v_2 N_2^* \geq 0.$

[14] $\frac{\partial \pi^*}{\partial v_1} = \frac{1}{v_2}\int_0^{N_2^*} (t(N_2^*) - t(k))\, dk \geq 0.$

[15] $\frac{\partial \pi^*}{\partial v_2} = \frac{N_2^*}{v_2}\left(1 - \frac{v_1}{v_2}\right)(t(N_2^*) + N_2^* t'(N_2^*)) + \frac{v_1}{v_2^2}\int_0^{N_2^*} t(k)dk \geq 0.$

Right table When computing the comparative statics for the case $v_1 \geq v_2$, two expressions based on re-organizing (15) in Proposition 1 are frequently used:

$$v_2 N_1 = t(N_2^*) - t(\underline{N}_2^*), \tag{25}$$

$$\frac{v_1}{v_2} = \frac{t(\underline{N}_2^*) + N_2^* t'(N_2^*)}{(N_2^* - \underline{N}_2^*)t'(N_2^*)}. \tag{26}$$

I now compute the comparative statics in the following order (starting from [16]):

	N_2^*	p_1^*	p_2^*	$p_1^* N_1$	$p_2^* N_2^*$	π^*
N_1	[19]+	[25]+	[16]−	[28]+	[22]−	[31]+
v_1	[20]+	[26]+	[17]−	[29]+	[23]−	[32]+
v_2	[21]+	[27] +/−	[18]+	[30] +/−	[24]+	[33]+

p_2^* is characterized by (15) in Proposition 1; its comparative statics is also based on (15):

[16] $\frac{\partial p_2^*}{\partial N_1} = \frac{-(v_1-v_2)+(v_1\underline{N}_2^*-(v_1-v_2)N_2^*)\frac{t''(N_2^*)}{t'(N_2^*)}}{2+\frac{v_1}{v_2}\left(\frac{t'(N_2^*)}{t'(\underline{N}_2^*)}-1\right)-\frac{t(\underline{N}_2^*)t''(N_2^*)}{(t'(N_2^*))^2}} \leq 0.$[43]

[17] $\frac{\partial p_2^*}{\partial v_1} = -\frac{\frac{N_2^*-\underline{N}_2^*}{v_2}t'(N_2^*)}{2+\frac{v_1}{v_2}\left(\frac{t'(N_2^*)}{t'(\underline{N}_2^*)}-1\right)-\frac{t(\underline{N}_2^*)t''(N_2^*)}{(t'(N_2^*))^2}} \leq 0.$

[18] $\frac{\partial p_2^*}{\partial v_2} = \frac{\left(\frac{v_1}{v_2^2}(N_2^*-\underline{N}_2^*)-\left(\frac{v_1}{v_2}-1\right)\frac{N_1}{t'(N_2^*)}\right)t'(N_2^*)+\left(\frac{v_1}{v_2^2}\underline{N}_2^*-\left(\frac{v_1}{v_2}-1\right)N_2^*\right)\frac{N_1 t''(N_2^*)}{t'(N_2^*)}}{2+\frac{v_1}{v_2}\left(\frac{t'(N_2^*)}{t'(\underline{N}_2^*)}-1\right)-\frac{t(\underline{N}_2^*)t''(N_2^*)}{(t'(N_2^*))^2}}$

$$= \frac{\frac{v_1}{v_2}\left((N_2^*-\underline{N}_2^*)t'(N_2^*)-v_2N_1\right)+v_2N_1\left(1-\left(\frac{v_1}{v_2}(N_2^*-\underline{N}_2^*)-N_2^*\right)\frac{t''(N_2^*)}{t'(N_2^*)}\right)}{v_2\left(2+\frac{v_1}{v_2}\left(\frac{t'(N_2^*)}{t'(\underline{N}_2^*)}-1\right)-\frac{t(\underline{N}_2^*)t''(N_2^*)}{(t'(N_2^*))^2}\right)}$$

$$= \frac{\frac{v_1}{v_2}\left(N_2^*-\underline{N}_2^*\right)\left(t'(N_2^*)-\frac{t(N_2^*)-t(\underline{N}_2^*)}{N_2^*-\underline{N}_2^*}\right)+v_2N_1\left(1-\frac{t(\underline{N}_2^*)t''(N_2^*)}{(t'(N_2^*))^2}\right)}{v_2\left(2+\frac{v_1}{v_2}\left(\frac{t'(N_2^*)}{t'(\underline{N}_2^*)}-1\right)-\frac{t(\underline{N}_2^*)t''(N_2^*)}{(t'(N_2^*))^2}\right)} \text{ (by (25) and (26))}$$

$$= N_1\frac{1+\frac{v_1}{v_2}\left(\frac{t'(N_2^*)}{\frac{t(N_2^*)-t(\underline{N}_2^*)}{N_2^*-\underline{N}_2^*}}-1\right)-\frac{t(\underline{N}_2^*)t''(N_2^*)}{(t'(N_2^*))^2}}{2+\frac{v_1}{v_2}\left(\frac{t'(N_2^*)}{t'(\underline{N}_2^*)}-1\right)-\frac{t(\underline{N}_2^*)t''(N_2^*)}{(t'(N_2^*))^2}} \text{ (by (25))}$$

$$\geq \frac{N_1\left(1-\frac{t(\underline{N}_2^*)}{t(N_2^*)}\right)}{2+\frac{v_1}{v_2}\left(\frac{t'(N_2^*)}{t'(\underline{N}_2^*)}-1\right)-\frac{t(\underline{N}_2^*)t''(N_2^*)}{(t'(N_2^*))^2}} \geq 0. \text{ (} t \text{ is convex and log-concave)}$$

As shown in (9), N_2^* can be expressed as a function of p_2^*. Hence, its comparative statics is based on that of p_2^* in [16]–[18]:

[19] $\frac{\partial N_2^*}{\partial N_1} = \frac{1}{t'(N_2^*)}\left(v_2 - \frac{\partial p_2^*}{\partial N_1}\right) \geq 0.$

[43] As shown in Appendix A.2, the denominator $2+\frac{v_1}{v_2}\left(\frac{t'(N_2^*)}{t'(\underline{N}_2^*)}-1\right)-\frac{t(\underline{N}_2^*)t''(N_2^*)}{\left(t'(N_2^*)\right)^2}$ is positive.

[20] $\frac{\partial N_2^*}{\partial v_1} = -\frac{1}{t'(N_2^*)}\frac{\partial p_2^*}{\partial v_1} \geq 0.$

[21] $\frac{\partial N_2^*}{\partial v_2} = \frac{1}{t'(N_2^*)}\left(N_1 - \frac{\partial p_2^*}{\partial v_2}\right)$

$$= \frac{N_1}{t'(N_2^*)}\frac{2+\frac{v_1}{v_2}\left(\frac{t'(N_2^*)}{t'(\underline{N}_2^*)}-1\right)-\frac{t(\underline{N}_2^*)t''(N_2^*)}{(t'(N_2^*))^2}-\left(1+\frac{v_1}{v_2}\left(\frac{t'(N_2^*)}{\frac{t(N_2^*)-t(\underline{N}_2^*)}{N_2^*-\underline{N}_2^*}}-1\right)-\frac{t(\underline{N}_2^*)t''(N_2^*)}{(t'(N_2^*))^2}\right)}{2+\frac{v_1}{v_2}\left(\frac{t'(N_2^*)}{t'(\underline{N}_2^*)}-1\right)-\frac{t(\underline{N}_2^*)t''(N_2^*)}{(t'(N_2^*))^2}}$$

$$= \frac{N_1}{t'(N_2^*)}\frac{1+\frac{v_1}{v_2}t'(N_2^*)\left(\frac{1}{t'(\underline{N}_2^*)}-\frac{1}{\frac{t(N_2^*)-t(\underline{N}_2^*)}{N_2^*-\underline{N}_2^*}}\right)}{2+\frac{v_1}{v_2}\left(\frac{t'(N_2^*)}{t'(\underline{N}_2^*)}-1\right)-\frac{t(\underline{N}_2^*)t''(N_2^*)}{(t'(N_2^*))^2}} \geq 0.$$

The comparative statics of $p_2^*N_2^*$ is based on those of p_2^* and N_2^* in [16]–[21]:

[22] $\frac{\partial p_2^*N_2^*}{\partial N_1} = \frac{\partial p_2^*}{\partial N_1}N_2^* + p_2^*\frac{\partial N_2^*}{\partial N_1} \leq 0.$

[23] $\frac{\partial p_2^*N_2^*}{\partial v_1} = \frac{\partial p_2^*}{\partial v_1}N_2^* + p_2^*\frac{\partial N_2^*}{\partial v_1} \leq 0.$

[24] $\frac{\partial p_2^*N_2^*}{\partial v_2} = \left(N_2^* - \frac{p_2^*}{t'(N_2^*)}\right)\frac{\partial p_2^*}{\partial v_2} + \frac{p_2^*N_1}{t'(N_2^*)}$

$$= \left(\frac{t(\underline{N}_2^*)+N_2^*t'(N_2^*)}{t'(N_2^*)}\right)N_1\frac{1+\frac{v_1}{v_2}\left(\frac{t'(N_2^*)}{\frac{t(N_2^*)-t(\underline{N}_2^*)}{N_2^*-\underline{N}_2^*}}-1\right)-\frac{t(\underline{N}_2^*)t''(N_2^*)}{(t'(N_2^*))^2}}{2+\frac{v_1}{v_2}\left(\frac{t'(N_2^*)}{t'(\underline{N}_2^*)}-1\right)-\frac{t(\underline{N}_2^*)t''(N_2^*)}{(t'(N_2^*))^2}} - \frac{t(\underline{N}_2^*)N_1}{t'(N_2^*)} \text{ (by (10))}$$

$$= \frac{N_1t(\underline{N}_2^*)}{t'(N_2^*)}\frac{-1+\frac{v_1}{v_2}t'(N_2^*)\left(\left(1+\frac{N_2^*t'(N_2^*)}{t(\underline{N}_2^*)}\right)\frac{1}{\frac{t(N_2^*)-t(\underline{N}_2^*)}{N_2^*-\underline{N}_2^*}}-\frac{1}{t'(\underline{N}_2^*)}\right)-\frac{N_2^*t''(N_2^*)}{t'(N_2^*)}}{2+\frac{v_1}{v_2}\left(\frac{t'(N_2^*)}{t'(\underline{N}_2^*)}-1\right)-\frac{t(\underline{N}_2^*)t''(N_2^*)}{(t'(N_2^*))^2}}$$

$$\geq \frac{N_1t(\underline{N}_2^*)}{t'(N_2^*)}\frac{-1+\frac{v_1}{v_2}t'(N_2^*)\left(\frac{1}{t'(N_2^*)}+\frac{N_2^*}{t(\underline{N}_2^*)}-\frac{\underline{N}_2^*}{t(\underline{N}_2^*)}\right)-\frac{N_2^*t'(N_2^*)}{t(N_2^*)}}{2+\frac{v_1}{v_2}\left(\frac{t'(N_2^*)}{t'(\underline{N}_2^*)}-1\right)-\frac{t(\underline{N}_2^*)t''(N_2^*)}{(t'(N_2^*))^2}} \text{ (t is convex)}$$

$$\geq \frac{N_1t(\underline{N}_2^*)}{t'(N_2^*)}\frac{\frac{t(\underline{N}_2^*)+N_2^*t'(N_2^*)}{(N_2^*-\underline{N}_2^*)t'(N_2^*)}\frac{t'(N_2^*)}{t(\underline{N}_2^*)}(N_2^*-\underline{N}_2^*)-\frac{N_2^*t'(N_2^*)}{t(N_2^*)}}{2+\frac{v_1}{v_2}\left(\frac{t'(N_2^*)}{t'(\underline{N}_2^*)}-1\right)-\frac{t(\underline{N}_2^*)t''(N_2^*)}{(t'(N_2^*))^2}} \text{ (by (26) and $v_1 \geq v_2$)}$$

$$= \frac{N_1t(\underline{N}_2^*)}{t'(N_2^*)}\frac{1+\frac{N_2^*t'(N_2^*)}{t(\underline{N}_2^*)}-\frac{N_2^*t'(N_2^*)}{t(N_2^*)}}{2+\frac{v_1}{v_2}\left(\frac{t'(N_2^*)}{t'(\underline{N}_2^*)}-1\right)-\frac{t(\underline{N}_2^*)t''(N_2^*)}{(t'(N_2^*))^2}} \geq 0. \text{ (t is increasing)}$$

As shown in Proposition 1, p_1^* is a function of N_2^*. Hence, its comparative statics is based on that of N_2^* in [19]–[21]:

[25] $\frac{\partial p_1^*}{\partial N_1} = v_1 \frac{\partial \underline{N}_2^*}{\partial N_1} - \frac{v_1}{v_2 N_1}(t(N_2^*) - t(\underline{N}_2^*))\frac{\partial N_2^*}{\partial N_1}$
$+\frac{v_1}{v_2 N_1}(N_2^* - \underline{N}_2^*)t'(N_2^*)\frac{\partial N_2^*}{\partial N_1} - \frac{v_1}{v_2 N_1^2}\int_{\underline{N}_2^*}^{N_2^*}(t(N_2^*) - t(k))dk$
$= \frac{v_1}{v_2 N_1}(N_2^* - \underline{N}_2^*)\left(v_2 - \frac{\partial p_2^*}{\partial N_1}\right) - \frac{v_1}{v_2 N_1^2}\int_{\underline{N}_2^*}^{N_2^*}(t(N_2^*) - t(k))dk$ (by (25))
$\geq \frac{v_1}{v_2 N_1^2}\left((t(N_2^*) - t(\underline{N}_2^*))(N_2^* - \underline{N}_2^*) - \int_{\underline{N}_2^*}^{N_2^*}(t(N_2^*) - t(k))dk\right)$ (by (25) and $\frac{\partial p_2^*}{\partial N_1} \leq 0$)
$= \frac{v_1}{v_2 N_1}\int_{\underline{N}_2^*}^{N_2^*}(t(k) - t(\underline{N}_2^*))dk \geq 0.$

[26] $\frac{\partial p_1^*}{\partial v_1} = \underline{N}_2^* + v_1 \frac{\partial \underline{N}_2^*}{\partial v_1} + \frac{1}{v_2 N_1}\int_{\underline{N}_2^*}^{N_2^*}(t(N_2^*) - t(k))dk$
$-\frac{v_1}{v_2 N_1}(t(N_2^*) - t(\underline{N}_2^*))\frac{\partial \underline{N}_2^*}{\partial v_1} + \frac{v_1}{v_2 N_1}(N_2^* - \underline{N}_2^*)t'(N_2^*)\frac{\partial N_2^*}{\partial v_1}$
$= \underline{N}_2^* + \frac{1}{v_2 N_1}\int_{\underline{N}_2^*}^{N_2^*}(t(N_2^*) - t(k))dk + \frac{v_1}{v_2 N_1}(N_2^* - \underline{N}_2^*)t'(N_2^*)\frac{\partial N_2^*}{\partial v_1} \geq 0.$

[27] $\frac{\partial p_1^*}{\partial v_2}$ can be positive as shown in Figures 2 and 3 of Appendix E. I now show with an example that $\frac{\partial p_1^*}{\partial v_2}$ can also be negative. Suppose $N_1 = v_1 = 1$ and define

$$\widehat{t}(k) = \begin{cases} 2k - \frac{1}{2} & \text{if } \frac{1}{2} \leq k \leq \overline{N}_2, \\ k & \text{if } 0 \leq k \leq \frac{1}{2}. \end{cases}$$

The stand-alone cost function $\widehat{t}$ does not satisfy all the imposed assumptions in Section 3.1 because it is not twice-differentiable. Yet, we can "perturb" $\widehat{t}$ a bit to a smooth function t, such that t satisfies all the imposed assumptions in Section 3.1, and $t(k) = \widehat{t}(k)$ for all $k \in [0, \overline{N}_2]$ except for those that are arbitrarily close to $\frac{1}{2}$. Given this stand-alone cost function t, by (15), we have

$$p_2^* = \frac{v_2}{2} - \frac{1}{2}, \quad N_2^* = \frac{1}{2} + \frac{v_2}{4}, \quad \underline{N}_2^* = \frac{1}{2} - \frac{v_2}{2}.$$

Hence, by Proposition 1, the platform's optimal group-1 price is $p_1^* = \frac{1}{2} - \frac{v_2}{16}$, which is decreasing in v_2.

The comparative statics of $p_1^* N_1$ is based on that of p_1^* in [25]–[27]:

[28] $\frac{\partial p_1^* N_1}{\partial N_1} = \frac{\partial p_1^*}{\partial N_1} + p_1^* \geq 0.$

[29] $\frac{\partial p_1^* N_1}{\partial v_1} = N_1 \frac{\partial p_1^*}{\partial v_1} \geq 0.$

[30] $\frac{\partial p_1^* N_1}{\partial v_2} = N_1 \frac{\partial p_1^*}{\partial v_2}$; the sign is ambiguous.

π^* is given by Proposition 1, and its comparative statics can be obtained by applying the envelope theorem:

[31] $\frac{\partial \pi^*}{\partial N_1} = v_1 N_2^* + \frac{v_2 p_2^*}{t'(N_2^*)} = v_1 \underline{N}_2^* + v_2 N_2^* \geq 0.$

[32] $\frac{\partial \pi^*}{\partial v_1} = N_1 \underline{N}_2^* + \frac{1}{v_2} \int_{\underline{N}_2^*}^{N_2^*} \left(t(N_2^*) - t(k)\right) dk \geq 0.$

To show $\frac{\partial \pi^*}{\partial v_2} \geq 0$, I first prove that $p_2^* \geq -\frac{v_1 N_1}{2}$ when $v_2 \to 0$. As shown in (15),

$$v_2 p_2^* = \left(v_1 t^{-1}(-p_2^*) - (v_1 - v_2)\, t^{-1}(v_2 N_1 - p_2^*)\right) t'(t^{-1}(v_2 N_1 - p_2^*)).$$

Given that both sides converge to zero when $v_2 \to 0$, by L'Hospital's rule, we can differentiate both sides of the above equation with respect to v_2 and obtain

$$p_2^* = \left(N_2^* - \frac{(v_1 - v_2)}{t'(N_2^*)} N_1\right) t'(N_2^*) + \left(v_1 \underline{N}_2^* - (v_1 - v_2)\, N_2\right) \frac{N_1 t''(N_2^*)}{t'(N_2^*)}.$$

As $v_2 \to 0$, both $\underline{N}_2^*$ and N_2^* converge to $t^{-1}(-p_2^*)$. The above equation becomes

$$v_1 N_1 = t(N_2^*) + N_2^* t'(N_2^*) \geq 2t(N_2^*) = -2p_2^*. \quad (t \text{ is convex})$$

Therefore, we have $p_2^* \geq -\frac{v_1 N_1}{2}$ as $v_2 \to 0$. By [18] (i.e., $\frac{\partial p_2^*}{\partial v_2} \geq 0$ when $v_1 \geq v_2$), we have $p_2^* \geq -\frac{v_1 N_1}{2}$ for all $v_1 \geq v_2$. I now show that $\frac{\partial \pi^*}{\partial v_2} \geq 0$.

[33] $\frac{\partial \pi^*}{\partial v_2} = \frac{v_1}{v_2^2} \int_{\underline{N}_2^*}^{N_2^*} \left(t(k) - t(\underline{N}_2^*)\right) dk + \frac{p_2^* N_1}{t'(N_2^*)}$
$\geq \frac{v_1}{v_2^2} \left(\int_{\underline{N}_2^*}^{N_2^*} \left(t(k) - t(\underline{N}_2^*)\right) dk + \frac{(t(N_2^*) - t(\underline{N}_2^*))^2}{2t'(N_2^*)} \right)$ (by (25))
$\geq \frac{v_1}{v_2^2} \left(\frac{(t(N_2^*) - t(\underline{N}_2^*))^2}{2t'(N_2^*)} - \frac{(t(N_2^*) - t(\underline{N}_2^*))^2}{2t'(N_2^*)} \right) = 0.$ (t is convex)

A.4 Corollary 1

I first prove that $\frac{\partial \pi^*}{\partial v_1} \leq \frac{\partial \pi^*}{\partial v_2}$ when $2v_1 \leq v_2$. When $v_1 \leq v_2$, $\frac{\partial \pi^*}{\partial v_1}$ and $\frac{\partial \pi^*}{\partial v_2}$ are given by [14] and [15] in Appendix A.3 respectively, i.e.,

$$\begin{aligned}
\frac{\partial \pi^*}{\partial v_2} - \frac{\partial \pi^*}{\partial v_1} &= \frac{N_2^*}{v_2}\left(1 - \frac{v_1}{v_2}\right)\left(t(N_2^*) + N_2^* t'(N_2^*)\right) + \frac{v_1}{v_2^2}\int_0^{N_2^*} t(k)dk - \frac{1}{v_2}\int_0^{N_2^*} \left(t(N_2^*) - t(k)\right) dk \\
&= \frac{1}{v_2}\left(N_2^* t'(N_2^*) + \frac{1}{N_2^*}\int_0^{N_2^*} t(k)dk - \frac{v_1}{v_2}\left(t(N_2^*) + N_2^* t'(N_2^*) - \frac{1}{N_2^*}\int_0^{N_2^*} t(k)dk\right)\right).
\end{aligned}$$

Hence,

$$\frac{\partial \pi^*}{\partial v_1} \leq \frac{\partial \pi^*}{\partial v_2} \quad \text{if and only if} \quad \frac{v_1}{v_2} \leq 1 - \frac{t(N_2^*) - \frac{2}{N_2^*}\int_0^{N_2^*} t(k)dk}{N_2^* t'(N_2^*) + t(N_2^*) - \frac{1}{N_2^*}\int_0^{N_2^*} t(k)dk}.$$

Note that

$$\begin{aligned}
\frac{t(N_2^*) - \frac{2}{N_2^*}\int_0^{N_2^*} t(k)dk}{N_2^* t'(N_2^*) + t(N_2^*) - \frac{1}{N_2^*}\int_0^{N_2^*} t(k)dk} &\leq \frac{t(N_2^*) - \frac{1}{2N_2^*}\int_0^{N_2^*} t(k)dk}{2t(N_2^*) - \frac{1}{N_2^*}\int_0^{N_2^*} t(k)dk} \quad (t \text{ is convex}) \\
&\leq \frac{1}{2}.
\end{aligned}$$

Therefore, $\frac{\partial \pi^*}{\partial v_1} \leq \frac{\partial \pi^*}{\partial v_2}$ when $2v_1 \leq v_2$. I now prove that $\frac{\partial \pi^*}{\partial v_1} \geq \frac{\partial \pi^*}{\partial v_2}$ when $v_1 \geq v_2$. When $v_1 \geq v_2$, $\frac{\partial \pi^*}{\partial v_1}$ and $\frac{\partial \pi^*}{\partial v_2}$ are given by [32] and [33] in Appendix A.3 respectively, i.e.,

$$\begin{aligned}
\frac{\partial \pi^*}{\partial v_1} - \frac{\partial \pi^*}{\partial v_2} &= N_1 \underline{N}_2^* + \frac{1}{v_2}\int_{\underline{N}_2^*}^{N_2^*} \left(t(N_2^*) - t(k)\right) dk - \frac{v_1}{v_2^2}\int_{\underline{N}_2^*}^{N_2^*} \left(t(k) - t(\underline{N}_2^*)\right) dk + \frac{p_2^* N_1}{t'(N_2^*)} \\
&= \frac{1}{v_2}\left(\begin{array}{c} v_2 N_1 \left(\frac{t(\underline{N}_2^*) + \underline{N}_2^* t'(N_2^*)}{t'(N_2^*)}\right) \\ + \int_{\underline{N}_2^*}^{N_2^*} \left(t(N_2^*) - t(k)\right) dk - \frac{v_1}{v_2}\int_{\underline{N}_2^*}^{N_2^*} \left(t(k) - t(\underline{N}_2^*)\right) dk \end{array}\right) \quad (\text{by } (14)) \\
&= \frac{1}{v_2}\left(\begin{array}{c} \left(t(N_2^*) - t(\underline{N}_2^*)\right)\left(\frac{t(\underline{N}_2^*) + \underline{N}_2^* t'(N_2^*)}{t'(N_2^*)} - (N_2^* - \underline{N}_2^*)\right) \\ + \int_{\underline{N}_2^*}^{N_2^*} \left(t(N_2^*) - t(k)\right) dk - \frac{v_1}{v_2}\int_{\underline{N}_2^*}^{N_2^*} \left(t(k) - t(\underline{N}_2^*)\right) dk \end{array}\right) \quad (\text{by } (25)) \\
&= \frac{1}{v_2}\left(\begin{array}{c} \left(\frac{v_1}{v_2} - 1\right)(N_2^* - \underline{N}_2^*)\left(t(N_2^*) - t(\underline{N}_2^*)\right) \\ + \int_{\underline{N}_2^*}^{N_2^*} \left(t(N_2^*) - t(k)\right) dk - \frac{v_1}{v_2}\int_{\underline{N}_2^*}^{N_2^*} \left(t(k) - t(\underline{N}_2^*)\right) dk \end{array}\right) \quad (\text{by } (26)) \\
&= \frac{1}{v_2}\left(\frac{v_1}{v_2}\int_{\underline{N}_2^*}^{N_2^*} \left(t(N_2^*) - t(k)\right) dk - \int_{\underline{N}_2^*}^{N_2^*} \left(t(k) - t(\underline{N}_2^*)\right) dk\right) \\
&\geq \frac{1}{v_2}\left(\int_{\underline{N}_2^*}^{N_2^*} \left(t(N_2^*) - t(k)\right) dk - \int_{\underline{N}_2^*}^{N_2^*} \left(t(k) - t(\underline{N}_2^*)\right) dk\right) \geq 0. \quad (t \text{ is convex})
\end{aligned}$$

A.5 Lemma 5

For a group-1 agent, if there are n_1^A group-1 agents (excluding himself) and n_2^A group-2 agents joining platform A, his payoff difference between joining platform A or B is

$$\begin{aligned} u_1^A(n_2^A, p_1^A) - u_1^B(n_2^B, p_1^B) &= v_1^A n_2^A - p_1^A - v_1^B n_2^B + p_1^B \quad \text{(by (16))} \\ &= (v_1^A + v_1^B) n_2^A - v_1^B N_2 - \Delta p_1. \end{aligned}$$

The corresponding difference in P is

$$P(n_1^A + 1, n_2^A | \Delta p_1, \Delta p_2) - P(n_1^A, n_2^A | \Delta p_1, \Delta p_2) = n_2^A - \frac{v_1^B N_2 + \Delta p_1}{v_1^A + v_1^B}. \quad \text{(by (18))}$$

Clearly, the change in the group-1 agent's payoff from unilaterally switching actions is proportional (with proportion $v_1^A + v_1^B$) to the change in P. The same logic applies to a group-2 agent (with proportion $v_2^A + v_2^B$ for him). Therefore, every subgame is a weighted potential game with the potential function given by (18).

B Mathematical Definition of Weighted Potential Games

The definition can be found in Monderer and Shapley (1996, p. 127–128). Let $I \equiv \{1, \ldots, N\}$ denote the set of players, A_i denote the set of actions for player i, and $u_i : A \to \mathbb{R}$ denote the payoff function for player i, where $A \equiv A_1 \times \cdots \times A_N$. A game $G \equiv (I, A, (u_i)_{i \in I})$ is a weighted potential game if there is a function $P : A \to \mathbb{R}$ and a vector $(w_i)_{i \in I} \in \mathbb{R}^N_{++}$ such that, for all $i \in I$ and $a_{-i} \in A_{-i}$,

$$u_i(a_i, a_{-i}) - u_i(a_i', a_{-i}) = w_i(P(a_i, a_{-i}) - P(a_i', a_{-i})), \quad \forall a_i, a_i' \in A_i.$$

C The Finite-Agent Model of Section 3.1 and the Proof of Lemma 3

In this appendix, first I present a finite-agent model that converges to the continuum-agent model in Section 3.1. Then, I show that the finite-agent model is a weighted potential game with the potential function converging to (11) in Lemma 3. The subsequent analysis for this limiting case of the finite-agent model is identical to that of the continuum-agent model in Section 3.2 and thus omitted.

In the finite-agent model, there are $\gamma N_1 \in \mathbb{N}$ identical group-1 agents and $\gamma \overline{N}_2 \in \mathbb{N}$ heterogeneous group-2 agents ($\gamma, N_1, \overline{N}_2 \in \mathbb{R}_{++}$). If the platform attracts $n_1 \equiv \sum_{k=1}^{\gamma N_1} a_1^k$ group-1 agents and $n_2 \equiv \sum_{k=1}^{\gamma \overline{N}_2} a_2^k$ group-2 agents, the payoffs of a group-1 agent and agent $k \in \{1, \ldots, \gamma \overline{N}_2\}$ from joining the platform are

$$u_1(n_2, p_1; \gamma) = \frac{v_1}{\gamma} n_2 - p_1, \quad u_2^k(n_1, p_2; \gamma) = \frac{v_2}{\gamma} n_1 - p_2 - t\left(\frac{k}{\gamma}\right), \tag{27}$$

where the function $t : \{\frac{1}{\gamma}, \frac{2}{\gamma}, \ldots, \overline{N}_2\} \to \mathbb{R}_+$ specifies each group-2 agent's stand-alone cost from joining the platform.[44]

In this finite-agent model, the platform's profit is

$$\pi(n_1, n_2, p_1, p_2; \gamma) = \frac{1}{\gamma}(p_1 n_1 + p_2 n_2).$$

The rest of the model setup is the same as that of the baseline model.

Clearly, this finite-agent model converges to the continuum-agent model in Section 3.1 as $\gamma \to \infty$. It remains to show that this finite-agent model is a weighted potential game with the potential function converging to (11) in Lemma 3 as $\gamma \to \infty$. I now prove that every subgame in stage 2 is a weighted potential game with the potential function

$$P(n_1, \mathbf{a}_2 | p_1, p_2; \gamma) = \frac{1}{\gamma^2}\left(n_1 n_2 - \frac{\gamma p_1}{v_1} n_1 - \frac{\gamma p_2}{v_2} n_2 - \frac{\gamma}{v_2} \sum_{k=1}^{\gamma \overline{N}_2} t\left(\frac{k}{\gamma}\right) a_2^k\right). \tag{28}$$

Proof. For a group-1 agent, if there are n_1 group-1 participants (excluding himself) and the group-2 agents' action profile is $\mathbf{a}_2$, his payoff difference between joining the platform or not is

$$u_1(n_2, p_1; \gamma) - 0 = \frac{v_1}{\gamma} n_2 - p_1. \quad \text{(by (27))}$$

The corresponding difference in P is

$$P(n_1 + 1, \mathbf{a}_2 | p_1, p_2; \gamma) - P(n_1, \mathbf{a}_2 | p_1, p_2; \gamma) = \frac{n_2}{\gamma^2} - \frac{p_1}{\gamma v_1}. \quad \text{(by (28))}$$

[44] Section 3.1 imposes some assumptions on the function t. Yet, these assumptions play no role in this appendix and thus omitted.

Clearly, the change in the group-1 agent's payoff from unilaterally switching actions is proportional (with proportion γv_1) to the change in P.

Similarly, for agent k, if there are n_1 group-1 participants and the group-2 agents' action profile (except himself) is $\mathbf{a}_2^{-k}$, his payoff difference between joining the platform or not is

$$u_2^k(n_1, p_2; \gamma) - 0 = \frac{v_2}{\gamma} n_1 - p_2 - t\left(\frac{k}{\gamma}\right). \quad \text{(by (27))}$$

The corresponding difference in P is

$$P(n_1, \mathbf{a}_2^{-k}, a_2^k = 1 | p_1, p_2; \gamma) - P(n_1, \mathbf{a}_2^{-k}, a_2^k = 0 | p_1, p_2; \gamma) = \frac{n_1}{\gamma^2} - \frac{p_2}{\gamma v_2} - \frac{1}{\gamma v_2} t\left(\frac{k}{\gamma}\right). \quad \text{(by (28))}$$

Clearly, the change in agent k's payoff from unilaterally switching actions is proportional (with proportion γv_2) to the change in P. Therefore, every subgame is a weighted potential game with the potential function given by (28). ■

Clearly, (28) converges to (11) in Lemma 3 as $\gamma \to \infty$. This completes the proof of Lemma 3. The subsequent analysis of this finite-agent model as $\gamma \to \infty$ is identical to that of the continuum-agent model in Section 3.2 and thus omitted.

D Two Benchmarks for the Model in Section 3.1

Appendix D.1 analyzes the model under Pareto-dominance selection while Appendix D.2 analyzes that under Pareto-dominated selection.

D.1 Pareto-Dominance Selection

Under Pareto-dominance selection, all relevant agents always join the platform whenever there are multiple equilibria. Given that group-1 agents are identical, the platform charges the highest possible group-1 price so that all group-1 agents will join the platform with zero surplus in stage 2, i.e., $p_1^* = v_1 N_2$.

It remains to derive the platform's optimal group-2 price p_2^*. From (8), the platform's profit-maximization problem in stage 1 becomes

$$\max_{p_2 \le v_2 N_1} v_1 N_2 N_1 + p_2 N_2. \tag{29}$$

Solving the above optimization problem gives us p_2^* and N_2^*:

$$p_2^* = v_2 N_1 - t(N_2^*) = N_2^* t'(N_2^*) - v_1 N_1. \tag{30}$$

As shown in the above expression, the platform's optimal group-2 price p_2^* is equal to the standard monopoly markup $N_2^* t'(N_2^*)$, adjusted downward by the network effects $v_1 N_1$ to group-1 agents. (30) actually appears in Armstrong's (2006, expression 3) paper.[45]

After identifying p_2^* and N_2^*, we can derive the platform's optimal group-1 price and its equilibrium profit from (29):

$$p_1^* = v_1 N_2^*, \quad \pi^* = (v_1 + v_2) N_1 N_2^* - N_2^* t(N_2^*).$$

As compared to Proposition 1, the equilibrium outcome under Pareto-dominance selection is very different from that under potential-maximizer selection.

D.2 Pareto-Dominated Selection

The second benchmark analyzes the model under Pareto-dominated selection, in which all relevant agents always coordinate on the Pareto-dominated equilibrium whenever there are multiple equilibria. To make a positive profit, the platform has to guarantee participation from one side by subsidizing that side and then monetizes the other side. Therefore, the platform either subsidizes group 1 and monetizes group 2, or subsidizes group 2 and monetizes group 1. I discuss these two strategies one by one.

Group-1 subsidy strategy For the first strategy, the platform provides free access for group 1 (i.e., $p_1^* = 0$) so that all group-1 agents will join the platform for sure. The coordination problem no longer exists among the agents: for any group-2 price $p_2 \leq v_2 N_1$ set by the platform in stage 1, the continuum $[0, N_2]$ of group-2 agents will join the platform for sure. As shown in (9), there is a one-to-one correspondence between $p_2 \leq v_2 N_1$ and $N_2 \in [0, \overline{N}_2)$.

[45] As mentioned in footnote 13, Armstrong imposes Pareto-dominance selection in his analysis. The terms N_2^* and $t'(N_2^*)$ in (30) correspond to $\phi_2(u_2)$ and $\frac{1}{\phi_2'(u_2)}$ respectively in his paper.

Hence, we can assume that the platform chooses $N_2 \in [0, \overline{N}_2)$ rather than $p_2 \leq v_2 N_1$ to maximize its profit:

$$\max_{N_2 \in [0, \overline{N}_2)} (v_2 N_1 - t(N_2)) N_2. \tag{31}$$

Solving the above optimization problem gives us p_2^* and N_2^*:

$$p_2^* = v_2 N_1 - t(N_2^*) = N_2^* t'(N_2^*).$$

Then, we can derive the platform's equilibrium profit from (31):

$$\pi^* = (v_2 N_1 - t(N_2^*)) N_2^*.$$

Group-2 subsidy strategy If the platform subsidizes group 2 and monetizes group 1, it strictly subsidizes group-2 participants such that the continuum $[0, \underline{N}_2]$ of group-2 agents, whose stand-alone costs are really low, will join the platform for sure. All other group-2 agents will not join the platform under Pareto-dominated selection. Therefore, the platform optimally sets $p_1^* = v_1 \underline{N}_2$ so that all group-1 agents will join the platform with zero surplus in stage 2. As shown in (10), there is a one-to-one correspondence between $p_2 \leq 0$ and $\underline{N}_2 \in [0, \overline{N}_2)$. Hence, we can assume that the platform chooses $\underline{N}_2 \in [0, \overline{N}_2)$ rather than $p_2 \leq 0$ to maximize its profit:

$$\max_{\underline{N}_2 \in [0, \overline{N}_2)} (v_1 N_1 - t(\underline{N}_2)) \underline{N}_2. \tag{32}$$

Solving the above optimization problem gives us p_2^* and $\underline{N}_2^*$:

$$p_2^* = -t(\underline{N}_2^*) = \underline{N}_2^* t'(\underline{N}_2^*) - v_1 N_1.$$

Then, we can derive the platform's optimal group-1 price and its equilibrium profit from (32):

$$p_1^* = v_1 \underline{N}_2^*, \quad \pi^* = (v_1 N_1 - t(\underline{N}_2^*)) \underline{N}_2^*.$$

By comparing the two optimization problems (31) and (32), it is clear that group-1 subsidy strategy yields higher profit than group-2 subsidy strategy if and only if $v_1 \leq v_2$, and it is irrespective of the number of group-1 agents N_1 and the exact form of group-2 agents' stand-alone cost function t. Thus, the equilibrium outcome under Pareto-dominated selection is summarized as follows.

Lemma 7 *Under Pareto-dominated selection, the platform adopts group-1 subsidy strategy when $v_1 \leq v_2$ and group-2 subsidy strategy when $v_1 \geq v_2$. Under group-1 subsidy strategy, the platform's group-1 price is $p_1^* = 0$, and the optimal group-2 price p_2^* and the equilibrium mass of group-2 participants N_2^* are implicitly given by*

$$p_2^* = v_2 N_1 - t(N_2^*) = N_2^* t'(N_2^*).$$

Under group-2 subsidy strategy, $p_1^* = v_1 \underline{N}_2^*$, where p_2^* and the equilibrium mass of group-2 participants $\underline{N}_2^*$ are implicitly given by

$$p_2^* = -t(\underline{N}_2^*) = \underline{N}_2^* t'(\underline{N}_2^*) - v_1 N_1.$$

As compared to Proposition 1, the equilibrium outcome under Pareto-dominated selection differs from that under potential-maximizer selection. Nevertheless, their equilibrium outcomes share some common features. For example, under both selection criteria, the platform monetizes group 1 and subsidizes group 2 if and only if $v_1 \geq v_2$. Still, the platform's optimal pricing strategy under Pareto-dominated selection is discontinuous and takes a "more extreme" form compared to that under potential-maximizer selection.

E Two Examples of the Model in Section 3.1

Appendix E.1 analyzes the model with linear stand-alone cost $t(k) = tk$ while Appendix E.2 analyzes that with quadratic stand-alone cost $t(k) = tk^2$.

E.1 Linear Stand-Alone Cost

By Proposition 1, the equilibrium outcome when $t(k) = tk$ is

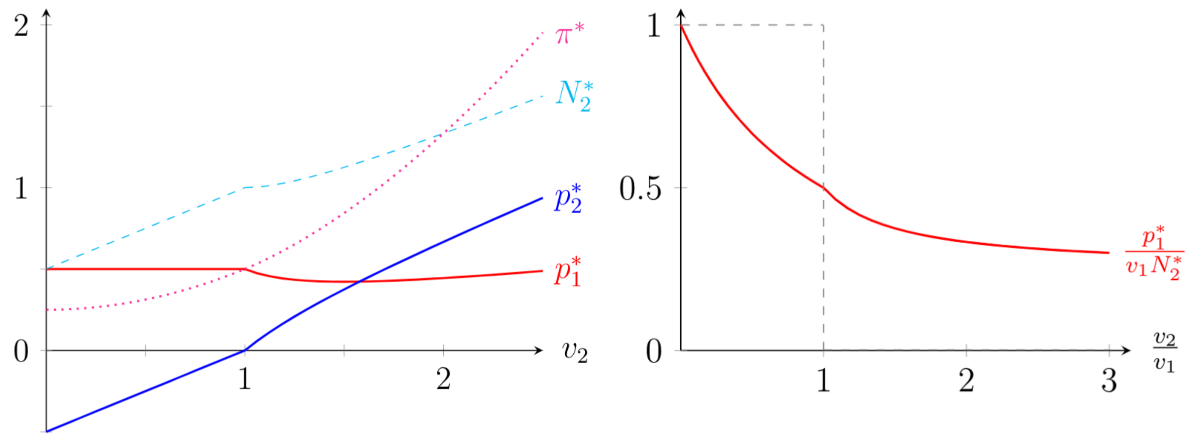

Figure 2: The equilibrium outcome (left) and the proportion of surplus extracted from group 1 (right) under linear stand-alone cost (with $v_1 = N_1 = t = 1$).

1. when $v_1 \leq v_2$:

$$p_1^* = \frac{v_1 v_2^3 N_1}{2t\left(2v_2 - v_1\right)^2}, \ p_2^* = \frac{(v_2 - v_1)v_2 N_1}{2v_2 - v_1}, \ N_2^* = \frac{v_2^2 N_1}{t(2v_2 - v_1)}, \ \pi^* = \frac{v_2^3 N_1^2}{2t\left(2v_2 - v_1\right)}; \quad (33)$$

2. when $v_1 \geq v_2$:

$$p_1^* = \frac{v_1^2 N_1}{2t}, \ p_2^* = -\frac{(v_1 - v_2)N_1}{2}, \ N_2^* = \frac{\left(v_1 + v_2\right) N_1}{2t}, \ \pi^* = \frac{\left(v_1^2 + v_2^2\right) N_1^2}{4t}. \quad (34)$$

Figure 2 (left) sketches the equilibrium outcome when $v_1 = N_1 = t = 1$. The platform's optimal group-1 price p_1^* is non-monotonic in v_2. Nevertheless, the proportion of surplus $\frac{p_1^*}{v_1 N_2^*}$ extracted from group 1 by the platform always decreases with v_2 irrespective of values of v_1, N_1, and t. To see this, by (33) and (34),

$$\frac{p_1^*}{v_1 N_2^*} = \begin{cases} \frac{\frac{v_2}{v_1}}{2\left(2\frac{v_2}{v_1} - 1\right)} & \text{if } v_1 \leq v_2, \\ \frac{1}{1 + \frac{v_2}{v_1}} & \text{if } v_1 \geq v_2. \end{cases}$$

As shown in the above expression, $\frac{p_1^*}{v_1 N_2^*}$ depends only on the ratio $\frac{v_2}{v_1}$ as sketched in Figure 2 (right). The dashed line represents the proportion of surplus extracted from group 1 by the monopoly platform in the baseline model: it extracts all surplus from group 1 when $v_1 \geq v_2$

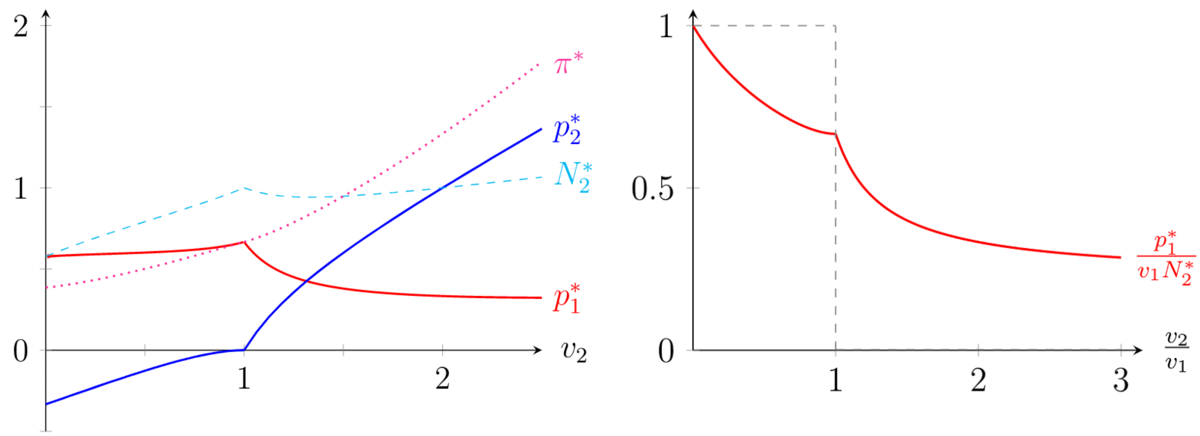

Figure 3: The equilibrium outcome (left) and the proportion of surplus extracted from group 1 (right) under quadratic stand-alone cost (with $v_1 = N_1 = t = 1$).

and leaves all surplus to group 1 when $v_1 \leq v_2$. By contrast, $\frac{p_1^*}{v_1 N_2^*}$ decreases gradually with $\frac{v_2}{v_1}$ in the current framework due to the smooth demand expansion effect on group 2.[46]

E.2 Quadratic Stand-Alone Cost

By Proposition 1, the equilibrium outcome when $t(k) = tk^2$ is

1. when $v_1 \leq v_2$:

$$p_1^* = \frac{2v_1 v_2^2}{3(3v_2 - 2v_1)^{\frac{3}{2}}}\sqrt{\frac{N_1}{t}},\ p_2^* = \frac{2(v_2 - v_1)v_2 N_1}{3v_2 - 2v_1},\ N_2^* = v_2\sqrt{\frac{N_1}{t(3v_2 - 2v_1)}},\ \pi^* = \frac{2v_2^2 N_1^{\frac{3}{2}}}{3\sqrt{t(3v_2 - 2v_1)}};$$

2. when $v_1 \geq v_2$:

$$p_2^* = -\frac{2N_1}{3}\frac{v_1\sqrt{v_1^2 - 2v_2^2 + 2v_1 v_2} + v_1^2 + 3v_2^2 - 5v_1 v_2}{4v_1 - 3v_2},$$

and other variables are functions of p_2^* as characterized by Proposition 1.

[46] Compared to the baseline model, group-2 agents incur additional stand-alone costs in the current framework. Therefore, the proportion of surplus extracted from group 2 cannot be compared meaningfully in a similar fashion.

Figure 3 (left) sketches the equilibrium outcome when $v_1 = N_1 = t = 1$. The equilibrium mass of group-2 participants N_2^* is non-monotonic in v_2. Similar to the previous example, the proportion of surplus extracted from group 1 $\frac{p_1^*}{v_1 N_2^*}$ depends only on the ratio $\frac{v_2}{v_1}$ as sketched in Figure 3 (right). Again, $\frac{p_1^*}{v_1 N_2^*}$ decreases gradually with $\frac{v_2}{v_1}$ in the current framework due to the smooth demand expansion effect on group 2.

F Formal Analysis of Armstrong's Original Model

Appendix F.1 presents Armstrong's (2006, Section 3) original model. Appendix F.2 analyzes the model. Appendix F.3 studies a special case of the model and derives further implications.

F.1 Model

In Armstrong's original model, there are a continuum $[0, \overline{N}_1]$ of heterogeneous group-1 agents and a continuum $[0, \overline{N}_2]$ of heterogeneous group-2 agents ($\overline{N}_1, \overline{N}_2 \in \mathbb{R}_{++}$). If the platform attracts $n_1 \equiv \int_0^{\overline{N}_1} a_1^k dk$ group-1 agents and $n_2 \equiv \int_0^{\overline{N}_2} a_2^k dk$ group-2 agents, the payoff from joining the platform for agent $k \in [0, \overline{N}_i]$ from group $i = 1, 2$ is

$$u_i^k(n_j, p_i) = v_i n_j - p_i - t_i(k),$$

where the function $t_i : [0, \overline{N}_i] \to \mathbb{R}_+$ specifies each group-i agent's stand-alone cost from joining the platform. I permute the agents such that t_1 and t_2 are increasing. I also assume that t_1 and t_2 are strictly convex, twice-differentiable, $t_1(0) = t_2(0) = 0$, and $t_1(\overline{N}_1), t_2(\overline{N}_2) \to \infty$. Under these assumptions, agents from each side are sufficiently heterogeneous in a smooth way. The rest of the model setup is the same as that in Section 3.1.

F.2 Analysis

Compared to the model in Section 3.1, demand expansion effects are now present on both sides. Hence, the equilibrium masses of group-1 and group-2 participants in stage 2 are simultaneously determined by the prices (p_1, p_2) set by the platform in stage 1. Similar to Section 3.2, there are two stable equilibria and an unstable equilibrium in stage 2 when both

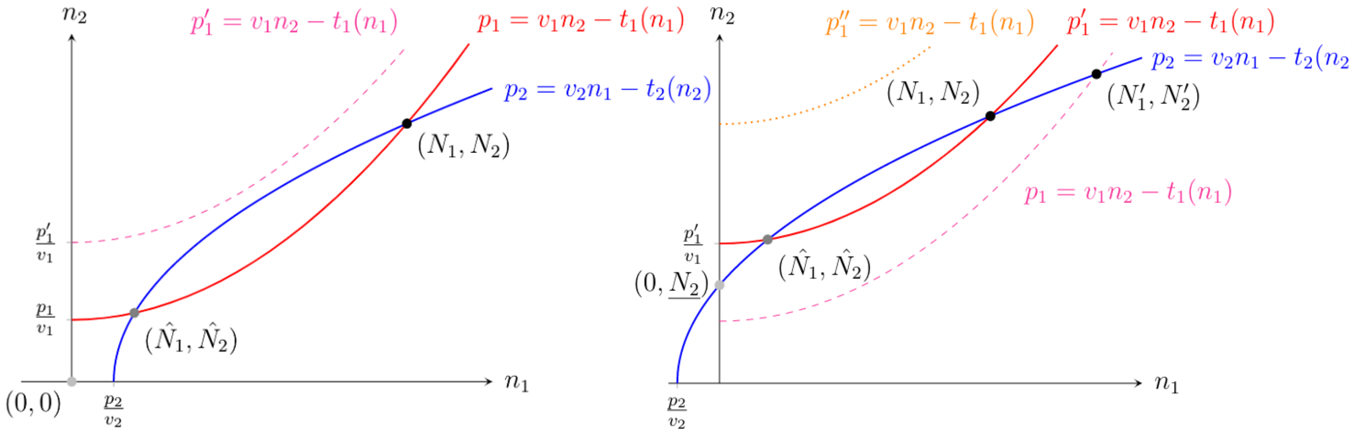

Figure 4: The equilibria of the subgame in stage 2 when $p_1, p_2 \geq 0$ (left) and $p_2 \leq 0 \leq p_1$ (right).

prices p_1 and p_2 are positive and sufficiently low. As shown in Figure 4 (left), the three equilibria in stage 2 are

1. Pareto-dominant equilibrium: $(\mathbf{a}_1^*, \mathbf{a}_2^*) = (\mathbf{1}_{[0,N_1]}, \mathbf{1}_{[0,N_2]})$;
2. unstable equilibrium: $(\mathbf{a}_1^*, \mathbf{a}_2^*) = (\mathbf{1}_{[0,\widehat{N}_1]}, \mathbf{1}_{[0,\widehat{N}_2]})$;
3. Pareto-dominated equilibrium: $(\mathbf{a}_1^*, \mathbf{a}_2^*) = (\mathbf{0}, \mathbf{0})$,

where (N_1, N_2) and $(\widehat{N}_1, \widehat{N}_2)$ are the solutions to the system of equations

$$p_1 = v_1 n_2 - t_1(n_1), \quad p_2 = v_2 n_1 - t_2(n_2). \tag{35}$$

As shown in the dashed line of Figure 4 (left), the Pareto-dominated equilibrium is the unique equilibrium if the platform's prices are too high.

Similar to Section 3.2, when one of the platform's price, say, p_2, is negative,[47] joining the platform is the (strictly) dominant strategy for agent $k \in [0, \underline{N}_2)$ from group 2, where $\underline{N}_2 \equiv t_2^{-1}(-p_2)$. As shown in Figure 4 (right), there are at most three equilibria in stage 2 when $p_2 \leq 0 \leq p_1$, and they are

[47] The analysis for $p_1 \leq 0$ is analogous to that of $p_2 \leq 0$ and thus omitted in this appendix. I also omit the case in which the platform sets negative prices on both sides.

1. Pareto-dominant equilibrium: $(\mathbf{a}_1^*, \mathbf{a}_2^*) = (\mathbf{1}_{[0,N_1]}, \mathbf{1}_{[0,N_2]})$;

2. unstable equilibrium: $(\mathbf{a}_1^*, \mathbf{a}_2^*) = (\mathbf{1}_{[0,\widehat{N}_1]}, \mathbf{1}_{[0,\widehat{N}_2]})$;

3. Pareto-dominated equilibrium: $(\mathbf{a}_1^*, \mathbf{a}_2^*) = (\mathbf{0}, \mathbf{1}_{[0,\underline{N}_2]})$.

As shown in the solid line of Figure 4 (right), given a fixed value of p_2, all three equilibria are present when p_1 is not too high or too low. The unstable equilibrium disappears if p_1 is too low as illustrated by the dashed line.[48] The Pareto-dominated equilibrium is the unique equilibrium if p_1 is too high as illustrated by the dotted line.

As explained in footnote 13, Armstrong (2006, Section 3) imposes Pareto-dominance selection in his analysis. Under Pareto-dominance selection, we can assume that the platform chooses the masses of participants $(N_1, N_2) \in [0, \overline{N}_1) \times [0, \overline{N}_2)$ rather than setting prices $(p_1, p_2) \in \mathbb{R}^2$ to maximize its profit. From (8) and (35), the platform's profit-maximization problem in stage 1 becomes

$$\max_{(N_1,N_2)\in[0,\overline{N}_1)\times[0,\overline{N}_2)} (v_1 N_2 - t_1(N_1))N_1 + (v_2 N_1 - t_2(N_2))N_2. \tag{36}$$

Solving the above optimization problem gives us (p_1^*, p_2^*) and (N_1^*, N_2^*):

$$\begin{aligned} p_1^* &= v_1 N_2^* - t_1(N_1^*) = N_1^* t_1'(N_1^*) - v_2 N_2^*, \\ p_2^* &= v_2 N_1^* - t_2(N_2^*) = N_2^* t_2'(N_2^*) - v_1 N_1^*. \end{aligned} \tag{37}$$

As shown in the above expressions, the platform's optimal prices p_1^* and p_2^* are equal to the standard monopoly markups $N_1^* t_1'(N_1^*)$ and $N_2^* t_2'(N_2^*)$, adjusted downward by the network effects $v_2 N_2^*$ and $v_1 N_1^*$ to the other side. Expressions (37) actually appear in Armstrong's (2006, expression 3) paper.[49]

I now analyze the model under potential-maximizer selection. First, I show that every subgame in stage 2 is a weighted potential game.

[48] In this case, the Pareto-dominant equilibrium becomes (N_1', N_2') in Figure 4 (right), and the Pareto-dominated equilibrium is still $(0, \underline{N}_2)$.

[49] The terms N_i^* and $t_i'(N_i^*)$ in (37) correspond to $\phi_i(u_i)$ and $\frac{1}{\phi_i'(u_i)}$ respectively in his paper.

Lemma 8 *Every subgame in stage 2 is a weighted potential game with the potential function*

$$P(\mathbf{a}_1, \mathbf{a}_2|p_1, p_2) = n_1 n_2 - \frac{p_1}{v_1} n_1 - \frac{p_2}{v_2} n_2 - \frac{1}{v_1}\int_0^{\overline{N}_1} t_1(k) a_1^k dk - \frac{1}{v_2}\int_0^{\overline{N}_2} t_2(k) a_2^k dk.$$

Proof. The proof is very similar to that of Lemma 3 in Section 3.2 and thus omitted. ■

Compared to Lemma 3, the extra term $\frac{1}{v_1}\int_0^{\overline{N}_1} t_1(k) a_1^k dk$ captures the total stand-alone cost incurred by group-1 participants.

After identifying the potential function, the next step is to identify the potential maximizer. It can be done so by following the same approach in proving Lemma 4 in Section 3.2 as shown below. Similar to Lemma 4, the unstable equilibrium is never selected under potential-maximizer selection.

Lemma 9 *Under potential-maximizer selection, the unique equilibrium of the subgame in stage 2 if there are multiple equilibria is*

1. *when* $p_1, p_2 \geq 0$*:*

$$\begin{aligned}
(\mathbf{a}_1^*, \mathbf{a}_2^*) &= (\mathbf{1}_{[0,N_1]}, \mathbf{1}_{[0,N_2]}) \quad \textit{if } \frac{1}{v_1}\int_0^{N_1}(p_1 + t_1(k))dk + \frac{1}{v_2}\int_0^{N_2}(p_2 + t_2(k))dk \leq N_1 N_2, \\
(\mathbf{a}_1^*, \mathbf{a}_2^*) &= (\mathbf{0}, \mathbf{0}) \qquad \textit{otherwise.}
\end{aligned} \tag{38}$$

2. *when* $p_2 \leq 0 \leq p_1$*:*

$$\begin{aligned}
(\mathbf{a}_1^*, \mathbf{a}_2^*) &= (\mathbf{1}_{[0,N_1]}, \mathbf{1}_{[0,N_2]}) \quad \textit{if } \frac{1}{v_1}\int_0^{N_1}(p_1 + t_1(k))dk + \frac{1}{v_2}\int_{\underline{N}_2}^{N_2}(p_2 + t_2(k))dk \leq N_1 N_2, \\
(\mathbf{a}_1^*, \mathbf{a}_2^*) &= (\mathbf{0}, \mathbf{1}_{[0,\underline{N}_2]}) \qquad \textit{otherwise.}
\end{aligned} \tag{39}$$

Proof. The proof is very similar to Appendix A.1. Here, I only prove the non-trivial part, i.e., to show that the unstable equilibrium is never the potential maximizer. I prove for the case $p_1, p_2 \geq 0$; the proof for the case $p_2 \leq 0 \leq p_1$ is analogous. By Lemma 8, the potential of the unstable equilibrium is

$$P(\mathbf{1}_{[0,\widehat{N}_1]}, \mathbf{1}_{[0,\widehat{N}_2]}|p_1, p_2) = \widehat{N}_1 \widehat{N}_2 - \frac{1}{v_1}\int_0^{\widehat{N}_1}(p_1 + t_1(k))dk - \frac{1}{v_2}\int_0^{\widehat{N}_2}(p_2 + t_2(k))dk.$$

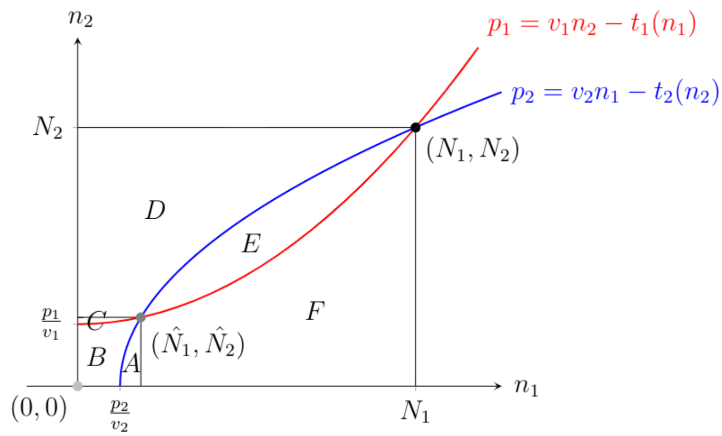

Figure 5: The equilibria of the subgame in stage 2 when $p_1, p_2 \geq 0$.

To prove that the unstable equilibrium is never the potential maximizer, it suffices to show that its potential is always less than that of the Pareto-dominated equilibrium, i.e., $\frac{1}{v_1}\int_0^{\widehat{N}_1}(p_1 + t_1(k))dk + \frac{1}{v_2}\int_0^{\widehat{N}_2}(p_2 + t_2(k))dk \geq \widehat{N}_1\widehat{N}_2$. This can be easily seen in Figure 5 which is edited from Figure 4 (left): the area of $A + B$ is $\frac{1}{v_1}\int_0^{\widehat{N}_1}(p_1 + t_1(k))dk$; the area of $B + C$ is $\frac{1}{v_2}\int_0^{\widehat{N}_2}(p_2 + t_2(k))dk$; the area of $A + B + C$ is $\widehat{N}_1\widehat{N}_2$.[50] ■

Similar to Lemma 4 in Section 3.2, the platform has to leave enough surplus to the participants by setting sufficiently low prices (p_1, p_2) in stage 1, so that all agents will coordinate on the Pareto-dominant equilibrium in stage 2. In other words, potential-maximizer selection essentially imposes an additional constraint (i.e., the inequalities in (38) and (39) of Lemma 9) to the platform's profit-maximization problem in (36). Sometimes this additional constraint does not bind in the equilibrium. As shown in Lemma 9, this happens when the platform's prices (p_1, p_2) and the participants' total stand-alone costs $\int_0^{N_1} t_1(k)dk$ and $\int_0^{N_2} t_2(k)dk$ are sufficiently low.[51] If this is the case, the equilibrium outcome under potential-

[50] By the same token, we can directly identify the potential maximizer in Figure 5. The area of $A+B+F$ is $\frac{1}{v_1}\int_0^{N_1}(p_1+t_1(k))dk$; the area of $B+C+D$ is $\frac{1}{v_2}\int_0^{N_2}(p_2+t_2(k))dk$; the area of $A+B+C+D+E+F$ is N_1N_2. Thus, when $p_1, p_2 \geq 0$, the potential maximizer is the Pareto-dominant (Pareto-dominated) equilibrium when $E \geq B$ ($E \leq B$).

[51] If $p_2 \leq 0 \leq p_1$, only the total stand-alone cost of group-2 participants who do not have a dominant strategy to join the platform matters, i.e., only $\int_{\underline{N}_2}^{N_2} t_2(k)dk$ but not $\int_0^{N_2} t_2(k)dk$ matters.

maximizer selection coincides with that under Pareto-dominance selection; otherwise, their equilibrium outcomes differ.

F.3 An Example

In what follows, I analyze an example in which the stand-alone cost functions take the form

$$t_1(k) = t_1 k^{\alpha}, \quad t_2(k) = t_2 k^{\alpha}. \quad (\alpha > 1)$$

Solving the system of equations in (37) gives us the platform's optimal prices (p_1^*, p_2^*) and the equilibrium masses of participants (N_1^*, N_2^*) under Pareto-dominance selection:

$$N_1^* = \left(\frac{1}{t_1^{\alpha} t_2}\right)^{\frac{1}{\alpha^2-1}} \left(\frac{v_1+v_2}{\alpha+1}\right)^{\frac{1}{\alpha-1}}, \quad N_2^* = \left(\frac{1}{t_1 t_2^{\alpha}}\right)^{\frac{1}{\alpha^2-1}} \left(\frac{v_1+v_2}{\alpha+1}\right)^{\frac{1}{\alpha-1}}, \tag{40}$$

$$p_1^* = \frac{\alpha v_1 - v_2}{\alpha+1}\left(\frac{1}{t_1 t_2^{\alpha}}\right)^{\frac{1}{\alpha^2-1}} \left(\frac{v_1+v_2}{\alpha+1}\right)^{\frac{1}{\alpha-1}}, \; p_2^* = \frac{\alpha v_2 - v_1}{\alpha+1}\left(\frac{1}{t_1^{\alpha} t_2}\right)^{\frac{1}{\alpha^2-1}} \left(\frac{v_1+v_2}{\alpha+1}\right)^{\frac{1}{\alpha-1}}. \tag{41}$$

The following lemma states the main result of this example.

Lemma 10 *The equilibrium outcome under potential-maximizer selection differs from that of the Pareto-dominant selection if and only if* $\frac{1}{\alpha} < \frac{v_1}{v_2} < \alpha$.

Proof. When $\frac{1}{\alpha} < \frac{v_1}{v_2} < \alpha$, p_1^* and p_2^* are positive under Pareto-dominance selection as shown in (41). By substituting (40) and (41) into the inequality in (38) and with some simplifications, we obtain

$$(\alpha v_1 - v_2)(v_1 - \alpha v_2) \leq 0. \tag{42}$$

The above inequality is violated if and only if $\frac{1}{\alpha} < \frac{v_1}{v_2} < \alpha$. In other words, the Pareto-dominant equilibrium is not the potential maximizer when $\frac{1}{\alpha} < \frac{v_1}{v_2} < \alpha$. This implies that the equilibrium outcomes under potential-maximizer selection and Pareto-dominance selection must differ.

By contrast, (42) holds when $\frac{v_1}{v_2} \geq \alpha$. Moreover, the term $\int_{\underline{N}_2}^{N_2}(p_2 + t_2(k))dk$ in (39) is less than the term $\int_0^{N_2}(p_2 + t_2(k))dk$ in (38) because, $\frac{v_1}{v_2} \geq \alpha$ implies $p_2^* \leq 0$ from (41). Hence,

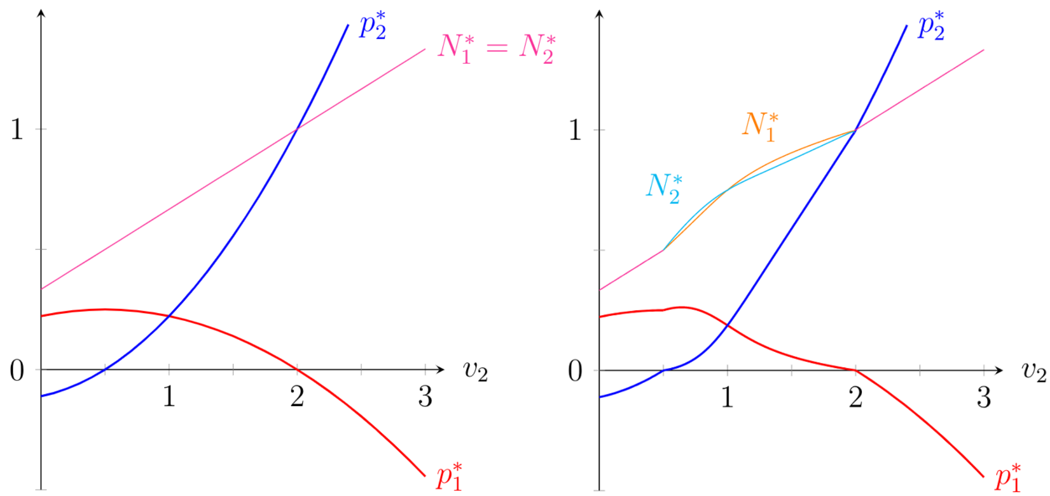

Figure 6: The equilibrium outcomes under Pareto-dominance selection (left) and potential-maximizer selection (right) when $v_1 = t_1 = t_2 = 1$ and $\alpha = 2$.

the Pareto-dominant equilibrium is the potential maximizer when $\frac{v_1}{v_2} \geq \alpha$. The same logic applies to the case $\frac{v_1}{v_2} \leq \frac{1}{\alpha}$. ■

In what follows, I characterize the equilibrium under potential-maximizer selection when $\frac{1}{\alpha} < \frac{v_1}{v_2} < \alpha$. It turns out that the inequality in (38) but not (39) binds under potential-maximizer selection. Hence, from (36) and with some simplifications, the platform's profit-maximization problem in stage 1 becomes

$$\max_{(N_1,N_2)\in[0,\overline{N}_1)\times[0,\overline{N}_2)} (v_1N_2 - t_1N_1^{\alpha})N_1 + (v_2N_1 - t_2N_2^{\alpha})N_2 \quad \text{s.t.} \quad \frac{t_1}{v_1}N_1^{\alpha+1} + \frac{t_2}{v_2}N_2^{\alpha+1} = \frac{\alpha+1}{\alpha}N_1N_2.$$

Solving the above optimization problem gives us the equilibrium masses of participants (N_1^*, N_2^*). There are closed-form solutions. Here, I present the solution when $v_1 = t_1 = t_2 = 1$ and $\alpha = 2$:

$$N_1^* = \left(\frac{v_2\left(3 - 3v_2 + \sqrt{7v_2^2 - 13v_2 + 7}\right)}{2 - v_2}\right)^{\frac{1}{3}} \frac{4v_2 + \sqrt{7v_2^2 - 13v_2 + 7} - 5}{6(v_2 - 1)},$$

$$N_2^* = \left(\frac{v_2\left(3 - 3v_2 + \sqrt{7v_2^2 - 13v_2 + 7}\right)}{2 - v_2}\right)^{\frac{2}{3}} \frac{4v_2 + \sqrt{7v_2^2 - 13v_2 + 7} - 5}{6(v_2 - 1)},$$

and the platform's optimal prices (p_1^*, p_2^*) can be computed from (35).

Figure 6 sketches the equilibrium outcomes under both Pareto-dominance selection and potential-maximizer selection. Their equilibrium outcomes coincide if and only if $v_2 \leq \frac{1}{2}$ or $v_2 \geq 2$. Under Pareto-dominance selection, the platform always attracts the same mass of group-1 and group-2 participants in the equilibrium. By contrast, under potential-maximizer selection, the platform attracts more (fewer) group-2 participants than group-1 participants when $\frac{1}{2} < v_2 < 1$ $(1 < v_2 < 2)$. Nevertheless, under both selection criteria, the platform's optimal group-1 price p_1^* and the equilibrium masses of participants N_1^* and N_2^* increase with v_2, while the optimal group-2 price p_2^* tends to decrease with v_2.

G Alternative Pricing Instruments

Appendix G.1 modifies the duopoly-platform model in Section 4 so that the competing platforms charge transaction fees instead of subscription fees. Appendix G.2 analyzes the model. Appendix G.3 demonstrates how the analysis can be extended to two-part tariffs.

G.1 Model

The platforms set transaction fees instead of subscription fees to the two groups. If a group-i agent joins platform m, he pays a transaction fee $p_i^m \in \mathbb{R}$ per each interaction with group-j agents who join the same platform. Thus, his payoff from joining platform m is

$$u_i^m(n_j^m, p_i^m) = (v_i^m - p_i^m)n_j^m. \tag{43}$$

If platform m attracts n_1^m group-1 participants and n_2^m group-2 participants, there is a total of $n_1^m n_2^m$ interactions within the platform. Platform m's profit is the total transaction fees collected from both sides, i.e.,

$$\pi^m(n_1^m, n_2^m, p_1^m, p_2^m) = (p_1^m + p_2^m)n_1^m n_2^m. \tag{44}$$

I assume that both platforms do not use any weakly dominated strategies, i.e., the sum of transaction fees on both sides $p_1^m + p_2^m$ is non-negative. The rest of the model setup is the same as that in Section 4.1.

G.2 Analysis

Pareto-dominance selection and Pareto-dominated selection remain not applicable for the same reason as in Section 4 while potential-maximizer selection remains valid. Note that the agent's payoff function (43) in this alternative model can be obtained by replacing v_i^m and p_i^m in the original model (16) with $v_i^m - p_i^m$ and 0 respectively. Hence, the current model is a weighted potential game, and its potential function and the potential maximizer can be obtained by applying the above replacements to Lemmas 5 and 6. Thus, we can immediately characterize the unique equilibrium in stage 2 under potential-maximizer selection.

Lemma 11 *When $p_i^m \leq v_i^m$ for all $m \in \{A, B\}$ and $i = 1, 2$,*[52] *the unique equilibrium of the subgame in stage 2 under potential-maximizer selection is*

$$\begin{array}{ll} \textit{all agents join platform } A & \textit{if } (v_1^A - p_1^A)(v_2^A - p_2^A) \geq (v_1^B - p_1^B)(v_2^B - p_2^B), \\ \textit{all agents join platform } B & \textit{otherwise.} \end{array}$$

Under potential-maximizer selection, all agents will coordinate on the platform that delivers a higher product of net per-interaction benefits $(v_1^m - p_1^m)(v_2^m - p_2^m)$ for the two sides in stage 2. Thus, stage 1 is analogous to the standard Bertrand competition as in Section 4.2. Generically and w.l.o.g., assume that A (B) is the dominant (dominated) platform in the equilibrium. Standard analysis for Bertrand competition implies that B charges the minimum prices to maximize $(v_1^B - p_1^B)(v_2^B - p_2^B)$. Under the non-negative profit constraint $p_1^B + p_2^B \geq 0$, B's equilibrium pricing strategy is

$$p_1^{B*} = \frac{v_1^B - v_2^B}{2}, \quad p_2^{B*} = \frac{v_2^B - v_1^B}{2}. \tag{45}$$

As shown in the above expression, B rebalances the net per-interaction benefits of the two sides by monetizing the side with higher per-interaction benefit and subsidizing the other side. The resulting net per-interaction benefit is $\frac{v_1^B + v_2^B}{2}$ for both sides. Standard analysis for

[52] The only interesting subgames are those with $p_i^m \leq v_i^m$ for all $m \in \{A, B\}$ and $i = 1, 2$ because both platforms will not charge a transaction fee higher than the respective per-interaction benefit in the equilibrium.

Bertrand competition also implies that A slightly undercuts B to capture the entire market. By Lemma 11 and (45), this implies that

$$(v_1^A - p_1^{A*})(v_2^A - p_2^{A*}) = \left(\frac{v_1^B + v_2^B}{2}\right)^2. \tag{46}$$

Under the non-negative profit constraint $p_1^A + p_2^A \geq 0$, it is easy to see that A can successfully undercut B if and only if $v_1^A + v_2^A > v_1^B + v_2^B$. Hence, by assuming $v_1^A + v_2^A > v_1^B + v_2^B$, A maximizes its profit by optimally allocating the prices to the two sides subject to the constraint in (46), i.e.,

$$\max_{\left\{(p_1^A, p_2^A) \in (-\infty, v_1^A] \times (-\infty, v_2^A] : p_1^A + p_2^A \geq 0\right\}} (p_1^A + p_2^A) N_1 N_2 \quad \text{s.t.} \quad (v_1^A - p_1^A)(v_2^A - p_2^A) = \left(\frac{v_1^B + v_2^B}{2}\right)^2.$$

Solving the above optimization problem shows that A adjusts the net per-interaction benefits of the two sides with transaction fees, such that the net per-interaction benefit is also $\frac{v_1^B + v_2^B}{2}$ for both sides, i.e.,

$$p_1^{A*} = v_1^A - \frac{v_1^B + v_2^B}{2}, \quad p_2^{A*} = v_2^A - \frac{v_1^B + v_2^B}{2}.$$

Hence, A's equilibrium profit is

$$\pi^{A*} = \left(v_1^A + v_2^A - v_1^B - v_2^B\right) N_1 N_2.$$

The equilibrium outcome under potential-maximizer selection is summarized as follows.

Proposition 3 *Generically and w.l.o.g., suppose $v_1^A + v_2^A > v_1^B + v_2^B$. Under potential-maximizer selection, there is a unique equilibrium in this model. Stage 1 is a Bertrand equilibrium with*

$$p_1^{A*} = v_1^A - \frac{v_1^B + v_2^B}{2}, \quad p_2^{A*} = v_2^A - \frac{v_1^B + v_2^B}{2}, \quad p_1^{B*} = \frac{v_1^B - v_2^B}{2}, \quad p_2^{B*} = \frac{v_2^B - v_1^B}{2}.$$

All agents join platform A in stage 2, and platform A's equilibrium profit is

$$\pi^{A*} = \left(v_1^A + v_2^A - v_1^B - v_2^B\right) N_1 N_2.$$

Similar to the model in Section 4, the market tips to a dominant platform. However, the dominant platform is now the one with a higher sum of per-interaction benefits $v_1^m + v_2^m$ rather than the product of them $v_1^m v_2^m$ as in the pure-subscription model. Following Section 4.2, I discuss the three key implications under the current framework.

Divide-and-conquer strategy As shown in Proposition 3, the dominated platform (B) monetizes the side with higher per-interaction benefit and subsidizes the other side. By contrast, the dominant firm may or may not monetize one side and subsidize the other side depending on how competitive the two platforms are. If the platforms are very competitive (say, $v_1^A + v_2^A \approx v_1^B + v_2^B$), the dominant platform (A) will divide and conquer; if one of the platforms is inferior (say, $v_1^B \approx v_2^B \approx 0$), the superior platform will monetize both sides.

Money/subsidy side The money/subsidy side of the dominated platform (B) depends only on whether its own per-interaction benefit v_1^B or v_2^B is larger. As explained, the dominant platform (A) might monetize both sides if its competitor is inferior. Nevertheless, when A divides and conquers, the money/subsidy side depends only on whether its own per-interaction benefit v_1^A or v_2^A is larger.[53] This is different from that of the model in Section 4, in which the money/subsidy side of the dominant platform also depends on the competitor's per-interaction benefits v_1^B and v_2^B.

Optimal design Given that the platforms can rebalance the net per-interaction benefits with transaction fees, the optimal design of the platforms is to maximize the sum of per-interaction benefits $v_1^m + v_2^m$. This differs from that of the pure-subscription model, in which the platforms maximize $v_1^m v_2^m$ instead. In other words, given the same set of parameter values, the dominant platform may differ under these two duopoly-platform models. Take (21) as an example; A is the dominant platform in the pure-subscription model but B is the dominant platform in the current model. In contrast to the pure-subscription model, all agents now always coordinate on the platform that delivers the higher social surplus $(v_1^m + v_2^m)N_1N_2$. Moreover, the optimal design of the platforms is to maximize $v_1^m + v_2^m$, and thus it also maximizes social surplus.

[53] As shown in Proposition 3, if A subsidizes group 1 ($p_1^{A*} < 0$) and monetizes group 2 ($p_2^{A*} > 0$), it implies that $v_1^A < v_2^A$, and this is irrespective of the competitor's per-interaction benefits v_1^B and v_2^B.

G.3 Two-Part Tariffs

The analysis can be easily extended to two-part tariffs. Suppose the agent's payoff from joining the platform takes the form

$$u_i^m(n_j^m, p_i^m, r_i^m) = (v_i^m - r_i^m)n_j^m - p_i^m, \tag{47}$$

where $p_i^m \in \mathbb{R}$ is the subscription fee and $r_i^m \in \mathbb{R}$ is the transaction fee. Similar to Appendix G.2, the agent's payoff function in (47) can be obtained by replacing v_i^m in the original model (16) with $v_i^m - p_i^m$. Hence, the current model is a weighted potential game, and we can immediately characterize the potential maximizer. The subsequent analysis is very similar to those of the previous duopoly-platform models and thus omitted.

H Same-Side Network Effects

Appendix H.1 extends the baseline model with negative same-side network effects on one side and shows that it is equivalent to the model in Section 3.1. By the same token, the model with negative same-side network effects on both sides is equivalent to Armstrong's original model. Appendix H.2 extends the baseline model with positive same-side network effects on one side. Extending the analysis to positive same-side network effects on both sides is straightforward and thus omitted. Likewise, the analysis of the model with positive same-side network effects on one side and negative same-side network effects on the other side is very similar to that of Appendix H.1, and thus also omitted. This appendix adopts the continuum-agent framework.

H.1 Negative Same-Side Network Effects

The model is identical to the model in Section 3.1 except the payoff of a group-2 agent from joining the platform is now

$$u_2(n_1, n_2, p_2) = v_2 n_1 - p_2 - t(n_2), \tag{48}$$

where $t : [0, \overline{N}_2] \to \mathbb{R}_+$ is an increasing function that measures the negative same-side network effects each group-2 participant suffers.

Similar to the model in Section 3.1, there is a demand expansion effect on group 2 because of the negative same-side network effects. More precisely, for any group-2 price $p_2 \leq v_2 N_1$ set by the platform in stage 1, there is at most a mass of N_2 group-2 participants in the equilibrium, where N_2 is given by (9). Similarly, if p_2 is negative, there is at least a mass of $\underline{N}_2$ group-2 participants in the equilibrium, where $\underline{N}_2$ is given by (10). Hence, the set of equilibria in this model is closely related to that of the model in Section 3.1. More precisely, there are two sets of stable equilibria and a set of unstable equilibria in stage 2 under Case 1 or Case 2 as defined on p. 18. The masses of participants (n_1^*, n_2^*) for each set of equilibria are characterized by Figure 1 in Section 3.2. For Case 1, the masses of participants for the three sets of equilibria are

1. Pareto-dominant equilibrium: $(n_1^*, n_2^*) = (N_1, N_2)$;
2. unstable equilibrium: $(n_1^*, n_2^*) = (\widehat{N}_1, \widehat{N}_2)$;
3. Pareto-dominated equilibrium: $(n_1^*, n_2^*) = (0, 0)$.

For Case 2, the masses of participants for the three sets of equilibria are

1. Pareto-dominant equilibrium: $(n_1^*, n_2^*) = (N_1, N_2)$;
2. unstable equilibrium: $(n_1^*, n_2^*) = (\widehat{N}_1, \widehat{N}_2)$;
3. Pareto-dominated equilibrium: $(n_1^*, n_2^*) = (0, \underline{N}_2)$.

From the platform's point of view, the set of equilibria in the current model is equivalent to that in Section 3.2: ultimately it only cares about the masses of participants but not the identity of each participant. It remains to show that the potential maximizers of these two models are also equivalent. First, I show that every subgame in stage 2 is a weighted potential game.

Lemma 12 *Every subgame in stage 2 is a weighted potential game with the potential function*

$$P(n_1, n_2 | p_1, p_2) = n_1 n_2 - \frac{p_1}{v_1} n_1 - \frac{p_2}{v_2} n_2 - \frac{1}{v_2} \int_0^{n_2} t(k) dk. \tag{49}$$

Proof. We can follow the same approach in proving Lemma 3 in Section 3.2 to prove this lemma. Alternatively, we can prove that every subgame in stage 2 is a continuum-agent weighted potential game as defined by Sandholm (2001, p. 85).[54] Similar to the definition of a finite-agent weighted potential game in Definition 1, a continuum-agent game is a continuum-agent weighted potential game if there exists a function P defined on the strategy space of the game, such that the change in any player's payoff from unilaterally switching actions is (positively) proportional to the corresponding differential change in P.

I now show that every subgame in stage 2 is a continuum-agent weighted potential game. Suppose there are a mass of n_1 group-1 participants and a mass of n_2 group-2 participants. The payoff difference between joining the platform or not for a group-1 agent is

$$u_1(n_2, p_1) - 0 = v_1 n_2 - p_1. \quad \text{(by (7))}$$

The corresponding differential change in P is

$$\frac{\partial P(n_1, n_2 | p_1, p_2)}{\partial n_1} = n_2 - \frac{p_1}{v_1}. \quad \text{(by (49))}$$

Similarly, the payoff difference between joining the platform or not for a group-2 agent is

$$u_2(n_1, n_2, p_2) - 0 = v_2 n_1 - p_2 - t(n_2). \quad \text{(by (48))}$$

The corresponding differential change in P is

$$\frac{\partial P(n_1, n_2 | p_1, p_2)}{\partial n_2} = n_2 - \frac{p_2}{v_2} - \frac{t(n_2)}{v_2}. \quad \text{(by (49))}$$

Clearly, the change in a group-i agent's payoff from unilaterally switching actions is proportional (with proportion v_i) to the differential change in P. Therefore, every subgame is a weighted potential game with the potential function given by (49). ■

The term $\frac{1}{v_2}\int_0^{\overline{N}_2} t(k) a_2^k dk$ in Lemma 3 is now replaced by $\frac{1}{v_2}\int_0^{n_2} t(k) dk$ in Lemma 12. Nevertheless, we can easily see that the respective potentials of the three sets of equilibria

[54] Sandholm (2001) only defines continuum-agent exact potential games, but the definition and the results can be naturally extended to continuum-agent weighted potential games. As proved by Sandholm (2001, Theorem 6.1), continuum-agent potential games are the limits of convergent sequences of the finite-agent potential games.

are equal to those of the three equilibria in Section 3.2. Hence, the potential maximizers of these two models are equivalent from the platform's point of view, and thus the subsequent analysis and the results are identical. In other words, there is no distinction between the agents' stand-alone costs and negative same-side network effects from the platform's point of view. Yet, group-2 participants are better off in the former because their total stand-alone cost is $\int_0^{N_2^*} t(k)dk$ while each of them suffers a cost of $t(N_2^*)$ in the latter.

H.2 Positive Same-Side Network Effects

The model is identical to the previous model in Appendix H.1 except the payoff of a group-2 agent from joining the platform is now

$$u_2(n_1, n_2, p_2) = v_2 n_1 - p_2 + t(n_2),$$

where $t : [0, N_2] \to \mathbb{R}_+$ is an increasing function that measures the positive same-side network effects each group-2 participant enjoys. Note that the mass of group-2 agents is N_2 (but not $\overline{N}_2$) in this model, and I do not impose any additional assumptions on the function t.

Unlike the previous model, there is no demand expansion effect on group 2 when the same-side network effects are positive. Thus, the set of equilibria is similar to that of the baseline model: there are two stable equilibria in stage 2 when $(p_1, p_2) \in [0, v_1 N_2] \times [0, v_2 N_1 + t(N_2)]$:[55]

1. all agents join the platform;

2. no one joins the platform.

By Lemma 12 in Appendix H.1 (with the sign of t reversed), every subgame in stage 2 is a weighted potential game with the potential function

$$P(n_1, n_2 | p_1, p_2) = n_1 n_2 - \frac{p_1}{v_1} n_1 - \frac{p_2}{v_2} n_2 + \frac{1}{v_2} \int_0^{n_2} t(k)dk.$$

[55]Multiple equilibria also exist when $(p_1, p_2) \in (-\infty, 0) \times [v_2 N_1, v_2 N_1 + t(N_2)] \cup (v_1 N_2, \infty) \times [0, t(N_2)]$, but clearly the platform will not set these prices in the equilibrium. There are also unstable equilibria when $(p_1, p_2) \in [0, v_1 N_2] \times [0, v_2 N_1 + t(N_2)]$, but these equilibria are never the potential maximizer for the same reason as before and thus ignored.

Following the same approach in proving Lemma 2 in Section 2.3.2, we can easily identify the potential maximizer when $(p_1, p_2) \in [0, v_1 N_2] \times [0, v_2 N_1 + t(N_2)]$, and it is

$$\begin{aligned} &\text{all agents join the platform} \quad \text{if } \frac{p_1}{v_1 N_2} + \frac{p_2}{v_2 N_1} \leq 1 + \frac{1}{v_2 N_1 N_2} \int_0^{N_2} t(k) dk, \\ &\text{no one joins the platform} \quad\quad \text{otherwise.} \end{aligned} \tag{50}$$

Hence, the platform's profit-maximization problem in stage 1 becomes

$$\max_{(p_1,p_2)\in[0,v_1N_2]\times[0,v_2N_1+t(N_2)]} p_1 N_1 + p_2 N_2 \quad \text{s.t.} \quad \frac{p_1}{v_1 N_2} + \frac{p_2}{v_2 N_1} \leq 1 + \frac{1}{v_2 N_1 N_2} \int_0^{N_2} t(k) dk.$$

Solving the above optimization problem gives us the platform's optimal pricing strategy:

1. when $v_1 < v_2$:
$$p_1^* = 0, \quad p_2^* = v_2 N_1 + \frac{1}{N_2} \int_0^{N_2} t(k) dk;$$
2. when $v_1 > v_2$:
$$p_1^* = v_1 N_2, \quad p_2^* = \frac{1}{N_2} \int_0^{N_2} t(k) dk.$$

Similar to the baseline model, the platform always divides and conquers. Moreover, the money/subsidy side is not affected by the additional positive same-side network effects. The platform now marks up the group-2 price by $\frac{1}{N_2} \int_0^{N_2} t(k) dk$ because of this additional positive same-side network effects they enjoy. Still, group-2 agents are better off in the current model because the additional benefit $t(N_2)$ each group-2 agent enjoys is higher than the price markup $\frac{1}{N_2} \int_0^{N_2} t(k) dk$.