Addictive Platforms*

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Abstract

We study competition for consumer attention in which platforms can sacrifice service quality for attention. A platform can choose the "addictiveness" of service; a more addictive platform yields consumers a lower utility of participation but a higher marginal utility of allocating attention. We provide conditions under which increased competition harms consumers. In particular, if attention is scarce, increased competition reduces consumer welfare because business stealing incentives induce platforms to choose high addictiveness. Restricting consumers' platform usage may reduce addictiveness and improve consumer welfare. A platform's ability to charge for service can decrease addictiveness, but the welfare implication is ambiguous.

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1 Introduction

Online platforms, such as Facebook, Google, and Twitter, monetize consumer attention. Because attention is finite, competition for attention may encourage firms to improve their services to attract consumers. At the same time, there is a growing concern for consumers and policymakers—that competition for attention could also incentivize a firm to sacrifice its service quality for attention. For example, a platform may adopt news feeds that display low quality content users are likely to watch; it may also adopt a certain user interface, such as an intrusive notification system or infinite scrolling (Scott Morton et al., 2019).[1]

We study a model of competition for consumer attention in which a platform can sacrifice service quality for attention. The model consists of a consumer and platforms. First, platforms choose the "addictiveness" of their services. Second, the consumer chooses the set of platforms to join, then allocates her attention. A more addictive platform yields the consumer a lower utility of participation but a higher marginal utility of allocating attention. As a result, the consumer prefers to join less addictive platforms, but after joining, she allocates more attention to more addictive platforms. The consumer incurs a cost of allocating attention. She also faces an attention constraint, which caps the maximum attention she can allocate. A platform provides the service for free and earns revenue that is increasing in the amount of attention the consumer allocates.

The addictiveness in our model is a platform's strategic variable that captures a choice to sacrifice service quality to make it more capable of capturing consumer attention. For example, a platform may design a content selection algorithm or user interface in a certain way, or collect sensitive individual data for personalization. We model such a choice as the shift of service utilities and marginal utilities provided to the consumer.

Our main question is whether competition for attention benefits consumers. We examine several notions of increased competition and provide conditions under which competition increases or decreases consumer welfare. Competition affects platforms' incentives in two ways. On the one hand, it encourages platforms to reduce addictiveness: If a consumer faces competing platforms,

[1]For example, Scott Morton and Dinielli (2020) argue that "another reduction in quality that Facebook's market power allows is the serving of addictive and exploitative content to consumers. Facebook deploys various methods to maintain user attention—so that it can serve more ads—using techniques that the medical literature has begun to demonstrate are potentially addictive."

she loses less by refusing to join a single platform and continuing to use other services. To attract the consumer, a platform needs to reduce addictiveness and offer high service quality. On the other hand, competition introduces business stealing incentives: A platform can increase addictiveness to capture attention the consumer would allocate to its rivals.

The countervailing incentives derive our main insight: Under a certain condition, increased competition could encourage platforms to sacrifice quality for attention, leading to lower service quality and consumer welfare. Such an outcome is likely in particular when attention is scarce: When the attention constraint is tight, higher addictiveness does not increase total attention but only changes how the consumer divides her attention across platforms. Competition then introduces business stealing incentives, leading to higher addictiveness and lower consumer welfare. Conversely if the consumer does not face a tight attention constraint, platforms that do not face competition set high addictiveness without discouraging consumer participation. Competition then incentivizes platforms to decrease addictiveness, leading to higher consumer welfare. We also show that when platforms incur a cost of increasing addictiveness (e.g., technological investment), increased competition can reduce consumer welfare regardless of the scarcity of attention. Our result highlights that when firms can sacrifice quality for attention, the effect of competition in improving consumer welfare could be limited in the attention economy.

As a policy remedy, we examine the impact of a digital curfew, which restricts the consumer's platform usage. For example, the Social Media Addiction Reduction Technology Act (the "SMART" Act) proposed in the US requests that companies limit the time a user may spend on their services.[2] We model a digital curfew as a reduction of the consumer's attention capacity. A digital curfew may increase consumer welfare by limiting a platform's incentive to increase addictiveness.

Finally we examine the role of a platform's revenue model. We compare the baseline model to a model of price competition, in which platforms earn revenue only by charging prices that are independent of the level of attention. Because platforms do not monetize attention, they set zero

[2] See https://www.congress.gov/bill/116th-congress/senate-bill/2314 (accessed on November 24, 2020). Several other countries have implemented some restrictions to protect young people from addictive games. In 2003 Thailand implemented a shutdown law that banned young people from playing online games between 22:00 and 06:00. In 2011, South Korea passed a similar legislature, known as the Youth Protection Revision Act. In 2007, China introduced the so-called "fatigue" system under which game developers need to reduce or stop giving out rewards (e.g., game items, experience value) in games after a player reached certain hours of play.

addictiveness and offer high service quality. However, when the market is sufficiently competitive, the consumer is better off under attention competition: The consumer faces high marginal utilities from addictive services, so she can earn a high incremental gain by refusing to join a platform and continuing to use other services. The better outside option encourages platforms to offer higher net utilities to the consumer under attention competition than price competition. The result indicates that the welfare impact of revenue models depends on the market structure.

In our baseline model, the consumer correctly perceives the level of addictiveness. In practice, consumers may systematically underestimate the addictive features of platforms. We study such a naive consumer and extend our insights. The naivete increases equilibrium addictiveness, decreases her welfare, and renders price competition more desirable than attention competition for the consumer.

Related literature The paper relates to the literature on platform competition, in particular competition for consumer attention (e.g., Rochet and Tirole 2003; Armstrong 2006; Anderson and De Palma 2012; Bordalo et al. 2016; Wu 2017; Evans 2017, 2019; Prat and Valletti 2019; Galperti and Trevino 2020; Anderson and Peitz 2020). Platforms in our model have a new strategic variable that captures a firm's choice to degrade quality for attention. We model such a choice as the increase in marginal utilities and the decrease in the level of utilities provided to consumers. As a result our model also relates to Armstrong and Vickers (2001), in which firms compete in utility space; in our model, platforms compete in the space of utilities and marginal utilities. The divergence between utilities and marginal utilities does not arise in competition on other dimensions, such as price or advertising load, in which they typically move in the same direction, or the allocation of attention is not explicitly modeled (e.g., Anderson and Coate 2005; de Corniere and Taylor 2020; Choi and Jeon 2020).[3]

Second, the paper contributes to the nascent literature on possible negative impacts of digital services on consumers (Allcott and Gentzkow, 2017; Allcott et al., 2020; Mosquera et al., 2020; Allcott et al., 2021). A recent discussion points out that technology companies may have an incentive to adopt features (e.g., user interfaces) that increase user engagement at the expense of

[3]We abstract away from the two-sided aspect of the market, so our result differs from that of two-sided markets, in which competition for one side could harm other sides (e.g., Tan and Zhou 2021).

their welfare (Alter, 2017; Scott Morton et al., 2019; Newport, 2019; Rosenquist et al., 2020). We contribute to this literature by examining interactions between competition for attention and the addictiveness of digital services. Although we later motivate our model based on habit formation with a time-inconsistent agent, we largely abstract away from dynamics and behavioral biases relevant to addiction (Becker and Murphy, 1988; Gruber and Köszegi, 2001; Orphanides and Zervos, 1995). The abstraction allows us to provide a general intuition.

Finally, the recent policy and public debates recognize the problem that a firm that monetizes attention could distort its service quality to capture consumer attention (Crémer et al., 2019; U.K. Digital Competition Expert Panel, 2019; Scott Morton and Dinielli, 2020). We contribute to the discussion by providing a new intuition—that competition may not mitigate the problem.

2 Model

There are $K \in \mathbb{N}$ platforms and a single consumer. We write K for the number and the set of the platforms. Suppose the consumer joins a set $J \subset K$ of platforms, and allocates attention $a_k \geq 0$ to each platform $k \in J$. If $J = \emptyset$, she receives a payoff of zero. Otherwise, her payoff is

$$\sum_{k \in J} u(a_k, d_k) - C\left(\sum_{k \in J} a_k\right). \tag{1}$$

In the first term, $u(a_k, d_k)$ is the utility from platform k's service. The utility $u(a_k, d_k)$ depends on the *addictiveness* $d_k \in \mathbb{R}_+$ of platform k. We impose the following assumption (see Figure 1).

Assumption 1. The function $u(\cdot, \cdot) : \mathbb{R}_+^2 \to \mathbb{R}$ is twice differentiable and satisfies the following:

(a) For every $d \geq 0$, utility $u(a, d)$ is strictly increasing and concave in a, and $u(0, 0) \geq 0$.

(b) For every $a \geq 0$, utility $u(a, d)$ is strictly decreasing in d, and
$\max_{a \geq 0}[u(a, d) - C(a)] < 0$ for some d.

(c) For every $a \geq 0$, the marginal utility for attention $\frac{\partial u}{\partial a}(a, d)$ is strictly increasing in d.

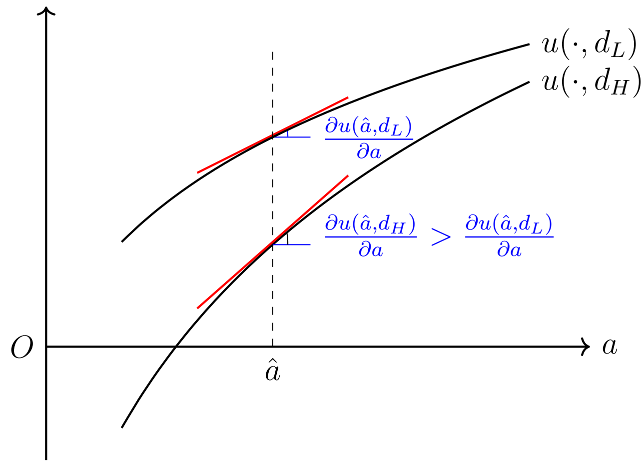

Figure 1: Utilities under d_L and $d_H > d_L$.

Points (b) and (c) imply that higher addictiveness decreases the consumer's utility of joining a platform but increases her marginal utility of allocating attention. Assumption 1 holds if, for example, $u(a, d) = 1 - e^{-\rho(a-d)}$ with $\rho > 0$ or $u(a, d) = v(a - d)$ with an increasing and concave $v(\cdot)$. Section 2.1 motivates the assumption.

The second term $C\left(\sum_{k \in J} a_k\right)$ of the consumer's payoff (1) is the *attention cost*—e.g., the opportunity and cognitive costs of using services. We impose the following assumption.

Assumption 2. $C(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ is strictly increasing, convex, and twice differentiable, and $\lim_{a \to 0} \left[\frac{\partial u}{\partial a}(a, 0) - C'(a)\right] > 0$.

The consumer also faces the *attention constraint*, which captures the scarcity of attention: The consumer can allocate the total attention of at most $\overline{A} \in \mathbb{R}_+ \cup \{\infty\}$ across platforms. A finite bound $\overline{A}$ comes from, for example, the consumer's preferences, physical constraints, and an exogenous restriction such as a digital curfew. To ensure an equilibrium exists, if $\overline{A} = \infty$, we assume that the primitives are such that the consumer has an optimal attention profile $(a_k)_{k \in J} \in \mathbb{R}^J_+$ given any $(d_1, \ldots, d_K)$ and any set $J \subset K$ of platforms she has joined (e.g., $u(\cdot, d)$ is bounded for each d).

If the consumer joins and allocates attention a to platform k, it earns a payoff of ra, where $r > 0$ is an exogenous value of attention to a platform. If the consumer does not join platform k, it receives a payoff of zero. For example, r is the unit price of attention in the (unmodeled) advertising market. Total surplus refers to the sum of the payoffs of the consumer and all platforms.

The timing of the game is as follows: First, each platform $k \in K$ simultaneously chooses d_k, which the consumer observes. Second, the consumer chooses which platforms to join and how much attention to allocate. In equilibrium the consumer solves

$$\max_{J \subset K, (a_k)_{k \in J}} \sum_{k \in J} u(a_k, d_k) - C\left(\sum_{k \in J} a_k\right) \tag{2}$$

$$\text{s.t.} \quad \sum_{k \in J} a_k \leq \overline{A} \quad \text{and} \quad a_k \geq 0, \forall k \in J.$$

Because $u(0, d) < 0$ for $d > 0$, the consumer's payoff of not joining platform k can differ from the payoff of joining but setting $a_k = 0$. Our solution concept is pure-strategy subgame perfect equilibrium, which we call *equilibrium*. Under monopoly, we study an equilibrium in which the platform breaks ties in favor of the consumer.[4]

2.1 Interpretation of Addictiveness d

The addictiveness d captures the choice of a firm that makes its service more capable of capturing attention at the expense of quality. We capture such choices as the changes of service utilities and marginal utilities provided to consumers. The paper is agnostic about a particular mechanism that cause such changes. However, we present two applications that illustrate how the changes of utilities and marginal utilities could occur.

2.1.1 Habit Formation

We motivate our utility specification using a three-period model of rational addiction with a time-inconsistent consumer (e.g., Becker and Murphy 1988; Gruber and Köszegi 2001). Given addictiveness $(d_1, \ldots, d_K)$, consider the following problem (see Figure 2). In $t = 1$, the consumer chooses the set $J \subset K$ of platforms to join. In $t = 2$, the consumer allocates attention $a_0 > 0$ and obtains utility $u_0 \geq 0$ on each platform in J. This period is a "pre-addiction" stage—i.e., the consumer has yet to be addicted, and the service utilities and the optimal amount of attention

[4] The tie-breaking arises if the platform incurs a small cost of choosing positive addictiveness, which can be a cost of technological investment or reputational loss. Section 5 more generally studies the costly choice of d, in which case the tie-breaking assumption can be dropped.

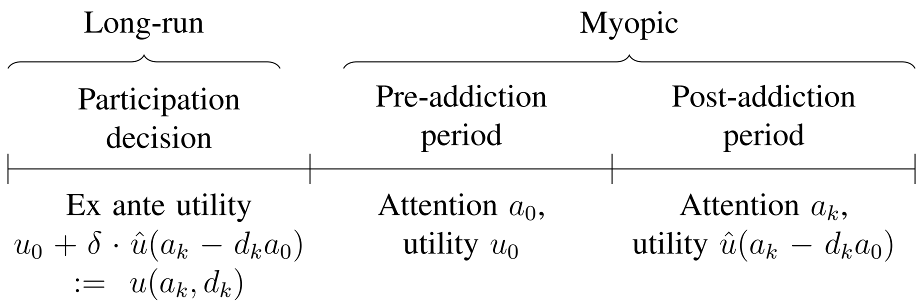

Figure 2: Three-period problem of the consumer

do not depend on $(d_1, \ldots, d_K)$.[5] In $t = 3$, the consumer allocates her attention across platforms in J. This period is a "post-addiction" stage: If the consumer allocates attention a to platform k, she receives $\hat{u}(a - a_0 d_k)$, where $\hat{u}(\cdot)$ is an increasing concave function with $\hat{u}(0) \geq 0$. The payoff $\hat{u}(a - a_0 d_k)$ captures linear habit formation (e.g., Rozen, 2010). Here, $a_0 d_k$ is the reference point against which the consumer evaluates service consumption of platform k in $t = 3$. We can interpret $\frac{1}{d_k}$ as the "rate of disappearance of the physical and mental effects of past consumption" (Becker and Murphy, 1988). A higher d_k imposes a greater harm on the consumer in $t = 3$, and she needs to increase her attention in $t = 3$ to ensure the same payoff as in $t = 2$. Allcott et al. (2021) empirically show that consumption of digital services could exhibit habit formation.

Motivated by dual-self models, we assume that the long-run self makes the participation decision and the short-run selves allocate attention (e.g., Thaler and Shefrin, 1981; Fudenberg and Levine, 2006). Specifically, in $t = 1$ the long-run self decides which platforms to join, anticipating the behavior of future selves: In $t = 2$ the short-run self allocates attention a_0 to each platform, then in $t = 3$ she allocates attention $(a_k^*)_{k \in J}$ to maximize $\sum_{k \in J} \hat{u}(a_k - a_0 d_k) - C\left(\sum_{k \in J} a_k\right)$. Assume the long-run self has discount factor δ. The consumer's participation decision is based on the service utility $u(a_k, d_k) := u_0 + \delta \hat{u}(a_k - a_0 d_k)$, which satisfies Assumption 1.

Our model is suitable when a consumer is susceptible to addictive features of digital services, but she recognizes it and may avoid joining platforms as a commitment device. The model is not

[5] We do not need to specify the utility function the consumer faces in $t = 2$. However, to derive our functional form, we need to assume that a_0 does not depend on the number of platforms the consumer has joined. One way to endogenize such an outcome is to assume that the consumer's utility from each platform in $t = 2$ is $v(a)$ that is maximized at an interior optimum $a_0 \leq \overline{A}/K$. Alternatively, we can assume that the utility function in $t = 2$ is $u(a, 0)$, the attention cost is linear ($C(a) = ca$), and the attention constraint does not bind in the pre-addiction stage, i.e., $\arg\max_{a \geq 0} u(a, 0) - ca \leq \overline{A}/K$.

suitable for a consumer who joins platforms but can use them cautiously to avoid addiction. Such a situation would correspond to the consumer who is forward-looking in periods 2 and 3.

2.1.2 Data Collection and Personalization

Our model can apply to a situation that does not feature a typical "addiction." Suppose that a platform requests consumers to provide their personal data upon registration. Let d denote the amount of data the platform requests. To provide data, consumers incur a privacy cost of ℓd with $\ell > 0$. It captures negative consequences of data collection, such as the risk of data leakage, identity theft, and discrimination. The platform can use their data to personalize offerings, which increases the value of the service from the base value $w(a)$ to $(1+d)w(a)$, where $w(\cdot)$ is increasing, concave, and bounded. A consumer's utility from joining the platform is $u(a,d) := (1+d)w(a) - \ell d$. If $\ell > \sup_{a\geq 0} w(a)$, $u(a,d)$ satisfies Assumption 1. A consumer perceives a platform as low quality when it collects more personal information upon registration, but after joining it, she has more incentive to spend time on a platform that has more information about her.

2.2 Other Modeling Assumptions

Multi-homing. For any $K \geq 2$, the consumer can multi-home—e.g., they may divide time across social media, video streaming, mobile applications, and online games, all of which monetize attention. If $K \geq 2$ but the consumer must single-home, all platforms set zero addictiveness in equilibrium.

Platform's revenue. We can generalize a platform's payoff function in two ways. First, the main insight holds even when a platform incurs a cost of raising d, which could be a cost of technological investment. Section 5 studies such a case. Second, most of the results continue to hold in the following setting: If the consumer allocates attention $(a_1, \ldots, a_K)$, platform k earns a payoff of $r_k(a_1, \ldots, a_K)$, where a function $r_k : \mathbb{R}_+^K \to \mathbb{R}$ is strictly increasing in a_k and depends arbitrarily on $(a_j)_{j\in K\setminus\{k\}}$. For example, a platform's payoff captures revenue in the advertising market, in which platforms can sell consumer attention at a market price.

Addictiveness reduces welfare. In practice, platforms may also adopt features that increase consumer attention and their welfare.[6] To incorporate such features, suppose the consumer's utility from a platform is $u(a, d, b)$, where $u(a, d, b)$ and $\frac{\partial u}{\partial a}(a, d, b)$ are increasing in $b \in [0, 1]$. Since a higher b encourages the consumer to join a platform and allocates more attention, we can redefine $u(a, d) = u(a, d, 1)$ and apply our model.

Representative consumer. A single consumer makes the model tractable and extendable to various considerations. It also helps us focus on our key forces with no potential confounding effects from consumer heterogeneity. However, consumers may be heterogeneous in various aspects. For example, a random component ε_k^i could be added to reflect individual "tastes" for platforms such that consumer i earns the utility $u_k^i := u(a_k^i, d_k) + \varepsilon_k^i$ from platform k. Consumers may also have heterogeneous attention costs and attention caps. Although these extensions are out of the scope of the paper, we expect that our main economic force would be relevant under modest heterogeneity in those aspects, as long as platforms are symmetric and compete under a common demand system. Yet, further studies await to see effects of consumer heterogeneity.

2.3 The First-Best Outcomes

As a benchmark, we characterize the outcome that maximizes total or consumer surplus, when the consumer acts to maximize her payoff. Let $CS(d)$ denote the value of the consumer's problem (2) when all platforms choose addictiveness $d_k = d$. Let $A(d)$ denote the total amount of attention she allocates to attain $CS(d)$. All omitted proofs are in the Appendix.

Claim 1. *Consumer surplus is maximized by $d_k = 0$ for all $k \in K$. Total surplus is maximized by $d^{TS} \in \arg\max_{d \geq 0} \; CS(d) + rA(d)$. If $A(0) < \overline{A}$, then for a sufficiently large r, we have $d^{TS} > 0$.*

The consumer-optimal outcome is $d_k = 0$ because higher addictiveness lowers service quality. In contrast, total surplus may be maximized by $d > 0$.[7] The consumer does not internalize the value of attention to platforms. For a large r, she chooses an inefficiently low level of attention, so positive addictiveness may increase total surplus.

[6] Hagiu and Wright (2020) study a model of dynamic competition with data-enabled learning. In one specification, higher past consumption leads to greater consumption utilities in the future, which resembles beneficial addiction.

[7] If the welfare-maximizing social planner could force the consumer to spend more attention than she would to maximize her utility, the planner would choose zero addictiveness and allocate consumer attention to maximize total surplus. In practice, a regulator would not have such control.

3 Equilibrium

3.1 Monopoly ($K = 1$)

A monopolist maximizes attention subject to the consumer's participation constraint. Let $d^P(\overline{A})$ denote the highest addictiveness that satisfies the participation constraint—i.e., $\max_{A\in[0,\overline{A}]}\ u(A,d) - C(A) = 0$. Let $A(d) := \arg\max_{A\geq 0}\ u(A,d) - C(A)$ denote the consumer's unconstrained choice of attention, which is independent of $\overline{A}$. We then define $d^A(\overline{A}) := \min\left\{d \in [0,\infty] : A(d) \geq \overline{A}\right\}$, which is the lowest addictiveness under which the consumer exhausts her attention. The following result characterizes the monopoly equilibrium and presents comparative statics with respect to attention cap $\overline{A}$.

Proposition 1. *In equilibrium the monopolist sets addictiveness* $\min\left\{d^A(\overline{A}), d^P(\overline{A})\right\}$*, which increases in* $\overline{A}$*. There is* $\overline{A}^M > A(0)$ *such that, as a function of* $\overline{A}$*, the consumer's equilibrium payoff is increasing on* $[0, A(0)]$*, decreasing on* $[A(0), \overline{A}^M]$*, and equal to zero on* $[\overline{A}^M, \infty]$*. In particular, if* $\overline{A} \leq A(0)$*, the equilibrium maximizes consumer and total surplus.*

In Figure 3, the blue solid line depicts the consumer's equilibrium payoff under monopoly as a function of her attention capacity $\overline{A}$. A monopolist's incentive depends on the scarcity of attention. If the attention constraint is tight, the consumer exhausts her attention capacity $\overline{A}$ at zero addictiveness. In such a case, the monopolist sets $d = 0$, which maximizes consumer and total surplus. As $\overline{A}$ increases beyond $A(0)$, the monopolist increases addictiveness to incentivize the consumer to spend more attention. Although a higher $\overline{A}$ relaxes the attention constraint, the increased addictiveness reduces the service utility and harms the consumer. For a large $\overline{A} \geq \overline{A}^M$, the monopolist raises addictiveness to increase consumer attention until she becomes indifferent between joining and not joining the platform.

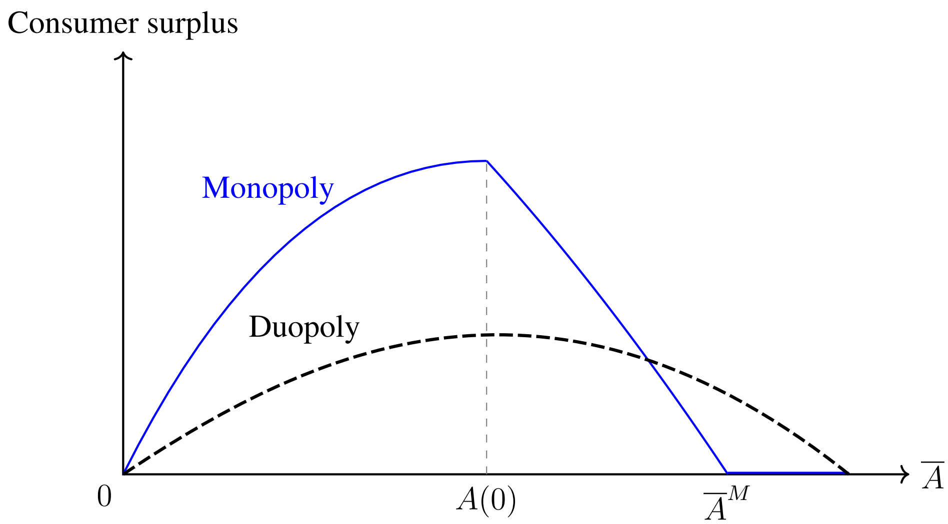

Figure 3: Consumer surpluses under monopoly and duopoly

Note: The graph uses $u(a,d) = 1 - \rho e^{-\rho(a-d)}$ and $C(a) = \frac{c}{2}a^2$ with $(\rho, c) = (2, 1)$.

3.2 Competition ($K \geq 2$)

For each $K \geq 2$, define

$$A_K(d) := \arg \max_{A \in [0, \overline{A}]} Ku\left(\frac{A}{K}, d\right) - C(A).$$

The consumer chooses total attention $A_K(d)$ if she joins K platforms with addictiveness d. The following result characterizes the equilibrium.

Proposition 2. *Fix any $K \geq 2$. In a unique equilibrium, all platforms choose positive addictiveness $d^* > 0$ that makes the consumer indifferent between joining and not joining each platform:*

$$K \cdot u\left(\frac{A_K(d^*)}{K}, d^*\right) - C\left(A_K(d^*)\right) = (K-1) \cdot u\left(\frac{A_{K-1}(d^*)}{K-1}, d^*\right) - C\left(A_{K-1}(d^*)\right). \quad (3)$$

The equilibrium never maximizes consumer surplus.

The intuition is as follows. Upon choosing addictiveness, each platform faces a trade-off. On the one hand, higher addictiveness renders its service less attractive to the consumer. On the other hand, conditional on joining, she will allocate more attention to more addictive services. Each platform then prefers to increase its addictiveness so long as the consumer joins it. The equilibrium addictiveness makes the consumer indifferent between joining and not joining each platform.

The equilibrium does not maximize consumer surplus because platforms choose positive addictiveness. In contrast, it is ambiguous whether the equilibrium addictiveness exceeds the welfare-maximizing level in Claim 1. The equilibrium addictiveness is determined by the consumer's indifference condition and is independent of r, but the welfare-maximizing level can depend on r. As a result, the equilibrium addictiveness can be higher or lower than the welfare-maximizing level. For example, if r is high but platforms decrease addictiveness, the consumer reduces total attention, which may decrease total surplus.

4 The Impact of Competition

We now turn to the main question: Does competition benefit the consumer? We consider the question by analyzing a few notions of increased competition: comparison of monopoly and duopoly, comparison between the equilibrium and the joint-profit maximizing outcome, and the analysis of the "limit economy."

4.1 Monopoly vs. Duopoly

To begin with, we compare monopoly to duopoly. At the same level of addictiveness, the consumer prefers duopoly because she can use more services. When platforms choose addictiveness, the impact of competition depends on the scarcity of attention. Recall that $A(0)$ is the consumer's attention choice on a monopoly platform with zero addictiveness, and $\overline{A}^M$ is a threshold such that for any $\overline{A} \geq \overline{A}^M$, consumer surplus is zero under monopoly.

Proposition 3. *Compare monopoly to duopoly. If attention is so scarce that $\overline{A} \leq A(0)$ holds, the consumer is strictly better off under monopoly. If $\overline{A} \geq \overline{A}^M$, the consumer is weakly better off under duopoly.*

Proof. If $\overline{A} \leq A(0)$ the monopolist chooses zero addictiveness (Proposition 1). Under duopoly the consumer's payoff equals the payoff from joining a single platform, which now chooses positive addictiveness. As a result, the consumer is strictly better off under monopoly. If $\overline{A} \geq \overline{A}^M$, the consumer receives a payoff of zero under monopoly but a nonnegative payoff under duopoly. $\square$

Figure 3 depicts consumer surpluses under monopoly and duopoly. When the attention constraint is tight, higher addictiveness does not increase total attention, so a monopoly platform sets zero addictiveness. Each of competing platforms, however, benefits from higher addictiveness because the consumer will allocate a greater fraction of her total attention $\overline{A}$ to more addictive services. As a result, competition for attention can increase addictiveness and decrease consumer surplus, despite the benefit of providing more services to the consumer.

4.2 Equilibrium vs. Joint-Profit Maximizing Outcome

In this section, we study the question by comparing the equilibrium to the joint-profit maximizing outcome, in which K platforms bundle their services and act to maximize the total profits. The joint-profit maximizing outcome captures the lack of competition without changing the number of platforms in the market. Unlike the comparison of equilibria with different numbers of platforms, the approach enables us to focus on how increased competition affects consumers through the choice of addictiveness but not through the increased service variety.

Definition 1. The *joint-profit maximizing outcome* is the equilibrium of the game in which the platforms collectively choose $(d_1, \ldots, d_K)$ to maximize the sum of their profits while breaking ties for the consumer, and she only chooses between joining all platforms and joining none.

Proposition 4. *The following holds.*

1. *For any $K \geq 2$, there is an $A^* > 0$ such that if $\overline{A} \leq A^*$, the consumer is strictly better off under the joint-profit maximizing outcome than the equilibrium.*

2. *Suppose $\lim_{x \to 0} \dfrac{\partial u}{\partial a}(x, 0) = \infty$. For any $\overline{A} \in \mathbb{R}_{++}$ there is a K^* such that if $K \geq K^*$, the consumer is strictly better off under the joint-profit maximizing outcome than the equilibrium.*

3. *The consumer is weakly better off under the equilibrium than the joint-profit maximizing outcome if $\overline{A}$ is above some threshold.*

Relative to the joint-profit maximizing outcome, competition affects the incentives of platforms in two ways: Competition encourages a platform to offer a higher service quality, because otherwise, the consumer can refuse to join a platform and use other services. At the same time, competition introduces business stealing incentives. Similar to Proposition 3, the business stealing incentives become a dominant force when the attention is scarce. As a result, competition lowers consumer welfare. Point 2 implies that even if we arbitrarily fix $\overline{A}$, the same welfare result holds for a large K under the Inada-type condition. A larger K works similarly as a smaller $\overline{A}$, because it tightens the attention constraint relative to the number of available services.

4.3 Increased Competition with a Fixed Market Size

This section captures increased competition by considering a sequence of markets that become growingly competitive but have the same size. By keeping the market size constant, we exclude the variety-expansion effect that mechanically favors competition.[8] This approach also enables us to characterize the "limit economy" and compare it to monopoly for all parameters, unlike the previous results in which the impact of competition can be ambiguous for an intermediate $\overline{A}$.

Consider a sequence of markets, $(\mathcal{E}_K)_{K\in\mathbb{N}}$. The market $\mathcal{E}_1$ consists of a monopolist that provides service utility $u(a,d)$. For each $K \geq 2$ the market $\mathcal{E}_K$ consists of K platforms, each of which provides service utility $\hat{u}(a,d) := \frac{1}{K}u(aK,d)$. The markets $(\mathcal{E}_K)_{K\in\mathbb{K}}$ have the same size: If the consumer allocates total attention A equally across K platforms, she obtains total utility $u(A,d)$ regardless of K. The converse is also true: In any market with such a property, each platform provides utility $\frac{1}{K}u(aK,d)$.[9] Thus so long as we focus on symmetric markets with a constant size, our choice is unique. In all markets $(\mathcal{E}_K)_{K\in\mathbb{K}}$, the consumer faces the same attention cost and constraint $(C(\cdot),\overline{A})$. Note that we can apply Proposition 2 to the current setting and conclude that each market $\mathcal{E}_K$ has a unique equilibrium. We define the consumer's best response as $A(d) := \arg\max_{A\in[0,\overline{A}]}[u(A,d) - C(A)]$.

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[8] A similar approach appears in papers on the relation between perfectly and imperfectly competitive equilibria in product markets (e.g., Novshek, 1980, 1985; Allen and Hellwig, 1986).

[9] If utility function $\hat{u}(a,d)$ has the property that the consumer obtains $u(A,d)$ by allocating A/K to each of K platforms, we have $K\hat{u}(\frac{a}{K},d) = u(a,d)$, which implies $\hat{u}(a,d) = \frac{1}{K}u(aK,d)$.

Proposition 5. *The equilibrium addictiveness in market $\mathcal{E}_K$ is weakly decreasing in $K \geq 2$. As $K \to \infty$ it converges to $d^\infty > 0$ that solves*

$$u\left(A(d^\infty), d^\infty\right) = A\left(d^\infty\right) \cdot \frac{\partial u}{\partial a}\left(A(d^\infty), d^\infty\right). \tag{4}$$

Proposition 5 states that once we go beyond duopoly, competition reduces addictiveness. However the equilibrium addictiveness remains positive even in the limit. If the market consists of many platforms, the consumer can avoid highly addictive services and allocate her attention to less addictive services. To attract the consumer who has better outside options, platforms need to reduce addictiveness and offer higher service quality. However, the business stealing incentives never vanish, so platforms set positive addictiveness in the limit. Equation (4) captures the intuition in the limiting case. The left-hand side $u\left(A(d^\infty), d^\infty\right)$ is the consumer's loss of not joining a platform, and the right-hand side $A\left(d^\infty\right) \cdot \frac{\partial u}{\partial a}\left(A(d^\infty), d^\infty\right)$ is the incremental gain of reallocating the saved attention. In equilibrium the two terms coincide.

Proposition 5 allows us to establish an analogue of Proposition 3: The consumer is better off under monopoly than the limit economy if the attention is scarce. To state the result, for any $\overline{A} > 0$ and $K \in \mathbb{N}$, let $CS_K(\overline{A})$ denote the consumer surplus in the equilibrium of $\mathcal{E}_K$, and let $CS_\infty(\overline{A})$ denote the one in the limit economy, i.e., $CS_\infty(\overline{A}) = \max_{A \in [0,\overline{A}]}[u\left(A, d^\infty\right) - C(A)]$. Recall that $A(0)$ is the consumer's choice of attention on a monopoly platform with zero addictiveness.

Corollary 1. *Compare monopoly to the limit economy.*

1. *If $\overline{A} \leq A(0)$, the consumer is strictly better off under monopoly: $CS_1(\overline{A}) > CS_\infty(\overline{A})$.*
2. *There is $\overline{A}^M > 0$ (defined in Proposition 1) such that if $\overline{A} \geq \overline{A}^M$, the consumer is weakly better off in the limit economy: $CS_1(\overline{A}) \leq CS_\infty(\overline{A})$.*

Proof. Point 1 holds because the monopoly platform chooses zero addictiveness for any $\overline{A} \leq A(0)$, whereas the limit outcome involves positive addictiveness. Point 2 holds because for $\overline{A} \geq \overline{A}^M$, the consumer's receivers a payoff of zero. □

To obtain stronger results, we now impose more structures on the consumer's payoffs. We present welfare implications in the main text and relegate the analytical expression of the equilibrium objects to the appendix. The following result presents a case in which we obtain a strict welfare comparison for almost all parameters (see Figure 4).

Proposition 6 (Quadratic Attention Cost and Exponential Utility). *Assume $C(a) = \frac{ca^2}{2}$ for some $c > 0$, and $u(a, d) = 1 - e^{-\rho(a-d)}$ for some $\rho > c$. Consumer surplus is lower under the limit economy than under monopoly if and only if $\overline{A} \leq \overline{A}^* := \frac{-c+\sqrt{c^2+4c\rho^2}}{2c\rho}$. The welfare comparison is strict whenever $\overline{A} \neq \overline{A}^*$.*

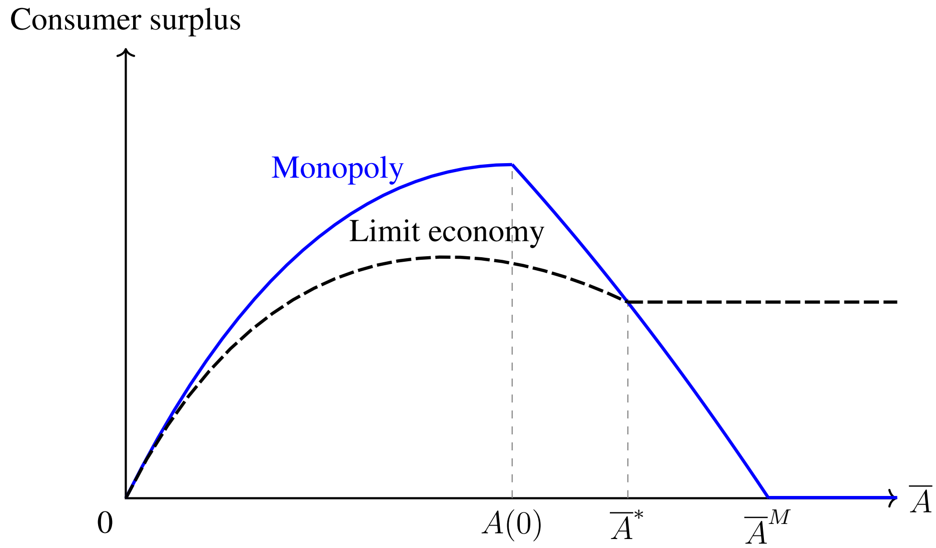

Figure 4: Consumer surpluses under the quadratic attention cost

Note: The graph uses $(\rho, c) = (2, 1)$, where $A(0) \approx 0.601$, $\overline{A}^* \approx 0.781$, and $\overline{A}^M = 1$.

The next result shows a case in which moving from monopoly to competitive markets weakly lowers consumer welfare across all $\overline{A}$.

Proposition 7 (Linear Attention Cost). *Assume $C(a) = ca$ for some $c > 0$, and $u(a, d) = v(a - d)$ for an increasing concave $v(\cdot)$ with $v'(0) > c$. Let $g = (v')^{-1}$ denote the inverse of v'.*

1. *If $\overline{A} < \frac{v(g(c))}{c}$, consumer surplus is strictly higher under monopoly than any market $(\mathcal{E}_K)_{K\geq 2}$.*
2. *If $\overline{A} \geq \frac{v(g(c))}{c}$, consumer surplus is zero in any market $(\mathcal{E})_{K\in\mathbb{N}}$.*

Figure 5 depicts the consumer's equilibrium payoffs under monopoly and the limit economy. Corollary 1 suggests that the consumer could benefit or lose from competition, depending on her attention capacity $\overline{A}$. Proposition 7 shows that the welfare comparison could be unambiguous, i.e., competition may weakly harm the consumer across all parameters.

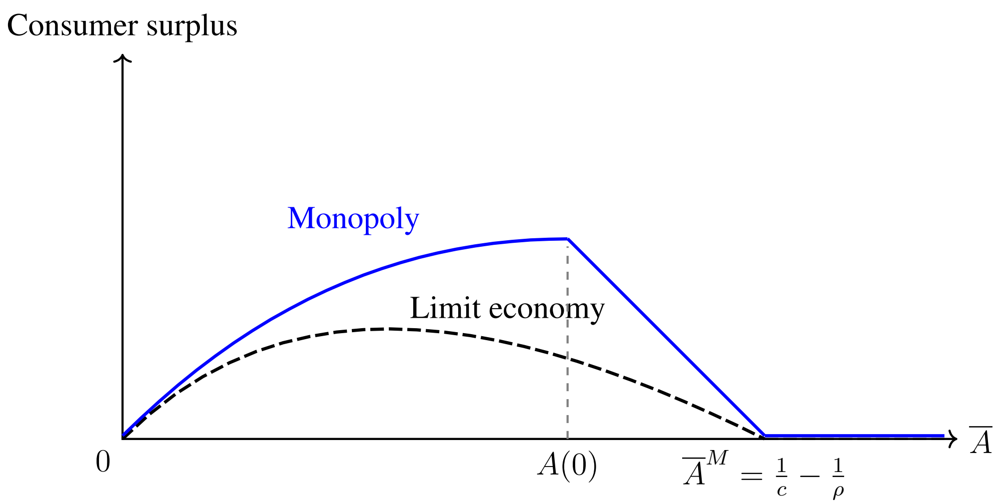

Figure 5: Consumer surpluses in the linear environment

Note: The graph uses $u(a,d) = 1 - e^{\rho(a-d)}$ and $C(a) = ca$ with $(\rho, c) = (2, 1)$, where $A(0) \approx 0.346$ and $\frac{1}{c} - \frac{1}{\rho} = 0.5$.

Recall that under monopoly, the consumer's equilibrium payoff is non-monotone in her attention capacity $\overline{A}$ (Proposition 1). The results of this section show that the same non-monotonicity holds in the limit economy. Competition may not eliminate a platform's incentive to raise addictiveness to influence the consumer's choice of total attention.

5 Costly Investment in Addictive Technology

In this section, we extend the analysis by assuming that a platform incurs a cost of raising addictiveness, such as the cost of technological investment. When platforms incur such costs, competition could reduce consumer welfare regardless of the scarcity of attention. Throughout the section, we impose the following assumptions.

Assumption 3. Each platform incurs a cost of $\kappa\gamma(d)$ to choose d with $\kappa > 0$. The function $\gamma : \mathbb{R}_+ \to \mathbb{R}_+$ is increasing, strictly convex, and differentiable, and satisfies $\lim_{d\to 0} \gamma'(d) = 0$ and $\lim_{d\to\infty} \gamma'(d) = \infty$. Assume $u(a,d) = v(a-d)$, and $g'(C'(x))C''(x)$ is weakly decreasing in x, where $g(\cdot)$ is the inverse of $v'(\cdot)$.

The last condition makes the profit of each platform concave in its addictiveness given the consumer's optimal behavior. The condition holds, for example, if $C(\cdot)$ is quadratic. The parameter κ captures how costly it is for a platform to increase addictiveness, and $\kappa = 0$ is our baseline model.

We have shown that for a large $\overline{A}$, competition benefits the consumer because a platform that has market power will choose a high d to increase attention. The costly choice of addictiveness can overturn the result.[10] The following result shows that if and only if the cost of increasing d is substantial, increased competition, in the sense we studied in Section 4.2, harms the consumer.

Proposition 8. *Assume each platform incurs cost $\kappa\gamma(d)$ to choose addictiveness d. There is a unique equilibrium in which all platforms choose the same addictiveness. For any $\overline{A} > 0$ including $\overline{A} = \infty$, there is a $\kappa^* \in \mathbb{R}_+$ such that consumer surplus is higher under the joint-profit maximizing outcome than the equilibrium if and only if $\kappa \geq \kappa^*$.*

The result does not contradict Proposition 4. For example, if $\overline{A}$ is small, the consumer is better off under the joint-profit maximizing outcome for any $\kappa \geq \kappa^* = 0$.

To see the intuition, suppose for simplicity that $\overline{A} = \infty$. If κ is low, under the joint-profit maximization, platforms choose a high d that yields the consumer a payoff of zero. In such a case, competition benefits the consumer for the standard reason: The consumer can choose to not join a low-quality platform and to use other services. The better outside option incentivizes platforms to decrease addictiveness, leading to a higher consumer welfare. In contrast, for a high κ, a platform's choice is determined not by the consumer's participation incentive but by the marginal calculus between the benefit of more attention and the cost of increasing d. In such a case, competing platforms, which have business stealing incentives, sacrifice quality for attention more than how they would in the absence of competition. The argument does not depend on the attention constraint.

6 Digital Curfew

How would a regulator increase consumer welfare when it cannot directly control a platform's choice of d? We examine the impact of a digital curfew, which restricts the consumer's platform

[10] The other result—i.e., increased competition reduces consumer welfare if $\overline{A}$ is small—continues to hold for any $\kappa \geq 0$.

usage. Under a digital curfew at A, the consumer's attention cap becomes $\overline{A} = A$. Recall $A(0)$ denotes the consumer's optimal attention on a monopoly platform with zero addictiveness.

Proposition 9. *The following holds.*

1. *In a monopoly market, a digital curfew at $\overline{A} = A(0)$ attains the consumer-optimal outcome.*

2. *For any $K \geq 2$, no digital curfew attains the consumer-optimal outcome.*

3. *Take any $K \geq 2$ and $\overline{A} = A$. Suppose the attention constraint holds with a strict inequality in equilibrium. Then a digital curfew at some $\overline{A} = A_D < A$ strictly benefits the consumer.*

The intuition is that a digital curfew reduces a platform's incentive to increase addictiveness to expand the consumer's total attention, but it does not eliminate business stealing incentives. For example, consider a digital curfew at $\overline{A} = A(0)$, which prevents the consumer from spending longer time on digital services than how much she would have spent if the services had zero addictiveness. Under monopoly, such a digital curfew makes it optimal for the platform to set zero addictiveness. The same digital curfew, however, does not eliminate business stealing incentives, because the consumer will allocate a greater fraction of her attention to more addictiveness platforms, even if the total attention is fixed.

Two remarks are in order. First, we have examined a policy that limits total attention across platforms, but it is not the only way to define a digital curfew. For example, suppose a regulator could require that the consumer spend at most $A_K(0)/K$ unit of attention (defined in Section 3.2) on *each* platform. Such a policy induces zero addictiveness and maximized consumer welfare.

Second, we assume a digital curfew is exogenous to the consumer, but we could ask whether consumers are willing to adopt a digital curfew voluntarily. Suppose that there is a continuum of consumers, each of whom $i \in [0, 1]$ chooses the maximum amount of attention $A_i \in [0, A_{max}]$ she can spend on platforms ($A_{max} > 0$ is an exogenous cap on possible attention constraints). After consumers choose $(A_i)_{i \in [0,1]}$, the original game of attention competition is played.[11] In equilibrium, all consumers choose the maximum attention A_{max}, because each consumer is atomless and her choice does not affect the behavior of platforms. Consumers cannot voluntarily enforce a digital curfew, even though they could benefit from collectively reducing A_i's.

[11] If platform k obtains attention a_k^i from each consumer i, then k's profit is $\int_{i \in [0,1]} a_k^i$.

7 The Role of Revenue Models

We have shown that competition can benefit or harm the consumer. The result depends on the revenue models of platforms. To highlight the idea, we study the following model of price competition. As in Section 4.3, we describe the model of price competition by keeping the market size constant. The game of price competition in market $\mathcal{E}_K$ is as follows. First, each platform $k \in K$ simultaneously chooses its addictiveness $d_k \geq 0$ and price $p_k \in \mathbb{R}$. The consumer observes $(d_k, p_k)_{k \in K}$, then chooses the set $J \subset K$ of platforms to join and how much attention to allocate. The consumer has to pay price p_k to join platform k. Each platform $k \in J$ receives a payoff of p_k, and any platform $k \notin J$ obtains a payoff of zero. The consumer receives a payoff of $\sum_{k \in J} \frac{1}{K} [u(Ka_k, d_k) - p_k] - C\left(\sum_{k \in J} a_k\right)$ if $J \neq \emptyset$ and zero if $J = \emptyset$ (recall that in $\mathcal{E}_K$, the service utility of each platform is $\frac{1}{K} u(Ka, d)$). In equilibrium, the consumer solves

$$\max_{J \subset K, (a_k)_{k \in J}} \sum_{k \in J} \left[\frac{1}{K} u(Ka_k, d_k) - p_k\right] - C\left(\sum_{k \in J} a_k\right) \tag{5}$$

$$\text{s.t.} \quad \sum_{k \in J} a_k \leq \overline{A} \quad \text{and} \quad a_k \geq 0, \forall k \in J,$$

where the objective is zero if $J = \emptyset$.

Under price competition, platforms do not monetize attention, and they charge prices that are independent of consumer attention. The model captures digital services not supported by advertising, such as Netflix and YouTube Premium.[12]

Lemma 1. *The game of price competition has a unique equilibrium, in which all platforms choose zero addictiveness and set the same positive price that makes the consumer indifferent between joining K and $K-1$ platforms. Consumer surplus is minimized under monopoly.*

Under price competition the profits of platforms do not depend on attention, so they prefer to decrease addictiveness and charge higher prices. In equilibrium all platforms set zero addictiveness, and price p^* equals the incremental contribution of each platform to the consumer's total

[12] We do not consider the endogenous choice of business models or richer pricing instruments that may use allocated attention to determine a price. For recent studies on business models in two-sided markets, see, e.g., Gomes and Pavan (2016), Lin (2020), Carroni and Paolini (2020), and Jeon et al. (2021).

payoff. In particular, a monopoly platform extracts full surplus. As a result, under price competition, increased competition examined in Section 4 benefits the consumer.

We now compare different business models from the consumer's perspective. The consumer can use services for free under attention competition, but the service quality is typically lower than under price competition. The following result provides sufficient conditions under which the consumer is better off under attention competition.

Proposition 10. *The consumer is strictly better off under attention competition than price competition in market $\mathcal{E}_K$ if the attention cost $C(\cdot)$ is strictly convex and the market is sufficiently competitive—i.e., K is greater than some cutoff $K^* \in \mathbb{N}$.*

The intuition is as follows. Under attention competition, platforms set positive addictiveness, so the consumer faces higher marginal utilities of allocating attention. The consumer then faces a higher gain of refusing to join a platform and continuing to use other $K-1$ platforms. For example, if the consumer spends total attention A on platforms, she can increase her attention to each platform $j \neq 1$ from $\frac{A}{K}$ to $\frac{A}{K-1}$ by not joining platform 1. The gain of doing so increases in d. If the consumer faces a higher gain of not joining each platform under attention competition, platforms must provide higher net utilities to ensure consumer participation. The actual proof is more subtle, because the consumer faces a steeper utility function under attention competition than price competition, but she evaluates these functions at different levels of attention. However, in a sufficiently competitive market, higher marginal utilities ensure a higher consumer surplus under attention competition.

8 Extension: Naive Consumer

In practice, consumers may be unaware of the (part of) addictive features of platforms. We now consider such a naive consumer.[13] We extend the model of Section 2 as follows (Appendix H provides details). In the first stage, all platforms simultaneously choose addictiveness, $(d_k)_{k \in K}$. In the second stage, the consumer decides which platforms to join, based on the *perceived addictiveness,*

[13]Our consumer naivete is similar to the uninformed myopes in Gabaix and Laibson (2006), whereby consumers do not recognize the full price of a product before making actual purchase decisions.

$(sd_k)_{k \in K}$. The parameter $s \in (0, 1]$ is exogenous and captures the degree of consumer sophistication. If (and only if) $s < 1$, the consumer falsely thinks that the addictiveness of each platform is lower than the true value. In the third stage, after joining platforms the consumer allocates attention to maximize her utility based on the true addictiveness. The consumer's problem is the same as (2) except she has chosen the set of platforms to join based on the perceived addictiveness.

If the consumer is naive, a platform finds it more profitable to increase its addictiveness, because it can capture more attention without much affecting the consumer's participation decision.

Proposition 11. *For any $K \geq 2$, in equilibrium, platforms choose addictiveness $\frac{d^*}{s} > 0$, where d^* is the equilibrium addictiveness under the sophisticated consumer in Proposition 2. The consumer's equilibrium payoff given the true addictiveness is increasing and the payoff of each platform is decreasing in s.*

Appendix H also shows that the consumer naivete tends to favor the game of price competition. Consumer surplus is increasing in s under attention competition but independent of s under price competition. As a result, for any $K \geq 2$, a sufficiently naive consumer is better off under price competition than attention competition when there are multiple platforms.

9 Conclusion

Competition for consumer attention could distort the kind of services provided to the market: Driven by business stealing incentives, platforms provide low quality services that are capable of capturing attention. In terms of the scarcity of attention and the cost investing in addictive technology, we provide a condition under which increased competition reduces consumer welfare. A digital curfew could improve consumer welfare by eliminating part of a platform's incentive to increase addictiveness. The negative impact of competition does not arise under the standard price competition. To our knowledge, the paper highlights the potential downside of competition in the attention economy that has been discussed in the public and policy debate but not been examined in the literature.

We close the paper with several directions for future research. First, the literature points to behavioral biases that are relevant to addiction, so it would be promising to incorporate them into a model of competition for attention in which firms may exploit behavioral biases to capture attention. Second, it appears worth studying platforms' choices of business models and their implications on addictiveness and welfare. For example, a firm may offer both ad-supported and subscription plans to screen consumers who have different preferences. From the empirical perspective, the most relevant question is what features of digital services correspond to the "addictiveness" of our model. Also, little seems known about how consumers allocate their attention across multiple digital services and how their attention responds to various features of platforms. Building upon our work, we anticipate further studies on various intriguing questions on theoretical and empirical fronts.

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Appendix

A Proof of Claim 1

Proof. Consumer surplus is maximized by $d_k = 0$ because service utilities decrease in addictiveness. To show $d^{TS} > 0$ for a large r, suppose all platforms choose $d = 0$. The consumer optimally joins all platforms. Denoting $TS(d) = CS(d) + rA(d)$, the envelope formula implies

$$TS'(0) = K\frac{\partial u}{\partial d}\left(\frac{A(0)}{K}, 0\right) + rA'(0).$$

Because $A'(0) > 0$, we have $TS'(0) > 0$ for a large r. For such an r, if all platforms choose a small but positive d, the consumer strictly increases her attention, and the total surplus strictly increases. □

B Proof of Proposition 1

Proof. Suppose the monopolist chooses $d^M(\overline{A}) := \min\left\{d^A(\overline{A}), d^P(\overline{A})\right\}$. Because $d^M(\overline{A}) \leq d^P(\overline{A})$, it is optimal for the consumer to join the platform. If $d^M(\overline{A}) = d^P(\overline{A})$ and the monopolist increases addictiveness, the consumer will not join it. If $d^M(\overline{A}) = d^A(\overline{A})$ and the monopolist increases addictiveness, the consumer will continue to choose $\overline{A}$ because her marginal utility of allocating attention is increasing in d. The monopolist then continues to earn the same payoff, $r\overline{A}$. In either case the monopolist does not strictly benefit from changing $d^M(\overline{A})$. Because $d^P(\overline{A})$ and $d^A(\overline{A})$ are increasing in $\overline{A}$, $d^M(\overline{A})$ is increasing in $\overline{A}$.

To show the comparative statics in $\overline{A}$, we say that *the participation constraint binds* if the consumer's equilibrium payoff is zero. We also say that *the attention constraint is slack* if the consumer chooses $A(d^M(\overline{A})) \leq \overline{A}$, that is, the consumer's unconstrained choice of attention at $d^M(\overline{A})$ satisfies the attention constraint. Note that the attention constraint can hold with equality and be slack at the same time.

First, take any $\overline{A}^1$ at which the participation constraint binds. If the attention constraint is not slack (i.e., $A(d^M(\overline{A}^1)) > \overline{A}^1$), the platform could slightly lower addictiveness to attain the same payoff $r\overline{A}^1$. This contradicts the tie-breaking rule of the monopolist (see Section 2). Thus if the participation constraint binds, the attention constraint is slack. As a result, we have $\max_{a\geq 0} u(a, d^M(\overline{A}^1))-$

$C(a) = 0$, i.e., the consumer's optimal payoff from the unconstrained problem is equal to zero. For any $\overline{A}^2 > \overline{A}^1$ we have

$$\max_{a\in[0,\overline{A}^2]} u(a, d^M(\overline{A}^2)) - C(a) \leq \max_{a\geq 0} u(a, d^M(\overline{A}^1)) - C(a) = 0,$$

because in the right-hand side of the inequality, the consumer does not face the attention constraint and the platform chooses lower addictiveness. As a result, the participation constraint also binds at $\overline{A}^2$. Thus there is some $\overline{A}^M$ such that the participation constraint binds if and only if $\overline{A} \geq \overline{A}^M$.

We show the comparative statics using $A(0)$ and $\overline{A}^M$. First, for any $\overline{A} \leq A(0)$ the consumer chooses $\overline{A}$ at $d^M = 0$, so it is indeed the monopolist's equilibrium choice. The consumer's equilibrium payoff is increasing in $\overline{A}$ whenever $\overline{A} \leq A(0)$ because the consumer faces the same addictiveness with a more relaxed attention constraint. Because the consumer earns a positive payoff at $d^M = 0$, we have $\overline{A}^M > A(0)$. Second, for any $\overline{A} \in [A(0), \overline{A}^M)$ the participation constraint is not binding. In such a case we have $d^M(\overline{A}) = d^A(\overline{A})$, i.e., the monopolist chooses the lowest addictiveness at which the consumer exhausts her attention. Given $d^A(\overline{A})$, the consumer's unconstrained choice $A(d^M(\overline{A}))$ satisfies the attention constraint with equality. To show that her payoff decreases on $[A(0), \overline{A}^M]$, take any $\overline{A}^3$ and $\overline{A}^4$ such that $A(0) \leq \overline{A}^3 < \overline{A}^4 < \overline{A}^M$. We have

$$\begin{aligned}
&\max_{a\in[0,\overline{A}^4]} u(a, d^M(\overline{A}^4)) - C(a) \\
=&\max_{a\geq 0} u(a, d^M(\overline{A}^4)) - C(a) \\
\leq&\max_{a\geq 0} u(a, d^M(\overline{A}^3)) - C(a) \\
=&\max_{a[0,\overline{A}^3]} u(a, d^M(\overline{A}^3)) - C(a)
\end{aligned}$$

Thus the consumer's payoff is lower under $\overline{A}^4$ than $\overline{A}^3$. Finally the consumer's payoff hits zero at $\overline{A} = \overline{A}^M$, which completes the proof. □

C Proof of Proposition 2

We first prove several lemmas.

Lemma C.1. *Take any increasing, strictly concave, and differentiable function, $u(\cdot) : \mathbb{R}_+ \to \mathbb{R}$. Then, $u(x) - xu'(x)$ is strictly increasing in x.*

Proof. For any x and $y > x$, we have

$$\frac{u(y) - u(x)}{y - x} > u'(y) \Rightarrow \; u(y) - u(x) > u'(y)(y - x) \Rightarrow \; u(y) - yu'(y) > u(x) - xu'(x).$$

$\square$

Lemma C.2. *For any $y > 0$, consider the problem*

$$\max_{A \in [0, \overline{A}]} y \cdot u\left(\frac{A}{y}, d\right) - C(A). \tag{A.1}$$

Let $A^*(y)$ and $V^*(y)$ denote the maximizer and the maximized value, respectively. Then, $A^*(y)$ is increasing in y, $\frac{A^*(y)}{y}$ is decreasing in y, $V^*(y)$ is strictly concave in y, and $\frac{dV^*}{dy}$ is decreasing in d.

Proof. We write u_1 and u_{11} for $\frac{\partial u}{\partial a}$ and $\frac{\partial^2 u}{\partial a^2}$. Define $V(A, y) := y \cdot u\left(\frac{A}{y}, d\right) - C(A)$. We have $\frac{\partial^2 V}{\partial A \partial y} = -\frac{A}{y^2} u_{11}\left(\frac{A}{y}, d\right) > 0$, and thus $A^*(y)$ is increasing in y. To show $\frac{A^*(y)}{y}$ is decreasing, we rewrite (A.1) as

$$\max_{a \in [0, \overline{A}/y]} y \cdot u\left(a, d\right) - C(ay). \tag{A.2}$$

The maximizer of (A.2) is $a^*(y) := \frac{A^*(y)}{y}$. If $A^*(y) < \overline{A}$, then $a^*(y)$ satisfies the first-order condition $u_1(a, d) - C'(ay) = 0$, whose solution is decreasing in y. If y is so large that $A^*(y) = \overline{A}$, then for any such y, we have $a^*(y) = \frac{\overline{A}}{y}$, which is decreasing in y. Because $\frac{A^*(y)}{y}$ is continuous in y, it is decreasing in y.

We now show that $V^*(y)$ is concave. The envelope theorem (e.g., Corollary 4 of Milgrom and Segal (2002)) implies

$$\frac{dV^*}{dy} = u\left(\frac{A^*(y)}{y}, d\right) - \frac{A^*(y)}{y} u_1\left(\frac{A^*(y)}{y}, d\right).$$

This expression is decreasing in y, because $u(x, d) - xu'(x, d)$ is increasing in x (Lemma C.1) and $\frac{A^*(y)}{y}$ is decreasing in y. Finally, $\frac{\partial^2 V^*}{\partial y \partial d} = u_2\left(\frac{A^*(y)}{y}, d\right) + y \cdot u_{12}\left(\frac{A^*(y)}{y}, d\right) \cdot \frac{\partial}{\partial y}\left(\frac{A^*(y)}{y}\right) < 0$. The cross derivative $\frac{\partial^2 V^*}{\partial y \partial d}$ is well-defined for all $y \neq y^*$. Thus, $\frac{\partial V^*}{\partial y} = \int_0^d \frac{\partial^2 V^*}{\partial y \partial d}(y, t) dt + c$ (with some constant c) is decreasing in d. $\square$

The next lemma says that the incremental gain of joining a platform decreases in the addictiveness of other platforms the consumer has joined.

Lemma C.3. *Fix any $d' \geq 0$ and $\overline{A} > 0$, and consider the problem*

$$U(x, y, d) := \max_{(A_x, A_y) \in \mathbb{R}^2} \; x \cdot u\left(\frac{A_x}{x}, d\right) + y \cdot u\left(\frac{A_y}{y}, d'\right) - C(A_x + A_y) \tag{A.3}$$
$$s.t. \quad A_x \geq 0, A_y \geq 0, A_x + A_y \leq \overline{A}.$$

Then, $U_2(x, y, d)$ is decreasing in d.

Proof. The envelope theorem implies $U_2(x, y, d) = u\left(\frac{A_y^*}{y}, d'\right) - \frac{A_y^*}{y} u_1\left(\frac{A_y^*}{y}, d'\right)$, where A_y^* is a part of the maximizer of (A.3). The objective function in (A.3) is supermodular in $(A_x, -A_y, d)$, so A_y^* is decreasing in d. Lemma C.1 implies $u(a, d') - a \cdot u_1(a, d')$ is increasing in a. Thus $U_2(x, y, d)$ is decreasing in d. □

The following result shows that the consumer faces a decreasing incremental gain of joining platforms for any choices of addictiveness.

Lemma C.4. *Take any $S, S' \subset K_{-1} := \{2, 3, \ldots, K\}$ such that $S' \subset S$. For any choice of addictiveness, the consumer's incremental gain of joining platform 1 is greater when she has already joined platforms S' than S. Formally, the following holds. Fix any $(d_1, \ldots, d_K) \in \mathbb{R}_+^K$. For any $y \in [0, 1]$ and $S \subset K_{-1}$, define*

$$V(y, S) := \max_{(a_k)_{k \in S \cup \{1\}}} \sum_{k \in S} u(a_k, d_k) + y \cdot u(a_1, d_1) - C\left(\sum_{k \in S \cup \{1\}} a_k\right) \tag{A.4}$$
$$s.t. \quad \sum_{k \in S \cup \{1\}} a_k \leq \overline{A} \quad and \quad a_k \geq 0, \forall k \in S \cup \{1\}.$$

Then for any $S', S \subset K_{-1}$ such that $S' \subsetneq S$,

$$\frac{\partial V}{\partial y}(y, S) \leq \frac{\partial V}{\partial y}(y, S'). \tag{A.5}$$

In particular, $V(1, S) - V(0, S) \leq V(1, S') - V(0, S')$. These inequalities are strict whenever the consumer allocates positive attention to every platform in S and S' upon solving (A.4).

Proof. We fix any $(d_1, \ldots, d_K)$. To simplify notation, we write $u(a,d)$ as $u(a)$ and $\frac{\partial u}{\partial a}$ as $u'(a)$. Let $a_1(y,S)$ denote the optimal value of a_1 in (A.4). By the envelope formula, we have

$$\frac{\partial V}{\partial y}(y,S) = u(a_1(y,S)).$$

To show (A.5), we first show $a_1(y,S) \leq a_1(y,S')$ for any $S' \subset S$.

Suppose to the contrary that $a_1(y,S) > a_1(y,S')$, which implies $u'(a_1(y,S')) > u'(a_1(y,S))$. Because $a_1(y,S) > 0$, we have $u'(a_1(y,S)) \geq u'(a_j(y,S))$ for all $j \in S$; otherwise, the consumer can increase her payoff by decreasing a_1 and increasing a_j. Similarly, for every $j \in S'$ such that $a_j(y,S') > 0$, we have $u'(a_j(y,S')) \geq u'(a_1(y,S'))$. These inequalities imply that for every $j \in S'$ with $a_j(y,S') > 0$, we have $u'(a_j(y,S')) > u'(a_j(y,S))$, or equivalently, $a_j(y,S) > a_j(y,S')$. Also, there is some $j \in S'$ with $a_j(y,S') > 0$ because of the last inequality in Assumption 2. We derive a contradiction. First, if we have $\sum_{k \in S' \cup \{1\}} a_k(y,S') = \overline{A}$, we obtain $\sum_{k \in S \cup \{1\}} a_k(y,S) > \overline{A}$, which is a contradiction. Second, if $\sum_{k \in S' \cup \{1\}} a_k(y,S') < \overline{A}$, then for any j with $a_j(y,S') > 0$, we have

$$u'(a_j(y,S)) < u'(a_j(y,S')) = C'\left(\sum_{k \in S' \cup \{1\}} a_k(y,S')\right) < C'\left(\sum_{k \in S \cup \{1\}} a_k(y,S)\right),$$

which is also a contradiction. As a result, we obtain $a_1(y,S) \leq a_1(y,S')$. Integrating both sides of (A.5) from $y = 0$ to $y = 1$, we have $V(1,S) - V(0,S) \leq V(1,S') - V(0,S')$. If the consumer allocates positive attention to every platform in S and S' upon solving (A.4), we can use the same argument to show that $a_1(y,S) \geq a_1(y,S')$ (i.e., weak inequality) leads to a contradiction. Thus we have $a_1(y,S) < a_1(y,S')$ and obtain (A.5) as a strict inequality. $\square$

Proof of Proposition 2. STEP 1:*There is a unique $d^* > 0$ that satisfies (3).* To show this, define

$$f(K,d) := Ku\left(\frac{A_K(d)}{K}, d\right) - C(A_K(d)) - \left[(K-1)u\left(\frac{A_{K-1}(d)}{K-1}, d\right) - C(A_{K-1}(d))\right].$$

The function $f(K,d)$ is the difference between payoffs when the consumer joins K platforms and when she joins $K-1$ platforms, given optimally allocating attention. Hereafter, we use the notation $V^*(y,d)$ for $V^*(y)$ of Lemma C.2 to make the dependence of $V^*(y)$ on d explicit. We can write $f(K,d) = V^*(K,d) - V^*(K-1,d)$. Lemma C.2 implies $V_1^*(y,d)$ is decreasing in d. Thus, $f(K,d) = \int_{K-1}^{K} V_1^*(y,d)dy$ is decreasing in d. Points (a) and (b) of Assumption 1 imply $f(K,0) > 0$ and $f(K,d) < 0$ for a large d. Thus, there is a unique d^* such that $f(K,d^*) = 0$, and d^* solves (3).

STEP 2: *There is an equilibrium in which each platform sets d^*.* Suppose all platforms choose d^*. First, we show that the consumer prefers to join all the platforms. Given $d_k = d^*$ for all k, the consumer's payoff from joining $J \leq K$ platforms is $V^*(J,d^*)$, which is strictly concave in J (Lemma C.2). We have $V^*(K,d^*) = V^*(K-1,d^*)$ by construction. As a result, $V^*(J,d^*)$ is strictly increasing in $y \leq K-1$. Thus, the consumer prefers to join all platforms.

Second, no platform has a profitable deviation. Consider the incentive of (say) platform 1. If it increases d_1, the consumer joins only platforms $2,\ldots,K$ to achieve the same payoff as without platform 1's deviation. Platform 1 does not benefit from such a deviation.

Suppose platform 1 decreases d_1 from d^* to d. The consumer joins platform 1. If she additionally joins other $y \leq K-1$ platforms, her payoff becomes $U(1,y,d)$ according to the notation of Lemma C.3 (with $d' = d^*$). Before the deviation (i,e., $d_1 = d^*$), $U(1,y,d^*)$ is maximized at $y = K-2$ and $y = K-1$. Because $U_{23}(1,y,d) < 0$ by Lemma C.3, the consumer's marginal gain from joining platforms increases after platform 1's deviation to $d < d^*$. As a result, $U(1,y,d)$ is uniquely maximized at $y = K-1$ across all $y \in \{1,\ldots,K-1\}$, so the consumer joins all platforms after the deviation. However, the consumer will then allocate a smaller amount of attention to platform 1 compared to without deviation, because platform 1 now offers a lower marginal utility. Thus, platform 1 does not strictly benefit from decreasing addictiveness.

STEP 3: *The uniqueness of equilibrium.* Take any pure-strategy subgame perfect equilibrium. Because any platform can set $d_k = 0$ to ensure participation, the consumer joins all platforms in equilibrium. First, we show all platforms choose the same addictiveness. Suppose to the contrary that there is an equilibrium in which platforms choose $(d_k^*)_{k\in K}$ such that (without loss) $d_2^* = \max_k d_k^* > \min_k d_k^* = d_1^*$. We show platform 1 has a profitable deviation. Suppose platform 1 deviates and increases its addictiveness to $d_1 = d_1^* + \varepsilon < d_2^*$. We show that the consumer joins platform 1 for a small ε. Suppose the consumer joins platform 2 after the deviation. Then she will also join platform 1; otherwise, she could obtain a strictly higher payoff by replacing platform 2 with 1. Suppose she does not join platform 2 after the deviation. Note that before the deviation, the consumer weakly prefers to join platform 1 when she joins other $K-1$ platforms. Lemma C.4 implies that before the deviation the consumer strictly prefers to join platform 1 when she does not join platform 2. As a result even after the deviation, the consumer strictly prefers to join platform 1 for a small $\varepsilon > 0$. In either case, platform 1 can profitably deviate to $d_1^* + \varepsilon$ with a small $\varepsilon > 0$ because the consumer joins platform 1 and allocates strictly greater attention. We obtain a contradiction because $(d_k^*)_{k \in K}$ is a part of equilibrium.

We have shown that all platforms choose the same addictiveness in any pure-strategy equilibrium. If the equilibrium addictiveness does not satisfy the consumer's indifference condition (3), then either (i) the left-hand side is strictly greater, in which case a platform prefers to deviate and increase its addictiveness, or (ii) the right-hand side is greater, in which case the consumer does not join at least one platform. In either case we obtain a contradiction. $\square$

D Proofs for Section 4: The Impact of Competition

Proof of Proposition 4. In equilibrium, platforms choose positive addictiveness. In contrast, if the consumer exhausts her attention $\overline{A}$ at zero addictiveness, platforms choose $d = 0$ at the joint-profit maximizing outcome. Thus we obtain Points 1 and 2 if we show that the consumer chooses $\sum_{k \in K} a_k = \overline{A}$ under the conditions stated there.

First, let $A_K(0)$ denote the total attention the consumer would choose on K platforms with zero addictiveness. If $\overline{A} \leq A_K(0)$, she chooses $\sum_{k \in K} a_k = \overline{A}$, so we have Point 1. Second, the Inada-type condition in Point 2 implies that for some $\Delta > 0$, we have $u_1(\Delta, 0) - C'(\overline{A}) > 0$. For any $K > \frac{\overline{A}}{\Delta}$, the consumer will choose $\sum_{k \in K} a_k = \overline{A}$. Otherwise, we have $a_k < \Delta$ for at least one k, in which case the consumer could benefit from increasing her attention on platform k.

For Point 3, it suffices to show that the consumer obtains zero payoff at the joint-profit maximizing outcome for a large $\overline{A}$. Let $(d_1, \ldots, d_K)$ denote the solution of the joint-profit maximizing outcome when $\overline{A} = \infty$. Let A denote the total attention she will choose. Suppose she faces attention capacity $\overline{A} = B > A$ but obtains a positive payoff. Then the attention constraint must hold with equality. However, the platforms can attain $B > A$ at the joint-profit maximizing outcome, which contradicts the construction of A. Thus the consumer obtains zero payoff for any

$\overline{A} > A$. $\square$

Proof of Proposition 5. To show the first part, fix any $K \geq 2$ and let d^* denote the equilibrium addictiveness in $\mathcal{E}_K$. Let $A_x(d)$ denote the unique maximizer of the problem

$$V^*(x,d) := \max_{A \in [0,\overline{A}]} x \cdot u\left(\frac{A}{x}, d\right) - C(A). \tag{A.6}$$

If the consumer joins K platforms with addictiveness d, she allocates total attention $A_1(d)$ (recall the normalization of service utility functions). If the consumers joins $K-1$ platforms, she allocates total attention $A_{\frac{K-1}{K}}(d)$. In equilibrium, the consumer is indifferent between joining K and $K-1$ platforms. Thus, we have

$$u(A_1(d^*), d^*) - C(A_1(d^*)) = \frac{K-1}{K} u\left(\frac{K}{K-1} A_{\frac{K-1}{K}}(d^*), d^*\right) - C\left(A_{\frac{K-1}{K}}(d^*)\right). \tag{A.7}$$

Suppose to the contrary that for some K, the equilibrium addictiveness weakly increases from d^* to d^{**} as we move from $\mathcal{E}_K$ to $\mathcal{E}_{K+1}$. Equation (A.7) implies that $V^*(1, d^*) = V^*(\frac{K-1}{K}, d^*)$. Because $V^*(x,d)$ is strictly concave in x, we have $V^*(1,d^*) < V^*(\frac{K}{K+1}, d^*)$, or equivalently,

$$u(A_1(d^*), d^*) - C(A_1(d^*)) < \frac{K}{K+1} u\left(\frac{K+1}{K} A_{\frac{K}{K+1}}(d^*), d^*\right) - C\left(A_{\frac{K}{K+1}}(d^*)\right). \tag{A.8}$$

If d^* increases, the left-hand side decreases more than the right-hand side. To see this, first, note that

$$\begin{aligned}
\frac{\partial}{\partial d} V^*(x,d) &= x u_2\left(\frac{A_x(d)}{x}, d\right), \\
\frac{\partial^2}{\partial x \partial d} V^*(x,d) &= u_2\left(\frac{A_x(d)}{x}, d\right) + x \cdot \frac{\partial}{\partial x}\left(\frac{A_x(d)}{x}\right) \cdot u_{12}\left(\frac{A_x(d)}{x}, d\right) < 0.
\end{aligned}$$

The inequality uses $\frac{\partial}{\partial x}\left(\frac{A_x(d)}{x}\right) < 0$, which follows from Lemma C.2. Now, we can write (A.8) as $\int_{\frac{K}{K+1}}^1 \frac{\partial}{\partial x} V^*(x, d^*) dx < 0$. Because $\frac{\partial}{\partial x} V^*(x,d)$ is decreasing in d, we have $\int_{\frac{K}{K+1}}^1 \frac{\partial}{\partial x} V^*(x, d^{**}) dx < 0$, or equivalently, $V^*(1, d^{**}) < V^*\left(\frac{K}{K+1}, d^{**}\right)$. As a result, we have

$$u(A_1(d^{**}), d^{**}) - C(A_1(d^{**})) < \frac{K}{K+1} u\left(\frac{K+1}{K} A_{\frac{K}{K+1}}(d^{**}), d^{**}\right) - C\left(A_{\frac{K}{K+1}}(d^{**})\right),$$

which contradicts that the consumer joins all platforms in equilibrium when there are $K+1$ platforms. Therefore we have $d^* \geq d^{**}$, i.e., the equilibrium addictiveness is weakly decreasing in $K \geq 2$.

We show the last part. We write the equilibrium addictiveness as d_x^* to emphasize that it depends on $x = \frac{K}{K+1}$, or equivalently, on K. we write (A.7) as

$$\frac{u(A_1(d_x^*), d_x^*) - C(A_1(d_x^*)) - \left[xu\left(\frac{1}{x}A_x(d_x^*), d_x^*\right) - C\left(A_x(d_x^*)\right)\right]}{1-x} = 0, \quad \forall x \in \left\{\frac{K}{K+1}\right\}_{K\in\mathbb{N}}. \tag{A.9}$$

Define

$$f_x(d) := \frac{u(A_1(d), d) - C(A_1(d)) - \left[xu\left(\frac{1}{x}A_x(d), d\right) - C\left(A_x(d)\right)\right]}{1-x}. \tag{A.10}$$

We can write equation (A.9) as $f_x(d_x^*) = 0$.

We make three observations. First, the equilibrium addictiveness is decreasing in $K \geq 2$. Thus, across all $x \in \left\{\frac{K}{K+1}\right\}_{K\in\mathbb{N}}$, the set of possible levels of equilibrium addictiveness is a subset of a compact set $[0, d_{\frac{1}{2}}^*]$, where $d_{\frac{1}{2}}^*$ is the one for duopoly. Second, for each x, $f_x(d)$ is continuous in d. As $x \to 1$, it converges pointwise to

$$\begin{aligned}
&\lim_{x\to 1} \frac{u(A_1(d), d) - C(A_1(d)) - \left[xu\left(\frac{1}{x}A_x(d), d\right) - C\left(A_x(d)\right)\right]}{1-x} \\
=&u\left(A_1(d), d\right) - A_1(d)u_1\left(A_1(d), d\right) \\
:=&f_1(d).
\end{aligned}$$

Here we use the envelope theorem. Third, the function $xu\left(\frac{1}{x}A_x(d), d\right) - C\left(A_x(d)\right)$ is concave in x (Lemma C.2). As a result, for any d, $f_x(d)$ is decreasing in x.

We have shown that $(f_x(\cdot))_x$ is a monotonically decreasing sequence of continuous functions defined on a compact set, and the sequence converges pointwise to $f_1(\cdot)$ as $x \to 1$. By Dini's Theorem, $f_x(\cdot)$ uniformly converges to $f_1(\cdot)$ (e.g., Theorem 7.13 in Rudin (1976)). Recall that we have $f_x(d_x^*) = 0$ for any x. Because $f_x(\cdot)$ uniformly converges to $f_1(\cdot)$ and $d_x^* \to d^\infty$, we have

$\lim_{x\to\infty} f_x(d_x^*) = f_1(d^\infty) = 0.$[14] As a result, we have $u\left(A_1(d^\infty), d^\infty\right) - A_1(d^\infty)u_1\left(A_1(d^\infty), d^\infty\right) = 0.$

Finally, we show $d^\infty > 0$. If $d^\infty = 0$, we have $u(A,0) - Au_1(A,0) = 0$ for $A = A(0)$, which implies $\dfrac{u(A,0)-u(0,0)}{A-0} = u_1(A,0)$. This contradicts $u(x,0)$ being strictly concave and $A > 0$. $\square$

Proof of Proposition 6. Consider the consumer's problem, $\max_{A\in[0,\overline{A}]} u(A,d) - C(A)$. First, given the quadratic attention cost and the exponential utility, the solution $A^U(d)$ of the consumer's unconstrained problem (i.e., $\max_{A\geq 0} u(A,d) - C(A)$) satisfies the first-order condition:

$$\rho e^{-\rho(A^U(d)-d)} = cA^U(d) \iff A^U(d) = g\left(\frac{c}{\rho}e^{-\rho d}\right), \quad \text{where} \quad g^{-1}(x) = \frac{e^{-\rho x}}{x}.$$

The solution of the consumer's constrained problem is

$$A^*(d,\overline{A}) := \min\left\{\overline{A},\ g\left(\frac{c}{\rho}e^{-\rho d}\right)\right\}.$$

By Proposition 5 the equilibrium addictiveness d^∞ satisfies

$$\begin{aligned}
& 1 - e^{-\rho(A^*(d^\infty,\overline{A})-d^\infty)} = A^*(d^\infty,\overline{A})\rho e^{-\rho(A^*(d^\infty,\overline{A})-d^\infty)} \\
\iff\quad & 1 = (1+\rho A^*(d^\infty,\overline{A}))\cdot e^{-\rho(A^*(d^\infty,\overline{A})-d^\infty)} \\
\iff\quad & d^\infty = A^*(d^\infty,\overline{A}) - \frac{1}{\rho}\ln\left(1+\rho A^*(d^\infty,\overline{A})\right).
\end{aligned}$$

Suppose $A^*(d^\infty,\overline{A}) = \overline{A}$ in equilibrium. Then,

$$d^\infty = \overline{A} - \frac{1}{\rho}\ln\left(1+\rho\overline{A}\right).$$

[14]If $f_n(\cdot)$ uniformly converges to a continuous function $f(\cdot)$ and x_n converges to x, then $f_n(x_n)$ converges to $f(x)$. Indeed, we have $|f_n(x_n) - f(x)| \leq |f_n(x_n) - f(x_n)| + |f(x_n) - f(x)|$. Then $|f_n(x_n) - f(x_n)| \to 0$ because of the uniform convergence, and $|f(x_n) - f(x)| \to 0$ because f is continuous.

The attention constraint indeed binds at d^∞ if and only if

$$
\begin{aligned}
& g\left(\frac{c}{\rho}e^{-\rho d^\infty}\right) \geq \overline{A} \iff \frac{c}{\rho}e^{-\rho d^\infty} \leq g^{-1}(\overline{A}) \iff \frac{c}{\rho}e^{-\rho d^\infty} \leq \frac{e^{-\rho\overline{A}}}{\overline{A}} \\
\iff & \frac{c}{\rho}e^{-\rho\left[\overline{A}-\frac{1}{\rho}\ln\left(1+\rho\overline{A}\right)\right]} \leq \frac{e^{-\rho\overline{A}}}{\overline{A}} \iff \frac{c}{\rho}\left(1+\rho\overline{A}\right) \leq \frac{1}{\overline{A}} \iff 0 \geq c\rho\overline{A}^2 + c\overline{A} - \rho \\
\iff & \overline{A} \leq \overline{A}^* := \frac{-c+\sqrt{c^2+4c\rho^2}}{2c\rho}. \qquad \text{(A.11)}
\end{aligned}
$$

The second inequality holds because g^{-1} is decreasing. As a result, if $\overline{A} \leq \frac{-c+\sqrt{c^2+4c\rho^2}}{2c\rho}$ the equilibrium in the limit economy is as follows:

$$
\begin{aligned}
A^\infty &= \overline{A} \\
d^\infty &= \overline{A} - \frac{1}{\rho}\ln\left(1+\rho\overline{A}\right), \quad \text{and} \\
CS^\infty &= 1 - \frac{1}{1+\rho\overline{A}} - \frac{c}{2}\overline{A}^2. \qquad \text{(A.12)}
\end{aligned}
$$

We now consider the other case: $\overline{A} > \frac{-c+\sqrt{c^2+4c\rho}}{2c\rho}$. Suppose the consumer's choice is interior. The addictiveness d^∞ satisfies

$$
\begin{aligned}
& 1 = (1+\rho A^U(d^\infty))e^{-\rho(A^U(d^\infty)-d^\infty)} \\
\iff \quad & 1 = (1+\rho A^U(d^\infty))\frac{cA^U(d^\infty)}{\rho} \\
\iff \quad & c\rho(A^U(d^\infty))^2 + cA^U(d^\infty) - \rho = 0.
\end{aligned}
$$

Thus,

$$
A^U(d^\infty) = \frac{-c+\sqrt{c^2+4c\rho^2}}{2c\rho} \qquad \text{(A.13)}
$$

The consumer surplus is

$$
\begin{aligned}
CS^{\infty} &= A^{\infty}(d,\overline{A})\rho e^{-\rho(A^{\infty}(d,\overline{A})-d)} - \frac{c}{2}A^{\infty}(d,\overline{A})^2 \\
&= cA^U(d)^2 - \frac{c}{2}A^U(d)^2 \\
&= \frac{c}{2}A^U(d)^2 \\
&= \frac{c}{2}\left[\frac{-c+\sqrt{c^2+4c\rho^2}}{2c\rho}\right]^2 > 0
\end{aligned}
$$

Finally, we show CS^{∞} is non-monotone in $\overline{A} \geq 0$. When the attention constraint binds, consumer surplus is (A.12). We have

$$
\frac{\partial CS^{\infty}}{\partial \overline{A}} = \frac{\rho}{(1+\rho\overline{A})^2} - c\overline{A}. \tag{A.14}
$$

Because the right-hand side is decreasing in $\overline{A}$, CS^{∞} is concave in $\overline{A}$ for $\overline{A} \in [0, \frac{-c+\sqrt{c^2+4c\rho^2}}{2c\rho}]$. To show CS^{∞} is non-monotone in $\overline{A}$ on $\left[0, \frac{-c+\sqrt{c^2+4c\rho^2}}{2c\rho}\right]$, it suffices to show $\frac{\partial CS^{\infty}}{\partial \overline{A}} < 0$ at the cutoff $\overline{A}^* = \frac{-c+\sqrt{c^2+4c\rho^2}}{2c\rho}$. Recall that the cutoff $\overline{A}^*$ satisfies $\frac{1}{1+\rho\overline{A}^*} = \frac{c\overline{A}^*}{\rho}$, so we have

$$
\frac{\partial CS^{\infty}}{\partial \overline{A}}\Big|_{\overline{A}=\overline{A}^*} = \rho \cdot \left(\frac{c\overline{A}^*}{\rho}\right)^2 - c\overline{A}^* = c\overline{A}^* \cdot \left(\frac{c\overline{A}^*}{\rho} - 1\right) < 0.
$$

Next, consider monopoly. Suppose the attention constraint binds and the monopolist sets positive addictiveness d^*. Note that d^* satisfies the consumer's first-order condition at $\overline{A}$:

$$
\rho e^{-\rho(\overline{A}-d^*)} = c\overline{A} \iff d^* = \overline{A} + \frac{1}{\rho}\ln\left(\frac{c\overline{A}}{\rho}\right).
$$

Consumer surplus is

$$
CS^M = 1 - \frac{c\overline{A}}{\rho} - \frac{c\overline{A}^2}{2}.
$$

As $\overline{A}$ increases, CS^M decreases, and it hits 0 at $\overline{A}$ that satisfies

$$\begin{aligned} &1 - \frac{c\overline{A}}{\rho} - \frac{c\overline{A}^2}{2} = 0 \\ \iff\ & c\rho\overline{A}^2 + 2c\overline{A} - 2\rho = 0 \\ \implies & \overline{A}^M = \frac{-c + \sqrt{c^2 + 2c\rho^2}}{c\rho}. \end{aligned}$$

Consumer surplus is positive (and thus the attention constraint binds) under monopoly if and only if $\overline{A} < \overline{A}^M$.

In the limit economy, if the attention constraint binds, we have (A.14). If the attention constraint does not bind, $\frac{\partial CS^\infty}{\partial \overline{A}} = 0$. Under monopoly, if the attention constraint binds,

$$\frac{\partial CS^M}{\partial \overline{A}} = -\frac{c}{\rho} - c\overline{A}. \tag{A.15}$$

As a result, if $\overline{A}$ is such that the attention constraint binds under monopoly, we have $\frac{\partial CS^\infty}{\partial \overline{A}} > \frac{\partial CS^M}{\partial \overline{A}}$.

We now establish the welfare comparison. If $\overline{A} \leq A(0)$, the monopolist sets $d = 0$, so the consumer is strictly better off under monopoly. If $A(0) < \overline{A} < \overline{A}^M$, consumer surplus under monopoly decreases faster than consumer surplus in the limit economy, because $\frac{\partial CS^\infty}{\partial \overline{A}} > \frac{\partial CS^M}{\partial \overline{A}}$. At $\overline{A}^M$, the consumer gets a payoff of zero under monopoly and a positive payoff in the limit economy. Thus there is a unique cutoff $A^{**} \in (A(0), \overline{A}^M)$ such that the consumer is better off under monopoly if and only if $\overline{A} \leq A^{**}$.

Finally, we show that $A^{**} = \overline{A}^*$, where $\overline{A}^*$ defined in (A.11) is the cutoff at which for any $\overline{A} \leq \overline{A}^*$, the consumer's attention constraint binds in the limit economy. First, we show $\overline{A}^* < \overline{A}^M$. We have

$$\begin{aligned} \overline{A}^* < \overline{A}^M \iff\ & \frac{-c + \sqrt{c^2 + 4c\rho}}{2c\rho} < \frac{-c + \sqrt{c^2 + 2c\rho^2}}{c\rho} \\ \iff\ & c + \sqrt{c^2 + 4c\rho^2} < 2\sqrt{c^2 + 2c\rho^2} \\ \iff\ & 1 + \sqrt{1 + 4x} < 2\sqrt{1 + 2x}, \quad \text{where} \quad x = \frac{\rho^2}{c} \\ \iff\ & 0 < 4x^2. \end{aligned}$$

Thus the cutoff at which the participation constraint binds under monopoly is strictly greater than the cutoff $\overline{A}^*$ at which the attention constraint binds in the limit economy. It implies that when $\overline{A} = \overline{A}^*$, the monopolist and platforms in the limit economy set the same addictiveness, i.e., all of them set the lowest addictiveness at which the attention constraint binds. Thus, the consumer obtains the same equilibrium payoff in the two cases when $\overline{A} = \overline{A}^*$. Thus we conclude $A^{**} = \overline{A}^*$. $\square$

Proof of Proposition 7. First we characterize the equilibrium in the limit economy. Let $A^U(d)$ denote the consumer's unconstrained choice of total attention when she faces platforms with addictiveness d. Because $A^U(d)$ solves the first-order condition $v'(a-d) = c$, we have

$$A^U(d) = d + g(c), \quad \text{where} \quad g = (v')^{-1}.$$

The consumer's objective is concave, so the consumer's constrained choice of total attention is $A^*(d,\ \overline{A}) := \min\left\{\overline{A},\ d + g(c)\right\}$. The equilibrium addictiveness d^∞ in the limit economy solves $u(A^*(d^\infty, \overline{A}),\ d^\infty) = A^*(d^\infty, \overline{A}) \cdot u_1(A^*(d^\infty, \overline{A}),\ d^\infty)$, which is equivalent to

$$\begin{aligned} & v(A^*(d^\infty, \overline{A}) - d) = A^*(d, \overline{A}) \cdot v'(A^*(d^\infty, \overline{A}) - d) \\ \iff & d^\infty = A^*(d^\infty, \overline{A}) - h\left(A^*(d^\infty, \overline{A})\right), \quad \text{where} \quad h = \left(\frac{v}{v'}\right)^{-1}. \end{aligned}$$

Suppose $A^*(d^\infty, \overline{A}) = \overline{A}$, which implies $d^\infty = \overline{A} - h(\overline{A})$. Then we have $A^U(d^\infty) = \overline{A} - h(\overline{A}) + g(c)$. The attention constraint indeed binds if and only if

$$\overline{A} - h(\overline{A}) + g(c) \geq \overline{A} \iff g(c) \geq h(\overline{A}) \iff \frac{v(g(c))}{c} \geq \overline{A}.$$

As a result, if $\frac{v(g(c))}{c} \geq \overline{A}$, the equilibrium total attention, addictiveness, and consumer surplus in the limit economy are as follows:

$$\begin{aligned} A^\infty &= \overline{A}, \\ d^\infty &= \overline{A} - h(\overline{A}), \quad \text{and} \\ CS^\infty &= v(\overline{A} - d^\infty) - c\overline{A} = v(h(\overline{A})) - c\overline{A}. \end{aligned}$$

We now consider the other case: $\frac{v(g(c))}{c} < \overline{A}$. Suppose the consumer's choice is interior given the equilibrium addictiveness d^∞. The addictiveness d^∞ satisfies

$$\begin{aligned} & d^\infty = d^\infty + g(c) - h(d^\infty + g(c)) \\ \iff & d^\infty = h^{-1}(g(c)) - g(c) = \frac{v(g(c))}{c} - g(c). \end{aligned}$$

Because $A^U(d^\infty) = \frac{v(g(c))}{c} < \overline{A}$, the consumer's choice is interior. As a result, if $\frac{v(g(c))}{c} < \overline{A}$, the equilibrium is as follows:

$$\begin{aligned} A^\infty &= \frac{v(g(c))}{c}, \\ d^\infty &= \frac{v(g(c))}{c} - g(c), \quad \text{and} \\ CS^\infty &= v(A^\infty - d^\infty) - c \cdot A^\infty = 0. \end{aligned}$$

We now turn to monopoly. Take any $\overline{A}$ and suppose the monopoly chooses d such that the consumer exhausts her attention:

$$\overline{A} = d + g(c) \iff d = \overline{A} - g(c).$$

The monopolist sets the addictiveness of $\max\{0,\ \overline{A} - g(c)\}$ to make the consumer choose $\overline{A}$. The consumer's payoff is then

$$v\left(\overline{A} - \max\{0,\ \overline{A} - g(c)\}\right) - c\overline{A}. \tag{A.16}$$

Because the consumer's payoff is positive for $\overline{A} < g(c)$, the payoff (A.16) becomes non-positive if and only if $\overline{A} \geq \frac{v(g(c))}{c}$, which is the same threshold at which the consumer's equilibrium payoff becomes zero in the limit economy.

We now compare consumer surpluses under monopoly and the limit economy. If $\overline{A} > \frac{v(g(c))}{c}$, the consumer's equilibrium payoff is zero in either case. Suppose $\overline{A} < \frac{u(g(c))}{c}$. If $\overline{A} \leq A(0)$, the monopolist sets zero addictiveness. Otherwise, the monopoly is strictly better if and only if

$$\begin{aligned} & v(g(c)) - c\overline{A} > v(h(\overline{A})) - c\overline{A} \\ \iff & \overline{A} < \frac{v(g(c))}{c}. \end{aligned}$$

Thus, for any $\overline{A} < \frac{v(g(c))}{c}$, the monopoly is strictly better. Because monopoly dominates the limit economy, it dominates any other market $\mathcal{E}_K$ with $K \geq 2$. $\square$

E Proof of Proposition 8: Costly Investment in Addictive Technology

The appendix consists of several parts. First, we characterize the consumer's optimal attention allocation for any profile of addictiveness. Second, we characterize the equilibrium and the joint-profit maximizing outcome. Third, we provide a sufficient condition under which the consumer is better off under the joint-profit maximizing outcome. Finally, we prove Proposition 8. Without loss of generality, we assume $r = 1$, so platform k's payoff is $a_k - \kappa\gamma(d_k)$.

Lemma E.1. *Suppose that the consumer joins all of the K platforms with $(d_1, \ldots, d_K)$. Define $D = \sum_{k \in K} d_k$. The consumer's optimal attention to platform k is*

$$a_k = \frac{1}{K}\left[\min\{A(D),\ \overline{A}\} - D\right] + d_k. \tag{A.17}$$

Here $A(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ is concave and uniquely solves $A(D) - D = Kg\left(C'\left(A(D)\right)\right)$, where $g(\cdot)$ is the inverse of $v'(\cdot)$.

Proof. Having joined K platforms, the consumer chooses $(a_k)_{k \in K} \in \mathbb{R}_+^K$ to maximize

$$\sum_{k \in K} v(a_k - d_k) - C\left(\sum_{k \in K} a_k\right) \quad \text{s.t.} \quad \sum_{k \in K} a_k \leq \overline{A}.$$

The objective is concave, so the first-order condition characterizes the *unconstrained* optimal choice:

$$v'(a_k - d_k) - C'\left(\sum_{k \in K} a_k\right) = 0 \iff a_k - d_k = g\left(C'\left(\sum_{k \in K} a_k\right)\right). \tag{A.18}$$

Let $A = \sum_{k \in K} a_k$ and $D = \sum_{k \in K} d_k$. Summing up equation (A.18) across all $k \in K$, we obtain

$$A - D = Kg\left(C'\left(A\right)\right).$$

The left-hand side is strictly increasing in A and the right-hand side is strictly decreasing in A. Also, the left-hand is smaller if $A \leq D$ and is bigger for a large A. Thus there is a unique

$A(D) > 0$ that solves the equation. Note that

$$A'(D) = \frac{1}{1 - Kg'\left(C'\left(A(D)\right)\right) \cdot C''\left(A(D)\right)} < 1. \tag{A.19}$$

The strict inequality in (A.19) holds because $g = (v')^{-1}$ is decreasing and $g'\left(C'\left(A(D)\right)\right) C''\left(A(D)\right)$ is negative. Under the assumption that $g'\left(C'\left(x\right)\right) \cdot C''\left(x\right)$ is decreasing in x, $A'(D)$ is decreasing in D.

We now show that the consumer's optimal total attention given the constraint is $\min\left\{A(D), \overline{A}\right\}$. Suppose $A(D) \leq \overline{A}$. We can directly verify that $a_k^* = \frac{1}{K}(A(D) - D) + d_k$ for each k satisfies the attention constraint and solves the consumer's first-order condition (A.18). Thus $(a_1^*, \ldots, a_K^*)$ is the optimal choice and satisfies $\sum_{k \in K} a_k^* = A(D)$. Suppose $A(D) > \overline{A}$. Suppose to the contrary that the consumer's optimal total attention A is strictly less than $\overline{A}$. Because $A < A(D)$, we have $A - D < Kg\left(C'\left(A\right)\right)$, which implies $v'(a_k - d_k) > C'(A)$ for some k. As a result, the consumer can slightly increase some a_k to increase her payoff, which is a contradiction. Thus $A = \overline{A}$. To sum up, the consumer's optimal total attention is $\min\left\{A(D), \overline{A}\right\}$.

The consumer's constrained choice solves $v'(a_k - d_k) - C'\left(\sum_{k \in K} a_k\right) - \lambda = 0$, where λ is the Lagrangian multiplier for the attention constraint. Because $a_k - d_k$ is constant across k at the optimum, we have $a_k - d_k = \frac{1}{K}(\sum_{k \in K} a_k - D)$. Thus $a_k = \frac{1}{K}\left[\min\left\{A(D), \overline{A}\right\} - D\right] + d_k$. $\square$

Lemma E.2. *There is a unique pure-strategy subgame perfect equilibrium in which all platforms choose the same addictiveness* $\min\{d^1(\kappa), d^2\}$. *Here,* $d^1(\kappa)$ *is a unique* d^1 *that satisfies*

$$d^1 \in \arg\max_{x \geq 0} \frac{1}{K}\left[\min\{A(x + (K-1)d^1),\ \overline{A}\} - x - (K-1)d^1\right] + x - \kappa\gamma(x), \tag{A.20}$$

and d^2 *is the equilibrium addictiveness at no cost benchmark, i.e.,* d^2 *makes the consumer indifferent between joining* K *and* $K-1$ *platforms. Also* $d^1(\kappa)$ *is decreasing,* $\lim_{\kappa \to 0} d^1(\kappa) = \infty$, *and* $\lim_{\kappa \to \infty} d^1(\kappa) = 0$. *Thus there is a* κ^E *such that the equilibrium addictiveness is* d^2 *if and only if* $\kappa \leq \kappa^E$.

Proof. We show that (A.20) has a unique solution d^1. For any $\kappa > 0$, define

$$\Pi(x, d) := \frac{1}{K}\left[\min\{A(x + (K-1)d),\ \overline{A}\} - x - (K-1)d\right] + x - \kappa\gamma(x), \tag{A.21}$$

which is a platform's profit when it chooses x, other platforms choose d, and the consumer joins all platforms and allocates attention optimally. Because $A(\cdot)$ is concave and $\gamma(\cdot)$ is strictly convex, $\Pi(x, d)$ is strictly concave in x and has decreasing differences in (x, d). Thus for each d, a platform has a unique best response $x(d)$ that is decreasing in d. If $\Pi_x(0, 0) \leq 0$, then we have $x(0) = 0$. If $\Pi_x(0, 0) > 0$, then we have $x(0) > 0$ and $x(d) < d$ for a sufficient large d because $\lim_{d\to\infty} \gamma'(d) = \infty$. As a result, there is a unique d^1 that satisfies $x(d^1) = d^1$. Addictiveness d^1 solves (A.20).

In a unique equilibrium, platforms choose $\min\{d^1(\kappa), d^2\}$. First suppose $d^1(\kappa) \leq d^2$. It is an equilibrium that all platforms choose $d^1(\kappa)$: If platform k unilaterally deviates and chooses $d_k > d^2$, the consumer does not join k. If it chooses $d_k \in (d^1(\kappa),\ d^2)$, the consumer joins k; however, the platform earns a lower payoff because $\Pi_x(d_k,\ d^1(\kappa)) < 0$. If it chooses $d_k < d^1(\kappa)$, then because $d_k < d^2$, we have $\Pi_x(d_k,\ d^1(\kappa)) > 0$, so platform k does not benefit from such a deviation. Second suppose $d^1(\kappa) > d^2$. By a similar argument, we can show that no platform has a profitable deviation from d^2. The uniqueness follows the same argument. For example, if all platforms choose $d \in (d^1(\kappa), d^2)$, one platform can profitably deviate by slightly decreasing d. Finally, the assumptions on $\gamma(\cdot)$ and equation (A.20) imply that $d^1(\kappa)$ is decreasing, $\lim_{\kappa\to 0} d^1(\kappa) = \infty$, and $\lim_{\kappa\to\infty} d^1(\kappa) = 0$. These properties ensure the existence of κ^E. □

Lemma E.3. *There is a unique joint-profit maximizing outcome, in which platforms choose the same addictiveness* $\min\{d_J^1, d_J^2\}$. *Here,* d_J^1 *solves*

$$\max_{d\geq 0} \left[\min\{A(Kd),\ \overline{A}\} - K\kappa\gamma(d)\right], \tag{A.22}$$

and d_J^2 *is the unique level of addictiveness at which the consumer obtains a payoff of zero by joining all platforms.*

Proof. Because $\gamma(\cdot)$ is strictly convex, d_J^1 is unique. The rest of the proof follows the same logic as Proposition 1. Even though the platforms' gross revenue depends only on $\sum_{k\in K} d_k$, the joint-profit maximizing outcome implies that all platforms choose the same addictiveness, because they incur a strictly convex cost of increasing d. □

Lemma E.4. *If the equilibrium addictiveness is* $d^1(\kappa)$ *(see Lemma E.2), the consumer is better off under the joint-profit maximizing outcome than the equilibrium.*

Proof. Suppose the primitives are such that the equilibrium addictiveness is $d^1(\kappa)$. We consider two cases. First, suppose that the consumer's total attention is $\overline{A}$ in equilibrium. At the joint-profit maximizing outcome, platforms do not choose strictly higher addictiveness than the minimum level of addictiveness at which the consumer exhausts her attention $\overline{A}$. As a result, platforms choose lower addictiveness under the joint-profit maximizing outcome than the equilibrium.

Second, suppose that the consumer's total attention is strictly less than $\overline{A}$ in equilibrium, i.e., $A(Kd^1(\kappa)) < \overline{A}$. Then we can rewrite (A.20) as the first-order condition:

$$\frac{1}{K}(A'(Kd^1(\kappa)) - 1) + 1 - \kappa\gamma'(d^1(\kappa)) = 0. \tag{A.23}$$

Similarly, the joint-profit maximizing outcome (that ignores the consumer's participation incentive) is $d^J > 0$ that solves

$$A'(Kd^J) - 1 + 1 - \kappa\gamma'(d^J) = 0. \tag{A.24}$$

Equation (A.19) implies $A'(Kd) - 1 < 0$, so for any d we have

$$\frac{1}{K}(A'(Kd) - 1) + 1 - \kappa\gamma'(d) \geq A'(Kd) - 1 + 1 - \kappa\gamma'(d),$$

which implies $d^1(\kappa) \geq d^J$. The addictiveness under the joint-profit maximizing outcome is at most d^J, so the consumer is better off under the joint-profit maximizing outcome. □

We now prove Proposition 8.

Proof of Proposition 8. Let $U^J(\kappa)$ and $U^E(\kappa)$ denote the consumer's payoffs at the joint-profit maximizing outcome and the equilibrium. We consider two cases. First, suppose that $U^J(0) \geq U^E(0)$. Note that $U^J(\kappa)$ is increasing in κ. Take any $\kappa' > 0$. Suppose that at κ', the equilibrium addictiveness is $d^1(\kappa')$. Lemma E.4 implies that the consumer is better off under the joint-profit maximizing outcome. Suppose that at κ', the equilibrium addictiveness is d^2, i.e., it is determined by the consumer's participation constraint. Then for any $\kappa < \kappa'$, the equilibrium addictiveness continues to be d^2. Because $U^J(0) \geq U^E(0)$ at $\kappa = 0$, we have $U^J(\kappa') \geq U^E(\kappa')$ at $\kappa = \kappa'$. To sum up, if $U^J(0) \geq U^E(0)$, then $U^J(\kappa) \geq U^E(\kappa)$ for all $\kappa \geq 0$, so we have $\kappa^* = 0$.

Second, suppose $U^J(0) < U^E(0)$. The equilibrium addictiveness at $\kappa = 0$ is d^2 by Lemma E.4.

Recall that κ^E is the cutoff such that $d^2 = d^1(\kappa^E)$. For any $\kappa \geq \kappa^E$, the equilibrium addictiveness is $d^1(\kappa)$, so Lemma E.4 implies $U^J(\kappa) \geq U^E(\kappa)$. For any $\kappa < \kappa^E$, $U^J(\kappa)$ is increasing in κ and $U^E(\kappa)$ is constant. Thus there is a $\kappa^* \leq \kappa^E$ such that $U^J(\kappa) \leq U^E(\kappa)$ if $\kappa \leq \kappa^*$ and $U^J(\kappa) \geq U^E(\kappa)$ if $\kappa \geq \kappa^*$. □

F Proof of Proposition 9: The Impact of Digital Curfew

Proof. Point 1 follows from Proposition 1, and Point 2 follows from Proposition 2. To show Point 3, for K, X, and d, let $A_K(X, d)$ denote the consumer's total attention when she faces attention cap $\overline{A} = X$ and joins K platforms with addictiveness d. Suppose the consumer initially faces the attention cap of $\overline{A} = A$. Let d^* denote the equilibrium addictiveness. Proposition 2 implies that the equilibrium addictiveness satisfies

$$K \cdot u\left(\frac{A_K(A, d^*)}{K}, d^*\right) - C\left(A_K(A, d^*)\right) = (K-1) \cdot u\left(\frac{A_{K-1}(A, d^*)}{K-1}, d^*\right) - C\left(A_{K-1}(A, d^*)\right).$$

Suppose that we decrease the attention cap to $X \in [A_{K-1}(A, d^*), A_K(A, d^*))$. We have $A_{K-1}(A, d^*) < A_K(A, d^*)$ because $A_K(A, d^*)$ is an interior solution by our assumption. Given $\overline{A} = X$ the attention constraint binds if she joins K platforms but not if she joins $K-1$ platforms. Thus we have

$$K \cdot u\left(\frac{X}{K}, d^*\right) - C\left(X\right) < (K-1) \cdot u\left(\frac{A_{K-1}(X, d^*)}{K-1}, d^*\right) - C\left(A_{K-1}(X, d^*)\right). \tag{A.25}$$

The incremental gain of joining a platform is decreasing in addictiveness (see the last part of Lemma C.2). Thus for any $d \geq d^*$, we have

$$K \cdot u\left(\frac{X}{K}, d\right) - C\left(X\right) < (K-1) \cdot u\left(\frac{A_{K-1}(X, d)}{K-1}, d\right) - C\left(A_{K-1}(X, d)\right). \tag{A.26}$$

Inequality (A.26) implies that if platforms increased addictiveness after a cap of X, the consumer joins at most $K-1$ platforms, which contradicts the equilibrium condition. Thus after a curfew the platforms set a strictly lower addictiveness.

Suppose a digital curfew decreases the attention cap from A to $A_D := A_{K-1}(A, d^*)$. If platforms continued to set d^*, this digital curfew would not change the consumer's payoff because she could join $K-1$ platforms and allocate attention $A_{K-1}(\overline{A}, d^*)$ optimally. After the cap, the platforms strictly decrease their addictiveness, so the consumer's payoff increases. □

G Proofs for Section 7: Price Competition and Attention Competition

Proof of Lemma 1. Throughout the proof, we fix K and write $\hat{u}(a,d) := \frac{1}{K}u(Ka,d)$, and let $A_K(d)$ denote the consumer's optimal total attention when she joins K platforms with addictiveness d. Define

$$p^* := K\hat{u}\left(\frac{A_K(0)}{K}, 0\right) - C(A_K(0)) - \left[(K-1)\hat{u}\left(\frac{A_{K-1}(0)}{K-1}, 0\right) - C(A_{K-1}(0))\right].$$

We show that the game of price competition has an equilibrium in which each platform k sets $d_k = 0$ and $p_k = p^*$. Suppose each platform k sets $(d_k, p_k) = (0, p^*)$. The consumer chooses the number K' of platforms to join to maximize

$$V(K') := \max_{A \in [0,\overline{A}]} K'\hat{u}\left(\frac{A}{K'}, 0\right) - C(A) - K'p^*.$$

Lemma C.2 implies that $V(K')$ is concave on $[0, K]$. Because p^* makes the consumer indifferent between joining K and $K-1$ platforms, it is optimal for her to join all platforms. By the same argument as the case of attention competition, we can show that a platform does not strictly benefit from deviating to $p \neq p^*$.

The above equilibrium is unique. To show this, take any equilibrium and suppose each platform k chooses (d_k^*, p_k^*). First, we show that the consumer joins all platforms in equilibrium. Fix $\hat{k} \in K$, and suppose platform $\hat{k}$ sets $(d_{\hat{k}}, p_{\hat{k}}) = (0, 0)$, which may or may not be a deviation. Take any $K' \subset K$ such that $\hat{k} \notin K'$. First, if $d_j^* > 0$ for some $j \in K'$, then the consumer strictly prefers joining $(K' \setminus \{j\}) \cup \left\{\hat{k}\right\}$ to joining K'. Second, if $d_j^* = 0$ for all $j \in K'$ or $K' = \emptyset$, then the consumer strictly prefers $K' \cup \left\{\hat{k}\right\}$ to K'. Thus, for any set K' of platforms such that $\hat{k} \notin K'$, we can find some set S of platforms such that $\hat{k} \in S$ and the consumer strictly prefers S to K'. As a result, for a sufficiently small $p_{\hat{k}} > 0$ and $d_{\hat{k}} = 0$, the consumer still joins platform $\hat{k}$. Because any platform has a strategy to earn a positive profit, the consumer joins all platforms in any equilibrium.

Second, we show all platforms set zero addictiveness in any equilibrium. Suppose to the contrary that $d_k^* > 0$ for some k. Suppose platform k deviates and chooses $(d_k, p_k) = (0, p_k^*)$. Before the deviation, the consumer weakly prefers joining all platforms to joining any set K' of platforms that does not contain k. Thus, after the deviation to $(0, p_k^*)$, the consumer strictly prefers to joining platform k. As a result, platform k can slightly increase its price while retaining the consumer. This is a contradiction.

We have shown that in any equilibrium, the consumer joins all platforms, which set zero addictiveness. The price of each platform makes the consumer indifferent between joining and not joining the platform; otherwise, the platform can deviate by slightly increasing its price. Therefore, $(d_k^*, p_k^*) = (0, p^*)$ is a unique equilibrium. □

Proof of Proposition 10. We use the notation in Proposition 5. In equilibrium, the consumer is indifferent between joining K and $K-1$ platforms that choose zero addictiveness. Thus, we have

$$u(A_1(0),0) - C(A_1(0)) - Kp^* = \frac{K-1}{K} u\left(\frac{K}{K-1} A_{\frac{K-1}{K}}(0), 0\right) - C\left(A_{\frac{K-1}{K}}(0)\right) - (K-1)p^*. \tag{A.27}$$

The equation implies

$$Kp^* = K(1-x) \cdot \frac{u(A_1(0),0) - C(A_1(0)) - \left[xu\left(\frac{A_x(0)}{x}, 0\right) - C\left(A_x(0)\right)\right]}{1-x} \tag{A.28}$$

for any $x \in \left\{\frac{K-1}{K}\right\}_{K\in\mathbb{N}}$. Now, define $f(x) := xu\left(\frac{A_x(0)}{x}\right) - C\left(A_x(0)\right)$. Since $K(1-x) = 1$ for any $x \in \left\{\frac{K-1}{K}\right\}_{K\in\mathbb{N}}$, the right-hand side of (A.28), as $K \to \infty$, converges to $f'(1)$. Corollary 4 of Milgrom and Segal (2002) implies $f'(1) = u(A_1(0),0) - A_1(0)u'\left(A_1(0),0\right)$. Thus, by taking $K \to \infty$, we obtain $\lim_{K\to\infty} Kp^* = u(A_1(0),0) - A_1(0)u'\left(A_1(0),0\right)$. Thus, the consumer's payoff converges to

$$u(A_1(0),0) - C(A_1(0)) - [u(A_1(0),0) - A_1(0)u'(A_1(0),0)] = A_1(0)u'(A_1(0),0) - C(A_1(0)).$$

Let $d^* > 0$ denote the equilibrium addictiveness in the limit economy under attention competition (Proposition 5). In the limit $K \to \infty$, the consumer's payoffs under attention competition and price competition are $A_1(d^*)u_1(A_1(d^*),d^*) - C(A_1(d^*))$ and $A_1(0)u'(A_1(0),0) - C(A_1(0))$, respectively.

To show $A_1(d^*)u_1(A_1(d^*), d^*) - C(A_1(d^*)) > A_1(0)u'(A_1(0), 0) - C(A_1(0))$, we consider three cases. Note that we always have $A_1(0) \leq A_1(d^*)$. First, suppose $A_1(d^*) < \overline{A}$. By the first-order conditions, these payoffs are respectively equal to $A_1(d^*)C'(A_1(d^*)) - C(A_1(d^*))$ and $A_1(0)C'(A_1(0)) - C(A_1(0))$. The function $xC'(x) - C(x)$ is increasing because its first derivative is $xC''(x) \geq 0$. As a result,

$$A_1(d^*)C'(A_1(d^*)) - C(A_1(d^*)) \geq A_1(0)C'(A_1(0)) - C(A_1(0)).$$

If $C''(\cdot) > 0$, the inequality is strict.

Second, suppose $A_1(0) = A_1(d^*) = \overline{A}$. Then, the consumer's payoffs under attention competition and price competition are $\overline{A}u_1(\overline{A}, d^*) - C(\overline{A})$ and $\overline{A}u_1(\overline{A}, 0) - C(\overline{A})$, respectively. The former is strictly greater than the latter as $u_{12} > 0$.

Third, suppose $A_1(0) < A_1(d^*) = \overline{A}$. Then, the consumer's payoffs under attention competition is $\overline{A}u_1(\overline{A}, d^*) - C(\overline{A}) \geq \overline{A}C'(\overline{A}) - C(\overline{A}) > A_1(0)C'(A_1(0)) - C(A_1(0))$. Thus, the consumer is strictly better off under attention competition in the limit, and the same welfare comparison holds for a sufficiently large K. $\square$

H Appendix for Section 8: Naive Consumer

We formally describe the timing of the game and the optimization problems of the naive consumer. First, each platform $k \in K$ simultaneously chooses its addictiveness, $d_k \geq 0$. Second, given the perceived addictiveness $(sd_k)_{k \in K}$, the consumer chooses the set $J \subset K$ of platforms to maximize the perceived indirect utility $V(J)$, where

$$V(J) := \max_{(a_k)_{k \in J} \in \mathbb{R}_+^J} \sum_{k \in J} u(a_k, sd_k) - C\left(\sum_{k \in J} a_k\right) \tag{A.29}$$
$$\text{s.t.} \quad \sum_{k \in J} a_k \leq \overline{A} \quad \text{and} \quad a_k \geq 0, \forall k \in J.$$

If $J = \emptyset$, all players obtain a payoff of zero, and the game ends. After joining platforms $J \neq \emptyset$, the consumer observes the true addictiveness of each platform, then allocates her attention. In equilibrium, the consumer solves

$$\max_{(a_k)_{k\in J}\in\mathbb{R}^J_+} \sum_{k\in J} u(a_k, d_k) - C\left(\sum_{k\in J} a_k\right) \tag{A.30}$$
$$\text{s.t.} \quad \sum_{k\in J} a_k \le \overline{A} \quad \text{and} \quad a_k \ge 0, \forall k \in J.$$

The above two maximization problems coincide if $s = 1$. Our solution concept continues to be pure-strategy subgame perfect equilibrium. Even if $s < 1$, we can use SPE by treating the consumer who solves (A.29) and the consumer who solves (A.30) as different players.

Proof of Proposition 11. Define $d^*_s := \frac{d^*}{s}$, where d^* is the equilibrium addictiveness of the original model (i.e., $s = 1$). Recall that d^* satisfies the sophisticated consumer's indifference condition, which we can rewrite as

$$K \cdot u\left(\frac{A_K(sd^*_s)}{K}, sd^*_s\right) - C\left(A_K(sd^*_s)\right) = (K-1)\cdot u\left(\frac{A_{K-1}(sd^*_s)}{K-1}, sd^*_s\right) - C\left(A_{K-1}(sd^*_s)\right). \tag{A.31}$$

The equation means that the consumer with s is indifferent between joining K and $K-1$ platforms that choose addictiveness d^*_s. Note that the participation decision uses the perceived addictiveness, sd^*_s. By the same argument as the proof of Proposition 2, we can use this indifference condition to show the following: (i) given addictiveness d^*_s, the consumer joins all platforms; (ii) if platform k deviates and increases its addictiveness, the consumer joins all platforms but k; and (iii) if platform k decreases its addictiveness, she joins all platforms and allocates less attention to k. Points (i), (ii), and (iii) imply that in equilibrium, all platforms choose $\frac{d^*}{s}$. The welfare result follows from $\frac{d^*}{s}$ being decreasing in s. □

Under price competition, the platforms first set addictiveness and prices. Then the consumer decides which platforms to join by maximizing $V^P(K')$, where

$$V^P(J) := \max_{(a_k)_{k\in J}\in\mathbb{R}^J_+} \sum_{k\in J} [u(a_k, sd_k) - p_k] - C\left(\sum_{k\in J} a_k\right) \tag{A.32}$$
$$\text{s.t.} \quad \sum_{k\in J} a_k \le \overline{A} \quad \text{and} \quad a_k \ge 0, \forall k \in J.$$

Note that the consumer now pays p_k to join platform k. After joining platforms, the consumer allocates her attention by solving (A.30). As before, the payoff of platform k is p_k and 0 if the consumer does and does not join platform k, respectively.

Claim H.1. *For any $K \geq 2$, there is an $s^* \in (0, 1]$ such that the following holds: The consumer is better off under price competition than attention competition if and only if $s \leq s^*$.*

Proof. For any $s \in (0, 1]$ the same argument as Lemma 1 implies that all platforms set zero addictiveness in a unique equilibrium under price competition. Thus the consumer's payoff is independent of s under price competition, and it is increasing in s under attention competition. Also, for a small s the consumer's payoff under attention competition is negative because of Point (b) of Assumption 1. As a result, price competition gives the consumer a greater payoff if s is below some $s^* \in (0, 1]$. $\square$