User Investment and Firm Value: Case of Internet Firms

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Abstract

This paper studies the mechanism of users’ online activities and their implications for the firm value, and quantifies them for Internet firms that have been recently growing fast. Users collectively spend billions of person-hours creating user generated contents which is then becomes an asset that produces a stream of value for fellow users, and of course, the host site itself, labeled as user capital. Using the firm-level characteristics and data on users’ online activities, we find that an Internet firm’s value is significantly positively associated with the users’ time spent on the firm’s own websites. Building and studying a dynamic general equilibrium model of users’ time allocation among competing websites, we find that improvements in either the website quality or the efficiency of user-capital investment cause an immediate increase in the Internet firm’s value by shifting up users’ time spent on the website in the future. The simulated results show that user capital account for about 60% of the Internet firm’s value.

Keywords: User-generated Contents, Firm Value, Intangibles, Internet Firms.
I. Introduction

Recent years have witnessed a rapid growth in households’ time spent at online. At the same time, many internet business firms have exhibited starkly high growth. For instance, the annual revenue of Facebook grew from $0.78 billion in 2009 to $3.71 billion in 2011. Interestingly, the recent high growth of Facebook’s revenue comes with the increased user activities on Facebook, about 200% annual growth rate, and without nearly as high growth in the expenditures on tangible assets and employees. By the very nature and as exemplified by the Facebook case, growth opportunities of many Internet firms are mainly determined by the user activities, in particular, the users’ contribution of time and contents. Users of websites collectively spend billions of person-hours creating “user generated capital” which is then becomes an asset that produces a stream of value for fellow users, and of course, the host site itself. This paper examines such a view both empirically and theoretically to address the questions as follows. What is the mechanism of the formation of user generated capital? How much does user-generated capital contribute to the value of an Internet firm? How can an Internet firm effectively elicit the users’ investments of time to increase the firm’s own value?

Compared to the traditional physical-asset-intensive firms, a new type of firms such as an Internet firm is affected more by intangible assets for their core engines of growth. (See, e.g., Zingales, 2000, Jorgenson and Stiroh, 2000, Brynjolfsson et al., 2002, Jorgenson et al., 2002, and Corrado et al., 2009.) Unfortunately, intangibles are difficult to measure: both their quantity and returns, which is a main stumbling block for the study of the formation process and valuation mechanism of intangibles. Accordingly, there has been recently new attempts to quantify

1 Note that almost all of the content generation on sites like Facebook, Youtube and Wikipedia comes from the efforts of their users rather than from their employees.
2 See, e.g., the ‘intellectual capital’ literature for categorizing and measuring various components of intangibles.
particular classes of intangibles, e.g., customer, R&D, organization, brand capital, etc. (See, Gupta et al. (2004), McGrattan and Prescott (2010), Lin (2012), Eisfeldt and Papanikolaou (2013), and Belo et al. (2013).) We extend this approach by defining and measuring a new category of intangibles: user-generated capital, which is believed by many experts to be crucial for an Internet firm’s business and its valuation. (See, e.g., Shirky (2011).) More specifically, we document stylized facts on the relationship between the user activities and Internet firm values and study and quantify it in a dynamic general equilibrium model.

We begin by analyzing the firm value and users’ online activities for a number of Internet and media companies. Using the COMPUSTAT annual files over the period 2000-2012 and data on users’ time spent on websites, we find that for the case of Internet exclusive media firms, the firm value is considerably higher, both in terms of the level of market value and Tobin’s q ratio, for the firms with the larger amount of hours spent by users on the firm’s own website than with the smaller amount of users’ hours. In addition, one-percentage increase in users’ hours spent on an Internet firm’s website is significantly associated with an increase in the firm’s market value by about 12%.

We, in turn, develop a model of the user’s time allocation among a number of competing websites. In the model, the household makes the decision on consumption of Internet vs. non-Internet good where the Internet good is a constant-elasticity-of-substitution (CES) aggregate of a continuum of varieties, i.e., differentiated websites. Importantly, the household determines how much time to spend on three types of activities: consumption, e.g., online surfing, and investment, e.g., uploading pictures, of Internet-good varieties and working. An Internet-good variety firm owns a unique website and monopolistically competes with other Internet firms, (see, Dixit and Stiglitz (1977)), to attract users’ attention/time by improving its intangible capital, i.e., the
quality of its website. An Internet firm’s production function is Cobb-Douglas of two inputs: tangible capital and the household’s consumption-purpose time spent on the firm’s own website, capturing the fact that an Internet firm’s revenue source is largely advertising. Two key features are introduced: (i) the household’s utility of per-hour consumption of an Internet-good variety is increasing in the level of user-generated capital, i.e., the amount of contents, and (ii) the next-period user-generated capital stock is increasing jointly in the amount of time spent by the household for investment purpose and the firm’s intangible capital, i.e., the website quality/efficiency.

In the model, the household’s consumption-purpose hours spent on an Internet-good variety are increasing in the level of user-generated capital specific to the variety, and the household’s investment-purpose hours are increasing in the level of the Internet firm’s intangible capital. Taking such decision rules of the household as given, an Internet firm optimally accumulates tangible and intangible capital. The model predicts that improvements in either the website quality or the efficiency of user-capital investment result in an immediate increase in the Internet firm’s value by shifting up users’ time spent on the website in the future.

The mechanism is then quantified by calibrating the model to the data on financials of Internet firms and households’ time uses by websites in the U.S. economy over the period 2000-2002. The simulated results show that an increase in the efficiency of user-capital investment specific to an individual website results in a shift of the household’s time allocation: both the current investment-purpose hours and the future consumption-purpose hours spent on the website increase greatly. The mechanism is as follows; the increased efficiency of user-capital investment shifts down the Internet firm’s cost of intangible investment and hence the firm increases its intangible investment greatly, resulting in the next-period levels of both user capital
and the household’s consumption-purpose hours in the future. As a result, the firm’s current (equity) value, assessed by the rational investors, jumps up immediately to a great extent, consistent with the recent observations for a successful Internet company as documented earlier.

Then the importance of user capital in accounting for the Internet firm’s observed market value is examined. More specifically, the contribution of user-generated capital to the firm value is measured by carrying out a counterfactual experiment in which an Internet-good firm’s user-generated capital is completely removed together with the firm’s efficiency of user-capital investment is reduced to a very low level, 50% of the steady-state value, so that such a firm’s user-generated capital stays in a low level for a long time. We find that in this case, the firm’s current value drops down substantially, by about 60 percent relative to before such an experiment. To the best of our knowledge, this paper is the first to quantify the contribution of user-generated capital to the Internet firm’s value by using a dynamic general equilibrium model.

The rest of paper proceeds as follows. In Section II, we discuss related approaches and previous studies in the literature on measuring the value of intangible capital and valuation of information technology (IT) firms. Section III develops the model and presents analytical results. Key implications of the model are empirically examined in Section IV. Section V calibrates the model and discusses the simulated results. Section VI concludes.

II. Literature Review

Our work is related to several strands of literature. Previous studies have investigated the link between intangible assets and firm value. It is shown that intangible capital is a dominant source of firm performance and rapid growth in the economy (see, e.g., Brynjolfsson et al. (2002); Corrado et al. (2009); Hall (2001); Jorgenson and Stiroh (2000); and Jorgenson et al. (2002);
McGrattan and Prescott (2000)). For instance, Brynjolfsson, Hitt, and Yang (2002) provide evidence that firms’ investments in IT-related intangible capital are associated with significant market value. They analyze how new intangible organizational assets complement IT capital, such as new organizational processes and structures, worker knowledge and incentive systems and therefore, increase market value of a firm.

Many studies are based on the approach that the market value represents the true intrinsic value of the firm at any time – an efficient market argument. Discrepancy between the book value and market value of a firm’s total assets is often referred to as compelling evidence supporting the existence of an intangible asset. More recently, Saunders and Brynjolfsson (2011) examined the value of intangible assets in the firm and found that intangible assets are correlated with significantly higher market values, about 30-55% premium. Tambe, Hitt and Brynjolfsson (2011) analyze both price fluctuations and quantity in measuring the value of IT-related intangible capital. They found that by 2006, IT-related assets had grown to about one-third of the level of physical assets.

The aforementioned equity market approach, quantifying the amount of intangible assets and relates their influence to the market value, however, leaves a number of relevant questions unanswered. In principle, the difference between the book and market values of an asset indicates the difference between the purchased price of capital when installed and the resale price of capital prevailing currently in the market. Not only the quantity of capital but also the price, market value of a capital, could explain this difference (see, e.g., Tambe et al. (2011)). In an extreme case, even in the absence of any intangible capital, the starkly higher market value of any capital, due to any reason that increase in the demand of and fixed supply of capital, can results in the same. The major drawback of an efficient market argument is that it assumes
investors having perfect information on the value of intangibles. It is, however, extremely difficult to measure the value of intangibles. A stream of studies identifies and defines intangibles of a firm that might influence market value and productivity.

Intangible capital, however, is a broad concept and evolves over time. Intangibles are often differentiated from tangible plant and equipment in the sense that their influence on the current output is not directly observable. In fact, intangibles such as firms’ knowledge or expertise, often lack physical embodiment therefore difficult to measure in units. Many scholars found different aspects of intangible assets. Corrado, Hulten, and Sichel (2006) denote the components of intangible assets as R&D expenditures, both scientific and non-scientific, software, worker-training, brand equity, and organizational development. Hall (2000) proposed a concept of e-capital, to characterize as a body of technical and organizational know-how. E-capital includes business methods and systems such as speed and accuracy in handling enormous flows of transactions based on computers instead of counting computers itself. He considered computers and software as rather ordinary plant and equipment. Cummins (2003) argues that intangible capital is not a distinct input to production like physical capital or labor but rather it is the glue that creates value from other inputs, which can be defined in terms of adjustment costs.

More new companies depends less on physical assets as their core engines of growth. Human capital, broadly defined, of employees’ and customers’ is emerging as the most crucial sources of the firm’s growth. Accordingly, more and more researchers in studies recognize companies’ non-financial stakeholders as an important and growing component of a firm’s intangible asset. For example, Gupta et al. (2004) show that the long-term value of customers can be a strong and stable determinant of firm value. Belo et al. (2012) examined the role of brand capital for firm valuation and risk in the cross-section of publicly traded firms. They found that the value of the
brand capital stock constructed from advertising expenditures data represents about 30% of firms’ market value. Gourio and Rudanko (2011) study the implications of customer capital for firm value and investment dynamics. As the more dominant a firm’s intangible asset itself becomes in determining the value of a firm in general in the current new era of economy, the more important become identifying a firm’s key intangible asset.

III. Model

This section lays out an economic environment in which user-generated capital is a key factor in generating an Internet firm’s cash flow. The model is a dynamic general equilibrium model of the household’s time allocation among competing websites. In the model, the household allocates time on three types of activities: consumption of user-generated capital, i.e., the available Internet contents, investment of user-generated capital, i.e., creation of new Internet contents such as uploading pictures, and working. An internet firm owns its unique website differentiated from others and generates the cash flow by using two input factors: the firm’s own tangible capital and the amount of time spent by the household for consuming user-generated capital on the firm’s own website. Importantly, an Internet firm can indirectly improve user-generated capital and hence its cash flow by increasing the level of its own intangible capital.

Environment

There are two goods: composite and Internet goods. The composite good is homogeneous and can be consumed and invested for formation of tangible and intangible capital, while the Internet goods are heterogeneous, differentiated among each other, i.e., a number of competing varieties of websites, and consumed by the household. The composite good is produced by the stand-in producer as well as by a number of Internet good producers, while a variety of an
Internet good is produced only by an individual Internet good producer. Thus, an Internet good producer serves dual roles: providing a differentiated Internet good via its own unique website as well as producing the composite good. The key is that in return to the supply of the Internet good, an Internet good producer earns its revenue indirectly by using the household/user’s time spent on the firm’s own website for producing the composite good rather than directly charging service fees for such an Internet good consumption.

Household

A representative household has preferences as:

\[
\text{Max}_{n_{i},c_{i}(i),h_{i}(i),h'_{i}(i))} \left\{ E \left[ \sum_{i=0}^{\infty} \beta^{i} \left( \frac{(n_{i})^{1-\sigma}}{1-\sigma} + \phi \frac{(v_{i})^{1-\sigma}}{1-\sigma} \right) \right] \right\} \quad \text{where} \quad v_{i} = \left[ \int_{0}^{1} \omega(i) \cdot [c_{i}(i)]^{(\eta-1)/\eta} di \right]^{-\eta/(\eta-1)}
\]

and \( \beta \in (0,1) \) refers to discount factor, \( \sigma > 0 \) the risk-aversion, \( n_{i} \) consumption of the composite good, \( v_{i} \) consumption, in a CES aggregate form, of a continuum of measure one of the Internet good varieties \( c_{i}(i) \) for \( i \in [0,1] \), \( \omega(i) \) the variety-level weight for preferences, and \( \phi \) the preferences factor for the utility of the Internet good consumption relative to that of the composite good. And \( \eta \geq 0 \) refers to the (constant) elasticity of substitution between the internet good varieties \( c_{i}(i) \) and \( c_{i'}(i') \) for \( i \neq i' \) where consumption of the internet good variety \( i \) is written as:

\[
c_{i}(i) = k^{U}_{i}(i)h_{i}(i)
\]

\( k^{U}_{i}(i) \) refers to user-generated capital for the internet good variety \( i \), of which law of motion will be discussed soon. Note that the user’s time spent on consumption of the internet
good variety $h^C_i(i)$ can be thought of as the utilization rate of the given user-capital stock, similar to the case of the variable utilization of physical capital in production as discussed in the literature (see, e.g., Greenwood et al. (1988)).

User capital $k^U_i(i)$ evolves as:

$$k^U_{i+1}(i) = [1 - \delta^U]k^U_i(i) + z^U_i(i)G\left(k^P_i(i), A_i h^I_i(i)\right), \quad \delta^U \in [0,1]$$  \hspace{1cm} (2)

where $\delta^U$ refers to the depreciation rate of user capital, $z^U_i(i)$ the efficiency of investment to user capital $i$, and $A_i$ the labor-augmenting technology growing at the constant rate of $\gamma > 0$: $A_i = (1 + \gamma)^t$, $\forall t \geq 0$. And $G\left(k^P_i(i), A_i h^I_i(i)\right)$ refers to the user-capital formation function with two inputs: the household/user’s time spent for accumulating the user capital $h^I_i(i)$ and the firm $i$’s intangible capital stock $k^P_i(i)$, and is Cobb-Douglas written as:

$$G\left(k^P_i(i), A_i h^I_i(i)\right) = \left(k^P_i(i)^{\alpha^P}\right)^{1-\alpha^P} \left(A_i h^I_i(i)\right)^{\alpha^P}, \quad \alpha^P \in (0,1).$$  \hspace{1cm} (3)

The household allocates one unit of time for three purposes every period: consumption of the internet good variety $h^C_i(i)$, investment to user-generated capital specific to the internet good variety $h^I_i(i)$, and earning labor income. And the household owns all of the firms and hence receives a total of dividend payments of those firms $\int_0^1 d_i(i) di$ every period. Thus, given the wage rate $w_i$ and the price of the composite good normalized to one, the household’s per-period budget constraint is written as:
\[ n_t = w_t \left[ 1 - \int_0^1 h_t^C(i) + h_t^I(i)di \right] + \int_0^1 d_t(i)di \]  

which says that the household uses labor income and a total of dividend payments \( \int_0^1 d_t(i)di \) in consuming the composite good.

**Technology**

The representative firm produces the composite good using the technology linear in labor\(^3\): \( o_t = A_t h_t \) where \( o_t \) refers to output of the composite good produced by hiring labor \( h_t \) given the labor-augmenting technology \( A_t \). By contrast, each of Internet firms produces the composite good by using the hours spent by the household on the firm’s own website rather than by hiring labor. That is, differently from the representative firm, Internet firms have their own know-hows in attracting and utilizing the household’s attention/time in running their businesses, which is discussed in detail below.

Each of Internet firm owns its unique website and provides the household with online contents, which has been labeled as ‘Internet-good variety’ earlier, differentiated from those for other Internet firms. Rather than charging services fees for such online contents, an Internet firm indirectly generates its revenue by utilizing the household’s attention/time spent on consumption of the firm’s internet good variety \( h_t^C(i) \) and combining it with tangible capital \( k_t^T(i) \) in producing the composite good. More specifically, letting \( y_t(i) \) denote the composite good

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\(^3\) For simplicity, the representative firm’s technology is assumed to be linear in labor. Alternatively, we could model the case in which the representative firm’s production function is of constant returns to scale for two inputs, tangible capital and labor, which would make no difference to the model results.
output produced by the firm $i$, i.e., the after-tax cash flow before investment expenditures in the data, we write the firm $i$’s production function of the composite good as:

$$y_i (i) = z_i (i) F (k^T_i (i), A, h^C_i (i)) - \tau$$

(5)

where $z_i (i)$ refers to firm $i$’s productivity, $\tau$ the fixed cost of operation, and $F (\cdot, \cdot)$ the firm $i$’s variable production function, which satisfies the standard properties.\(^4\) Firm $i$’s dividend payment, which could be either negative or positive, is then written as: $d_i (i) = y_i (i) - x^T_i (i) - x^P_i (i)$ where $x^T_i (i)$ refers to firm $i$’s investment to tangible capital, and $x^P_i (i)$ the expenditures on accumulating the firm’s own intangible capital $k^P_i (i)$ where intangible capital $k^P_i (i)$ in this paper specifically refers to intangible assets that are related with and restricted to the firm’s website quality/efficiency such as usability, friendliness of access, visibility of contents to other users and so on so that the household/user’s one hour input can create larger effective units of online contents. Thus, $x^P_i (i)$ also refers to the expenditures for the specific purpose of improving the quality/efficiency of its website such as developing and maintaining the user-friendly online platform.

Firm $i$’s tangible capital stock evolves as:

$$k^T_{i,t+1} (i) = [1 - \delta^T] k^T_i (i) + x^T_i (i), \quad \delta^T (i) \in [0, 1]$$

(6)

where $\delta^T$ refers to the depreciation rate of tangible capital. Similarly, firm $i$’s intangible capital stock $k^P_i (i)$ evolves as:

\(^4\) $F (k^T, h^C)$ is homogeneous of degree one w.r.t. $(k^T, h^C) \in \mathbb{R}^2_+$: $F (\lambda k^T, \lambda h^C) = \lambda \cdot F (k^T, h^C), \forall \lambda > 0, \forall (k^T, h^C) \in \mathbb{R}^2_+$, concave, twice continuously differentiable, and satisfies that the marginal product of $k^T$ and $h^C$ is positive and decreasing, respectively.
\[ k_{i,1}^P(i) = [1 - \delta^P]k_i^P(i) + x_i^P(i), \quad \delta^P(i) \in [0,1] \]  

where \( \delta^P \) refers to the depreciation rate of intangible capital that is, as discussed earlier, used in the formation of user-capital \( k_{i,1}^U(i) \) as: \( k_{i,1}^U(i) = [1 - \delta^U]k_i^U(i) + z_i^U(i)G(k_i^P(i), A_i h_i^I(i)) \).

Note that firm \( i \)'s business is exposed to the two exogenous shocks: the cash-flow productivity \( z_i(i) \) and the efficiency of user-capital investment \( z_i^U(i) \) where \( z_i^U(i) \) directly affects the next-period user capital \( k_{i,1}^U(i) \) and hence would indirectly affect firm \( i \)'s future cash flow via the household’s consumption-purpose hours \( h_{i,1}^C(i) \) due to the complementary effect of \( k_{i,1}^U(i) \) on the household’s utility from \( h_{i,1}^C(i) \). Below we discuss the stochastic process of \((z_i(i), z_i^U(i))\). First, the cash-flow productivity \( z_i(i) \) and the efficiency of user-capital investment \( z_i^U(i) \) are independent of each other, and each of \( z_i(i) \) and \( z_i^U(i) \) follows its own first-order Markov process and independent across \( i \). Second, both the distribution of \( z_i(i) \) and that of \( z_i^U(i) \) are constant over time.\(^5\)

**Firm Value**

For simplicity, we assume that Internet firms are financed entirely by equity but not by debt. Given the constant and common per-period equity discount rate \( r > 0 \), the Internet firm \( i \)'s beginning-of-period equity value \( V_i(i) \) is the sum of the current and discounted expected future dividend payments and written as:

\(^5\)This property would be formally derived, due to the law of large numbers, from the assumption that the initial cash-flow productivity \( z_o(i) \) is independently drawn from a common distribution and that the initial user-capital investment efficiency \( z_o^U(i) \) is also independently drawn from another common distribution.
\[ V_i(i) = d_i(i) + \frac{1}{1 + r} E \left[ \sum_{t=r+1}^{\infty} d_i(i) \right] \]  

which is referred to as the (current) firm \( i \)'s value throughout this paper given that the firm is entirely equity financed. The discount rate \( r > 0 \) is formally written as: \( r = r_f + \pi \) where \( r_f \) refers to the constant risk-free rate and \( \pi \) the constant risk premium.

**Market Structure**

Below we discuss the market structure for the two goods, composite and Internet goods. For the case of the composite good, the market is of perfect competition (homogeneity of the composite good). By contrast, the market for the Internet good is monopolistically competitive mainly due to the differentiated websites. More specifically, given that each of Internet good firms owns its own unique website, again differentiated from the other websites, we assume that the Internet good sector is monopolistically competitive in terms of getting and managing the attention/time of the household/user in the sense of Dixit and Stiglitz (1977); taking the decision rules of the household/user and the other firms, an individual Internet firm controls its own resources, e.g., tangible and intangible capital, to maximize its own value.

**Resource Constraint**

The resource constraint for the composite good is written as:

\[
n_i + \int_0^1 x_i^c(i) + x_i^p(i) \, di = A_i \left[ 1 - \int_0^1 \left( h_i^c(i) + h_i^p(i) \right) \, di \right] + \int_0^1 \left[ z_i^y(i) F(k_i^c(i), A_i h_i^c(i)) - \tau \right] \, di \tag{9}
\]

which says that the composite good is produced by the representative producer and a continuum of measure one of Internet producer and used for either consumption or investment.
**Discussion: User Capital vs. Intangible Capital**

User capital, which refers to the Internet contents available online, indirectly increases the value of an Internet firm. For consuming the Internet goods/services available online, the household/user must pay his/her attention/time to the website in spite of no monetary payment to the website-owning firm, which results in the website-owning firm’s cash flow being generated, i.e., the advertising revenue. The higher the level of user capital available on an Internet firm’s website, the larger the household/user’s hours spent on the firm’s own website and hence the greater the firm’s value. Thus, user capital can be broadly thought of as an intangible asset in the sense that it increases the firm’s value in spite of no increase in the firm’s own tangible asset. Importantly, user capital differs from the other intangibles that have been discussed in the literature, i.e., human, organization, customer, and brand capital, in the sense that investment to user capital is mainly expensed by and hence under control of users rather than the firm itself. An Internet firm can at most indirectly manage and control the formation of user capital by increasing the quality/efficiency of the firm’s own website, but still subject to the users’ decisions; an online user wants to consume contents voluntarily created by the user itself or the other users but not by people hired by the firm.

Nonetheless, an Internet firm can still control, even though in an indirect way, user capital by investing to improve the quality/efficiency of its website. The better the quality of an Internet firm’s website, the higher the level of user capital and hence the larger the firm’s value holding all else constant. Thus, an Internet firm’s website quality can be also thought of as intangibles and labeled as *platform capital* in the sense that it contributes to the improvement of the firm value mainly by, and to the extent of, promoting users’ online activities to take place on the firm’s own website but not elsewhere.
Equilibrium

We rewrite variables in order for the model economy to be stationary; let the hatted variable denote the ratio of a variable to the labor-augmenting technology $A_t = (1 + \gamma)^T$:

$$
\hat{n}_t = \frac{n_t}{A_t}, \hat{v}_t = \frac{v_t}{A_t}, \hat{k}_r^U (i) = \frac{k_r^U (i)}{A_t}, \hat{r}_r (i) = \frac{r_r (i)}{A_t}, \hat{d}_r (i) = \frac{d_r (i)}{A_t}, \hat{V}_t (i) = \frac{V_t (i)}{A_t}, \hat{k}_r^T (i) = \frac{k_r^T (i)}{A_t},
$$

$$
\hat{k}_r^R (i) \equiv \frac{k_r^R (i)}{A_t}, \hat{x}_r^R (i) \equiv \frac{x_r^R (i)}{A_t}, \hat{x}_r^P (i) \equiv \frac{x_r^P (i)}{A_t}, \hat{w}_r = \frac{w_r}{A_t}.
$$

(10)

Thus, all of the hatted variables are stationary. Let $\hat{z} (i)$ denote the vector of firm $i$’s productivity and user-capital investment efficiency: $\hat{z} (i) \equiv (z (i), z^U (i))$. Then we write the internet-good variety firm $i$’s value function $\hat{V} (\cdot)$, which is also the firm’s current beginning-of-period equity value relative to $A_t$, in recursive form as:

$$
\hat{V} (\hat{k}_r^R (i), \hat{k}_r^T (i), \hat{k}_r^P (i), \hat{z} (i)) =
$$

$$
\max_{\hat{k}_r^R (i), \hat{k}_r^T (i), \hat{k}_r^P (i)} \left\{ z (i) \cdot F (\hat{k}_r^T (i), h^C (i, \cdot)) - \hat{x}_r^R (i) - \hat{x}_r^P (i) + \frac{1 + \gamma}{1 + \rho} E \left[ \hat{V} (\hat{k}_r^R (i), \hat{k}_r^T (i), \hat{k}_r^P (i), \hat{z} (i) | \hat{z} (i) ) \right] \right\}
$$

(11)

subject to the laws of motion for tangible, intangible and user capital written as:

$$
[1 + \gamma] \hat{k}_r^T (i) = [1 - \delta^T] \hat{k}_r^T (i) + \hat{x}_r^T (i),
$$

(12)

$$
[1 + \gamma] \hat{k}_r^P (i) = [1 - \delta^P] \hat{k}_r^P (i) + \hat{x}_r^P (i),
$$

(13)

$$
[1 + \gamma] \hat{k}_r^U (i) = [1 - \delta^U] \hat{k}_r^U (i) + z^U (i) \hat{G} \left( \hat{k}_r^r (i), h^l (i, \cdot) \right),
$$

(14)
and the household’s equilibrium policy functions of $h_t^c(i,·)$ and $h_t^f(i,·)$ taken as given.

We study a stationary recursive equilibrium in the sense that (i) both the household and firms take into their consideration all of the available information in making their decisions, (ii) the household and numeraire good producer take prices and law of motion for aggregate state variables as given, (iii) an internet-good variety firm monopolistically competes with other firms by taking as given the policy functions of the household and other firms as in Dixit and Stiglitz (1977), (iv) aggregate variables grow along the balanced growth path so that the hatted aggregate variables should be constant over time, and (v) the decision rules of firms and the household are in recursive form and in terms of the hatted variables.\(^6\)

The first three conditions of the equilibrium (i)-(iii) define the substantive property of the equilibrium, in particular, the monopolistic competition among internet-good variety firms, while the last two conditions (iv)-(v) essentially say that the equilibrium is stationary and in recursive form. More specifically, by the stationary equilibrium outcome, we mean that the distribution of key variables across the internet good variety is constant even though there could be changes in the level of the individual internet good variety, which is similar to the (stationary) industry equilibrium studied in the literature.\(^7\) (See, e.g., Hopenhayn (1992) and Miao (2005).)

**A Firm’s Decision Rules**

\(^6\) The formal definition of the equilibrium is available in the online appendix.
\(^7\) More precisely, an individual (internet good) variety firm is of measure zero and hence changes in key variables of an individual variety firm are also of measure zero so that the distribution of key variables across the internet good variety firms is still constant almost everywhere in the space of internet good variety firms.
Given the state of \((\hat{k}^p(i), \hat{k}^u(i), \hat{k}^T(i), \bar{z}(i))\), firm \(i\)'s (equilibrium) decision rules are characterized as follows. First, the firm \(i\)'s decision for the next-period tangible capital \(k^T\cdot(i)\) is, as the usual firm-side Euler equation, written as:

\[
1 = \frac{1}{1 + r} E\left[1 - \sigma^T + z'(i)F_1(\hat{k}^T\cdot(i), h^C\cdot(i))\right]
\]

(15)

where \(F_i(\hat{k}^T, \cdot) = \partial F(\hat{k}^T, \cdot) / \partial \hat{k}^T\) for \(\hat{k}^T > 0\) refers to the first-order partial derivative of \(F(\hat{k}^T, h^C)\) with respect to \(\hat{k}^T > 0\), i.e., the marginal product of tangible capital given the cash-flow productivity. The optimality condition for \(\hat{k}^T\cdot(i)\) above says that the firm chooses its \(\hat{k}^T\cdot(i)\) such that the next-period (expected) marginal product of capital net depreciation discounted by firm’s (equity) discount rate \(r\), i.e., the marginal benefit of \(\hat{k}^T\cdot(i)\), should equal the reduction in the current dividend payment, i.e., the marginal cost of \(\hat{k}^T\cdot(i)\). Second, the firm \(i\)'s decision for the next-period intangible capital stock \(\hat{k}^p\cdot(i)\) is written as:

\[
1 = \frac{1 - \delta^U}{1 + r} + \frac{1 + \gamma}{[1 + r]^2} E\left[z''(i)F_2(\hat{k}^T\cdot(i), h^C\cdot(i)) \frac{\partial h^C\cdot(i, \cdot)}{\partial \hat{k}^p\cdot(i, \cdot)} - \left(\frac{\partial \hat{k}^u\cdot(i, \cdot)}{\partial \hat{k}^p\cdot(i, \cdot)} \frac{\partial \hat{k}^p\cdot(i, \cdot)}{\partial h^I\cdot(i, \cdot)} \right)\right]
\]

(16)

where double-primed variables refer to the variables in two periods later from now. The optimality condition for \(\hat{k}^p\cdot(i)\) above says that the marginal cost of \(\hat{k}^p\cdot(i)\), which is a reduction in the current dividend payment to the investors, must be the same with its marginal benefit, which consists of the two components: first, the direct benefit of intangibles net depreciation in the next period, and second, the induced increase in the cash flow in two periods later from now via the increased user capital stock \(\hat{k}^u\cdot(i, \cdot)\).
The Household’s Decision Rules

We turn to discuss the household’s decision rules. First, the household’s consumption-purpose time allocation across internet good varieties is characterized by:

\[
\frac{h^C(i)}{h^C(\tilde{i})} = \left( \frac{\omega(i)}{\omega(\tilde{i})} \right)^\eta \left( \frac{\hat{k}^U(i)}{\hat{k}^U(\tilde{i})} \right)^{\eta^{-1}}
\]  

(17)

which says that consumption-purpose time spent on the internet good variety \(i\) is proportional to that for the reference variety \(\tilde{i}\) by the factor of the user capital ratio \(\left( k^U(i)/k^U(\tilde{i}) \right)^{\eta^{-1}}\) given the preferences ratio \(\left( \omega(i)/\omega(\tilde{i}) \right)^\eta\). We impose the restriction that relative to the reference variety \(\tilde{i}\), the consumption-purpose time spent on the other variety \(i\) must be increasing in the relative user-capital stock \(k^U(i)/k^U(\tilde{i})\) but at the decreasing rate, which is essentially governed by the elasticity-of-substitution parameter \(\eta\) as:

**Assumption 1:** \(1 < \eta < 2\).

Note that \(h^C(i)/h^C(\tilde{i})\) is proportional to \(\left( \hat{k}^U(i)/\hat{k}^U(\tilde{i}) \right)^{\eta^{-1}}\). Thus, the condition that \(h^C(i)/h^C(\tilde{i})\) is increasing in \(k^U(i)/k^U(\tilde{i})\) but at the decreasing rate is equivalent to:

\(0 < \eta - 1 < 1\), which is the case of Assumption 1 above. The second property on the decreasing rate of an increase in \(h^C(i)/h^C(\tilde{i})\) w.r.t. \(k^U(i)/k^U(\tilde{i})\) can be interpreted as the decreasing
marginal utility of the variety-level consumption of internet goods. Second, the household’s investment-purpose time allocation across internet good varieties \( h^I(i) \) is written as:

\[
(\hat{n})^{-\sigma} = \beta [1 + \gamma]^{1-\sigma} \cdot \frac{1}{1 + \gamma} \frac{z^U(i) \partial G(k^P(i), h^I(i))}{\partial h^I(i)} \cdot E \left[ (\hat{n})^{-\sigma} \frac{h^C(i)}{k^U(i)} \right]
\]  

(18)

which says that at the optimum, the marginal cost of the investment-purpose time, measured by the marginal utility of the numeraire-good consumption today, must be equal to its marginal benefit, measured by the (discounted) marginal utility of the numeraire-good consumption in the next period multiplied by the marginal product of the investment-purpose time in increasing the next-period user capital stock and utilization rate of such a user capital.

**Analytic Results**

In this section, we discuss the analytic results to understand well the key mechanism. Given that we have already discussed the household’s decision on the allocation of consumption-purpose hours earlier, we begin by showing that the household’s allocation of investment-purpose hours is increasing in the level of the corresponding internet good firm’s intangible capital. From now on, for the purpose of exposition, we assume no uncertainty such that \((z(i), z^U(i))\) is deterministic for every variety \( i \), and discuss the key mechanism for the case of \( 1 < \eta < 2 \) being assumed as discussed earlier, see Proposition 1, 2, and 3 below.

---

8 Rearranging terms, we can also characterize \( h^C(i) \) in terms of the variety \( i \)'s own user capital as:

\[
(\hat{n})^{-\sigma} = \phi(\hat{\nu})^{-\sigma} (\hat{\nu})^{1/\phi} \omega(i) \left[ \frac{z^U(i)}{\hat{\nu}} \right]^{\phi} \left[ \frac{h^C(i)}{\hat{\nu}} \right]^{1-\phi}
\]

where the equilibrium condition for the wage rate \( \hat{\nu} = 1 \) has been already incorporated. This equation will be used later in calibrating the preferences parameter \( \phi \).
**Proposition 1**: For a given variety $i$, the household’s allocation of investment-purpose hours $n^i(i)$ is increasing in the level of the corresponding (current) intangible capital $\hat{k}^p(i)$:

$$\frac{dh^i(i)}{d\hat{k}^p(i)} > 0.$$  

(proof) See the Mathematical Appendix.

As shown in Proposition 1, the higher the level of an Internet good firm’s intangible capital, i.e., the better the website quality, the larger the amount of the household’s investment-purpose hours spent on that particular website. The main reason is the complementarity between the Internet firm’s own intangible capital and the household’s investment-purpose hours in accumulating user capital; an increase in the current level of an Internet good firm’s intangible capital shifts up the marginal product of the household’s investment-purpose hours in creating user capital, which leads to an increase in the household’s current amount of investment-purpose hours via the *inter-temporal substitution* channel: moving the marginal hours from working to creating user capital in response to the currently increased efficiency of user-capital investment, the household can improve his welfare by increasing the future utility high enough to compensate more than the decreased current utility.

Note that the results in Proposition 1 does not necessarily imply that an Internet good firm can be always better off by simply investing more on its intangible capital, even though increasing intangible capital does help the firm to increase the future level of user capital and hence the future cash flow and the current firm value. The reason is that investment is, of course, costly and hence the optimal amount of the firm’s investment on intangible capital would be determined by equating the increase in its discounted future cash flow to the current cost at the
margin. Below we focus on discussing the mechanism on the (marginal) increase in an Internet firm’s discounted future cash flow via increased intangible capital and then the (marginal) cost.

**Lemma 1**: For a given variety $i$, an increase in the next-period level of user capital $\hat{k}^U(i)$ causes increases in the concurrent consumption-purpose hours, tangible capital and cash flow in the next period:

$$\frac{dh^C(i)}{d\hat{k}^U(i)} > 0, \frac{\hat{k}^T(i)}{d\hat{k}^U(i)} > 0, \frac{dF(\hat{k}^T(i), h^C(i))}{d\hat{k}^U(i)} > 0.$$  

(proof) See the Online Appendix.

Results of Lemma 1 essentially say that the household’s consumption-purpose time allocation is increasing in the concurrent level of user capital, which, combined with the indirect effect on the concurrent level of tangible capital, also applies to the concurrent level of the Internet firm’s output.

**Proposition 2**: For a given variety $i$, an increase in the current level of intangible capital $\hat{k}^P(i)$ results in immediate increases in the current level of firm value as well as subsequent increases in the level of user-capital, consumption-purpose hours, tangible capital and cash flow in the next period:

$$\frac{d\hat{V}(i)}{d\hat{k}^P(i)} > 0, \frac{\hat{k}^U(i)}{d\hat{k}^P(i)} > 0, \frac{dh^C(i)}{d\hat{k}^P(i)} > 0, \frac{\hat{k}^T(i)}{d\hat{k}^P(i)} > 0, \frac{dF(\hat{k}^T(i), h^C(i))}{d\hat{k}^P(i)} > 0.$$  

(proof) See the Online Appendix.

Results of Proposition 2 essentially says that an increase in the current level of an Internet firm’s intangible capital boosts up the future level of user capital available on and serviced by the
firm $d\hat{k}^u(i)/d\hat{k}^p(i) > 0$ and hence induces large amount of consumption-purpose hours spent on the firm’s own website by the household, which results in the growth in the future cash flow, i.e., more advertising revenue earned in the future, and thus the immediate increase in the current value of the firm. The expected increase in the household’s future consumption-purpose hours also shifts up the marginal return to the firm’s tangible investment, which induces the firms to increase its current tangible investment and hence results in the firm’s future cash flow being boosted up furthermore.

**Proposition 3:** For a given variety $i$, an increase in the current efficiency of user-capital investment $z^u(i)$ results in immediate increases in the current level of firm value as well as subsequent increases in the level of user-capital, consumption-purpose hours, tangible capital and cash flow in the next period:

\[
\frac{d\hat{V}(i)}{dz^u(i)} > 0, \quad \frac{d\hat{k}^u(i)}{dz^u(i)} > 0, \quad \frac{dh^c(i)}{dz^u(i)} > 0, \quad \frac{d\hat{k}^r(i)}{dz^u(i)} > 0, \quad \frac{dF(\hat{k}^r(i), h^c(i))}{dz^u(i)} > 0.
\]

(proof) See the Online Appendix.

Results of Proposition 3 are essentially the same with those of Proposition 2 except that the efficiency of user-capital investment now replaces the role of intangible capital in Proposition 2. The reason is as follows. The improved efficiency of user-capital investment effectively shifts up the marginal products of both the firm’s own intangible capital and the household’s investment-purpose hours in accumulating user capital; as a result, even in the absence of increases in the firm’s intangible-capital investment and/or the household’s investment-purpose hours, the future
level of user capital increases, which induces, in the future, subsequent increases in the household’s consumption-purpose hours, firm’s cash flow, and the immediate increases in the current firm value and tangible investment. Note that for the case of an improvement of the efficiency of user-capital investment, such an increase in the current firm value can be brought on without necessarily large cost of intangible-capital investment, which suggests that the magnitude of such an effect on the current firm value would be greater in this case than for the case of an increase in the intangible-capital investment in the absence of the efficiency of such an investment. This quantitative property is interesting and will be examined in a later quantitative analysis section.

IV. Empirical Analysis

Data

Our primary data sources for Internet usage is Alexa.com, which provides daily page views per user and time spent on websites by the website URLs. Accounting data on publicly listed US companies is from the COMPUSTAT annual industrial files. We focus on Internet exclusive media companies. To uncover any systematic differences in the role of users’ investments on the firm value, we compare the influence of user-generated capital between the two groups: Internet-exclusive media companies vs. other media companies. With annual data over the period 2000-2012, we have a total of 1,400 observations for the 196 Internet-exclusive media firms (hereafter, referred to as Internet firms). For comparison, we have also collected financial variables for all other firms from the same COMPUSTAT database. All variables are deflated by the US CPI provided by the BLS website so as to be in real terms where the CPI is
constructed such that the deflated variables, which are in real terms, are equal to their nominal counterparts on average during the “base-years” period 1982-1984.

For both groups of Internet-exclusive and other media firms, each firm owns multiple websites to interact with their users. We collected 459 number of website URLs for 87 number of Internet-exclusive media companies, and 1,011 number of URLs for 79 number of other media companies existing in 2012. For each of such collected website URLs, we collected web traffic information that shows Internet users’ interests, total number of page views, and time spent on the website. By analyzing such data sets, we aim to enhance understanding how the recent increase in the amount of time spent on Internet has affected the value of Internet-exclusive media companies.

We begin by discussing descriptive statics for Internet-exclusive media firms and all other firms. It will be shown that the cash-flow aspect of Internet firms starkly differs from that of all other firms, in particular, the growth rate of the market value and the level of the market value relative to tangible assets. Then we proceed to documenting empirically what factors are systematically relevant to such stylized facts via the regression analysis.

Table 1: Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>Internet exclusive Media Firms</th>
<th>All Other Small Firms</th>
<th>All Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm-level Growth rate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tangible Assets (PPENT)</td>
<td>2.48 (188.8)</td>
<td>2.48 (10.38)</td>
<td>12.20 (2,817.9)</td>
</tr>
<tr>
<td>Growth rate</td>
<td>0.30</td>
<td>0.06</td>
<td>0.25</td>
</tr>
<tr>
<td>Total Assets (AT)</td>
<td>35.44 (1,777.5)</td>
<td>20.31 (176.6)</td>
<td>73.17 (8,231.3)</td>
</tr>
<tr>
<td>Growth rate</td>
<td>0.51</td>
<td>0.13</td>
<td>0.28</td>
</tr>
<tr>
<td>Market Value (MKVALT)</td>
<td>75.87 (6,350.0)</td>
<td>34.91 (403.1)</td>
<td>117.53 (20,701.4)</td>
</tr>
<tr>
<td>Growth rate</td>
<td>0.29</td>
<td>0.15</td>
<td>0.21</td>
</tr>
<tr>
<td>Operating Cash Flow (OCF)</td>
<td>117.27 (1,031.9)</td>
<td>99.40 (146.6)</td>
<td>137.86 (3,154.6)</td>
</tr>
<tr>
<td>Growth rate</td>
<td>0.037</td>
<td>0.004</td>
<td>0.010</td>
</tr>
</tbody>
</table>
Table 1 presents key financial variables of Internet-exclusive media firm in comparison to all other firms, which do not belong to Internet exclusive media firms, over the sample period 2000-2012 where for each variable, the cross-sectional median value for a given year is taken and then averaged over the sample period. In order to control for the possible size effects on both growth rate and level of key variables, we sort by size and select all other firms such that their median firm size is equal to the median firm size of Internet firms where firm size is measured as the book value of tangible assets (the COMPUSTAT item ‘PPENT’). In addition, statistics for non-sorted all firms are also provided.

Most notable, the growth rate of the market value for Internet firms is as about twice high as that for all other small (same-sized) firms. For the level of the market value relative to the firm size, we can see that the Tobin’s Q ratio is also much higher for Internet firms than for all other firms, by about 33%, implying the effective amount of intangibles for Internet firms being larger than for all other firms. It is interesting to ask what factors are behind such a fact on the larger effective amount of intangibles for Internet firms relative to all other firms. We consider the main hypothesis that users’ activities are more important as a value-creation engine and contribute to a larger part of the firm value for Internet firms than for all other firms. The reason is, of course, that an Internet firm distinctly differs from all other firms in its unique business
model such that users’ activities taking place on the Internet firm’s website are the main source of revenue, e.g., cash generation by the users’ clicking the advertising banners. Thus, we proceed to measuring the importance of users’ online activities in explaining the Internet firm’s value.

Figure 1: Tobin’s Q, Market Value, Tangible Assets and Salary expenditure Ratios (Median)
Figure 1 plots time-series of key financial variables for three groups of firms by expanding the sample dating back to 1995 so that the overall trend, if any, can be more easily seen than for the sample period only. Note that the sample period of 2000-2012 is still of main interest in the sense that it has not been until 2000 that users actively engage in online activities, at least in terms of their contribution to the value creation of Internet firms, by spending considerable amount of hours and generate contents available online. As shown by Figure 1, all of key variables exhibit significant differences between the two groups, Internet vs. all other firms, except for the level of tangible assets (because tangible assets, by construction of the sample of all other firms, must be comparable between the two groups). The market value-to-tangible assets ratio clearly shows the 1999-2000 dotcom bubble incident as well as a jump to a high level since 2003, which guides us to use the period of 2000-2002 as the sample period for calibrating the model as discussed later. Below we proceed to documenting key factors, e.g., the amount of users’ online activities, that are likely to have driven such a dramatic changes in the market value for Internet firms relative to that for all other firms.

Table 2: Summary of Users’ Participation on Website

<table>
<thead>
<tr>
<th>URLs owned by Internet exclusive Media Firms</th>
<th>URLs owned by Media Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (S.D)</td>
<td></td>
</tr>
<tr>
<td>Total hours per site (hours)</td>
<td></td>
</tr>
<tr>
<td>402,400 (42,889,889)</td>
<td></td>
</tr>
<tr>
<td>Total page views (thousands)</td>
<td></td>
</tr>
<tr>
<td>18,164 (187,411)</td>
<td></td>
</tr>
<tr>
<td>Time spent on site per user (min.)</td>
<td></td>
</tr>
<tr>
<td>4.028</td>
<td></td>
</tr>
<tr>
<td>Page views per user</td>
<td></td>
</tr>
<tr>
<td>3.782 (0.0052)</td>
<td></td>
</tr>
<tr>
<td>Number of users (thousands)</td>
<td></td>
</tr>
<tr>
<td>1,492 (11,830)</td>
<td></td>
</tr>
<tr>
<td>Number of site URLs: 459</td>
<td></td>
</tr>
<tr>
<td>Total page views (thousands)</td>
<td></td>
</tr>
<tr>
<td>213.9 (1,208)</td>
<td>3.9</td>
</tr>
<tr>
<td>Page views per user</td>
<td>2.436 (1.739)</td>
</tr>
<tr>
<td>2.020</td>
<td></td>
</tr>
<tr>
<td>Number of users (thousands)</td>
<td>74.0 (359)</td>
</tr>
<tr>
<td>2.685</td>
<td>0.06</td>
</tr>
<tr>
<td>Number of site URLs: 1,011</td>
<td>5,857</td>
</tr>
</tbody>
</table>
Table 2 summarizes statistics, along several dimensions, of users’ activities on websites owned by Internet-exclusive media firms in comparison to other media firms. *Alexa.com*, the source of our data set, provides information on total and per-user page views. Table 2 shows that both total number of users and page views are at least 9 times higher on websites owned by Internet-exclusive media firms than by other media firms. In short, users participate more actively and spend more time on websites for Internet-exclusive media firms than for other media firms. Naturally, we are interested in how such difference in the amount of users’ involvement/activities between the two groups of firms is linked to the relative firm-value aspect between them, which is summarized in Table 3.

Table 3 provides key statistics in the aspect of building blocks of asset side of Internet-exclusive media firms conditional on the amount of users’ activities; 87 Internet-exclusive media firms are categorized into the two groups: high vs. low amount of users’ time spent on the firm’s own websites. The firm value is considerably higher, both in terms of the Tobin’s Q ratio and level of the market value, for the high user-time group than for the low user-time group, even

---

For total page views, we adjust its scale by taking into consideration that *at least* total 36 billion pages are available online on average during the sample period. The indexed web contains at least 3.65 billion pages based on any search engine. In this paper, we take it as benchmark to assume 36 billion webpages existing based on the lower bound of the Google search engine ([http://www.worldwidewebsize.com/](http://www.worldwidewebsize.com/)).
though all of the building blocks of a firm’s asset side, e.g., intangible vs. tangible mixture, investment rate, etc, are almost identical between the two groups. Thus, we conclude that for Internet-exclusive media firms, the amount of users’ activities is crucial in determining the Internet firm’s value, and hence a number of interesting questions arise as follows. What is the mechanism of the amount of users’ activities? What are the Internet firm’s optimal strategies to manage, at least indirectly, users’ activities? How large the quantitative importance of the amount of users’ activities in terms of the contribution to the Internet firm’s observed value? These questions call for the quantitative study of the model and will be addressed in the quantitative analysis Section.

**Anecdotal Analysis**

This section examines the influence of users’ participation/activities on the observed market value of an Internet firm. For this purpose, we carry out regression analysis for the two samples of Internet-exclusive vs. other media firms. The regression results will show that the amount of users’ activities taking place on an Internet firm’s website can account for a considerable part of the firm’s observed market value. The regression equation of a firm’s market value \( MV_i \) is written, for the two specifications, as:

\[
\log( MV_i ) = \beta_0^i + \beta_1^i \log( Tang_i ) + \beta_2^i \log(\text{Intang}_i ) + \beta_3^i \log(\text{TotHours}_i ) + \epsilon_i
\]  

(19)

\[
\log( MV_i ) = \beta_0^i + \beta_1^i \log( Tang_i ) + \beta_2^i \log(\text{Intang}_i ) + \beta_3^i \log(\text{PageViews}_i ) + \beta_4^i \log( User_i ) + \epsilon_i
\]  

(20)

where the independent variable \( Tang_i \) refers to the book value of tangible assets, \( \text{Intang}_i \) the intangibles, \( \text{TotHours}_i \) the total hours spent by users, \( \text{PageViews}_i \) the per-user page views, and
the number of users participating the firm’s own website. Both dependent and independent variables are logged.

Table 4: Regression Results

<table>
<thead>
<tr>
<th>Market Value (Log variables)</th>
<th>Internet exclusive Media Firms</th>
<th>Internet exclusive Media Firms</th>
<th>Media Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>2.276*** (0.0000)</td>
<td>2.021*** (0.0000)</td>
<td>1.456*** (0.0005)</td>
</tr>
<tr>
<td>Tangible Assets</td>
<td>0.559*** (0.0000)</td>
<td>0.547*** (0.0000)</td>
<td>0.402*** (0.0000)</td>
</tr>
<tr>
<td>Intangible Assets</td>
<td>0.227*** (0.0008)</td>
<td>0.231*** (0.0004)</td>
<td>0.401*** (0.0000)</td>
</tr>
<tr>
<td>Total Hours per site</td>
<td>0.124** (0.001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pageviews per User</td>
<td></td>
<td>0.481** (0.038)</td>
<td>0.353* (0.089)</td>
</tr>
<tr>
<td>Number of Users</td>
<td></td>
<td>0.106** (0.012)</td>
<td>0.120** (0.007)</td>
</tr>
<tr>
<td>Adjusted_R sq.</td>
<td>0.825</td>
<td>0.837</td>
<td>0.923</td>
</tr>
<tr>
<td>Number of Firms</td>
<td>87</td>
<td>87</td>
<td>79</td>
</tr>
<tr>
<td>Number of Websites</td>
<td>459</td>
<td>459</td>
<td>1,011</td>
</tr>
</tbody>
</table>

Table 4 summarizes the estimation results of the regression equations (19) and (20). In general, both tangible and intangible are significant in their relationships with the firm’s market value for both the Internet-exclusive and other media firm groups. Comparing samples, we see a considerable surge in the slope coefficient of intangible assets on the market value for the other media firms compared to the Internet-exclusive media firms. In turn, the regression coefficient of the per-user page views is higher for the Internet-exclusive media firms than for other media firms. Thus, we conclude that the predicted market value of an Internet-exclusive media firm is more sensitive to and increasing in the amount of users’ activities taking place on their own websites than for other media firms.

Table 5: Decomposition of the Predicted Market-Value Variation
Table 5 presents the results of decomposing the variation of the predicted market value into the variations and coefficients of independent variables. More specifically, we multiply the one standard deviation of an individual independent variable with its estimated coefficient, calculate its percentage relative to the sum of such values across all independent variables, and take it as the measure of the importance of the individual independent variable in accounting for the firm’s predicted market value. For instance, a standard deviation change of intangible assets accounts for about 39% of total change in the market value of other media firms, while it only explains about 22% for Internet-exclusive media firms. In turn, the standard-deviation change in the amount of users’ activities, including per-user page views and number of users, accounts for about 20% of the change in the predicted firm value for Internet-exclusive firms, while it does about 16% for the case of other media firms. These results suggest that it is important to understand correctly the return to the amount of users’ activities as one of intangibles, which again calls for the quantitave study.

V. Quantitative Analysis

This section parameterizes the stochastic processes for cash-flow productivity $z_i(i)$ and efficiency of user-capital investment $z_i^u(i)$ and production function $F(k^T(i), h(i))$, and calibrates parameters of the model. The model’s quantitative properties are then numerically studied.

Parameterization
The cash-flow productivity is constant equal to one: \( z_i = 1, \forall t, \forall i \in [0,1] \), while the logged efficiency of user-capital investment \( z_i^U (i) \) evolves according to the AR(1) process as:

\[
\log( z_{i+1}^U (i) ) = (1 - \rho) \cdot \log( \bar{z}^U ) + \rho \cdot \log( z_i^U (i) ) + \sigma^e (i) \cdot \epsilon_{i+1}(i), \quad |\rho| < 1, \quad \sigma^e (i) \geq 0
\]

(21)

where \( \rho \) represents the persistence of the above mean-reverting process, \( \bar{z}^U = 1 \) the unconditional average of \( z_i^U (i) \), \( \epsilon_{i+1}(i) \) the i.i.d. standard normal random variable, and \( \sigma^e (i) \) the volatility of the i.i.d. error term. Note that for a stationary equilibrium, the distribution of \( z_i^U (i) \) over the internet good variety \( i \) is of first-order importance. For simplicity, we focus on the homogeneous case in which \( z_i^U (i) \) is constant equal to \( \bar{z}^U \) and \( \sigma^e (i) = 0 \) almost everywhere over the space of internet good variety \( i \) except that \( z_i^U (i) \neq \bar{z}^U \) and \( \sigma^e (i) > 0 \) for at most a finite number of internet good varieties. And the variety-level weight for preferences is homogeneous across varieties: \( \omega(i) = 1, \forall i \in [0,1] \).

Firm \( i \)'s production function \( F(\hat{k}^T (i), h(i)) \) is Cobb-Douglas and written as:

\[
F(\hat{k}^T (i), h^T (i)) = (\hat{k}^T (i))^\alpha^T \cdot (h^T (i))^{1 - \alpha^T}, \quad \alpha^T \in (0,1).
\]

(22)

Calibration

This section calibrates the parameters of the model. Assuming that the model economy is in the deterministic steady state, we choose values of parameters so that the model matches the long-run averages of key variables in the data. Note that in the steady state, all of the key variables are homogeneous across the internet-good variety firms essentially due to productivities \( (z(i), z_i^U (i)) \) being assumed homogeneous: \( \bar{z}^U \) and \( \sigma^e (i) = 0 \) for every Internet firm.
The benchmark parameter values for this case are listed in the table below. Then discussion of the procedure of calibrating parameter values follows.

Table 6: Benchmark Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free interest rate</td>
<td>( r_f )</td>
<td>4%</td>
</tr>
<tr>
<td>Risk premium on stock returns</td>
<td>( \pi )</td>
<td>3%</td>
</tr>
<tr>
<td>Growth rate of per-capita GDP</td>
<td>( \gamma )</td>
<td>2%</td>
</tr>
<tr>
<td>Depreciation rate of tangible capital</td>
<td>( \delta^T )</td>
<td>10%</td>
</tr>
<tr>
<td>Depreciation rate of intangible capital</td>
<td>( \delta^P )</td>
<td>15%</td>
</tr>
<tr>
<td>Depreciation rate of user capital</td>
<td>( \delta^U )</td>
<td>15%</td>
</tr>
<tr>
<td>Preferences: discount factor</td>
<td>( \beta )</td>
<td>0.9615</td>
</tr>
<tr>
<td>Preferences: risk-aversion</td>
<td>( \sigma )</td>
<td>2.0</td>
</tr>
<tr>
<td>Preferences: elasticity of substitution among internet-good varieties</td>
<td>( \eta )</td>
<td>1.5</td>
</tr>
<tr>
<td>Preferences: weight to internet-good utility</td>
<td>( \varphi )</td>
<td>0.0298</td>
</tr>
<tr>
<td>Tangible capital share in output</td>
<td>( \alpha^T )</td>
<td>0.0754</td>
</tr>
<tr>
<td>Intangible capital share in user-capital investment</td>
<td>( \alpha^P )</td>
<td>0.7454</td>
</tr>
<tr>
<td>Fixed cost of operation</td>
<td>( \tau )</td>
<td>0.1276</td>
</tr>
<tr>
<td>Efficiency of user-capital investment</td>
<td>( z^U )</td>
<td>1.0</td>
</tr>
<tr>
<td>Firm-level productivity</td>
<td>( z )</td>
<td>1.0</td>
</tr>
</tbody>
</table>

First, given \((\gamma = 0.02, \delta^P = 0.15, \delta^U = 0.15, \delta^T = 0.10, \delta^T = 0.10, r = 0.07)\) based on direct estimates in the data or standard values in the literature, the (subjective) discount factor \( \beta = 0.9615 \) is chosen given the annual risk-free rate of about 4% in the data, the risk-aversion parameter \( \sigma \) is set to 2.0 as in the business cycle literature, and the (constant) elasticity of substitution among the internet good varieties \( \eta = 1.5 \) is chosen as benchmark given the restriction of \( 1.0 < \eta < 2.0 \) so that the household’s variety-level consumption-purpose time \( h^C(i) \) should be, relative to the other variety, increasing in the user capital stock \( k^U(i) \) at a decreasing rate. Second, the optimality condition for the household’s investment-purpose time spent on internet good varieties \( h' \) yields:
\[
\frac{h'}{h^c} = [1 - \alpha^p] \cdot \left[ \gamma + \delta^T \right] \frac{\beta (1 + \gamma)^{1-\sigma}}{(1 + \gamma)}
\]  

(23)

where \( h^c \) denotes the steady-state household’s consumption-purpose time spent on internet good varieties and is characterized by:

\[
\left( [1-h^c-h'] + [\hat{k}^p]^\alpha [h^c]^{1-\alpha} - \tau - [\gamma + \delta^p] [\hat{k}^p] - [\gamma + \delta^T] [\hat{k}^T] \right)^{-\sigma} = \phi \cdot [\hat{k}^u]^{1-\sigma} [h^c]^{-\sigma}.
\]  

(24)

Guided by the steady-state two equations above, we choose \( \alpha^p = 0.7454 \) so that the model can match the target \( (h' = 0.0105, h^c = 0.2622) \) and, for the same reason, \( \phi = 0.0298 \) given the model-determined intangible and tangible capital stocks \( (\hat{k}^p, \hat{k}^T) \). Third, \( \alpha^T = 0.0754 \) is chosen to match the targeted internet-good firm’s output-to-tangible capital ratio \( \text{Output} / \hat{k}^T \), about 2.25 in the data, that is characterized in the model as:

\[
1 + \gamma = \frac{1 + \gamma}{1 + r} E \left[ 1 - \delta^T + \alpha^T \cdot \frac{\text{Output}}{\hat{k}^T} \right]
\]  

(25)

where output again refers to the after-tax cash flow before investment expenditures and fixed cost of operation. Given that \( \text{Output} / \hat{k}^T = [h^c / \hat{k}^T]^{1-\alpha} \) in the model, it immediately follows that \( \hat{k}^T = 0.1089 \) given the targeted value of \( h^c = 0.2622 \). Then \( \hat{k}^p = 0.0836 \) is solved for from the internet-good firm’s optimality condition for \( \hat{k}^p \):

\[
1 + r = 1 - \delta^p + \frac{1 + \gamma [\gamma + \delta^u]}{1 + \gamma} \cdot [\hat{k}^p]^{1-\alpha^p} \cdot [\eta - 1].
\]  

(26)

Fourth, \( z^u(i) \) is normalized to one, implying the user capital stock \( \hat{k}^u = 0.2898 \) according to the law of motion: \( [\gamma + \delta^u] \hat{k}^u = z^u [\hat{k}^p]^\alpha [h^c]^{1-\alpha} \). Fifth, the fixed cost of operation \( \tau \) is set to
0.1276 by targeting the market value of the firm-to-tangible asset ratio, about 17.9 in the data; \( \tau \) is about 52% of output in the model and interpreted as expenses related with maintaining the firm’s competitive edge in its business such as advertising and selling expenses; for comparison, advertising and selling expenses are about 43% of output and staff expenses, part of which is likely to expenditures on improving the website quality, are about 42% of output in the data.

### Table 7: Key Statistics

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Online hours: consumption purpose/(working &amp; online hours)</td>
<td>26.20%</td>
<td>26.20%</td>
</tr>
<tr>
<td>Online hours: investment purpose/(working &amp; online hours)</td>
<td>1.05%</td>
<td>1.05%</td>
</tr>
<tr>
<td>Output-to-tangible capital ratio</td>
<td>2.25</td>
<td>2.25</td>
</tr>
<tr>
<td>Output: volatility</td>
<td>n.a.</td>
<td>3.2%</td>
</tr>
<tr>
<td>Output: auto correlation</td>
<td>0.83</td>
<td>0.86</td>
</tr>
<tr>
<td>Equity value-to-tangible capital ratio</td>
<td>17.91</td>
<td>17.79</td>
</tr>
<tr>
<td>Equity value: volatility</td>
<td>n.a.</td>
<td>3.1%</td>
</tr>
<tr>
<td>Equity value: auto correlation</td>
<td>0.87</td>
<td>0.88</td>
</tr>
<tr>
<td>Intangibles-to-tangible capital ratio</td>
<td>0.87</td>
<td>0.77</td>
</tr>
</tbody>
</table>

Note: this table presents the model-generated statistics for an Internet firm.

Calibration results for the key statistics are listed in Table 7. The model matches well the targeted as well as non-targeted statistics. We, in turn, proceed to calibrating the shock process for the efficiency of user-capital investment for a particular individual firm \( i \): we need to calibrate the two parameters \((\rho, \sigma^e(i))\) where \((1 - \rho)\) refers to the weight to mean-reverting relative to the previous state and \(\sigma^e(i)\) the standard deviation of the i.i.d. error term. We choose \(\rho = 0.868\) by targeting the autocorrelation of the market value of an Internet firm and \(\sigma^e(i) = 0.02\) as benchmark, which results in, as will be discussed later, the simulated firm
value’s autocorrelation being 0.88 and the firm value’s (annual) volatility being about 3.1 percentage points.

**Simulated Results**

In this section, the calibrated model is used as a laboratory to implement an experiment of imposing various shocks to the model economy, in particular, a shock to an individual firm’s efficiency of user-capital investment. First, the effects of temporary increases in the efficiency of user-capital investment are discussed. Second, importance of user capital for the firm value is discussed, in particular, results of the counterfactual experiment of removing user capital.

Before presenting the simulated results, we discuss the simulation method. Since the equilibrium of the model is too complicated to be solved analytically, numerical technique is used for solving for the equilibrium. More specifically, the state space of \((\hat{k}_t^u(i), \hat{k}_t^u(i), \hat{k}_t^T(i), z_t(i))\) is discretized, over which the equilibrium policy functions are approximated by using the first-order finite element method (FEM) as illustrated in McGrattan (1996) and then solved by the iteration method.\(^\text{10}\)

**Benchmark** We impose temporary shocks to \(z^u(i)\) on the calibrated economy so that the efficiency of user-capital investment \(z^u(i)\) for firm \(i\) keeps increasing for a number of periods holding all else constant. Figure 2 displays the simulated key variables.\(^\text{11}\) As shown by Figure 2, in response to temporary positive shocks to \(z^u(i)\), firm \(i\)’s intangible-capital investment \(\hat{x}^F(i)\)

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\(^\text{10}\) The numerical errors between the guessed and updated policy functions are, except for the firm’s own value function, about 0.00000001 percentage points relative to the steady-state values uniformly over the state space, while the numerical error between the guessed and updated firm’s own value function is about 0.1 percentage points relative to the steady-state value uniformly over the state space. See Mathematical Appendix for the equilibrium policy functions that are numerically solved for.

\(^\text{11}\) For the case of shocks to \(z^u(i)\) being drawn randomly according to the calibrated AR(1) process, see Figure 3 for one particular sample of simulated time-series of key variables.
increases greatly so as to utilize the increased efficiency of user-capital investment and hence so does firm $i$’s intangible capital stock $\hat{k}^r(i)$, which, combined by the household’s increased investment-purpose hours $h^i(i)$, induces the household’s increased consumption-purpose hours $h^c(i)$ mainly due to the increased user-capital stock; as a result, the firm’s value $\hat{V}(i)$ increase greatly. In particular, the firm’s intangible-capital investment peaks up during the early part of periods, while the tangible-capital investment starts to grow up slowly, which is broadly consistent with the recent observation that a successful internet firm has increased its expenditures on both intangible and tangible assets than other firms.

Figure 2: Simulated Key Variables: Impulse-Response

Note: this figure presents the simulated results of temporary positive shocks to the efficiency of investment to user capital specific to the Web site owned by firm $i$; all variables are in percentage terms relative to those in the initial state, which is at period 0. The top-left panel presents the time series of the efficiency of investment to user capital specific to the Web site owned by firm $i$ (impulse), the top-right panel presents the simulated user capital (response), and the top-middle displays the simulated two variables: firm $i$’s intangible capital (solid line) and tangible capital (dashed line). The bottom-left panel presents the simulated two variables: the household’s time spent on investment to user capital (solid line) and on consumption of user capital (dashed line). The bottom-middle panel presents the simulated two variables: firm $i$’s investment to (solid line) and to tangible capital (dashed line). The bottom-right panel presents the simulated two variables: firm $i$’s equity value (solid line) and output (dashed line).
Quantification of the Value of User Capital  In this section, we quantify how much user capital contributes to the Internet firm’s value; for this purpose, we carry out a counterfactual experiment of removing user capital in place. More specifically, we consider the case of setting \( \hat{k}^U (i) \) to zero and lowering the efficiency of user-capital investment \( z^U (i) \) to a low level, 0.50, which is 50% relative to the steady state; the reason that we set \( z^U (i) \) to such a low level is essentially to capture the case of user capital being depressed for a long time because otherwise user capital would be restored to the steady-state level quickly via the firm’s highly increased intangible capital.

Table 8: Value of User Capital

<table>
<thead>
<tr>
<th></th>
<th>( \hat{k}^P (i) )</th>
<th>( \hat{k}^U (i) )</th>
<th>( h^I (i) )</th>
<th>( h^C (i) )</th>
<th>( \hat{V} (i) )</th>
<th>( z^U (i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady State</td>
<td>0.080</td>
<td>0.280</td>
<td>0.010</td>
<td>0.26</td>
<td>1.86</td>
<td>1.0</td>
</tr>
<tr>
<td>Case: ( \hat{k}^U (i) = 0, z^U (i) = 0.5 )</td>
<td>0.193</td>
<td>0.027</td>
<td>0.019</td>
<td>0.00</td>
<td>0.81</td>
<td>0.5</td>
</tr>
<tr>
<td>Case: ( z^U (i) = 0.5 )</td>
<td>0.002</td>
<td>0.252</td>
<td>0.004</td>
<td>0.26</td>
<td>1.40</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Note: this table presents the results of counterfactual experiment of setting \( \hat{k}^U (i) \) to zero together with imposing shocks to \( z^U (i) \). The line headed ‘Steady State’ refers to the calibrated case, i.e., no shocks to \( \hat{k}^U (i) \) and no shocks to \( z^U (i) \), while the two cases of imposing shocks are as follows: the line headed ‘Case: \( \hat{k}^U (i) = 0, z^U (i) = 0.5 \)’ refers to the case of \( \hat{k}^U (i) \) being set to zero and \( z^U (i) \) being set to 0.5, and the line headed ‘Case: \( z^U (i) = 0.5 \)’ the case of \( z^U (i) \) being set to 0.5 and no shocks to \( \hat{k}^U (i) \). The column headed ‘\( \hat{k}^P (i) \)’ refers to the firm’s choice of the next-period intangible capital stock, ‘\( z^U (i) \)’ the next-period state of user capital, ‘\( \hat{V} (i) \)’ the current-period equity value of the firm, and ‘\( h^I (i) \)’ and ‘\( h^C (i) \)’ the current-period response of the household’s investment- and consumption-purpose hours, respectively.

Table 8 presents the experiment results of removing user capital in place by controlling the efficiency of user-capital investment at the same time. Note that for the benchmark case of setting \( \hat{k}^U (i) \) to zero and \( z^U (i) \) to 0.5, the firm value \( V (i) \) drops down substantially, by about 57% relative to before the experiment. In this case, user capital is prolonged to be of a low level, for instance, the next-period level of user capital is about 10% relative to before the experiment, see, the column headed ‘\( \hat{k}^U (i) \)’ in Table 8; consequently, the household’s consumption purpose
hours $h^c_{(i)}$ are also prolonged of a low level and hence the firm’s cash flow is doomed for a long time, resulting in the firm’s current value $\hat{V}_{(i)}$ dropping sharply. Note that in response of the sudden removal of user capital, the firm attenuates the reduction in the firm’s value by sharply increasing the intangible investment so that the next-period level of user capital can be restored to a certain level above zero. (See, the firm’s next-period intangible capital stock $\hat{k}^r_{(i)}$ jumps to about two and half times relative to before the experiment.)

For comparison, lowering $z^u_{(i)}$ to 0.5 alone but without removal of $\hat{k}^u_{(i)}$ also reduces the firm value to an extent still sizable, by about 25% relative to before the experiment: in this case, the level of user capital decreases slowly over time, (the next-period level of user capital $\hat{k}^u_{(i)}$ being about 90% relative to before the experiment), and hence so does the household’s consumption-purpose time spending on this firm’s website, resulting in the moderate reduction in the firm’s current value.

VI. Conclusion

This paper studies the growing importance of users’ online activities for the value of an Internet firm. Using the data on firm-level financials and households’ time spent on websites, we find that the amount of users’ activities on an Internet firm’s own website is significantly positively associated with the firm’s market value. This suggests that users’ online activities contribute to forming an important part of an Internet firm’s asset, namely the user-generated capital. Moreover, we also find that the market value of a firm relative to both book values of intangibles and tangibles is much higher for an Internet firm than for other firms, suggesting that user-generated capital is important quantitatively in accounting for the Internet firm’s value.
Thus, this paper studies the mechanism of user-generated capital and quantifies the contribution of user-generated capital to the Internet firm’s value by building and calibrating a dynamic general equilibrium model of the users’ time allocation among competing websites. The model predicts that improvements in either an Internet firm’s website quality or the efficiency of investment to user-generated capital increase the Internet firm’s future cash flows and hence the current value of the firm by boosting up the users’ hours spent on the firm’s own website. The simulated results show that such effects are sizable. Moreover, the counterfactual experiment of completely removing user-generated capital and lowering the efficiency of investment to user-generated capital shows that user-generated capital account for about 60% of the Internet firm’s value.

The competition among Internet firms and their strategic behaviors are modeled to be simple as in the standard framework of monopolistic competition among a continuum of firms. It would be interesting to study a number of competing Internet firms’ strategic behaviors to pouch the dominant online platform, which we leave for a future work.

VII. References


I. Appendix
Mathematical Appendix

The household’s equilibrium policy functions for \( h^C(i) \) and \( h^I(i) \) are characterized by:

\[
h^C(i) = h^C \left( \frac{k^U(i)}{k^U} \right)^{q-1}, \tag{A1}
\]

\[
h^I(i) = h^I \cdot \left( \frac{k^U(i)}{k^U} \right)^{(q-2)} \left( \frac{z^U(i)}{z^U} \right)^{1/a^p} \tag{A2}
\]

where variables \((h^C, h^I, z^U, \hat{k}^U)\) refer to the counterparts of the other (homogeneous) firms of which states are the same as for the steady-state values. The firm \(i\)’s equilibrium policy functions for \( \hat{k}^T(i) \) and \( \hat{k}^P(i) \) are written as:

\[
[k^T(i)]^{1-a^T} = \frac{\alpha^T}{r + \delta^T} E \left[ h^C(i) \right]^{1-a^T}, \tag{A3}
\]

\[
[k^P(i)]^{1-a^P} = \frac{1 - \alpha^T \eta - 1}{r + \delta^P} \left[ \frac{h^C}{[k^U]^{q-1}} \right]^{1-a^P} E \left[ z^U(i) h^I(i) \right]^{1-a^p} \left[ \frac{[\hat{k}^T(i)]^{a^T}}{[\hat{k}^P(i)]^{1-(q-1)(1-a^T)}} \right] \tag{A4}
\]

where the expectation operator \( E[\cdot] \) in both the two equations above is taken with respect to the firm \(i\)’s next-period efficiency of user-capital investment \( z^U(i) \) conditional on its current efficiency \( z^U(i) \).