Sharing Economics: Procuring Third-Party WiFi Capacity for Mobile Data Offloading *

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Abstract

The unprecedented growth of cellular traffic driven by web surfing, video streaming, and cloud-based services poses challenges for cellular service providers to meet the demand for bandwidth. To minimize the costs of unmet demand (e.g., dissatisfied customers, or churn), service providers are experimenting paying WiFi hotspots to serve excess demand. In the present study, we propose an optimal procurement mechanism with contingent contracts for service providers to leverage the advantages of both cellular and WiFi resources. Unlike conventional cellular communication technologies, WiFi hotspots provide data rates with a more limited coverage. Our present work contributes to the existing literature by developing an analytical model, which considers this unique challenge of integrating the long-range cellular resource and short-range WiFi hotspots. We show the procedure of computing the optimal procurement mechanism with a tight integration of economics and computational technology. Simulation using cellular network data from a large U.S. service provider suggests that the proposed procurement mechanism significantly outperforms the standard Vickrey-Clarke-Groves (VCG) auction in terms of the service provider's expected payoff.

Keywords: sharing economy, mechanism design

1 Introduction

We are witnessing an explosion of mobile data traffic driven by web surfing, video streaming, and online gaming largely due to the increasing popularity of smartphones. Global mobile data traffic grew 70 percent in 2012 and will increase thirteen-fold between 2012 and 2017 (Cisco 2013)[1]. The huge amount of mobile data traffic poses a challenge to the network infrastructure: Cellular networks are overloaded and congested during peak hours because of insufficient capacity. Network congestion can lead to poor user experience and churn.

Researchers have proposed several solutions from both technical and economic aspects: (1) increasing the number of cellular towers or deploying the cell-splitting technology; (2) upgrading the network to fourth-generation (4G) networks such as Long Term Evaluation (LTE), High Speed Packet Access (HSPA) and WiMax; (3) expanding capacity by acquiring the spectrum of other networks, such as the attempted purchase of T-Mobile USA by AT&T; (4) adopting smart data pricing mechanisms (e.g. usage-based and app-based pricing plans) to constrain the heaviest mobile data users, instead of using flat-rate pricing plans with unlimited data (Sen et al. 2012); and (5) offloading data traffic to WiFi networks (Bulut and Szymanski 2012).

Although all these solutions help alleviate the problem, each has its disadvantages. The first two solutions require heavy investments, and getting government approval for building new cell towers can take two years.[2] From the economic perspective, it is extremely expensive to increase the number of cellular base stations just for peak traffic demands.[3] As a result, all cellular networks augment the first two solutions with other approaches to expand capacity. The third solution suffers from regulatory constraints. Cramton, Skrzypacz, and Wilson (2007) showed that an important market failure arises in spectrum auctions with dominant

[1] According to this white paper, in 2012, a typical smartphone generated 50 times more mobile data traffic than a typical non-smartphone, and global mobile data traffic reached 885 petabytes per month at the end of 2012, up from 520 petabytes per month at the end of 2011.

[2] See http://www.businessweek.com/technology/content/aug2009/tc20090823_412749.htm.

[3] "While cell-splitting provides capacity benefits, it could be quite expensive and economically infeasible since in addition to the base station hardware/deployment cost, each of the new bases needs to be provided with backhaul connectivity either via wireline access or microwave links." (Balachandran et al., 2008)

incumbents. They suggest that the Federal Communications Commission (FCC) should place limits on how much spectrum AT&T and Verizon are allowed to buy.[4] Although the average net benefits realized under congestion-based pricing tend to be higher than the average net benefits realized under flat-rate pricing (Gupta et al., 2011), usage based plans may also backfire by singling out the smartphone users who have the highest potential for future revenue.

Because of these technical, economic and regulatory constraints, the fifth solution, using WiFi hotspots for mobile data traffic offloading, seems to be one of the most promising approaches in augmenting solutions (1) and (2). A straightforward approach is for the cellular service providers to build and manage their own hotspots. In fact, we have seen some pilot projects for self-managed hotspots (Aijaz et al. 2013). Even though the option of service providers directly managing hotspots is often available, it is quite expensive (Iosifidis et al., 2013) and thus may not be cost-effective. Paul et al. (2011) found that 28% of subscribers generate traffic only in a single hour during peak hours in a day. Clearly, building and managing hotspots just for that peak hour is not efficient. Offloading traffic to third party hotspots overcomes the obstacle of managing a hotspot and ensures the high availability of WiFi resources. This strategy could potentially be a win-win solution: The cellular service provider achieves significant savings by not building more cellular base stations or hotspots just for the peak traffic demands. The WiFi hotspots gain additional revenue by sharing their otherwise wasted spare capacity. Indeed, such practice of sharing unused capacity is gaining traction in the industry (e.g., Airbnb, Uber, etc.) thanks to the advancement in technology and the study of such sharing economy is also on the rise (Weber 2013, 2015). We follow this paradigm of sharing economy and focus on offloading mobile traffic to third-party WiFi hotspots owned by entities such as local restaurants, bookstores, and hotels. The purpose of this paper is to introduce an innovative economic mechanism to integrate third-party WiFi

[4] This concern is also reflected in the action taken by the FCC to block the recent merger between AT&T and T-Mobile. Another example of regulation is the Net Neutrality Rules that have become a subject under fierce debate (Cheng, Bandyopadhyay, and Guo 2011).

hotspots with existing cellular resources.

Cellular service providers have shown great interests in this approach. For example, KDDI Corporation, a principal telecommunication provider in Japan, has cooperated with about 100,000 commercial WiFi hotspots by March 2012 (Aijaz et al., 2013). However, offloading data traffic to third-party WiFi hotspots is not purely a technology augmenting the existing cellular network. Considering the economic incentives of third-party WiFi hotspots, WiFi offloading is also a practical mechanism design problem and requires the combination of both computing technology and economic theory to effectively leverage WiFi capacity (Bichler, Gupta, and Ketter 2010). Because WiFi capacity is a type of product with quite standardized characteristics, competitive bidding should be a better way to select the lowest cost bidder than negotiations.[5]

There are several challenges in the design of such procurement auction system. First, the longer range cellular resource introduces coupling between the shorter range WiFi hotspots. In reality, WiFi networks usually have a more limited range than cellular resources. In our model, we partition the range of a cellular tower into several WiFi regions. The cellular capacity can serve data traffic in any region, whereas the WiFi resource can only serve local traffic. Buying more resources from one local WiFi hotspot frees up more in-house cellular capacity to serve unsatisfied demand in other WiFi regions. Second, the data traffic is uncertain and changes frequently over time. It is critical to provide real-time support for computing the optimal contract. This paper aims to combine analytical modeling and simulations with real data in order to analyze a new procurement mechanism with contingent contracts to meet these challenges in a realistic environment. Simulation results show that, compared with a Vickrey-Clarke-Groves (VCG) type auction for mobile data offloading (Dong et al., 2012), the optimal procurement auction with contingent contracts proposed in this paper can significantly improve the cellular network's expected payoff.

[5]See for example, Bajari et al. (2009), who considered several determinants that may influence the choice of auctions versus negotiations. For complex projects, auctions may stifle communication between the buyer and the contractor. Clearly, WiFi capacity satisfies the standard assumption of well-defined products in auction literature.

Our insights also apply more generally to optimal mechanism design in a class of supply chain problems. Conceptually, the key problem in the purchase of WiFi capacity is to determine the optimal procurement strategy in the presence of product flexibility and information asymmetry between suppliers (WiFi hotspots) and the downstream firm (cellular service provider). This procurement problem in the wireless industry can be extended to a more general setting where (1) the downstream firm owns in-house capacity (cellular capacity) that can be used for multiple products (the wireless service in different WiFi regions); (2) The product-flexible capacity is limited, and the firm needs to procure products from multiple upstream suppliers; and (3) Each supplier is specialized and can only produce one product (each WiFi hotspot can only serve local traffic). Given the presence of limited product-flexible capacity (in-house capacity) and upstream suppliers, the downstream firm needs to design an optimal procurement auction when the customer demand is volatile and unpredictable. This procurement auction design becomes complicated when the downstream firm faces product flexibility and information asymmetry. This procurement scenario is common when companies are investing in product-flexible capacity that entails the ability to produce multiple products on the same capacity, and the ability to reallocate capacity between products (Goyal and Netessine 2011). Many manufacturing and service companies use flexible capacity to hedge against uncertainty in future demand (Fine and Freund 1990; Van Mieghem 1998, 2004).[6]

2 Literature Review

The technology aspect and implementation of this study are clearly related to the vast literature in computer science on mobile data offloading (Balasubramanian et al. 2010; Dong et al. 2012; Iosifidis et al. 2013). We refer interested readers to Aijaz et al. (2013) for an overview of the technical and business perspectives of mobile data offloading and to

[6] In the automotive industry, the plants for most of the automobile companies are much more flexible than before: Ford's Rouge Plant can manufacture nine different products (Goyal and Netessine 2011).

Kang et al. (2013) for a discussion on mobile data offloading through a third-party WiFi. The theoretical aspect of this study is mostly related to two streams of literature: supply chain management and auction design.

Several literature on supply chain management focused on scenarios where adding product-flexible capacity is beneficial (Goyal and Netessine 2011). Janakiraman, Nagarajan, and Veeraraghavan (2009) considered a firm that produces multiple products each period, using a shared resource with limited capacity, in a periodically reviewed stochastic inventory model. Simchi-Levi and Wei (2012) studied the performance of flexibility designs when a chain of partial flexibility is implemented. A natural question is, with limitations on product-flexible capacity, how a downstream firm should design its procurement auction mechanism. Federgruen and Yang (2011) analyzed the downstream firm's optimal procurement strategy with unreliable suppliers. Their analytical model is formulated as a single-agent optimization problem. The underlying assumption is the symmetric information between suppliers and the downstream firm. In our study, we introduce a game-theoretical model with asymmetric information in the presence of product-flexible capacity. The downstream firm procures products from the suppliers before the actual demand is known, and optimally allocates its in-house capacity to produce different products when the demand is realized. In this process, the downstream firm makes the following decisions: How to allocate its product-flexible capacity to produce different products? How much quantity should be procured from each supplier? What is the corresponding payment scheme for each supplier? Our theoretical model provides an auction framework to answer these questions in the context of the wireless industry. In this study, the theoretical results complement the existing literature on product line designs when the product-flexible capacity is limited (Simchi-Levi and Wei 2012; Netessine and Taylor 2007). Netessine, Dobson, and Shumsky (2002) analytically characterized the critical effects of increasing demand correlation between products on the flexible capacity decisions. We also find that the demand correlation as well as the level of in-house capacity plays a crucial role in the optimal design of procurement mechanisms.

When the demand correlation is highly positive or the in-house capacity is relatively large, the optimal procurement mechanism is a global auction including all upstream suppliers; otherwise, it is optimal to hold separate auctions for each product.[7]

The present study is closely related to the literature on auction design. Dasgupta and Spulber (1989) extended the standard fixed quantity auction and studied a quantity auction that allows the quantity of the goods purchased to be endogenously based on the submitted bids. In many procurement situations, the buyer cares about other attributes in addition to price when evaluating the submitted bids. In a multi-attribute scoring auction, suppliers submit multidimensional bids, and the contract is awarded to the supplier who submitted the bid with the highest score according to a scoring rule. Che (1993) developed a scoring procurement auction in which suppliers bid on two dimensions of the good. This scoring auction allows only sole sourcing. However, offloading data traffic to multiple WiFi hotspots is naturally done in our procurement setting. Duenyas, Hu, and Beil (2013) showed that a simple version of the open-descending auction can implement the optimal procurement mechanism for a newsvendor problem. Chu and Sappington (2009) examined optimal procurement contracts that prevail in practice. Auctions with contingent contracts have been widely studied in economics literature.[8] Hansen (1985) studied an auction with contingent payments. Chen et al. (2010) showed that the procurement auctions with contingent contracts can manage the project failure risk of suppliers and significantly improve both social welfare and the buyer's payoff. The model in our study differs from such auctions because of the unique challenge in our application setting.

[7] In our wireless context, a separate auction refers to a local auction within a WiFi region.

[8] A contingent contract is a type of forward contract that depends on the realizations of some uncertain events. For example, a contract can be contingent on the uncertain demand or the future spot market price.

3 Model

3.1 Model Setup

A cellular network provides service to its customers who demand bandwidth. We model the user demand as a random variable X with a cumulative distribution function $G(X)$ in the support $[0,1]$[9]. When user demand X is below a certain threshold X_B, the cellular service provider faces no additional cost except the sunk cost of buying the spectrum and keeping the system running. Here, the threshold X_B is the cellular capacity[10] owned by the service provider. The standard metrics used in the telecommunications industry to measure quality of service (QoS), such as Erlang B formula and Kleinrock delay formula, depend on the difference between user demand and capacity or their ratio (Pinto and Sibley 2013). In our problem setting, $X - X_B$ is the difference between user demand and capacity. Note that capacity should not be interpreted as a strict output limit, but rather as a factor in maintaining QoS. When user demand X exceeds X_B, the cellular service provider incurs a cost of $C_0(X - X_B)$. The cost function $C_0(\cdot)$ is strictly increasing and strictly convex, which captures the rapidly rising cost of congestion (e.g., dissatisfied customers, or churn). A similar convex cost function has been used in modeling the congestion cost of the Internet (Dong et al. 2012). Apparently, we have $C_0(x) = 0$ for any $x \leq 0$. Denote $c_0(x) = C_0'(x)$ as the marginal cost of congestion.

Given the unprecedented growth rate of mobile data demand and the high cost associated with congestion, the cellular network is interested in procuring spare resources from third-party WiFi hotspots. Although both can be used to meet the user demand, cellular resources and WiFi resources have different spatial coverages. In suburban areas, a typical cellular

[9] Note that the assumption of the support is essentially saying that demand is bounded, which is without loss of generality for any realistic situation. Apparently, the interpretation of 1 will be different for different scenarios. For example, 1 could be interpreted as 1 terabyte per second or 10 terabytes per second depending on specific scenarios.

[10] X_B is interpreted as the channel capacity stated by the Shannon–Hartley theorem (Kennington et al. 2011). The theorem shows that when the information transmitted rate is less than X_B, the probability of error at the receiver can be made arbitrary small. When the information transmitted rate is greater than X_B, the probability of error increases as the information transmitted rate is increased.

base station covers 1-2 miles (2-3 km) and in dense urban areas, it may cover one-fourth to one-half mile (400-800 m). A typical WiFi network has a range of 120 feet (32 m) indoors and 300 feet (95 m) outdoors.[11] To model this unique feature of bandwidth supply, we partition a cell sector into several regions so that WiFi hotspots within the same region are relatively close together. In particular, we assume the cell sector can be divided into M WiFi regions.[12] Cellular resources can serve traffic in any region m, whereas WiFi hotspots in region m can only serve local traffic. We assume the same congestion cost function of the cellular service provider for all regions. A unique challenge in the procurement auction is that the longer range cellular resource introduces coupling between the shorter range WiFi hotspots. The procurement problem in one WiFi region is not independent of the procurement problem in another region, because purchasing more WiFi capacity from a local WiFi hotspot in one region frees up more cellular capacity that can be used to serve the demand in another region. We denote the demand in region m by X_m and assume the demand vector $(X_1, X_2, \cdots, X_M)$ has a joint distribution function $G(X_1, X_2, \cdots, X_M)$.

Serving mobile demand for the cellular network provider incurs cost to a hotspot which differs among hotspots and is private information to each hotspot. We assume the cost function for hotspot i to provide capacity Q to the cellular network is

$$C(Q, \theta_i) \equiv \int_0^Q c(q, \theta_i) dq, i = 1, 2, ..., n.$$

where $c(q, \theta_i) \geq 0$ is the marginal cost function for hotspot i, and θ_i reflects each hotspot's private information about the cost of bandwidth provision which might differ among different hotspots. We assume $c_q(q, \theta_i) \geq 0$ to capture the fact that the marginal cost of providing capacity for each hotspot increases as more capacity is provided to the cellular network.

[11] See http://en.wikipedia.org/wiki/Wifi, and http://en.wikipedia.org/wiki/Cell_site.

[12] A WiFi hotspot might be on the boundary of two regions. In Section 4, we generate regions by clustering the WiFi hotspots using k-means method. Note that for simplicity, we assume that cellular capacity can be reallocated seamlessly from one WiFi region to another. In practice, some cellular capacity can be redirected (e.g., core processing for the base station), and some capacity cannot be redirected (e.g., radio capacity for directional antennas – these cover only a certain direction and angular range).

Marginal costs are increasing and convex in the cost parameter, $c_\theta \geq 0$, $c_{\theta\theta} \geq 0$. Also, we assume $c_{q\theta} \geq 0$. Hotspots' cost parameters are independently and identically distributed with a continuously differentiable cumulative distribution function $F(\cdot)$ defined on $[\underline{\theta}, \bar{\theta}]$ which is common knowledge. We assume $H(\theta) \equiv F(\theta)/F'(\theta)$ is an increasing function of θ which is satisfied by common distribution functions such as the uniform distribution.

The cellular service provider follows a two-step decision procedure: In the first stage, it purchases WiFi capacity from hotspots in different regions. In the second stage, the cellular service provider adjusts the allocation of cellular resources across regions.

3.2 Non-Contingent Procurement Auction

We first examine the optimization problem in the second stage. Suppose the cellular service provider has purchased Y_m units of bandwidth from hotspots in region m. Let $\mathbb{E}_G$ denote expectation taken over $X_1, X_2, \cdots, X_M$. The expected congestion cost is then

$$\begin{aligned} J(Y_1 \cdots, Y_M) = \quad & \mathrm{Min}_{y_1,\cdots,y_M} \mathbb{E}_G \left[\sum_{m=1}^{M} C_0(X_m - Y_m - y_m) \right] \\ s.t. \quad & \sum_{m=1}^{M} y_m = X_B, y_m \geq 0, \text{for } m = 1, 2, ...M, \end{aligned} \tag{3.1}$$

where y_m is the amount of cellular capacity allocated to region m. Without procuring any bandwidth from hotspots, the expected congestion cost is simply $J(0) \equiv \mathbb{E}_G[C_0(X - X_B)]$.

Define

$$X \equiv X_1 + X_2 + \cdots + X_M, \ \bar{X} \equiv \frac{X}{M}, \ \bar{X}_B \equiv \frac{X_B}{M}, \ \bar{Y} \equiv \frac{Y_m}{M}.$$

Because $C_0(\cdot)$ is convex, using Jensen's inequality, we have

$$\sum_{m=1}^{M} C_0(X_m - Y_m - y_m) \geq MC_0 \left(\frac{1}{M} \sum_{m=1}^{M} (X_m - Y_m - y_m) \right) = MC_0 \left(\bar{X} - \bar{X}_B - \bar{Y} \right). \tag{3.2}$$

Hence, the optimal allocation of cellular resources should be $y_m^* = \bar{X}_B + (X_m - \bar{X}) - (Y_m - \bar{Y})$

and the resulting expected congestion cost is

$$J(Y_1, \cdots, Y_M) = M \cdot \mathbb{E}_G\big[C_0(\bar{X} - \bar{X}_B - \bar{Y})\big].$$

For this optimal allocation to be feasible, we need $y_m^* \geq 0$, or equivalently,

$$\bar{X}_B \geq \big(Y_m - \bar{Y}\big) - \big(X_m - \bar{X}\big) \tag{3.3}$$

for $m = 1, 2, ..., M$, and for all possible realizations of private cost parameters (θ_i, θ_{-i}). Clearly, the condition is more likely to be satisfied if X_B is relatively large. Alternatively, the condition is more likely to be satisfied if more hotspot bandwidth supply is available in regions with more bandwidth demand (i.e., X_m and Y_m are positively correlated), which we believe is reasonable because the economic incentive to supply bandwidth is larger in regions with high demand. In this section, we assume inequality 3.3 is always satisfied. We relax this assumption in Section 3.4.

Because $J(Y_1, \cdots, Y_M)$ is only a function of $X_1, \cdots, X_M$ through $\bar{X}$, we denote the distribution of $\bar{X}$ as $\bar{G}$ and denote $\mathbb{E}_{\bar{G}}$ as the expectation over $\bar{X}$. The valuation of procuring Y_m amount of bandwidth from region m, $m = 1, \cdots, M$, is the expected reduction of congestion cost:

$$V(Y_1, \cdots, Y_M) = J(0) - J(Y_1, \cdots, Y_M) = J(0) - M \int_{\bar{X}_B + \bar{Y}}^{1} C_0(\bar{X} - \bar{X}_B - \bar{Y}) d\bar{G}(\bar{X}). \tag{3.4}$$

Clearly, $V(Y_1, \cdots, Y_M)$ is a function of $Y_1, \cdots, Y_M$ only through $\bar{Y}$, With slight abuse of notation, we may write $V(Y_1, \cdots, Y_M)$ as $V(\bar{Y})$. Because

$$V'(\bar{Y}) = M \int_{\bar{X}_B + \bar{Y}}^{1} C_0'(\bar{X} - \bar{X}_B - \bar{Y}) dG(\bar{X}) > 0 \tag{3.5}$$

and

$$V''(\bar{Y}) = -M \int_{\bar{X}_B+\bar{Y}}^{1} C_0''(\bar{X} - \bar{X}_B - \bar{Y}) dG(\bar{X}) - MC_0'(0) g(\bar{X}_B + \bar{Y}) < 0,$$

where $g(\cdot)$ is the density function of $\bar{X}$. Hence, $V(\bar{Y})$ is strictly increasing and strictly concave.

When condition 3.3 is satisfied, WiFi resource in one region is perfect substitute of WiFi resource in another region from the perspective of the cellular service provider. Essentially, we are dealing with a variable quantity procurement auction with multiple winners which is studied in Dasgupta and Spulber (1989). In the first stage, the cellular service provider's optimization problem is characterized as a direct revelation game in which hotspots announce their types and truthful revelation is a Bayes-Nash equilibrium. We adopt the notational convention of writing $\theta_{-i} = (\theta_1, ..., \theta_{i-1}, \theta_{i+1}, ..., \theta_n)$. The optimal allocation for the cellular service provider can be implemented via a direct revelation mechanism where

- The cellular service provider announces a payment-bandwidth schedule $P_i = P(\theta_i, \theta_{-i})$, and a bandwidth allocation schedule $q_i = Q(\theta_i, \theta_{-i})$;
- Hotspot i reports the private cost parameter θ_i given $P(\theta_i, \theta_{-i})$ and $Q(\theta_i, \theta_{-i})$;
- Hotspot i provides bandwidth $q_i = Q(\theta_i, \theta_{-i})$ to the cellular service provider and its payment is $P_i = P(\theta_i, \theta_{-i})$.

Following Dasgupta and Spulber (1989), the optimal mechanism $(P^*(\theta_i, \theta_{-i}), Q^*(\theta_i, \theta_{-i}))$ for the cellular service provider is given by the following proposition.

Proposition 1 *In the optimal direct revelation mechanism, all hotspots truthfully announce their cost parameters θ. The optimal procurement quantity schedule $q_i = Q^*(\theta_i, \theta_{-i})$, for $i = 1, 2, ...n$ is determined by*

$$\frac{\partial}{\partial q_i} V\left(\frac{1}{M}\sum_{i=1}^{n} q_i\right) = \int_{\bar{X}_B+\bar{Y}}^{1} C_0'(\bar{X} - \bar{X}_B - \bar{Y}) dG(\bar{X}) = c(q_i, \theta_i) + c_\theta(q_i, \theta_i) H(\theta_i).$$

where $Y_i = q_i$. The optimal payment schedule $P_i = P^*(\theta_i, \theta_{-i})$, for $i = 1, 2, ...n$ is given by:

$$P_i = C(q_i, \theta_i) + \int_{\theta_i}^{\theta^*} C_\theta(q_i, \theta) d\theta.$$

The cellular service provider's expected gain from the procuement auction is

$$\mathbb{E}\left[V\left(\frac{1}{M}\sum_{i=1}^{n} q_i\right) - \sum_{i=1}^{n} C(q_i, \theta_i) - \sum_{i=1}^{n} C_\theta(q_i, \theta_i) H(\theta_i)\right].$$

The proofs of all propositions can be found in the Appendix.

In the direct revelation game, hotspot i announces its cost parameter θ_i. The capacity it needs to provide is $q_i = Q^*(\theta_i, \theta_{-i})$, and its payment is $P_i = P^*(\theta_i, \theta_{-i})$. This optimal mechanism is a global auction including all hotspots from different regions. Note that launching separate auctions within each region is not optimal because the cellular resource can serve traffic in any region. The intuition is that procuring more WiFi resources in one region frees up more cellular resources, and the cellular service provider can allocate the cellular resources to other regions. In equilibrium, the virtual marginal costs $c(q_i, \theta_i) + c_\theta(q_i, \theta_i)H(\theta_i)$ are equalized across hotspots in different regions, and the marginal benefits of procuring WiFi capacity should be equalized across regions as well. In addition, the number of hotspots might be small in some specific regions. The global auction effectively creates the inter-region competition among the hotspots when the intra-region competition is limited. Under our procurement mechanism, the network becomes more resilient because the peak data traffic can be seamlessly offloaded to some nearby hotspots with minimal service disruption. The procedure of computing the optimal procurement auction is included in Appendix.

3.3 Contingent Procurement Auction

In the previous section, the procurement mechanism is implemented before the demand is realized. In this sense, there is an ex-post inefficiency: The cellular service provider might purchase either too much or too little bandwidth. We explore the use of contingent contracts to mitigate such problem. Note that in our model, we do not focus on the delivery risk that is discussed in Tang, Gurnani, and Gupta (2014).

A prerequisite for a contingent contract is that the uncertain demand should be contractable, which means the realized demand must be one that both cellular service provider and hostpots can observe and measure and that neither side can covertly manipulate. An increasingly important response to cost pressure in supply chains is the information sharing between retailers and suppliers (Aviv 2001). Emerging technologies, such as Electronic Data Interchange (EDI) and Radio Frequency Identification (RFID), facilitate sales data-sharing and make the design of contingent contracts more practical and reliable. In our problem settings, the cellular service provider can directly observe the demand information, but the hotspots cannot observe it. In this section, we show that the cellular service provider does not have incentive to misreport the private demand information. Therefore, the design of a procurement auction with contingent contracts is practical.[13]

From equation 3.4, the expected reduction of congestion cost for the cellular service provider after the procurement of hotspot bandwidth given the realization of the demand $\vec{x} \equiv (x_1, x_2, \cdots, x_M)$ is $U\left(\overline{Y}, \bar{x}\right) \equiv J(0) - M \cdot C_0(\bar{x} - \bar{X}_B - \overline{Y})$, where $\bar{x} = (x_1 + \cdots + x_M)/M$. $U(\overline{Y}, \bar{x})$ is also increasing and concave in $\overline{Y}$. When the cellular service provider observes the realized demand $\vec{x}$, it announces the demand information $\bar{x}$ to implement the mechanism which is quite similar to the non-contingent one. Assuming that inequality 3.3 always holds, the optimal mechanism $(P^*(\theta_i, \theta_{-i}, \bar{x}), Q^*(\theta_i, \theta_{-i}, \bar{x}))$ for the cellular service provider is summarized by the following proposition:

Proposition 2 *In the equilibrium of the procurement auction with contingent contracts, the cellular service provider announces the true demand $\bar{x}$ and implements the optimal mecha-*

[13] Sharing demand information with hotspots is a type of open book policy for a cellular service provider. The continuing interaction between a cellular service provider and hotspots makes contingent contracts more reasonable and attractive.

nism where $q_i = Q^*(\theta_i, \theta_{-i}, \bar{x})$, *for* $i = 1, 2, ...n$ *is given by:*

$$\frac{\partial}{\partial q_i} U\left(\frac{1}{M}\sum_{i=1}^{n} q_i, \bar{x}\right) = c_0\big(\bar{x} - \bar{X}_B - \frac{1}{M}\sum_{i=1}^{n} q_i\big) = c(q_i, \theta_i) + c_\theta(q_i, \theta_i)H(\theta_i), \tag{3.6}$$

and the optimal payment schedule $P_i = P_i^*(\theta_i, \theta_{-i}, \bar{x})$, *for* $i = 1, 2, ...n$ *is determined by:*

$$P_i = C(q_i, \theta_i) + \int_{\theta_i}^{\theta^*} C_\theta(q_i, \theta)\, d\theta.$$

The cellular service provider's expected gain from the procuement auction is

$$\mathbb{E}_{\bar{G}}\Big[\mathbb{E}\big[U\big(\frac{1}{M}\sum_{i=1}^{n} q_i, \bar{x}\big) - \sum_{i=1}^{n} C(q_i, \theta_i) - \sum_{i=1}^{n} C_\theta(q_i, \theta_i)H(\theta_i)\big]\Big].$$

This proposition shows that in equilibrium the cellular service provider will truthfully report the demand information. The intuition is that if the cellular service provider misreports the demand information, it distorts the bandwidth provision of WiFi hotspots and reduces the expected payoff of the cellular service provider. The optimal mechanism can be similarly implemented as the non-contingent mechanism except that the auction rule depends on the realized demand.

3.4 Integrating Global Auction and Local Auction

In Section 3.2, we have assumed that $y_m^* \geq 0$ for all m, or equivalently, the cellular capacity X_B is sufficiently large such that for all m and all possible realizations of cost parameters (θ_i, θ_{-i}) drawn from the distribution $F(\cdot)$, condition 3.3 is always satisfied:

$$\bar{X}_B \geq \big(Y_m - \bar{Y}\big) - \big(X_m - \bar{X}\big)$$

We call it the feasibility condition. Under a contingent procurement mechanism, the equilibrium quantity purchased in region m, $Y_m = \sum_{i \in \Psi_m} Q^*(\theta_i, \theta_{-i}, \bar{x})$, where Ψ_m is the set of hotspots in region m. Note that condition 3.3 may hold for some realizations of cost parameters (θ_i, θ_{-i}) but not for some others. Our feasibility condition requires that condition 3.3 holds for every realization of cost parameters (θ_i, θ_{-i}).

In this section, we introduce a modified contingent procurement mechanism to explore the optimal procurement mechanism when the feasibility assumption is relaxed. We start with a simple scenario with two WiFi regions (i.e., $M = 2$).

To gain some intuitions about the feasibility condition, we depict two illustrating examples in Figure 1. We assume that there are two WiFi regions ($M = 2$), and that each region has four hotspots ($n = 8$). The congestion cost functions for the service provider and WiFi hotspots are simple: $C_0(x) = 2x^2$, and $C(x, \theta_i) = \left(\frac{1}{2} + \theta_i\right) x^2$, where the private cost parameters for hotspots, θ_i, is drawn from a uniform distribution $U[0, 1]$ for 1,000 times. The data traffic for each region, X_m, $m = 1, 2$, is drawn from independent standard uniform distributions $U[0, 1]$ for 1,000 times. In the figure, The blue "X"s indicate that the feasibility condition is always satisfied when the demand is (X_1, X_2), the red dots indicate that condition 3.3 is violated for some realizations of cost parameters (θ_i, θ_{-i}) drawn from the distribution $F(\cdot)$, and the black stars indicate that condition 3.3 is violated for all possible realizations of cost parameters (θ_i, θ_{-i}) drawn from the distribution $F(\cdot)$.

When the feasibility condition is always satisfied (the blue $\times$), the optimal procurement mechanism is the global auction we discussed in Section 3.3. Although service from one hotspot is not a direct substitute for service from a different, far-away hotspot due to the fact that a hotspot can only serve customers who are physically nearby, the cellular service provider can make them indirectly substitutable by adjusting the allocation of cellular resources among regions. For example, even though a hotspot in region A cannot serve customers in region B, by serving customers in region A, it can free up some cellular capacity which can then be used to serve customers in region B. Thus, a single global auction to obtain bandwidth from all hotspots should outperform multiple local auctions which reduce competition among hotspots.

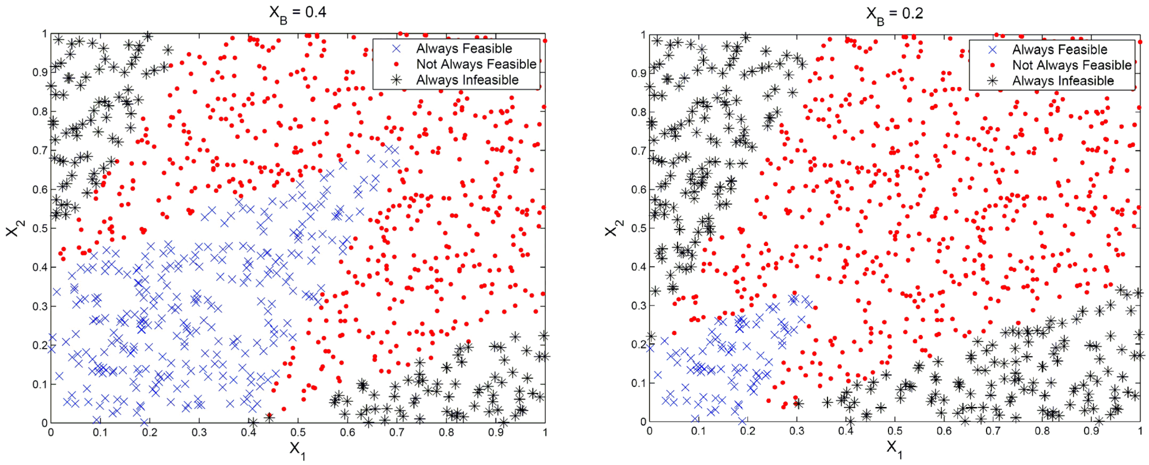

Figure 1: Illustrating Examples of the Feasibility Condition. The cellular capacity, X_B, is set to be 0.4 in the left panel and 0.2 in the right panel. The feasibility condition is more likely to be violated when the demands are unbalanced or X_B is small.

When the feasibility condition is always violated (the black stars), the marginal benefits of procuring WiFi capacity for the cellular service provider cannot be equalized across different regions because all cellular resources have been allocated to the region experiencing a surge in demand. In this case, a separate local auction for each region is optimal.

Our extended mechanism focuses on the non-trivial scenario (the red dots): the condition 3.3 is violated for some realizations of cost parameters (θ_i, θ_{-i}). We modify the original procurement mechanism by integrating local auctions with global auctions.

Let $\hat{y}_m$ be the optimal amount of cellular capacity allocated to region m without the nonnegativity constraint $y_m \geq 0$. Hence,

$$\begin{aligned}
\hat{y}_m &= \bar{X}_B + (x_m - \bar{x}) - (Y_m - \bar{Y}) \\
&= \bar{X}_B + (x_m - \bar{x}) - \left[\sum_{i \in \Psi_m} Q^* \left(\theta_i, \theta_{-i}, x_1, x_2\right) - \frac{1}{2} \sum_{i=1}^{n} Q^* \left(\theta_i, \theta_{-i}, x_1, x_2\right) \right],
\end{aligned}$$

where $Q^* \left(\theta_i, \theta_{-i}, x_1, x_2\right)$ is given by equation 3.6 when $M = 2$.

Define

$$\lambda_m(x_1, x_2) = \begin{cases} 0, \text{ if } \hat{y}_m < 0, \\ \hat{y}_m / X_B, \text{ if } 0 \leq \hat{y}_m \leq X_B, \\ 1, \text{ if } \hat{y}_m > X_B, \end{cases}$$

The optimal mechanism (P_i^{**}, q_i^{**}) for the cellular service provider is given by the following proposition:

Proposition 3 *Suppose $M = 2$. The optimal quantity function is given by*

$$q_i^{**} = Q^*(\theta_i, \theta_{-i}, x_1, x_2), i \in \Psi_m.$$

if $\lambda_m = \hat{y}_m / X_B$, q_i^{**}, and is given by

$$C_0'\Big(x_m - \lambda_m X_B - \sum_{i \in \Psi_m} q_i^{**}\Big) = c(q_i^{**}, \theta_i) + c_\theta(q_i^{**}, \theta_i) H(\theta_i), i \in \Psi_m \tag{3.7}$$

if $\lambda_m = 0$ or 1, q_i^{**}. The optimal payment schedule $P_i^{**}(\theta_i, \theta_{-i}, x_1, x_2)$, for $i = 1, 2, ...n$, is given by:

$$P_i^{**}(\theta_i, \theta_{-i}, x_1, x_2) = C(q_i^{**}, \theta_i) + \int_{\theta_i}^{\theta^*} C_\theta(q_i^{**}, \theta)\, d\theta. \tag{3.8}$$

The intuition is that when the feasibility condition is satisfied, the modified mechanism is equivalent to the optimal mechanism described in Proposition 2 which is essentially a *global* auction that includes all hotspots from different regions. When the feasibility condition is not satisfied, an optimal mechanism is to allocate all cellular capacity to one region, and then organize one *local* auction for each region. This integration of global auction and local auction in the optimal procurement auction is very interesting. It is the consequence of two unique features of procuring WiFi capacity for mobile traffic offloading: 1) the coupling of local auction because of the existence of the more flexible cellular capacity; 2) the heterogeneity of demand for mobile bandwidth and supply of WiFi capacity in different regions.

With more than two regions, the basic idea of integrating multiple local auctions and one global auction remains the same although the optimal pairing of the two become more involved. Mathematically, the problem of optimal cellular resource allocation is the following

$$\begin{aligned}
\min_{y_m \geq 0, Y_m} \quad & \sum_{m \in \{1, \cdots, M\}} C_0(x_m - Y_m - y_m) \\
s.t. \quad & \sum_m y_m = X_B.
\end{aligned}$$

where $Y_m = \sum_{i \in \Psi_m} Q^* (\theta_i, \theta_{-i}, \vec{x})$ is the optimal procurement quantity for region m. We denote by R_l those regions where the non-negativity constraints for y_m are binding and denote by R_g the regions where the non-negativity constraints for y_m are not binding. The optimal auction involves local auctions for regions in R_l (one auction for each region) and one global auction for regions in R_g. The key is to *optimally* divide the set of regions into R_g and R_l. This is non-trivial because whether y_m will be binding depends on the procured quantities $\{Y_1, \cdots, Y_M\}$ which in turn depend on the construction of R_g and R_l. A brute-force approach will result in combinatorial explosion with large number of regions. The main insight from our next result is that we can use a sequential procedure to construct R_g and R_l which takes linear time.

To describe the procedure, we first introduce the k-th stage congestion cost minimization problem. Let y_{mk} be the solution to the following optimization problem.

$$\begin{aligned}
\min_{y_{mk}, m \in R_g^k} \quad & \sum_{m \in R_g^k} C_0(x_m - Y_{mk} - y_{mk}) \\
s.t. \quad & \sum_{m \in R_g^k} y_{mk} = X_B.
\end{aligned}$$

where $Y_{mk} = \sum_{i \in \Psi_m} Q_k^* (\theta_i, \theta_{-i}, \vec{x})$ is the optimal procurement quantity for region $m \in R_g^k$, and $Q_k^* (\theta_i, \theta_{-i}, \vec{x})$ is given by equation 3.6 with $i \in \cup_{j \in R_g^k} \Psi_j$ and $\bar{x}$ is the average demand across regions in R_g^k. Let $R_+^k \equiv \{m \in R_g^k | y_{mk} \geq 0\}$, and $R_-^k \equiv \{m \in R_g^k | y_{mk} < 0\}$

Proposition 4 *Suppose $M \geq 2$. The optimal quantity schedule q_i^{**} is given by*

$$\begin{aligned} c_0\Big(\frac{1}{|R_g|}\sum_{m\in R_g} x_m - \bar{X}_B - \frac{1}{|R_g|}\sum_{i\in\Psi_m, m\in R_g} q_i^{**}\Big) &= c(q_i^{**},\theta_i) + c_\theta(q_i^{**},\theta_i)H(\theta_i), \forall i \in \Psi_m, m \in R_g \\ C_0'\big(x_m - \sum_{i\in\Psi_m} q_i^{**}\big) &= c\left(q_i^{**},\theta_i\right) + c_\theta(q_i^{**},\theta_i)H(\theta_i), \forall i \in \Psi_m, m \in R_l \end{aligned}$$

where R_g and R_l is constructed through the following iterative procedure:

- *Step 0: Let $k = M$, $R_g^M = \{1, 2, \cdots, M\}$, and $R_l^M = \emptyset$.*
- *Step 1: If $R_-^k = \emptyset$, let $R_g = R_g^k$ and $R_l = R_l^k$. Stop the procedure.*
- *Step 2: If $R_-^k \neq \emptyset$, let $R_g^{k-1} = R_+^k$ and $R_l^{k-1} = R_l^k \cup R_-^k$. Decrease k by 1 and go back to Step 1.*

The optimal payment schedule P_i^{**}, for $i = 1, 2, ...n$ is given by:

$$P_i^{**} = C\left(q_i^{**},\theta_i\right) + \int_{\theta_i}^{\theta^*} C_\theta\left(q_i^{**},\theta\right)d\theta. \tag{3.9}$$

4 Simulation

Applying our model to the network data from one of the largest U.S. service providers, we address the following question in this section: As compared with the standard VCG auction, how much can our optimal procurement auction improve the cellular network's expected payoff? The Monte Carlo simulation results demonstrate that, as compared with the standard VCG auction, our contingent procurement auction significantly improves the cellular network's expected payoff. We also evaluate the impact of the cellular capacity and the relative cost of deploying cellular resources on the performance difference between these two mechanisms.

Before we do the comparison, we will first review the multi-unit VCG auction for procurement in our context. The following list describes the VCG procurement auction:

- Invite each hotspot to report its cost parameter θ. Denote the submitted cost parameters as $\{\theta_1, \theta_2, \cdots, \theta_n\}$.

- Under the VCG mechanism, the socially efficient allocation minimizes the sum of the expected congestion cost of the cellular service provider and the cost of hotspots. According to equation 3.2, we have the sum of the expected congestion cost, and the minimization problem is formalized as follows:

$$\begin{aligned}
\min_{q_1, q_2, \ldots, q_k} \quad & M\mathbb{E}_G\left[C_0(\bar{X} - \bar{X}_B - \bar{Y})\right] + \sum_{i=1}^{n} C(q_i, \theta_i) \\
s.t.\ q_i \quad & \geq \quad 0, \ \text{for } i = 1, 2, ..., n, \\
\bar{Y} \quad & = \quad \frac{1}{M}\sum_{i=1}^{M} Y_i = \frac{1}{M}\sum_{i=1}^{M} q_i.
\end{aligned}$$

- Let $\pi\left(\theta_1, \theta_2, \cdots, \theta_k\right)$ be the optimal value of the objective function, and let $(q_1^*, q_2^*, \cdots, q_n^*)$ be an optimal solution to the cost minimization problem. Let $\pi_{-i}\left(\theta_{-i}\right)$ be the optimal value of the objective function with the additional constraint $q_i = 0$ (i.e., hotspot i does not participate in the auction).

- The cellular service provider will pay hotspot i according to the following:

$$P_i = \pi_{-i}\left(\theta_{-i}\right) - \pi\left(\theta_1, \theta_2, \cdots, \theta_n\right) + C(q_i^*, \theta_i) \tag{4.1}$$

  where $\pi_{-i}\left(\theta_{-i}\right) - \pi\left(\theta_1, \theta_2, \cdots, \theta_n\right)$ is the bonus payment to hotspot i, representing the positive externality that hotspot i is imposing on the cost minimization problem. The cellular service provider pays hotspot i its cost $C(q_i^*, \theta_i)$, plus its contribution to the cost minimization problem. This payment internalizes the externality.

- Hotspot i provides capacity q_i^* and receives payment P_i.

Note that the VCG auction is both truth-telling and socially efficient by standard arguments. All hotspots bid their cost parameters truthfully, irrespective of other hotspots' bids. The VCG mechanism guarantees the minimum total cost. However, it leads to an overpayment to hotspots that is shown in the simulation.[14]

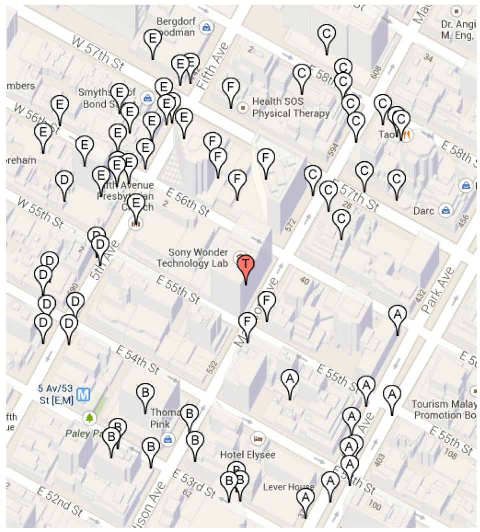

Figure 2: Area Map of A Typical Cell Sector

In our simulations, we consider a typical urban neighborhood in New York City, as shown in Figure 2. We define a cell sector as the range of the cell tower. Our dataset consists of the location information of 14,576 cell towers from a large cellular provider in the U.S. In our simulation study, we pick a cell tower in New York City from the full list of cell towers and simulate the mobile data demand in this sector. In Figure 2, the cell tower is represented by the marker labelled with the letter "T", and the 69 WiFi hotspots in the given cell sector are represented by other markers.[15] Following Dong et al. (2012), we set the communication range for a cell tower as 250m, and set the communication range for Wi-Fi as 100m. The following steps describe the procedure of simulations:

- Generating traffic demands in the given cell sector: To gain a sense of the population density in the coverage area of the cell tower, we use 2010 census data, which contains the land area coverage and population density of each zip code. Combining the market

[14]Note that this VCG mechanism is not contingent on the realized demand. We also simulate the performance of a contingent VCG mechanism. The basic results of performance comparison remain unchanged.
[15]Locations of commercial WiFi hotspots are from http://wigle.net.

share of this service provider for the first quarter 2013[16], we estimate the number of users in the given cell sector. On average, smartphone users consume about 1GB data per month, but the usage patterns of mobile data is highly uneven.[17] Paul et al. (2011) and Jin et al. (2012) found that a small number of heavy users contribute to a majority of data usage in the network. To consider the heterogeneity of data usage and the effects of peak hours, we simulate individual data usage from the byte distribution in Jin et al. (2012).[18]

- Generating WiFi regions in the cell sector: Dong et al. (2012) showed that the appropriate number of WiFi regions in a cell sector is six. Following their approach, we generate six WiFi regions by clustering the WiFi hotspots using k-means. In Figure 2, Region A, Region B, ... , and Region F indicate which region the WiFi hotspots belong to.

- Generating traffic demands in each WiFi region: We use two different methods to place users in the cell sector and assign them to the corresponding WiFi regions according to their locations. (1) All users are randomly placed in the cell sector. (2) All users are placed according to the densities of the hotspots.[19] After placing all the users, a nearest hotspot is calculated for each user location. If the distance between the nearest hotspot found and the user location is less than the hotspot range (100m), the user is counted as one of the regional population according to the WiFi region; otherwise, the user is considered as in the region with no hotspots (region 0). We run 1,000

[16]See http://www.talkandroid.com/159929-t-mobile-loses-market-share-while-verizon-and-att-continue-to-dominate.

[17]See http://www.fiercewireless.com/special-reports/average-android-ios-smartphone-data-use-across-tier-1-wireless-carriers-thr-1#ixzz2ZSpDoS5Z.

[18]We obtain the quantiles of the byte distribution from Jin et al. (2012) and generate inidvidual usage using the Johnson System. We also adjust the usage by considering the effect of peak hours, see http://chitika.com/browsing-activity-by-hour.

[19]To calculate the densities of the hotspots for different locations, we divide the square circumscribing the cell sector into a 20 by 20 array of grids. By default, each grid has a weight of 1, except the grids whose centers are not in the range of the tower. The grid's weight is increased by the number of hotspots whose locations are inside the grid. Then, a list of grid indices is created according to the weight of each grid. Finally, for each user, a grid index is first uniformly chosen from the list, and then the location of the user is uniformly chosen from the range of the grid with the grid index just picked.

simulations to generate traffic demands in each WiFi region.

- Generating cell tower capacity: The cell tower capacity is set to three carriers, that is, three times 3.84 MHz (Dong et al. 2012). Data spectral efficiency varies across towers from 0.5 to 2 bps/Hz.[20] We set spectral efficiency to be 1 by default and then vary the spectral efficiency to evaluate its impact. Note that when the user demand for mobile data is below 80% of the cell tower capacity, the cellular service provider faces no congestion cost.

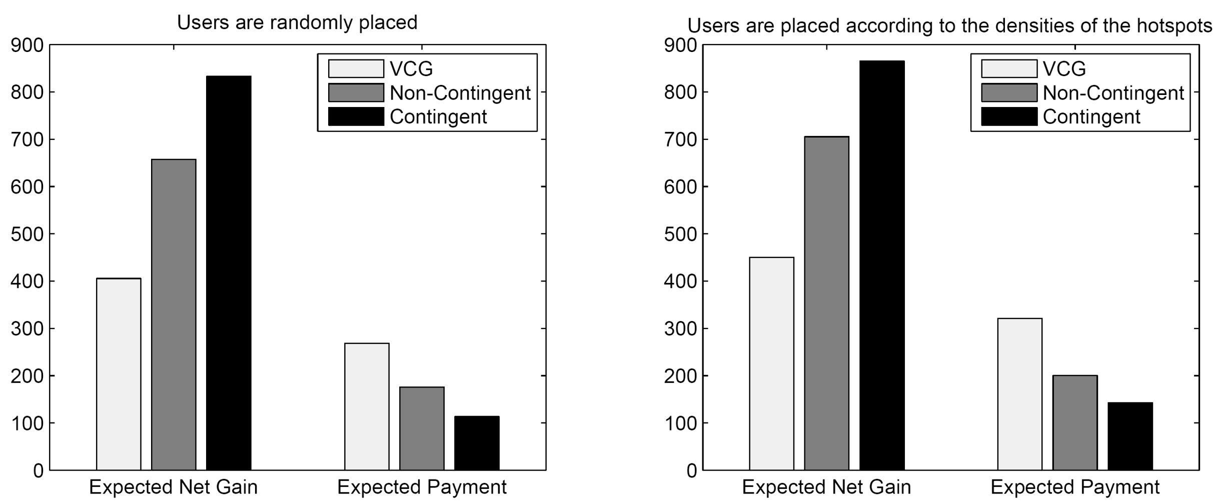

Figure 3: Performance Comparison of the Procurement Mechanisms for the Service Provider

Using the algorithms in Section 3.2 and Section 3.4, we conduct a variety of simulations to compute the corresponding allocation under the VCG mechanism, the non-contingent procurement auction described in Section 3.2, and our contingent procurement auction (CPA). The relative cost of deploying cellular resources as compared with WiFi resources affects the bandwidth allocation result. Dong et al. (2012) assumed that spectrum cost is always higher than WiFi and that WiFi is always preferred when the cellular service provider is overloaded. Joseph et al. (2004) assumed that the relative cost of deploying cellular resources as compared with WiFi resources is 4:1. We follow their assumptions and set the parameter values:

[20] See http://www.rysavy.com/Articles/2011_05_Rysavy_Efficient_Use_Spectrum.pdf

$C_0(x) = 0.5 \cdot ax^2$, and $C(x, \theta_i) = (0.5 + \theta_i)\, x^2$, where $a = 4$, by default. In the simulation, we vary a to evaluate its impact. A hotspot's private cost parameters θ_i is drawn from a standard uniform distribution $U[0, 1]$ for 1,000 times.

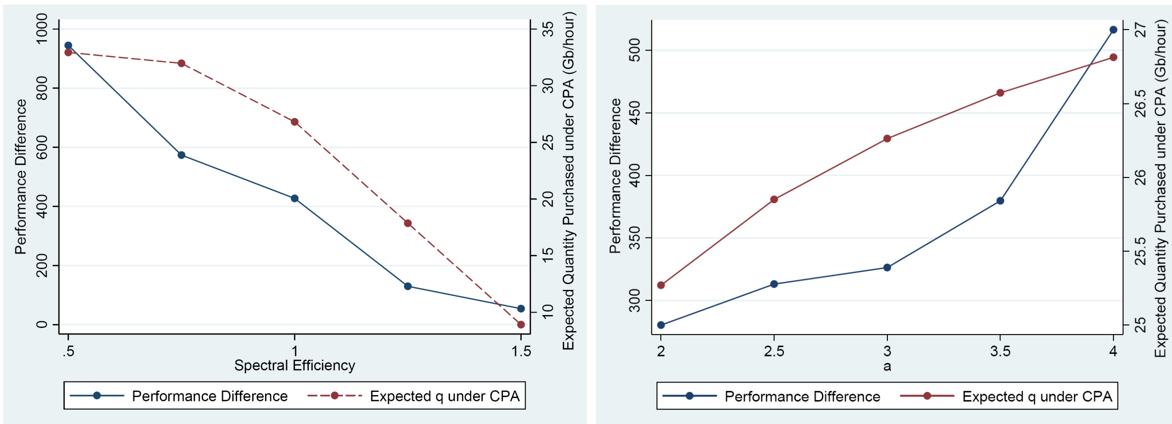

Figure 4: Performance Difference and Cell Tower Capacity (Left); Performance Difference and Relative Cost of Deploying Cellular Resources (Right)

The simulation result of the performance comparison is shown in Figure 3. In the left panel, the users are randomly placed in the cell sector. In the right panel, the users are placed according to the densities of the hotspots. The two panels show similar results: our non-contingent procurement auction significantly outperforms the VCG mechanism in terms of the expected net gain of the cellular service provider (the expected net gain = the reduction of congestion cost - the payment to hotspots). The contingent arrangements can further improve the expected gain of the cellular service provider. Note that both of the panels suggest that the VCG mechanism leads to an overpayment to hotspots. Our contingent mechanism reduces procurement cost by 57.7% in the left panel and by 55.4% in the right panel compared to the VCG mechanism.

Data spectral efficiency varies across cell towers using different wireless technologies. An increase in spectral efficiency significantly contributes to tower capacity (Dong et al. 2012). The left panel of Figure 4 evaluates the impact of spectral efficiency (cell tower capacity) on the performance difference, which is defined as the difference between the service provider's expected net gain under the proposed CPA system and the gain under the VCG mechanism.[21] Note that the unit of the performance difference is normalized, and we are only interested in the trend. We find that as the cellular capacity increases, the advantage of our CPA system, in comparison with the VCG mechanism, decreases. This is because the bandwidth purchased from the WiFi hotspots also decreases with the cellular capacity (see the dashed line in the left panel of Figure 4). The service provider is less willing to purchase WiFi resources when it owns a relatively large cellular capacity, and the overpayment problem in the VCG mechanism is thus less detrimental to the service provider's expected gain. This simulation result suggests that the proposed CPA system is particularly useful when the cell tower capacity is relatively small.

We also vary the relative cost of deploying cellular resources as compared with WiFi resources to evaluate its impact. The right panel of Figure 4 shows that as the relative cost parameter a increases, the advantage of our CPA system as compared with the VCG mechanism increases. When the relative cost of deploying cellular resources is high, the service provider is more willing to procure from the WiFi hotspots, which exacerbates the overpayment problem in the VCG mechanism. Therefore, the advantage of our CPA system increases with the relative cost parameter a.

5 Conclusion and Discussion

In the present study, we designed an optimal procurement auction with contingent contracts for mobile data offloading. The integration of both cellular and WiFi resources significantly improves mobile bandwidth availability. A unique challenge in this procurement auction is that the longer-range cellular resource introduces coupling between the shorter range WiFi hotspots. We solved for the optimal auction mechanism and provide computational methods for the corresponding contingent contract. The simulation results showed that our

[21] The simulation results are similar when the users are randomly placed or are placed according to the densities of the hotspots, so here we only present the result when the users are randomly placed.

procurement auction significantly outperforms the standard VCG auction.

The actual auctions and offloading to WiFi would need to be integrated with the policy management infrastructure, which is able to supply some of the key variables in the auction valuation: (1) the currently offered data traffic, (2) the capacity of each cell tower, and (3) the congestion cost when offered traffic exceeds capacity (e.g., in terms of rejected sessions or excessive delay). This procurement auction relies on automation technology and becomes a type of information systems: completely integrate all relevant information into the supply chain through wireless networks. Our procurement mechanism extends beyond the limits of service providers' cellular resource to interconnect multiple hotspots in different regions by allowing for real-time and accurate data sensing. This leads to a more precise monitoring and control of mobile data offloading. The conventional data offloading is on the basis of the access network discovery and selection function (ANDSF)[22] that processes static WiFi offload policies. Recently, the intelligent mobile solution company, Tekelec, Inc., has developed its Mobile Policy Gateway (MPG)[23] to implement complex WiFi offload policies. The Tekelec MPG enables support for our smart data offloading based on the real-time auction approach.

In the real-time procurement auctions, fast computation of the corresponding contingent contract is critical to ensure the cellular network's expected gain. Recent advances in real-time database technology, such as Spark,[24] makes it possible to compute and implement a huge number of contingent contracts — a task that was once considered computationally prohibitive.

Even though our procurement mechanism was a static model, it can apply to dynamic real-world settings by using a real-time auction. In a dynamic model, we assume that the cost parameter of hotspot i at time t, θ_{it}, is drawn from a distribution with a cumulative

[22]The purpose of the ANDSF is to assist user equipment to discover and select non-3GPP networks such as WiFi and WiMax.

[23]See http://www.tekelec.com/2012-press-releases/tekelec-and-roke-partner-to-deliver-policyonthemobile-solutions.aspx.

[24]Spark is an open source cluster computing system that aims to make data analytics fast. It provides primitives for in-memory cluster computing: Data can be loaded into memory and be queried repeatedly much more quickly than with disk-based systems, like RDBMS and Hadoop/MapReduce.

distribution function $F_t(\cdot)$. If t' denotes peak hours and t'' denotes off-peak hours, we have $F_{t'}(\cdot)$ fist-order stochastically dominates $F_{t''}(\cdot)$. The process flow for a dynamic model is shown in Figure 5. Step 1 computes the optimal mechanism including the optimal payment schedule, $P^*(\theta_i, \theta_{-i}, x_1, x_2, \cdots, x_M)$, and the optimal bandwidth allocation schedule, $Q\left(\theta_i, \theta_{-i}, x_1, x_2, \cdots, x_M\right)$, according to Proposition 2. We call Step 1 the pre-computing stage. After data traffic is generated at time t, an auction system automatically bids for hotspots given θ_{it}, which may depend on the instantaneous bandwidth demand a hotspot faces from its own users. The functional forms are specified by hotspots in advance, but the value of θ_{it} varies over time. Our system finds the corresponding contingent contract: $P^*(\theta_{it}, \theta_{-it}, x_1, x_2, \cdots, x_M)$ and $Q\left(\theta_{it}, \theta_{-it}, x_1, x_2, \cdots, x_M\right)$ given the data traffic at time t, $x_t = (x_{1t}, x_{2t}, \cdots, x_{Mt})$, and the auction results are immediately executed. We call Step 2 - Step 4 the real-time auction stage. Like the display advertising auctions (McAfee, 2011), speed is of the essence in our real-time procurement auction, because the slow process of showing the auction results would sacrifice the cellular service provider's profit. Bichler, Gupta, and Ketter (2010) also addressed the need for real-time intelligence in dynamic markets. At time $t+1$, we repeat the real time stage and show the corresponding auction results when the data traffic is $x_{t+1} = (x_{1t+1}, x_{2t+1}, \cdots, x_{Mt+1})$.

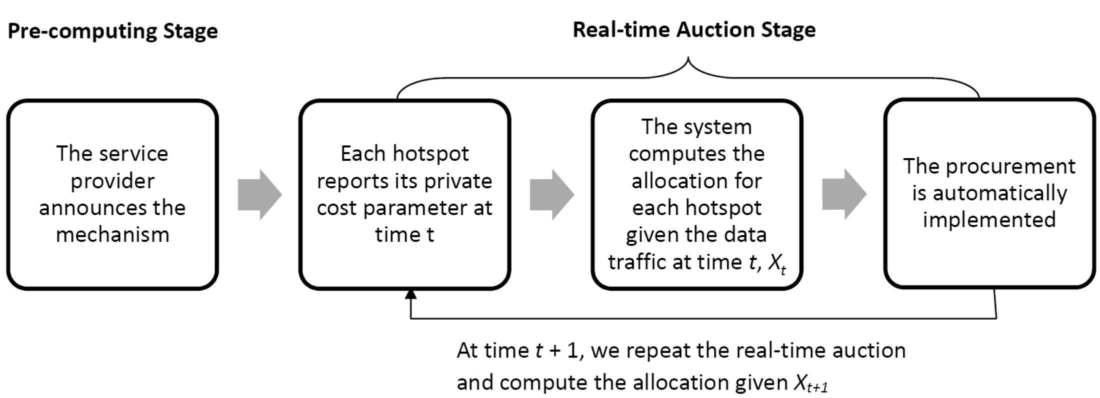

Figure 5: The Process Flow for the Automated Auction System

The model in the present study can also be useful for a general supply chain problem.

The independent management of procuring multiple products could be inefficient in the presence of limited product-flexible capacity (Demirel 2012). Van Mieghem and Rudi (2002) studied newsvendor networks allowing for multiple products. In our theoretical model, the wireless service in different WiFi regions can be thought of as different products in the supply chain problem. When we consider the procurement of third party WiFi capacity, the service provider owns the cellular capacity that can serve traffic in all WiFi regions, whereas each WiFi hotspot can only serve local traffic. Consider a firm that produces multiple products using a shared resource (in-house capacity) that is common to products 1 and 2. Because of capacity limitations, the firm also may need to procure the products from different suppliers. In this example, suppliers 1 only produces product 1; suppliers 2, 3, and 4 only produce product 2. Because the in-house capacity is a shared resource that can be used for all products, we cannot decompose this supply chain problem into two independent procurement problems. Our theoretical model provides an auction framework for the downstream firm to optimally integrate the upstream capacity with its own product-flexible capacity.

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A Online Appendix

A.1 The Procedure of Computing the Optimal Procurement Auction

- Invite each of the n hotspots to report its cost parameter θ. Denote the submitted cost parameters as $\{\theta_1, \theta_2, \cdots, \theta_n\}$.
- Define the map $q : \Theta^n \to R^n$ as follows:
  - For each $i = 1, 2, \cdots, n$ and $x \geq 0$, let $\phi_i(x)$ be the implicit function satisfying the following equation $c(\phi_i(x), \theta_i) + c_\theta(\phi_i(x), \theta_i)H(\theta_i) = x$. Because the left-hand-side of the equation is increasing in $\phi_i(x)$, given a value of x, $\phi_i(x)$ can be easily solved using bisection in the interval $[0, \bar{q}_i]$ where $\bar{q}_i$ is a positive number large enough so that the value of left-hand-side exceeds x.
  - From equation 3.4, $V'(q)$ can be written as

$$V'(q) = \int_{\bar{Y}}^{1} c_0(\bar{X} - \bar{X}_B - \bar{Y})d\bar{G}(\bar{X}) = \int_{q/M}^{1} c_0(\bar{X} - q/M)d\bar{G}(\bar{X}).$$

    Let q^* be the solution to the following equation: $\sum_{i=1}^{n} \phi_i\left(V'(q)\right) = q$. Again, because the left-hand-side is decreasing in q, we can easily solve for q^* using bisection in the interval $[0, M]$.[25]
  - Let $\vec{q} \equiv (q_1, q_2, \cdots, q_n) \equiv (\phi_1(V'(q^*)), \phi_2(V'(q^*)), \cdots, \phi_m(V'(q^*)))$.
- Define payment plan P_i as

$$P_i \equiv P_i(\theta_1, \cdots, \theta_n) \equiv C(q_i, \theta_i) + \int_{\theta_i}^{\theta^*} C_\theta(q_i(\theta, \theta_{-i}), \theta)d\theta,$$

  where θ^* is a threshold cost parameter to be determined.

[25] When $q = 0$, the left-hand-side is positive. When $q = M$, the left-hand-side is nonpositive. More generally, q^* can be found in the interval $[0, M\bar{\bar{X}}]$ where $\bar{\bar{X}}$ is the upper bound of $\bar{X}$.

- Hotspot i will provide capacity q_i and receive payment P_i.

- The expected gain of the cellular service provider before the auction is

$$W(\theta^*) = \mathbb{E}\left[V(\frac{q^*}{M}) - \sum_{i=1}^{n} C(q_i, \theta_i) - \sum_{i=1}^{n} C_\theta(q_i, \theta_i)H(\theta_i)\right]$$

- The optimal procurement auction can be obtained by searching over $[\underline{\theta}, \bar{\theta}]$ for the optimal threshold value θ^* that yields the highest value of $W(\theta^*)$.

A.2 Proof of Proposition 1

Proof. The proof below is adapted from the proof of Proposition 5 in Dasgupta and Spulber (1989). The expected profit of a hotspot provider with cost parameter θ_i reporting parameter θ' is

$$\pi(\theta', \theta_i) = \mathbb{E}_{-i}\big[P(\theta', \theta_{-i}) - C\big(Q(\theta', \theta_{-i}), \theta_i\big)\big].$$

Define $\pi(\theta) = \pi(\theta, \theta)$. Incentive compatibility implies that

$$\pi(\theta, \theta) - \pi(\theta, \theta') \geq \pi(\theta, \theta) - \pi(\theta', \theta') \geq \pi(\theta', \theta) - \pi(\theta', \theta')$$

or equivalently,

$$\mathbb{E}_{-i}\big[C\big(Q(\theta, \theta_{-i}), \theta_i'\big) - C\big(Q(\theta, \theta_{-i}), \theta_i\big)\big] \geq \pi(\theta, \theta) - \pi(\theta', \theta') \geq \mathbb{E}_{-i}\big[C\big(Q(\theta', \theta_{-i}), \theta_i'\big) - C\big(Q(\theta', \theta_{-i}), \theta_i\big)\big].$$

Dividing both sides by $\theta - \theta'$ and taking limits as $\theta' \to \theta$, we have

$$\frac{d\pi(\theta)}{d\theta} = -\mathbb{E}_{-i}\big[C_\theta\big(Q(\theta, \theta_{-i}), \theta\big)\big].$$

Integrating both sides from θ_i to θ^* and using the fact that $\pi(\theta^*) = 0$, we have

$$\pi(\theta_i) = \int_{\theta_i}^{\theta^*} \mathbb{E}_{-i}\big[C_\theta\big(Q(\theta, \theta_{-i}), \theta\big)\big] d\theta = \mathbb{E}_{-i}\left[\int_{\theta_i}^{\theta^*} C_\theta\big(Q(\theta, \theta_{-i}), \theta\big) d\theta\right]$$

Hence, the expected payment a hotspot provider with cost parameter θ_i will receive is

$$\mathbb{E}_{-i}[P(\theta_i, \theta_{-i})] = \mathbb{E}_{-i}\left[C\big(Q(\theta_i, \theta_{-i}), \theta_i\big) + \int_{\theta_i}^{\theta^*} C_\theta\big(Q(\theta, \theta_{-i}), \theta\big) d\theta\right].$$

From the buyer's perspective, the expected payment to any hotspot provider is

$$\begin{aligned}
\mathbb{E}_i\big[\mathbb{E}_{-i}[P(\theta_i, \theta_{-i})]\big] &= \mathbb{E}\big[C\big(Q(\theta_i, \theta_{-i}), \theta_i\big)\big] &+& \int_{\underline{\theta}}^{\theta^*} \left(\int_{\theta_i}^{\theta^*} \mathbb{E}_{-i}\big[C_\theta\big(Q(\theta, \theta_{-i}), \theta\big)\big] d\theta\right) dF(\theta_i) \\
&= \mathbb{E}\big[C\big(Q(\theta_i, \theta_{-i}), \theta_i\big)\big] &+& F(\theta_i) \int_{\theta_i}^{\theta^*} \mathbb{E}_{-i}\big[C_\theta\big(Q(\theta, \theta_{-i}), \theta\big)\big] d\theta \bigg|_{\underline{\theta}}^{\theta^*} \\
& &+& \int_{\underline{\theta}}^{\theta^*} F(\theta_i)\mathbb{E}_{-i}\big[C_\theta\big(Q(\theta_i, \theta_{-i}), \theta_i\big)\big] d\theta_i \\
&= \mathbb{E}\big[C\big(Q(\theta_i, \theta_{-i}), \theta_i\big)\big] &+& \int_{\underline{\theta}}^{\theta^*} \mathbb{E}_{-i}\big[F(\theta_i) C_\theta\big(Q(\theta_i, \theta_{-i}), \theta_i\big)\big] d\theta_i \\
&= \mathbb{E}\big[C\big(Q(\theta_i, \theta_{-i}), \theta_i\big) &+& C_\theta\big(Q(\theta_i, \theta_{-i}), \theta_i\big) H(\theta_i)\big]
\end{aligned}$$

where $H(\theta) \equiv F(\theta)/F'(\theta)$.

Let $Q_i(\vec{\theta}) \equiv Q(\theta_i, \theta_{-i})$. The cellular service provider's optimization problem can be written as the following convex optimization problem:

$$\max_{Q_i(\vec{\theta}), i=1,\cdots,n} \Pi = \mathbb{E}\left[V\left(\sum_{i=1}^n Q_i(\vec{\theta})\right) - \sum_{i=1}^n C(Q_i(\vec{\theta}), \theta_i) - \sum_{i=1}^n C_\theta(Q_i(\vec{\theta}), \theta_i) H(\theta_i)\right].$$

Clearly, the "virtual" marginal costs must be equalized across hotspots at the optimal:

$$V'\left(\sum_{i=1}^n Q_i(\vec{\theta})\right) = c(Q_i(\vec{\theta}), \theta_i) + c_\theta(Q_i(\vec{\theta}), \theta_i) H(\theta_i).$$

for $i = 1, 2, ...n$, which determines the optimal quantity functions $Q_i(\vec{\theta})$, $i = 1, \cdots, n$. Because $c_\theta \geq 0$, $c_{\theta\theta} > 0$, and $H(\theta)$ is increasing in θ, it is straightforward to see that $Q(\theta_i, \theta_i)$ is decreasing in θ_i. ■

A.3 Proof of Proposition 2

Proof. Notice that announcing a demand vector other than the true demand vector in the mechanism will only result in a possible deviation of procured quantity from the optimal one. Clearly, the cellular service provider cannot benefit from such misreport because its valuation function remains unchanged regardless of what demand information it announces, after all, it has to meet the true demand. Hence, the proof is a straightforward extension of the proof of Proposition 1 with the mechanism contingent on the realized demand summarized in $\bar{x}$.
■

A.4 Proof of Proposition 3

Proof. Following an argument similar to that in the proof of Proposition 1, it is easy to see that incentive compatibility is ensured by the payment schedule. The revelation principle implies that we only need to find the quantity schedule $Q(\theta_i, \theta_{-i})$ that maximizes the service provider's gain from the procurement auction. To this end, we divide the space of Θ into two components,

$$\Theta_1 \equiv \{\vec{\theta} = (\theta_1, \theta_2) \big| 0 \leq \hat{y}_m \leq X_B\}, \Theta_2 \equiv \{\vec{\theta} = (\theta_1, \theta_2) \big| \hat{y}_m < 0 \text{ or } \hat{y}_m > X_B\}.$$

Suppose $\vec{\theta} \in \Theta_1$. We have: $\lambda_m X_B = \hat{y}_m = (x_m - \bar{x}) - (Y_m - \bar{Y}) + \bar{X}_B$, in which case the quantity and the payment are exactly the same as those discussed in Proposition 2. Because the mechanism described in Proposition 2 is optimal for the cellular service provider when $0 \leq \hat{y}_m \leq X_B$, the proposed mechanism is also optimal when $0 \leq \hat{y}_m \leq X_B$.

Suppose $\vec{\theta} \in \Theta_2$. By definition of Θ_2 and $\hat{y}_m$, with procured WiFi bandwidth Y_1 and Y_2 and demand realization x_1 and x_2, the solution to the following congestion cost minimization problem is a corner solution with either $(y_1^*, y_2^*) = (X_B, 0)$, or $(y_1^*, y_2^*) = (0, X_B)$.

$$\begin{aligned} \min_{y_1, y_2} \quad & \sum_{m=1}^{2} C_0(x_m - Y_m - y_m) \\ s.t. \quad & \sum_{m=1}^{M} y_m = X_B, y_m \geq 0. \end{aligned}$$

Hence, the value of procuring (Y_1, Y_2) from the two regions is

$$\begin{aligned} V(Y_1, Y_2) = \mathbb{E}\Bigg[J(0) \quad & - \quad C_0(x_1 - Y_1 - X_B) - C_0(x_2 - Y_2) \\ & - \quad \sum_{i=1}^{n} C(Q_i(\vec{\theta}), \theta_i) - \sum_{i=1}^{n} C_\theta(Q_i(\vec{\theta}), \theta_i) H(\theta_i) \Bigg| \theta \in \Theta_2 \Bigg] \end{aligned}$$

which is exactly the total values of the cellular service provider when it organizes two separate local auctions with $\lambda_1 = 1$ and $\lambda_2 = 0$. Therefore, with $\theta \in \Theta_2$, the optimal mechanism is to allocate all cellular resource to one region and then organize two separate local auctions. ■

A.5 Proof of Proposition 4

Proof. The key is to show that the order of picking out regions for local auction does not matter when we construct the optimal R_g and R_l. Mathematically, we want to show that if $m \in R_k^-$ and $R_g^{k-1} = R_g^k \backslash \{s\}$, where $s \in R_k^-$, then $m \notin R_{k-1}^+$.

If $s = m$, it is trivial to show that $m \notin R_{k-1}^+$. Thus, we focus on the non-trivial case: $s \in R_k^-$ and $s \neq m$. Without loss of generality, we set s to be k for the notational simplicity. Consider a k-region global auction. The optimal allocation schedule of WiFi capacity, $q_i^k = Q^*(\theta_i, \theta_{-i}, \bar{x}_k)$, is given by:

$$C_0'(\bar{x}_k - X_B/k - \bar{Y}_k) = c(q_i^k, \theta_i) + c_\theta(q_i^k, \theta_i) H(\theta_i), \text{ for all } i \in \cup_{m=1}^k \Psi_m, \tag{A.1}$$

where $\bar{x}_k = \frac{x_1 + x_2 + \cdots + x_k}{k}$, and $\bar{Y}_k = \frac{1}{k} \sum_{i \in \cup_{m=1}^k \Psi_m} q_i^k$. In a $(k-1)$ -region global auction, q_i^{k-1}, is given by the following equation:

$$C_0'\left(\bar{x}_{k-1} - X_B/\left(k-1\right) - \bar{Y}_{k-1}\right) = c(q_i^{k-1}, \theta_i) + c_\theta(q_i^{k-1}, \theta_i)H(\theta_i), \text{ for all } i \in \cup_{m \in R_g^{k-1}} \Psi_m, \tag{A.2}$$

where $\bar{x}_{k-1} = \frac{x_1+x_2+\cdots+x_{k-1}}{k-1}$, and $\bar{Y}_{k-1} = \frac{1}{k-1}\sum_{i\in\cup_{m\in R_g^{k-1}}\Psi_m} q_i^{k-1}$. We show that there exists $i \in \cup_{m\in R_g^{k-1}}\Psi_m$, such that $q_i^{k-1} \geq q_i^k$, by contradiction. Suppose that $q_i^{k-1} < q_i^k$ for all $i \in \cup_{m\in R_g^{k-1}}\Psi_m$. By equation A.1 and A.2, $\bar{x}_{k-1} - X_B/\left(k-1\right) - \bar{Y}_{k-1} < \bar{x}_k - X_B/k - \bar{Y}_k$. Let the optimal allocation of cellular capacity across regions in a $(k-1)$-region global auction be $y_{m,k-1}$, and we have

$$\begin{aligned} y_{m,k-1} &= X_B/\left(k-1\right) + \left(x_m - \bar{x}_{k-1}\right) - \left(Y_{m,k-1} - \bar{Y}_{k-1}\right) \\ &> X_B/k + \left(x_m - \bar{x}_k\right) - \left(Y_{mk} - \bar{Y}_k\right) = y_{mk}, \text{ for } m = 1, 2, ..., k-1. \end{aligned}$$

where $Y_{mk} = \sum_{i\in\Psi m} q_i^k$, and $Y_{m,k-1} = \sum_{i\in\Psi_m} q_i^{k-1}$. Because $\sum_{m=1}^{k} y_{mk} = X_B$, and $y_{ks} = y_{kk} < 0$, $\sum_{m=1}^{k-1} y_{m,k-1} > \sum_{m=1}^{k-1} y_{mk} > X_B$, which contradicts $\sum_{m=1}^{k-1} y_{m,k-1} = X_B$ in a global auction with $k-1$ regions. Therefore, there exists $i \in \cup_{m\in R_g^{k-1}}\Psi_m$, such that $q_i^{k-1} \geq q_i^k$. By equation A.1 and A.2, $\bar{x}_{k-1} - X_B/\left(k-1\right) - \bar{Y}_{k-1} \geq \bar{x}_k - X_B/k - \bar{Y}_k$, and then we can obtain that $y_{m,k-1} \leq y_{mk}$ for all $m = 1, 2, ..., k-1$. In other words, if $y_{mk} < 0$, then $y_{m,k-1} < 0$. Therefore, the order of elimination does not matter. ■