Vertical Relationships within Platform Marketplaces

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Abstract

In two-sided markets a platform allows consumers and sellers to interact by creating sub-markets within the platform marketplace. For example, Amazon has sub-markets for all the products sold on its site and smartphones have sub-markets for different types of apps. Using micro foundations, consumer demand for products sold in these sub-markets is developed. This generates the network effects between consumers and sellers. These network effects depend on the mode of competition within sub-markets: more competition between sellers lowers product prices. This increases the surplus consumers receive from a sub-market and makes platform membership more desirable for consumers. However, more competition also lowers profits for a seller which makes platform membership less desirable for a seller. This dynamic between seller competition within a sub-market and agent network benefits leads to platform pricing strategies, participation decisions by consumers and sellers, and welfare results that depend on the mode of competition. Thus, the sub-market structure is important when investigating platform marketplaces.

Keywords: platforms and two-sided markets, platform sub-markets, platform marketplaces, online marketplaces, digital marketplaces, network effects.
JEL Classifications: L42, L14, D40.

1 Introduction

Over the last ten years smartphones have become ubiquitous. In 2013, the smartphone market reached 1 billion total sales to consumers worldwide.[1] In addition to being an important consumer good, smartphones also create new sub-markets that would not exist without the smartphone. For example, there exist individual sub-markets for gaming apps, weather apps, map apps, etc. These apps are often provided by an outside developer that does not produce the smartphone, and they can only be used by consumers who own a smartphone. That is, the smartphone is the platform connecting the two sides, consumers and app providers, in a two-sided market with sub-markets for individual app types.

This structure exists in many economic markets: video game consoles with consumers and game developers; websites like Youtube, Spotify, Netflix, and Hulu that connect viewers with media producers; credit cards that connect consumers with merchants through purchases; and online marketplaces like Amazon that connect consumers and sellers. In this paper I consider platform pricing strategies and the welfare that is generated when consumers purchase products from sellers within sub-markets on a platform.

Competition among sellers within each sub-market affects the network benefit for consumers. More competition leads to lower product prices within sub-markets which increases consumer surplus. Thus, the network benefit to consumers' increases and joining the platform is more desirable. All consumers prefer more products being available on the platform as this allows them to make more purchases. However, consumers are heterogeneous in their demand for the amount of products on the platform. This leads to consumers differing in the number of product purchases they make, a common feature of platform marketplaces.

Competition within each sub-market also affects the network benefit to sellers. More competition leads to a lower product price which decreases seller profits. Sellers always prefer more consumers to join the platform as this raises demand for their products. However, consumer heterogeneity implies that the marginal consumer that joins the platform is the

[1]For more statistics on smartphones, see Bouchard et al. (2014).

least likely to purchase their product. Hence, sellers have decreasing marginal benefits from consumer participation on the platform.[2]

Participation by consumers and the availability of products on the platform is endogenously determined through the membership prices that the platform sets to consumers and sellers. This implies that given platform prices, more competition within a sub-market results in fewer sub-markets (i.e. fewer products available to consumers) since the reduction in profits for sellers leads to exit on the seller side. However, more competition implies less market power for a seller which reduces the deadweight loss within a sub-market. Thus, there exists a tradeoff for changes in competition within a sub-market. More competition implies that less deadweight loss is created within each sub-market, which increases total surplus, but more competition also implies that there will be fewer products or sub-markets available on the platform, which decreases total surplus. This implies that the platform pricing strategies, the amount of participation by consumers and sellers, and the welfare generated by the platform depend on the mode of competition that exists within each sub-market.

The tradeoff between deadweight loss and the number of products available to consumers leads to interesting pricing strategies by the platform. I find that when sellers have more market power within a sub-market, the platform lowers its membership prices to consumers and sellers so that there is more participation on each side of the platform. As participation is what generates surplus on the platform, this pricing strategy by the platform leads to several interesting results regarding welfare.

I find that less competition within each sub-market causes an increase in the number of sub-markets available on the platform because less competition increases the revenue a seller receives from joining the platform and because less competition lowers platform prices. This leads to greater entry by consumers and a greater number of sub-markets. Furthermore, this added surplus from additional sub-markets is enough to overcome the additional deadweight

[2] This differs from the previous literature, including Rochet and Tirole (2003, 2006) and Armstrong (2006) and subsequent work, including more recent papers by Shy and Wang (2011) and Jeitschko and Tremblay (2015).

loss from increased seller market power. In other words, the total deadweight loss across all the sub-markets from less competition at the seller level is overcome by the additional surplus generated from more sub-markets being made available on the platform resulting in greater welfare. These findings show the importance of the sub-market structure when investigating pricing and welfare on a two-sided platform.

Many of the original papers on platforms and two-sided markets, Rochet and Tirole (2003, 2006), Hagiu (2006), and Armstrong (2006), and subsequent work assume that agents have homogeneous network effects. Furthermore, the magnitude of the network effect on each side of the market is independent of the network effect on the other side of the market. To this end, the platform literature has abstracted away from the sub-market structure that generates the network effects between consumers and sellers.

When solving for equilibria when agents have homogeneous network effects on each side of the market, restrictive assumptions on participation decisions of agents must be made. Also homogeneous network effects do not coincide with the empirical evidence found by Lee (2013) and Bresnahan et al. (2014), who find that the network benefits consumers receive from video games and apps vary across consumers. Thus, allowing for heterogeneity is important in modelling platforms. Jeitschko and Tremblay (2015) develop a model with heterogeneous consumers and find equilibria that correspond to many platform markets, including smartphones and video game consoles. However, they do not consider the pricing relationship that exists between consumers and sellers.[3] This paper allows for heterogeneity and aims to illustrate the importance of the sub-market structure that exists within a platform in analyzing network effects, platform pricing strategies, and platform welfare.

The relationship that exists between a platform and its sellers relates to the traditional models on vertical relationships. When a wholesaler and a retailer each have market power, Spengler (1950) shows that this leads to double marginalization — final prices in excess of the monopoly price — where each firm along the supply chain adds a market-power markup. He

[3]Deltas and Jeitschko (2007) also show how heterogeneity on the consumer side plays a critical role in the platform profit maximization problem.

finds that a vertical merger between wholesalers and retailers improve efficiency by lowering the final price while increasing profits and consumer surplus.

The research on platforms relating to vertical relationships is very limited. Lee (2013) investigates empirically exclusive deals, which can be interpreted as vertical integration. He shows how exclusive deals between video game platforms and video game developers enable entry into the market for video game consoles. Lee finds that vertical integration helps a platform enter the market and compete with an incumbent platform. However, his model does not lend itself to investigating the effect of vertical relationships on prices.

Vertical relationships in a two-sided market for credit cards is investigated in a different model by Shy and Wang (2011). The focus of their work is on the fee structures for consumers and merchants used by credit card companies. They find that greater merchant competition leads to lower prices for consumers and increased welfare when there is a monopoly platform. This is consistent with the usual double marginalization result. However, Shy and Wang assume homogeneous agents on each side of the market and a network benefit structure that does not coincide with the empirical findings of Lee (2013) and Bresnahan et al. (2014). Furthermore, participation on the consumer side of the market is exogenously given: consumers do not choose whether or not to join the platform. This paper is not limited by these assumptions.

The remainder of the paper is organized as follows. In Section 2 the general model of consumers, sellers within sub-markets, and the platform are introduced. In Section 3, the equilibrium for the entire two-sided platform and the effects of changes in competition within sub-markets on equilibrium participation and welfare are determined. A model that allows for an investigation of different modes of competition between sellers within a sub-market is developed in Section 4. It describes consumers' demand for products within an individual sub-market, sellers within a sub-market, and the platform's aggregation of sub-markets. The equilibria for the entire two-sided platform for different modes of competition within sub-markets are also determined in Section 4. Section 5 concludes, followed by an appendix containing the proofs of all the formal findings.

2 The Model

There are three types of players: a platform, consumers who join the platform on one side, and sellers who make up the other side. Consumers benefit by purchasing products that are available on the platform. To reach consumers, sellers must join the platform to make their products available to consumers. The platform earns profits by charging consumers and sellers to join the platform.[4]

2.1 The Platform and Timing of Play

The platform connects consumers with sellers. Every consumer pays a membership fee, P_1, to join the platform. For example, this would be the monthly fee consumers pay to join Netflix or Hulu, or the retail price that consumers pay to purchase a smartphone or video game console. Similarly, the platform charges a membership fee, P_2, to sellers. This gives sellers access to the consumers that join the platform.

The platform then maximizes profits with respect to prices P_1 and P_2. Platform profits are given by:

$$\Pi = N_1 \cdot P_1 + n \cdot N_2 \cdot P_2, \tag{1}$$

where N_1 is the number of consumers that join the platform, n is the number of sellers in each sub-market, and N_2 is the number of sub-markets so that $n \cdot N_2$ is the total number of sellers that join the platform. For simplicity, I will assume that the platform's marginal and fixed costs are zero.

The timing of play is as follows. First, the number of sellers in each sub-market, n, is given and observed by all agents. The type of competition that exists between sellers in

[4] I will use the terminology "join the platform." Depending on the type of platform market, consumers either join the platform (e.g. Netflix, Amazon, Pandora, etc.) or consumers purchase the platform (e.g. video game consoles, smartphones, digital devices, etc.).

sub-markets is common knowledge: homogeneous product oligopolies, differentiated product oligopolies, or something more general. Then the platform sets prices, P_1 and P_2, which can be less than zero. Next, consumers and sellers decide whether or not to join the platform based on expected gains from joining the platform and the prices that are set by the platform. For many apps and games the quality is unclear upon release to the market and it takes time for consumers to experience and review a new product. Thus, the popularity of products are realized after sunk participation decisions are made. Lastly, the resulting payoffs are realized.

2.2 Consumer and Sellers

Consider a two-sided market with consumers on Side 1 and sellers on Side 2. Fist, I introduce the seller or product side of the market, Side 2, as this leads to a more natural development in this kind of marketplace.

On Side 2 there exists products and their sellers. Product categories are indexed by $\theta \in [0, \infty)$. I use θ to represent both the product and its sub-market.[5] For example, sub-markets on online marketplaces include the books sub-market, the clothing/apparel sub-market, the tools sub-market, and the DVD sub-market; and sub-markets on smartphones include the map app sub-market, the weather app sub-market, and the fitness app sub-market. The number of products (sub-markets) that are available on the platform is denoted by N_2.

Within each sub-market there are n sellers of product θ that compete within that sub-market and sellers are single-product firms. When $n = 1$, each product is being sold to consumers by a monopoly seller and the market becomes more competitive as n increases. I assume n is the same for all sub-markets. Thus, the total mass of sellers that join the platform is the number of sellers within every sub-market times the number of sub-markets, $n \cdot N_2$.

[5]Thus, more products on the platform coincides with more sub-markets on the platform.

The type of competition between the n sellers within each sub-market is defined by $\mathcal{C}$. Each seller of product θ receives profits from a consumer of type τ given by $\pi(\tau, \theta, n, \mathcal{C})$. That is, profitability differs across sub-markets and across consumers, and profitability depends on the competitive structure and the number of sellers within the sub-market.

On Side 1 there exists a mass of consumers, normalized to 1, with individual consumers indexed by $\tau \in [0, 1]$. The number of consumers that decide to join the platform is denoted by $N_1 \in [0, 1]$. The consumer surplus for consumer τ from sub-market θ when there are n sellers whose competition structure is defined by $\mathcal{C}$ is given by $cs(\tau, \theta, n, \mathcal{C})$.[6]

Consumers are heterogeneous in their likelihood to value a sub-market. There are some products that consumers are not interested in. For example, teens may have many apps on their smartphone relative to their parents; whereas, their parents may have a few apps that they value. Consumer τ is only interested in a particular sub-market θ with probability $(1-\tau)$. Thus, τ captures the probability that a consumer has any interest in a given product.[7] Formally, this is captured in Assumption 1.

Assumption 1. *Consumers are heterogeneous so that for product θ:*

$$\begin{aligned} cs(\tau, \theta, n, \mathcal{C}) > 0 \quad & \textit{with probablity } (1-\tau) \\ cs(\tau, \theta, n, \mathcal{C}) = 0 \quad & \textit{with probablity } (\tau)\,. \end{aligned} \tag{2}$$

Assumption 1 implies consumers are heterogeneous in their probability of having positive consumer surplus for a given product. Thus, consumers with lower τ have a positive value for more products (teens with apps) than consumers with higher τ (parents fewer with apps), in expectation. Thus, the expected consumer surplus for consumer τ from product θ is:

$$E[cs(\tau, \theta, n, \mathcal{C})] = cs(\theta, n, \mathcal{C}) \cdot (1-\tau), \tag{3}$$

[6] Consumer surplus and seller profits being functions of θ allows for differences in demand across sub-markets.

[7] This is common in platform marketplaces where consumers vary in the number of products that interest them.

where $cs(\theta, n, \mathcal{C})$ is the expected surplus a consumer receives from product θ, given that the consumer is interested in the product.

Given a consumer is interested in a product θ does not imply they will purchase the product as the price may be to high; hence, $cs(\theta, n, \mathcal{C})$ is an expected surplus. Depending on the sub-market structure, seller profit and consumer surplus depend on the price or prices of products in the sub-market. I am assuming that all of these features are captured by n and $\mathcal{C}$. More specifically, I assume that the profit and consumer surplus functions are twice differentiable in n for a given competitive structure, $\mathcal{C}$.

For sellers, Assumption 1 implies that some consumers simply will not make a purchase as they are not interested in the product. Thus, the expected profits for a seller of product θ from a consumer τ is given by

$$E[\pi(\tau, \theta, n, \mathcal{C})] = \pi(\theta, n, \mathcal{C}) \cdot (1 - \tau), \tag{4}$$

where $\pi(\theta, n, \mathcal{C})$ is the expected profit a seller makes from a consumer that is interested in the product.[8] This implies that when there are N_1 available consumers on the platform, a seller of product θ will have profits given by:

$$\int_0^{N_1} \pi(\theta, n, \mathcal{C}) \cdot (1 - \tau) d\tau = \pi(\theta, n, \mathcal{C}) \cdot \left(1 - \frac{N_1}{2}\right) N_1. \tag{5}$$

Notice how seller profits change in the number of consumers that join the platform. Sellers always prefer more consumers to join the platform, $\frac{\partial \pi}{\partial N_1} > 0$, as this raises demand for their products. However, consumer heterogeneity implies that the marginal consumer that joins the platform is the least likely to purchase their product. Hence, sellers have decreasing marginal benefits from consumer participation on the platform, $\frac{\partial^2 \pi}{\partial N_1^2} < 0$. This differs from the previous literature, including Rochet and Tirole (2003, 2006) and Armstrong (2006) and

[8]Similar to consumer surplus, $cs(\theta, n, \mathcal{C})$, this is an expected profit as some consumers that are interested in the product may not purchase it.

subsequent work, including more recent papers by Shy and Wang (2011) and Jeitschko and Tremblay (2015).

Consumers and sellers take expectations of the popularity of a product as these are experienced goods. Thus, the expected consumers surplus and the expected profit for a seller from a given product are given by:

$$\int_0^{N_2} cs(\theta, n, \mathcal{C}) \cdot (1-\tau) d\theta = cs(n, \mathcal{C}) \cdot (1-\tau), \tag{6}$$

$$\int_0^{N_2} \pi(\theta, n, \mathcal{C}) \left(1 - \frac{N_1}{2}\right) N_1 d\theta = \pi(n, \mathcal{C}) \left(1 - \frac{N_1}{2}\right) N_1, \tag{7}$$

where $cs(n, \mathcal{C})$ is the expected consumer surplus across products that consumer τ receives when it is interested in a product and $\pi(n, \mathcal{C})$ is the expected profit from a consumer across product popularity, given a consumer is interested in the product.

When consumers join the platform they have access to all of the products that are available on the platform, N_2. Thus, by joining the platform consumer τ's expected utility is given by:

$$u_1(\tau, n, \mathcal{C}) = cs(n, \mathcal{C}) \cdot (1-\tau) \cdot N_2 - P_1, \tag{8}$$

where N_2 is the number of sub-markets that are available on the platform and P_1 is the price a consumer pays to join the platform. Every consumer has a reservation utility that is normalized to zero. Thus, a consumer τ joins the platform when $u_1(\tau) \geq 0$.

To allow for endogenous entry of sub-markets, sellers have different sunk costs. Let $c \cdot \theta$ be the sunk cost to developing product θ. That is, low θ-type sellers have lower sunk costs than high θ-sellers. Sellers earn profits by joining the platform and selling their product. The marginal cost of production is set to zero.

Thus, a seller of product θ has expected utility from joining the platform which is given by:

$$u_2(\theta, n, \mathcal{C}) = \pi(n, \mathcal{C}) \left(1 - \frac{N_1}{2}\right) N_1 - c \cdot \theta - P_2, \tag{9}$$

where P_2 is the price that a seller pays to join the platform. Every seller has a reservation utility that is normalized to zero. Thus, a seller of type θ joins the platform when $u_2(\theta) \geq 0$.

3 Equilibrium

The aim of this paper is to determine how the amount of competition between sellers within sub-markets, characterized by the number of sellers and the competitive structure, affects the platform's pricing strategies and the welfare generated on the platform. In principle, consumer surplus and seller profits move in opposite directions as they are splitting the total surplus generated within a sub-market. However, this change need not be one to one. For example, when sellers of homogeneous products compete *a la* Cournot then an increase in the number of sellers increases total surplus by eliminates deadweight loss. Similarly, if sellers have differentiated products within the sub-market then an increase in the number of sellers can increase consumer surplus more than it decreases sellers' profit. This leads to the following assumption:

Assumption 2. *For all* $\mathcal{C}$*,*

$$\frac{\partial cs(n,\mathcal{C})}{\partial n} \geq 0 \quad and \quad \frac{\partial \pi(n,\mathcal{C})}{\partial n} \leq 0. \tag{10}$$

Assumption 2 implies that for any competition structure within sub-markets, when the number of sellers increases within a sub-market then consumer surplus for each consumer increases and each seller's profit decreases. This provides a general characterization of sub-markets that allows for many types of competitive structures.

Given the number of sellers and the competitive structure with each sub-market, $\langle n, \mathcal{C}\rangle$, consumers and sellers anticipate the size of the network, N_1 and N_2, given the platform prices and the platform sets prices P_1 and P_2 to maximize profit given by Equation (1). However, using Equations (29) and (30) the platform solves for each price as a function of N_1 and N_2 since $u_1(N_1, n, \mathcal{C}) \equiv 0$ and $u_2(N_2, n, \mathcal{C}) \equiv 0$ identify the last agent type, $\tau = N_1$ and $\theta = N_2$, to join the platform on each side. Thus, the platform's profit is a function of N_1 and N_2. For ease of exposition, the $\mathcal{C}$ in $\pi(n, \mathcal{C})$ and $cs(n, \mathcal{C})$ will be suppressed so that we have $\pi(n)$ and $cs(n)$.

Solving the platform's problem gives the equilibrium of the entire game:[9]

$$N_1^* = \frac{n\pi(n) + cs(n)}{n\pi(n) + 2cs(n)}, \tag{11}$$

$$N_2^* = \frac{[n\pi(n) + cs(n)]^2}{4cn[n\pi(n) + 2cs(n)]}, \tag{12}$$

$$P_1^* = \frac{cs(n)^2 \cdot [n\pi(n) + cs(n)]^2}{8c[n\pi(n) + 2cs(n)]^2}, \tag{13}$$

$$P_2^* = \frac{n\pi(n) + cs(n)}{8[n\pi(n) + 2cs(n)]^2} \cdot [3n^2\pi(n)^2 + 9n\pi(n)cs(n) - 2cs(n)]. \tag{14}$$

By examining this equilibrium we have the following result:

Theorem 1. *The equilibrium number of consumers that the platform serves is decreasing in the number of sellers within sub-markets, $\frac{\partial N_1^*}{\partial n} < 0$. And, the equilibrium number of sub-markets the platform makes available is decreasing in the number of sellers within sub-markets unless the change in consumer surplus from more sellers within sub-markets is relatively large, $\frac{\partial N_2^*}{\partial n} < 0$ unless $\frac{\partial cs(n)}{\partial n} > \frac{cs(n)}{2} + \left(-\frac{\partial \pi(n)}{\partial n}\right) \cdot \left(\frac{n^2\pi(n)}{2cs(n)} + \frac{3n}{2}\right)$.*

Theorem 1 states two general results regarding the affects of the sub-market on the platform. First, an increase in the number of sellers within a sub-market always results in the platform lowering consumer participation. Network effects play an important role in this result. When the number of sellers within a sub-market increases, each seller receives less profit from a given consumer; thus, the platform has less of an incentive to provide sellers with additional consumers. Furthermore, when the number of sellers within a sub-market increases then each consumer receives more consumer surplus from joining the platform. This creates an incentive for the platform to charge a higher consumer price which reduces

[9] The second-order conditions hold for profit maximization.

consumer participation.

Second, it is likely that the number of sub-markets that the platform provides will decrease in the number of sellers per sub-market. The case where an increase in the number of sellers leads to the platform providing more sub-markets requires consumer surplus to increase substantially relative to the other parameters. In a market that is "emerging," with strong consumer gains and only a few sellers within sub-markets, this is a possibility.[10]

Given this equilibrium, we also investigate welfare and the affects on welfare from changes in the sub-markets. Total welfare generated by the platform is given by:

$$W^* = \frac{[n\pi(n) + cs(n)]^4}{32cn[n\pi(n) + 2cs(n)]^3} \cdot [3n\pi(n) + 10cs(n)]. \tag{15}$$

In platform markets, surplus is generated by the interaction between the two sides of the market. Thus, holding network benefits fixed, a reduction in the number of consumers or in the number of sub-markets on the platform results in less surplus, i.e. less welfare. With this in mind, consider how an increase in the number of sellers in a sub-market will affect total welfare generated by the platform. From Theorem 1 we know that the number of consumers will decrease which lowers welfare. Furthermore, the number of sub-markets decreases unless the increase in consumer surplus is relatively large. Thus, when the number of sellers in sub-markets, n, increases then total welfare generated by the platform, W^*, will decrease unless the increase in consumer surplus, $cs'(n)$, is relatively large.

Unfortunately, the exact comparative static on equilibrium welfare with respect to the number of sellers in a sub-market is not clean. Because of this I state what is essentially required for welfare to increase when the number of sellers increases in the following proposition and leave the complete parametrization in the proof.

Proposition 1. *The total welfare generated by the platform is increasing in the number of sellers in a sub-market,* $\frac{\partial W^*}{\partial n} > 0$*, only when the increase in consumer surplus from more*

[10] To this point, we have not made assumptions regarding the second derivative of $\pi(n)$ and $cs(n)$. This kind of "emerging" market requires adequate concavity of $cs(n)$.

sellers in a sub-market, $\frac{\partial cs(n)}{\partial n}$, is relatively large.

Proposition 1 implies that we can expect the platform to generate more welfare when the sub-markets are considered emerging markets. That is, when the number of sellers in a sub-market, n, is relatively low and the gains to consumers from additional sellers in a sub-market, $cs'(n)$, is relatively large.

In this section a general formalisation of how the sub-market structure affects the platform was analyzed. In order to analyze this relationship further, several specific sub-market structures are investigated in the following section and the effects of the sub-market structure on network effect and the platform are determined.

4 Sub-Market Competition Structures

In this section assumptions are made that provide a simple model of the platform and sub-market structure so that consumers have linear demand for the product within a sub-market. This allows for the analysis of different types of competition between sellers within sub-markets, $\langle n, \mathcal{C} \rangle$. ???CITE Cloud Paper and Tax Paper here.???

4.1 Consumers and Sellers

Suppose consumers have unit demands for each product that is available. Let $p_\theta(n, \mathcal{C})$ be the price of product θ when there are n sellers in the sub-market and the competitive structure of is $\mathcal{C}$. Let $\upsilon_{\tau,\theta}$ be the reservation value that consumer τ has for product θ so that products are homogeneous within each sub-market. Consumer τ has a positive reservation value for a given product θ, $\upsilon_{\tau,\theta} > 0$, with probability $(1 - \tau)$. That is, $\upsilon_{\tau,\theta} = cs(\theta, n, \mathcal{C})$ and τ again captures the probability that a consumer has any interest in a given product. This implies that Assumption 1 becomes the following:

Assumption 3. *Consumers are heterogeneous so that for product θ:*

$$\begin{aligned} \upsilon_{\tau,\theta} > 0 \quad & \textit{with probablity } (1-\tau) \\ \upsilon_{\tau,\theta} = 0 \quad & \textit{with probablity } (\tau) \,. \end{aligned} \tag{16}$$

Assumption 3 implies consumers are heterogeneous in their probability of having a positive value for a given product. Thus, consumers with lower τ have a positive value for more products (teens with apps) than consumers with higher τ (parents fewer with apps), in expectation.

Assumption 4. *Given consumer τ has a positive value for product θ, $\upsilon_{\tau,\theta} > 0$, let*

$$\upsilon_{\tau,\theta} \sim U[0, \sigma_\theta], \tag{17}$$

where σ_θ depends on product θ.[11]

Assumption 4 implies that all consumers that have a positive value for product θ ($\upsilon_{\tau,\theta} > 0$), their value is distributed uniformly between 0 and σ_θ, where σ_θ is a parameter that captures the potential value (i.e. the popularity) of product θ.

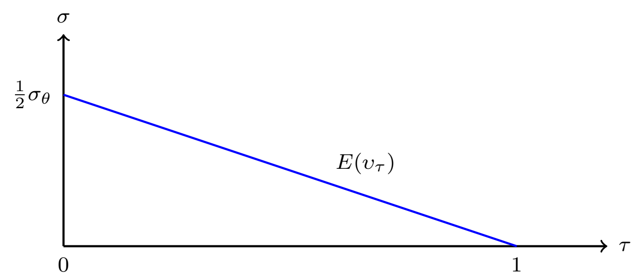
Assumptions 3 and 4 determine consumer demand for product θ. Demand depends on the number of consumers that are available to sellers, i.e. demand depends on the number consumers that join the platform. Given that N_1 consumers join the platform, the demand for product θ is given by:

$$\begin{aligned} Q_\theta(p_\theta(n,\mathcal{C})) = \int_0^{N_1} \Pr(\tau \text{ buys}|p_\theta(n,\mathcal{C}))d\tau &= \int_0^{N_1} \left(1 - \frac{p_\theta(n,\mathcal{C})}{\sigma_\theta}\right)(1-\tau)d\tau \\ &= \left(1 - \frac{p_\theta(n,\mathcal{C})}{\sigma_\theta}\right)\left(1 - \frac{N_1}{2}\right) \cdot N_1. \end{aligned} \tag{18}$$

Thus, consumer demand for a product within a sub-market is linear as $\left(1 - \frac{p_\theta(n,\mathcal{C})}{\sigma_\theta}\right)$ is linear in $p_\theta(n,\mathcal{C})$. Furthermore, consumer demand for a product is increasing in the number

[11]The uniform distribution gives linear demand within sub-markets.

Figure 1: Expected Reservation Value for Product θ.



of consumers that join the platform, $\left(1 - \frac{N_1}{2}\right) \cdot N_1$ is increasing in N_1.

This linear demand stems from Assumptions 3 and 4 where consumers are heterogeneous in their likelihood of having a positive value of a given product. However, conditional on two consumers having a positive value for product θ, the expected value is the same, $E[\upsilon_{\tau,\theta}|\upsilon_{\tau,\theta} > 0]$ is $\frac{\sigma_\theta}{2}$.[12] Figure 1 illustrates the expected utility across consumers for product θ which is decreasing in τ since a consumer with a higher τ is less likely to value the product.

Using Assumptions 3 and 4 expected consumer surplus for consumer τ from product θ when the sub-market price is $p_\theta(n)$ is given by:[13]

$$cs(\theta, \tau, p_\theta(n, \mathcal{C})) = \frac{\sigma_\theta}{2}\left(1 - \frac{p_\theta(n, \mathcal{C})}{\sigma_\theta}\right)^2 \cdot (1 - \tau)\,. \tag{19}$$

Thus, $cs(\theta, \tau, p_\theta(n, \mathcal{C}))$ gives the expected gains that consumer τ receives from having product θ available at price $p_\theta(n, \mathcal{C})$. Note that consumer surplus is decreasing in τ. This means that consumers who have a positive value for more products are the consumers that receive the greatest consumer surplus. In addition, $cs(\theta, \tau, p_\theta(n, \mathcal{C}))$ is increasing in σ_θ. This implies that a highly valued product generates the most surplus for consumers.

When consumers join the platform they have access to all of the products that are available on the platform, N_2. Thus, by joining the platform consumer τ's expected utility

[12] Assumptions 3 and 4 imply that consumers of different types purchase a different number of products. This closely resembles what is seen in the market for smartphones [Bresnahan et al. (2014)] and video games [Lee (2013)]; and likely also for online marketplaces.

[13] The expectation is with respect to the probability of consumer τ making a purchase given Assumptions 3 and 4.

is given by:

$$u_1(\tau, n, \mathcal{C}) = \int_0^{N_2} cs(\theta, \tau, p_\theta(n, \mathcal{C}))d\theta - P_1, \tag{20}$$

where N_2 is the number of independent sub-markets that are available on the platform and P_1 is the price a consumer pays to join the platform. Every consumer has a reservation utility that is normalized to zero. Thus, a consumer τ joins the platform when $u_1(\tau) \geq 0$.

To allow for endogenous entry in sub-markets, sellers have different sunk costs of product development. Let $c \cdot \theta$ be the sunk cost to developing product θ. That is, low θ-type sellers have lower sunk costs than high θ-sellers. Sellers earn profits by joining the platform and selling their product. The marginal cost of production is set to zero.

Sellers earn profits by selling their product to consumers and they also incur a sunk costs of $c \cdot \theta$ as in the general model. Thus, a seller of product θ has expected utility from joining the platform which is given by:

$$u_2(\theta, n, \mathcal{C}) = \int_0^{N_1} \pi(\tau, \theta, p_\theta(n, \mathcal{C}))d\tau - c \cdot \theta - P_2, \tag{21}$$

where $\pi(\tau, \theta, p_\theta(n, \mathcal{C}))$ gives the expected profit for product θ from consumer τ when the price of product θ is given by $p_\theta(n, \mathcal{C})$ and P_2 is the price that a seller pays to join the platform. Every seller has a reservation utility that is normalized to zero. Thus, a seller of type θ joins the platform when $u_2(\theta) \geq 0$.

Consumers and sellers anticipate the size of the network, N_1 and N_2, given the platform prices that they observe. To solve for consumer and seller participation, the network effects must be determined. At this point the mode of competition between sellers within the sub-markets, $\mathcal{C}$, must be specified. This model is general to any form of competition among sellers that has a symmetric equilibrium. I focus on two fundamental models. First, I investigate the case when the sellers compete *à la* Cournot, and then I investigate the case where sellers compete *à la* Bertrand.

4.2 Cournot Competition

With n sellers competing *à la* Cournot, the inverse demand function is given by Equation (18) so that we have:

$$p_\theta(n) = \sigma_\theta - \frac{\sigma_\theta}{\left(1 - \frac{N_1}{2}\right) \cdot N_1} \cdot Q_\theta. \tag{22}$$

Solving the Cournot model with n symmetric sellers gives the Nash equilibrium price and quantities:

$$p_\theta(n) = \frac{\sigma_\theta}{n+1}, \tag{23}$$

$$q_\theta^i = \frac{1}{n+1} \cdot \left(1 - \frac{N_1}{2}\right) \cdot N_1, \tag{24}$$

for $i = 1, ..., n$.

I assume σ_θ has distribution $G(\cdot)$ with support $[\underline{\sigma}, \overline{\sigma}]$ and expected value $E[\sigma_\theta] = \sigma$. Since σ_θ is not realized until after membership decisions are made, consumers and sellers form expectations regarding valuations, $E[\sigma_\theta] = \sigma$. Thus, forward looking sellers have expected profit functions:

$$\pi^i(N_1, n) \equiv E\left[p_\theta(n) \cdot q_\theta^i\right] = \frac{\sigma}{(n+1)^2} \cdot \left(1 - \frac{N_1}{2}\right) \cdot N_1, \tag{25}$$

for all $i = 1, 2, ..., n$.

Equation (25) implies that the marginal benefit, or network effect, that sellers receive from an additional consumer is given by:

$$\frac{\partial \pi^i}{\partial N_1} = \frac{\sigma}{(n+1)^2} \cdot (1 - N_1). \tag{26}$$

Notice that an increase in the number of consumers, N_1, leads to a smaller network benefit for sellers, $\frac{\partial^2 \pi^i}{\partial N_1^2} < 0$. In other words, there are diminishing marginal returns from consumer participation. The marginal consumer who joins the platform is less likely to have a high willingness to pay for a given product; hence, as N_1 increases, the marginal benefit that sellers receive from an additional consumer decreases.

Similarly, there exists a network benefit on the consumer side. First, given equilibrium prices in Equation (23), the surplus that consumer τ receives when N_2 products are available on the platform with n competing Cournot sellers is:

$$cs(\tau, n) := \int_0^{N_2} cs(\theta, \tau, p_\theta(n)(n))d\theta = \frac{\sigma}{2}\left(\frac{n}{n+1}\right)^2 \cdot (1-\tau) \cdot N_2. \tag{27}$$

Thus, the network benefit or marginal gain that consumers receive from an additional product available is given by:

$$\frac{\partial cs(\tau, n)}{\partial N_2} = \frac{\sigma}{2}\left(\frac{n}{n+1}\right)^2 \cdot (1-\tau)\,. \tag{28}$$

This implies that an increase in the number of Cournot competitors, n, leads to an increase in the network benefit for consumers, $\frac{\partial^2 cs}{\partial n \partial N_2} > 0$. More competing sellers lowers the price of the product which increases the network benefit for consumers.

The network effects have an important relationship with the number of sellers, n. On the seller side, an increase in the number of Cournot competitors leads to a decrease in the network benefit to sellers, $\frac{\partial^2 \pi^i}{\partial n \partial N_1} < 0$. With perfect competition, which occurs when n approaches infinity, the profits for sellers, π^i, goes to zero. In this case, an additional consumer generates zero benefit to sellers. However, an increase in the number of Cournot competitors leads to an increase in the network benefit for consumers, $\frac{\partial^2 cs}{\partial n \partial N_2} > 0$. With monopoly sellers for each product, $n = 1$, network benefits are positive on both sides of the market since consumers are receiving some surplus from the products available on the platform and sellers are making monopoly profits from consumers.

The connection between the two network effects through the number of competitors plays a key role in the model. This dependence across network effects exists in many examples of platforms and two-sided markets where two sides of the market exchange products or services through a platform. Hence, this relationship is key to understanding many platforms.

Now all the elements that are needed to identify the net payoffs for consumers and sellers are defined. The net expected utility a consumer τ receives from joining the platform is given by:

$$u_1(\tau, n) = cs(\tau, n) - P_1. \tag{29}$$

This is the total network gain from the products available, $cs(\tau, n)$, minus the price consumers pay to join the platform, P_1.

On the seller side, the net expected payoff that a seller of product θ receives for joining the platform is given by:

$$u_2(\theta, n) = \pi^i(N_1, n) - c \cdot \theta - P_2. \tag{30}$$

Seller payoff is the profits that a seller receives, $\pi^i(N_1, n)$,[14] minus the sunk cost associated with product θ, $c \cdot \theta$, minus the price that sellers pay to the platform, P_2.

The platform sets prices P_1 and P_2 to maximize profit given by Equation (1). However, using Equations (29) and (30) the platform solves for each price as a function of N_1 and N_2 since $u_1(N_1, n) \equiv 0$ and $u_2(N_2, n) \equiv 0$ identify the last agent type, $\tau = N_1$ and $\theta = N_2$, to join the platform on each side. Thus, the platform's profit is a function of N_1 and N_2.

Solving the platform's problem gives the equilibrium of the entire game:[15]

$$N_1^C = \frac{n+2}{2(n+1)}, \tag{31}$$

$$N_2^C = \frac{\sigma}{16c} \cdot \frac{(n+2)^2}{(n+1)^3}, \tag{32}$$

$$P_1^C = v + \frac{\sigma^2}{64c} \cdot \frac{n^3(n+2)^2}{(n+1)^6}, \tag{33}$$

$$P_2^C = \frac{\sigma}{16} \cdot \frac{n+2}{(n+1)^4}(3n + 2 - n^2). \tag{34}$$

Superscript C denotes the Cournot equilibrium.

This model produces several interesting results. First, notice that the platform's price to

[14] This is sometimes called the variable profit, i.e. the profits without fixed costs.
[15] The second-order conditions hold for profit maximization.

sellers, P_2^C, is positive for $n \leq 3$ and is negative for $n \geq 4$. When n increases the individual seller profits from sales are reduced. In order to induce more participation of sellers, resulting in more products being made available to consumers, the platform subsidizes them. Thus, when there exists more than three sellers per product, the platform subsidizes the seller side of the market so that in order to provide the optimal number of sub-markets.

Second, the optimal number of consumers is between $1/2$ and $3/4$ for all n, $N_1^C \in [1/2, 3/4]$.[16] Furthermore, the platform's optimal number of consumers, N_1^C, is decreasing in the number of competing sellers, n, $\frac{\partial N_1^C}{\partial n} < 0$. This means that more competition within each market induces the platform to restrict consumer participation on the platform. As n increases, consumer surplus increases for consumers of low τ; the platform extracts this surplus with a much higher price resulting in less consumer participation.

Third, on the seller side of the market the number of sub-markets made available on the platform, N_2^C, is also decreasing in the number of competing sellers, n, $\frac{\partial N_2^C}{\partial n} < 0$. As n increases, the platform begins to subsidize sellers which is costly to the platform so it reduces the number of products it makes available on the platform, N_2^C. This leads to the following corollary:

Corollary 1. *The number of consumers that join the platform, N_1^C, and the number of sub-markets available on the platform, N_2^C, are decreasing in the number of competing sellers in each sub-market, n.*

Thus, as competition becomes stronger (n increases) the amount of participation on both the consumer side and the product side of the market is reduced.

Given this equilibrium, I investigate the effect of competition within each sub-market on welfare. Platform profit (Π), total consumer surplus across all sub-markets (CS), total seller surplus across all markets (SS), and total welfare generated by the platform (W) are given

[16] This also implies that there is always an interior solution. This is not surprising since the marginal benefit to sellers when N_1 is close to 1 is nearly zero for all n.

by:

$$\Pi^C = \frac{\sigma^2}{256c} \cdot \frac{n \cdot (n+2)^4}{(n+1)^6}, \tag{35}$$

$$CS^C \equiv \int_0^{N_1} u_1(\tau)d\tau = \frac{\sigma^2}{256c} \cdot \frac{n^2(n+2)^4}{(n+1)^7}, \tag{36}$$

$$SS^C \equiv n \cdot \int_0^{N_2} u_2(\theta)d\theta = \frac{\sigma^2}{512c} \cdot \frac{n(n+2)^4}{(n+1)^6}, \tag{37}$$

$$W^C \equiv \Pi + CS + SS = \frac{\sigma^2}{512c} \cdot \frac{n \cdot (n+2)^4 \cdot (5n+3)}{(n+1)^7}. \tag{38}$$

Now the effects on welfare from changes in competition within each sub-market, characterized by the n Cournot sellers are derived. An increase in n generates three forms of surplus destruction. First, with more sellers the platform uses its market power to limit the number of sub-markets available to consumers to make purchases. As the addition of products and their sub-markets is the means of generating surplus in this market, a restriction on the number of sub-markets leads to a loss in surplus.

Second, market power at the seller level leads to a price markup which generates deadweight loss within each sub-market on the platform. Within the sub-markets, when sellers have more market power, then the total surplus captured within the sub-market is reduced. This is the case across all sub-markets, creating a deadweight loss.

Lastly, for each additional seller of product θ there is an additional sunk cost that reduces the total surplus being generated, $-c\theta$. This is simply a redundant sunk cost for a sub-market that is provided.

Theorem 2 summarizes these effects on welfare.

Theorem 2 (Welfare with Cournot Competition)**.** *For any n, platform profits (Π^C), total consumer surplus (CS^C), total seller surplus (SS^C), and total welfare (W^C) generated by the platform across all sub-markets are decreasing in n.*

An increase in n leads to greater surplus for consumers from each sub-market. However, as n increases, supporting sub-markets becomes more difficult for the platform and this leads to a reduction in the total consumer surplus generated by the platform.

Theorem 2 implies that as n gets large the surplus destruction from sunk cost redundancies becomes to large and leads to a marketplace with platform prices that are close to marginal costs. In this case the network is so small that it generates few gains to all agents. Thus, this form of surplus destruction has the largest effect in this market. To understand the other forms of surplus destruction, platform market power and seller market power, I investigate Bertrand competition with n sellers. By comparing the two models one can tease out the relative magnitudes of these forms of surplus destruction.

4.3 Bertrand Competition

With Bertrand competition there are only two equilibrium prices that will occur over all n: $n = 1$ implies the monopoly price and $n \geq 2$ implies the competitive price. By solving this problem for all n we account for the cumulative sunk costs. This allows for a comparison of the Bertrand and Cournot equilibrium results. When $n = 1$ the outcome is the same as in the Cournot case with $n = 1$ where sellers charge monopoly prices for their products. In this section the equilibrium for all $n \geq 2$ with Bertrand competition is investigated. Suppose there exists $n \geq 2$ sellers for every product θ that compete in prices. Since the marginal cost to a seller is zero, the equilibrium price for product θ is zero, $p_\theta(n) = 0$. Equations (19) and (18) imply that the expected network benefits are $cs(\tau, n) = \frac{\sigma}{2}(1 - \tau)$ and $\pi(\theta, \tau, n) = 0$. Thus, the utility functions for consumers and sellers with Bertrand competition are given by:

$$u_1(\tau) = v + \frac{\sigma}{2}(1 - \tau) \cdot N_2 - P_1, \tag{39}$$

$$u_2(\theta) = 0 - c \cdot \theta - P_2. \tag{40}$$

Using these utility functions, the platform maximizes profits which leads to the following equilibrium for $n \geq 2$:

$$N_1^B = \frac{1}{2} \text{ and } N_2^B = \frac{\sigma}{16nc}, \tag{41}$$

$$P_1^B = v + \frac{\sigma^2}{64nc} \quad \text{and} \quad P_2^B = -\frac{\sigma}{16n}. \tag{42}$$

Superscript B denotes the Bertrand equilibrium.

Notice that with Bertrand competition there are discreet jumps in prices and participation levels when n changes from 1 to 2. In moving from a monopoly to a competitive setting, Equations (31) and (41) indicate that consumer participation, (N_1^B) jumps from 3/4 down to 1/2 and the membership fee to sellers, P_2^B, goes from being positive (monopoly $P_2^B \equiv P_2^C(n=1) = \frac{3\sigma}{64}$) to being a subsidy that depends on the number of sellers in each sub-market, $P_2^B = -\frac{\sigma}{16n} < 0$.

Given this equilibrium, welfare generated by the platform for $n \geq 2$ is

$$\Pi^B = \frac{\sigma^2}{256nc}, \tag{43}$$

$$CS^B = \frac{\sigma^2}{256nc}, \tag{44}$$

$$SS^B = \frac{\sigma^2}{512nc}, \tag{45}$$

$$W^B = \frac{5\sigma^2}{512nc}. \tag{46}$$

This implies the following.

Corollary 2 (Welfare with Bertrand Competition)**.** *Platform profits (Π^B), total consumer surplus (CS^B), total seller surplus (SS^B), and total welfare (W^B) generated by the platform across all sub-markets are decreasing in n for all $n \geq 2$.*

With Bertrand competition, an increase in n does not make sub-markets more competitive. Thus, surplus is not increasing within each sub-market as it did with Cournot competition. Corollary 2 confirms our findings following Theorem 2: redundant sunk costs are the main source of surplus destruction. However, a comparison between the Cournot and Bertrand models is needed to clearly determine the effects of the other sources of surplus destruction.

4.4 Comparison Between Cournot and Bertrand Equilibria

To determine the surplus gains from additional sub-markets relative to the deadweight loss within each sub-market from a decrease in the amount of seller competition, n, a comparison between the Cournot equilibrium and the Bertrand equilibrium is needed. A direct comparison between the Cournot equilibrium and the Bertrand equilibrium implies greater participation with Cournot competition:

Theorem 3 (Participation Comparison)**.** *For all $n \geq 2$, the number of consumers and the number of sub-markets on the platform are greater with Cournot competition, $N_i^C > N_i^B$ for $i = 1, 2$.*

Thus, when sellers compete *a la* Cournot the platform sets prices so that more consumers join the platform and more products are made available to those consumers than when sellers compete *a la* Bertrand. With Cournot both consumers and sellers have positive gains from platform membership resulting in greater total participation. Even though the price of products is higher with Cournot competition, it is possible that the total surplus generated within each sub-market is larger with Cournot competition than with Bertrand competition because there are more consumers within each sub-market with Cournot competition. If this is the case, then clearly welfare is greater with Cournot competition; however the following theorem shows that this is not the case.

Theorem 4 (Sub-Market Surplus Comparison)**.** *Even though Cournot competition results in more consumers participating in each sub-market, $N_1^C > N_1^B$, the surplus generated within each sub-market is greater when sellers compete* a la *Bertrand than when they compete* a la *Cournot for $n \geq 2$.*

Theorems 3 and 4 illustrate the two effects on welfare that differ with the competition structure. With Cournot competition there is more participation, as shown in Theorem 3, but the additional consumers do not create enough additional surplus within an individual sub-market to overcome the surplus destruction from price markups with Cournot competition as shown in Theorem 4. However, with Cournot competition there are additional sub-markets that also contribute surplus to the market and this additional surplus overcomes the surplus lost from price markups leading to greater welfare under Cournot than Bertrand. This result is formalized in the following theorem.

Theorem 5 (Welfare Comparison)**.** *For any $n \geq 2$, total welfare generated on the platform is greater when sellers compete* a la *Cournot compared to Bertrand competition. Furthermore, platform profits and total seller surplus are greater with Cournot than Bertrand for any $n \geq 2$ and total consumer surplus is greater with Cournot, $CS^C > CS^B$, for any $n \geq 3$.*

When sellers have market power they create a deadweight loss but additional consumers and additional sub-markets join the platform. Each additional consumer generates additional surplus in two ways. First, the additional consumer makes purchases in the existing sub-markets on the platform which generates consumer surplus and seller profits. Second, it gives product sellers an incentive to join the market which creates additional sub-markets. Similarly, an additional sub-market provides an additional product to the platforms existing customers and makes the platform more attractive for the marginal consumer. Theorem 5 states that holding cost redundancies fixed, that is n fixed, some market power for sellers (the Cournot outcome) leads to greater welfare than when there is no price markups at the seller level (the Bertrand outcome). That is, the deadweight loss generated from seller market power is overcome by the additional surplus from increased participation on each side of the platform. This results in greater welfare and implies that even without sunk cost redundancies there is greater welfare with more market power at the seller level.

When n is small, $n = 2$, Bertrand competition leads to more consumer surplus even with fewer sub-markets available on the platform. This is not surprising. With $n = 2$ in a Cournot setting, the consumer surplus from each sub-market is less than Bertrand as there are higher prices. Theorem 5 implies this reduced consumer surplus from each sub-market is not overcome by the additional sub-markets until $n \geq 3$.

Theorems 2 and 5 imply that the redundancies in sunk costs are the largest source of surplus distortion. The second largest source of distortion is the platform's power to influence participation on each side of the market. The smallest source of distortion is the deadweight loss created by seller market power. These results show the importance of the relationship between network benefits on each side of the market in investigating the vertical relationship between a platform and its sub-markets.

To understand the relationship between these three sources of surplus destruction, integration by vertically merging the platform and its sellers is explored. In many platform markets such an integration exists. For example, Amazon has many sub-markets with individual sellers; however, many products are also sold by Amazon itself. Furthermore, with video game consoles it is often the case that console developers also provide games; similarly, smartphones come with preloaded apps.[17]

4.5 Integration between the Platform and Sellers

In this section, integration between the platform and the seller side of the market is investigated.[18] Ideally, the platform would like each product to be provided by a single seller with prices set equal to marginal cost. This mitigates both the redundancies in sunk costs and the deadweight loss generated by seller price markups. Thus, the platform only lets one seller produce and it sets the price of every product equal to marginal cost which is zero (i.e. $n = 1$ and $p_\theta(n) = 0$).

The platform now faces the following problem:

$$\max_{N_1, N_2} \Pi = N_1 \cdot P_1 - \int_0^{N_2} c \cdot \theta \, d\theta \tag{47}$$

$$\text{s.t.} \quad P_1 = v + CS(N_1). \tag{48}$$

[17] See Hagiu and Wright (2014) and Johnson (2014) for more details.

[18] While many platform markets are not fully integrated; this investigation works as a base case for understanding the effects of platform and seller integration.

By maximizing platform profits with respect to N_1 and N_2 the following equilibrium emerges:

$$N_1^I = \frac{1}{2}, \tag{49}$$

$$N_2^I = \frac{\sigma}{8c}, \tag{50}$$

$$P_1^I = v + \frac{\sigma^2}{32c}. \tag{51}$$

Superscript I denotes the integration equilibrium.

Comparing Equations (33), (42), and (51), notice that the platform's price to consumers is higher, $P_1^I > max\{P_1^C(n), P_1^B(n)\}$ for all n. Thus, when the platform and sellers are integrated, the consumers will always face a higher price. Furthermore, the amount of consumer participation is not enhanced by integration since $N_1^I = 1/2 = N_1^B(n) \leq N_1^C(n)$ for all n.

On the seller side the opposite is true. Consumers have access to more sub-markets when the platform and the seller side are integrated: $N_2^I > N_2^C(n), N_2^B(n)$ for all n. This is not surprising since integration mitigates the distortions on the seller side resulting in the platform adding sub-markets at marginal cost, $c\theta$.

Since the number of consumers that join the platform is the same for integration as with Bertrand competition, $N_1^I = N_1^B = 1/2$, and product prices equal marginal cost in both equilibria, the amount of surplus generated within each sub-market is the same. This extends Theorem 4.

Theorem 6 (Sub-Market Surplus Comparison II)**.** *Even though Cournot competition results in more consumers participating in each sub-market, $N_1^C > N_1^B$, the surplus generated within each sub-market is greater when sellers compete* a la *Bertrand than when they compete* a la *Cournot for $n \geq 2$. The surplus generated within each sub-market is the same under integration and Bertrand competition.*

Equilibrium platform profits, total consumer surplus, total seller surplus, and total welfare generated on the platform are given by:

$$\Pi^I = \frac{\sigma^2}{128c}, \tag{52}$$

$$CS^I = \int_0^{N_1^I} u_1(\tau)d\tau = \frac{\sigma^2}{64c}, \tag{53}$$

$$W^I = \Pi^I + CS^I = \frac{3\sigma^2}{128c}, \tag{54}$$

Comparing welfare generated with integration to the welfare results from Section 3 leads to the following theorem.

Theorem 7 (Integration Welfare Comparison)**.** *For all n, platform profits, total consumer surplus, and total welfare generated on the platform are higher with integration than with Cournot or Bertrand competition even though there are fewer consumers with integration.*

Theorem 7 states that all parties are better off with integration between the platform and the seller side. Notice that consumer surplus is larger even though there are fewer consumers. The consumers that join the platform are the consumers with the largest gains from trade. Thus, the additional sub-markets generate additional surplus which overcomes the reduced number of consumers.

With integration the only type of surplus destruction present is through the platform's pricing power on the consumer side of the market. Redundant sunk costs and deadweight loss through seller price markups are avoided. Theorem 7 shows how eliminating these distortions increases not only the platform's profits but also consumer surplus and total welfare. This is important from a policy perspective, since it shows that there can be welfare gains from this form of vertical integration where products are provided by the platform.

5 Conclusion

In this paper the relationship between a platform and its sub-markets is considered. The mode of competition that exists within sub-markets affects the network effects that exist between consumers and sellers, which in turn affect agents participation decisions and platform pricing strategies. The network benefits that consumers and sellers receive from joining the platform are determined by consumer demand and the competitive structure that exists among sellers for these products. If there is less competition within a sub-market, then the price of the product will be relatively high. This leads to greater network gains for sellers but lower network gains for consumers. However, the size of the network also matters. More consumers on the platform increases demand for a product and more products available on the platform makes participation on the platform more desirable for consumers.

When the number of sellers increases, making the sub-markets more competitive, I find that the platform reduces consumer participation, and this result is robust to many types of competitive structures within sub-markets. When the number of sellers within a sub-market increases, each seller receives less profit from a given consumer; thus, the platform has less of an incentive to provide sellers with additional consumers resulting in the platform serving fewer consumers.

I find that if each product is provided by a monopoly seller, then welfare is greater than if products were provided by any number of sellers that compete *a la* Cournot or *a la* Bertrand. More generally, when the number of sellers increases, welfare generated by the platform decreases unless the gains to consumer surplus are significant. For example, if sellers within sub-markets sell differentiated products and these sub-markets have only a few sellers then the increase in consumer surplus from additional sellers may be sufficiently large so that welfare increases when the number of sellers increase. However, it is often the case that more competition within sub-markets results in less welfare generated by the platform.

Furthermore, by comparing Cournot competition with Bertrand competition, I find that welfare is always greater with Cournot. This implies that the platform's creation of deadweight loss from restricting the number of consumers and sub-markets on the platform is of a higher magnitude than the sellers' creation of deadweight loss within each sub-market through price markups. That is, having more consumers that make purchases within each sub-market and having more products available to consumers generates more surplus than the total deadweight loss across all sub-markets that is created by less competition within each sub-market. This provides the intuition for the welfare results found in this paper.

In many platform markets, mergers between the platform and the product or seller side of the market are common. For example, on its online marketplace, Amazon connects consumers with sellers but it is often a seller itself. Similarly, many video games are developed by the console developers. I find that efficiency increases when the platform integrates with the seller side of the market. With integration two forms of surplus destruction are mitigated. First, the platform only has one seller of each product; hence, no surplus is destroyed through redundant sunk costs. Second, the platform sets the price of each product equal to marginal cost so that there is no distortion in product provision. The only distortion that remains is the platform's market power on the consumer side of the market. I find that integration leads to greater platform profits, total consumer surplus, and total welfare generated on the platform. Even though full integration is unlikely to occur in many platform markets, this serves as a base case for policy makers where such integration is a concern.

Appendix

Proof of Theorem 1: Taking the derivative of Equation (11) with respect to n implies $\frac{\partial N_1^*}{\partial n} < 0$ if and only if $\frac{\pi'(n)}{\pi(n)} < \frac{cs'(n)}{cs(n)}$ which holds since $cs'(n) \geq 0 \geq \pi'(n)$ and $cs(n), \pi(n), \geq 0$. Taking the derivative of Equation (12) with respect to n implies $\frac{\partial N_2^*}{\partial n} > 0$ if and only if $\frac{\partial cs(n)}{\partial n} > \frac{cs(n)}{2} + \left(-\frac{\partial \pi(n)}{\partial n}\right) \cdot \left(\frac{n^2 \pi(n)}{2cs(n)} + \frac{3n}{2}\right)$. □

Proof of Proposition 1: Taking the derivative of Equation (15) with respect to n implies

$\frac{\partial W^*}{\partial n} > 0$ if and only if

$$4n \cdot c'(n)[n^2\pi(n)^2 + 4n\pi(n)c(n) + 10c(n)^2] + n\pi(n)[3n^2\pi(n)^2 + 34n\pi(n)c(n) + 10c(n)^2] >$$
$$> n^2\pi(n)^2[19c(n) - \pi'(n)6n^2] - \pi'(n)34n^3\pi(n)c(n) + c(n)^2[36n\pi(n) - \pi'(n)56n + 10c(n)],$$

where $c(n) = cs(n)$ and $c'(n) = cs'(n)$. Essentially need $cs'(n)$ to be large relative to $n\pi(n)$ and $cs(n)$ while $n\pi'(n)$ is not to negative. □

Proof of Theorem 2: Taking derivatives of Equations (35), (36), (37), and (38) gives:

$$\frac{d\Pi^C}{dn} = -\frac{\sigma^2}{256c} \cdot \frac{(n+2)^3 \cdot (n^2+5n-2)}{(n+1)^7} < 0,$$

$$\frac{dCS^C}{dn} = -\frac{\sigma^2}{256c} \cdot \frac{n(n+2)^3 \cdot (n^2+4n-4)}{(n+1)^8} < 0,$$

$$\frac{dSS^C}{dn} = -\frac{\sigma^2}{512c} \cdot \frac{(n+2)^3 \cdot (n^2+5n-2)}{(n+1)^7} < 0,$$

$$\frac{dW^C}{dn} = -\frac{\sigma^2}{512c} \cdot \frac{(n+2)^3 \cdot (5n^3+26n^2+n-6)}{(n+1)^8} < 0,$$

for all $n \geq 1$. □

Proof of Theorem 3: The result follows by comparing Equation (31) and (32) with Equations (41) to see that $N_i^C > N_i^B$ for $i = 1, 2$ for all n. □

Proof of Theorem 4: For $n \geq 2$, Equation (39) implies the surplus within a sub-market with Bertrand competition is $\int_0^{N_1^B} \frac{\sigma}{2}(1-\tau)d\tau = \frac{3\sigma}{16}$. From Equations (25) and (27) the surplus with Cournot competition is $\int_0^{N_1^C} \frac{\sigma}{2}\left(\frac{n}{n+1}\right)^2(1-\tau)d\tau + \frac{\sigma}{(n+1)^2}\left(1-\frac{N_1^C}{2}\right)N_1^C = \frac{\sigma}{16}\frac{(n^2+2)(3n+2)(n+2)}{(n+1)^4}$. By comparing the surpluses over all $n \geq 2$ it is clear that surplus is always greater with Bertrand competition. □

Proof of Theorem 5: The result follows by comparing Equation (38) with Equation (46), which implies that $W^C(n) > W^B(n)$ for all n; by comparing Equation (35) with Equation (43), which implies that $\Pi^C(n) > \Pi^B(n)$ for all n; by comparing Equation (37) with Equation

(45), which implies that $SS^C(n) > SS^B(n)$ for all n; and by comparing Equation (36) with Equation (44), which implies that $CS^C(n) > W^C S(n)$ for all $n \geq 3$. □

Proof of Theorem 6: The surplus within a sub-market with Bertrand competition is $\int_0^{N_1^I} \frac{\sigma}{2}(1-\tau)d\tau = \frac{3\sigma}{16}$ which is equivalent to the surplus generated with Bertrand competition from the proof of Theorem 4. □

Proof of Theorem 7: Comparing equilibrium results gives:

$$\Pi^I = \frac{\sigma^2}{128c} > \frac{\sigma^2}{256c} \cdot \frac{n \cdot (n+2)^4}{(n+1)^6} = \Pi^C,$$

$$\Pi^I = \frac{\sigma^2}{128c} > \frac{\sigma^2}{256nc} = \Pi^B,$$

$$CS^I = \frac{\sigma^2}{64c} > \frac{\sigma^2}{256c} \cdot \frac{n^2(n+2)^4}{(n+1)^7} = CS^C,$$

$$CS^I = \frac{\sigma^2}{64c} > \frac{\sigma^2}{256nc} = CS^B,$$

$$W^I = \frac{3\sigma^2}{128c} > \frac{\sigma^2}{512c} \cdot \frac{n \cdot (n+2)^4 \cdot (5n+3)}{(n+1)^7} = W^C,$$

$$W^I = \frac{3\sigma^2}{128c} > \frac{5\sigma^2}{512nc} = W^B.$$

□

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