Competing by Restricting Choice: The Case of Matching Platforms^{*}

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Abstract

We show that a two-sided matching platform can successfully compete by limiting the number of choices it offers to its customers, while charging higher prices than platforms with unrestricted choice. We develop a stylized model of online dating where agents with different outside options match based on how much they like each other. Starting from these micro-foundations, we derive the strength and direction of indirect network effects, and show that increasing the number of potential matches has a positive effect due to larger choice, but also a negative effect due to competition between agents on the same side. Agents resolve the trade-off between these competing effects differently, depending on their outside options. For agents with high outside options, the choice effect is stronger than the competition effect, leading them to prefer an unrestricted-choice platform. The opposite is the case for agents with low outside options, who then have higher willingness to pay for a platform restricting choice, as it also restricts the choice set of their potential matches. Moreover, since only agents with low outside options self-select into the restricted choice platform, the competition effect is mitigated further. This allows multiple platforms offering different number of choices to coexist without the market tipping.

Keywords: matching platform; indirect network effects; limits to network effects

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1 Introduction

In markets with network effects, consumers' utility for a product is partly determined by how many other consumers use that product (Katz and Shapiro, 1994; Ambrus and Argenziano, 2009), either because a consumer directly values the presence of other consumers (direct network effects), or indirectly through complements (indirect network effects).¹ Therefore, in markets with indirect network effects, firms are often advised to provide customers with a large choice of complements, without sacrificing quality, as this will allow them to charge higher prices. However, in certain markets with network effects we observe firms which thrive by actively reducing the number of choices, while charging higher prices. Consider, for example, the online heterosexual dating industry, where users seek to match with candidates of the opposite gender. Most sites, such as Match.com, compete by building the largest user base possible, and providing users with access to unlimited profiles on the platform. Others, such as eHarmony.com, pursue user growth with the same intensity, but allow users to only view and contact a limited number of others on the platform.² Despite the limited choice, eHarmony's customers on average pay 25% more to use the platform than Match's customers do to use theirs.

In this paper, we explain the ability to charge more despite offering less by exploring the interplay between two opposite effects that arise when a matching platform offers access to more candidates.³ These effects are especially prevalent when the two sides of the platform receive symmetric treatment, and the platform offers smaller or larger number of candidates to both sides of the market. On the one hand, agents are more likely to find more desirable candidates on a platform offering more choice. We call this the *choice effect*. On the other hand, these candidates have more potential matching options, which increases the probability that the candidate picks someone else as their best potential match, which decreases the probability that the agent will be accepted by the candidate he or she likes the most. We call this the *competition effect*.

We build a model where we derive properties of indirect network effects, and show that agents resolve the trade-off between the two opposing forces differently depending on their utility from staying alone, should they fail to match. Those who have high utility from staying alone do not

¹Seminal papers on network effects (e.g., Katz and Shapiro, 1985, 1994) define a positive network effect to exist when the value of joining the platform increases with the number of agents participating. This definition applies to both direct and indirect network effects. For the indirect network effect, in a two-sided platform, a positive network effect means that a consumer obtains higher utility from participating in a platform with larger number of agents on the same side. Such a benefit arises indirectly: having more agents on the same side increases the participation of agents on the other side. With more agents on the other side, the platform can offer a higher expected utility coming from a better match or larger variety.

²For details on eHarmony and Match, see Piskorski et al. (2009).

 $^{^{3}}$ As Baldwin and Woodard (2009) point out, the term "platform" is used in three distinct but related fields: product development, technology strategy, and industrial economics. We use it in the industrial economics sense to refer to a meeting place of consumers facilitated by a seller (Hagiu, 2014).

find rejection as costly, but want a broad choice of candidates who they like more than their high outside option.⁴ As a consequence, they will gravitate to platforms, such as Match, which offer a larger selection of candidates. Agents with low utility from staying alone find rejection by their matching candidates costly. Thus, they will avoid platforms with unrestricted access where competition effects are strong, and instead will gravitate to platforms such as eHarmony, where the likelihood of being accepted is higher, at the cost of seeing fewer candidates.⁵ There, they can improve the probability of being accepted for two reasons: (i) candidates they encounter have less choice, (ii) candidates attracted to the limited-choice platform also have a lower utility from being alone, which means that they are even more likely to accept the match. In fact, as we show later, these agents are willing to pay more to join the limited-choice platform to drive away the agents and candidates with high utility from being alone, whose presence would reduce the probability of being accepted. Therefore, in equilibrium the limited-choice platform can coexist in the market with a platform offering more choice, and charge higher prices.

We obtain this result only if we depart from typical assumptions in the literature on network effects and platform competition in two ways. First, prior literature on the topic has mostly assumed the presence of positive network effects, with some papers exploring the possibility of negative effects.⁶ By deriving the network effects from the matching environment, we show that the strength and direction of the network effect may vary in the same environment. For example, in our model, when a platform offers only a few candidates, there is little competition among agents on the same side, allowing the choice effect to dominate, and leading to a positive network effect. As the choice set increases, competition among agents on the same side increases and the competition effect becomes larger than the choice effect, resulting in a negative network effect.

Second, prior literature has paid little attention to differences in agents' outside options, and hence preferences for choice. Our model explicitly recognizes that agents have different outside options which changes how sensitive they are to the choice and competition effects. This means that the point at which positive network effect turns into a negative one is different for different agents. We show that the heterogeneity in agents' outside options allows for coexistence of platforms competing with different business models: those that offer more choice, and others that actively limit choice. Different offerings appeal to different types of customers.

⁴While the model presented here is a one-period model where agents differ in their utility from staying alone at the end of the period, the results remain the same if we interpret it as a dynamic model where agents differ in their patience. Agents with low utility from being alone correspond to agents who feel greater immediacy to find a match. Less patient agents join the platform with limited choice because it increases their chance to find a match sooner.

⁵This interpretation is consistent with eHarmony's advertising which focuses on people who want to get married, for whom the utility of being alone is low, as compared to those who want to date, but do not seek a long-term commitment to being in a relationship.

⁶Literature related to our model is discussed more extensively in Section 2.

Our model critically assumes the presence of same-side competition effect on both sides of the market. When this condition is met, the model and the insights it generates can be applied outside the dating example we use. Consider, for example, a labor market in which employers compete with each other to hire workers, and workers compete with each other to get jobs. In such a market, we observe headhunters that present employers and candidates with a limited number of choices, while charging a higher price, successfully co-exist with (online) platforms that offer a large pool of candidates and job posts. Our model suggests that even if headhunters provided no other services than limiting choice, they would still be able to coexist with other platforms, despite charging higher prices, as long as there is a segment of employers or candidates for whom the cost of not finding an acceptable match quickly is high. Empirical evidence documenting prevalence of headhunter use in such segments of the labor market is consistent with this explanation (Khurana, 2004).

Similarly, in real estate markets, we observe competition between sellers to find a buyer, and competition between buyers to secure a property. There, we observe costly real estate agents who usually limit the number of properties shown to clients, and vice versa, co-exist with unrestricted matching environments, such as For Sale By Owner (FSBO) or Zillow. Again, empirical evidence shows that people who value closing a real estate transaction quickly are more likely to opt for a real estate agent, as opposed to FSBO, consistently with the predictions from our model (Hendel et al., 2009).

At the same time, same-side competition on both sides of the platform is not always present. Consider, for example, video gaming platforms. Even though game producers compete with each other for gamers' money and attention, when a gamer buys a video game, she does not prevent others from buying a copy of the same game. Since same-side competition does not exist between gamers, our model would not apply in this context. As a consequence, a different explanation would be needed to explain why during certain periods of the history of the video game industry some platforms allowed unrestricted access of game developers to players, while others restricted it (Casadesus-Masanell and Hałaburda, 2014).

Our model also assumes that the platform changes access to candidates on both sides of the market in the same direction, and thereby excludes scenarios in which access to one side is restricted, but increased on the other side. Such co-movement on both sides is not a contrived element of our model, but rather an inherent property of markets with indirect network effects if the platform does not actively restrict the access. This is because by the standard logic of indirect network effects, increasing the number of agents on one side makes the platform more attractive to the other side, resulting in an increase in the number of agents on the other side, which benefits the agents on

side one. Thus, agents indirectly benefit from increasing the number of agents on the same side, because it induces the co-movement on the other side.

Finally, our model does not rely on any assumption of psychological aversion to abundant choices to explain why users may prefer limited choice platforms. Neither do we presume that matching platforms that restrict choice have any ability to recognize which matches may be better than others.⁷ Of course, our model does not preclude these alternative explanations. Indeed, if a matching platform that restricts choice also offers more compatible candidates, or if people have a distaste for excessive choice, it may be even more successful than our model predicts.

The remainder of the paper is structured in the following way: Section 2 provides a review of the related literature. Section 3 sets up the model and then analyzes the strength of and the limit to network effects, and how they depend on an agent's utility of staying alone. We show that as the number of candidates on both sides increases, positive network effects disappear and turn negative for agents with lower utility from being alone. Section 4 investigates a market with a matching platform, and shows that an equilibrium always exists where agents pay to participate in a platform that offers fewer candidates than the outside market, which is accessible for free. Section 4.1 focuses on the analysis of a monopolist platform whereas Section 4.2 shows that the findings are also true when two strategic platforms compete. Section 5 discusses the importance of key assumptions for the results and Section 6 concludes.

2 Related Literature

Our paper analyzes network effects and platform competition. Previous work on network effects has by and large assumed that the presence of other agents on the platform exogenously increases utility, usually in a linear form (e.g., Rochet and Tirole, 2003). As a consequence, every additional agent on the platform increases the overall payoff to others, no matter how many other agents are already available. For this reason, when platforms compete with each other, the one offering the largest choice should take over the market (e.g., Katz and Shapiro, 1985). In this paper, we derive the magnitude or direction of the network effects from micro-foundations of a particular matching environment. In this environment, we identify not only the positive (opposite-side) choice effect, but also a negative (same-side) competitive effect; and we study how the trade-off between the two effects leads to both positive and negative network effects in the same environment, which allows for coexistence of platforms with different business models.

⁷eHarmony claims to have an algorithm for generating superior matches between its users. Although the algorithm alone could generate higher willingness to pay and hence the price premium, our model shows that even without a superior algorithm, eHarmony could charge a higher price solely by limiting choice.

More recently, a few papers examined the trade-off between the positive opposite-side effect and the negative same-side effect to show how multiple firms can coexist in environments with network effects. For example, Ellison and Fudenberg (2003) and Ellison et al. (2004) study of co-existence of auction platforms of different sizes. Similarly to our paper, they assume that agents are heterogeneous. In contrast, however, their agents choose platforms before they know their type. while ours are aware of their type prior to choosing their platform. Furthermore, they assume that the clearing price on every platform is determined by the ratio of buyers to sellers. Then, they show that multiple auction platforms of different sizes can coexist as long as they have the same buyerto-seller ratio. However, the model does not generalize well outside the auction environment, and neither does it explain (or seek to do so) why some platforms may reduce the number of matching options and charge more for the reduced choice. Several other economists have identified the costs that arise from increasing the number of options in a network. Calvó-Armengol and Zenou (2005), for example, suggested that in the context of a labor market, being connected to too many agents in a random matching network can result in frictions and reduce the probability of a match in a job network. That study, however, does not identify the limits of positive network effects for each agent; instead, it arbitrarily assumes the same limit for all.

Our model is closest to Damiano and Li (2007), who examine why a revenue-maximizing monopolist would establish many matching platforms with different access prices. In their model, agents differ in productivities, and join a platform where they are presented with a single candidate to consider. They show that the matching platforms use prices to pool similar agents together, just as in our model. Their model, however, cannot explain why it would be optimal for the matching platforms to offer different number of candidates. Even if allowed, it would never be beneficial in their model for any of the platforms to offer more candidates.⁸

An emerging literature in strategy explores competitive interaction between organizations with different business models. Casadesus-Masanell and Ghemawat (2006) and Economides and Katsamakas (2006), for example, study duopoly models in which a profit-maximizing competitor interacts with an open-source competitor. Casadesus-Masanell and Hervas-Drane (2010) study competitive interaction between a high-quality incumbent and a low-quality ad-sponsored competitor. Finally, Casadesus-Masanell and Zhu (2010) analyze competitive interactions between a free peerto-peer file-sharing network and a profit-maximizing firm that sells the same content at a positive price, and distributes digital files through an efficient client-server architecture. In our paper, firms could be seen as competing with different business models, as one matching platform deliberately limits the choice (to all its customers) while competing against one that offers unlimited choice

⁸Section 6 offers more in-depth comparison of the results.

within its database. We study forces in the market that allow such competition to be successful.

Our study also relates to the literature on *co-opetition* (e.g., Brandenburger and Nalebuff, 1996; MacDonald and Ryall, 2004). In their widely-cited book, Brandenburger and Nalebuff (1996) argue that seemingly competitive product offerings may in fact act like complements and yield positive network effects. In many ways, the two forces we describe in our study, namely the competition and choice effects point out the complementary and substitutionary characteristics of candidates on a platform. Each candidate simultaneously acts as a complement for agents on the opposite side and a substitute for agents on the same side of the market, resonating with the arguments made by Brandenburger and Nalebuff (1996) and MacDonald and Ryall (2004). By deriving network effects in our model from micro-foundations, we show how the interplay between the competitive and cooperative forces affects the properties of the network effect. Moreover, we study how those forces affect the successful strategies of competing firms.

With our model, we show why some agents may prefer an environment with less choice. The reasons why rational agents would make such decisions might be of relevance to the branch of behavioral economics and psychology dealing with the negative outcomes of increasing choice. Work in this area suggests that providing a larger number of choices might eventually decrease the satisfaction and happiness levels of consumers, suggesting behavioral mechanisms such as decision fatigue, choice overload, and cognitive costs (e.g., Iyengar et al., 2006; Schwartz and Ward, 2004). Our study shows that even in the absence of behavioral considerations, there is an economic explanation for why some agents may obtain lower overall utility in environments offering more choice.

3 Matching Environment

We use a stylized example of a two-sided heterosexual dating market for stability of reference, and call one side "men" (denoted by the letter m) and the other side "women" (denoted by the letter w). Each agent can match to at most one agent on the other side. If they do not match, we call their outside option "staying alone." Agents are heterogeneous with respect to how much utility they receive from being alone, denoted by variable \mathfrak{a} with cumulative distribution function $G(\mathfrak{a})$ on the interval [0, 1]. The value \mathfrak{a} is private information for each agent.

There are two stages in the matching game. In the first stage, every agent (on either side of the market) meets some fixed number of N agents from the other side of the market. The number of candidates, N, is the comparative statics parameter in this model; we investigate how an increase in this parameter influences the expected payoff of agents. Note that the platform offers the same number of candidates to agents on both sides of the market.⁹ In the second stage, all agents simultaneously make at most one offer. The one-offer assumption made throughout the paper simplifies the analysis and the intuition behind the results.¹⁰ A match between man m and woman w happens if m made his offer to w and w has made her offer to m (i.e., the offer has been "reciprocated" or "accepted").

Let $\Lambda^m(w)$ represent how much the man m likes being with the woman w, and $\Lambda^w(m)$ represent how much the woman w likes being with the man m. We assume that both the woman's and the man's liking functions are drawn from some generalized distribution with the distribution $G(\Lambda)$ on the interval [0, 1].¹¹ When a man m meets a woman w,¹² he learns $\Lambda^m(w) \in [0, 1]$, i.e., how much he will like being in a relationship with her. Similarly every woman w learns $\Lambda^w(m) \in [0, 1]$ about every man m she meets.

For a man m_i with \mathfrak{a}^{m_i} to make an offer to a woman w_i , two conditions must be satisfied. First, he must like woman w_i more than staying alone $(\Lambda^{m_i}(w_i) > \mathfrak{a}^{m_i})$. Second, he must like w_i more than the other N-1 women he meets $(\Lambda^{m_i}(w_i) > \Lambda^{m_i}(w_j), \forall j = 1, 2, ..., N, j \neq i)$. For a successful match, the same must hold for the woman w_i ; she must like m_i more than she likes being alone $(\Lambda^{w_i}(m_i) > \mathfrak{a}^{w_i})$, and more than the other N-1 men she meets $(\Lambda^{w_i}(m_i) > \Lambda^{w_i}(m_j), \forall j =$ $1, 2, ..., N, j \neq i)$. When all of these conditions are satisfied, offers of m_i and w_i are reciprocated and a successful match takes place. If their offers are reciprocated, agents receive their respective payoffs of $\Lambda^{m_i}(w_i)$ and $\Lambda^{w_i}(m_i)$. If an offer was not reciprocated (i.e., it is "rejected") the agent who made the offer remains unmatched and receives his or her respective utility from \mathfrak{a} . The game ends with these payoffs.

An important assumption in our framework is the independence of Λ from other values of Λ and \mathfrak{a} . This implies that the function Λ is subjective in our model: the utility from w from being matched to m, $\Lambda^w(m)$, is intrinsic to w and is privately known by her and does not depend on \mathfrak{a}^m . In other words, our model assumes that agents differentiate their dating preferences "horizontally" rather than "vertically". This assumption has three consequences. First, how much two agents like each other is not correlated. This implies that the extent to which a man likes a particular woman

 $^{^{9}}$ We consider markets where the two sides are treated symmetrically. Platforms literature has shown the potential of asymmetric treatment of the two-sides (e.g., Parker and Van Alstyne, 2005). However, in many markets firms are restricted to treating both sides symmetrically, for legal or technical reasons.

¹⁰The assumption that limits agents to only one offer is meant to reflect the fact that people are able to pursue only limited number of possible relationships. This restriction applies also to other matching markets. In labor market, for example, although the employers screen dozens of applicants, they may have capacity for a much smaller number of interviews. Because this is a potentially restrictive assumption, Appendix A.1 considers tentative offer-making procedures while searching for a potential match and shows that the results hold also under more realistic procedures.

¹¹Where there is no risk of confusion, the notation is simplified by dropping superscripts. For example, $\Lambda^m(w)$ may be simplified to Λ .

¹²If m meets w, then it must be that w meets m.

is independent of how much she likes him. Second, how much a man (woman) likes a woman (man) is independent of how much the other men (women) like her (him). Finally, an agent can like two different agents at different rates. That is, how much m likes w_1 is also independent of how much he likes another woman w_2 .

The independence assumption differentiates our model from many matching models. Existing literature mostly focuses on agents' attributes that are desired by all potential partners. Such attributes can be characterized as measures of objective "quality" (e.g., Becker, 1973; McAfee, 2002; Damiano and Li, 2007, 2008). We want to study markets where preferences are more subjective—how much an agent likes a potential romantic partner is different from other agent's liking. In the main model, we assume full subjectivity. This can be justified when considering candidates within a certain category (e.g., the same education, status, sense of humor). For a more generalized approach, it is more appropriate to allow for partial correlation between agents' preferences.¹³

Lemma 1 identifies important characteristics of the described matching market.

Lemma 1 In a market with N candidates:

(i) For every agent the probability of being rejected by a candidate on the other side of the market is

$$Pr(rej|N) = \frac{N}{N+1}$$

That is, the probability of being accepted is $\frac{1}{N+1}$.

(ii) An agent \mathfrak{a} matches successfully with probability

$$(1 - Pr(rej|N)) (1 - G^N(\mathfrak{a}))$$
.

(iii) For an agent \mathfrak{a} the expected value of a match, conditional on being accepted, is

$$N\int_{\mathfrak{a}}^{1}G^{N-1}(\Lambda)g(\Lambda)\Lambda d\Lambda.$$

(iv) The total expected payoff for agent \mathfrak{a} is

$$EU(\mathfrak{a}|N) = \underbrace{\left[1 - (1 - Pr(rej|N))\left(1 - G^{N}(\mathfrak{a})\right)\right]}_{prob. of \text{ not matching}} \mathfrak{a} + \underbrace{\left(1 - Pr(rej|N)\right)}_{prob. of acceptance} \cdot \underbrace{N \int_{\mathfrak{a}}^{1} G^{N-1}(\Lambda)g(\Lambda)\Lambda d\Lambda}_{exp. payoff if accepted}$$

¹³In Appendix A.2, we use simulation to explore a matching environment where Λ 's are correlated with \mathfrak{a} 's. The results from the simulation show the qualitative insights from the main model hold for less-than-perfect correlations.

Proof: See Appendix B, page 32.

When a man makes an offer to a woman, he does not know her \mathfrak{a} or how much she likes him versus the other men she has met. A priori, she is equally likely to make an offer to any of the men, or not make an offer at all. Therefore, the probability that the offer is reciprocated by the woman is $\frac{1}{N+1}$. This is equivalent to the probability of being rejected $\frac{N}{N+1}$. The probability of being rejected increases with N, because the agent's candidates also have a larger choice set. This result is captured in part (i) of Lemma 1.

Whether an agent matches successfully depends on two factors: whether he wants to make an offer and whether the offer is reciprocated. Whether the agent wants to make an offer depends on his \mathfrak{a} . From part (i) of Lemma 1, the agent's offer is reciprocated with probability $\frac{1}{N+1}$. Because the expected probability of rejection is the same for all candidates, the optimal strategy is to make an offer to the best candidate, if that candidate is above the agent's \mathfrak{a} . With probability $G^N(\mathfrak{a})$, all N candidates are liked less than the outside option of the agent, \mathfrak{a} . With the remaining probability $1 - G^N(\mathfrak{a})$, the Λ the best candidate yields is above \mathfrak{a} . Combining the probability of making an offer and the probability that the offer is reciprocated, the probability of successfully matching is captured in part (ii) of Lemma 1.

If the offer is reciprocated, it means that the agent has matched with the highest Λ among the N candidates and this highest Λ was above his utility from being alone, \mathfrak{a} . The expected value of a match is formalized in part (iii) of Lemma 1; it is equivalent to the expected value of the maximum Λ , given that it is above \mathfrak{a} .

Part (iv) of Lemma 1 puts together all the previous three parts and formalizes the expected payoff of an agent \mathfrak{a} in a market with N candidates. We say that *positive* network effects are present if increasing the size of the accessible network, N, increases an agent's expected payoff. In contrast, when increasing the size of accessible network decreases an agent's expected payoff, we consider the network effects to be *negative*.

A number of properties follow directly from Lemma 1. Two of them, choice effect and competition effect, characterized in Corollaries 1 and 2, play especially important roles in our analysis.

Corollary 1 (Choice Effect) For any a < 1, expected value of a match, conditional on successfully matching, is nondecreasing with the number of candidates (N):

$$\frac{\partial \left(N \int_{\mathfrak{a}}^{1} G^{N-1}(\Lambda) g(\Lambda) \Lambda d\Lambda\right)}{\partial N} \ge 0.$$

Proof: See Appendix B, page 33.

Corollary 1 states that the expected value from a successful match is non-decreasing when an agent meets more candidates. As N increases, conditionally on a successful match, the agent can expect to match with a woman of his higher liking. We refer to this effect as the *choice effect*. This would suggest that an agent can achieve higher expected utility when dating in a market with more candidates, and in many environments, this effect is the driver of the positive network effects. However, we also need to take into account the *competition effect*, stated in Corollary 2 below. The probability that an agent \mathfrak{a} 's offer will be accepted is decreasing with N. With more candidates, each woman has more men to choose from, i.e., every man has more competition. Notice that increasing your own choice set also increases the choice set of your candidates. This in turn decreases the probability that a woman w wants to match with man m, when m wants to match with w.

Corollary 2 (Competition Effect) For every agent a < 1, the probability of being rejected is increasing in N.

Proof: Follows directly from part (i) of Lemma 1.

So, does a market offering a larger number of candidates make the agents better off? Corollaries 1 and 2 document effects going in opposite directions: the expected pay-off for an agent joining the platform depends on both the choice and the competition effect. If the expected payoff for an agent increases as N increases, there is a positive indirect network effect: having more agents on the same side increases the agent's utility because it is tied to increasing the number of candidates on the other side of the market. This is because expanding your own choice set at the same time also expands the choice set on the other side, and lowers the probability of a match. Proposition 1 shows that there are positive network effects, but they reach their limit, and then turn negative, as N increases. The limit to network effects emerges because some agents gain more from decreasing the choice set of the candidates they meet than they lose by reducing their own choice set.

Since the choice and the competition effects affect different agent types asymmetrically, the limit to the network effect is different for different types. The optimal size of the choice set is larger for agents with a higher utility from staying alone.

Proposition 1 (Limits to Positive Network Effects)

(i) For every \mathfrak{a} , there exists $\overline{N}(\mathfrak{a})$ such that $EU(\mathfrak{a}|N+1) - EU(\mathfrak{a}|N)$ is positive for $N < \overline{N}(\mathfrak{a})$, and negative for $N \ge \overline{N}(\mathfrak{a})$.

(ii) $\overline{N}(\mathfrak{a})$ is non-decreasing in \mathfrak{a} .

Proof: See Appendix B, page 34.



Figure 1: Limits to the network effect as a function of \mathfrak{a} .

Proposition 1 coins the first main insight of this paper: for every agent, there exists a limit beyond which there are no positive network effects. The choice effect, stated in Corollary 1, declines in strength as N increases. Each additional candidate increases the expected value of a successful match by a smaller amount than the previous one. At the same time, the competition effect, stated in Corollary 2, increases in N. The agent is less likely to be accepted as his or her candidates also have a larger set to choose from. With these two opposing forces, the positive network effect experienced by agent \mathfrak{a} declines in strength as N increases, until it reaches its limit at $\overline{N}(\mathfrak{a})$. Above that level, an increase of N decreases agent's expected payoff: above $\overline{N}(\mathfrak{a})$ the network effect is negative. The agent gains less by having more candidates than he loses when his candidates have more choices.

Additionally, part (ii) of Proposition 1 states that for agents with higher \mathfrak{a} 's, $\overline{N}(\mathfrak{a})$ is larger as illustrated in Figure 1. For agents with low \mathfrak{a} , it is likely that a few candidates already provide matching a value above \mathfrak{a} . Meeting more candidates does not increase this probability enough to offset the increased probability of having the offer rejected. However, for agents with high \mathfrak{a} , the increase in the probability that at least one candidate is better than \mathfrak{a} offsets the increased probability of having an offer rejected, as the agent meets an additional candidate. That is, agents with low \mathfrak{a} gain less by having more choices, and lose more by their candidates having more choices. Conversely, agents with high \mathfrak{a} gain more from a larger choice set than they lose by their candidates having a larger choice set. If the number of candidates N is large enough, agents with low \mathfrak{a} prefer lowering N, while agents with high \mathfrak{a} prefer increasing N even more. That is, agents with low \mathfrak{a} feel that they are in a market with "too many candidates." This property is driven mainly by the assumption that agents can court a limited number of candidates.¹⁴ The larger the pool, the smaller the probability that the agent is within the limited number of courted candidates.

Our analysis so far implies that network effects (both the strength and direction) depend not only on the agent's type, \mathfrak{a} , but also on how much competition the market allows, N. In contrast, most of the literature on platform competition assumes that the number of agents on the other side of the market enter the payoff function linearly, i.e., every agent on the other side of the network contributes the same amount to the expected payoff (e.g., Rochet and Tirole, 2003). In those papers, if the competitive effect is included, every additional agent on the same side of the market also affects the payoff linearly. The positive network effect is present when the choice effect outweighs the competition effect. But since the effects are assumed to be constant, the positive network effect never weakens or disappears, and it is always better for agents when the size of the accessible network increases (i.e., there are more choices and more competition). In such a set-up it is always profit maximizing for a platform to offer access to all the agents who have joined.¹⁵ Moreover, it would not be possible for a platform that restricts choice to attract agents away from a market with more choices and more competition. As we have shown in this section, departing from the linearity assumption, and deriving network effects from micro-foundations in a particular environment lets us identify a more nuanced interplay of choice and competition effects that gives rise to network effects that differ in strength and direction.

In the next section, we show that given the described properties of the matching market and limits to network effects, a platform can successfully operate in a market by offering fewer choices to all its customers compared to the market outside of the platform, which offers more choices to everyone.

4 Matching Platforms

The previous section considered the effect of the market offering a smaller or larger number of candidates on agents' payoffs in an environment where all agents are in the same market. In this section, we analyze an environment where agents can choose between multiple platforms that offer different numbers of candidates. As we expect, because of the forces identified in the previous section, different types of agents prefer to join platforms offering different numbers of candidates.

¹⁴Our main model assumes that an agent can court only one candidate. In Appendix A.1, we show that the results hold when every agent can court an arbitrary but fixed number of candidates.

¹⁵Notice that in our environment the platform could have many more agents joining but offer access to N candidates.

This self-selection of types has a further effect on performance of each platform, as agents of different types are, on average, more or less likely to reject a candidate.

4.1 Matchmaking Platform and Non-strategic Outside Market

The trade-off outlined in the previous section demonstrates that some agents prefer an environment offering fewer candidates, as it also reduces competition. We have shown that the trade-off varies with the type of agent, \mathfrak{a} . In this section, we explore strategic opportunities that those properties offer to a matching platform. In particular, we focus on the fact that a platform may earn positive profits when providing *fewer* candidates than the outside market to its customers, as long as it offers fewer candidates to agents on both sides.

Let the *outside market* be a decentralized, non-strategic market, where each agent meets Ω candidates and pays no fee. There is also a matching platform offering $N < \Omega$ candidates, and charging a positive participation fee f. Agents decide whether to participate in the platform or stay in the outside market.

The strategy of providing fewer candidates restricts the choice, but also reduces the competition, which results in a lower rejection probability. A restricted-choice platform will attract those agents who lose relatively little when reducing their own choice, but gain a lot from reducing the choice of their candidates. Hence, from the earlier analysis, we know that only agents with a sufficiently low \mathfrak{a} prefer to participate in the platform at a given positive fee. Agents with higher \mathfrak{a} prefer to stay in the outside market. In other words, agents for whom the competition effect is large compared to the choice effect are willing to pay a positive fee to participate in such a platform. Therefore, candidates that can be met in the restricted-choice platform have \mathfrak{a} 's drawn from a truncated distribution. The resulting self-selection further influences the rejection probabilities in the platform and in the outside market.

To characterize equilibria in such a market, we start with a situation in which *all* agents stay in the outside market, which is always an equilibrium. However, there also always exist equilibria where some agents participate in the platform. We focus our investigation on the equilibria where the platform is active (i.e., some agents participate in the platform). Specifically, we show that for $N < \Omega$, there always exists an equilibrium where some agents pay a positive price to participate in the platform. To characterize this equilibrium, we start by considering how an agent's willingness to pay for joining the platform changes with his or her type, \mathfrak{a} . An agent is willing to pay up to the additional utility that the platform provides above the outside market, i.e., $WTP(\mathfrak{a}) =$ $EU(\mathfrak{a}|N) - EU(\mathfrak{a}|\Omega)$. When an agent makes his individual decision about whether to join the platform, he takes others' actions as given; thus, the platform's fee and rejection probabilities are constant from the point of view of the agent. (In an equilibrium, the rejection probabilities are determined by all agents' participation decisions.) Whereas the rejection probabilities and the fee are the same for all agents, the expected payoff depends on \mathfrak{a} .

Lemma 2 For any given Pr(rej|N) and $Pr(rej|\Omega) > Pr(rej|N)$ and for any Ω and $N < \Omega$, the willingness to pay $WTP(\mathfrak{a}) = EU(\mathfrak{a}|N, Pr(rej|N)) - EU(\mathfrak{a}|\Omega, Pr(rej|\Omega))$ is positive and decreasing for $\mathfrak{a} \in [0, \tilde{\mathfrak{a}})$, where $0 < \tilde{\mathfrak{a}} \leq 1$. For $\mathfrak{a} \in (\tilde{\mathfrak{a}}, 1]$, $WTP(\mathfrak{a})$ is negative.

Proof. See Appendix B, page 35.

An agent prefers to join the platform only if the benefit of joining outweights the fee, i.e., $WTP(\mathfrak{a}) > f$. Therefore, it follows from Lemma 2 that for any Pr(rej|N) and $Pr(rej|\Omega) > Pr(rej|N)$ and some positive fee f,¹⁶ there exists $\hat{\mathfrak{a}} \in (0,1)$ such that agents with $\mathfrak{a} \in [0, \hat{\mathfrak{a}})$ strictly prefer to join the platform at f, agents with $\mathfrak{a} \in (\hat{\mathfrak{a}}, 1]$ prefer to stay outside, and agents $\hat{\mathfrak{a}}$ are indifferent. The properties characterized by Lemma 2 guarantee also that $\hat{\mathfrak{a}}$ is unique.

Under this circumstance, the probability of rejection for an agent is affected by the fact that \mathfrak{a} of candidates is not drawn from the whole distribution, but from a subinterval $[\underline{\mathfrak{a}}, \overline{\mathfrak{a}}] \subset [0, 1]$. We can no longer rely on the rejection probability characterized in Lemma 1. Instead, when taking self-selection into account, the probability of rejection is

$$Pr\left(rej|N, \mathfrak{a} \in [\underline{\mathfrak{a}}, \overline{\mathfrak{a}}]\right) = 1 - \frac{1}{N} + \frac{1}{N(N+1)} \cdot \frac{G^{N+1}(\overline{\mathfrak{a}}) - G^{N+1}(\underline{\mathfrak{a}})}{G(\overline{\mathfrak{a}}) - G(\underline{\mathfrak{a}})} \,.$$

Notice that $Pr(rej|N, \mathfrak{a} \in [0, \widehat{\mathfrak{a}})) < Pr(rej|\Omega, \mathfrak{a} \in (\widehat{\mathfrak{a}}, 1])$ for any $\Omega, N < \Omega$ and $\widehat{\mathfrak{a}}$. This comes from two separate forces working in the same direction: (i) because $N < \Omega$, and (ii) because lower \mathfrak{a} 's join the platform. With $Pr(rej|N) < Pr(rej|\Omega)$, the premise of Lemma 2 is satisfied. It is also worth noting that when the rejection probability is higher in the platform than in the outside market, i.e., $Pr(rej|N) > Pr(rej|\Omega)$, no agent joins the platform at any positive f.¹⁷ Therefore, there does not exist an equilibrium with an active platform and $Pr(rej|N) > Pr(rej|\Omega)$.

The platform sets its fee, f, with the objective of maximizing its profit.¹⁸ In equilibrium, it must be that $EU(\hat{\mathfrak{a}}|N, Pr(rej|N, \mathfrak{a} \in [0, \hat{\mathfrak{a}}))) - EU(\hat{\mathfrak{a}}|\Omega, Pr(rej|\Omega, \mathfrak{a} \in (\hat{\mathfrak{a}}, 1])) = f$. This condition characterizes the threshold $\hat{\mathfrak{a}}$ on which the market settles for any f chosen by the platform. Moreover, $\hat{\mathfrak{a}}$ uniquely characterizes the rejection probabilities, for a given N and Ω . Therefore, we can solve the problem as if the platform was choosing $\hat{\mathfrak{a}}$ directly instead of choosing f.

¹⁶As long as the fee is not prohibitively high, i.e., $f < EU(\mathfrak{a} = 0|N, Pr(rej|N)) - EU(\mathfrak{a} = 0|\Omega, Pr(rej|\Omega)).$

¹⁷This can be shown by the same reasoning as in the proof of Lemma 2. See Corollary B.1 in Appendix B, page 36. ¹⁸We assume that all the costs for the platform are fixed costs, and the marginal cost is 0. Thus, the profit maximization is equivalent to revenue maximization.

Platform's profit is $G(\hat{\mathfrak{a}}) \cdot f(\hat{\mathfrak{a}})$. Unsurprisingly, for higher fees fewer agents find it worthwhile to participate in the platform, and more agents join at lower fees. Nobody joins (i.e., $\hat{\mathfrak{a}} = 0$) when f rises to 1; to capture the whole market, the platform needs to set $f = 0.^{19}$ However, for intermediate fees (i.e., $\hat{\mathfrak{a}} \in (0,1)$) the profit is positive. Therefore, for any Ω and $N < \Omega$, there exists an equilibrium with an active platform.

Proposition 2 Suppose that in the outside market agents meet Ω candidates, and that there is a platform offering $N < \Omega$ candidates. For any Ω and $N < \Omega$, there exists an equilibrium where the platform maximizes its profit by charging a positive fee f. In this equilibrium there exists a threshold $\mathfrak{a}^* \in (0,1)$ such that agents with $\mathfrak{a} \in [0, \mathfrak{a}^*)$ join the platform, agents with $\mathfrak{a} \in (\mathfrak{a}^*, 1]$ stay in the outside market, and agents with $\mathfrak{a} = \mathfrak{a}^*$ are indifferent.

Proof. See Appendix B, page 37.

Proposition 2 establishes that there is a profitable strategy of limiting the number of candidates. When the platform provides fewer candidates than the outside market, a non-empty interval of agents joins the platform at a positive fee. Interestingly, the platform does not find it profitable to serve the whole market. The rejection probability in the platform is lower than the rejection probability in the outside market for two reasons. First, agents face less competition in the platform, due to $N < \Omega$. Second, agents in the platform are more likely to make and accept an offer, since they have lower utility from being alone. Notice that the first effect comes from the fact that the platform restricts not only the agent's own choice set, but also the choice sets of their candidates. However, the second effect comes directly from the fact that the agent's choice set is restricted). This is because a smaller choice set on the agent's own side attracts candidates with lower \mathfrak{a} on the other side. And it affects the rejection probability separately from the first effect. Observe that the second effect was not present in the analysis of the previous section, as it appears only when agents can choose which platform to join.

The outside market offers more candidates. A larger number of candidates increases the expected value of a match if matching is successful, while decreasing the probability of matching, as the candidates also have more choices. The probability of rejection in the outside market also increases due to the selection effect: the outside market attracts agents with higher \mathfrak{a} , who are more likely to reject all candidates. Nonetheless, for agents with a sufficiently high utility from being alone, the positive choice effect outweighs even this exacerbated negative competition effect. Those agents prefer the outside market, which offers more choice and more competition. Agents

¹⁹See Corollary B.2 in Appendix B, page 37.

with a lower utility from being alone, however, prefer to join the platform, where they have less competition, but also fewer choices.

4.2 Competing Platforms

The previous section analyzed the optimal strategy of a matching platform facing a non-strategic outside market. In this section, we investigate the equilibrium in a market where there are two platforms setting their access fees to maximize their profits. We show that a platform offering fewer candidates can profitably coexist in the market with a platform offering a larger number of candidates. Moreover, the platform with fewer candidates charges a higher price.

Suppose that one platform offers M_1 candidates to all its customers, and the other offers $M_2 > M_1$. We use M_i to denote both the platform and the number of candidates it offers. Each platform i = 1, 2 charges f_i to maximize its profit. We maintain the assumption of single-homing. An agent who does not join either of the platforms remains unmatched.²⁰

Each agent decides which platform to join, if any, given the decisions of everyone else. That is, from the point of view of an individual agent the fees charged by the platforms, and the respective rejection probabilities are constant. (In equilibrium, the agents' participation decisions determine them.)

Lemma 3 For any given $Pr(rej|M_2)$, $EU(\mathfrak{a}|M_2, Pr(rej|M_2)) - \mathfrak{a}$ is positive and decreasing in \mathfrak{a} . Moreover, for $\mathfrak{a} = 1$, $EU(\mathfrak{a}|M_2, Pr(rej|M_2)) - \mathfrak{a} = 0$.

Proof. See Appendix B, page 38.

For a given probability of rejection $Pr(rej|M_2)$, consider a positive fee f_2 .²¹ By Lemma 3, there exists $\hat{\mathfrak{a}}_2 \in (0,1)$, such that agent $\hat{\mathfrak{a}}_2$ is indifferent between joining platform M_2 at f_2 and staying unmatched, i.e., $EU(\hat{\mathfrak{a}}_2|M_2, Pr(rej|M_2)) - f_2 = \hat{\mathfrak{a}}_2$. All agents $\mathfrak{a} > \hat{\mathfrak{a}}_2$ strictly prefer staying unmatched to joining M_2 , while agents $\mathfrak{a} < \hat{\mathfrak{a}}_2$ strictly prefer joining M_2 to staying unmatched.

Applying Lemma 2 to $M_1 < M_2$ and any $Pr(rej|M_1)$ and $Pr(rej|M_2)$, such that $Pr(rej|M_1) < Pr(rej|M_2)$ yields Corollary 3.

Corollary 3 For any given $Pr(rej|M_1)$ and $Pr(rej|M_2)$, such that $Pr(rej|M_1) < Pr(rej|M_2)$, $EU(\mathfrak{a}|M_1, Pr(rej|M_1)) - EU(\mathfrak{a}|M_2, Pr(rej|M_2))$ is positive and decreasing for $\mathfrak{a} \in [0, \bar{\mathfrak{a}})$, where $0 < \bar{\mathfrak{a}} \leq 1$. Moreover, $EU(\bar{\mathfrak{a}}|M_1, Pr(rej|M_1)) - EU(\bar{\mathfrak{a}}|M_2, Pr(rej|M_2)) = 0$

 $^{^{20}}$ We make this assumption because the point of this section is to show the interaction between the two platforms, and the assumption allows for mathematical simplicity of the proofs.

²¹As long as the fee is not prohibitively high, i.e., we only consider $f_2 < EU(\mathfrak{a} = 0|M_2, Pr(rej|M_2))$.

Consider now any $f_1 > f_2$.²² By Corollary 3, there exists $\hat{\mathfrak{a}}_1 \in (0,1)$, such that agent $\hat{\mathfrak{a}}_1$ is indifferent between joining platform M_1 at f_1 and joining M_2 at f_2 , i.e., $EU(\hat{\mathfrak{a}}_1|M_1, Pr(rej, M_1))$ $f_1 = EU(\hat{\mathfrak{a}}_1|M_2, Pr(rej, M_2)) - f_2$. All agents $\mathfrak{a} > \hat{\mathfrak{a}}_1$ strictly prefer M_2 to M_1 , and all agents $\mathfrak{a} < \hat{\mathfrak{a}}_2$ strictly prefer M_1 to M_2 .

If $\hat{\mathfrak{a}}_1 > \hat{\mathfrak{a}}_2$, then no agent chooses to join platform M_2 . This is because agents with $\mathfrak{a} < \hat{\mathfrak{a}}_2$ prefer M_1 to M_2 or staying unmatched; agents with $\mathfrak{a} > \hat{\mathfrak{a}}_1$ prefer staying unmatched rather than joining M_2 or M_1 ; for agents with $\hat{\mathfrak{a}}_2 < \mathfrak{a} < \hat{\mathfrak{a}}_1$ both staying unmatched and joining platform M_1 are more attractive than M_2 .

When $\hat{\mathfrak{a}}_1 < \hat{\mathfrak{a}}_2$, then agents with $\mathfrak{a} < \hat{\mathfrak{a}}_1$ choose M_1 (they prefer M_2 to staying unmatched, but prefer M_1 to M_2 ; agents with $\mathfrak{a} \in (\hat{\mathfrak{a}}_1, \hat{\mathfrak{a}}_2)$ choose M_2 (they prefer it both to M_1 and to staying unmatched); agents with $\mathfrak{a} > \hat{\mathfrak{a}}_2$ choose staying unmatched to either of the platforms. Notice also, that in such a case the resulting rejection probabilities are indeed $Pr(rej|M_1, \mathfrak{a} \in [0, \hat{\mathfrak{a}}_1)) < 0$ $Pr(rej|M_2, \mathfrak{a} \in (\hat{\mathfrak{a}}_1, \hat{\mathfrak{a}}_2)).$

Given the decisions of the agents, platforms decide on their strategies, i.e., setting the fees. Notice, however, that f_1 and f_2 uniquely characterize $\hat{\mathfrak{a}}_1(f_1, f_2)$ and $\hat{\mathfrak{a}}_2(f_1, f_2)$; moreover, $\hat{\mathfrak{a}}_1$ and $\hat{\mathfrak{a}}_2$ uniquely characterize $Pr(rej|M_1,\mathfrak{a}\in[0,\hat{\mathfrak{a}}_1))$ and $Pr(rej|M_1,\mathfrak{a}\in(\hat{\mathfrak{a}}_1,\hat{\mathfrak{a}}_2))$. Therefore, we can think of the platforms as choosing $\hat{\mathfrak{a}}_i^*$ given $\hat{\mathfrak{a}}_i^*$, instead of f_i given f_j .

Platforms' profits are a product of their fees and the measure of agents who join them. First, notice that platform M_1 would never set $\hat{\mathfrak{a}}_1 = 1$, as it would require $f_1 = 0$, and would result in 0 profits, while positive profits for other $\hat{\mathfrak{a}}_1$'s are available. Similarly, M_1 never sets $\hat{\mathfrak{a}}_1 = 0$, as it also results in 0 profits. However, for $\hat{\mathfrak{a}}_1 \in (0,1)$, M_1 's profits are positive.

Similarly, platform M_2 would never set $\hat{\mathfrak{a}}_2 \leq \hat{\mathfrak{a}}_1$, as it would bring it 0 profit. Also, setting $\hat{\mathfrak{a}}_2 = 1$ would require $f_2 = 0$, and would result in 0 profits. But $\hat{\mathfrak{a}}_2 \in (\hat{\mathfrak{a}}_1, 1)$ yields a positive profit for M_2 . Thus, in an equilibrium, $0 < \hat{\mathfrak{a}}_1 < \hat{\mathfrak{a}}_2 < 1$. In the proof of Proposition 3 , we show that such an equilibrium exists.²³

Proposition 3 Suppose that in the market there are two matching platforms that offer M_1 and $M_2 > M_1$ candidates, respectively. For any M_1 and $M_2 > M_1$, there exists an equilibrium where platforms charge positive fees f_1 and $f_2 < f_1$, respectively, and there are two thresholds $\hat{\mathfrak{a}}_1$ and $\hat{\mathfrak{a}}_2$, such that $0 < \hat{\mathfrak{a}}_1 < \hat{\mathfrak{a}}_2 < 1$, and agents with $\mathfrak{a} \in [0, \hat{\mathfrak{a}}_1)$ participate in platform M_1 , agents with $\mathfrak{a} \in (\hat{\mathfrak{a}}_1, \hat{\mathfrak{a}}_2)$ participate in platform M_2 , and agents with $\mathfrak{a} \in (\hat{\mathfrak{a}}_2, 1]$ remain unmatched. Agents with $\mathfrak{a} = \hat{\mathfrak{a}}_1$ are indifferent between M_1 and M_2 , and agents with $\mathfrak{a} = \hat{\mathfrak{a}}_2$ are indifferent between platform M_2 and remaining unmatched.

²²As long as the fee is not prohibitively high, i.e., $f_1 < EU(\mathfrak{a} = 0|M_1, Pr(rej|M_1)) - EU(\mathfrak{a} = 0|M_2, Pr(rej|M_2)) + C(\mathfrak{a} = 0|M_2, Pr(rej|M_2))$ $f_{2.}$ ²³We do not exclude the existence of other equilibria.

Proof. See Appendix B, page 38.

Proposition 3 establishes that two strategic matching platforms can profitably coexist in the market. By the same logic as in the proposition, we can see that a larger number of such platforms—each offering a different number of candidates—could profitably coexist in the market. As the number of matching platforms operating in the market increases, each attracts a smaller interval of agents, thus earning smaller profits. Positive fixed costs of operation or entry costs may discourage more firms from entering the market, as they would not be able to cover those costs. Without fixed costs, and with a continuum of agents, there could be an infinite number of platforms profitably operating in the market—with platforms offering fewer candidates charging higher access fees.

These results bear some resemblance to the results in Damiano and Li (2007). They show how different types of agents self-select into different "meeting places," where they meet similar agents. The tool of separation between the meeting places is the price: Only some types find it worthwhile to pay a higher price. In both their paper and ours, meeting agents of a similar type increases the efficiency of matching. Many assumptions in Damiano and Li's (2007) model differ from our model (see Section 5 for discussion). Most importantly, they do not investigate the network effects. In every meeting place, every agent meets exactly one candidate. In our result, there are two effects. One—the self-selection—is similar to Damiano and Li's (2007), but the other—preferences over the number of candidates the platform offers to all its customers—is not captured by their model.

5 Discussion

This section focuses on two major assumptions of the model, and discusses the significance of these assumptions for the results.

5.1 Heterogeneous Value of Being Alone, a

Many papers in matching literature (e.g., Damiano and Li, 2007, 2008) assume that agents receive zero utility if they remain unmatched. Sometimes this assumption is relaxed by allowing agents to receive some other value when unmatched, but it is usually assumed that this value the same for all agents. However, in many markets, agents differ in the payoff they obtain when unmatched. It is not a trivial assumption, since the equilibria in the market change when we allow agents to differ in their utility from being alone.

Suppose that in our model the value of being alone is 0 for all agents. Then, every agent prefers as few candidates as possible, because this limits competition. A market with more candidates and more competition increases the probability of being rejected and staying alone. With a payoff of being alone of 0, the increase in the expected value of the best candidate does not offset the increased probability of being rejected.

An assumption setting the utility from being alone equal to 0 is an extreme assumption. Suppose that the value of being alone is some $\tilde{\mathfrak{a}}$ from the interval (0, 1), but that it is the same for all agents. Since agents are all the same when they make a decision whether to join the platform, they all make the same decision. For some values of parameters Ω , N and $\tilde{\mathfrak{a}}$, there exists an equilibrium with an active matching platform. In this equilibrium all agents join the matching platform. There always exists an equilibrium where no agent joins the platform. There are no other equilibria. Specifically, there does not exist an equilibrium in which some of agents strictly prefer to participate in the platform and other agents prefer to stay in the outside market.

5.2 Subjective Value of a Candidate, Λ

In a matching market, individuals may value their mates' characteristics differently. Boyd et al. (2013) documents such a situation in the context of a labor market. Employers may value the different characteristics of employees differently. Also employees may value different employers subjectively; for example, teachers' preferences for schools depend on the location. We could expect that such subjectivity plays an even larger role in the dating market, where the taste for the partners is even more idiosyncratic. Several decades of studies in economics, sociology, and psychology suggest that people differ in their valuation of characteristics of an ideal romantic partner (Eastwick et al., 2011). Being kind, understanding, and intelligent are equally desired characteristics by both men and women (Figueredo et al., 2006). However, people differ on who they perceive as kind, understanding and intelligent. And ultimately they differ in their assessment of which candidates are the most desirable to them. Other studies focusing on assortative mating find only small positive correlations across romantic partners at the value of 0.20 (Buss and Barnes, 1986).

Formally, individuals match both on horizontal and vertical properties. In this dichotomy, characteristics that are valued differently by different agents constitute horizontal attributes, and characteristics that are similarly valued by everyone are vertical attributes. Most studies, particularly those that study dating and marriage markets using empirical methods, assume some combination of vertical and horizontal preferences (Hitsch et al., 2010a,b; Banerjee et al., 2013; Gomes and Pavan, 2016). Interestingly, whether an attribute is considered vertical or horizontal may depend on the side of the market, i.e., may vary between men and women.

Other studies demonstrate that the importance of vertical and horizontal attributes follow a

sequential order such that once candidates are sorted based on a vertical attribute, the preferences among them remain relatively horizontal (Abramitzky et al., 2011; Banerjee et al., 2013). Bruch et al. (2016) study the search patterns of users in an online dating platform and demonstrate that once women are sorted in an age group that is acceptable to a man, men's preference for age becomes horizontal.

While in most matching studies using theoretical modeling, agents are endowed with a vertical attribute, which may be interpreted as objective "quality" (e.g., McAfee, 2002; Damiano and Li, 2007, 2008; Hoppe et al., 2011), some recent theoretical studies assume the other extreme—fully horizontal preferences (e.g., Ashlagi et al., 2013). In our model, we also focus only on horizontal preferences: the values of Λ are independent. That means that when two men meet a woman, the extent to which one man likes the woman is independent of how much the other man likes her.

Unsurprisingly, the predictions of our model change if we impose vertical preferences, i.e., perfect correlation between Λ 's and \mathfrak{a} 's. It turns out that in such a case, all agents are indifferent between meeting fewer or more candidates. Under perfectly correlated \mathfrak{a} and Λ , an agent \mathfrak{a} would only match with an agent of the same type, gaining as much from the matching as staying alone, $\Lambda = \mathfrak{a}$. To see this, note that a man \mathfrak{a}^m would not want to match with $\mathfrak{a}^w < \mathfrak{a}^m$, because with perfect correlation, he likes her less than being alone $\Lambda^m(w) = \mathfrak{a}^w < \mathfrak{a}^m$. He would like to match with a woman that he likes more, $\Lambda^m(w) > \mathfrak{a}^m$, but she would prefer to be alone than to match with him, as $\Lambda^w(m) = \mathfrak{a}^m < \Lambda^m(w) = \mathfrak{a}^w$, so $\Lambda^w(m) < \mathfrak{a}^w$. Thus, no matter how many candidates they meet, or whether they stay unmatched, agents get the same payoff.

As documented by the the empirical studies above, matching markets involve a mixture of vertical and horizontal preferences. Modeling the whole market with purely vertical or purely horizontal preferences is not a realistic assumption. However, the analysis of purely horizontal preferences is justified by the empirical literature showing a sequential order, where once sorted according to vertical preferences (as age or education), agents' preferences become horizontal.

It is more important, however, to investigate whether the results of our analysis apply directly to an environment characterized by a mixture of vertical and horizontal preferences. Such a mixture would be manifested in our model by an *imperfect* correlation between Λ 's and \mathfrak{a} 's. And indeed, simulations in Section A.2 show that once we step away from perfect correlation (i.e., purely vertical differences), the forces analyzed in our main model come into play, and the results hold even for mixed environments, where Λ and \mathfrak{a} are imperfectly correlated.

6 Conclusions

Theoretical literature on network effects suggests that agents should be attracted to a larger rather than to a smaller network, because larger networks offer them access to a broader set of candidates, which in turns allows the agents to find better matches. However, in practice, we observe that platforms that restrict choice exist and prosper alongside platforms that offer more choice. To explain why this can happen, we propose a model which recognizes that on a platform that offers more choices, agents also face more competition, as their candidates also enjoy a larger choice set. This gives rise to two opposing forces. On the one hand, an agent is more likely to find an attractive match on a platform that offers more candidates. On the other hand, she is less likely to be accepted by her chosen match on such a platform.

These two opposing forces are resolved differently by agents with different outside options, which has implications for the kinds of platforms they will choose. Platforms that restrict choice appeal primarily to agents who are impatient or who have a disutility from being alone. This is for two reasons, (i) the candidates the agent encounters have a smaller choice set in the restrictedchoice platform, and (ii) due to self-selection, the candidates are also impatient or have a disutility from being alone. Both work in the same direction and increase the probability of being accepted. Agents with more patience or a higher utility from being alone would rather use platforms that offer more choices. The effect resulting from self-selection also explains why a platform limiting choice is able to charge a higher price than the competitor offering more choices. A higher price makes joining the restricted platform even less worthwhile for agents with a higher utility from being alone. But keeping them out increases the probability of acceptance on the restricted-choice platform; therefore, it increases the willingness to pay of the agents with a low utility from being alone for participating in the restricted-choice platform. The larger the difference in fees charged, the greater are the differences, on average, between participants on the different platforms.

Finally, our analysis has implications for managers seeking to enter into or compete in industries with strong network effects. While prevailing wisdom suggests that offering a large choice set to consumers on their platforms should benefit all consumers, our model shows that this intuition may not always hold. In matching markets, when people significantly differ in their outside options, and when preferences are subjective, managers may have more flexibility in how to compete and may want to enter the market as a restricted-choice platform. While our paper captures the stylized facts of the online dating industry, it generalizes more broadly to other matching environments, such as real estate or labor markets.

A Appendix

A.1 Tentative Offers

The main model assumes that agents can make only a single offer. The goal of this assumption is to reflect the fact that people are able to pursue only a limited number of possible relationships. Limiting the number to one is extreme. This section shows that the qualitative results of the model hold for some other, more realistic, offer-making procedures.

In this section, we analyze a two-step offer-making procedure for an environment where agents can pursue multiple relationships. After the agents meet their candidates and observe how much they like them, they proceed to making offers. In the first stage they can send a fixed number of *tentative offers*. Simultaneously, other agents send their tentative offers. Every agent observes the tentative offers he or she has received, before sending one final offer in the second stage. The final offers are also sent simultaneously. As before, it is only if the final offer is reciprocated that the relationship is formed. Otherwise, both agents remain unmatched. We assume that if agents are indifferent between sending an offer (tentative or final) or not, they do not send it. This eliminates a possible situation where agents send tentative offers to candidates that they like less than being alone, but are sure to be rejected by.

We show here that even with the two-step offer-making procedure there are limits to network effects through the same forces as in the base model. Adding a tentative offer to the procedure increases the overall probability of a successful match. However, when the number of feasible tentative offers is fixed, but the number of candidates increases, agents with a lower utility from being alone prefer markets with fewer candidates, while agents with a higher utility from staying alone prefer markets with more candidates. A fixed number of tentative offers reflects in a more realistic way the limitations to how many potential relations people can pursue.²⁴ This section illustrates this point through an example of a market with two tentative offers allowed. However, the results can be extended for any fixed number of tentative offers.

Consider an equilibrium where every agent makes the tentative offers to his two best candidates, provided that at least two candidates are above the reservation threshold. Otherwise, the agent makes a tentative offer to the best candidate—if the best candidate is above the reservation threshold—or to no candidates, if no candidates are above the threshold. If an agent received a tentative offer from his best candidate, he makes the final offer to this candidate. If an agent did not receive a tentative offer from the best candidate, but got one from the second-best candidate, then the agent makes the final offer to the second-best candidate. If the agent did not receive a

²⁴In a labor market, it reflects the fact that an agent can go to only a limited number of interviews. In the case of an auction site, an agent can follow only a limited number of ongoing auctions.

tentative offer from the best nor from the second-best candidate, he does not make a final offer and remains unmatched.²⁵

For the purpose of the comparative statics we are looking for, we need to find the expected payoff of agent \mathfrak{a} when everyone meets $N \geq 2$ candidates. An agent gets a tentative offer from a particular candidate when he is either the first or second choice of this candidate, and he is above the candidate's reservation value. An agent is the first choice of a candidate (and above the reservation value) with a probability

$$Pr(best|N) = \frac{1}{N+1}.$$

An agent is the second choice of a candidate (and above the reservation value) with a probability

$$Pr(2nd|N) = \int_0^1 \int_{\mathfrak{a}}^1 (N-1)(1-G(\Lambda))G^{N-2}(\Lambda)g(\Lambda)g(\mathfrak{a})d\Lambda d\mathfrak{a} = \frac{N-1}{N(N+1)}$$

Thus, the probability that the agent gets a tentative offer from a particular candidate is

$$Pr(tentative|N) = Pr(best|N) + Pr(2nd|N) = \frac{2N-1}{N(N+1)}$$

An agent makes the final offer to the best candidate when he has received a tentative offer from this candidate and he likes the candidate more than being alone. However, it may be that the agent did not get a tentative offer from the best candidate, but he got one from the second-best candidate. If this is the case, and the second-best candidate is above the reservation threshold, the agent makes the final offer to the second-best candidate.

The agent gets the final offer when he is the most-preferred candidate, or when he is the secondbest candidate, but the best candidate did not make a tentative offer. Moreover, the agent gets the final offer from a candidate only if both he and the candidate made tentative offers to each other. The probability that the candidate makes a tentative offer is already incorporated in the probability of getting the final offer. But we need to remember that the agent makes a tentative offer to the best or second-best candidate only if the candidate is above the reservation value \mathfrak{a} .

²⁵There are also other equilibria possible. All have the following structure: Let Λ^{MAX} be the Λ of the best candidate. If agent \mathfrak{a} got a tentative offer from a candidate whose Λ is at least $x(\Lambda^{MAX})$, he makes the final offer to the best of such candidates, even if he did not make a tentative offer to this candidate. If the agent did not get a tentative offer from any of the candidates above $x(\Lambda^{MAX})$, he makes the final offer to his best candidate, even though he did not receive a tentative offer from this candidate. The additional probability of successfully matching in such equilibrium is very small and decreasing with the number of candidates. Therefore, it does not change the qualitative results of this section.

That is, the probability of getting both the tentative and the final offers is

$$Pr(final|N) = \left[Pr(best|N) + Pr(2nd|N) \cdot \left(1 - Pr(tentative|N)\right)\right] = \frac{2N-1}{N(N+1)} \cdot \frac{N^2+1}{N(N+1)}.$$

The agent matches with the best candidate when he received a tentative and final offer from that candidate and the candidate was better than being alone. The probability that the best candidate out of N is above \mathfrak{a} is $1 - G(\mathfrak{a})^N$. Therefore, the agent matches with the best candidate with probability

$$Pr(match \ best|N,\mathfrak{a}) = \frac{2N-1}{N(N+1)} \cdot \frac{N^2+1}{N(N+1)} \left(1 - G^N(\mathfrak{a})\right) \,.$$

The agent matches with the second best candidate when he received a tentative and final offer from that candidate, the second-best candidate was better than being alone, and he did not receive a tentative offer from the best candidate. The probability that the second-best candidate is above \mathfrak{a} is

$$N(N-1)\int_{\mathfrak{a}}^{1} G^{N-2}(\Lambda)(1-G(\Lambda))g(\Lambda)d\Lambda = 1 - G^{N}(\mathfrak{a}) - N \cdot G^{N-1}(\mathfrak{a})(1-G(\mathfrak{a})).$$

Thus, the agent matches with the second-best candidate with probability

$$\begin{aligned} \Pr(match \ 2nd|N, \mathfrak{a}) &= \left(1 - \Pr(tentative|N)\right) \cdot \Pr(final|N) \cdot \left(1 - G^{N}(\mathfrak{a}) - N \cdot G^{N-1}(\mathfrak{a})(1 - G(\mathfrak{a}))\right) = \\ &= \left(1 - \frac{2N-1}{N(N+1)}\right) \frac{2N-1}{N(N+1)} \cdot \frac{N^{2}+1}{N(N+1)} \cdot \left(1 - G^{N}(\mathfrak{a}) - N \cdot G^{N-1}(\mathfrak{a})(1 - G(\mathfrak{a}))\right) \,. \end{aligned}$$

With the remaining probability of

$$1 - Pr(match \ best|N, \mathfrak{a}) - Pr(match \ 2nd|N, \mathfrak{a}) =$$

$$= 1 - Pr(final|N) \Big(1 - G^{N}(\mathfrak{a}) + \Big(1 - Pr(tentative|N) \Big) \Big(1 - G^{N}(\mathfrak{a}) - N \cdot G^{N-1}(\mathfrak{a})(1 - G(\mathfrak{a})) \Big) =$$

$$1 - \frac{2N-1}{N(N+1)} \cdot \frac{N^{2}+1}{N(N+1)} \left(1 - G^{N}(\mathfrak{a}) + \left(1 - \frac{2N-1}{N(N+1)} \right) \Big(1 - G^{N}(\mathfrak{a}) - N \cdot G^{N-1}(\mathfrak{a})(1 - G(\mathfrak{a})) \Big) \right)$$

the agent remains unmatched and receives the payoff of \mathfrak{a} .

The expected payoff from matching with the best candidate out of N is

$$EU(match \ best|\mathfrak{a}, N) = N \int_{\mathfrak{a}}^{1} G^{N-1}(\Lambda) \Lambda g(\Lambda) d\Lambda = 1 - \mathfrak{a} G^{N}(\mathfrak{a}) - \int_{\mathfrak{a}}^{1} G^{N}(\Lambda) d\Lambda$$

The expected payoff from matching with the second-best candidate out of N is

$$\begin{split} EU(match\ 2nd|\mathfrak{a},N) &= \int_{\mathfrak{a}}^{1} N(N-1)G^{N-2}(\Lambda)(1-G(\Lambda))g(\Lambda)\Lambda d\Lambda \\ &= N\underbrace{\int_{\mathfrak{a}}^{1}(N-1)G^{N-2}(\Lambda)g(\Lambda)\Lambda d\Lambda}_{EU(match\ best|\mathfrak{a},N-1)} - (N-1)\underbrace{\int_{\mathfrak{a}}^{1}NG^{N-1}(\Lambda)g(\Lambda)\Lambda d\Lambda}_{EU(match\ best|\mathfrak{a},N)} \\ &= N\left(1-\mathfrak{a}G^{N-1}(\mathfrak{a}) - \int_{\mathfrak{a}}^{1}G^{N-1}(\Lambda)d\Lambda\right) - (N-1)\left(1-\mathfrak{a}G^{N}(\mathfrak{a}) - \int_{\mathfrak{a}}^{1}G^{N}(\Lambda)d\Lambda\right) \end{split}$$

Therefore, the expected payoff for agent \mathfrak{a} in a market where two tentative offers are allowed and there are N candidates is

$$\begin{split} EU(\mathfrak{a}|N) &= \\ &= Pr(final|N) \cdot EU(match \ best|\mathfrak{a}, N) + \left(1 - Pr(tentative|N)\right) Pr(final|N) \cdot EU(match \ 2nd|\mathfrak{a}, N) \\ &+ \mathfrak{a} \cdot \left[1 - Pr(final|N)\left(1 - G^{N}(\mathfrak{a}) + \left(1 - Pr(tentative|N)\right)\left(1 - G^{N}(\mathfrak{a}) - NG^{N-1}(\mathfrak{a})(1 - G(\mathfrak{a}))\right)\right] \\ &= \frac{(2N-1)(N^{2}+1)}{N^{2}(N+1)^{2}} \left[\left(2 - \frac{2N-1}{N(N+1)}\right)(1 - \mathfrak{a}) + \left((N-1)\left(1 - \frac{2N-1}{N(N+1)}\right) - 1\right)\int_{\mathfrak{a}}^{1} G^{N}(\Lambda)d\Lambda \\ &- \left(1 - \frac{2N-1}{N(N+1)}\right)\int_{\mathfrak{a}}^{1} NG^{N-1}(\Lambda)d\Lambda\right] + \mathfrak{a} \end{split}$$

Consider the difference $EU(\mathfrak{a}|N+1) - EU(\mathfrak{a}|N) \ge 0$. We can use the same approach as in the proof of Proposition 1 to show that there are limits to network effects. Albeit, the function corresponding to function F(x) in Lemma B.1 is much more complicated for tentative offers. But the relevant properties still hold.

Figure A.1 graphically shows this result for uniform distribution. The shaded region is where $EU(\mathfrak{a}|N+1) - EU(\mathfrak{a}|N) \ge 0$ given \mathfrak{a} and N. We can see that all agents prefer N = 3 to N = 2, but it is no longer true for larger N's. Agents with lower \mathfrak{a} 's (the white region) prefer a market with fewer candidates (N) than a market with more candidates (N+1). Thus, with two tentative offers, the basic trade off between the choice and the competition effect plays out in the same way as in the base model. Similarly to Proposition 1, the optimal number of candidates is weakly increasing with the utility from being alone.

Interestingly, if there is no limit on tentative offers (i.e., one can always make tentative offers to all candidates above the reservation value, as the number of candidates increases), then the probability of matching with someone above the reservation value increases with the number of candidates. There is no trade-off, and all agents always prefer to meet more candidates.



Figure A.1: Region where $EU(\mathfrak{a}|N+1) - EU(\mathfrak{a}|N) \ge 0$ holds for uniform distribution. Agents in the shaded region prefer a market with N + 1 candidates to a market with N candidates.

A.2 Correlation between Outside Options and Preferences

In the main body of the paper, we assumed that an agent's outside option (\mathfrak{a}) has no effect on how much others like him (Λ) . This assumption allows us to claim that agents always want to match with those whom they like the most and enables us to develop a closed-form solution to our model.

There are, however, a number of reasons to believe that an agent's outside option may be correlated with how much others like the agent. If these two values are positively correlated, one can no longer assume that agents will want to make an offer to the candidate whom they like the most. With positive correlation, a man knows that his top choice woman is likely to have a high outside option and that other men will rank her highly as well. Whether the man will make an offer to his first choice woman will depend on his own outside option, \mathfrak{a}^m . If his outside option \mathfrak{a}^m is high, he knows that his first choice woman derives a high value from being matched with him, as \mathfrak{a}^m correlates with her Λ^w for him. He will prefer to make an offer to her since she is unlikely to reject him. In contrast, if the man's outside option is low, he knows that his top choice woman is likely to derive a low value from a match with him and, therefore, she will likely reject his offer. He may, therefore, consider making an offer to his 2^{nd} or 3^{rd} choice women instead. Even though these lower-ranked choices give him lower value from a match, his probability of rejection with these candidates is also likely to be lower compared to his first-choice woman. As long as the decline in his value from a successful match is offset by the increase in his probability of a match, he may be better off making an offer to his 2^{nd} or 3^{rd} choice rather than his first choice.

Unfortunately, there is no closed form solution in this setup. To overcome this issue, we generate insights based on a numerical simulation. The simulation extends the main model by allowing for correlation (denoted by h) between agents' outside options (\mathfrak{a}) and how much others like them (Λ), and gives us insights about what happens to the expected utilities and network effects with positive correlation.

A.2.1 Simulation Setup

The expected utility for an agent \mathfrak{a} from making an offer to a candidate relies on two pieces of information: the probability that the candidate is going to accept this offer, and how much he likes the candidate. Accordingly, the numerical simulation we build first aims to establish a "lookup table," which is an approximation to a probability of acceptance matrix. This table lists the probability that an agent with outside option \mathfrak{a} will have his offer reciprocated by a candidate he likes at Λ . More specifically, discretizing the [0,1] space in increments of 0.1, we generate an 11×11 matrix, where the rows indicate the range in which the man's outside option falls $\mathfrak{a} \in \{[0,0.1) \cup [0.1,0.2) \cup [0.2,0.3) \dots \cup [0.9,1) \cup [1]\}$ and the columns indicate the Λ values of the candidate met, $\Lambda \in \{[0,0.1) \cup [0.1,0.2) \cup [0.2,0.3) \dots \cup [0.9,1) \cup [1]\}$.²⁶ Element *ij* of the matrix indicates the probability that a man (woman) of outside option \mathfrak{a} will receive an offer from a candidate he (she) likes at Λ , conditional on *h* and *N*. In the second part of the simulation, using the lookup table, we generate the expected utility for a representative agent with outside option $\mathfrak{a} = \{0, 0.1, 0.2, ..., 1\}$ from being on the platform and check for the limits to positive network effects.

The economic forces behind Proposition 1 are the main driving forces of all the following results. Therefore, the aim of the basic simulation is to check whether the results of Proposition 1 hold also under positive correlation between values of \mathfrak{a} and Λ . To do that, we manipulate the environment by changing $N \geq 1$ and the correlation between agents' outside options and how much others like them, in increments of 0.2 (h = 0, 0.2, 0.4, 0.6, 0.8). This implies that the restricted platform gives agents N candidates each of whom meets N-1 other candidates. We simulate the lookup table and the expected utility for an agent from having N = 1, 2,Y candidates, where Y is the maximum number of agents tested.²⁷

The lookup table is generated with the help of an outer and an inner loop. The simulation starts by setting some initial numbers in the lookup table. We set the initialization values to 1/N. In each iteration of the outer loop, we update the lookup table using the results from the inner

²⁶Since the probability of drawing an \mathfrak{a} or a Λ value equal to 1, although empirically possible in any single iteration of the simulation, approaches zero both in theory and in the simulation.

 $^{^{27}}$ In our simulations, we let this number go as high 150.

loop. In the inner loop, we simulate the choices of the candidates met in order to backtrack the probabilities of acceptance for an agent. More specifically, in each iteration of the inner loop, we simulate the choices of N men and N women, given the lookup table. We draw the outside options of the men and women $(\mathfrak{a}^m, \mathfrak{a}^w)$, and conditional on their outside options and the correlation in the market, we also draw how much each agent with an outside option \mathfrak{a} likes each candidate met, where $\Lambda \sim U[\mathfrak{a}h, 1 - h(1 - \mathfrak{a})]$. Recall that the correlation between Λ and \mathfrak{a} is determined by a population level parameter h, which is assumed to vary from zero to one. In particular, for each candidate, a Λ value is drawn from a uniform distribution between $\mathfrak{a}h$ and $1 - h(1 - \mathfrak{a})$. Notice that when h = 0, Λ is drawn from a standard uniform distribution between 0 and 1; thus, it is independent of the candidate's outside option, \mathfrak{a} (this construction reflects our main model). In contrast, for example if h = 0.5, how much a candidate is liked (Λ) is drawn from a uniform distribution between 0 and 1; thus, it is independent of the candidate's outside option, \mathfrak{a} (this construction reflects our main model). In contrast, for example if h = 0.5, how much a candidate is liked (Λ) is drawn from a uniform distribution between 0.5 \mathfrak{a} and 0.5 + 0.5 \mathfrak{a} , implying that the Λ value depends on \mathfrak{a} . In the extreme case, when h = 1, how much others like the candidate is fully determined by the outside option of this agent, equaling \mathfrak{a} . In this case, the agent's payoff does not depend on N, as it is always equal to the agent's outside options, as explained in Section 5.2. Therefore, we do not run the simulation for h = 1.

Using the lookup table and how much each candidate is liked, the agent \mathfrak{a} chooses the candidate who maximizes his expected utility. These choices are tracked in the inner loop. When the iterations for the inner loop are complete,²⁸ we update the look up table conditional on the number of times a man \mathfrak{a} in row *i* was chosen by a woman he liked at value Λ in column *j*, by dividing the number of times he was chosen by the woman with the number of times he met a candidate he liked at Λ in column *j*. After the inner loop is complete, we update the lookup table before going to the next run of the inner loop. With this procedure, we aim to reach a steady state in the lookup table.²⁹

In the second part of the simulation, using the lookup table generated in the first part of the simulation, we simulate the utility for a representative agent at a = 0 to a = 0.9, in increments of $0.1.^{30}$ The utility values from each iteration are noted, and averaged over a number of iterations.

A.2.2 Results

Our objective of the simulation is to test whether the limits to positive network effects exist when the desirability of a candidate is correlated with his outside options. The results demonstrate that for the tested levels of correlation (0 to 0.8, in increments of 0.2), the curvilinear relationship

 $^{^{28}}$ The iterations ranged between 300–1000, depending on the computational time it took for a run. For higher values of N, the run time is longer and, therefore, we used a lower number of iterations.

²⁹We also ran the outer loop 300–1000 times depending on N. For larger N, we set a smaller number of iterations to reduce the computational time.

³⁰An agent with a = 1 is expected to receive a utility of 1 independent of N and h, and although we generate the simulation results to confirm this expectation, we are not reporting a = 1 in the results.

between the number of candidates a platform offers and an agent's expected utility from participating in the platform is maintained. Specifically, echoing part (i) of Proposition 1, we find that agents derive utility from each additional candidate offered to them on the platform, as long as the number of candidates offered to them does not exceed a specific threshold. Beyond this threshold, agents prefer (a platform with) fewer choices. This result confirms that the qualitative results from our model hold even when the agents' outside options influence how much others like them. Put differently, under correlated preferences, there are still limits to positive network effects and a platform which restricts choice can still provide higher utility compared to another platform which does not. Figures A.2–A.4 visually represent the limits to positive network effects for h = 0, 0.4, 0.8.³¹



Figure A.2: Limits to Positive Network Effects, h = 0

Second, we find that the threshold at which agents begin to prefer platforms offering fewer choices over platforms that offer more choices increases in the agent's outside option value. For example, when h = 0.4, an agent with an outside-option value $\mathfrak{a} = 0.4$ would rather have the platform offering two candidates than three candidates. And an agent with a relatively higher outside option such as $\mathfrak{a} = 0.6$ prefers four candidates on the platform to any other number of candidates. This finding is very consistent with the intuition from part (ii) of Proposition 1. Recall

³¹Graphs for h = 0.2 and 0.6 are similar and available upon request.



Figure A.3: Limits to Positive Network Effects, h = 0.4



Figure A.4: Limits to Positive Network Effects, h = 0.8

that agents with low outside options care about finding a match who may reciprocate their offer. For these agents, a platform offering fewer candidates, where they will have less competition with same side agents, is attractive. In contrast, agents with high outside options care about obtaining a high match value. For them, a platform offering a higher number of candidates, of which one can exceed their high outside option, is more attractive. Those forces still hold for positive correlation.³²

Appendix B: Proofs

Proof of Lemma 1:

- (i) With N candidates, a woman that the man meets has N + 1 possible actions: to make an offer to one of the N candidates and to make no offer (when \mathfrak{a}^w is larger than any of the relevant Λ 's). All Λ 's and \mathfrak{a}^w are drawn independently from the same distribution. Therefore, without knowing \mathfrak{a}^w , each of the actions is equally likely.
- (ii) The agent makes an offer to the best Λ , if the highest Λ is above \mathfrak{a} . The highest Λ is above \mathfrak{a} with probability $1 G^N(\mathfrak{a})$. Independently, the best Λ makes an offer to agent \mathfrak{a} with probability $\frac{1}{N+1}$ (from point (1) of this Lemma).
- (iii) Unconditional expected value of a match is Pr(accepted) · E(max Λ | max Λ > a). Thus, the value of matching, conditional on being accepted is E(max Λ | max Λ > a). To find the conditional expected value of E(max Λ | max Λ > a), we first characterize the distribution function of max Λ under N candidates. Notice that the c.d.f. of max Λ is Pr(max Λ < x) = G^N(x). Thus, the pdf is ∂G^N(x)/∂x = NG^{N-1}(x)g(x). Using the probability density, we calculate the expected value of max Λ, given that max Λ > a:

$$\int_{\mathfrak{a}}^{1} NG^{N-1}(x)g(x)\cdot xdx = N\int_{\mathfrak{a}}^{1} G^{N-1}(x)g(x)xdx\,.$$

(iv) Follows directly from parts (i), (ii) and (iii) of the Lemma.

This completes the proof of Lemma 1. \Box

 $^{^{32}}$ We have tested the limits to network effects also in an environment with two competing platforms where the restricted platform offers one less candidate to agents than the non-restricted platform, and charges a higher fee. The results from the simulation show that findings in this environment when h > 0 are qualitatively similar to the results of Proposition 2. Since this more involved simulation requires a lengthy description but does not offer new insights, these results are available upon request.

Proof of Corollary 1: Using integration by parts,

$$N\int_{\mathfrak{a}}^{1} G^{N-1}(x)g(x)xdx = G^{N}(x)x\Big|_{\mathfrak{a}}^{1} - \int_{\mathfrak{a}}^{1} G^{N}(x)dx =$$
$$= G^{N}(1) \cdot 1 - G^{N}(\mathfrak{a})\mathfrak{a} - \int_{\mathfrak{a}}^{1} G^{N}(x)dx = 1 - G^{N}(\mathfrak{a})\mathfrak{a} - \int_{\mathfrak{a}}^{1} G^{N}(x)dx$$

Since $G(x) \leq 1$ for $0 \leq x \leq 1$, $G^N(x)$ is nonincreasing with N for $0 \leq x \leq 1$ and $\int_{\mathfrak{a}}^{1} G^N(x) dx$ is nonincreasing with N, $1 - G^N(\mathfrak{a})\mathfrak{a} - \int_{\mathfrak{a}}^{1} G^N(x) dx$ is nondecreasing with N. \Box

Lemma B.1 Consider an arbitrary $\hat{\mathfrak{a}} \in [0, 1)$.

(i) When
$$EU(\hat{\mathfrak{a}}|N+1) - EU(\hat{\mathfrak{a}}|N) \ge 0$$
, then for all $\mathfrak{a} \in (\hat{\mathfrak{a}}, 1)$, $EU(\mathfrak{a}|N+1) - EU(\mathfrak{a}|N) > 0$.

(ii) When $EU(\hat{\mathfrak{a}}|N+1) - EU(\hat{\mathfrak{a}}|N) \leq 0$, then for all $\mathfrak{a} \in [0, \hat{\mathfrak{a}})$, $EU(\mathfrak{a}|N+1) - EU(\mathfrak{a}|N) < 0$.

Proof of Lemma B.1: Notice that

$$EU(\mathfrak{a}|N+1) - EU(\mathfrak{a}|N) = \frac{1}{(N+1)(N+2)} \int_{\mathfrak{a}}^{1} \left[G^{N}(x) - 1 + (N+1)G^{N}(x)(1-G(x)) \right] dx \,.$$

Let's identify the sign of $\int_{a}^{1} F(x) dx$, where

$$F(x) = G^{N}(x) - 1 + (N+1)G^{N}(x)(1 - G(x)).$$

It is useful to learn the shape of F(x) to determine the sign of $\int_{\mathfrak{a}}^{1} F(x) dx$. For x = 0, F(x) = -1, and for x = 1, F(x) = 0. Moreover, it is single peaked: increasing for $x < \hat{x}$ and decreasing for $x > \hat{x}$, with maximum at \hat{x} s.t. $G(\hat{x}) = \frac{(N+1)^2 - 1}{(N+1)^2}$.

Since for $x \in (\hat{x}, 1]$, F(x) decreases and F(1) = 0, then $F(\hat{x}) > 0$. Moreover, F(0) = -1 and for $x \in [0, \hat{x})$, F(x) increases. Therefore, $\exists \hat{x} \in (0, \hat{x})$ s.t. $F(\hat{x}) = 0$.

Now, suppose $\int_{\hat{\mathfrak{a}}}^{1} F(x) dx \ge 0$. Take $\mathfrak{a} > \hat{\mathfrak{a}}$. Then

$$\int_{\mathfrak{a}}^{1} F(x)dx = \int_{\hat{\mathfrak{a}}}^{1} F(x)dx - \int_{\mathfrak{a}}^{\hat{\mathfrak{a}}} F(x)dx.$$

If $\mathfrak{a} > \hat{x}$, then F(x) > 0 for all $x > \mathfrak{a}$, so $\int_{\mathfrak{a}}^{1} F(x) dx > 0$. If $\mathfrak{a} \le \hat{x}$, then F(x) < 0 for all $x \in [\hat{\mathfrak{a}}, \mathfrak{a})$, so $\int_{\hat{\mathfrak{a}}}^{\mathfrak{a}} F(x) dx < 0$ and

$$\int_{\mathfrak{a}}^{1} F(x)dx = \int_{\hat{\mathfrak{a}}}^{1} F(x)dx - \int_{\hat{\mathfrak{a}}}^{\mathfrak{a}} F(x)dx > \int_{\hat{\mathfrak{a}}}^{1} F(x)dx \ge 0.$$

For the second part of the lemma, suppose that $\int_{\hat{\mathfrak{a}}}^{1} F(x) dx \leq 0$, and take $\mathfrak{a} < \hat{\mathfrak{a}}$. For $\int_{\hat{\mathfrak{a}}}^{1} F(x) dx \leq 0$ it must be that $\hat{\mathfrak{a}} < \hat{x}$. This is because for all $y > \hat{x}$, $\int_{y}^{1} F(x) dx > 0$. Then $\int_{\mathfrak{a}}^{\hat{\mathfrak{a}}} F(x) dx < 0$, and so $\int_{\mathfrak{a}}^{1} F(x) dx < 0$. This completes the proof of the lemma. \Box

Proof of Proposition 1: Let $\Delta(\mathfrak{a}|N) = EU(\mathfrak{a}|N+1) - EU(\mathfrak{a}|N)$

Step 1. Function F(x), defined and characterized in the proof of Lemma B.1, is decreasing in N, for any x. We show that F(x, N) - F(x, N+1) > 0.

$$\begin{aligned} G^{N}(x) &-1 + (N+1)G^{N}(x)(1-G(x)) - G^{N+1}(x) + 1 - (N+2)G^{N+1}(x)(1-G(x)) = \\ &= G^{N}(x)(1-G(x)) + (N+1)G^{N}(x)(1-G(x)) - (N+2)G^{N+1}(x)(1-G(x)) = \\ &= (N+2)G^{N}(x)(1-G(x)) - (N+2)G^{N+1}(x)(1-G(x)) = \\ &= (N+2)G^{N}(x)(1-G(x))(1-G(x)) > 0 \end{aligned}$$

Step 2. $\Delta(\mathfrak{a}|N)$ is decreasing in N, for any \mathfrak{a} . The fact that F(x, N+1) < F(x, N) may not be enough to prove

$$\underbrace{\Delta(\mathfrak{a}|N+1)}_{=\int_{\mathfrak{a}}^{1}\frac{F(x,N+1)}{(N+2)(N+3)}dx} < \underbrace{\Delta(\mathfrak{a}|N)}_{=\int_{\mathfrak{a}}^{1}\frac{F(x,N)}{(N+1)(N+2)}dx}$$

However, we can show that for any x

$$\frac{F(x,N+1)}{(N+2)(N+3)} < \frac{F(x,N)}{(N+1)(N+2)}\,,$$

because F(x, N + 1) < F(x, N) and $\frac{1}{N+3} < \frac{1}{N+1}$. Since at every point the integrated function is smaller, the integral also needs to be smaller. Alternatively:

$$\begin{split} \Delta(\mathfrak{a}|N+1) - \Delta(\mathfrak{a}|N) &= \int_{\mathfrak{a}}^{1} \frac{F(x,N+1)}{(N+2)(N+3)} dx - \int_{\mathfrak{a}}^{1} \frac{F(x,N)}{(N+1)(N+2)} dx = \\ &= \int_{\mathfrak{a}}^{1} \left(\frac{F(x,N+1)}{(N+2)(N+3)} - \frac{F(x,N)}{(N+1)(N+2)} \right) dx < 0 \,, \end{split}$$

because at any point x the integrated function is negative.

Step 3. For any \mathfrak{a} , there exists a finite N, such that $\Delta(\mathfrak{a}|N) < 0$. Suppose that $\Delta(\mathfrak{a}|1) > 0$ (otherwise $\overline{N}(\mathfrak{a}) = 1$ and the lemma is satisfied). For every $x \in (0, 1)$, $F(x) \to_{N \to \infty} -1$. Hence, as N goes to infinity, $\int_{\mathfrak{a}}^{1} F(x) dx \to -(1 - \mathfrak{a}) < 0$. Then, there must be an N, such that $\Delta(\mathfrak{a}|N) < 0$. Let $\overline{N}(\mathfrak{a})$ be the smallest N, such that $\Delta(\mathfrak{a}|N) < 0$. Therefore, for every \mathfrak{a} there exists such $\overline{N}(\mathfrak{a})$. From Step 2, we know that $\Delta(\mathfrak{a}|N)$ is (strictly) decreasing in N for any \mathfrak{a} . Therefore, for any $N < \overline{N}(\mathfrak{a}), \Delta(\mathfrak{a}|N)$ is positive, and for any $N \ge \overline{N}(\mathfrak{a})$ it is negative.

Step 4. $\bar{N}(\mathfrak{a})$ is non-decreasing in \mathfrak{a} . For any \mathfrak{a}' and \mathfrak{a}'' such that $\mathfrak{a}'' > \mathfrak{a}'$, we show that $\bar{N}(\mathfrak{a}'') \geq \bar{N}(\mathfrak{a}')$. Let $N' \equiv \bar{N}(\mathfrak{a}')$. That is $\Delta(\mathfrak{a}'|N) > 0$ for N < N' and $\Delta(\mathfrak{a}'|N) < 0$ for $N \geq N'$. Now consider $\mathfrak{a}'' > \mathfrak{a}'$. According to the previous lemma, when $\Delta(\mathfrak{a}'|N) > 0$, then $\Delta(\mathfrak{a}''|N) > 0$. Therefore, for N < N', $\Delta(\mathfrak{a}''|N) > 0$. Since for every \mathfrak{a} there exists $\bar{N}(\mathfrak{a})$ (Step 3), it must be that for \mathfrak{a}'' , $\bar{N}(\mathfrak{a}'') \geq N'$.

Lemma B.2 For $\Omega > N$, $\frac{G^N(\mathfrak{a}) - G^\Omega(\mathfrak{a})}{1 - G^N(\mathfrak{a})}$ is strictly increasing on $\mathfrak{a} \in (0, 1)$.

Proof of Lemma B.2: Consider only $\mathfrak{a} \in [0,1]$. Let $x = \Omega - N > 0$. The derivative of $\frac{G^N(\mathfrak{a}) - G^\Omega(\mathfrak{a})}{1 - G^N(\mathfrak{a})} = \frac{G^N(\mathfrak{a}) - G^{N+x}(\mathfrak{a})}{1 - G^N(\mathfrak{a})}$ with respect to \mathfrak{a} is then

$$\frac{[NG^{N-1}(\mathfrak{a})g(\mathfrak{a}) - (N+x)G^{N+x-1}(\mathfrak{a})g(\mathfrak{a})](1 - G^{N}(\mathfrak{a})) + NG^{N-1}(\mathfrak{a})g(\mathfrak{a})G^{N}(\mathfrak{a})(1 - G^{x}(\mathfrak{a}))}{(1 - G^{N}(\mathfrak{a}))^{2}} = \frac{G^{N-1}(\mathfrak{a})g(\mathfrak{a})}{(1 - G^{N}(\mathfrak{a}))^{2}} \left[[N - (N+x)G^{x}(\mathfrak{a})](1 - G^{N}(\mathfrak{a})) + NG^{N}(\mathfrak{a})(1 - G^{x}(\mathfrak{a})) \right] = \frac{G^{N-1}(\mathfrak{a})g(\mathfrak{a})}{(1 - G^{N}(\mathfrak{a}))^{2}} \underbrace{[N - (N+x)G^{x}(\mathfrak{a}) + xG^{N+x}(\mathfrak{a})]}_{X(N,x,\mathfrak{a})}$$

The sign of the derivative is the same as the sign of $X(N, x, \mathfrak{a})$. We claim that $X(N, x, \mathfrak{a})$ is positive. First, notice that for $\mathfrak{a} = 1$, X(N, x, 1) = 0. Moreover, the derivative of $X(N, x, \mathfrak{a})$ with respect to \mathfrak{a} is negative:

$$\underbrace{x(N+x)}_{+}\underbrace{G^{x-1}(\mathfrak{a})g(\mathfrak{a})}_{+}\underbrace{(G^{N}(\mathfrak{a})-1)}_{-}<0.$$

This is enough to establish that $X(N, x, \mathfrak{a})$ is positive. In addition notice that $X(N, x, \mathfrak{a} = 0) = N > 0$.

Therefore $\frac{G^N(\mathfrak{a}) - G^\Omega(\mathfrak{a})}{1 - G^N(\mathfrak{a})}$ is strictly increasing for $\Omega > N$. \Box

Proof of Lemma 2: Agent \mathfrak{a} 's willingness to pay to join platform N is equal to the additional expected payoff that the agent can get by joining the platform, i.e., $WTP(\mathfrak{a}) = EU(\mathfrak{a}|N) - EU(\mathfrak{a}|\Omega)$,

where

$$\begin{split} EU(\mathfrak{a}|N) &= [G^{N}(\mathfrak{a}) + (1 - G^{N}(\mathfrak{a}))Pr(rej|N)]\mathfrak{a} + (1 - Pr(rej|N)) \cdot N \int_{\mathfrak{a}}^{1} G^{N+1}(x)g(x)xdx \\ &= 1 + Pr(rej|N)(\mathfrak{a}-1) - (1 - Pr(rej|N)) \int_{\mathfrak{a}}^{1} G^{N}(x)dx \\ EU(\mathfrak{a}|\Omega) &= [G^{\Omega}(\mathfrak{a}) + (1 - G^{\Omega}(\mathfrak{a}))Pr(rej|\Omega)]\mathfrak{a} + (1 - Pr(rej|\Omega)) \cdot N \int_{\mathfrak{a}}^{1} G^{\Omega+1}(x)g(x)xdx \\ &= 1 + Pr(rej|\Omega)(\mathfrak{a}-1) - (1 - Pr(rej|\Omega)) \int_{\mathfrak{a}}^{1} G^{\Omega}(x)dx \end{split}$$

Then,

$$\begin{split} WTP(\mathfrak{a}) &= EU(\mathfrak{a}|N) - EU(\mathfrak{a}|\Omega) = \\ &= (1-\mathfrak{a})[Pr(rej|\Omega) - Pr(rej|N)] - [1 - Pr(rej|N)] \int_{\mathfrak{a}}^{1} G^{N}(x)dx + [1 - Pr(rej|\Omega)] \int_{\mathfrak{a}}^{1} G^{\Omega}(x)dx = \\ &= [1 - Pr(rej|\Omega)] \int_{\mathfrak{a}}^{1} [G^{\Omega}(x) - G^{N}(x)]dx + [Pr(rej|\Omega) - Pr(rej|N)] \cdot \left(1 - \mathfrak{a} - \int_{\mathfrak{a}}^{1} G^{N}(x)dx\right) \end{split}$$

Notice that for $\mathfrak{a} = 1$, $WTP(\mathfrak{a} = 1) = 0$; for $\mathfrak{a} = 0$, WTP may be positive or negative.

$$\frac{\partial WTP(\mathfrak{a})}{\partial \mathfrak{a}} = (1 - Pr(rej|\Omega)) \cdot [G^N(\mathfrak{a}) - G^\Omega(\mathfrak{a})] - (Pr(rej|\Omega) - Pr(rej|N)) \cdot (1 - G^N(\mathfrak{a}))$$

$$\frac{\partial WTP(\mathfrak{a})}{\partial \mathfrak{a}} < 0 \iff \frac{G^{N}(\mathfrak{a}) - G^{\Omega}(\mathfrak{a})}{1 - G^{N}(\mathfrak{a})} < \frac{Pr(rej|\Omega) - Pr(rej|N)}{1 - Pr(rej|\Omega)}$$

We consider $N < \Omega$. Then $G^{N}(\mathfrak{a}) - G^{\Omega}(\mathfrak{a}) > 0$. From Lemma B.2 we know that $\frac{G^{N}(\mathfrak{a}) - G^{\Omega}(\mathfrak{a})}{1 - G^{N}(\mathfrak{a})}$

is strictly increasing. Moreover, it takes value 0 for $\mathfrak{a} = 0$, and $\frac{\Omega - N}{N} > 0$ as $\mathfrak{a} \to 1$. If $Pr(rej|N) < Pr(rej|\Omega)$, then $\frac{Pr(rej|\Omega) - Pr(rej|N)}{1 - Pr(rej|\Omega)}$ is a positive constant. When $\frac{Pr(rej|\Omega) - Pr(rej|N)}{1 - Pr(rej|\Omega)} > \frac{\Omega - N}{N}$, the $WTP(\mathfrak{a})$ is decreasing on the whole interval $\mathfrak{a} \in [0, 1)$, and hence everywhere positive. When $\frac{Pr(rej|\Omega) - Pr(rej|N)}{1 - Pr(rej|\Omega)} < \frac{\Omega - N}{N}$, the $WTP(\mathfrak{a})$ is decreasing for small \mathfrak{a} 's, and increasing for large \mathfrak{a} 's. But since for $\mathfrak{a} = 1$, WTP = 0, WTP must increase to 0 from negative values. Therefore, for \mathfrak{a} 's where $WTP(\mathfrak{a}) > 0$, WTP is strictly decreasing.

Corollary B.1 For any given Pr(rej|N) and $Pr(rej|\Omega) < Pr(rej|N)$ and for any Ω and $N < \Omega$, the willingness to pay $WTP(\mathfrak{a}) = EU(\mathfrak{a}|N, Pr(rej|N)) - EU(\mathfrak{a}|\Omega, Pr(rej|\Omega))$ is non-positive for $\mathfrak{a} \in [0,1].$

Proof of Corollary B.1: Consider WTP from the proof of Lemma 2, and suppose $N < \Omega$. If

 $Pr(rej|N) > Pr(rej|\Omega)$, then $\frac{Pr(rej|\Omega) - Pr(rej|N)}{1 - Pr(rej|\Omega)} < 0$ while $\frac{G^N(\mathfrak{a}) - G^\Omega(\mathfrak{a})}{1 - G^N(\mathfrak{a})} > 0$. Therefore $\frac{\partial WTP(\mathfrak{a})}{\partial \mathfrak{a}} > 0$, i.e., WTP is strictly increasing everywhere. And since $WTP(\mathfrak{a} = 1) = 0$, then $WTP(\mathfrak{a})$ is negative for $\mathfrak{a} \in [0, 1)$. Thus, no agent has a positive willingness to pay to join a platform offering fewer candidates and higher probability of rejection. \Box

Corollary B.2 When fee f = 0, then $\hat{\mathfrak{a}}(f=0) = 1$. That is, all agents prefer to join the platform if the fee is the same as for participating on the outside market.

Proof of Corollary B.2: It follows from the fact that $EU(\hat{\mathfrak{a}}|N) - EU(\hat{\mathfrak{a}}|\Omega)$ is positive on the interval [0,1). Agents with $\mathfrak{a} \in [0,1)$ prefer to join when f = 0, and agents with $\mathfrak{a} = 1$ are indifferent. \Box

Proof of Proposition 2: Let $N < \Omega$. Suppose that agents then expect $Pr(rej|N) < Pr(rej|\Omega)$. From Lemma 2, we know that in such a case, on the interval $\mathfrak{a} \in [0, \overline{\mathfrak{a}})$ for $\overline{\mathfrak{a}} < 1$ willingness to pay is positive and decreasing (and continuous). Therefore, the WTP is highest for $\mathfrak{a} = 0$, $WTP(\mathfrak{a} = 0) > 0$. Thus, for any fee $f < WTP(\mathfrak{a} = 0)$ there exists \mathfrak{a}' such that $f = WTP(\mathfrak{a}')$. All agents with $\mathfrak{a} < \mathfrak{a}'$ have higher willingness to pay than f. Thus, they will pay the fee and join the platform, and the platform collects profits $G(\mathfrak{a}') \cdot WTP(\mathfrak{a}')$. In an equilibrium, the expectations need to be fulfilled. Since for given f agents with $\mathfrak{a} < \mathfrak{a}'$ join the platform, and those with $\mathfrak{a} > \mathfrak{a}'$ stay outside:

$$\begin{aligned} Pr(rej|N, \mathfrak{a} \in [0, \mathfrak{a}')) &= 1 - \frac{1}{N} + \frac{G^N(\mathfrak{a}')}{N(N+1)} \\ Pr(rej|\Omega, \mathfrak{a} \in (\mathfrak{a}', 1]) &= 1 - \frac{1}{\Omega} + \frac{1}{\Omega(\Omega+1)} \frac{1 - G^{\Omega+1}(\mathfrak{a}')}{1 - G(\mathfrak{a}')} \end{aligned}$$

Notice that $Pr(rej|\Omega, \mathfrak{a} \in (\mathfrak{a}', 1]) > Pr(rej|N, \mathfrak{a} \in [0, \mathfrak{a}'))$

$$\begin{split} Pr(rej|\Omega,\mathfrak{a}\in(\mathfrak{a}',1]) - Pr(rej|N,\mathfrak{a}\in[0,\mathfrak{a}')) > 0\\ 1 - \frac{1}{\Omega} + \frac{1}{\Omega(\Omega+1)} \frac{1 - G^{\Omega+1}(\mathfrak{a}')}{1 - G(\mathfrak{a}')} - \left(1 - \frac{1}{N} + \frac{G^N(\mathfrak{a}')}{N(N+1)}\right) > 0\\ \frac{1}{N} - \frac{1}{\Omega} + \frac{1}{\Omega(\Omega+1)} \frac{1 - G^{\Omega+1}(\mathfrak{a}')}{1 - G(\mathfrak{a}')} - \frac{G^N(\mathfrak{a}')}{N(N+1)}) > 0\\ \Omega(\Omega+1)(N+1) - N(N+1)(\Omega+1) + N(N+1) \frac{1 - G^{\Omega+1}(\mathfrak{a}')}{1 - G(\mathfrak{a}')} - \Omega(\Omega+1)G^N(\mathfrak{a}') > 0\\ \Omega(\Omega+1)N - N(N+1)(\Omega+1) + N(N+1) \frac{1 - G^{\Omega+1}(\mathfrak{a}')}{1 - G(\mathfrak{a}')} + \Omega(\Omega+1)\left(1 - G^N(\mathfrak{a}')\right) > 0\\ N(\Omega+1)\underbrace{(\Omega-N-1)}_{\geq 0} + N(N+1)\frac{1 - G^{\Omega+1}(\mathfrak{a}')}{1 - G(\mathfrak{a}')} + \Omega(\Omega+1)\left(1 - G^N(\mathfrak{a}')\right) > 0 \end{split}$$

All other terms are strictly positive. Moreover, notice that for $\mathfrak{a} = 0$ and for $\mathfrak{a} = \overline{\mathfrak{a}}$, the profit $G(\mathfrak{a}) \cdot WTP(\mathfrak{a}) = 0$. But for $\mathfrak{a} \in (0, \overline{\mathfrak{a}})$ both \mathfrak{a} and $WTP(\mathfrak{a})$ are positive, so $G(\mathfrak{a}) \cdot WTP(\mathfrak{a}) > 0$. Let \mathfrak{a}^* be the value that maximizes platform's profit. Then it must be that $\mathfrak{a}^* \in (0, \overline{\mathfrak{a}})$ and $G(\mathfrak{a}^*) \cdot WTP(\mathfrak{a}^*) > 0$. \Box

Proof of Lemma 3: We obtain the result by differentiating

$$EU(\mathfrak{a}|M_2, Pr(rej|M_2)) - \mathfrak{a} = 1 - Pr(rej|M_2)(\mathfrak{a} - 1) - (1 - Pr(rej|M_2)) \int_{\mathfrak{a}}^{1} G^{M_2}(x) dx - \mathfrak{a} = 0$$

with respect to \mathfrak{a} :

$$\frac{\partial (EU(\mathfrak{a}|M_2, Pr(rej|M_2)) - \mathfrak{a})}{\partial \mathfrak{a}} = Pr(rej|M_2) + [1 - Pr(rej|M_2)] G^{M_2}(\mathfrak{a}) - 1 = [1 - Pr(rej|M_2)] \underbrace{\left[G^{M_2}(\mathfrak{a}) - 1\right]}_{\mathbf{a}} < 0.$$

Thus, $EU(\mathfrak{a}|M_2, Pr(rej|M_2))$ is decreasing on the whole range of \mathfrak{a} .

Moreover, $EU(\mathfrak{a}|M_2, Pr(rej|M_2)) - \mathfrak{a}$ evaluated at $\mathfrak{a} = 1$ is:

$$1 + Pr(rej|M_2) \cdot 0 - (1 - Pr(rej|M_2)) \cdot 0 - 1 = 0.$$

Proof of Proposition 3: Suppose $M_1 < M_2$. Lemma 3 and Corollary 3 help characterize the agents' decisions about which platform to join, if any, given f_1 , $f_2 < f_1$, $Pr(rej|M_1)$, $Pr(rej|M_2) > Pr(rej|M_2)$.

Agents with $\mathfrak{a} < \hat{\mathfrak{a}}_2$ prefer M_2 to being unmatched, and those with $\mathfrak{a} > \hat{\mathfrak{a}}_2$ prefer being unmatched to M_2 . Agents with $\mathfrak{a} < \hat{\mathfrak{a}}_1$ prefer M_1 to M_2 , and those with $\mathfrak{a} > \hat{\mathfrak{a}}_1$ prefer M_2 to M_1 .

If $\hat{\mathfrak{a}}_1 > \hat{\mathfrak{a}}_2$, no agent chooses to join platform M_2 .

When $\hat{\mathfrak{a}}_1 < \hat{\mathfrak{a}}_2$, agents with $\mathfrak{a} < \hat{\mathfrak{a}}_1$ choose M_1 , agents with $\mathfrak{a} \in (\hat{\mathfrak{a}}_1, \hat{\mathfrak{a}}_2)$ choose M_2 , and agents with $\mathfrak{a} > \hat{\mathfrak{a}}_2$ stay unmatched. When this is the case, then the resulting rejection probabilities are indeed $\Pr(rej|M_1, \mathfrak{a} \in [0, \hat{\mathfrak{a}}_1)) < \Pr(rej|M_2, \mathfrak{a} \in (\hat{\mathfrak{a}}_1, \hat{\mathfrak{a}}_2))$.

Thresholds $\hat{\mathfrak{a}}_1$ and $\hat{\mathfrak{a}}_2$ depend on f_1 and f_2 , which are set by the platforms. Platforms take into account the resulting decisions of agents when setting their fees. Notice that f_1 and f_2 uniquely characterize $\hat{\mathfrak{a}}_1(f_1, f_2)$ and $\hat{\mathfrak{a}}_2(f_1, f_2)$. Therefore, we can think of the platforms as effectively choosing \mathfrak{a}_i^* given \mathfrak{a}_i^* . Platforms' profits are a product of their fees and the measure of agents who join them. First, notice that platform M_1 would never set $\hat{\mathfrak{a}}_1 = 1$, as it would require $f_1 = 0$ (to attract $\mathfrak{a} = 1$), and would result in 0 profits, while positive profits for other $\hat{\mathfrak{a}}_1$ are available. Similarly, platform M_1 never sets f_1 so high that $\hat{\mathfrak{a}}_1 = 0$, as it also results in 0 profits.

Next, notice that platform M_2 would never set $\hat{\mathfrak{a}}_2 \leq \hat{\mathfrak{a}}_1$, as it would bring it 0 profit. Also, setting $\hat{\mathfrak{a}}_2 = 1$ would require $f_2 = 0$, and would result in 0 profits, therefore, is suboptimal for M_2 .

Thus, in an equilibrium $0 < \hat{\mathfrak{a}}_1 < \hat{\mathfrak{a}}_2 < 1$. To show that such an equilibrium exists, we turn to analyzing platforms' best response curves. The profits are

$$\pi_1(\hat{\mathfrak{a}}_1|\hat{\mathfrak{a}}_2) = G(\hat{\mathfrak{a}}_1) \cdot f_1(\hat{\mathfrak{a}}_1, \hat{\mathfrak{a}}_2)$$

$$\pi_2(\hat{\mathfrak{a}}_2|\hat{\mathfrak{a}}_1) = (G(\hat{\mathfrak{a}}_2) - G(\hat{\mathfrak{a}}_1)) \cdot f_2(\hat{\mathfrak{a}}_1, \hat{\mathfrak{a}}_2)$$

where f_1 and f_2 are characterized by the indifference conditions

$$\begin{aligned} f_2(\hat{\mathfrak{a}}_1, \hat{\mathfrak{a}}_2) &= EU(\hat{\mathfrak{a}}_2 | M_2) - \hat{\mathfrak{a}}_2 \\ f_1(\hat{\mathfrak{a}}_1, \hat{\mathfrak{a}}_2) &= EU(\hat{\mathfrak{a}}_1 | M_1) - EU(\hat{\mathfrak{a}}_1 | M_2) + f_2 = \\ &= EU(\hat{\mathfrak{a}}_1 | M_1) - EU(\hat{\mathfrak{a}}_1 | M_2) + EU(\hat{\mathfrak{a}}_2 | M_2) - \hat{\mathfrak{a}}_2 \end{aligned}$$

The best responses $\hat{\mathfrak{a}}_1(\hat{\mathfrak{a}}_2)$ and $\hat{\mathfrak{a}}_2(\hat{\mathfrak{a}}_1)$ satisfy first order conditions³³

$$\begin{split} \frac{\partial \pi_1}{\partial \hat{\mathfrak{a}}_1} = & G'(\hat{\mathfrak{a}}_1) \left(EU(\hat{\mathfrak{a}}_1|M_1) - EU(\hat{\mathfrak{a}}_1|M_2) + EU(\hat{\mathfrak{a}}_2|M_2) - \hat{\mathfrak{a}}_2 \right) \\ &+ G(\hat{\mathfrak{a}}_1) \left(\frac{\partial \left[EU(\hat{\mathfrak{a}}_1|M_1) - EU(\hat{\mathfrak{a}}_1|M_2) \right]}{\partial \hat{\mathfrak{a}}_1} + \frac{\partial EU(\hat{\mathfrak{a}}_2|M_2)}{\partial \hat{\mathfrak{a}}_1} \right) = 0 \\ \frac{\partial \pi_2}{\partial \hat{\mathfrak{a}}_2} = & G'(\hat{\mathfrak{a}}_2) \left(EU(\hat{\mathfrak{a}}_2|M_2) - \hat{\mathfrak{a}}_2 \right) + \left(G(\hat{\mathfrak{a}}_2) - G(\hat{\mathfrak{a}}_1) \right) \left(\frac{\partial EU(\hat{\mathfrak{a}}_2|M_2)}{\partial \hat{\mathfrak{a}}_2} - 1 \right) = 0 \,. \end{split}$$

We don't know the exact shape of the best response curves. But we still can characterize certain aspects of them. First, consider the best response of platform M_1 to $\hat{\mathfrak{a}}_2$ set by M_2 . When $\hat{\mathfrak{a}}_2 = 0$, M_1 is de facto a monopolist, where the outside option for the agents is to stay unmatched. The optimal $\hat{\mathfrak{a}}_1(\hat{\mathfrak{a}}_2 = 0) \in (0, 1)$. When $\hat{\mathfrak{a}}_2 = 1$ (i.e., $f_2 = 0$), then M_1 's situation is as in Section 4.1, with the outside market offering M_2 candidates. The optimal $\hat{\mathfrak{a}}_1(\hat{\mathfrak{a}}_2 = 1) \in (0, 1)$ as well. Moreover, for all other values of $\hat{\mathfrak{a}}_2$, $\hat{\mathfrak{a}}_1(\hat{\mathfrak{a}}_2)$ is continuous.

Next, consider the best response of M_2 to $\hat{\mathfrak{a}}_1$. When $\hat{\mathfrak{a}}_1 = 0$, M_2 is defacto a monopolist, and the optimal $\hat{\mathfrak{a}}_2(\hat{\mathfrak{a}}_1 = 0) \in (0, 1)$. And for $\hat{\mathfrak{a}}_1 \to 1$, $\hat{\mathfrak{a}}_2(\hat{\mathfrak{a}}_1 \to 1) \to 1$ (because $\hat{\mathfrak{a}}_1 < \hat{\mathfrak{a}}_2(\hat{\mathfrak{a}}_1) < 1$). And

³³Note that $\frac{\partial EU(\hat{\mathfrak{a}}_2|M_2)}{\partial \hat{\mathfrak{a}}_1} \neq 0$, because $\hat{\mathfrak{a}}_1$ affects $Pr(rej|M_2, \mathfrak{a} \in (\hat{\mathfrak{a}}_1, \hat{\mathfrak{a}}_2))$, and the platform is aware of it when calculating its best response.

since $\hat{\mathfrak{a}}_1(\hat{\mathfrak{a}}_2 \to 1) < 1$, and both curves are continuous, they must intersect at least once for interior values of \mathfrak{a}_i^* (see the Figure B.1). Hence an equilibrium exists. And since $\hat{\mathfrak{a}}_1 < \hat{\mathfrak{a}}_2(\hat{\mathfrak{a}}_1) < 1$, the inequality $\hat{\mathfrak{a}}_1 < \hat{\mathfrak{a}}_2$ holds in this equilibrium. \Box



Figure B.1: We don't know the exact shape of the best response curves. But by the characteristics of the "endpoints", and the fact that both best response curves are continuous (from the first order conditions of platforms' profit maximization problems), they must intersect. And since $\hat{\mathfrak{a}}_2(\hat{\mathfrak{a}}_1) > \hat{\mathfrak{a}}_1$, it assures the properties of the equilibrium.

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