The importance of consumer multi-homing (joint purchases) for market performance: mergers and entry in media markets

Simon P. Anderson
University of Virginia
sa9w@virginia.edu

Øystein Foros
NHH Norwegian School of Economics
oystein.foros@nhh.no

Hans Jarle Kind
NHH Norwegian School of Economics
hans.kind@nhh.no

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Abstract: Consumer "multi-homing"(watching two TV channels, or buying Time and Newsweek) has surprisingly important effects on market equilibrium and performance in (two-sided) media markets. Indeed, consumer multi-homing also significantly impacts one-sided markets. To explore this, we first introduce consumer multi-homing into the classic circle model of product differentiation for a traditional one-sided market. We then introduce an advertiser sector to analyze a two-sided market setting. When consumers multi-home (attend more than one media platform), media firms can charge only incremental value prices to advertisers. We emphasize the implications for entry and optimal/equilibrium product variety, and merger. Results for media market performance can be quite different from the standard single-homing consumer model set-up.

Introduction

Readers who subscribe to more than one newspaper are called multi-homers. Multi-homers abound for streaming services like Netflix and HBO. Consumers with a multi-purpose tablet like Apple's iPad also frequently have Amazon's Kindle e-book reader. Facing multi-homers may dramatically change platforms' competitive strategies compared to when all consumers single-home. Amazon's CEO/founder Jeff Bezos illustrates (Amazon Press release, December 27th, 2010):

"We're seeing that many of the people who are buying Kindles also own an LCD tablet (e.g. an iPad). (…). They report preferring Kindle for reading because it weighs less, eliminates battery anxiety with its month-long battery life, and has the advanced paper-like Pearl e-ink display that reduces eye-strain, doesn't interfere with sleep patterns at bedtime, and works outside in direct sunlight, an important consideration especially for vacation reading."

If consumers buy either an iPad or a Kindle, their reservation prices are the standalone value for iPad and Kindle. Bezos focuses on the value of having a Kindle in addition to an iPad, i.e. the value of a Kindle for (potential) multi-homers. The maximal price - the reservation price - these consumers are willing to pay is the incremental value of a having Kindle. Then the incremental pricing principle applies (Anderson et al. 2017).

Incremental pricing is also important in ad-financed media platforms, which operate in two-sided markets. Such platforms sell eyeballs to advertisers, and the value of an ad increases with the audience size. But the value also depends on whether the audience can be reached elsewhere. If they cannot, each platform has exclusive market power in delivering its consumers to its advertisers. However, if a share of the audience visits two (or more) platforms, the overlapping consumers cannot be sold to the advertisers for a higher price than the extra value of reaching them more than once. Again, the incremental pricing principle applies (Anderson, Foros and Kind, 2018a, Ambrus et al., 2016, Athey et al., 2018): each platform is able to price to consumers and advertisers only the incremental value of the platform over the rival platform.

We analyze how the presence of multi-homing consumers change media platforms' entry and merger incentives compared to the standard single-homing consumer model set-up (Anderson and Coate, 2005, and subsequent papers). We show that entry and merger incentives sharply differs from outcomes where it is assumed that all consumers are single-homing.

A model of multi-homing consumers

To make life interesting, we need more than two platforms before a merger (to avoid a complete monopoly after the merger). Let us allow for $n \geq 2$ platforms, located on a circle of unit circumference (Vickrey, 1964; Salop, 1979). We restrict the analysis to outcomes with market coverage and where all firms have positive sales. Consumers are uniformly distributed around the circle, and for simplicity we assume that they are ad neutral. Then, there is no direct effect on consumers' utility from the ad side of the market.

Platforms are located symmetrically around the circle. In Figure 1 we illustrate with $n = 3$; platform 1 is located at noon (we hold this location fixed, independent of the number of firms), platform 2 at four o'clock, and platform 3 at eight o'clock.
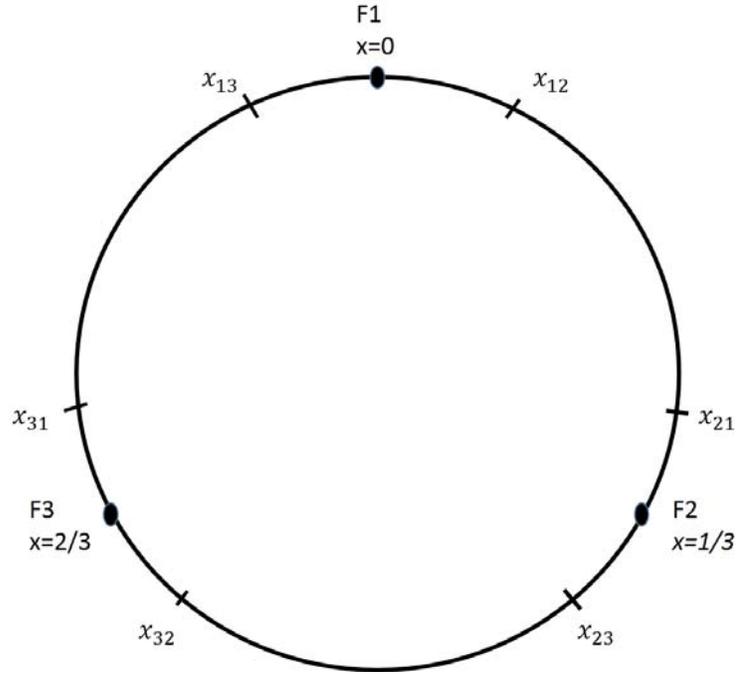
Figure 1: *Salop locations.*

Let the utility for a consumer located at x of buying from *only* platform i be $u_i = v - t|x_i - x| - p_i$, where x_i is the location of platform i. If the consumer is a multi-homer - meaning that he buys from more than one platform - the perceived value of good i might be lower than its stand-alone value. We follow Anderson et al. (2017) in assuming that the incremental value of buying from platform i in addition to platform j equals $u_{ji} = \theta(v - t|x_i - x|) - p_i$, where $\theta \in [0,1]$. A utility maximizing consumer will multi-home as long as $u_{ji} > 0$.

Let x_{ji} denote the location of the consumer who is indifferent between buying only good j and buying both good i and j. We find x_{ji} by solving $u_{ji} = \theta(v - t|x_i - x_{ji}|) - p_i = 0$. As an illustration, consider platform 1. The location of consumer x_{12}, who is indifferent between buying only from platform 1 and buying both from platform 1 and 2 (cf. Figure 1), is given by

$$u_{12} = 0 \Rightarrow x_{12} = \frac{1}{n} - \frac{v\theta - p_2}{t\theta}. \qquad \#$$

Solving $u_{1n} = 0$ we likewise find the location of the consumer who is indifferent between buying only from platform 1 and buying both from platform 1 and n :

$$u_{1n} = 0 \Rightarrow x_{1n} = 1 - \left(\frac{1}{n} - \frac{v\theta - p_n}{t\theta} \right). \qquad \#$$

The number of exclusive consumers for platform 1 is thus equal to $x_{12} + (1 - x_{1n})$. We shall focus on equilibria where platforms are symmetrically located, and where each platform has some multi-homers and some exclusive consumers (conditions for this to be the case are discussed in Section 2.2). This implies that no consumer will buy more than two goods; one from each of the platforms closest to him. Thus, we can distinguish between two groups of consumers for platform i; those who only buy from that platform, and those who buy from platform i as well as from *either* platform $i - 1$ *or* $i + 1$. From (ref: u12) and (ref: u1n) we can deduce that the number of exclusive consumers for platform i equals (superscript e)

$$x_i^e = \frac{2}{n} - \frac{2\theta v - p_{i-1} - p_{i+1}}{\theta t}. \qquad \#$$

Other things being equal, the number of exclusive consumers is consequently decreasing in the number of platforms and increasing in the prices charged by the (two closest) rivals.

Let D_i denote total demand faced by platform i. For platform 1, this is equal to the shorter arc distance between x_{31} and x_{21} in Figure 1, where x_{31} and x_{21} solve $u_{31} = 0$ and $u_{21} = 0$, respectively. This yields $D_1 = \frac{2}{t}\left(\frac{\theta v - p_1}{\theta}\right)$. For an arbitrary platform i we consequently have

$$D_i = \frac{2}{t}\left(\frac{\theta v - p_i}{\theta}\right), \qquad \#$$

so that we have a downward-sloping demand curve. Interestingly, since the *incremental* value of platform i is unaffected both by the number of platforms in the market and of the prices charged by the rivals, D_i is independent of n and p_j ($j \neq i$).

Subtracting (ref: x e) from (ref: Di) we find that the number of multi-homers (superscript *mh*) on platform i equals

$$x_i^{mh} = D_i - x_i^e = \frac{4\theta v - 2p_i - p_j - p_k}{\theta t} - \frac{2}{n} \qquad \#$$

Summing up, platform i's own pricing behavior does not affect its number of exclusive consumers, only how many multi-homers it will have. This is directly observed from Figure 1. A reduction in p_1 moves x_{21} clockwise and x_{31} counter-clockwise, thereby increasing the total demand and the number of multi-homers for platform 1, while x_{12} and x_{13}, determining the number of exclusive consumers, are not affected.

Below, we first consider a one-sided market; not even all media markets are multi-sided, and the presence of multi-homing consumers may affect merger incentives also in a such a market.

A one-sided market

We set all costs to zero, and write profit for platform i as $\pi_i = p_i D_i$. Since total demand for each platform is independent of prices charged by the rivals, profit-maximizing prices depend solely on the incremental value each good offers (given that the platforms cannot discriminate between single-homers and multi-homers). Inserting for D_i from equation (ref: Di) into the profit function and maximizing with respect to p_i we find

$$p_i = \theta v/2.$$

This price reflects the incremental pricing principle shown by Anderson et al. (2017) in a Hotelling duopoly set-up.

We have shown that platform i's total demand is independent of the number of rivals in the market. Since the same is true for the equilibrium price, we obviously have strong entry incentives when some (but not all) consumers multi-home. Note also that the profitability of establishing a new platform (outlet) is identical for an incumbent and a potential entrant.

These results differ sharply from what we have when consumers are restricted to single-homing. Under single-homing, entry reduces aggregate profit due to intensified price competition. Incumbents have lower incentives to set up a new outlet than a newcomer, since they take into account the response from the rival on its current business. This relates to the literature on preemptive investments in new goods, where Schmalensee (1978) and Eaton and Lipsey (1979) are seminal papers. Judd (1985) shows that such preemptive investments may not be credible, and that a multi-product incumbent may want to withdraw a product when a

newcomer enters the market to prevent reduced profit from the other products he offers. Such a defensive response makes entry more attractive for potential entrants. However, this effect is not present if incremental pricing is a reasonable description of the market behavior. Thus, other things equal, we may expect to observe more multi-product incumbents if some consumers multi-home than if we only have single-homers. This result is consistent with the observation that many digital hardware and software platforms, where incremental pricing seems reasonable, introduce a lot of new versions and products.

By the same token, if two firms merge in a Salop-Vickrey circle set-up under single-homing, the merged company may have incentives to close one outlet. While this does not happen in the context of linear transport costs (assumed for the main model), it will if we replace them with quadratic transport costs. To illustrate, assume single-homing consumers and that platforms 1 and 2 in Figure 1 merge. After the merger they will have incentives to shut down one of the outlets, and relocate to six o'clock. This follows from the seminal paper of d'Aspremont, Gabszewicz and Thisse (1979), who show that maximal differentiation on a Hotelling line is the duopoly equilibrium. Martinez-Giralt and Neven (1988) show for the circle model with multiple outlets that firms want to relocate their own neighboring products such that the distance to rivals is maximized, in order to relax price competition.

It is well known from the merger literature that without merger-specific efficiency gains, non-merging firms may benefit more from a merger than the merging firms (Deneckere and Davidson, 1985, and Farrell and Shapiro, 1990). This property holds for the Vickrey-Salop model under the conventional assumption of single-homing consumers. Depending on modeling details of merger decisions, this could create a hold-up problem and hinder profit-enhancing mergers from taking place. The results above show that the hold-up problem vanishes when we have multi-homing consumers. In the one-sided market version above, there is no spill-over on other firms, but no incentive to merge either. However, once we introduce the second side of the market, the neutrality result is tempered: there remains no spill-over on others, but there is an incentive to merge. In this sense, we may therefore expect mergers without efficiency gains to be more common under multi-homing than under single-homing.

Finally, before we move to the two-sided market set up, let us scrutinize an efficiency gain from a merger which is present under multi-homing but not under single-homing. We have assumed that firms cannot set different prices to exclusive and shared consumers. However, a merged company will have more accurate knowledge about its consumers locations than an independent firm. Zoom in on firm 1, located at noon. Total demand is given by D_1 (the sum of the intervals $[x_{31}, 0]$ and $[0, x_{21}]$ in Figure 1). However, platform 1 does not know whether a given consumer is an exclusive consumer, shared with platform 2 or shared with platform 3. Assume now that platform 1 and 2 merge. The merged company knows that a consumer who buys both good 1 and good 2 is in the interval $[x_{12}, x_{21}]$. Its also knows that a consumer buying only good 1 from the merged company is located between x_{13} and x_{12} (see Figure 1), while a consumer who only buys good 2 from the merged company is located between x_{21} and x_{23}. Such an informational advantage over platform 3 may be used by the merged company to customize its products. Below, we show that in a two sided market, more accurate information about consumers' location makes it possible to better match advertisers with the consumers they want to reach.

A two-sided market

We now allow platforms to place ads on their platforms, and to analyze two-sided pricing we follow Anderson et al. (2018b) and Foros et al. (2018). We hold on to the assumption that

we have partial multi-homing ($x_i^e > 0$ and $x_i^{mh} > 0$), see below.

The platforms decide a price per ad. Advertisers only place one advert per platform. Demand for ads is perfectly elastic, with a mass A of homogenous advertisers. We set $A \equiv 1$, so we need not to make a distinction between price per ad and total ad revenue.

We follow Anderson et al. (2018a) and assume that each advertiser is willing to pay b per ad per exclusive consumer reached, and σb per multi-homing consumer. A third impressions is worth nothing to advertisers. We let $0 \leq \sigma \leq 1$, so that the value of re-reaching the same consumer is (weakly) lower than the value of a first impression.

Profit for platform i is now equal to $\pi_i = p_i D_i + b x_i^e + \sigma b x_i^{mh}$. Inserting for demand from equations (ref: x e) - (ref: x s) and maximizing with respect to p_i we find that the equilibrium price and total sales per platform equal

$$p = \frac{\theta v - \sigma b}{2} \text{ and } D = \frac{1}{t}\left(\frac{\theta v + \sigma b}{\theta} \right). \qquad \#$$

The consumer price is decreasing in the incremental value of selling ads, as measured by σb, but independent of the number of platforms. Notice that, consistent with Armstrong (2006), the extra profit per mhc, σb, is akin to a negative marginal cost. Consequently, if an entrant sets up another platform, the consumer price is not affected. Consumers are nonetheless better off, since total industry volume increases (from nD to $(n+1)D$) and the price is below the consumers' reservation price.

With n platforms, it follows from (ref: x e) - (ref: x s) and (ref: p*) that the number of multi-homers and exclusive consumers on each of them is

$$x_n^{mh} = 2\left(D - \frac{1}{n} \right) \text{ and } x_n^e = \frac{2}{n} - D. \qquad \#$$

Clearly, all consumers will multi-home if the number of platforms is sufficiently large. Note also that the platforms charge lower prices the more profitable it is to sell ads and the smaller the consumers' incremental value of buying from a second platform. Thereby it follows that $dx_n^e/d(\sigma b) < 0$ and $dx_n^e/d\theta > 0$. We restrict attention to outcomes $D \in (D^{\min}, D^{\max})$, where $D^{\min} \equiv \frac{1}{n}$ and $D^{\max} \equiv \frac{2}{n}$. That is, we consider

$$D = \frac{1}{t}\left(\frac{\theta v + \sigma b}{\theta} \right) \in \left(\frac{1}{2n}, \frac{1}{n} \right). \qquad \#$$

This ensures partial multi-homing.

We label the profit level of each of the n platforms as π_n:

$$\pi_n = pD + b x_n^e + \sigma b x_n^{mh}. \qquad \#$$

Entry does not affect the incumbents' profit on the consumer side of the market, but reduces their profit from the ad side by increasing the fraction of multi-homing consumers.

Instead of entry from an outside firm, suppose that one of the incumbents sets up an additional platform, and that it places the new platform next to itself on the Salop circle (we maintain the assumption that all platforms are located equidistantly from each other). The two-platform company may then charge $b(1 + \sigma)$ for the eyeballs they have in common, because they control access completely to these consumers. As before, they charge b for exclusive eyeballs and $b\sigma$ for eyeballs they share with rival platforms.

With no loss of generality, let us assume that platform 1 establishes platform 2. If we assume that there were $n - 1$ platforms at the outset, there will be n platforms after platform 2 is established. The number of shared consumers between platform 1 and 2 is given by

$$x_{21} - x_{12} = \left(\frac{2v\theta - p_1 - p_2}{t\theta} - \frac{1}{n} \right)$$

A two-platform company may thus capture $b(1 + \sigma)(x_{21} - x_{12})$ when selling these eyeballs to advertisers. However, as shown by Anderson et al. (2018b), this does not affect the consumer price. The reason is easily seen from Figure 1. A slight reduction of p_1 will turn a previously exclusive consumer on platform n, located at x_{n1}, into a multi-homer ($n = 3$ in Figure 1). The consumer at x_{n1} may be sold to advertisers for σb regardless of the number of rivals. By the same token, at x_{21}, a consumer previously exclusive to platform 2, is turned into a multi-homer. The two-platform company can now charge advertisers $b(1 + \sigma)$ for the marginal consumer at x_{21}. However, the incremental value is still given by σb, since this consumer where without a price reduction of p_1 sold as an exclusive consumer to platform 2 for b. Consequently, a multi-platform company sets the same consumer price as a single-platform firm (Anderson et al., 2018b). The price is given by (ref: Pi n). By the same token, a merger between two platforms does not affect the consumer price.

Since the consumer price is identical for a two-platform company and a single-platform company, the number of jointly shared consumers is given by $x_n^{mh}/2$. Consequently, joint profit for a two-platform company, consisting of platform i and j, is given by:

$$\pi_{i+j} = 2pD + 2bx_n^e + 2\sigma bx_n^{mh} + b(1 - \sigma)\frac{x_n^{mh}}{2} = 2\pi_n + b(1 - \sigma)\frac{x_n^{mh}}{2} \qquad \#$$

To find the incremental profit for an incumbent from establishing an additional platform we need to compare (ref: Pi i+j) with π_{n-1}. Defining $\Delta^{incumbent} = \pi_{i+j} - \pi_{n-1}$ we find

$$\Delta^{incumbent} = pD + bx_n^e + \sigma bx_n^{mh} + b(1 - \sigma)\frac{x_n^{mh}}{2} - [b(x_{n-1}^e - x_n^e) + \sigma b(x_{n-1}^{mh} - x_n^{mh})]$$

The three first terms equals an entrant's profit given by (ref: Pi n). Using equations (ref: x e) and (ref: x s) we further find that the term in the square bracket equals

$$I(D) \equiv b(1 - \sigma)\left(D - \frac{n+1}{n(n-1)}\right).$$

We now have

$$\Delta^{incumbent} = \pi_n + I(D). \qquad \#$$

If $I > 0$, an incumbent has greater incentives than a potential newcomer to set up a new platform, and vice versa if $I < 0$.

Assume first that $D \to D^{min} = 1/n$; the number of multi-homers approaches zero. This implies

$$I(D^{min}) = -\frac{2b(1 - \sigma)}{n(n - 1)} < 0.$$

Hence, an incumbent has lower incentives than an entrant to set up an additional platform if the fraction of multi-homers is sufficiently low. In contrast, assume that $D \to D^{max} = 2/n$; the number of exclusive consumers approaches to zero. Then,

$$I(D^{max}) = \frac{(n - 3)b(1 - \sigma)}{n(n - 1)} > 0.$$

Hence, if the demand approaches complete multi-homing, the incumbent may have higher incentives than an entrant if $n > 3$.

Let us go back to mergers. In our simple set-up a merger hurts the advertisers, with no impact for the consumers. Holding the number of platforms fixed, two neighbor platforms have incentives to merge in order to capture a higher price from their common multi-homers.

However, a merger may have other effects. As emphasized in the one-sided market section, a merged company will have more accurate information about its consumers location. More accurate information about consumers' locations may help the merged company to extract more revenues from advertisers. Advertising-financed media models typically assume that advertiser willingness to pay for impressions are uncorrelated to the tastes of the consumers for the medium. Anderson et al. (2018c) introduce correlation between the two, and draw out the implications for performance and equilibrium ad levels and ad pricing when media content matches advertisers to their prospective audience. Evans and Schmalensee (2015) provide an example. Hoist is a magazine with content specified towards overhead crane operators, and the magazine then offers ads to firms that want to sell equipment to this audience. For the advertisers of crane-related products it is more efficient and probably cheaper to have ads in Hoist compared to a more general magazine or newspaper.

Optimum and equilibrium firm numbers

The classic Salop-Vickrey circle model, with single-homing consumers and zero-profits for firms, delivers that the equilibrium has twice the optimum number of firms when transport costs are linear (Vickrey, 1964, Salop, 1979).

We now address this question in the two-sided market context with partially multi-homing consumers (phmc). Notice that a phmc regime requires some parameter restrictions. Namely, as noted above (see (ref: D-range)), that

$$D = \frac{1}{t}\left(\frac{\theta v + \sigma b}{\theta}\right) \in \left(\frac{1}{2n}, \frac{1}{n}\right).$$

When we come to free-entry, we shall require that the fixed cost, K, delivers such an outcome for n.

We first determine the free-entry (symmetric) equilibrium number of platforms, and then we find the social welfare function (the sum of consumer, producer, and advertiser surplus) to determine the welfare derivative when evaluated at the equilibrium number of firms.

For profits, the key ingredients on the consumer side (from (ref: p*)) are the equilibrium subscriber price, $p = \frac{\theta v - \sigma b}{2}$, and equilibrium demand as given above. The product represents subscription revenues. There are also the advertising revenues. These are b on the exclusive consumers, and σb on the mhc for each firm. Using (ref: x's in equil), the fractions of each type are $x_n^{mh} = 2(D - \frac{1}{n})$ and $x_n^e = \frac{2}{n} - D$ (so if $D \to \frac{1}{n}$ all consumers are exclusives, and if $D \to \frac{2}{n}$ they are all shared two ways).

We can then write the profit per firm (see (ref: Pi n)) as

$$\pi_n = \frac{(\theta v - \sigma b)D}{2} + b\left(\frac{2}{n} - D\right) + 2\sigma b\left(D - \frac{1}{n}\right).$$

Setting this equal to entry cost, K, yields the free-entry equilibrium number of firms as

$$n = \frac{2b(1 - \sigma)}{K - \frac{(\theta v - \sigma b)D}{2} + bD(1 - 2\sigma)}.$$

Consider next the social optimum. The sum of surpluses is as follows. Total profits with n firms are n times the profit expression above, minus the nK in entry costs. Thus the total profit derivative with respect to n is:

$$\frac{(\theta v - \sigma b)D}{2} - Db + 2\sigma bD - K;$$

evaluating this where profits are zero yields

$$\frac{d(n\pi_n)}{dn}\Big|_{\pi_n=K} = \frac{2b}{n}(\sigma - 1) < 0$$

as the profit externality on other firms from entry (where the relevant n solves $\pi_n = K$). This is the business stealing effect in this two-sided market context. The other externalities are the consumer surplus one and the advertiser surplus one. The former is more intricate when consumers multi-home, while the latter is particular to the two-sided market.

First, advertiser surplus is only earned on the multi-homing consumers, because firms can extract full value on the exclusives (single-homers). The multi-homers are worth $b(1 + \sigma)$ but the advertisers pay only $2\sigma b$ for them (σb to each firm providing a particular platform multi-homing pair), for a surplus of $b(1 - \sigma)$ each. Notice that each firm has $2(D - \frac{1}{n})$ multi-homers, so the total number of them is $n/2$ times this amount since each one is delivered by 2 platforms. Hence the total advertiser surplus is

$$nb(1 - \sigma)\left(D - \frac{1}{n}\right)$$

which is increasing in n at rate $b(1 - \sigma)D$. Combining with the profit externality, we have so far the total producer surplus externality as

$$\frac{d(n\pi_n + AS)}{dn}\Big|_{\pi_n=K} = b(1 - \sigma)\left(D - \frac{2}{n}\right) < 0, \tag{#}$$

where we can sign the expression under the restriction (see (ref: D-range)) that $D \in (\frac{1}{2n}, \frac{1}{n})$. As far as the full producer side is concerned, entry is excessive, despite the benefit to advertisers.

Therefore we need to turn to the consumer side to resolve whether entry can be insufficient or excessive. We next determine consumer surplus.

The first, traditional, component of consumer surplus accrues on "first"-purchases, that is, the more preferred product. To this we must add the extra surplus accruing on second-preference product. The first part is the closest product, the second is the second closest one. For first choices, the average "distance" travelled is $1/4n$ (to the closer product of the two bought, from he two neighboring firms). Because transport costs are linear, at rate t, the average distance disutility suffered on the first choices is then $t/4n$; the market is covered by first choice products (all consumers buy at least their best choice). Hence the consumer surplus on first choices is

$$v - \frac{t}{4n} - \frac{\theta v - \sigma b}{2}.$$

For the second choice products, the minimal distance travelled is $1/2n$ and the maximal one is $D/2$ (for the consumer indifferent between adding the second product). So the average distance travelled, conditional on multi-purchase, is $\frac{1}{2}(\frac{1}{2n} + \frac{D}{2})$. Such products are valued by their buyers at a gross surplus of θv minus $t\theta$ times the distance cost, so that the average surplus per multi-homer's second purchase is

$$\theta v - \frac{t\theta}{2}\left(\frac{1}{2n} + \frac{D}{2}\right) - \frac{\theta v - \sigma b}{2}$$

(where the last term is again the price).

Now, the mass of such consumers is $(nD - 1) \in (0, 1)$ (recall that each such consumer is shared twice). Therefore the total consumer surplus is

$$CS = v - \frac{t}{4n} - \frac{\theta v - \sigma b}{2} + (nD - 1)\left(\theta v - \frac{t\theta}{2}\left(\frac{1}{2n} + \frac{D}{2}\right) - \frac{\theta v - \sigma b}{2}\right)$$

$$= v - \frac{t}{4n} + nD\left(\frac{\theta v + \sigma b}{2}\right) + (nD - 1)\left(-\frac{t\theta}{2}\left(\frac{1}{2n} + \frac{D}{2}\right)\right) - \theta v.$$

The derivative is

$$\frac{dCS}{dn} = \frac{t}{4n^2} + D\frac{\theta v + \sigma b}{2} - D\frac{t\theta}{2}\left(\frac{1}{2n} + \frac{D}{2}\right) + (nD - 1)\frac{t\theta}{2}\left(\frac{1}{2n^2}\right),$$

which is positive by its construction.

The leading question is whether this can overturn the classic result noted above that entry is excessive. If we add the above to the other expression, can the answer be positive? The answer is affirmative. Recalling (from (ref: producer externality)) that $\frac{d(n\pi_n + AS)}{dn}\big|_{\pi_n = K} = b(1 - \sigma)(D - \frac{2}{n})$, with $D = \frac{1}{t}(\frac{\theta v + \sigma b}{\theta})$, then if $\sigma \to 1$ (or indeed if $b \to 0$) then the producer-side externality vanishes. It also vanishes as we approach fmhc (i.e., $D \to \frac{2}{n}$). This leaves the consumer side, which is positive, so that there is then under-entry.

On the other side of the coin, when do we now see over-entry? The welfare derivative is

$$\frac{d(n\pi_n + AS + CS)}{dn}\Big|_{\pi_n = K} = \frac{t}{4n^2} + D\frac{\theta v + \sigma b}{2} - D\frac{t\theta}{2}\left(\frac{1}{2n} + \frac{D}{2}\right) + (nD - 1)\frac{t\theta}{2}\left(\frac{1}{2n^2}\right) + b(1 - \sigma)\left(D - \frac{2}{n}\right)$$

$$= \frac{t}{4n^2} - (\theta v + \sigma b)\frac{1}{4}\left(\frac{1}{n} - D\right) + \left(D - \frac{1}{n}\right)\frac{t\theta}{2}\left(\frac{1}{2n}\right) + b(1 - \sigma)\left(D - \frac{2}{n}\right).$$

If we take $D \to \frac{1}{n}$ (i.e., take the corresponding K value ensuring this is so), then the derivative reduces to

$$\frac{d(n\pi_n + AS + CS)}{dn}\Big|_{\pi_n = K} = \frac{t}{4n^2} - \frac{b(1 - \sigma)}{n},$$

so that this is negative for $\frac{t}{4n} < b(1 - \sigma)$. Over-entry is associated with high advertiser value (but low second impression value), and low loyalty cost, t.

Equilibrium existence for pricing game

Salop (1979) noted the inverse demand curve facing an individual firm kinks down as competition moves from facing the outside good to facing a competitor. This feature can lead to multiple (asymmetric) price equilibria. What is different in the present context is that there is an upward kink in the demand curve as firms transition from the single-homing to the pmhc regime. To see this, note that the (inverse) demand curve slope under single-homing is $-1/2t$. However, under a pmhc regime it is $-1/\theta t$. Such upward kinks imply a jump up in marginal revenue, and so potentially two local maxima in profit. This feature may jeopardize the existence of the pmhc equilibrium candidate price, and is more contentious the smaller the number of pmhc.

Location analysis

We have chosen above a symmetric set of locations for firms. However, each firm in a pmhc regime is locally indifferent to moving. To see this, note that any small move does not change its price as long as it faces pmhc on both sides (see Figure 1). A small move then loses a consumer on one side, with advertising loss σb, but gains one on the other side, with benefit σb. Such a change is profit neutral. Hence any set of locations with firms facing pmhc on both sides constitutes an equilibrium. That is, each pair of locations around the circle must be more

than D apart, but less than $2D$. We selected the symmetric set of locations.

More interesting is the result that location incentives do not change if a firm merges with its neighbor. It has no strict incentive to move, and nor do its neighbors, and so the equi-spaced configuration prevails. To see this, suppose that Firms 1 and 2 are merged and move closer to Firm 3, such that one more consumer on each side of Firm 3 will now also read paper 1/paper 2 in addition to newspaper 3. The merged unit will gain $2\sigma b$. However, two consumers who previously bought both 1 and 2 will now buy only one of them. The loss from this is $2(b + \sigma b) - 2b = 2\sigma b$. So there is no change in profit if the merged firm moves its outlets.

Generalizations

The advertiser demand curve was deliberately simple. However, the results still hold with a more general advertiser demand curve, with the only difference being that the equilibrium ad level is just the monopoly one on the advertiser demand curve. Assume that each advertiser has a value $r(a)$ per unique impression, with advertisers ranked from high to low willingness to pay (as is standard). Then assume that they value 2 impressions at $(1 + \sigma)br(a)$. Then the advertiser demand curve just pivots up when there are multiple types of viewer in the basket offered by a platform. Then the equilibrium ad level is the monopoly one against the demand curve, a_m, independently of the composition of exclusive and mhc. The price of the bundle does depend on the composition: it is $b(x^e r(a_m) + \sigma x^{mh} r(a_m))$.

On the demand side we have used the traditional linear transport cost formulation. The key property for the pricing equilibrium to be independent of n is that competition among firms should be with the outside good at the margin of the indifferent pmhc. This continues to hold with other transport cost functions. For example, under the often-used quadratic transport cost assumption, we have the marginal consumer in the phmc regime given (in inverse demand form) $p = \theta v - tx^2$. Qualitative results are unaffected.

A more complex extension is to consider i.i.d. preferences across goods (non-localized competition). Then each platform faces competition with both the outside good and all other products for the second choice good. In the case of a merger, this suggests that a merged firm's price will be higher than when unmerged because it internalizes the effect on its sibling product when merged. Results are then more nuanced.

We have analyzed a pmhc regime above. However, if there are enough firms, there will be full multi-homing. That is, each consumer in each inter-firm interval will buy from its two closest firms. we can readily determine the corresponding equilibrium price candidate. Competition, at the margin, now moves to the marginal multi-homing consumer, who is located atop the next rival's location, and this competition is with the outlet two firms over. Thus for the case of a one-sided market, the traditional Salop price, t/n is modified by replacing n by $n/2$ and t by θt. So then the price becomes $2\theta t/n$. Whether prices are higher or lower in the market then depend on the size of θ. Indeed, if $\theta > 1/2$, prices are higher with multi-homing. This idea extends clearly to when consumers may buy more than one extra product.

For a two-sided market, the single-homing consumer price is $\frac{t}{n} - b$, because of value of advertisers is competed away (recall that the Salop price can be interpreted as the mark-up, so the claimed result follows). With a fmhc regime, we make the changes above, with the additional change that the value of an advertiser goes from b to σb. So the price under fmhc becomes $\frac{2\theta t}{n} - \sigma b$. The lower value of advertisers to firms provides an additional fillip to raising price.

The models have deliberately closed down ad nuisance in order to retain simplicity, as well as to focus on markets where nuisance does not have a first-order effect (newspapers perhaps, as opposed to television). Ad nuisance intertwines effects, but in a rather interesting way. The case of shc is quite straightforward: merger does not change ad levels. This is because ad levels are determined by the condition $R'(a) = \gamma$ to get the ad level (marginal revenue per consumer in the ad market equals nuisance cost per ad: see e.g. Anderson and Coate, 2005, and Anderson and Jullien, 2017 for further discussion). There are then only subscriber price effects to consider, but then it is a standard circle analysis (albeit asymmetric due to the merger), so the insights noted above apply that advertising benefits in the analysis accrue as if they were negative marginal costs, and the same for all firms. Things are more involved when there are pmhc. Then the composition effects (of the viewer basket) impinge, so that the $R'(a) = \gamma$ condition becomes a weighted average condition $x^e R'(a) + x^{mh} \sigma R'(a) = D\gamma$. This condition defines the relation between a and p as the firm varies the latter.

Conclusions

Two key properties are at play in the result that merger under pmhc impacts only advertisers, and not consumers nor other firms (and the ancillary result that there i.e. no relocation incentive). The first stems from incremental pricing to the marginal consumer: a firm does not change a rival's (or sibling's) consumer base when it changes its subscription price. This property implies there is no spill-over on the consumer side from merger. Secondly, incremental value pricing in advertising implies that switching a rival's customer from being that rival's exclusive to a shared customer gets the firm the incremental value (from the pricing of ads) σb; while converting a sibling's exclusive customer to a mutually shared one delivers the firm $b(1 + \sigma)$ in place of b. So the economic incentive is the same for conversions.

Any model with the two properties would give the consumer-price neutrality result, and have the merger fall just on advertisers. It would, for example, hold with a downward-sloping ad demand as described in the previous section. On the consumer side of the tally, any model where the monopoly incremental pricing result holds would work (i.e., not just a circle).

It was shown in Anderson, Foros, and Kind (2018b) that the shc two-sided model gives the impact of a merger only on consumers, and not on advertisers. The same is true for entry of a new firm. For a merger (in a context with three outlets, and two merging), the merged firm charges higher consumer prices and earns more than pre-merger (though the remaining firm's profit rises by more). Advertiser prices remain at b per consumer, because all consumers are reached only once under single-homing. This is a example of a (weak) see-saw effect (see Anderson and Peitz, 2017, for see-saw effects in media markets under shc): a change in circumstances that affects market participants on one side in the opposite direction to those on the other side. Another (again weak) see-saw effect occurs *in the opposite direction* under pmhc. Then the full brunt (for merger) or benefit (for entry) is borne by advertisers.

In order to deliver some continuity between these extreme cases, we could ask what happens if we have some fraction of consumers who are potential multi-homers. That is, θv is high for a fraction κ and prohibitively low for the rest, and we vary κ to put more or less weight on the pmhc segment. Then the impact of a change (merger or entry) will fall more on the advertisers the larger is κ. That is, a larger fraction of shc will cause a bigger consumer price response. There is no see-saw effect because both sides of the market are affected the same way, with the incidence depending on κ.

References

Ambrus, A., E. Calvano, and M. Reisinger (2016): Either or both competition: A "two-sided" theory of advertising with overlapping viewerships. American Economic Journal: Microeconomics, 8(3), 189-222.

Ambrus, Attila, and Marcus Reisinger (2005). Platform competition and welfare: media markets reconsidered. mimeo.

Anderson, S. and S. Coate (2005): Market provision of broadcasting: A welfare analysis. Review of Economic Studies, 72, 947-972.

Anderson, S., Ø. Foros, and H. J. Kind (2017): Product functionality, competition, and multi-purchasing. International Economic Review, 58(1) 183-210.

Anderson, S., Ø. Foros, and H. J. Kind (2018a): Competition for advertisers and for viewers in media markets. The Economic Journal. 128, 34-54.

Anderson, S., Ø. Foros, and H. J. Kind (2018b): Price effects in two-sided media mergers, work in progress.

Anderson, S., Ø. Foros, and H. J. Kind (2018c): Advertiser matching to Media, work in progress.

Anderson, S. P., and Gabszewicz, J. J. (2006). The media and advertising: a tale of two-sided markets. Handbook of the Economics of Art and Culture, 1, 567-614.

Anderson, S. P., and Jullien, B. (2015). The advertising-financed business model in two-sided media markets. In Handbook of Media Economics (Vol. 1, pp. 41-90). North-Holland.

Armstrong, M. (2006). Competition in two-sided markets. The RAND Journal of Economics, 37(3), 668-691.

Athey, S., E. Calvano, and J. Gans (2018): The Impact of Consumer Multi-Homing on Advertising Markets and Media Competition. Management Science. Forthcoming.

Cabral, Luis M. B. (2003) Horizontal mergers with free-entry: why cost efficiencies may be a weak defense and asset sales a poor remedy. International Journal of Industrial Organization 21 (5), 607-623.

Choi, J. P. (2006). Broadcast competition and advertising with free entry: Subscription vs. free-to-air. Information Economics and Policy, 18(2), 181-196.

Crampes, C., Haritchabalet, C., and Jullien, B. (2009). Advertising, competition and entry in media industries. The Journal of Industrial Economics, 57(1), 7-31.

d'Aspremont, C., Gabszewicz, J. J. and Thisse, J. F. (1979): On Hotelling's "Stability in Competition." *Econometrica* 17, 1145-1151.

Deneckere, J. and C. Davidson (1985): Incentives to form coalitions with Bertrand competition. Rand Journal of Economics, 16(4), 473-486.

Eaton, B.C. and R. G. Lipsey (1980): The Theory of Market Preemption: The Persistence of Excess Capacity and Monopoly in Growing Spatial Markets. Economica, 46, 149-158.

Eaton, B. C., and Wooders, M. H. (1985). Sophisticated entry in a model of spatial competition. The Rand Journal of Economics, 282-297.

Evans, D. S. and R. Schmalensee (2016): Matchmakers. The New Economics of Multisided Platforms, Harvard Business Review Press, USA.

Fan, Y. (2013): Ownership consolidation and product characteristics: A study of the US daily newspaper market. American Economic Review, 103(5), 1598-1628.

Farrell, J. and Shapiro, C. (1990): Horizontal mergers: An equilibrium analysis. American

Economic Review, 80(1), 107-126.

Foros, Ø., Kind, H. J., and Sørgard, L. (2015). Merger policy and regulation in media industries. In Handbook of Media Economics (Vol. 1, pp. 225-264). North-Holland.

Foros, Ø., H. J. Kind and T. Wyndham (2018): Tax-free digital news? International Journal of Industrial Organization, forthcoming (???)

Gentzkow, M. (2007): Valuing New Goods in a Model with Complementarity: Online Newspapers. American Economic Review, 97(3), 713-744.

Gentzkow, M., J. Shapiro and M. Sinkinson (2011): The Effect of Newspaper Entry and Exit on Electoral Politics. American Economic Review, 101(7), 2980-3018.

Gentzkow, M., J. Shapiro and M. Sinkinson (2014): Competition and Ideological Diversity: Historical Evidence from US Newspapers. American Economic Review, 104(10), 3073-3114.

Hotelling, H. (1929): Stability in Competition. Economic Journal, 39, 41-57.

Judd, K. (1985): Credible Spatial Preemption. RAND Journal of Economics, 16(2), 153-166.

Martinez-Giralt, Xavier and Neven, Damien J. (1988). Can price competition dominate market segmentation? Journal of Industrial Economics, 431-442.

Salop, S.C. 1979. Monopolistic competition with outside goods, The Bell Journal of Economics, 10(1), 141-156.

Schmalensee, R. (1978): Entry Deterrence in the Ready-to-Eat Breakfast Cereal Industry. Bell Journal of Economics, 9, 305-327.

Vickrey, W. S., Anderson, S. P., and Braid, R. M. (1999). Spatial competition, monopolistic competition, and optimum product diversity. International Journal of Industrial Organization, 17(7), 953-963.

Vickrey, W. (1964): Microstatics. Harcourt, Brace, New York.