Network Structures and Entry into Platform Markets

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Abstract

While some platform markets exhibit strongly interconnected network structures in which a buyer is interested in purchasing services from most providers, many platform markets consist of local clusters in which a buyer is primarily interested in purchasing services from providers within the same cluster. We examine how network structures affect interactions between an incumbent platform serving multiple markets and an entrant platform seeking to enter one of these markets. We find that having more mobile buyers, which increases interconnectivity among markets, reduces the incumbent's incentive to fight and increases the entrant's incentive to expand. Incumbent profits increase with interconnectivity. When advertising is inexpensive and mobile buyers consume in both local markets and the markets they visit, greater interconnectivity increases the entrant's profit, thus encouraging entry.

Key words: network structure, clustering, platform competition, entry barrier

1. INTRODUCTION

Platforms have become increasingly important in our economy today (e.g., Rochet and Tirole 2003; Iansiti and Levien 2004; Parker and Van Alstyne 2005; Panico and Cennamo 2015; McIntyre and Srinivasan 2017). Examples of popular platforms include Uber in the transportation industry, Airbnb in the accommodation industry, Craigslist in the classifieds market, and Groupon in the local daily deals market. All these platforms exhibit two-sidedness in that they facilitate matching and transactions between consumers and service providers in their markets, but the structures of their businesses vary considerably. Airbnb's network structure, for example, exhibits high interconnectivity between several local markets: travelers do not care about the number of hosts in their home cities; they care about the number of hosts in the cities they wish to visit. In contrast, Uber's network structure consists of local network clusters with some interconnectivity between them: riders transact with drivers in their own city, and except for frequent travelers, they care mostly about the local availability of Uber drivers. We observe similar local clusters for group buying platforms such as Groupon, classifieds sites such as Craigslist, food delivery platforms such as Grubhub, restaurant-reservation platforms such as OpenTable, and marketplaces that match freelance labor with local demand such as TaskRabbit, Instacart, and Rover.

The network structure of a platform market has important implications for the profitability and defensibility of incumbent platforms. When the network structure is strongly interconnected, it is difficult for a new entrant to compete, particularly when consumers in one local market only purchase services when they visit other markets. A platform that enters one city to compete with Airbnb, for example, would waste a significant amount of marketing resources to build awareness among local hosts and local residents without generating many transactions because local residents are only interested in transacting with hosts in cities they visit. Thus, a competitor would have to enter on a global scale, creating brand awareness in many cities simultaneously, in order to attract the critical mass of hosts and travelers necessary to build a liquid marketplace. Entry for a new platform is, therefore, very costly. Many of Airbnb's potential competitors exited the market after exhausting their marketing budgets. In contrast, when consumers and service providers mostly transact within their local clusters, it is relatively easy for a new platform to enter as it can specialize in one local cluster and build awareness from there. In the ride-sharing industry, many entrants have successfully challenged market leaders in local markets. In New York City, for example, Uber faces competitors such as Lyft, Juno, and Via. In China, the incumbent Didi faces competitors in several cities.

In this paper, we take a game-theoretical approach to examine how network structures affect competitive interactions between an incumbent platform and an entrant platform. The incumbent platform has an installed base of buyers and service providers in multiple local markets. The entrant is interested in entering one of these markets. To capture interconnectivity between local markets, we assume that some buyers are mobile: they travel between markets, purchasing services in each. In the first stage, the entrant invests money to build brand awareness in one of these markets. In the second stage, the incumbent and the entrant set prices for buyers and wages for service providers in that market. In the third stage, buyers and service providers in that market choose one platform on which to conduct transactions.

Our model yields several interesting results. First, even if there is no cost for the entrant to advertise, the entrant may not want to make every user in a local market aware of its service as doing so may trigger a competitive response from the incumbent. Second, when there is a greater number of mobile buyers (i.e., greater interconnectivity between markets), the entrant will advertise more aggressively. Buyers who come from other markets are only aware of the incumbent, and the incumbent thus has monopoly power over these buyers. As a result, the more mobile buyers there are, the less motivated the incumbent will be to initiate a price war with the entrant. The entrant thus has incentive to expand further into the incumbent's territory. Third, stronger interconnectivity across markets may or may not make the incumbent more defensible: when advertising is not costly and mobile buyers consume in both their local markets and the markets they visit, a large number of mobile buyers will increase the entrant's profitability, making it difficult for the incumbent to deter entry; when advertising is costly and/or mobile buyers only consume in the markets they travel to, a large number of mobile buyers will help the incumbent deter entry. Fourth, when there are heterogeneous number of mobile buyers in these markets, if advertising is inexpensive, the entrant should pick the market with the highest number of incoming mobile buyers to enter, but if advertising is costly, the entrant should pick the market with the smallest number of incoming mobile buyers to enter. Finally, targeting technologies increases the entrant's profit, while the presence of network effects is likely to decrease its profit.

Our paper adds to the literature examining entry in platform markets. Studies have identified a number of factors that drive the success or failure of entrants in platform markets, such as the strength of network effects (e.g., Zhu and Iansiti 2012; Llanes et al. 2016), platform quality (e.g., Liebowitz 2002; Tellis et al. 2009), multi-homing (e.g., Cennamo and Santalo 2013; Anderson et al. 2018) and exclusivity (e.g., Corts and Lederman 2009). These studies all assume a fully interconnected network structure. As pointed out in Afuah (2013), this assumption does not reflect the network structures in most industries. Our study extends this literature by examining how network structures affect incumbent and entrant performance.

Our study is also related to studies that examine how network structures affect product diffusion (e.g., Abrahamson and Rosenkopf 1997; Suarez 2005; Sundararajan 2007; Tucker 2008).

These studies typically focus on social networks, like instant messaging platforms, and examine questions related to issues such as seeding within these networks (e.g., Galeotti and Goyal 2009; Manshadi, Misra, and Rodilitz 2018), pricing policies to facilitate product diffusion (e.g., Campbell 2013; Leduc et al. 2017), local bias (e.g., Lee, Lee, and Lee 2006), market segmentation (e.g., Banerji and Dutta 2009), social distance in influencing incumbent advantage (e.g., Lee, Song, and Yang 2016), and network characteristics that result in the rapid decline of a network when its users start to leave (Knudsen and Belik 2018). These networks have more complicated structures because they depend on individuals' own social networks and, as a result, all these studies rely on simulations. We take a different perspective to focus on how interconnectivity between local markets affects market entry, and we derive closed-form solutions.

The rest of the paper is organized as follows: In Section 2, we introduce the model and analyze the competitive interactions between the incumbent and an entrant. We then examine extensions to our main models in Section 3. We conclude in Section 4 by discussing the implications and potential future research. All proofs are provided in a technical appendix.

2. MODEL

2.1 Model setup

Assume there are multiple local markets each with N buyers that are currently using the incumbent's platform (denoted as I) for transactions. A fraction of buyers in each market are mobile—r percent of them travel between markets. Assume the movement is random, so that in equilibrium, in each market, rN buyers visit other markets and rN additional buyers come from other markets to make purchases. Hence, r measures the interconnectivity between these markets. Each mobile buyer places one order for the service in his local market and another order when he travels. For example, riders use ride-sharing services in their local markets, and when they travel, they use ride-sharing services in other markets.[1] Each service provider fulfills at most one order. To accommodate these mobile buyers, we let each market have $(1+r)N$ service providers.

Before an entrant (denoted as *E*) enters one of these markets, the incumbent serves the market as a monopoly and all the users (i.e., both service providers and buyers) are aware of the incumbent.[2] Neither the buyers nor the service providers are aware of the entrant, but the entrant can advertise to build awareness.

The game proceeds as follows: In the first stage, the entrant invests to build brand awareness among users in the local market. Advertising is costly, and it costs the entrant $L(n)$ to reach n potential users. The entrant decides on θ, a fraction of the potential users reached through advertising. Because we have N buyers and $(1+r)N$ service providers, $n = \theta(2N + rN)$. Following the literature (Thompson and Teng 1984; Tirole 1988; Esteves and Resende 2016; Jiang and Srinivasan 2016), we assume the advertising cost is a (weakly) increasing and convex function of n: $L'(n) \geq 0$ and $L''(n) \geq 0$. Note that even with digital technologies, it is costly to build awareness. While some platforms may be able to attract their first tranche of customers relatively inexpensively, through word of mouth or other low-cost strategies, the cost typically starts to escalate when the platform begins to look for new and somewhat different customers through search advertising, referral fees, and other marketing strategies.[3] As a result, many platforms exit the market after burning too much money on customer acquisition. In our model, we allow advertising cost to vary. We also assume that the entrant is not able to separate buyers from service

[1] We consider the scenario in which mobile buyers do not consume in their local markets in an extension.
[2] This assumption is relaxed in an extension of the model in which not all buyers and sellers are aware of the incumbent.
[3] See, for example, "Unsustainable customer acquisition costs make much of ecommerce profit proof," Steve Dennis, *Forbes*, August 31, 2017.

providers when it advertises. For example, it can be hard to identify and separate riders and drivers when advertising in the ride-sharing market. We relax this assumption in an extension.

In the second stage, the incumbent sets the price to each buyer, denoted as p_I, and the wage to each service provider, denoted as w_I, in the local market. The entrant also sets the price to the service buyers, denoted as p_E, and the wage to the service providers, denoted as w_E. Instacart, for example, decides on prices to users and wages to shoppers. Uber decides on rates to riders and commissions it takes before passing on the revenue from riders to drivers, which effectively determines the wages for the drivers. Consistent with the practice, we allow firms to set different prices and wages in different markets, but they do not price discriminate based on whether a buyer is local or mobile within a market. We denote each buyer's willingness to pay for the service as v. We normalize the value of outside options to zero and the service providers' marginal cost to zero. We assume that p_I, p_E, w_I, and w_E are all non-negative numbers. Hence, without the entrant, as a monopoly, the incumbent will choose $p_I = v$ and $w_I = 0$.

In the third stage, the rN mobile buyers from other markets arrive. Buyers and service providers choose one platform on which to conduct transactions. Mobile buyers are not exposed to the entrant's advertisements and are therefore only aware of the incumbent. Hence, the entrant and the incumbent compete for buyers and service providers from the local market, but the mobile buyers will only use the incumbent platform.

The $(1 - \theta)$ portion of users in the local market is only aware of the incumbent and will buy or provide the service on the incumbent platform as long as they receive a non-negative utility from the incumbent. Specifically, a buyer will buy the service as long as $p_I \leq v$, and a service provider will provide the service as long as $w_I \geq 0$. Because $p_I \leq v$ and $w_I \geq 0$ should always hold, these users will always use the incumbent's platform.

The θ portion of users in the local market becomes aware of both the incumbent and the entrant and will select the platform that provides the higher utility. If a user elects to use the entrant's platform, there is a switching cost that varies across users. We denote this cost for a service provider, i, as c_i and for a buyer, j, as a_j. Similar to Ruiz-Aliseda (2016), we assume both c_i and a_j follow a uniform distribution between 0 and m, where m captures the difficulty in switching to a new service in the market. To be consistent with real world scenarios, we assume that m is sufficiently large (i.e., there are some users whose switching cost is sufficiently large) so that, in equilibrium, the entrant will not take away the entire segment of users who are aware of both platforms.[4]

Among the θ portion of service providers, a service provider, i, will choose the entrant if the utility from using the entrant's platform ($U_{Ei}^S = w_E - c_i$) is greater than the utility from using the incumbent's platform ($U_{Ii}^S = w_I$). The solution to the equation $U_{Ei}^S = U_{Ii}^S$ is $c^* = w_E - w_I$, describing the switching cost of the indifferent service provider. Thus, service providers with $c_i < c^*$ will choose the entrant and those with $c_i \geq c^*$ will choose the incumbent. Let N_I^S denote the number of service providers selecting the incumbent and N_E^S denote the number selecting the entrant. We have the following two equations:

$$N_I^S = \left(1 - \frac{c^*}{m}\theta\right)(1 + r)N \tag{1}$$

[4] Mathematically, this assumption requires that the distribution of the switching cost be sufficiently sparse, i.e., $m > \frac{9(1+r)v}{16(2+r)}$.

$$N_E^S = \frac{c^*}{m}\theta(1+r)N. \tag{2}$$

Similarly, a buyer, j, will choose the entrant if the utility from using the entrant's platform ($U_{Ej}^B = v - p_E - a_j$) is greater than the utility from using the incumbent's platform ($U_{Ij}^B = v - p_I$). The solution to the equation $U_{Ej}^B = U_{Ij}^B$ is $a^* = p_I - p_E$. Thus, buyers with $a_j < a^*$ will choose the entrant and those with $a_j \geq a^*$ will choose the incumbent. Let N_I^B denote the number of service buyers selecting the incumbent and N_E^B denote the number selecting the entrant. We have the following two equations:

$$N_I^B = \left(1 - \frac{a^*}{m}\theta + r\right)N. \tag{3}$$

$$N_E^B = \frac{a^*}{m}\theta N. \tag{4}$$

We can then derive the incumbent profit, π_I, and the entrant profit, π_E, from the local market as follows:

$$\pi_I = \min(N_I^S, N_I^B)\,(p_I - w_I). \tag{5}$$

$$\pi_E = \min(N_E^S, N_E^B)\,(p_E - w_E) - L\big(\theta(2N + rN)\big). \tag{6}$$

It is possible that under some prices and wages of the two platforms, the number of buyers is not the same as the number of service providers. In such cases, either some buyers' orders are not fulfilled, or some service providers will not serve any buyers and hence earn no income.

2.2 Equilibrium analysis

Before we derive the optimal prices and wages, we show that, in equilibrium, the incumbent and the entrant will always choose their prices and wages so that the number of service providers using a platform equals the number of buyers using the same platform: $N_I^S = N_I^B$ and $N_E^S = N_E^B$. Lemma 1 states this result (proofs of all lemmas and propositions are provided in the appendix).

Lemma 1. The incumbent and the entrant will set their prices and wages so that the number of service providers using a platform equals the number of buyers using the same platform.

The intuition for Lemma 1 is that if the numbers on the two sides are not balanced, a firm can adjust its price or wage to get rid of excess supply or demand to increase its profitability. The lemma suggests that $a^* = (1+r)c^*$. Hence, $(p_I - p_E) = (1+r)(w_E - w_I)$. We can re-write the profit functions as follows:

$$\pi_I = \left(1 - \frac{p_I - p_E}{m}\theta + r\right) N \left(p_I + \frac{p_I - p_E}{1+r} - w_E\right). \tag{7}$$

$$\pi_E = \frac{p_I - p_E}{m}\theta N \left(p_E - \frac{p_I - p_E}{1+r} - w_I\right) - L\big(\theta(2N + rN)\big). \tag{8}$$

We can then derive each platform's optimal price and profit given the entrant's advertising decision, as shown in Proposition 1.

Proposition 1. Given the entrant's choice of $\boldsymbol{\theta}$, the optimal prices, number of buyers and service providers, and platform profits can be determined as follows:

(i) If $\boldsymbol{0 \le \theta \le min\left(\frac{2m(2+r)}{3v}, 1\right)}$***, then*** $\boldsymbol{p_I^* = v, w_I^* = 0, p_E^* = \frac{(3+r)v}{2(2+r)}, w_E^* = \frac{v}{2(2+r)}, N_I^{B^*} = N_I^{S^*} = \frac{N(1+r)}{2}\left(2 - \frac{\theta v}{m(2+r)}\right), N_E^{B^*} = N_E^{S^*} = \frac{N(1+r)\theta v}{2m(2+r)}, \pi_I^*(\theta) = \frac{N(1+r)v}{2}\left(2 - \frac{\theta v}{m(2+r)}\right)}$, ***and*** $\boldsymbol{\pi_E^*(\theta) = \frac{N(1+r)\theta v^2}{4m(2+r)} - L(\theta(2N + rN))}$.

(ii) If $\boldsymbol{min\left(\frac{2m(2+r)}{3v}, 1\right) < \theta \le 1}$ ***, then*** $\boldsymbol{p_I^* = \frac{2(2+r)m}{3\theta}, w_I^* = 0, p_E^* = \frac{(3+r)m}{3\theta}, w_E^* = \frac{m}{3\theta}, N_I^{B^*} = N_I^{S^*} = \frac{2N(1+r)}{3}, N_E^{B^*} = N_E^{S^*} = \frac{N(1+r)}{3}, \pi_I^*(\theta) = \frac{4Nm(1+r)(2+r)}{9\theta}}$***, and*** $\boldsymbol{\pi_E^*(\theta) = \frac{Nm(1+r)(2+r)}{9\theta} - L(\theta(2N + rN))}$.

When θ is smaller than a certain threshold, we find that the incumbent platform chooses not to respond to the entrant. It continues to charge the monopoly price, v, and offer the monopoly wage, 0, although its profit does decreases as θ increases because it loses market share to the entrant. The entrant platform incentivizes some buyers and service providers to switch by charging a lower price and offering a higher wage.

The threshold for θ (weakly) increases with m and decreases with v. When m is large, it is more difficult for users to switch. When v is small, the buyers become less valuable. In both cases, the incumbent has less to lose to the entrant and hence has lower incentive to respond. The threshold also (weakly) increases with r because mobile buyers are only aware of the incumbent platform (i.e., the incumbent platform has monopoly power over them), and their existence reduces the incumbent's incentive to respond to the entrant. It is thus not surprising that the entrant can take advantage of this lack of incentive and increase its advertising intensity. The number of transactions hosted on the incumbent platform increases with r because of the mobile buyers from other markets, even though the incumbent loses more transactions from local buyers to the entrant when r increases. The incumbent platform's profit increases with r because of the increases in transactions at the same monopoly price it charges. The number of transactions the entrant serves also increases with r because it can advertise more aggressively without triggering a competitive response from the incumbent. The entrant's profit increases with r without taking the advertising cost into account. If advertising cost increases significantly with r, the entrant's profit may decrease with r, a scenario which will be examined later.

When θ is larger than the threshold, however, the entrant platform has the potential to steal a large market share from the incumbent. The incumbent platform chooses to respond by lowering its price to buyers. The entrant platform thus lowers its price to buyers as well. Notice that the wages offered by the entrant decrease with θ. This is because, even though many service providers are reached, there is not a demand for all the service providers because of the competitive response from the incumbent on the buyer side, allowing the entrant to offer lower wages.

We again find that because mobile buyers reduce the incumbent's incentive to fight, both the incumbent and the entrant can charge (weakly) higher prices to buyers while maintaining the same wages as r increases. They both have more transactions when r increases. The incumbent profit increases with r, while the entrant profit increases with r when its advertising cost does not increase too much with r.

Note that when θ is larger than the threshold, as θ increases, both platforms' profits decrease due to intense competition. We thus expect the entrant's optimal choice of θ to be no more than $min\left(\frac{2m(2+r)}{3v}, 1\right)$. That is, it is in the best interest of the entrant not to trigger the incumbent's competitive response.

Corollary 1. The entrant's optimal choice of $\boldsymbol{\theta}$ always satisfies $\boldsymbol{\theta \leq min\left(\frac{2m(2+r)}{3v}, 1\right)}$.

The exact optimal level of θ for the entrant depends on the cost of advertising, $L(n) = L(\theta(2N + rN))$. Following the literature (e.g., Thompson and Teng 1984; Tirole 1988; Esteves and Resende 2016; Jiang and Srinivasan 2016), we assume a quadratic cost function, $L(n) = kn^2$, where $k \geq 0$. A large k suggests that advertising is costly, while a small k suggests it is inexpensive.[5]

*** Figure 1 about here ***

Figure 1 shows how the entrant's profit changes with the choice of θ for different values of *k*. We notice that for a given level of *k*, the entrant profit increases and then decrease with θ. Even if advertising has no cost (i.e., $k = 0$), there is an optimal advertising level for the entrant. As k increases (i.e., advertising becomes more expensive), the optimal θ, θ^*, decreases. The incumbent's profit, however, always decreases with θ and is independent of *k*. The following proposition formalizes the relationship between the optimal θ, θ^*, and the value of *k*.

Proposition 2. The optimal θ, θ^*, depends on the value of k.

(i) ***If $k \geq max\left(\frac{3(1+r)v^3}{16m^2N(2+r)^4}, \frac{(1+r)v^2}{8mN(2+r)^3}\right)$, then $\theta^* = \frac{(1+r)v^2}{8(2+r)^3kNm}$, which increases with v and decreases with m and r. The entrant's profit is $\frac{(1+r)^2v^4}{64km^2(2+r)^4}$ and the incumbent's profit is $\frac{N(1+r)v}{2}\left(2 - \frac{(1+r)v^3}{8kNm^2(2+r)^4}\right)$.***

(ii) ***If $0 \leq k < max\left(\frac{3(1+r)v^3}{16m^2N(2+r)^4}, \frac{(1+r)v^2}{8mN(2+r)^3}\right)$, then $\theta^* = min\left(\frac{2m(2+r)}{3v}, 1\right)$, which weakly increases with m and r and weakly decreases with v. When $\frac{2m(2+r)}{3v} < 1$, the***

[5] If there is no cost for the entrant to reach its fans, we can modify the cost function to be $L(n) = k(max(n-z, 0))^2$, where *z* is the total number of fans. Our results hold qualitatively.

entrant's profit is $\boldsymbol{\frac{N(1+r)v}{6}-\frac{4kN^2m^2(2+r)^4}{9v^2}}$ ***and the incumbent's profit is*** $\boldsymbol{\frac{2Nv(1+r)}{3}}$**.**

When $\boldsymbol{\frac{2m(2+r)}{3v}\geq 1}$ ***, the entrant's profit is*** $\boldsymbol{\frac{N(1+r)v^2-4kN^2m(2+r)^3}{4m(2+r)}}$ ***and the incumbent's profit is*** $\boldsymbol{\frac{N(1+r)v}{2}\left(2-\frac{v}{m(2+r)}\right)}$**.**

We have two cases. When k is large, advertising is costly. In this case, the optimal θ, $\theta^* \leq min\left(\frac{2m(2+r)}{3v},1\right)$. When k is small, advertising is inexpensive, and the entrant platform thus has an incentive to have a large θ. The entrant's profit increases with θ until $\theta = min\left(\frac{2m(2+r)}{3v},1\right)$. When $\frac{2m(2+r)}{3v}\geq 1$, the entrant will advertise to everyone in the market. Otherwise, the entrant's profit first increases as θ increases up to $\frac{2m(2+r)}{3v}$ and then, because of the competitive response from the incumbent discussed in Corollary 1, decreases with θ afterwards. The entrant will thus choose $\theta^* = \frac{2m(2+r)}{3v}$.

The discussion above leads to the following corollary:

Corollary 2. Even if the advertising cost is zero (i.e., $\boldsymbol{L(n)}$ ***= 0 or k = 0), the entrant will not necessarily advertise to the entire market but instead choose*** $\boldsymbol{\theta^* = \frac{2m(2+r)}{3v}}$ ***when*** $\boldsymbol{\frac{2m(2+r)}{3v} < 1}$**.**

*** Figure 2 about here ***

Notice also that in the two cases in Proposition 2, the effects of *v* and *m* on θ^* are in opposite directions, as illustrated in Figure 2. When *k* is large (e.g., *k* = 0.0014 in Figure 2), θ^* is below the threshold at which the incumbent starts to respond. θ^* is determined by the profit-maximization function of the entrant. When *v* is high, buyers are more valuable, and the entrant becomes more aggressive in advertising. When *m* is large, it is more difficult to incentivize users to switch, making advertising less effective, and the entrant platform prefers to advertise less. Therefore, θ^* increases with *v* and decreases with *m* in this case.

When k is small (e.g., $k = 0$ in Figure 2), θ^* is determined by the threshold at which the incumbent starts to respond. When *m* is large (i.e., it is difficult for users to switch from the incumbent to the entrant) and *v* is low (the value of buyers is low), the incumbent has less incentive to respond to retain its users. Hence, θ^* increases with *m* and decreases with *v* in this case.

It is possible that for an intermediate value of k, as m or v changes, the optimal θ would switch between the two cases in Proposition 2. The relationship between θ^* and *m* or *v* becomes a hybrid of the two cases, as illustrated with the $k = 0.0002$ example in Figure 2. Thus, the optimal advertising level by the entrant can be a non-monotonic function of both *m* and *v*.

We thus have the following corollary:

Corollary 3. The effect of v and m on θ^* depends on k, and for intermediate value of k, θ^* can be a non-monotonic function of v and m.

*** Figure 3 about here ***

We then examine how the fraction of mobile buyers, r, affects the optimal θ and the platform profits in the two cases in Proposition 2. Figure 3 illustrates the relationships given different values of k. When k is large (e.g., $k = 0.0004$ in Figure 3), as r increases, the total number of buyers and service providers, $(2N + rN)$, increases in the market, and the likelihood that advertising is wasted on the service providers without matched buyers also increases. With a large k, it is optimal for the entrant to reduce θ^* to reduce its advertising cost, $L(\theta^*(2N + rN))$, even if a large r reduces the incumbent's incentive to respond. Because the entrant advertises to fewer buyers, the entrant's profit also decreases with r.

In contrast, when *k* is small, θ^* (weakly) increases with *r*. This is because when advertising is inexpensive, the advertising wasted on unmatched service providers becomes a lesser issue and the entrant wants to take advantage of the incumbent's disincentive to respond instead. The impact of *r* on the entrant's profit is also positive as long as *k* is sufficiently small.[6] This result is consistent with Proposition 1, where we have shown that if the advertising cost is small for the entrant, the entrant's profit will increase with *r* regardless of θ. When we use *k* to capture the cost of advertising, as long as *k* is small enough (e.g., *k* = 0 in Figure 3b), the entrant's profit increases with *r*. The result shows that when the incumbent has more captive buyers and therefore less incentive to fight, the entrant could be more profitable when advertising is not costly. The result also suggests that when *k* is not sufficiently small, the entrant's profit may decrease with *r* once *r* exceeds a certain value. It is also possible that when k continues to increase (e.g., *k* = 0.0002 in Figure 3), the optimal θ would switch between the two cases in Proposition 2 as *r* changes. In both cases (i.e., *k* is intermediate), we observe a non-monotonic relationship between entrant profit and *r*. Regardless of the value of *k*, incumbent profit always increases with *r*, because it has more captive buyers when *r* is larger (as shown in Figure 3c).

The two corollaries below summarize the relationship between the entrant's advertising intensity and the fraction of mobile buyers (Corollary 4) and the relationship between the platform profit and the fraction of mobile buyers (Corollary 5):

Corollary 4. When k is small, as the fraction of mobile buyers, r, increases, the entrant has incentive to advertise more (higher θ^*) until it reaches the entire market. Conversely, when k is large, as the fraction of mobile buyers, r, increases, the entrant has incentive to reduce

[6] When $\frac{2m(2+r)}{3v} < 1$, $\theta^* = \frac{2m(2+r)}{3v}$ and the entrant's profit increases with *r* as long as $k < \frac{3v^3}{32Nm^2(2+r)^3}$. When $\frac{2m(2+r)}{3v} \geq 1$, $\theta^* = 1$ and the entrant's profit increases with *r* as long as $k < \frac{v^2}{8mN(2+r)^3}$.

advertising (lower θ^*). For intermediate values of k, the optimal θ^* can be a non-monotonic function of r.

Corollary 5. The incumbent's profit always increases with the fraction of mobile buyers, r. How the fraction of mobile buyers, r, affects the entrant's profit depends on the value of k. When k is small, the entrant's profit increases with r, and conversely, when k is large, the entrant's profit decreases with r. For intermediate values of k, the entrant's profit can be a non-monotonic function of r.

3. EXTENSIONS

3.1 Heterogeneous markets with different fractions of mobile buyers

In our analysis, we have assumed that all markets are homogenous. As a result, the entrant could start by entering any one of these markets. In this extension, we consider a scenario in which different markets have different fractions of mobile buyers visiting from other markets. Assuming the entrant enters one market only, which market should the entrant choose to enter?

Proposition 3. When k is small, the entrant should choose the market with the highest fraction of mobile buyers from other markets, r, to enter; when k is large, the entrant should choose the market with the lowest fraction of mobile buyers from other markets, r, to enter. For an intermediate value of k, the entrant may choose a market where r is also intermediate.

The proposition follows directly from Corollary 5, where we find that the entrant's profit increases with r when k is small, decreases with r when k is large, and has a non-monotonic relationship with r for an intermediate value of k. Hence, when k is small, the entrant should pick the market with the highest r, and when k is large, the entrant should pick the market with the lowest r. For an intermediate value of k, the entrant may pick the market that has an intermediate r that yields the highest profit. The proposition suggests that the entrant's optimal choice of location is a function of its advertising cost and the market's fraction of mobile buyers. For example, if Google wants to offer ride-sharing services because it already has many users from its current services and can build awareness at a low cost (k is small), Google should start offering these services in cities with a large fraction of travelers. But for a new startup to enter a market like this, when advertising is costly, it should target cities with a small fraction of travelers.

3.2 The incumbent does not own the whole market

We have also assumed that the incumbent owns the whole market (i.e., all potential buyers and service providers are aware of the incumbent) before the entrant emerges. In reality, it is possible that not every user in the local market is aware of the incumbent. It is thus possible for the entrant to attract users that are not aware of the incumbent. We consider this possibility in this extension. Assume the incumbent's market share before the entrant arrives is s, where $0 < s < 1$. We have the following proposition:

Proposition 4. The results from our main model are qualitatively the same when $\boldsymbol{s \geq \frac{m(2+r)}{2m+mr+v+rv}}$***. If*** $\boldsymbol{s < \frac{m(2+r)}{2m+mr+v+rv}}$***, both platforms charge buyers*** $\boldsymbol{p_I^* = p_E^* = v}$ ***and offer service providers*** $\boldsymbol{w_I^* = w_E^* = 0}$***.***

Our results from the main model remain qualitatively the same as long as s is sufficiently large. But when s is below a certain threshold, the results differ from our main results. When the incumbent has a small share of the market, the entrant and the incumbent can effectively avoid competing by targeting different segments of that market. Hence, both will charge monopoly prices and offer monopoly wages. No buyers and service providers will switch from the incumbent to the entrant.

3.3 Mobile buyers only consume when they travel

In our model, mobile buyers purchase services in both their local markets and the markets they visit. This assumption fits with markets such as in the ride-sharing industry, where riders hail cars in their own markets and in other markets when they travel, or daily local deal markets, where consumers buy deals in their own markets and in other markets when they travel. The assumption, however, may not hold for markets such as the accommodation market, where buyers typically only consume when they travel. In this extension, we examine the scenario where mobile buyers do not consume in their local markets. We obtain the following result under this assumption:

Proposition 5. The results from the main model are qualitatively the same when mobile buyers do not consume in their local markets, except that the entrant's profit under the optimal θ always decreases with r.

When mobile buyers do not consume in a local market, a local market with a larger fraction of mobile buyers will have fewer potential buyers for the entrant. Although the entrant can continue to take advantage of the incumbent's disincentive to fight and advertise more aggressively, its demand decreases, and hence its profit decreases with r. This result explains why it is more difficult to challenge an incumbent platform like Airbnb, compared to Uber.

3.4 Targeted advertising by the entrant

We have assumed that the entrant is not able to separate buyers from service providers when it advertises. This assumption is likely to hold for firms operating in the sharing economy, which facilitate peer-to-peer transactions. We now relax this assumption and assume that the entrant has the ability to identify buyers and service providers in the local market and advertise to them separately. In equilibrium, because the entrant needs to balance demand and supply, the entrant will advertise to exactly θN buyers and θN service providers. Note that targeted advertising allows the entrant to separate buyers and service providers in the local market but does not allow the entrant to advertise to mobile buyers in other markets, who are much more difficult to target.

Proposition 6. When the entrant can advertise to the buyers and service providers separately, the results from the main model are qualitatively the same except the followings:

- ***a) When k is large, the entrant's optimal advertising level, θ^*, is not affected by the fraction of mobile buyers, r.***
- ***b) When k is large or r is large, the entrant's profit is not affected by r.***

Comparing part a) of Proposition 6 and Corollary 4, we find that when k is large, with targeted advertising, the optimal advertising level, θ^*, no longer decreases with the fraction of mobile buyers, *r*. In this case, because advertising is costly, the optimal advertising level for the entrant is low, and the incumbent has no incentive to respond. Without targeted advertising, when the fraction of mobile buyers increases, the entrant wastes more advertising expenditure on the service providers without matched buyers. With targeted advertising, the entrant can balance demand and supply and hence will not change its advertising level based on the fraction of mobile buyers.

Comparing part b) of Proposition 6 and Corollary 5, we again find that when k is large, with targeted advertising, the entrant's profit no longer decreases with the fraction of mobile buyers, r, because in this case, the entrant no longer wastes advertising expenditure on unmatched service providers (as in our main model).

Proposition 6 shows that targeted advertising improves the entrant's advertising efficiency, making the entrant more difficult for the incumbent to deter.

3.5 The presence of network effects

In our baseline model, we have assumed that every buyer is matched to a service provider. As a result, similar to other matching models (e.g., Zhang et al. 2018), we do not explicitly model network effects. This approach allows us to separate the network-structure effect from network effects, but network effects may have an impact on matching quality or speed. In the case of ride-hailing services, for example, a large number of drivers on a platform can reduce the wait time for riders. Likewise, a large number of riders reduces the idle time for drivers. In the accommodation market, a large number of hosts and travelers on a platform increase the chances that each traveler and each host is matched with a party close to his or her personal preference. To capture such benefits, we add a utility to capture network effects in the buyers' and service providers' utility functions and allow this utility to increase with the number of users on the other side of the same platform:

$$U_I^B = eN_I^S + v - p_I. \tag{9}$$

$$U_E^B = eN_E^S + v - p_E - a_i. \tag{10}$$

$$U_I^S = eN_I^B + w_I. \quad (11)$$

$$U_E^S = eN_E^B + w_E - c_i. \quad (12)$$

Here, we use parameter e ($e \geq 0$) to capture the strength of network effects. To avoid multiple equilibria due to network effects, we assume e to be small compared to the value of the transaction itself.[7] This assumption is reasonable because in such markets most benefits to buyers or service providers come from the transaction itself. Given this assumption, our main results are qualitatively unchanged, as summarized in the following proposition:

Proposition 7. The results from the main model are qualitatively the same in the presence of network effects when the strength of network effects is small.

We also examine the impact of the strength of network effects on the entrant's and incumbent's profits. Given the computational complexity, we explore this effect as the strength of network effects, e, approaches 0. We find that as long as m is sufficiently large (e.g., $m > v$), the result confirms the intuition that because the incumbent has a larger market share, network effects make the incumbent more attractive to users, reducing users' tendencies to switch to the entrant. Hence, as network effects become stronger, the entrant's profit decreases and the incumbent's profit increases.

4. DISCUSSION AND CONCLUSION

Extant studies in the platform literature typically assume that each participant on one side of a market is connected to every participant on the other side of the market. Our paper departs from

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[7] Mathematically, we need $e < \min\left(\frac{v}{2N}, \frac{m}{4N}\right)$.

this assumption to explore the heterogeneous network structures across platform markets and how this heterogeneity affects the defensibility of an incumbent with a presence in multiple markets against an entrant that seeks to enter one of those markets.

*** Figure 4 about here ***

As shown in Figure 4, our model captures network structures from isolated network clusters (r = 0) to a strongly connected network ($r = 1$). When we have isolated local clusters (i.e., no mobile buyers), as our results show, an incumbent has low profitability. Examples of such network structure include Handy, a marketplace for handyman services, and Instacart, a platform that matches consumers with personal grocery shoppers. In such markets, consumers only buy services in their local markets and do not typically use such services when they travel. At the other end of spectrum, we have a strongly connected network structure. This is the case for Airbnb, through which travelers can transact with any hosts outside their local clusters, and Upwork, an online outsourcing marketplace, where any clients and freelancers can initiate projects. Between the two extreme scenarios, we have network structures that consist of local clusters with some interconnectivities. In the case of Uber, Grubhub, and Groupon, consumers primarily use their services in their local clusters but also use such services when they travel.

We find that the greater the interconnectivity, the lower the incumbent's incentive to respond, and hence, the stronger the entrant's incentive to reach more users in a local market. While we find that incumbent profits always increase with interconnectivity, entrant profits do not always increase with interconnectivity. When advertising is inexpensive and mobile buyers consume in both their local markets and the markets they travel to, high interconnectivity between markets also increases the entrant's profit, making it difficult for the incumbent to deter entry; when advertising is costly and/or mobile buyers only consume in the markets they travel to, high interconnectivity reduces the entrant's profit, helping the incumbent deter entry. We also find that targeting technologies benefit the entrant, but the presence of network effects harms the entrant. Overall, these results help explain barriers to entry in platform markets and the resulting performance heterogeneity among platform firms in different markets.

These results corroborate empirical observations of many platform markets with local network structures. For example, we show that it is optimal for an entrant not to trigger incumbent responses. The founders of Fasten, an entrant into the ride-hailing market in Boston, were very clear from the beginning that they did not want to trigger Uber's response by strategically minimizing their advertising activities.[8] Indeed, although Fasten grew rapidly in Boston during 2015–2017, Uber and Lyft did not change their prices or wages to compete. As a counterexample, when Meituan—a major player in China's online-to-offline services such as food delivery, movie ticketing, and travel bookings—entered the ride-hailing business, it was able to build awareness of its service at almost no cost through its existing app, which had an extensive user base. Meituan's entry into the Shanghai ride-hailing market triggered strong responses from the incumbent, Didi, leading to a subsidy war between the two companies. Meituan subsequently decided to halt ride-hailing expansion in China.

Our results also suggest that Airbnb's and Booking.com's business models are more defensible than Uber's because most of their customers are travelers and do not typically use the service in their local markets, while Uber consumers primarily use its services in their local markets. The difference in defensibility is a key aspect of why both Airbnb and Booking.com are profitable, while Uber is still hemorrhaging money.

[8] Based on the authors' interviews with the founders.

Our study offers important managerial implications to platform owners. We find that an incumbent's profit increases with interconnectivity, so incumbent platforms should seek to build strong interconnectivity in their network structures. In our model, the level of interconnectivity is given exogenously, but in practice, how firms design their platforms can influence interconnectivity. For example, while Craigslist is a local classifieds service, its housing and job services attract users from other markets. Our research suggests that such services are important sources of Craigslist's profitability, and so Craigslist should strategically devote more resources to grow these services. As another example, many social networking platforms such as Facebook and WeChat allow companies or influencers to create public accounts that any user can connect with. Such moves increase interconnectivity between their local network clusters.

Our research suggests that an entrant needs to conduct thorough network analysis to understand the interconnectivity between different markets, the strength of network effects, and whether mobile users consume in their local markets or not. These factors, together with the cost of reaching users and the entrant's ability to target users, can help inform its location choice and how aggressively it should build awareness in a new market. The entrant needs to realize that even if advertising incurs little cost, it is not always optimal for it to advertise to every user. The entrant should advertise to the extent that it does not trigger competitive responses from the incumbent. Equally important, it is not always the case that an entrant should choose a market with low interconnectivity. When advertising is inexpensive and mobile buyers consume in local markets, it could be more profitable to enter a market with high interconnectivity.

As one of the first papers that explicitly models network structures of platform markets, our paper opens a new direction for future research on platform strategies. For example, our model allows an entrant to enter only one market. Entrants with sufficient resources, such as one large platform trying to envelop an adjacent, smaller platform (Eisenmann et al. 2011), typically choose to enter multiple markets at once. How their location choices are affected by network structures is an interesting question for future research.

In addition, our research focuses on an entrant's entry strategy, and our model only allows the incumbent to react through pricing. Future research could consider the incumbent's perspective and examine its other strategies for entry deterrence or expansion into additional local markets.

To focus on the impact of network structures, we abstract away many other factors that could influence competitive interactions between incumbents and entrants. For example, in the ride-sharing industry, riders may not care much about vehicle features. However, in the accommodation industry, travelers are likely to care about features of properties, making it easier for an entrant into the accommodation industry to differentiate itself from an incumbent, reducing the competitive intensity. Future research could explore how these factors affect competitive interactions.

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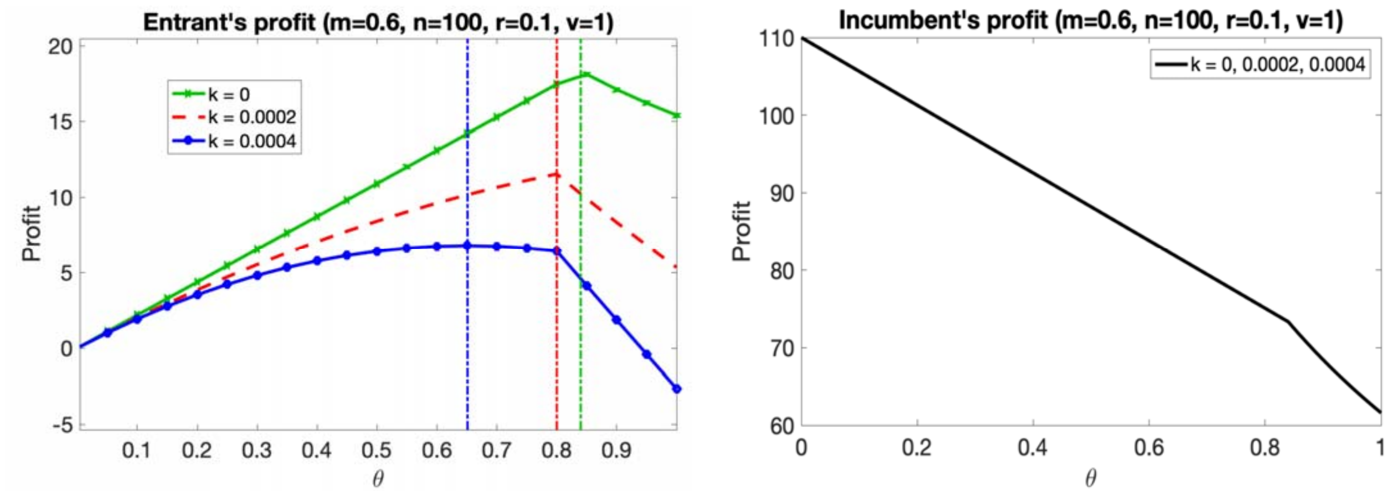
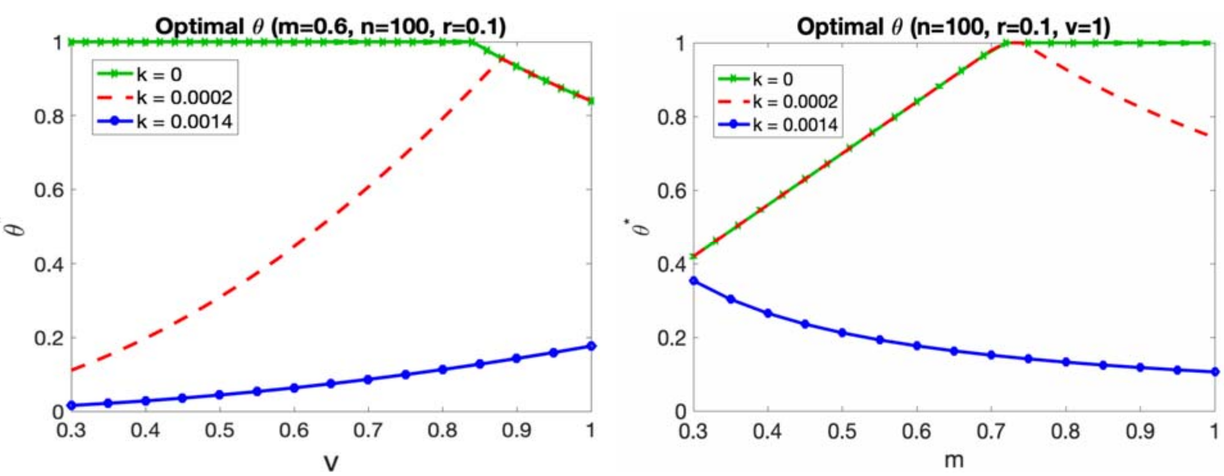


(a) (b)

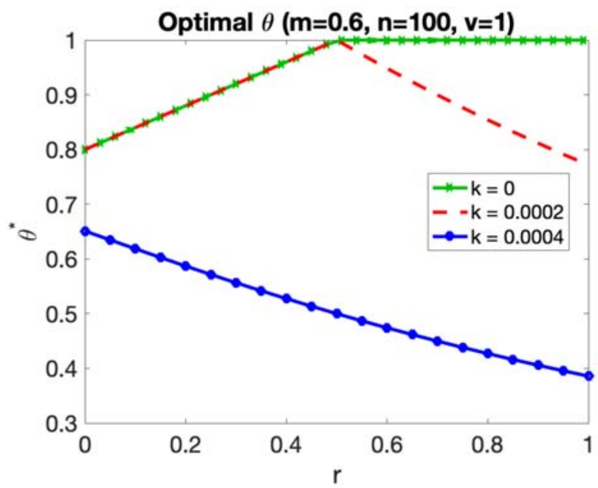
The vertical lines indicate the optimal θ for each scenario.

Figure 1: Firms' profits vs. θ

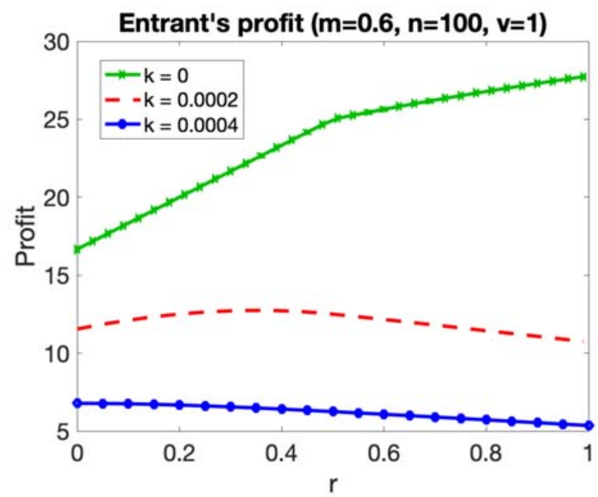


(a) (b)

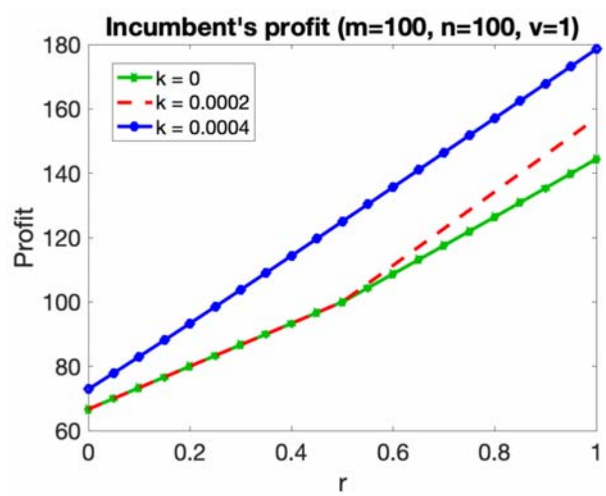
Figure 2: Optimal θ, θ^{*}, under different values of *v* and *m*



(a)



(b)



(c)

Figure 3: The entrant's optimal θ and the firms' profits under different values of r

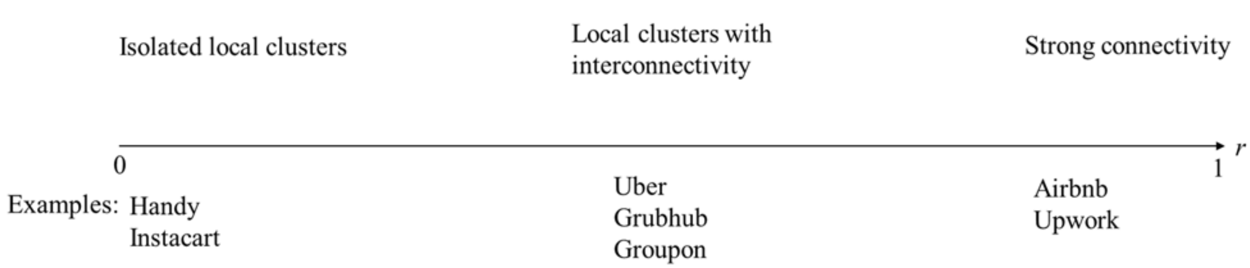


Figure 4: Markets with different interconnectivities

APPENDIX: PROOFS

Proof of Lemma 1. Given the entrant's prices, p_E and w_E, the incumbent's best response is always to choose p_I and w_I such that the number of service buyers equals the number of service providers on the incumbent's platform. Otherwise, the incumbent can always increase profit by increasing p_I or decreasing w_I so that the profit margin $(p_I - w_I)$ goes up without affecting the matched demand (i.e., $\min(N_I^S, N_I^B)$). Similarly, given the incumbent's prices, p_I and w_I, the entrant's best response is always to choose p_E and w_E such that the number of service buyers equals the number of service providers on the entrant's platform.

Proof of Proposition 1. We first solve for the optimal prices for the interior equilibrium where $0 < a_i^* < m$. We first confirm that the second order derivatives are both negative: $\frac{\partial^2 \pi_I}{\partial {p_I}^2} = \frac{\partial^2 \pi_E}{\partial {p_E}^2} = -\frac{2N(1+\frac{1}{1+r})\theta}{m} < 0$. We then derive the first order conditions:

$$\frac{\partial \pi_I}{\partial p_I} = N\left(2 + r + \frac{(p_E(3+r) - 2p_I(2+r) + (1+r)w_E)\theta}{m(1+r)}\right) = 0 \quad \text{(A1)}$$

$$\frac{\partial \pi_E}{\partial p_E} = \frac{N(p_I(3+r) - 2p_E(2+r) + (1+r)w_I)\theta}{m(1+r)} = 0 \quad \text{(A2)}$$

And we further obtain the following:

$$p_I = \frac{1}{2}\left(\frac{p_E(3+r) + (1+r)w_E}{2+r} + \frac{m(1+r)}{\theta}\right) \quad \text{(A3)}$$

$$p_E = \frac{(3+r)p_I + (1+r)w_I}{4+2r} \quad \text{(A4)}$$

Solving (A3) and (A4) together with $(p_I - p_E) = (1+r)(w_E - w_I)$ (according to Lemma 1), we get $p_I^* = \frac{2m(2+r)}{3\theta} + w_I$, $p_E^* = \frac{m(3+r)}{3\theta} + w_I$, and $w_E^* = \frac{m}{3\theta} + w_I$. The number of buyers and service providers using each platform under the optimal prices are ${N_I^B}^* = {N_I^S}^* = \frac{2N(1+r)}{3}$ and ${N_E^B}^* = {N_E^S}^* = \frac{N(1+r)}{3}$. The profits under the optimal prices are $\pi_I^*(\theta) = \frac{4mN(1+r)(2+r)}{9\theta}$ and $\pi_E^*(\theta) = \frac{mN(1+r)(2+r)}{9\theta} - L(\theta(2N + rN))$.

For this interior equilibrium to hold, we need to make sure that the incumbent has no incentive to deviate from this equilibrium by charging such a high price $p_I = v$ that no one from the overlapped market will transact on its platform but that it gets the most profit from the users who are not aware of the entrant.[9] The highest possible deviation profit the incumbent gets in this case is $(1 - \theta + r)Nv$, whereas the incumbent's equilibrium profit is $\frac{4mN(1+r)(2+r)}{9\theta}$. To guarantee that the latter is higher (i.e., $\frac{4mN(1+r)(2+r)}{9\theta} > (1 - \theta + r)Nv$) for all values of θ, we assume $m > \frac{9(1+r)v}{16(2+r)}$. This condition also ensures that the incumbent will never completely give up the overlapped market. That is, the entrant will never get the entire overlapped market. This is

[9] This deviation is not properly captured by the optimization process above because its calculation automatically assigns a negative profit to the overlapped market if $p_I - p_E > m$.

quite realistic because, in practice, there are always users who have a sufficiently high switching cost that they would rather stay with their current platform.

For this interior equilibrium to hold, we must also have $p_I^* = \frac{2m(2+r)}{3\theta} + w_I < v$. Because the choice of w_I does not affect either platform's profit and any border solution is inferior to the interior solution, the incumbent has incentive to make sure that $p_I^* < v$ holds as much as possible by setting $w_I^* = 0$. As a result, $p_I^* = \frac{2m(2+r)}{3\theta}$, $p_E^* = \frac{m(3+r)}{3\theta}$, and $w_E^* = \frac{m}{3\theta}$, and the condition $p_I^* < v$ requires $\theta > \frac{2m(2+r)}{3v}$.

When $\theta \leq \min\left(\frac{2m(2+r)}{3v}, 1\right)$, even with $w_I^* = 0$, the optimal p_I^* cannot satisfy $p_I^* < v$. The incumbent's price is thus bounded at $p_I^* = v$ to the buyers. Then based on (A4), $w_I^* = 0$ and $p_I - p_E = (1+r)(w_E - w_I)$, we get $p_E^* = \frac{(3+r)v}{2(2+r)}$ and $w_E^* = \frac{v}{2(2+r)}$. The number of buyers and service providers using each platform under the optimal prices are $N_I^{B^*} = N_I^{S^*} = \frac{N(1+r)}{2}\left(2 - \frac{v\theta}{m(2+r)}\right)$ and $N_E^{B^*} = N_E^{S^*} = \frac{N(1+r)v\theta}{2m(2+r)}$. The profits under the optimal prices are $\pi_I^*(\theta) = \frac{N(1+r)v}{2}\left(2 - \frac{v\theta}{m(2+r)}\right)$ and $\pi_E^*(\theta) = \frac{N(1+r)v^2\theta}{4m(2+r)} - L\big(\theta(2N + rN)\big)$.

Proof of Corollary 1. When $\theta > \frac{2m(2+r)}{3v}$, $\pi_E^*(\theta) = \frac{mN(1+r)(2+r)}{9\theta} - L\big(\theta(2N + rN)\big)$, which decreases with θ. Thus, the entrant never has incentive to increase θ once $\theta > \frac{2m(2+r)}{3v}$. Therefore, the optimal choice of θ always satisfies $\theta \leq \min\left(\frac{2m(2+r)}{3v}, 1\right)$.

Proof of Proposition 2. According to Corollary 1, the entrant always chooses $\theta \leq \min\left(\frac{2m(2+r)}{3v}, 1\right)$, and then according to Proposition 1, $\pi_I^*(\theta) = \frac{N(1+r)v}{2}\left(2 - \frac{v\theta}{m(2+r)}\right)$ and $\pi_E^*(\theta) = \frac{N(1+r)v^2\theta}{4m(2+r)} - L\big(\theta(2N + rN)\big)$. Given $L(n) = kn^2$, $\pi_E^*(\theta) = \frac{N(1+r)v^2\theta}{4m(2+r)} - kN^2(2+r)^2\theta^2$. We first confirm that the second-order derivative is negative: $\pi_E^{*\prime\prime}(\theta) = -2k\,N^2(2+r)^2 < 0$. We then derive the first-order condition:

$$\pi_E^{*\prime}(\theta) = \frac{N(1+r)v^2 - 8kN^2m(2+r)^3\theta}{4m(2+r)} = 0 \quad \text{(A5)}$$

This gives us the following:

$$\theta^* = \frac{(1+r)v^2}{8kNm(2+r)^3} \quad \text{(A6)}$$

Because the optimal choice of θ is bounded by $\theta \leq \frac{2m(2+r)}{3v}$ and $\theta \leq 1$, we compare $\frac{(1+r)v^2}{8kNm(2+r)^3}$ with the two bounds and get $\frac{(1+r)v^2}{8kNm(2+r)^3} \leq \frac{2m(2+r)}{3v}$ if $k \geq \frac{3(1+r)v^3}{16m^2N(2+r)^4}$ and $\frac{(1+r)v^2}{8kNm(2+r)^3} \leq 1$ if $k \geq \frac{(1+r)v^2}{8mN(2+r)^3}$. We thus derive the following two cases:

(i) When $k \geq max\left(\frac{3(1+r)v^3}{16m^2N(2+r)^4}, \frac{(1+r)v^2}{8mN(2+r)^3}\right)$, both $\frac{(1+r)v^2}{8kNm(2+r)^3} \leq \frac{2m(2+r)}{3v}$ and $\frac{(1+r)v^2}{8kNm(2+r)^3} \leq 1$ hold. Then $\theta^* = \frac{(1+r)v^2}{8kNm(2+r)^3}$, and it is easy to check that $\frac{\partial\theta^*}{\partial v} > 0$, $\frac{\partial\theta^*}{\partial m} < 0$, and $\frac{\partial\theta^*}{\partial r} < 0$. By replacing θ with $\frac{(1+r)v^2}{8kNm(2+r)^3}$ in $\pi_I^*(\theta) = \frac{N(1+r)v}{2}\left(2 - \frac{v\theta}{m(2+r)}\right)$ and $\pi_E^*(\theta) = \frac{N(1+r)v^2\theta}{4m(2+r)} - L\left(2N\left(\theta + \frac{\theta r}{2}\right)\right)$, we get $\pi_I^* = \frac{N(1+r)v}{2}\left(2 - \frac{(1+r)v^3}{8kNm^2(2+r)^4}\right)$ and $\pi_E^* = \frac{(1+r)^2v^4}{64km^2(2+r)^4}$.

(ii) When $k < max\left(\frac{3(1+r)v^3}{16m^2N(2+r)^4}, \frac{(1+r)v^2}{8mN(2+r)^3}\right)$, either $\frac{(1+r)v^2}{8kNm(2+r)^3} \leq \frac{2m(2+r)}{3v}$ or $\frac{(1+r)v^2}{8kNm(2+r)^3} \leq 1$ does not hold. Then $\theta^* = min\left(\frac{2m(2+r)}{3v}, 1\right)$, and it is easy to check that $\frac{\partial\theta^*}{\partial v} \leq 0$, $\frac{\partial\theta^*}{\partial m} \geq 0$, and $\frac{\partial\theta^*}{\partial r} \geq 0$. When $\frac{2m(2+r)}{3v} < 1$, by replacing θ with $\frac{2m(2+r)}{3v}$ in $\pi_I^*(\theta) = \frac{N(1+r)v}{2}\left(2 - \frac{v\theta}{m(2+r)}\right)$ and $\pi_E^*(\theta) = \frac{N(1+r)v^2\theta}{4m(2+r)} - L\left(2N\left(\theta + \frac{\theta r}{2}\right)\right)$, we get $\pi_I^* = \frac{2Nv(1+r)}{3}$ and $\pi_E^* = \frac{N(1+r)v}{6} - \frac{4kN^2m^2(2+r)^4}{9v^2}$. When $\frac{2m(2+r)}{3v} \geq 1$, by replacing θ with 1 in $\pi_I^*(\theta) = \frac{N(1+r)v}{2}\left(2 - \frac{v\theta}{m(2+r)}\right)$ and $\pi_E^*(\theta) = \frac{N(1+r)v^2\theta}{4m(2+r)} - L\left(2N\left(\theta + \frac{\theta r}{2}\right)\right)$, we get $\pi_I^* = \frac{N(1+r)v}{2}\left(2 - \frac{v}{m(2+r)}\right)$ and $\pi_E^* = \frac{N(1+r)v^2 - 4kN^2m(2+r)^3}{4m(2+r)}$.

Proofs of Corollaries 2, 3, 4 and 5. These corollaries follow directly from Proposition 2.

Proof of Proposition 3. If $\frac{2m(2+r)}{3v} < 1$, then the following is true:

(i) When $0 \leq k < \frac{3v^3}{32Nm^2(2+r)^3}$, $\pi_E^* = \frac{N(1+r)v}{6} - \frac{4kN^2\,m^2(2+r)^4}{9v^2}$ increases with *r*.

(ii) When $\frac{3v^3}{32Nm^2(2+r)^3} < k < \frac{3(1+r)v^3}{16Nm^2(2+r)^4}$, $\pi_E^* = \frac{N(1+r)v}{6} - \frac{4kN^2m^2(2+r)^4}{9v^2}$ decreases with *r*.

(iii) When $k > \frac{3(1+r)v^3}{16Nm^2(2+r)^4}$, $\pi_E^* = \frac{(1+r)^2v^4}{64km^2(2+r)^4}$ decreases with *r*.

Given a value of k, it is possible that as *r* increases, we switch from case (i) to case (ii), yielding a non-monotonic relationship between π_E^* and *r*.

If $\frac{2m(2+r)}{3v} \geq 1$, then the following is true:

(i) When $0 \leq k < \frac{v^2}{8Nm(2+r)^3}$, $\pi_E^* = \frac{N(1+r)v^2 - 4kmN^2(2+r)^3}{4m(2+r)}$ increases with *r*.

(ii) When $\frac{v^2}{8Nm(2+r)^3} < k < \frac{(1+r)v^2}{8Nm(2+r)^3}$, $\pi_E^* = \frac{N(1+r)v^2 - 4kN^2m(2+r)^3}{4m(2+r)}$ decreases with *r*.

(iii) When $k > \frac{(1+r)v^2}{8Nm(2+r)^3}$, $\pi_E^* = \frac{(1+r)^2v^4}{64km^2(2+r)^4}$ decreases with *r*.

Again, given a value of k, it is possible that as r increases, we switch from case (i) to case (ii), yielding a non-monotonic relationship between π_E^* and r.

Summarizing the two cases yields Proposition 3.

Proof of Proposition 4. We can similarly obtain the demand function as follows:

$N_I^S = \left(1 - \frac{c^*}{m}\theta\right)(1+r)sN$	(A7)
$N_E^S = \frac{c^*}{m}\theta(1+r)sN + \theta(1-s)N$	(A8)
$N_I^B = \left(1 - \frac{a^*}{m}\theta + r\right)sN$	(A9)
$N_E^B = \frac{a^*}{m}\theta sN + \theta(1-s)N$	(A10)

Following the same procedure as in the main analysis, we can obtain the following two main propositions with a similar assumption on switching cost ($m > \frac{3v}{5}$)[10]:

Proposition 1A: *Given the entrant's choice of θ, the optimal prices, transaction quantity, and profits for the two platforms are as follows:*

(i) If $s \geq \frac{m(2+r)}{2m+mr+v+rv}$

a. *If* $0 \leq \theta \leq min\left(\frac{2m(1+r)(2+r)s}{3(1+r)sv - m(2+r)(1-s)}, 1\right)$, *then* $p_I^* = v$, $p_E^* = \frac{1}{2}\left(m\left(\frac{1}{s} - 1\right) + \frac{(3+r)v}{2+r}\right)$, $w_E^* = \frac{1}{2}\left(\frac{v}{2+r} - \frac{m(1-s)}{(1+r)s}\right)$, $w_I^* = 0$, $N_I^{B^*} = N_I^{S^*} = \frac{N}{2}\left(\theta + s\left(2 + 2r - \theta - \frac{(1+r)v\theta}{m(2+r)}\right)\right)$, $N_E^{B^*} = N_E^{S^*} = \frac{N}{2}\left(1 - s\left(1 - \frac{(1+r)v}{m(2+r)}\right)\right)\theta$, $\pi_I^*(\theta) = \frac{Nv(m(2+r)(s(2+2r-\theta)+\theta) - (1+r)sv\theta)}{2m(2+r)}$, *and* $\pi_E^*(\theta) = \frac{N(m(2+r)(1-s) + (1+r)sv)^2\theta}{4m(1+r)(2+r)s} - L\big(\theta(2N+rN)\big)$.

b. *If* $min\left(\frac{2m(1+r)(2+r)s}{3(1+r)sv - m(2+r)(1-s)}, 1\right) < \theta \leq 1$, *then* $p_I^* = \frac{m(2+r)(s(2+2r-\theta)+\theta)}{3(1+r)s\theta}$, $p_E^* = \frac{m}{3}\left(\frac{(3+2r)(1-s)}{(1+r)s} + \frac{3+r}{\theta}\right)$, $w_E^* = \frac{m}{3}\left(\frac{1}{\theta} - \frac{1-s}{(1+r)s}\right)$, $w_I^* = 0$, $N_I^{B^*} = N_I^{S^*} = \frac{N(s(2(1+r)-\theta)+\theta)}{3}$, $N_E^{B^*} = N_E^{S^*} = \frac{N(s(1+r-2\theta)+2\theta)}{3}$, $\pi_I^*(\theta) = \frac{mN(2+r)(s(2+2r-\theta)+\theta)^2}{9(1+r)s\theta}$, *and* $\pi_E^*(\theta) = \frac{mN(2+r)(s(1+r-2\theta)+2\theta)^2}{9(1+r)s\theta} - L\big(\theta(2N+rN)\big)$.

(ii) If $s < \frac{m(2+r)}{2m+mr+v+rv}$

a. $p_I^* = v$, $p_E^* = v$, $w_E^* = 0$, $w_I^* = 0$, $N_I^{B^*} = N_I^{S^*} = (1+r)sN$, $N_E^{B^*} = N_E^{S^*} = (1-s)N\theta$, $\pi_I^*(\theta) = (1+r)sNv$, *and* $\pi_E^*(\theta) = (1-s)Nv\theta - L\big(\theta(2N+rN)\big)$.

Endogenizing θ, we obtain the following proposition.

Proposition 2A: *The optimal θ^* depends on the value of k and the value of s.*

[10] The same assumption on m is also used in the proof of the propositions in the other extensions.

(i) If $s \geq \frac{m(2+r)}{2m+mr+v+rv}$

a. *If* $k \geq max\left(\frac{(m(2+r)(1-s)+(1+r)sv)^2(3(1+r)sv-m(2+r)(1-s))}{16m^2N(1+r)^2(2+r)^4s^2}, \frac{(m(2+r)(1-s)+(1+r)sv)^2}{8mN(1+r)(2+r)^3s}\right)$, *then* $\theta^* = \frac{(m(2+r)(1-s)+(1+r)sv)^2}{8kmN(1+r)(2+r)^3s}$, *which increases with v and decreases with m, r, and s. The entrant's profit is* $\frac{(m(2+r)(1-s)+(1+r)sv)^4}{64km^2(1+r)^2(2+r)^4s^2}$ *and the incumbent's profit is* $\frac{v\left(m^3(2+r)^3(1-s)^3-m(1+r)^2(2+r)(1-s)s^2v^2-(1+r)^3s^3v^3+m^2(1+r)(2+r)^2s\left(16kN(1+r)(2+r)^2s+(1-s)^2v\right)\right)}{16km^2(1+r)(2+r)^4s}$.

b. *If* $0 \leq k < max\left(\frac{(m(2+r)(1-s)+(1+r)sv)^2(3(1+r)sv-m(2+r)(1-s))}{16m^2N(1+r)^2(2+r)^4s^2}, \frac{(m(2+r)(1-s)+(1+r)sv)^2}{8mN(1+r)(2+r)^3s}\right)$, *then* $\theta^* = min\left(\frac{2m(1+r)(2+r)s}{3(1+r)sv-m(2+r)(1-s)}, 1\right)$, *which weakly increases with m and r and weakly decreases with v and s. When* $\frac{2m(1+r)(2+r)s}{3(1+r)sv-m(2+r)(1-s)} < 1$, *the entrant's profit is* $\frac{N\left((m(2+r)(1-s)+(1+r)sv)^2(3(1+r)sv-m(2+r)(1-s))-8km^2N(1+r)^2(2+r)^4s^2\right)}{2\left(3(1+r)sv-m(2+r)(1-s)\right)^2}$ *and the incumbent's profit is* $\frac{2N(1+r)^2s^2v^2}{3(1+r)sv-m(2+r)(1-s)}$. *When* $\frac{2m(1+r)(2+r)s}{3(1+r)sv-m(2+r)(1-s)} \geq 1$, *the entrant's profit is* $\frac{N(m(2+r)(1-s)+(1+r)sv)^2}{4m(2+r)(1+r)s} - k\left((2+r)N\right)^2$ *and the incumbent's profit is* $\frac{Nv(m(2+r)(1+s+2rs)-(1+r)sv)}{2m(2+r)}$.

(ii) If $s < \frac{m(2+r)}{2m+mr+v+rv}$,

a. *If* $k \geq \frac{(1-s)v}{2N(2+r)^2}$, *then* $\theta^* = \frac{(1-s)v}{2kN(2+r)^2}$, *which increases with v and decreases with s and r. The entrant's profit is* $\frac{(1-s)^2v^2}{4k(2+r)^2}$ *and the incumbent's profit is* $(1+r)sNv$.

b. *If* $0 \leq k < \frac{(1-s)v}{2N(2+r)^2}$, *then* $\theta^* = 1$, *the entrant's profit is* $N\left((1-s)v-kN(2+r)^2\right)$ *and the incumbent's profit is* $(1+r)sNv$.

Proof of Proposition 5. In this case we only need *N* service providers to match the *N* orders in each local market. We can obtain the demand functions as:

$$N_I^S = \left(1 - \frac{c^*}{m}\theta\right)N \quad \text{(A11)}$$

$$N_E^S = \frac{c^*}{m}\theta N \quad \text{(A12)}$$

$$N_I^B = \left(1 - \frac{a^*}{m}\theta\right)(1-r)N + rN \quad \text{(A13)}$$

$$N_E^B = \frac{a^*}{m}\theta(1-r)N \quad \text{(A14)}$$

We then follow the same procedure as in our main analysis to derive the following two main propositions given $m > \frac{3v}{5}$:

Proposition 1B: *Given the entrant's choice of* θ, *the optimal prices, number of buyers and service providers, and profits are as follows:*

(i) *If* $0 \le \theta \le min\left(\frac{2(2-r)m}{3(1-r)v}, 1\right)$*, then* $p_I^* = v$, $p_E^* = \frac{(3-2r)v}{2(2-r)}$, $w_E^* = \frac{(1-r)v}{2(2-r)}$, $w_I^* = 0$, $N_I^{B^*} = N_I^{S^*} = \frac{N(2(2-r)m-(1-r)\theta v)}{2(2-r)m}$, $N_E^{B^*} = N_E^{S^*} = \frac{(1-r)N\theta v}{2(2-r)m}$, $\pi_I^* = \frac{Nv(2(2-r)m-(1-r)\theta v)}{2(2-r)m}$, *and* $\pi_E^* = \frac{(1-r)N\theta v^2}{4(2-r)m} - L(2\theta N)$.

(ii) *If* $min\left(\frac{2(2-r)m}{3(1-r)v}, 1\right) < \theta \le 1$*, then* $p_I^* = \frac{2(2-r)m}{3(1-r)\theta}$, $p_E^* = \frac{(3-2r)m}{3(1-r)\theta}$, $w_E^* = \frac{m}{3\theta}$, $w_I^* = 0$, $N_I^{B^*} = N_I^{S^*} = \frac{2N}{3}$, $N_E^{B^*} = N_E^{S^*} = \frac{N}{3}$, $\pi_I^*(\theta) = \frac{4N(2-r)m}{9(1-r)\theta}$, *and* $\pi_E^*(\theta) = \frac{N(2-r)m}{9(1-r)\theta} - L(2\theta N)$.

We again confirm that the entrant's optimal choice of θ to be no more than $min\left(\frac{2(2-r)m}{3(1-r)v}, 1\right)$. That is, it is in the best of the entrant not to trigger the incumbent's competitive response. Endogenizing θ, we obtain the following proposition.

Proposition 2B: *The optimal* θ^* *depends on the value of k:*

(i) *If* $k \ge max\left(\frac{3(1-r)^2v^3}{64m^2N(2-r)^2}, \frac{(1-r)v^2}{32mN(2-r)}\right)$*, then* $\theta^* = \frac{(1-r)v^2}{32kNm(2-r)}$*, which increases with v and decreases with m and r. The entrant's profit is* $\frac{(1-r)^2v^4}{256km^2(2-r)^2}$ *and the incumbent's profit is* $Nv - \frac{(1-r)^2v^4}{64km^2(2-r)^2}$.

(ii) *If* $0 \le k < max\left(\frac{3(1-r)^2v^3}{64m^2N(2-r)^2}, \frac{(1-r)v^2}{32mN(2-r)}\right)$*, then* $\theta^* = min\left(\frac{2(2-r)m}{3(1-r)v}, 1\right)$*, which weakly increases with m and r and weakly decreases with v. When* $\frac{2(2-r)m}{3(1-r)v} < 1$*, the entrant's profit is* $\frac{Nv}{6} - \frac{16kN^2m^2(2-r)^2}{9(1-r)^2v^2}$ *and the incumbent's profit is* $\frac{2Nv}{3}$*. When* $\frac{4m}{3v} \ge 1$*, the entrant's profit is* $\frac{N(1-r)v^2}{4m(2-r)} - 4kN^2$ *and the incumbent's profit is* $\frac{Nv}{2}\left(2 - \frac{(1-r)v}{m(2-r)}\right)$.

One difference between Proposition 2B and Proposition 2 is here $\pi_E^*(\theta)$ always decreases with *r*.

Proof of Proposition 6. In this case, the entrant will advertise to exactly θN buyers and θN service providers. We can obtain the demand function as follows:

$N_I^S = \left(1 - \frac{c_i^*}{m}\theta + r\right)N$	(A15)
$N_E^S = \frac{c_i^*}{m}\theta N$	(A16)
$N_I^B = \left(1 - \frac{a_i^*}{m}\theta + r\right)N$	(A17)
$N_E^B = \frac{a_i^*}{m}\theta N$	(A18)

Following the same procedure as in the main analysis, we can derive the profit functions as follows:

$\pi_I = \left(1 - \frac{p_I - p_E}{m}\theta + r\right)N(p_I + p_I - p_E - w_E)$	(A19)
$\pi_E = \left(\frac{p_I - p_E}{m}\theta N\right)(p_E - (p_I - p_E) - w_I) - L(2\theta N)$	(A20)

We can then obtain the following two main propositions given $m > \frac{3v}{5}$:

Proposition 1C: *Given the entrant's choice of* θ*, the optimal prices, transaction quantity, and profits for the two platforms are as follows:*

(iii) If $0 \leq \theta \leq min\left(\frac{4m(1+r)}{3v}, 1\right)$*, then* $p_I^* = v$, $w_I^* = 0, p_E^* = \frac{3v}{4}$, $w_E^* = \frac{v}{4}$, $N_I^{B^*} = N_I^{S^*} = N\left(1 + r - \frac{\theta v}{4m}\right)$, $N_E^{B^*} = N_E^{S^*} = \frac{N\theta v}{4m}$, $\pi_I^*(\theta) = Nv\left(1 + r - \frac{\theta v}{4m}\right)$*, and* $\pi_E^*(\theta) = \frac{N\theta v^2}{8m} - L(2\theta N)$.

(iv) If $min\left(\frac{4m(1+r)}{3v}, 1\right) < \theta \leq 1$*, then* $p_I^* = \frac{4(1+r)m}{3\theta}, w_I^* = 0, p_E^* = \frac{(1+r)m}{\theta}, w_E^* = \frac{(1+r)m}{3\theta}$, $N_I^{B^*} = N_I^{S^*} = \frac{2N(1+r)}{3}$, $N_E^{B^*} = N_E^{S^*} = \frac{N(1+r)}{3}$, $\pi_I^*(\theta) = \frac{8Nm(1+r)^2}{9\theta}$*, and* $\pi_E^*(\theta) = \frac{2Nm(1+r)^2}{9\theta} - L(2\theta N)$.

We again confirm that the entrant's optimal choice of θ to be no more than $min\left(\frac{4m(1+r)}{3v}, 1\right)$. That is, it is in the best interest of the entrant not to trigger the incumbent's competitive response. Endogenizing θ, we obtain the following proposition.

Proposition 2C: *The optimal* θ^* *depends on the value of k and the value of s.*

(iii) *If* $k \geq max\left(\frac{3v^3}{256m^2N(1+r)}, \frac{v^2}{64mN}\right)$*, then* $\theta^* = \frac{v^2}{64kNm}$*, which increases with v and decreases with m. The entrant's profit is* $\frac{v^4}{1024km^2}$ *and the incumbent's profit is* $N(1 + r)v - \frac{v^4}{256km^2}$.

(iv) *If* $0 \leq k < max\left(\frac{3v^3}{256m^2N(1+r)}, \frac{v^2}{64mN}\right)$*, then* $\theta^* = min\left(\frac{4m(1+r)}{3v}, 1\right)$*, which weakly increases with m and r and weakly decreases with v. When* $\frac{4m(1+r)}{3v} < 1$*, the entrant's profit is* $\frac{N(1+r)v}{6} - \frac{64kN^2m^2(1+r)^2}{9v^2}$ *and the incumbent's profit is* $\frac{2Nv(1+r)}{3}$*. When* $\frac{4m(1+r)}{3v} \geq 1$*, the entrant's profit is* $\frac{Nv^2}{8m} - 4kN^2$ *and the incumbent's profit is* $Nv\left(1 + r - \frac{v}{4m}\right)$.

According to Proposition 2C, when k is small $\left(0 \leq k < max\left(\frac{3v^3}{256m^2N(1+r)}, \frac{v^2}{64mN}\right)\right)$, $\theta^* = min\left(\frac{4m(1+r)}{3v}, 1\right)$ increases with r until it reaches 1. In this case, if r is small $\left(\frac{4m(1+r)}{3v} < 1\right)$, the entrant's profit $\left(\frac{N(1+r)v}{6} - \frac{64kN^2m^2(1+r)^2}{9v^2}\right)$ increases with r, but if r is large $\left(\frac{4m(1+r)}{3v} \geq 1\right)$, the entrant's profit $\left(\frac{Nv^2}{8m} - 4kN^2\right)$ is independent of r.

When k is large $\left(k \geq max\left(\frac{3v^3}{256m^2N(1+r)}, \frac{v^2}{64mN}\right)\right)$, $\theta^* = \frac{v^2}{64kNm}$ is independent of r, and the entrant's profit $\left(\frac{v^4}{1024km^2}\right)$ is also independent of r.

Proof of Proposition 7. Assume a^* is the switching cost of the indifferent user and c^* is the switching cost of the indifferent service provider. Then Equations (1) – (4) define the number of buyers and service providers selecting the entrant and the incumbent, respectively. Then given the utility functions in Equations (9)-(12), we can derive a^* and c^* by solving the following two equations simultaneously:

$$e\left(1-\frac{c^*}{m}\theta\right)(1+r)N+v-p_I=e\frac{c^*(1+r)}{m}\theta N+v-p_E-a^* \qquad \text{(A21)}$$

$$e\left(1-\frac{a^*}{m}\theta+r\right)N+w_I=e\frac{a^*}{m}\theta N+w_E-c^* \qquad \text{(A22)}$$

We thus obtain that $a^*=\frac{m\left(m(p_I-p_E-eN(1+r))+2eN(1+r)\theta\left(w_E-w_I-e\,N\,(1+r)\right)\right)}{m^2-4e^2N^2(1+r)\theta^2}$ and $c^*=\frac{m\left(m(w_E-w_I-eN(1+r))+2eN\theta(p_I-p_E-eN(1+r))\right)}{m^2-4e^2N^2(1+r)\theta^2}$. We can then prove that Lemma 1 holds in this extension, that is, the incumbent and the entrant will always choose their prices and wages so that $a^*=(1+r)c^*$, as long as $e<\frac{m}{4N}$.

We then follow the same procedure as in our main analysis to derive the two main propositions and find that our key results hold qualitatively under conditions $m>\frac{3v}{5}$ and $e<\frac{v}{2N}$.

Proposition 1D: *Given the entrant's choice of θ, the optimal prices, number of buyers and service providers, and profits are as follows:*

(i) If $0\le\theta\le min\left(\frac{4m(2+r)}{16eN(1+r)+3v+\sqrt{16e^2N^2(1+r)^2+9v^2}},1\right)$, *then* $p_I^*=\frac{2m(2+r)(eN(1+r)+v)-eN(1+r)(6eN(1+r)+8v)\theta}{2m(2+r)-7eN(1+r)\theta}$, $p_E^*=\frac{m(eN(1+r)^2+(3+r)v)-eN(1+r)(eN(1+r)+6v)\theta}{2m(2+r)-7eN(1+r)\theta}$, $w_E^*=\frac{m(eN(1+r)(3+2r)+v)-eN(1+r)(5eN(1+r)+2v)\theta}{2m(2+r)-7eN(1+r)\theta}$, $w_I^*=0$, $N_I^{B^*}=N_I^{S^*}=\frac{N(1+r)(2m(2+r)-(6eN(1+r)+v)\theta)}{2m(2+r)-7eN(1+r)\theta}$, $N_E^{B^*}=N_E^{S^*}=\frac{N(1+r)(v-eN(1+r))\theta}{2m(2+r)-7eN(1+r)\theta}$, $\pi_I^*=\frac{2N(1+r)(2m(2+r)-(6eN(1+r)+v)\theta)(m(2+r)(eN(1+r)+v)-eN(1+r)(3eN(1+r)+4v)\theta)}{(2m(2+r)-7eN(1+r)\theta)^2}$, *and* $\pi_E^*=\frac{N(1+r)\left(v-eN(1+r)\right)^2\theta(m(2+r)-4eN(1+r)\theta)}{(2m(2+r)-7eN(1+r)\theta)^2}-L\left(\theta(2N+rN)\right)$.

(ii) If $min\left(\frac{4m(2+r)}{16eN(1+r)+3v+\sqrt{16e^2N^2(1+r)^2+9v^2}},1\right)<\theta\le1$, *then* $p_I^*=\frac{2m(2+r)}{3\theta}-2eN(1+r)$, $p_E^*=\frac{m^2(2+r)(3+r)-3meN(1+r)(8+3r)\theta+12eN(1+r)(2eN(1+r))\theta^2}{3\theta(m(2+r)-4eN(1+r)\theta)}$, $w_E^*=\frac{m^2(2+r)-meN(4-r)(1+r)\theta}{3\theta(m(2+r)-4eN(1+r)\theta)}$, $w_I^*=0$, $N_I^{B^*}=N_I^{S^*}=\frac{2N(1+r)(m(2+r)-3eN(1+r)\theta)}{3(m(2+r)-4eN(1+r)\theta)}$, $N_E^{B^*}=N_E^{S^*}=\frac{N(1+r)(m(2+r)-6eN(1+r)\theta)}{3(m(2+r)-4eN(1+r)\theta)}$, $\pi_I^*(\theta)=\frac{4N(1+r)(m(2+r)-3eN(1+r)\theta)^2}{9\theta(m(2+r)-4eN(1+r)\theta)}$, *and* $\pi_E^*(\theta)=\frac{N(1+r)(m(2+r)-6eN(1+r)\theta)^2}{9\theta(m(2+r)-4eN(1+r)\theta)}-L\left(\theta(2N+rN)\right)$.

We again confirm that the entrant's optimal choice of θ to be no more than $min\left(\frac{4m(2+r)}{16eN(1+r)+3v+\sqrt{16e^2N^2(1+r)^2+9v^2}},1\right)$. That is, it is in the best interest of the entrant not to trigger the incumbent's competitive response.

Therefore, the entrant will select θ to maximize $\pi_E^*(\theta) = \frac{N(1+r)\left(v-eN(1+r)\right)^2\theta(m(2+r)-4eN(1+r)\theta)}{(2m(2+r)-7eN(1+r)\theta)^2} - k\left(\theta(2N+rN)\right)^2$ under the constraint that $\theta \leq min\left(\frac{4m(2+r)}{16eN(1+r)+3v+\sqrt{16e^2N^2(1+r)^2+9v^2}}, 1\right)$. Because the first term of $\pi_E^*(\theta)$ increases with θ and the second term of $\pi_E^*(\theta)$ decreases with θ, we can conclude that there exists a k^* so that the two scenarios in Proposition 2 hold qualitatively. That is, 1) when $k \geq k^*$, the optimal θ^* is the solution to $\frac{\partial \pi_E^*(\theta)}{\partial \theta} = 0$, and $\theta^* \leq min\left(\frac{4m(2+r)}{16eN(1+r)+3v+\sqrt{16e^2N^2(1+r)^2+9v^2}}, 1\right)$; and 2) when $k < k^*$, $\theta^* = min\left(\frac{4m(2+r)}{16eN(1+r)+3v+\sqrt{16e^2N^2(1+r)^2+9v^2}}, 1\right)$.