Quality Regulation on Two-Sided Platforms:
Exclusion, Subsidy, and First-party Application

Abstract

Managing the quality of complementary applications is vital to the success of a two-sided platform. While prior research has focused solely on limiting platform access based on a quality threshold, we consider three quality regulation strategies: 1) exclusion, in which the platform excludes access to low quality complementors, 2) subsidy, in which the platform provides a fixed subsidy to high quality complementors, and 3) first-party application, in which the platform develops high quality applications by itself in addition to those developed by third-party complementors. Our analyses reveal that the widely adopted exclusion strategy is a special case of the subsidy strategy, and it does not always benefit the platform. In contrast, both subsidy and first-party application strategies always make the platform owner better off, with greater profits, higher average quality, larger network sizes and higher access fees levied on both sides of the platform, but only subsidy always improves social welfare. In addition, the tradeoff between subsidy and first-party application strategies depends on the development cost of first-party applications, as well as the fraction of high quality third-party complementors, but the relationship is not monotonic. Interestingly, our results demonstrate that the platform does not have to sacrifice application variety for higher application quality. With the right choices of quality regulation strategy, the platform can profitably improve both measures simultaneously. This research provides concrete guidelines to help platform managers make these decisions.

1. Introduction

Firms in the technology industries often build their product or service offerings around a platform, consisting of a set of core elements that are used in common across implementations and interchangeable, complementary components that enhance the value of the platform (Boudreau 2010). This mechanism of value co-creation gives rise to the model of platform ecosystems where the success of a platform depends critically on coordinating third-party complementary innovations (Ceccagnoli et al. 2012, Gawer and Cusumano 2002). However, to orchestrate such a platform ecosystem, firms face significant governance challenges, such as balancing platform openness and control (Boudreau 2010), designing boundary resources (Ghazawneh and Henfridsson 2013), or managing intellectual properties within the ecosystem (Huang et al. 2013, Parker and Alstyne forthcoming). A burgeoning body of literature has examined a variety of issues involved in the governance of technology platforms, particularly in the context of those serving two or more distinct user groups in the presence of network effects (Eisenmann et al. 2009, Gawer and Henderson 2007, Hagiu 2014, Parker and Van Alstyne 2005, Song et al. 2018, Tiwana et al. 2010).

Despite progress, one of the understudied but fundamentally important challenges that remain in platform governance is the regulation of the quality of complementary goods (Hagiu 2009a). The importance of quality regulation is highlighted by the collapse of the videogame market in the early 1980s, where unrestricted entry resulted in a market for "lemons" flooded by poor quality games, leading to the bankruptcy of over 90 percent of videogame developers as well as the dominant videogame platform at the time, Atari (Boudreau and Hagiu 2009). In contrast, the later success of Nintendo was partly attributed to its restrictive platform access strategy in which it used a security chip to lock out unlicensed, low quality game developers. Recent technology platforms have witnessed a number of more subtle quality regulation strategies: While denying the access to low quality complementors is still being widely adopted (such as in the case of Apple's iOS platform), some have embraced a strategy that subsidizes high quality complementors. For example, Google aimed to attract high quality complementors by giving $10 million in prizes to developers of the best apps in several categories in early stages of its Android platform,[1] and Facebook created fbFund in partnership with venture capitalists that awarded seed grants up to $250 thousand to selected startups dedicated to developing Facebook applications.[2] In addition, many two-sided platform owners, such as manufacturers of video game consoles (e.g., PlayStation or Xbox) and media streaming service providers (e.g., Netflix, Hulu or Amazon Prime Video), often create their own high quality applications or content – also known as first-party applications – on top of their platforms (Hagiu and Spulber 2013).[3] These exclusive, hit applications, sometimes offered as part of a product bundle, play an important role in attracting an initial critical mass of platform adopters as well as winning the battle with competing platforms, especially when third-party applications are subject to multihoming (Hagiu and Spulber 2013, Rochet and Tirole 2003).

Although some scholars have started to tackle the issue of quality regulation in the presence of network effects, research in this area so far has focused exclusively on the strategy of exclusion based on a quality threshold (Hagiu 2009a, Zheng and Kaiser 2013). Given the varied quality regulation strategies employed by recent platforms, there is a notable gap in understanding the relative effectiveness and limitations of these strategies. We aim to address this gap by analyzing a model under the setting of a profit-maximizing, two-sided platform where indirect network effect

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[1] http://googlepress.blogspot.com/2007/11/google-announces-10-million-android_12.html

[2] https://techcrunch.com/2007/09/17/facebook-launches-fbfund-with-accel-and-founders-fund-to-invest-in-new-facebook-apps/

[3] Some examples of first-party applications/content include the *Halo* franchise by Xbox, the *Uncharted* franchise by PlayStation, the web TV series *House of Cards* by Netflix, *The Handmaid's Tale* by Hulu, and the movie *Manchester by the Sea* by Amazon Prime Video.

on the consumer side depends not only on the variety of complementary applications,[4] but also on their average quality. In our model, applications developed by complementors differentiate from one another both vertically and horizontally, and the platform owner collects its revenue by charging entry fees to both sides of the market. We compare three quality regulation mechanisms: 1) the platform excludes low quality complementors using a quality threshold, 2) the platform provides a fixed amount of subsidy to high quality complementors, and 3) the platform has the option of producing certain amount of high quality, first-party applications/content at a cost and therefore improves the average quality of applications in the platform ecosystem.

Our analyses reveal several important observations. First, we show that the widely adopted exclusion strategy is a special case of the quality subsidy strategy; i.e., for every optimal exclusion strategy there is an equivalent quality subsidy strategy that achieves the same level of profit. On the other hand, there exist conditions under which exclusion is strictly dominated by quality subsidy strategy, which is more flexible due to its mechanism of price discrimination. Second, compared to the benchmark scenario without platform owner intervention, both quality subsidy and first-party application strategies make the platform owner better off, with greater profits, higher average quality, larger network sizes and higher entry fees levied on both sides of the market, but only subsidy always improves social welfare. An important insight is that, in contrast to the exclusion strategy (Hagiu 2009a), the adoption of the other two strategies does not require sacrificing quantity in order to improve quality (or vice versa); in fact, both strategies can achieve greater quantity and quality at the same time. Third, the tradeoff between quality subsidy and first-party application strategies depends on the development cost of first-party applications, as well as the fraction of high quality complementors, but the relationship is not monotonic. In addition, comparing the two, the winning strategy is always associated with larger network sizes on both sides of the network. Finally, we discuss the limitation of each quality regulation strategy: for

[4] In this work we use application variety and application quantity interchangeably.

subsidy, the disadvantage becomes more evident when the fraction of high quality complementors is particularly low or high, which leads to cost inefficiency and limited effectiveness in improving quality. For first-party applications, the limitation lies in its inability to internalize the development cost, primarily due to the freeriding of low quality complementors.

This study makes a number of novel contributions to the extant literature on platform governance. First, in contrast to a large body of platform literature dedicated to two-sided pricing strategies (Armstrong 2006, Bernard and Jullien 2003, Parker and Van Alstyne 2005, Rochet and Tirole 2003), the issue of managing the quality of complementary applications has received only scant attention. As Boudreau and Hagiu (2009) noted, ‘getting the price right’ is not a sufficient condition that guarantees the success of a multi-sided market. As such, our work builds on Hagiu (2009a) and contributes directly to the discourse on the quality vs. quantity tradeoff in platform governance. Different from Hagiu (2009a) that focuses solely on the strategy of exclusion, we compare three different forms of strategy that have seen widespread adoption in the technology industry, and show that the subsidy strategy subsumes exclusion. Second, we also add to an emerging literature on first-party content (e.g., Hagiu and Spulber 2013, Lee 2013) by showing that such a strategy has important implications for platform adoption, contributing to indirect network effects not only by increasing application variety but also by meeting consumers’ quality preferences. However, while effective in improving application quality and platform profit, first-party application strategy is not always socially desirable. Third, although some prior studies alluded to the use of subsidy strategy in a two-sided platform setting (Economides and Katsamakas 2006, Eisenmann et al. 2006, Gawer and Cusumano 2008, Parker and Van Alstyne 2005), the primary consideration was to attract initial adoption; i.e., to get one side of the market on board so as to solve the chicken-and-egg dilemma when the platform is first launched (Bernard and Jullien 2003, Parker et al. 2016). We take one step further and examine this strategy from a quality regulation perspective. Therefore, the strategy we consider is one of selective subsidy conditional on quality level, instead of indiscriminately subsidizing all players on one side of the market.

2. Related Literature

Our study is directly related to the literature on quality management in two-sided markets. Researchers have long recognized that indirect network effects depend not only on the variety of complementary goods but also on their quality (Kim et al. 2014). Earlier work suggests that information asymmetry likely leads to certain types of market failure with suboptimal quality levels, and minimum quality standard often provides a solution that results in social desirable outcomes (Akerlof 1970, Leland 1979). Ronnen (1991) further shows that a minimum quality standard strategy not only resolves the 'underprovision' of quality but also reduces excessive quality differentiation, therefore improves social welfare even in the absence of network externalities.

A number of studies also examined the effect of exclusive distribution on content quality. For example, in a model where two distributors bargain with a content producer for distribution rights, Stennek (2014) shows that exclusive distribution may encourage investments in quality, and force the competitor to reduce price, therefore benefiting all viewers. Under the context of media platforms, D'Annunzio (2017) demonstrates that a content provider always prefers granting premium content exclusively to a single distribution platform; however, a vertically integrated content provider (i.e., a situation similar to first-party content) has lower incentives to invest in quality than an independent one.

Some researchers have studied the effect of open access on one side of the market on quality provision. For example, Jeon and Rochet (2010) shows that under an open access model, a for-profit journal tends to publish more low quality articles in order to increase profit from author fees. Surprisingly, quality degradation occurs even when the journal is not-for-profit and aims to maximize readers' welfare. Casadesus-Masanell and Llanes (2015) compare incentives to invest in platform quality between open-source and proprietary platforms. They show that under certain conditions, an open platform may lead to higher investment than a proprietary platform, and opening up one side of a proprietary platform may lower incentives to invest in platform quality.

The work that most closely relates to ours is the stream of literature that studies the tradeoff between quantity and quality of complementary goods in a two-sided market (Hagiu 2009a, Zheng and Kaiser 2013). In particular, Hagiu (2009a) proposes a model where users value the quality of complementary goods in addition to their variety, and such preference is incorporated into the indirect network effect. He concludes that the incentive to exclude low quality complementors depends on the relative preference for quality vs. for quantity, and on the fraction of high quality complementors. Building on his framework, Zheng and Kaiser (2013) study the determination of optimal quality threshold for limiting entry, where complementors with a quality lower than the threshold will be denied admission into a two-sided farmers' market. It is notable that in both studies the focus is placed solely on the exclusion strategy.

3. The Benchmark Model

We consider a two-sided platform with indirect network effects where one side of the market can join to offer their applications or content that enhance the value of the platform and the other side can join to consume the applications or content. For the purpose of exposition, we call the former "developers" and the latter "consumers". The platform charges a fixed access fee p_d to a developer, and a fixed access fee p_c to a consumer. Such a model setup can accommodate a wide range of applications, including technology platforms such as a digital game distribution platform (e.g., Steam by Valve Corporation) or an online market intermediary (e.g., HomeAdvisor), as well as non-technology platforms such as a job fair. For simplicity, we assume each developer offers only one application. The applications differ vertically, with quality being either high or low. We assume that a fraction $\lambda \in [0,1]$ of the developers are of high quality $q_h > 0$, and $1-\lambda$ of the developers are of low quality q_l. Without loss of generality, we normalize q_l to zero. As customary, we assume that the platform has superior information than consumers regarding application quality (Hagiu 2009a): The platform observes the quality of each developer, but consumers only observe the value of λ (and therefore the average quality level of applications), and they are not able to tell the quality of a specific developer.

Consider the case where there are n developers and m consumers who join the platform. Let $\bar{q}$ be the average quality of the n developers on the platform. The utility of a consumer joining the platform is given by

$$V(\theta_j) = w + (\alpha_c + \beta\bar{q})n - p_c - \theta_j,$$

where w is the stand-alone utility of joining the platform, $\alpha_c + \beta\bar{q}$ is the indirect network effect parameter on the consumer side, and θ_j is a consumer-specific horizontal differentiation parameter that is uniformly distributed on $[0, \theta_c]$. Note that the consumer side network effect depends on both application variety and quality: α_c is indirect network effect only related to the quantity of the applications, and $\beta\bar{q}$ is the component of indirect network effect that is also related to the applications' average quality. Therefore, β can be viewed as a measure of consumer quality preference. We assume $\beta > 0$ throughout the paper.

Because consumers know λ, they can infer the average quality of the n developers as $\bar{q} = \lambda q_h$. Therefore, the utility of a consumer joining the platform can be written as

$$V(\theta_j) = w + (\alpha_c + \beta\lambda q_h)n - p_c - \theta_j.$$

A consumer with parameter θ_j will join the platform if $V(\theta_j) \geq 0$. Because θ_j follows a uniform distribution on $[0, \theta_c]$, the demand function of the consumer side can be expressed as

$$m = \frac{w + (\alpha_c + \beta\lambda q_h)n - p_c}{\theta_c}. \quad (1)$$

Or equivalently, the inverse demand function of the consumer side can be written as

$$p_c = w + (\alpha_c + \beta\lambda q_h)n - m\theta_c. \quad (2)$$

To simplify the exposition, we assume that the marginal costs of offering applications on the platform are zero for developers. The utility of a developer joining the platform with m consumers is given by

$$U(\theta_i) = \alpha_d m - p_d - \theta_i,$$

where α_d is the indirect network effect parameter on the developer side, and θ_i is a developer-specific horizontal differentiation parameter that is uniformly distributed on $[0, \theta_d]$. As such, the demand function of the developer side is

$$n = \frac{\alpha_d m - p_d}{\theta_d}, \tag{3}$$

and the inverse demand function of the developer side is

$$p_d = \alpha_d m - n\theta_d. \tag{4}$$

We assume that the platform incurs an operating cost that is proportional to the overall network size; i.e., with n developers and m consumers, the platform's operating cost is ηmn. Similar to Rochet and Tirole (2003), here mn can be interpreted as the volume of "transactions" between consumers and developers. The profit of the platform can be written as

$$\Pi^0 = p_d n + p_c m - \eta mn.$$

where the first term is the total access fees collected from the n developers, the second term is the total access fees collected from m consumers, and the last term is the operating cost of the platform. Let's denote the numbers of high quality developers and low quality developers joining the platform as n_h and n_l, respectively. Note that we have $n_h = \lambda n$, $n_l = (1-\lambda)n$, and $n = n_h + n_l$ in the benchmark model.

Substituting (2) and (4) into the above profit function, the platform's profit optimization problem can be formulated as

$$\max_{m\geq 0, n\geq 0} \Pi^0 = wm + (\alpha_d + \alpha_c + \beta\lambda q_h - \eta)mn - \theta_c m^2 - \theta_d n^2.$$

Define $\xi = \alpha_d + \alpha_c + \beta\lambda q_h - \eta$. Lemma 1 summarizes the platform's equilibrium outcome in the benchmark model. We assume $\alpha_c + \alpha_d - \eta \geq 0$ throughout the paper to ensure that the number of developers joining the platform is non-negative.

Lemma 1

When the platform does not regulate quality of the developers,

(1) the optimal developer and consumer network sizes are

$$m^{0*} = \frac{2\theta_d w}{4\theta_d\theta_c - \xi^2}$$

and

$$n^{0*} = \frac{w\xi}{4\theta_d\theta_c - \xi^2},$$

respectively. The corresponding average quality is $\bar{q}^{0*} = \lambda q_h$, *and the optimal developer and consumer access fees are*

$$p_d^{0*} = \frac{\theta_d w(\alpha_d - \alpha_c + \eta - \beta\lambda q_h)}{4\theta_d\theta_c - \xi^2}$$

and

$$p_c^{0*} = \frac{2w\theta_d\theta_c - w\xi(\alpha_d - \eta)}{4\theta_d\theta_c - \xi^2},$$

respectively. The optimal profit for the platform is

$$\Pi^{0*} = \frac{\theta_d w^2}{4\theta_d\theta_c - \xi^2};$$

(2) when $\alpha_c - \alpha_d \geq \eta - \beta\lambda q_h$, *the platform offers free access to developers, or* $p_d^{0*} = 0$*; when* $\xi(\alpha_d - \eta) \geq 2\theta_d\theta_c$, *the platform offers free access to consumers, or* $p_c^{0*} = 0$.

Proof: All proofs are presented in Appendix 2.

It is not difficult to see that both the optimal profit of the platform and the optimal network sizes increase in network effects α_d and α_c, consumers' quality preference β, and the fraction of high quality developers λ, but decrease in operating cost coefficient η, and the horizontal differentiation parameters of developers and consumers θ_d and θ_c.

Prior research shows that due to indirect network effects, platform pricing strategies often involve granting free access to one side of the market and recovering the loss on the other side (Caillaud and Jullien 2003). Part (2) of Lemma 1 implies that the platform is willing to offer free access to developers and make money from consumers when the platform has a strong consumer network effect α_c and a weak developer network effect α_d, because under such conditions growing developer network is more profitable than growing consumer network. In contrast, it is willing to grant free access to consumers and make money from the developer side with a stronger network effect on the developer side α_d, because a large consumer base makes the platform highly attractive to developers. As expected, the chance of platform offering free access to either side is greater with lower operating cost coefficient η.

4. Quality Regulation Strategies

Because consumers derive greater utilities with higher average quality of the applications on the platform, the platform has incentives to implement some quality regulation strategies to improve the average quality when such strategies lead to higher profit. In this section, we consider three widely used quality regulation strategies: exclusion, subsidy, and first-party application. We characterize the equilibrium outcomes under each strategy, and compare them with the benchmark case where no quality regulation strategy is employed.

4.1 Exclusion

With exclusion, the platform simply uses a quality threshold to exclude low quality developers from joining the platform (Hagiu 2009a, Zheng and Kaiser 2013). In our model with two quality levels, the strategy dictates that only high quality developers are granted access to the platform. As a result, the average quality of developers on the platform under exclusion is $\bar{q}^E = q_h$, which is clearly higher than the average quality of developers in the benchmark model $\bar{q}^{0*} = \lambda q_h$.

The developer utility function and the consumer utility function remain the same as the ones in the benchmark model. Because only high quality developers are allowed to join under exclusion, the demand function of the developer side is

$$n = \lambda \frac{\alpha_d m - p_d}{\theta_d}. \tag{5}$$

The inverse demand function of the developer side is

$$p_d = \alpha_d m - \frac{n\theta_d}{\lambda}. \tag{6}$$

Using the average quality $\bar{q}^E = q_h$ under exclusion, the demand function of the consumer side can be written as

$$m = \frac{w + (\alpha_c + \beta q_h)n - p_c}{\theta_c}. \tag{7}$$

Comparing to the demand functions in the benchmark model (3) and (1), we can see the tradeoff under exclusion clearly from the demand functions (5) and (7). On the one hand, excluding low quality developers raises the average quality of applications on the platform to q_h, which makes the platform more attractive to consumers, everything else being equal. On the other hand, exclusion leads to a lower number of developers *n,* everything else being equal, which reduces the attractiveness of the platform to consumers. Therefore, the net effect of exclusion on consumer network size can be either positive or negative.

The inverse demand function of the consumer side is

$$p_c = w + (\alpha_c + \beta q_h)n - m\theta_c. \tag{8}$$

The profit of the platform has a similar form as

$$\Pi^E = p_d n + p_c m - \eta m n.$$

Using (6) and (8), the profit optimization problem for the platform can be formulated as

$$\max_{m\geq 0, n\geq 0} \Pi^E = wm + \xi_1 mn - \theta_c m^2 - \frac{\theta_d n^2}{\lambda},$$

where we define $\xi_1 = \alpha_d + \alpha_c + \beta q_h - \eta$.

We derive the optimal developer and consumer network sizes, the corresponding optimal developer and consumer access fees, and the optimal profit for the platform in Appendix 1. The following proposition summarizes the properties of exclusion as compared to the benchmark model without quality regulation.

Proposition 1

As compared to the benchmark model without quality regulation,

(1) exclusion always increases average application quality, $\bar{q}^{E*} > \bar{q}^{0*}$. *However, exclusion may not always increase the platform's profit or the consumer network size. Particularly, if there is a scarcity of high quality developers, i.e.,* $\lambda < (\alpha_d + \alpha_c - \eta)^2/(\beta q_h)^2$, *exclusion leads to lower platform profit and smaller consumer network size,* $\Pi^{E*} < \Pi^{0*}$ *and* $m^{E*} < m^{0*}$*;*

(2) under exclusion, it is possible that even high quality developers are worse off, $U_h^{E*} < U_h^{0*}$, *and the equilibrium number of high quality developer is lower than that in the benchmark, i.e.,*

$n^{E*} < n_h^{0*}$;

(3) developers always pay less under exclusion, $p_d^{E*} \leq p_d^{0*}$; in addition, when exclusion improves the profit of the platform, $\Pi^{E*} > \Pi^{0*}$, it leads to larger consumer network size $m^{E*} > m^{0*}$, and higher access fee to consumers, $p_c^{E*} > p_c^{0*}$;

(4) when $\alpha_c - \alpha_d \geq \eta - \beta q_h$, the platform offers free access to developers, $p_d^{E*} = 0$; when $\xi_1 \lambda(\alpha_d - \eta) \geq 2\theta_d \theta_c$, the platform offers free admission to consumers, $p_c^{E*} = 0$. Under exclusion the platform is more likely to offer free access to developers, and less likely to offer free access to consumers.

According to part (1) of Proposition 1, while exclusion always increases the average quality of applications on the platform, it doesn't necessarily result in more consumers because it also reduces the number of developers on the platform, making the platform less attractive to consumers. Smaller consumer and developer network sizes would lead to lower profit for the platform. The condition in part (1) suggests that this is more likely to happen if the percentage of high quality developers λ is low. Under such a condition, exclusion would prevent a large number of developers from participating, which significantly weakens the network sizes of the platform. This is also likely to happen when the network effects on both sides α_d and α_c are high, the operating cost η is low, and the consumer quality preference β and the value of the high quality q_h are low. Under these scenarios, consumers and the developers as well as the platform all prefer larger network sizes (or quantities) over higher quality. As a result, exclusion would not benefit the platform. Some of the above findings are consistent with the ones in Hagiu (2009a) with a similar context.

Part (2) of Proposition 1 indicates that exclusion might not even attract more high quality developers to join the platform due to reduced consumer network size. When this happens, the benefits of quality improvement are not sufficient to compensate for the loss of application quantity. As a result, exclusion significantly hurt the welfare of developers, regardless of their quality.

Part (3) suggests that to compensate for the reduced developer network size, the platform always charges a lower access fee to developers under exclusion to encourage more high quality developers to join. This reduces the revenue from the developer's side, which implies that the platform has to make up the difference from the consumer side. Indeed, we find that when exclusion is beneficial to the platform, the platform charges a higher access fee to a larger consumer network to make higher profit than it does in the benchmark model. In other words, when exclusion is profit improving for the platform, the underlying mechanism is to build a smaller, "elite" developer network which allows the platform to charge a high access fee to a larger number of consumers that have strong preference for higher quality.

Comparing the conditions in part (2) of Lemma 1 in the benchmark model and part (4) of the above proposition, we can see that the platform is more likely to offer free access to developers and to make money from consumers under exclusion. This is because $\alpha_c - \alpha_d \geq \eta - \beta q_h$ can be satisfied more easily than $\alpha_c - \alpha_d \geq \eta - \beta\lambda q_h$, the condition under which developers are granted free access under the benchmark case. On the other hand, because $\xi \geq \xi_1\lambda$, it can be deduced that $\xi_1\lambda(\alpha_d - \eta) \geq 2\theta_d\theta_c$ is more difficult to be satisfied than $\xi(\alpha_d - \eta) \geq 2\theta_d\theta_c$, the condition under which consumers are granted free access under the benchmark case. In other words, under exclusion consumers are more likely to be the "money" side from which the platform profits.

4.2 Subsidy

In a two-sided market, subsidy has been shown to be a particularly effective mechanism to attract platform adoption and build market momentum (Gawer and Cusumano 2008). In order to improve overall application quality, the platform can also subsidize high quality developers to create incentives for them to join the platform. We consider a strategy under which the platform offers a subsidy $\gamma \geq 0$ to each high quality developer who joins the platform. Such practices are commonly seen in platform markets: for example, when Uber launched in Seattle, to attract high end ride providers, it subsidized town car participation by paying drivers even when they weren't transporting customers.[5] Note that by providing a subsidy to some developers but not others, the platform is able to implement a price discrimination strategy; i.e., it essentially charges different access fees to high quality and low quality developers.

Under subsidy, the utility functions for low quality developers and high quality developers are different, because high quality developers earn a subsidy γ, which can simply be viewed as a quality premium. We define U_h, U_l as the utility of a high quality developer and that of a low quality developer, respectively. The utility function for low quality developers stays unchanged as $U_l(\theta_i) = \alpha_d m - p_d - \theta_i$, and the utility function for high quality developers with subsidy γ becomes

$$U_h(\theta_i) = \alpha_d m + \gamma - p_d - \theta_i.$$

Given the utility functions, the number of low quality developer joining the platform is $n_l = (1 - \lambda)(\alpha_d m - p_d)/\theta_d$, and the number of high quality developers joining is $n_h = \lambda(\alpha_d m + \gamma - p_d)/\theta_d$. Therefore, the total number of developers in the market $n = n_l + n_h$ can be written as

$$n = \frac{\alpha_d m - p_d}{\theta_d} + \frac{\gamma\lambda}{\theta_d}. \tag{9}$$

Comparing (9) to (3), with everything else being equal, we can see that subsidy attracts more high quality developers joining the platform, leading to an increase in the total number of developers by $\gamma\lambda/\theta_d$. The inverse demand function of the developer side can be written as

$$p_d = \alpha_d m + \lambda\gamma - n\theta_d. \tag{10}$$

Comparing (10) to (4) in the benchmark model, while the platform offers subsidy γ to high quality developers, it also bumps up the access fee to low quality developers by $\lambda\gamma$.

The average quality under subsidy can be calculated as $\bar{q}^S = (n_h q_h + n_l q_l)/n$. Substituting n_h, n_l and n, we obtain

$$\bar{q}^S = \lambda q_h + \frac{\gamma\lambda(1-\lambda)q_h}{\theta_d n}. \tag{11}$$

[5] See https://www.huffingtonpost.com/alex-moazed/7-strategies-for-solving-_b_6809384.html.

Recall that in the benchmark model, we have $\bar{q}^{0*} = \lambda q_h$. We can see that subsidy increases the average quality of developers, everything else being equal.

The utility of a consumer has the same form as in the benchmark model. Substituting (11), the demand function of the consumer side can be expressed as

$$m = \frac{w + (\alpha_c + \lambda\beta q_h)n - p_c}{\theta_c} + \frac{\gamma\lambda\beta(1-\lambda)q_h}{\theta_d\theta_c}. \tag{12}$$

Comparing (12) to (1), with everything else being equal, subsidy allows the platform to attract $\gamma\lambda\beta(1-\lambda)q_h/(\theta_d\theta_c)$ additional consumers due to higher average quality. The inverse demand function of consumers is

$$p_c = w + (\alpha_c + \beta\bar{q}^S)n - m\theta_c. \tag{13}$$

Hence, the profit of the platform under subsidy can be written as

$$\Pi^S = p_d n + p_c m - \eta mn - \gamma n_h^S,$$

where the last term is total subsidy paid by the platform to high quality developers. Using (10) and (13), the profit optimization problem for the platform can be formulated as

$$\max_{m\geq 0, n\geq 0, \gamma\geq 0} \Pi^S = wm + \xi mn - \theta_c m^2 - \theta_d n^2 + \frac{\beta\gamma\lambda(1-\lambda)}{\theta_d} q_h m - \frac{\gamma^2\lambda(1-\lambda)}{\theta_d}.$$

In Appendix 1, we characterize the optimal subsidy, developer and consumer network sizes, the corresponding optimal developer and consumer access fees, and the optimal profit for the platform under subsidy. We define $\tau = \lambda(1-\lambda)\beta^2 q_h^2 \geq 0$, and assume $4\theta_d\theta_c - \xi^2 - \tau > 0$ to ensure that an equilibrium exists.

Given the optimal subsidy γ^* and access fee to developers p_d^{S*}, we define $p_{dh}^{S*} \triangleq p_d^{S*} - \gamma^*$ as the effective access fee charged to high quality developers. The following proposition characterizes the properties of γ^*, p_d^{S*}, and p_{dh}^{S*}, and Figure 1 illustrates the properties.

Proposition 2

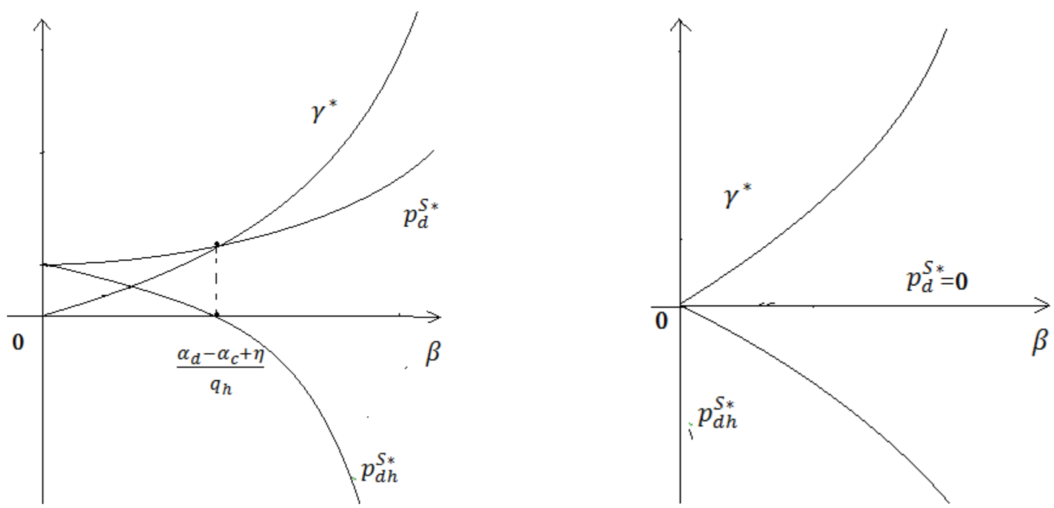
When the platform offers subsidy to high quality developers,

(1) the optimal subsidy is

$$\gamma^* = \frac{\theta_d w\beta q_h}{4\theta_d\theta_c - \xi^2 - \tau};$$

(2) while both the optimal subsidy, γ^* and the optimal access fee to developers, p_d^{S*} are increasing in consumer quality preference β, the effective access fee to high quality developers, p_{dh}^{S*} is decreasing in consumer quality preference β;

(3) if consumer quality preference is sufficiently high, i.e., $\beta > (\alpha_d - \alpha_c + \eta)/q_h$, it is optimal for the platform to subsidize high quality developers more than the optimal access fee so that high quality developers effectively get paid to join the platform, i.e., $\gamma^* > p_d^{S*}$ or $p_{dh}^{S*} < 0$.



(a) $\alpha_d - \alpha_c + \eta > 0$ (b) $\alpha_d - \alpha_c + \eta \leq 0$

Figure 1: The optimal subsidy γ^*, access fee p_d^{S*}, and effective access fee for high quality developers p_{dh}^{S*} as functions of consumer quality preference.

In essence, subsidy is a form of price discrimination which allows the platform to charge differential access fees to developers with different quality levels, specifically p_d^{S*} to low quality developers and $p_{dh}^{S*} = p_d^{S*} - \gamma^*$ to high quality ones. The price discrimination enables the platform to achieve desired average quality level and network size on the developer side more efficiently than under uniform pricing. As consumers' quality preference increases (i.e., with higher β), the platform wants more high quality developers but less low quality developers to join, leading to greater extent of price discrimination (represented by γ^*). According to part (2) of Proposition 2, the platform can achieve the goal by increasing the access fee p_d^{S*} while simultaneously decreasing the effective access fee to the high quality developers p_{dh}^{S*} (by making the subsidy γ^* sufficiently large to offset p_d^{S*}). Increasing access fee p_d^{S*} would discourage low quality developers who are not desirable to the platform. At the same time, with a sufficiently high subsidy γ^*, the reduced effective access fee to high quality developers p_{dh}^{S*} would attract more of them to join.

High quality developers benefit directly from the subsidy. When consumer quality preference β is high enough, as part (3) of Proposition 2 shows, high quality developers become so desirable that the platform is willing to offer such a high subsidy that the effective access fee to them can be negative. In other words, the high quality developers could actually get paid by the platform to join. This will also happen when $(\alpha_d - \alpha_c + \eta) \leq 0$. Under such a condition, growing the developer network is highly profitable that even low quality developers are granted free access, as we see in Figure 1(b).

The following proposition compares the equilibrium parameters between the subsidy strategy and the benchmark model.

Proposition 3

As compared to the benchmark model without quality regulation,

(1) subsidy always increases the average developer quality, the platform profit, the developer network size, and the consumer network size, i.e., $\bar{q}^{S*} > \bar{q}^{0*}$, $\Pi^{S*} > \Pi^{0*}$, $n^{S*} > n^{0*}$, *and* $m^{S*} > m^{0*}$*;*

(2) while subsidy attracts more high quality developers, $n_h^{S*} > n_h^{0*}$, *it may also attracts more low quality developers,* $n_l^{S*} > n_l^{0*}$, *if* $4\theta_d\theta_c < \xi(1-\lambda)\beta q_h - \xi^2$*;*

(3) subsidy leads to higher access fees for both developers and consumers, $p_d^{S*} > p_d^{0*}$, $p_c^{S*} > p_c^{0*}$*; Even with subsidy, the effective access fee for high quality developers can be higher* $p_{dh}^{S*} > p_d^{0*}$, *if* $4\theta_d\theta_c < (2\alpha_d - \xi)\lambda\beta q_h + \xi^2$*;*

(4) if $\alpha_c - \alpha_d \geq \eta$, *the platform offers free access to all developers,* $p_d^{S*} = 0$. *With subsidy, the platform is less likely to offer free access to developers, and equally likely to offer free access to consumers.*

Proposition 3 demonstrates that subsidy is a powerful quality regulation strategy, and reveals the mechanisms through which it benefits the platform. According to part (1) of Proposition 3, subsidy always increases average quality of the developers and improves the profit of the platform. Intuitively, the benchmark model without quality regulation is a special case of subsidy with $\gamma = 0$. Subsidy also leads to both larger developer network size and larger consumer network size for the platform. It is not surprising that subsidy grows the developer network size as a whole and attracts more high quality developers especially. But, as part (2) shown, the network size effect of subsidy can be so strong that it could also attract more low quality developers to join the platform, although they don't qualified for the subsidy and have to pay a higher access fee.

With bigger networks sizes, the platform can charge higher access fees to both sides. As we discuss in Proposition 2, with the subsidy, the effective access fee for high quality developers could even be negative. However, this does not necessarily happen all the time. As part (3) of Proposition 3 indicates, there are cases where even with subsidy, the effective access fee to high quality developers is higher than the one in the benchmark model. In these cases, the platform would enjoy higher fees on bigger network sizes, thereby much higher revenues. In addition, as compared to the benchmark model, subsidy is less likely to set the access fee to developers to be zero as indicated in Part (4), because doing so encourages a large number of low quality developers to join, making subsidy less effective in improving quality. In sum, subsidy increases network size on both sides of the market, and leads to higher average quality so that higher access fees, especially to consumers, can be charged to improve profit for the platform.

4.3 First-party Application

First-party applications, often seen in the video game or the media streaming services industries, provide a mechanism for platform providers to integrate into content development and publishing. Such content or applications are usually exclusively distributed on the native platform and therefore add to the appeal of the platform by creating differentiation (Lee 2013). Here, we consider another strategic use of first-party applications; i.e., in order to improve average application quality, the platform can develop and offer high quality first-party applications directly. In our setting, unlike third-party developers, the platform can develop and offer multiple applications if it wants to. We assume that the platform incurs a development cost of kx^2 for producing x high quality first-party applications.

Consider the case where there are n developers and m consumers who join the platform, and the platform develops x first-party applications. With x first-party applications, the total number of applications offered on the platform is $n + x$, and the total number of high quality applications is $n_h + x$, where $n_h = \lambda n$. The average quality of applications on the platform is given as

$$\bar{q}^F = \frac{(\lambda n + x)q_h}{n + x}. \tag{14}$$

Recall that in the benchmark model, $\bar{q}^{0*} = \lambda q_h$. It is not difficult to see that $\bar{q}^F \geq \bar{q}^{0*}$, that is, first-party application increases the average quality.

The consumer utility with first-part application is given by

$$V(\theta_j) = w + (\alpha_c + \beta\bar{q}^F)(n + x) - p^c - \theta_j. \tag{15}$$

Substitute (14) into (15), we get the demand function of the consumer side as

$$m = \frac{w + (\alpha_c + \beta\lambda q_h)n + (\alpha_c + \beta q_h)x - p_c}{\theta_c}.$$

Compared with (1), with x first-party applications, the platform can attract $(\alpha_c + \beta q_h)x/\theta_c$ more consumers. The inverse demand function of the consumer side is

$$p_c = w + (\alpha_c + \beta\lambda q_h)n + (\alpha_c + \beta q_h)x - m\theta_c. \tag{16}$$

The utility, demand function, and the inverse demand function on the developer side remain the same as the ones in the benchmark model.

With $n + x$ applications on the platform, the operating cost of the platform becomes $\eta m(n + x)$. Hence, the profit of the platform with x first-party applications is

$$\Pi^F = p_d n + p_c m - \eta m(n + x) - kx^2.$$

where the last term is the development cost for the first-party applications.

Substituting (4) and (16) into Π^F, the platform's profit optimization problem when it develops first-party applications can be formulated as

$$\max_{m\geq 0, n\geq 0, x\geq 0} \Pi^F = \xi mn + wm + (\alpha_c + \beta q_h)xm - \eta mx - m^2\theta_c - n^2\theta_d - kx^2.$$

To simplify exposition, we define $\delta = (\theta_d(\alpha_c + \beta q_h - \eta)^2)/k \geq 0$. We assume $4\theta_d\theta_c - \xi^2 - \delta > 0$ to guarantee an equilibrium exists.

We characterize the optimal developer and consumer network sizes, the corresponding optimal developer and consumer access fees, and the optimal profit for the platform with first-party application in Appendix 1. The following proposition explains some properties of the optimal number of first-party applications. Figure 2 illustrate these properties visually.

Proposition 4

When the platform develops first-party applications,

(1) the optimal number of first-party applications the platform should develop is

$$x^* = \frac{\theta_d w(\alpha_c + \beta q_h - \eta)}{k(4\theta_d\theta_c - \xi^2 - \delta)}.$$

Particularly, if $\alpha_c + \beta q_h < \eta$*, then it is not in the interest of the platform to develop any first-party applications,* $x^* = 0$ *;*

(2) when it is beneficial for the platform to develop first-party applications, i.e., $x^* > 0$*, the optimal number of first-party applications* x^* *is always increasing in the fraction of the high quality developers,* λ *(or equivalently, decreasing in* $1 - \lambda$*). However, the ratio of first-party applications over third-party applications,* $\frac{x^*}{n^{F*}}$ *decreases with* λ*;*

(3) when it is beneficial for the platform to develop first-party applications, i.e., $x^* > 0$*, the optimal number of first-party applications* x^* *is always increasing in consumer quality preference* β*. In addition, the ratio of first-party applications over third-party applications,* $\frac{x^*}{n^{F*}}$ *is also increasing in* β*.*

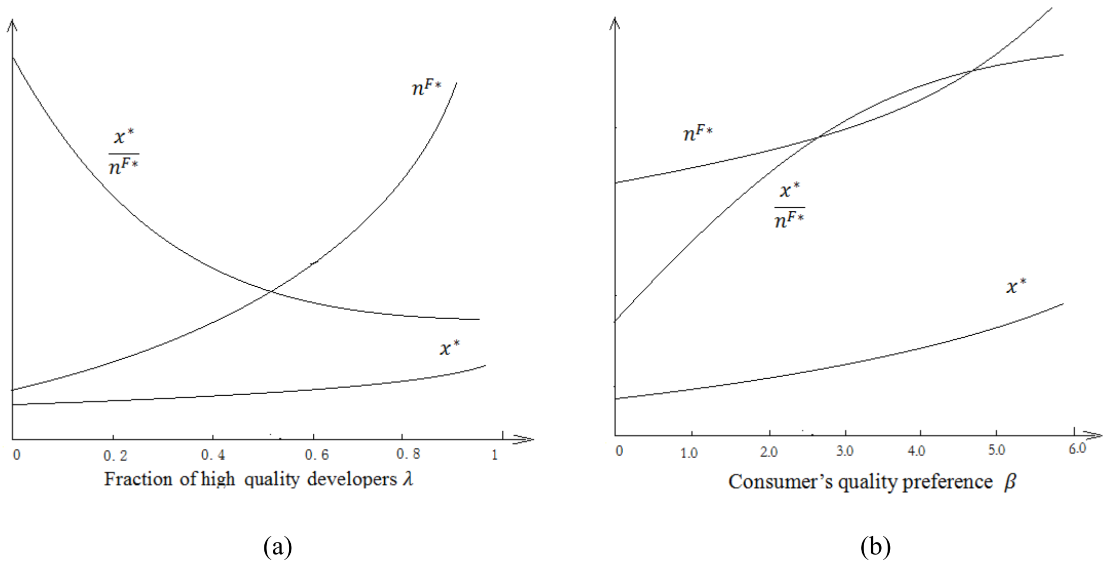


Figure 2. The number of developers, n^{F*}, first-party applications, x^*, and their ratio, x^*/n^{F*} as functions of the fraction of high quality developers, λ (shown in (a)), and as functions of consumer's quality preference, β (shown in (b)). $q_h=0.4$, $\theta_c=\theta_d=2$, $\alpha_c=\alpha_d=0.6$, $w=1$, $\eta=0.6$, and $k=27$ in both (a) and (b). $\beta=2.7$ in (a), and $\lambda=0.55$ in (b).

Proposition 4 implies that if the consumer network effect α_c, the consumer quality preference β, or the value of the high quality q_h is too low, first-party application is not a desirable strategy. In addition, if the platform operating cost η is too high, the platform should also not develop any first-party applications. Under these scenarios, the added value by providing more high quality applications is low because the increase in network effect (through both application variety and quality) is not strong enough to offset the platform maintenance cost. Otherwise, investments in developing first-party applications will lead to higher platform profit.

Intuitively, when the fraction of low quality developers, $1-\lambda$ is high, one might expect that the platform offers more first-party applications to compensate for the low average quality. Surprisingly, part (2) of Proposition 4 suggests the opposite: the platform should offer *fewer* first-party applications when the fraction of low quality developers, $1-\lambda$ is high. Note that with first-party application, the platform is unable to completely internalize the development cost because it lacks the ability to price discriminate and has to set a uniform access fee to both high quality and low quality developers. Charging a high access fee to developers will discourage high quality developers and therefore weakens the effectiveness of improving quality, but charging a low access fee will allow low quality developers to freeride the quality improvement (and the resulting larger consumer base) brought by the first-party applications, which dilutes the effect of improving quality. Therefore, when there is a large fraction of low quality developers, greater externality deters the platform from creating more first-party applications. Part (2) also suggests that although the optimal number of first-party applications x^* is increasing in the fraction of high quality developers λ, the equilibrium number of third-party applications n^{F*} increases at a much faster rate. This is because when λ increases, the average application quality improvement comes from both increase in the number of high quality third-party developers (a first order effect), and from more first-party applications offered by the platform (a second order effect through x^*), leading to much higher incentives for developers to join. As a result, with a large λ, the platform is more likely to be dominated by third-party applications.

Part (3) indicates that when consumers have higher quality preference, not surprisingly, the platform is willing to develop more first-party applications. With a high β, the platform is better able to recover a large part of the development costs from the consumer side, taking advantage of the consumers' willingness to pay for quality. While the number of third-party developers n^{F*} also increases with consumer quality preference, its rate of increase is lower than that of first-party applications, because the average quality of third-party applications is lower than that of first-party applications (due to the presence of low quality developers). As a result, with a high value of β the platform is more likely to be dominated by first-party applications.

The following proposition compares the equilibrium parameters between the first-party application strategy and the benchmark model.

Proposition 5

As compared to the benchmark model without quality regulation,

(1) first-party application always increases average application quality, platform profit, both developer and consumer network sizes, and access fees to both developer and consumer, i.e.,

$\bar{q}^{F*} > \bar{q}^{0*}$, $\Pi^{F*} > \Pi^{0*}$, $n^{F*} > n^{0*}$, $m^{F*} > m^{0*}$, $p_d^{F*} > p_d^{0*}$, and $p_c^{F*} > p_c^{0*}$.

(2) with first-party application, if $\xi(\alpha_d - \eta) \geq 2\theta_d\theta_c + \theta_d\eta(\alpha_c + \beta q_h - \eta)/kw$, the platform offers free access to consumers, $p_c^{F*} = 0$. With first-party application, the platform is less likely to offer free access to consumers, and equally likely to offer free access to developers.

Proposition 5 shows that first-party application is also an effective quality regulation strategy for the platform. With the introduction of high quality first-party applications, both application variety and the average quality are higher for certain. The benchmark model without quality regulation is a special case of first-party application with $x = 0$. Therefore, the platform will certainly do better under first-party application. As average quality improves, the platform attracts more consumers to join, which would in turn attract more developers to join, creating a positive feedback between the two sides of the market through network effects. Due to higher attractiveness to both sides, the platform is able to charge higher access fees to both sides to increase revenues. The increased revenues from access fees would be sufficient to offset the development cost for the first-party applications to increase profit for the platform.

As compared to the benchmark model, the platform is less likely to offer free access to consumers and make money from developers. This is because the introduction of first-party applications creates significant value for consumers through both greater application variety and quality, leading to higher willingness to pay by the consumers.

5. Optimal Quality Regulation Strategy

In this section, we investigate the platform's optimal choice of quality regulation strategy, and compare the social welfare under the different strategies. We also discuss the relative advantages and limitations of the strategies.

5.1 Exclusion vs. Subsidy

In the following proposition, we compare exclusion and subsidy from the platform's perspective.

Proposition 6

(1) Subsidy is the dominant choice over exclusion to the platform, i.e., $\Pi^{S*} \geq \Pi^{E*}$*. In fact, exclusion is a special case of subsidy, that is, for every optimal exclusion strategy, there always exists an equivalent subsidy strategy.*

(2) Exclusion achieves higher average developer quality than subsidy does, $\bar{q}^{E*} \geq \bar{q}^{S*}$*;*

(3) Subsidy leads to larger network sizes and higher access fees for both developers and consumers than exclusion, $n_h^{S*} > n^{E*}$, $m^{S*} > m^{E*}$, $p_d^{S*} > p_d^{E*}$, and $p_c^{S*} > p_c^{E*}$.

Proposition 6 states that exclusion is dominated by subsidy as a quality regulation strategy for the platform, because it is a special case of subsidy. It is easy to see why this is the case: With subsidy, the platform can always set the developer access fee p_d^S sufficiently high so that all low quality developers choose not to join, and then adjust the subsidy γ accordingly to offset the high access fee to attract the desired amount of high quality developers to join, achieving the same effect as exclusion. Therefore, subsidy is a more general and flexible quality regulation strategy as compared to exclusion.

However, as indicated by part (2) of Proposition 6, an advantage of exclusion is that it does achieve higher average quality than subsidy (and higher than first-party application as well). Recall that the average quality under exclusion is $\bar{q}^{E*} = q_h$, which is simply the highest average quality can be possibly achieved by the platform in our model setting. However, highest level of quality is not always preferable to a platform, which explains why subsidy dominates exclusion: with subsidy the platform can balance between quantity and quality, while exclusion is more rigid with a constant quality level. Exclusion does have some appeal: If the objective is to achieve a high (or the highest as in our model) average quality, exclusion is a more effective and direct strategy that is simpler to implement than many others such as subsidy and first-party application. This might explain why exclusion is commonly used in practice although it is not necessarily the profit-optimizing strategy.

Except for average quality, subsidy dominates every other front according to part (3) of

Proposition 6. Subsidy leads to larger network sizes on both developer and consumer sides, which allows the platform to charge higher fees on both sides, thereby generating higher revenue that is enough to offset the cost of subsidy to earn higher profit.

In light of the fact that exclusion is a special case of the subsidy strategy, the platform's optimal choice of quality regulation strategy is between subsidy and first-party application, which will be the focus of the rest of this section.

5.2 Optimal Choice of Quality Regulation Strategy

The following proposition characterizes the platform's optimal choice of quality regulation strategy.

***Proposition** 7*

The platform's optimal choice of quality regulation strategy between subsidy and first-party content can be characterized as:

(1) *When the first-party application development cost coefficient k is sufficiently low, i.e.,* $k < 4\theta_d(\alpha_c + \beta q_h - \eta)^2/(\beta^2 q_h^2)$, *first-party application is optimal,* $\Pi^{F*} > \Pi^{S*}$*;*

(2) *When first-party application development cost coefficient k is high,* $k > 4\theta_d(\alpha_c + \beta q_h - \eta)^2/(\beta^2 q_h^2)$, *there exist two thresholds* $0 < \underline{\lambda} < \overline{\lambda} < 1$ *(defined in the proof) so that subsidy is optimal, or* $\Pi^{S*} > \Pi^{F*}$*, when* $\underline{\lambda} < \lambda < \bar{\lambda}$*; whereas first-party application is optimal, or* $\Pi^{F*} > \Pi^{S*}$*, when* $0 < \lambda < \underline{\lambda}$ *or* $\overline{\lambda} < \lambda < 1$.

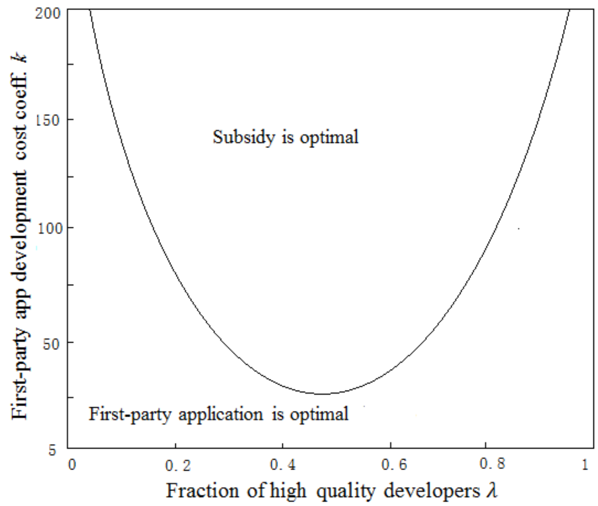


Figure 3. The platform's optimal quality regulation strategy. $\beta=2.7$, $q_h=0.4$, $\theta_c=\theta_d=2$, $\alpha_c=\alpha_d=0.6$, $w=1$, and $\eta=0.6$.

Intuitively, when the first-party application development cost is sufficiently low, first-party application should be the optimal choice for the platform, which is confirmed by part (1) of Proposition 7. Surprisingly, we find that even when the first-party application development cost is high, first-party application can still outperform subsidy. Part (2) of Proposition 7 shows that this happens when the percentage of high quality developers λ is either sufficiently low or sufficiently high. The reason is that when λ is either low or high, subsidy may not work effectively and cost efficiently to achieve the desired average quality level (and therefore network sizes). When λ is too low, there simply aren't enough high quality developers out there for the platform to subsidize in order to achieve the desired average quality level without sacrificing developer network size significantly (recall that under subsidy the platform also raises the access fee to low quality developers). When λ is too high, the cost of subsidy becomes substantial, and the subsidy strategy only achieves very limited improvement in average quality because most developers joining are high quality anyway. In contrast, first-party application does not suffer from these limitations, because the number of first-party applications to offer is fully under the discretion of the platform. Thus, in these situations, developing its own first-party applications proactively is the strategy of choice for the platform to improve the average quality level and profit. Conversely, when λ is moderate, the condition is just right for subsidy to fully leverage its advantages to become the optimal strategy for the platform.

Figure 3 illustrates the platform's optimal choice between subsidy and first-party application as its quality regulation strategy graphically on the plane of the first-party development cost, *k* and the fraction of high quality developers, λ.

The following proposition provides further insight on the optimal quality regulation strategy for the platform.

Proposition 8

When the platform chooses between subsidy and first-party application,

(1) the optimal strategy always leads to larger developer and consumer network sizes, but not necessarily higher average quality;

(2) when the platform is indifferent between the two strategies, or $\Pi^{S*} = \Pi^{F*}$*, it charges a higher access fee to low quality developers under subsidy,* $p_d^{S*} > p_d^{F*}$*, and a lower effective access fee to high quality developers under subsidy,* $p_{dh}^{S*} < p_d^{F*}$*. It also charges a lower access fee to consumers under subsidy,* $p_c^{S*} < p_c^{F*}$*.*

(3) when the platform is indifferent between the two strategies, or $\Pi^{S*} = \Pi^{F*}$*, low quality developers are better off under first-party application, i.e.,* $U_l^{F*} > U_l^{S*}$*, while high quality developers are better off under subsidy, i.e.,* $U_h^{S*} > U_h^{F*}$*. Consumer utilities are the same under both strategies, i.e.,* $V^{S*} = V^{F*}$*.*

Part (1) of Proposition 8 reveals that the platform prefers a quality regulation strategy (between subsidy and first-party application) that can enable it to grow the network sizes on both sides rather than achieving the highest average quality. In other words, with quality regulation, the platform's ultimate goal is to become a larger platform with higher application variety and more consumers so that it can charge a higher access fee to consumers or developers to improve its profitability. In Figure 4, we illustrate the platform's optimal choice of quality regulation strategy when its objective is to maximize average application quality, and contrast this choice to the profit maximizing choice described in Proposition 7.

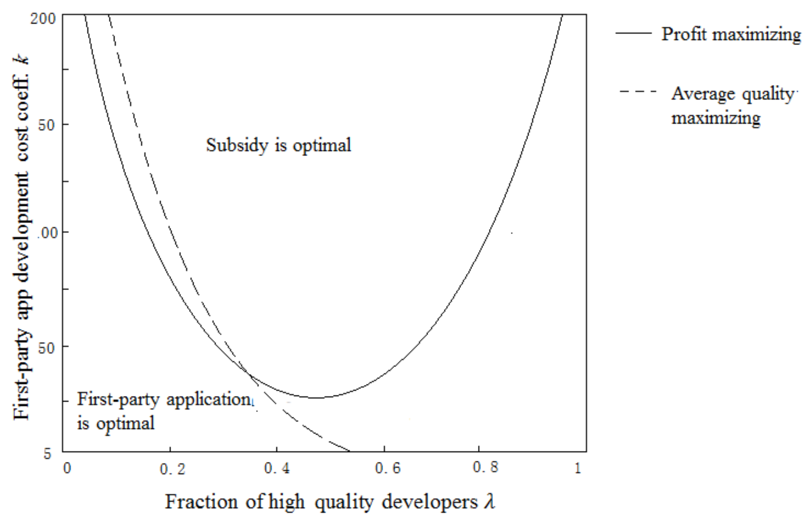


Figure 4. Profit maximizing strategy vs. average quality maximizing strategy. β*=2.7,* q_h*=0.4,* $\theta_c = \theta_d$*=2,* $\alpha_c = \alpha_d$*=0.6,* w*=1,* and η*=0.6.*

Part (2) and part (3) of Proposition 8 show the differences in the underlying mechanisms between the two strategies when both are equally attractive to the platform. Particularly, we observe that when subsidy strategy is adopted, the platform raises access fee to deter low quality developers, due to its ability to price discriminate. As a result, high quality developers, who pay a lower effective access fee, are better off at the expense of low quality developers. However, when first-party application strategy is adopted, low quality developers benefit more from larger consumer network size and improvement in quality at the expense of high quality developers. As we discussed before, first-party application leads to the freeriding of low quality developers, because the platform lacks the ability to price discriminate and has to set a uniform access fee to both high quality and low quality developers.

Proposition 8 also shows that consumers are neutral to platform quality regulation strategy choice when both strategies achieve the same profit level. When the two strategies lead to the same profit for the platform, they result in the same consumer network size and third-party developer network size. However, the overall number of applications (or variety) is higher by x^* under first-party application. The resulting increase in consumer utility is completely internalized by the platform through a higher access price charged to consumers under first-party application.

5.3 Social Welfare

We have studied how different quality regulation strategies improve the platform's profit. We now shift to understanding their impacts on social welfare. For each of the models $t \in \{0, E, S, F\}$, the social welfare is the sum of total consumer utility V^*, total developer utility U^*, and the platform's profit Π^{t*}, defined as

$$W^{t*} = \int_0^n U(\theta_i)^{t*} di + \int_0^m V(\theta_j)^{t*}\, dj + \Pi^{t*}.$$

The optimal social welfare of the models, benchmark W^{0*}, exclusion W^{E*}, subsidy W^{S*}, and first-party application W^{F*}, are characterized in Appendix 1. The properties and the implications of the optimal social welfare are summarized in the following proposition.

Proposition 9

(1) Subsidy always improves both the platform's profit and the social welfare, $\Pi^{S*} \geq \Pi^{0*}$ *and* $W^{S*} \geq W^{0*}$. *While first-party application always improves the platform's profit,* $\Pi^{F*} \geq \Pi^{0*}$*, it does not necessarily improve the social welfare. Exclusion does not necessarily improve either.*

(2) *When subsidy is the optimal strategy for the platform,* $\Pi^{S*} \geq \Pi^{F*}$*, it always leads to higher social welfare than first-party application does,* $W^{S*} > W^{F*}$. *However, the opposite is not necessarily true. Therefore, a social planner would more likely to choose subsidy over first-party content than the platform would.*

We have discussed in the previous section that exclusion does not necessarily improve the platform's profit, while both subsidy and first-party application do. According to part (1) of Proposition 9, subsidy also surely improves the social welfare, because the subsidy is just an internal transfer between the platform and high quality developers. In contrast, first-party application might not always increase social welfare, because the first-party application development cost is an extra cost to the platform and the society as whole. A profit-maximizing platform may have the incentive to over-invest in first-party applications even when it is not as efficient as third-party developers, which hurts the social welfare. Recall that exclusion reduces the network sizes, especially the developer network size, increases the access fee to consumers, and could lower the profit for the platform, which are all detrimental to social welfare.

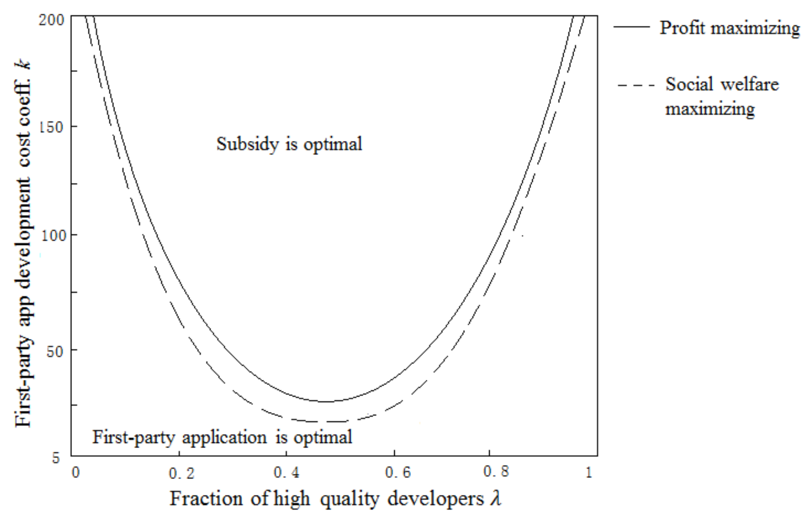


Figure 5. Profit-maximizing quality regulation strategy vs. social welfare-maximizing quality regulation strategy. $\beta=2.7,\ q_h=0.4,\ \theta_c=\theta_d=2,\ \alpha_c=\alpha_d=0.6,\ w=1,$ and $\eta=0.6.$

Hence, subsidy, in addition to being profit improving, is the most social welfare friendly quality regulation strategy among the three. In fact, when subsidy is optimal or profit-maximizing for the platform, it is always social welfare-maximizing. However, when first-party application is optimal for the platform, it may not be social welfare-maximizing, which suggests that a welfare-maximizing social planner prefers subsidy more often than the platform. Figure 5 shows the difference between the platform's choice and the social planner's choice between the two strategies. As we can see, the area under which subsidy is optimal is larger for the social planner and subsumes that for the platform, implying that the social planner would be more likely to choose subsidy as the optimal quality regulation strategy than the platform would.

6. Discussion and Conclusions

With platforms becoming an increasingly ubiquitous business model in the technology industry, the role of a platform company transits from coordinating internal economic activities to also include providing boundary resources to outside complementors as well as regulating the conduct of firms within its platform ecosystem (Boudreau and Hagiu 2009). While prior literature has provided many insights into the pricing strategies in a two-sided market (Hagiu 2006, 2009b, Jeon and Rochet 2010), in this study we focus on a non-pricing aspect in platform governance – the regulation of the quality of complementary applications – which has so far received little research attention. We compare three strategies that are widely employed in practice: excluding access to low quality complementors, providing a subsidy to high quality ones, and developing high quality first-party applications. Our analyses reveal that it is imperative for platforms to understand the mechanisms behind quality regulation strategies, because under a wide range of scenarios implementing one of these strategies will lead to higher platform profit, and will often result in greater social welfare as well. Interestingly, strategies aimed at increasing application variety and those aimed at improving application quality need not be in conflict with one another as suggested by prior research (Hagiu 2009a); instead, both objectives can be achieved simultaneously if the platform makes smart choices.

We show that each of the three strategies we study has its unique advantages and limitations. Under exclusion, a platform is able to achieve a high quality level with a relatively straightforward implementation. However, being the least flexible among the three, exclusion does not necessarily improve either platform profit or social welfare. In contrast, providing subsidy to high quality developers does improve both due to its power of price discrimination, and is a particularly attractive choice if the platform faces a high first-party development cost. However, subsidy becomes increasingly ineffective if the platform is fraught with low quality developers, and is not cost efficient when third-party developers are predominantly of high quality. Under these conditions, first-party application strategy works particularly well if platform development cost is low, but such a strategy may suffer from over-provision of first-party applications to the extent that it hurts social welfare. In addition, under first-party application the platform faces challenge in internalizing development cost due to the freeriding of low quality developers, and the issue is most prominent when quality provision by third-party is more evenly distributed.

Our research also points to a number of important managerial implications for practitioners. For example, although the strategy of exclusion appears to be intuitively appealing, it may lead to unintended consequences under certain contexts, and therefore its adoption should be carefully weighed against other alternatives. In contrast, platform designs that involve subsidizing high quality complementors, like the actions taken by Google's Android platform, or setting differential platform access fees based on application quality can often make the platform more profitable and socially desirable at the same time. Moreover, with many platforms – such as Netflix – start integrating into content provision and investing aggressively in the development of their exclusive first-party applications, managers need to carefully evaluate whether choosing such a strategy is advantageous, taking into consideration factors such as their cost efficiency in relative to outside developers, and the quality distribution among third-party applications. Our study here provides some concrete guidelines to help managers make these decisions.

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Quality Regulation on Two-Sided Platforms:
Exclusion, Subsidy, and First-party Application

Appendix 1: Equilibrium Outcomes under Benchmark and Quality Regulation

(a) The Benchmark Model

Differentiating Π^0 with respect to m, n, respectively, we obtain first order conditions

$$w + (\alpha_d + \alpha_c + \beta\lambda q_h - \eta)n - 2\theta_c m = 0,$$

$$(\alpha_d + \alpha_c + \beta\lambda q_h - \eta)m - 2\theta_d n = 0.$$

Define $\xi = \alpha_d + \alpha_c + \beta\lambda q_h - \eta$. We get the optimal m and n

$$m^{0*} = \frac{2\theta_d w}{4\theta_d\theta_c - \xi^2},$$

$$n^{0*} = \frac{w\xi}{4\theta_d\theta_c - \xi^2}.$$

Substituting m^{0*}, n^{0*} into Π^0 yields

$$\Pi^{0*} = \frac{\theta_d w^2}{4\theta_d\theta_c - \xi^2}.$$

The corresponding optimal access fees to developer and consumer are

$$p_d^{0*} = \frac{\theta_d w(\alpha_d - \alpha_c + \eta - \beta\lambda q_h)}{4\theta_d\theta_c - \xi^2} \tag{A1}$$

and

$$p_c^{0*} = \frac{2w\theta_d\theta_c - w\xi(\alpha_d - \eta)}{4\theta_d\theta_c - \xi^2}.$$

The corresponding optimal average quality is

$$\bar{q}^{0*} = \lambda q_h. \tag{A2}$$

In the benchmark model, welfare can be formulated as

$$W^0 = \int_0^n (\alpha_d m - p_d - \theta_i)di + \int_0^m \left(w + \alpha_C(\bar{q})n - p_c - \theta_j\right) dj + \Pi^0$$
$$= wm + \xi mn - \frac{\theta_d n}{2} - \frac{\theta_c m}{2}.$$

Substituting n^{0*}, m^{0*} into W^0 yields the optimal social welfare in the benchmark model as

$$W^{0*} = \frac{8w^2\theta_d^2\theta_c}{(4\theta_d\theta_c - \xi^2)^2} - \frac{\theta_d w\left(\theta_c + \frac{1}{2}\xi\right)}{4\theta_d\theta_c - \xi^2}.$$

□

(b) Exclusion

Differentiating Π^E with respect to m and n, respectively, we obtain first order conditions

$$w + (\alpha_d + \alpha_c + \beta q_h - \eta)m = 2\theta_c m \tag{A3}$$

and

$$(\alpha_d + \alpha_c + \beta q_h - \eta)n = \frac{2\theta_d n}{\lambda}. \tag{A4}$$

Define $\xi_1 = \alpha_d + \alpha_c + \beta q_h - \eta$. Solving equations (A3) and (A4), we obtain

$$m^{E*} = \frac{2\theta_d w}{4\theta_d\theta_c - \lambda\xi_1^2}$$

and

$$n^{E*} = \frac{\lambda w \xi_1}{4\theta_d\theta_c - \lambda\xi_1^2}.$$

Substituting n^{E*}, m^{E*} into Π^E yields

$$\Pi^{E*} = \frac{\theta_d w^2}{4\theta_d\theta_c - \lambda\xi_1^2}.$$

The corresponding optimal access fees to developer and consumer are

$$p_d^{E*} = \frac{\theta_d w(\alpha_d - \alpha_c + \eta - \beta q_h)}{4\theta_d\theta_c - \lambda\xi_1^2}$$

and

$$p_c^{E*} = \frac{2w\theta_d\theta_c - w\xi_1\lambda(\alpha_d - \eta)}{4\theta_d\theta_c - \lambda\xi_1^2}.$$

The corresponding optimal average quality is

$$\bar{q}^{E*} = q_h. \tag{A5}$$

Under exclusion, welfare can be formulated as

$$W^E = \int_0^{\lambda n} (\alpha_d m - p_d - \theta_i)di + \int_0^m \left(w + \alpha_C(\bar{q}^E)n - p_c - \theta_j\right)dj + \Pi^E$$

$$= wm + \xi_1 mn - \frac{\theta_d\lambda n}{2} - \frac{\theta_c m}{2}.$$

Substituting n^{E*}, m^{E*} into W^E yields the optimal social welfare under exclusion as

$$W^{E*} = \frac{8\theta_d^2\theta_c w^2}{(4\theta_d\theta_c - \lambda\xi_1^2)^2} - \frac{\theta_d w\left(\frac{1}{2}\lambda^2\xi_1 + \theta_c\right)}{4\theta_d\theta_c - \lambda\xi_1^2}.$$

□

(c) Subsidy

Differentiating Π^S with respect to $m, n,$ and γ, respectively, we obtain first order conditions

$$w + \xi n + \frac{\beta\gamma\lambda(1-\lambda)}{\theta_d} q_h - 2\theta_c m = 0, \tag{A6}$$

$$\xi m - 2\theta_d n = 0, \tag{A7}$$

$$\beta q_h m = 2\gamma. \tag{A8}$$

Solving equations (A6), (A7) and (A8), and let $\tau = \lambda(1-\lambda)\beta^2 q_h^2$, we get optimal m, n, γ

$$m^{S*} = \frac{2\theta_d w}{4\theta_d\theta_c - \xi^2 - \tau},$$

$$n^{S*} = \frac{w\xi}{4\theta_d\theta_c - \xi^2 - \tau},$$

$$\gamma^* = \frac{\theta_d w \beta q_h}{4\theta_d\theta_c - \xi^2 - \tau}.$$

Because the number of low quality developer joining the platform is $n_l = (1-\lambda)(\alpha_d m - p_d)/\theta_d$, and the number of high quality developers joining is $n_h = \lambda(\alpha_d m + \gamma - p_d)/\theta_d$, we get the optimal number of high quality developers and low quality developers, respectively,

$$n_h^{S*} = \frac{w\lambda(\xi + \lambda\beta q_h)}{4\theta_d\theta_c - \xi^2 - \tau}, \tag{A9}$$

$$n_l^{S*} = \frac{w(1-\lambda)(\alpha_d + \alpha_c - \eta)}{4\theta_d\theta_c - \xi^2 - \tau}. \tag{A10}$$

Substituting n^{S*}, m^{S*}, γ^* into Π_S yields

$$\Pi^{S*} = \frac{\theta_d w^2}{4\theta_d\theta_c - \xi^2 - \tau}.$$

The corresponding optimal access fees to developer and consumer are

$$p_d^{S*} = \frac{\theta_d w(\alpha_d - \alpha_c + \eta)}{4\theta_d\theta_c - \xi^2 - \tau}$$

and

$$p_c^{S*} = \frac{2w\theta_d\theta_c - w\xi(\alpha_d - \eta)}{4\theta_d\theta_c - \xi^2 - \tau}.$$

For given n and γ, the average quality under subsidy is

$$\bar{q}^S = \lambda q_h + \frac{\gamma\lambda(1-\lambda)q_h}{\theta_d n}. \tag{A11}$$

According to $\bar{q}^S = (n_h q_h + n_l q_l)/n$ the corresponding optimal average quality is

$$\bar{q}^{S*} = \lambda q_h + \frac{\lambda(1-\lambda)\beta q_h^2}{\xi}. \tag{A12}$$

Under subsidy, welfare can be formulated as

$$\begin{aligned} W^S &= \int_0^{n_l} (\alpha_d m - p^d - \theta_i) di + \int_0^{n_h} (\alpha_d m + \gamma - p^d - \theta_i) di \\ &+ \int_0^m \left(w + \alpha_C(\bar{q}^S)n - p^c - \theta_j\right) dj + \Pi^S \\ &= wm + \xi mn + \frac{\beta\gamma\lambda(1-\lambda)q_h m}{\theta_d} - \frac{\theta_d n}{2} - \frac{\theta_c m}{2}. \end{aligned}$$

Substituting n_S^*, m_S^*, γ^* into W^S yields the optimal social welfare under subsidy as

$$W^{S*} = \frac{8\theta_d^2\theta_c w^2}{(4\theta_d\theta_c - \xi^2 - \tau)^2} - \frac{\theta_d w\left(\theta_c + \frac{1}{2}\xi\right)}{4\theta_d\theta_c - \xi^2 - \tau}.$$

□

(d) First-party Applications

Differentiating Π^F with respect to $m, n,$ and x, respectively, we obtain first order conditions

$$\xi n + (\alpha_c + \beta q_h - \eta)x + w = 2m\theta_c, \tag{A13}$$

$$\xi m = 2n\theta_d, \tag{A14}$$

$$(\alpha_c + \beta q_h - \eta)m = 2kx. \tag{A15}$$

Solving equations (A13), (A14) and (A15), and define $\delta = \theta_d(\alpha_c + \beta q_h - \eta)^2/k$, we get the optimal $m, n,$ and x

$$m^{F*} = \frac{2\theta_d w}{4\theta_d\theta_c - \xi^2 - \delta},$$

$$n^{F*} = \frac{w\xi}{4\theta_d\theta_c - \xi^2 - \delta},$$

$$x^* = \frac{\theta_d w(\alpha_c + \beta q_h - \eta)/k}{4\theta_d\theta_c - \xi^2 - \delta}.$$

Substituting $m^{F*} n^{F*}, x^*$ into Π^F yields

$$\Pi^{F*} = \frac{\theta_d w^2}{4\theta_d\theta_c - \xi^2 - \delta}.$$

The corresponding optimal access fees to developer and consumer are

$$p_d^{F*} = \frac{\theta_d w(\alpha_d - \alpha_c + \eta - \beta\lambda q_h)}{4\theta_d\theta_c - \xi^2 - \delta}$$

and

$$p_c^{F*} = \frac{2w\theta_d\theta_c - w\xi(\alpha_d - \eta) + \theta_d\eta(\alpha_c + \beta q_h - \eta)/k}{4\theta_d\theta_c - \xi^2 - \delta}.$$

For given n and x, the average quality under first-party application is

$$\bar{q}^F = \frac{(\lambda n + x)q_h}{n + x}. \quad \text{(A16)}$$

Substituting n^{F*} and x^*, the corresponding optimal average quality is

$$\bar{q}^{F*} = \lambda q_h + \frac{(1-\lambda)q_h\theta_d(\alpha_c + \beta q_h - \eta)}{k\xi + \theta_d(\alpha_c + \beta q_h - \eta)}. \quad \text{(A17)}$$

Under first-party content, welfare can be formulated as

$$W^F = \int_0^n (\alpha_d m - p^d - \theta_i)di + + \int_0^m \left(w + \alpha_C(\bar{q}^F)n - p^c - \theta_j\right) dj + \Pi^F$$
$$= wm + \xi mn - \frac{\theta_d n}{2} - \frac{\theta_c m}{2} - kx^2 + (\alpha_c + \beta q_h - \eta)mx.$$

Substituting n^{F*}, m^{F*}, x^* into W^F yields

$$W^{F*} = \frac{8\theta_d^2\theta_c w^2 - \theta_d w^2\delta}{(4\theta_d\theta_c - \xi^2 - \delta)^2} - \frac{\theta_d w\left(\theta_c + \frac{1}{2}\xi\right)}{4\theta_d\theta_c - \xi^2 - \delta}.$$

□

Appendix 2: Proofs

Proof of Lemma 1:

(1) See Appendix 1(a).

(2) Directly follow from the expressions of $p_c^{0*}$and p_d^{0*} in Appendix 1(a).

□

Proof of Proposition 1

(1) Recall that $\bar{q}^{E*} = q_h$ and $\bar{q}^{0*} = \lambda q_h$. Therefore, we have $\bar{q}^{E*} \geq \bar{q}^{0*}$ because $\lambda \leq 1$. Note

that Π^{0*} and Π^{E*} have the same numerator. The difference between the denominators of Π^{0*} and Π^{E*} is

$$\lambda\xi_1^2 - \xi^2 = (1-\lambda)(\lambda(\beta q_h)^2 - (\alpha_d + \alpha_c - \eta)^2).$$

Thus, when $\lambda < (\alpha_d + \alpha_c - \eta)^2/(\beta q_h)^2$, the above difference is negative, which implies that $\Pi^{E*} < \Pi^{0*}$. Use the same logic, we can prove that if $\lambda < (\alpha_d + \alpha_c - \eta)^2/(\beta q_h)^2$, then $m^{E*} < m^{0*}$.

(2) Consider the case where $\lambda > (\alpha_d - \alpha_c + \eta)/(\beta q_h)$ and $\lambda < (\alpha_d + \alpha_c - \eta)^2/(\beta q_h)^2$. Under such conditions, according to lemma 1 and part (3) of Proposition 1, $p_d^{0*} = 0$, $p_d^{E*} = 0$, and according to part (1) of Proposition 1 $m^{E*} < m^{0*}$. Since $U_h^{0*}(\theta_i) = \alpha_d m^{0*} - p_d^{0*} - \theta_i$ and $U_h^{E*}(\theta_i) = \alpha_d m^{E*} - p_d^{E*} - \theta_i$, we have $U_h^{E*}(\theta_i) < U_h^{0*}(\theta_i)$. As a result, for some high quality developers, it is possible that $U_h^{0*}(\theta_i) > 0$ and $U_h^{E*}(\theta_i) < 0$, leading to $n^{E*} < n_h^{0*}$.

(3) Note that

$$p_d^{0*} - p_d^{E*} = \frac{(1-\lambda)(\alpha_d - \alpha_c + \eta - \beta q_h)(\alpha_c + \alpha_d - \eta)^2 + \beta(1-\lambda)q_h(4\theta_d\theta_c - \lambda\xi_1^2)}{(4\theta_d\theta_c - \lambda\xi_1^2)(4\theta_d\theta_c - \xi^2)}.$$

When $\alpha_d - \alpha_c + \eta - \beta q_h > 0$, because $4\theta_d\theta_c - \lambda\xi_1^2 > 0$, we have $p_d^{0*} > p_d^{E*}$. When $\alpha_d - \alpha_c + \eta - \beta q_h \le 0$, we have $p_d^{E*} = 0$, and therefore $p_d^{0*} \ge p_d^{E*}$. When $\Pi^{E*} > \Pi^{0*}$, it implies $\lambda\xi_1^2 > \xi^2$. Combining $\lambda\xi_1 < \xi$ with $\lambda\xi_1^2 > \xi^2$, we see that $p_c^{E*} > p_c^{0*}$.

(4) The conditions for free access can be obtained by solving $p_d^{E*} = 0$ or $p_c^{E*} = 0$. Comparing the conditions to the free access conditions in the benchmark model in Lemma 1(2), we can see that $\alpha_d + \eta \le \alpha_c + \beta q_h$ can be satisfied more easily than $\alpha_d + \eta \le \beta\lambda q_h + \alpha_c$, which implies that the platform is more likely to offer free access to developers under exclusion than in the benchmark; Because $\xi \ge \xi_1\lambda$, it can be deduced that $\xi_1\lambda(\alpha_d - \eta) \ge 2\theta_d\theta_c$ is more difficult to be satisfied than $\xi(\alpha_d - \eta) \ge 2\theta_d\theta_c$, which implies that the platform is less likely to offer free access to consumers under exclusion.

□

Proof of Proposition 2

(1) See Appendix 1(c).

(2) Differentiating p_d^{S*} and γ^* with respect to β, we obtain

$$\frac{d\gamma^*}{d\beta} = \frac{\theta_d w q_h(4\theta_d\theta_c - \xi^2 - \lambda(1-\lambda)\beta^2 q_h^2 + 2\lambda\beta q_h(\alpha_c + \alpha_d - \eta + \beta q_h))}{(4\theta_d\theta_c - \xi^2 - \tau)^2} > 0$$

and

$$\frac{dp_d^{S*}}{d\beta} = \frac{2\theta_d w\lambda\beta q_h(\alpha_d - \alpha_c + \eta)(\alpha_c + \alpha_d - \eta + \beta q_h)}{(4\theta_d\theta_c - \xi^2 - \tau)^2} > 0.$$

Note that

$$p_{dh}^{S*} = p_d^{S*} - \gamma^* = \frac{\theta_d w(\alpha_d - \alpha_c + \eta - \beta q_h)}{4\theta_d\theta_c - \xi^2 - \tau}. \qquad \text{(A18)}$$

Differentiating p_{dh}^{S*} with respect to β, we get

$$\frac{dp_{dh}^{S*}}{d\beta} = \frac{\theta_d w q_h G(\beta)}{(4\theta_d\theta_c - \xi^2 - \tau)^2},$$

where $G(\beta) = 2\lambda(\xi + (1-\lambda)\beta q_h)(\alpha_d - \alpha_c + \eta - \beta q_h) - (4\theta_d\theta_c - \xi^2 - \tau)$. Note that the sign of $\frac{dp_{dh}^{S*}}{d\beta}$ is the same as the sign of $G(\beta)$. We now show $G(\beta) < 0$.

When $\beta = (\alpha_d - \alpha_c + \eta)/q_h$, we have

$$G\left(\frac{\alpha_d - \alpha_c + \eta}{q_h}\right) = -(4\theta_d\theta_c - \xi^2 - \tau) < 0.$$

When $\beta > (\alpha_d - \alpha_c + \eta)/q_h$, the first term in $G(\beta)$ becomes negative so that $G(\beta) < 0$. When $\beta < (\alpha_d - \alpha_c + \eta)/q_h$, differentiating $G(\beta)$, we have $G'(\beta) = 2q_h\lambda(1 + 2(1-\lambda))(\alpha_d - \alpha_c + \eta - \beta q_h) > 0$, which implies $G(\beta)$ is increasing in β. Thus, if $\beta < (\alpha_d - \alpha_c + \eta)/q_h$, then $G(\beta) < G((\alpha_d - \alpha_c + \eta)/q_h) < 0$. Therefore, $G(\beta)$ is always negative, which implies $\frac{dp_{dh}^{S*}}{d\beta} < 0$.

(3) From p_{dh}^{S*} expression, it follows directly that if $\beta > (\alpha_d - \alpha_c + \eta)/q_h$, then $p_{dh}^{S*} < 0$.

□

Proof of Proposition 3

(1) Because $\tau \geq 0$, it is straightforward to see that $\Pi^{S*} \geq \Pi^{0*}$ and $m^{S*} \geq m^{0*}$. We have $\bar{q}^{S*} \geq \bar{q}^{0*}$ because $\lambda(1-\lambda)\beta q_h^2/\xi \geq 0$.

(2) We have

$$n_h^{0*} = \lambda n^{0*} = \frac{\lambda w \xi}{4\theta_d\theta_c - \xi^2},$$

and

$$n_l^{0*} = (1-\lambda)n^{0*} = \frac{(1-\lambda)w\xi}{4\theta_d\theta_c - \xi^2}.$$

Comparing to (A9), clearly $n_h^{S*} > n_h^{0*}$, because $\tau \geq 0$ and $\lambda\beta q_h \geq 0$. Using (A10), we have

$$n_l^{0*} - n_l^{S*} = \frac{(1-\lambda)w\xi}{4\theta_d\theta_c - \xi^2} - \frac{w(1-\lambda)(\alpha_d + \alpha_c - \eta)}{4\theta_d\theta_c - \xi^2 - \tau}$$
$$= \frac{w(1-\lambda)\lambda\beta q_h(4\theta_d\theta_c - \xi(1-\lambda)\beta q_h + \xi^2)}{(4\theta_d\theta_c - \xi^2)(4\theta_d\theta_c - \xi^2 - \tau)}.$$

If $4\theta_d\theta_c < \xi(1-\lambda)\beta q_h - \xi^2$ holds, we have $n_l^{0*} - n_l^{S*} < 0$, or $n_l^{S*} > n_l^{0*}$.

(3) Because $\tau \geq 0$, it is clear that $p_c^{0*} < p_c^{S*}$ and $p_d^{0*} < p_d^{S*}$. Using (A1) and (A18), we obtain

$$p_{dh}^{S*} - p_d^{0*} = \frac{\theta_d w((2\alpha_d - \xi)\lambda\beta q_h + \xi^2 - 4\theta_d\theta_c)}{(4\theta_d\theta_c - \xi^2 - \tau)(4\theta_d\theta_c - \xi^2)}.$$

Thus, if $4\theta_d\theta_c < (2\alpha_d - \xi)\lambda\beta q_h + \xi^2$, then $p_{dh}^{S*} - p_d^{0*} > 0$, or $p_{dh}^{S*} > p_d^{0*}$.

(4) From the expression of p_d^{S*} in Appendix 1(c), it follows directly that if $\alpha_d + \eta \leq \alpha_c$, then $p_d^{S*} = 0$. The corresponding condition in the benchmark model for $p_d^{0*} = 0$ is $\alpha_d + \eta \leq \alpha_c + \beta\lambda q_h$. Therefore, the condition $\alpha_d + \eta \leq \alpha_c$ under subsidy is less likely to hold than the condition under the benchmark $\alpha_d + \eta \leq \alpha_c + \beta\lambda q_h$, because $\beta\lambda q_h \geq 0$.

□

Proof of Proposition 4

(1) The optimal x^* is derived in Appendix 1(d). Clearly, if $\alpha_c + \beta q_h \leq \eta$, then $x^* \leq 0$. Because the number of application is constrained to be non-negative, if $\alpha_c + \beta q_h \leq \eta$, it implies that $x^* = 0$.

(2) Differentiating x^* with respect to λ, we have

$$\frac{dx^*}{d\lambda} = \frac{2\xi\beta q_h\theta_d w(\alpha_c + \beta q_h - \eta)}{k(4\theta_d\theta_c - \xi^2 - \delta)^2}.$$

When $x^* > 0$, it is true that $\alpha_c + \beta q_h - \eta \geq 0$, which implies $\frac{dx^*}{d\lambda} \geq 0$.

Substituting n^{F*} and x^* from Appendix 1(d), we get

$$\frac{x^*}{n^{F*}} = \frac{\theta_d(\alpha_c + \beta q_h - \eta)}{k\xi}.$$

Because $\xi = \alpha_d + \alpha_c + \beta\lambda q_h - \eta \geq 0$, it follows that $\frac{x^*}{n^{F*}}$ is decreasing in λ.

(3) Differentiating x^* with respect to β, we have

$$\frac{dx^*}{d\beta} = \frac{q_h\theta_d w(2\lambda\xi(\alpha_c + \beta q_h - \eta) + 4\theta_d\theta_c - \xi^2 - \delta)}{k(4\theta_d\theta_c - \xi^2 - \delta)^2} \geq 0.$$

Differentiating x^*/n^{F*} with respect to β, we have

$$\frac{d(x^*/n^{F*})}{d\beta} = \frac{\theta_d q_h((1-\lambda)(\alpha_c + \alpha_d - \eta) + \lambda\alpha_d)}{k\xi^2} \geq 0.$$

Therefore, both x^* and $x^*/n^{F*}$are increasing in β.

□

Proof of Proposition 5

(1) Using (A2) and (A17), we have

$$\bar{q}^{F*} - \bar{q}^{0*} = \frac{\lambda q_h\theta_d(\alpha_c + \beta q_h - \eta)}{k\xi + \theta_d(\alpha_c + \beta q_h - \eta)}.$$

If $\alpha_c + \beta q_h - \eta > 0$, then $\bar{q}^{F*} > \bar{q}^{*0}$. If $\alpha_c + \beta q_h - \eta \leq 0$, then $x^* = 0$ so that $\bar{q}^{F*} = \bar{q}^{0*}$. Therefore, $\bar{q}^{F*} \geq \bar{q}^{0*}$.

From the expressions of Π^{F*} and Π^{0*} in Appendix 1, it is clear that $\Pi_F^* > \Pi_0^*$ because $\delta > 0$. Similarly, we can see that $m^{F*} > m^{0*}, n^{F*} > n^{0*}, p_c^{F*} > p_c^{0*}$, and $p_d^{F*} > p_d^{0*}$.

(2) Directly follows from p_c^{F*} in Appendix 1(d), and the comparison between the conditions for free access to consumers in the benchmark model and under first-party application using the fact $\theta_d\eta(\alpha_c + \beta q_h - \eta)/kw \geq 0$.

□

Proof of Proposition 6

(1) From the expressions of Π^{E*} and Π^{S*} in Appendix 1, we can see that they have the same numerator. Because $\xi^2+\tau-\lambda\xi_1^2=(1-\lambda)(\alpha_d+\alpha_c-\eta)^2\geq 0$, the denominator of Π^{E*} is larger than that of Π^{S*}. Therefore, $\Pi^{S*}\geq\Pi^{E*}$.

For a given subsidy γ, the optimal profit of the platform can be written as

$$\Pi^{S*}(\gamma)=\frac{\theta_d\left(w\theta_d+\frac{\beta\gamma\lambda(1-\lambda)q_h}{\theta_d}\right)^2}{4\theta_d\theta_c-\xi^2}-\frac{\lambda(1-\lambda)\gamma^2}{\theta_d}.$$

We compare optimal profit under subsidy γ, $\Pi_S^*(\gamma)$ and optimal profit under exclusion Π_E^*

$$\Pi^{S*}(\gamma)-\Pi^{E*}=\frac{\theta_d\left(w\theta_d+\frac{\beta\gamma\lambda(1-\lambda)q_h}{\theta_d}\right)^2}{4\theta_d\theta_c-\xi^2}-\frac{\lambda(1-\lambda)\gamma^2}{\theta_d}-\Pi^{E*}.$$

Substituting Π^{E*}, and simplifying, we get

$$\Pi^{S*}(\gamma)-\Pi^{E*}=-\frac{\lambda(1-\lambda)(4\theta_d\theta_c-\xi^2-\lambda(1-\lambda)\beta^2q_h^2)}{\theta_d(4\theta_d\theta_c-\xi^2)}\gamma^2+\frac{2w\beta\lambda(1-\lambda)q_h}{4\theta_d\theta_c-\xi^2}\gamma$$

$$+\frac{\theta_d w^2}{4\theta_d\theta_c-\xi^2}-\frac{\theta_d w^2}{4\theta_d\theta_c-\lambda\xi_1^2}.$$

If $4\theta_d\theta_c-\lambda\xi_1^2>(\lambda(1-\lambda)^2)/\theta_d^2$, the equation $\Pi^{S*}(\gamma)-\Pi^{E*}=0$ has two positive roots; If $4\theta_d\theta_c-\lambda\xi_1^2\leq(\lambda(1-\lambda)^2)/\theta_d^2$, it has only one positive root. Therefore, for any given Π^{E*}, there always exists at least one γ to satisfy $\Pi^{S*}(\gamma)=\Pi^{E*}$.

(2) From (A5) and (A12), we have

$$\bar{q}^{E*}-\bar{q}^{S*}=\frac{(1-\lambda)\beta q_h(\alpha_d+\alpha_c-\eta)}{\xi}\geq 0,$$

which implies $\bar{q}^{E*}\geq\bar{q}^{S*}$.

Because $\xi^2+\tau-\lambda\xi_1^2=(1-\lambda)(\alpha_d+\alpha_c-\eta)^2\geq 0$, we see that $m^{S*}>m^{E*}$, $n_h^{S*}>n^{E*}$, $p_d^{S*}>p_d^{E*}$, and $p_c^{S*}>p_c^{E*}$.

□

Proof of Proposition 7

Π^{S*} and Π^{F*} have the same numerator. The difference between their denominators is

$$\tau - \delta = \lambda(1-\lambda)\beta^2 q_h^2 - \frac{\theta_d(\alpha_c + \beta q_h - \eta)^2}{k},$$

which is concave and quadratic in λ.

(1) When $k < 4\theta_d(\alpha_c + \beta q_h)^2/(\beta^2 q_h^2)$, the quadratic equation $\tau - \delta = 0$ has no root. Because $\tau - \delta$ is concave, we have $\tau - \delta < 0$, which implies $\Pi^{S*} < \Pi^{F*}$.

(2) When $k \geq 4\theta_d(\alpha_c + \beta q_h)^2/(\beta^2 q_h^2)$, the quadratic equation $\tau - \delta = 0$ has two roots $\underline{\lambda}$ and $\overline{\lambda}$ ($0<\underline{\lambda}<\overline{\lambda}<1$) as $\underline{\lambda} = 1/2 - \sqrt{1/4 - \delta/\beta^2 q_h^2}$ and $\overline{\lambda} = 1/2 + \sqrt{1/4 - \delta/\beta^2 q_h^2}$. Therefore, when $\underline{\lambda} < \lambda < \overline{\lambda}$, we have $\tau - \delta > 0$, which implies $\Pi^{S*} > \Pi^{F*}$. When $0 < \lambda < \underline{\lambda}$ or $\overline{\lambda} < \lambda < 1$, we have $\tau - \delta < 0$, which implies $\Pi^{S*} < \Pi^{F*}$.

□

Proof of Proposition 8

(1) If subsidy is optimal, i.e., $\Pi^{S*} > \Pi^{F*}$, it implies that $\tau > \delta$. When $\tau > \delta$, we can observe that $m^{S*} > m^{F*}$and $n^{S*} > n^{F*}$. If first-party application is optimal, i.e., $\Pi^{F*} > \Pi^{S*}$, it implies that $\tau < \delta$. When $\tau < \delta$, we can observe that $m^{F*} > m^{S*}$and $n^{F*} > n^{S*}$.

From (A12) and (A17), we have

$$\bar{q}^{F*} - \bar{q}^{S*} = \frac{(1-\lambda)(\theta_d(\alpha_c + \beta q_h - \eta)(\alpha_c + \alpha_d - \eta) - \lambda\beta q_h k\xi)}{\left(k\xi + \theta_d(\alpha_c + \beta q_h - \eta)\right)\xi}.$$

Under $\tau > \delta$ condition (which implies subsidy is optimal), when $k < (\theta_d(\alpha_c + \beta q_h - \eta)(\alpha_c + \alpha_d - \eta))/\lambda\beta q_h \xi$, we have $\bar{q}^{F*} > \bar{q}^{S*}$; under $\tau < \delta$ condition (which implies first-party application is optimal), when $k > (\theta_d(\alpha_c + \beta q_h - \eta)(\alpha_c + \alpha_d - \eta))/\lambda\beta q_h \xi$, we have $\bar{q}^{F*} < \bar{q}^{S*}$. Therefore, the optimal strategy does not necessarily lead to higher average quality.

(2) When $\Pi^{S*} = \Pi^{F*}$ (which implies $\tau = \delta$), it is obvious that $p_d^{S*} > p_d^{F*}, p_{dh}^{S*} < p_d^{F*}, p_c^{F*} > p_c^{S*}$.

(3) The utilities of the two types of developers under subsidy are $U_l^S(\theta_i) = \alpha_d m - p_d - \theta_i$ and $U_h^S(\theta_i) = \alpha_d m + \gamma - p_d - \theta_i$, respectively, and under first-party application are $U_h^F(\theta_i) = U_l^F(\theta_i) = \alpha_d m - p_d - \theta_i$. Substituting $m = (w + (\alpha_c + \beta\lambda q_h)n - p_c)/\theta_c$ and $p_d =$

$\alpha_d m - n\theta_d$, we have $U_h^S(\theta_i) = n^{S*}\theta_d + \gamma(1-\lambda) - \theta_i$, $U_l^S(\theta_i) = n^{S*}\theta_d - \lambda\gamma - \theta_i$, and $U_h^F(\theta_i) = U_l^F(\theta_i) = n^{F*}\theta_d - \theta_i$. When $\Pi^{S*} = \Pi^{F*}$ (which implies $\tau = \delta$), we can see that $n^{S*} = n^{F*}$. Therefore, we have $U_l^F(\theta_i) > U_l^S(\theta_i)$ and $U_h^S(\theta_i) > U_h^F(\theta_i)$.

The consumer's utilities under subsidy is $V^S(\theta_j) = w + (\alpha_c + \beta\bar{q}^S)n - p_c - \theta_j$. Substitute (A11), $n^S = (\alpha_d m - p_d)/\theta_d + \gamma\lambda/\theta_d$ and $p_c^S = w + (\alpha_c + \beta\bar{q}^S)n - m\theta_c$ into $V^S(\theta_j)$, we have $V^{S*}(\theta_j) = m^{S*}\theta_c - \theta_j$. The consumer's utilities under first-party applications is $V(\theta_j) = w + (\alpha_c + \beta\bar{q}^F)(n + x) - p^c - \theta_j$. Substitute (A16), $n^F = (\alpha_d m - p_d)/\theta_d$ and $p_c^F = w + (\alpha_c + \beta\lambda q_h)n + (\alpha_c + \beta q_h)x - m\theta_c$ into $V^F(\theta_j)$, we have $V^{F*}(\theta_j) = m^{F*}\theta_c - \theta_j$. When $\Pi^{S*} = \Pi^{F*}$, we have $m^{S*} = m^{F*}$. Therefore, $V^{S*}(\theta_j) = V^{F*}(\theta_j)$.

□

Proof of Proposition 9

(1) Because $4\theta_d\theta_c - \xi^2 - \lambda(1-\lambda)\beta^2 q_h^2 \leq 4\theta_d\theta_c - \xi^2$, it is clear that $W^{0*} \leq W^{S*}$.

Comparing exclusion and the benchmark, we have

$$W^{E*} - W^{0*} = \frac{8\theta_d\theta_c w(8\theta_d\theta_c - 2\xi_1^2) - \left(\theta_c + \frac{1}{2}\xi_1\right)(4\theta_d\theta_c - \xi_1^2)(4\theta_d\theta_c - \lambda\xi_1^2)}{(4\theta_d\theta_c - \xi^2)^2(4\theta_D\theta_C - \lambda\xi_1^2)^2}.$$

We can see that if $8\theta_d\theta_c w(8\theta_d\theta_c - 2\xi_1^2) > (or\ \leq)\left(\theta_c + \frac{1}{2}\xi_1\right)(4\theta_d\theta_c - \xi_1^2)(4\theta_d\theta_c - \lambda\xi_1^2)$, then $W^{E*} > (or\ \leq) W^{0*}$.

Similarly, comparing first-party application and the benchmark, we can show that if $(12\theta_d\theta_c + \xi^2)/\delta - 8\theta_d\theta_c/(4\theta_d\theta_c - \xi^2) > (or\ \leq)\left(\theta_c + \frac{\xi}{2}\right)(4\theta_d\theta_c - \xi^2 - \delta)$, then $W^{F*} > (or\ \leq) W^{0*}$.

(2) When $\Pi^{S*} = \Pi^{F*}$, it implies $\delta = \tau$. From W_S^* and W^{F*} expressions, we can see that if $\delta = \tau$, $W^{F*} < W^{S*}$.

□