Best or Right? - Positioning and Authentication in Online Matching Platforms

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Abstract

A firm seeking a business partner, or an individual searching for a life partner, can use an online matching platform not only to efficiently search for available candidates, but also to address two related challenges. First, a match-seeker may not know what candidates would be compatible with them. And second, particularly in the online setting, candidates may misrepresent their credentials. In this paper, we model and analyze whether an online matching platform's decisions should enhance search with a *positioning* capability that helps match-seekers determine the subjective compatibility of potential matches (horizontal differentiation), and also whether it should offer an *authentication* service that enables match-seekers to reliably signal their objective quality (vertical differentiation). We analyze the equilibrium behavior of match-seekers in the presence of uncertainty about both compatibility and quality of potential matches, and show how this behavior impacts the optimal strategy of the platform with respect to positioning and authentication. For instance, positioning and authentication reinforce each other (act as complements) for some levels of market quality and the platform's positioning capability, while they detract from each other (act as substitutes) in others. These results also help us develop guidelines for the platform's pricing decisions. Our findings provide valuable practical insights for owners and operators of match-making platforms, by helping them understand the interplay between these two important and orthogonal features in online matching.

Keywords: matching platforms, subjective preferences, authentication, online markets, game theory

1 Introduction

A firm seeking a business partner, or an individual seeking a life partner, faces two kinds of uncertainty about potential matches. First, the match-seeker (firm or individual) may not be sure what subjective criteria they should focus on. Second, the match-seeker may be unsure of the objective quality of each potential match. As a result, it may be difficult for the match-seeker to obtain a suitable match, and may require them to consider multiple candidates over time. For instance, while the firm wants a high-quality supplier, it may start out looking for a supplier that offers a low price and may not realize that in its specific circumstance it may be more important to find a supplier that meets relevant scheduling constraints. Match-seekers can thus benefit from resources to not only improve the efficiency of their search, but also mitigate these uncertainties.

Online matching platforms have emerged as a useful resource for match-seekers in both the consumer and business space. Such platforms can aggregate demand and enable match-seekers to find available candidates more efficiently. A number of matching platforms recognize the importance of match-seekers' subjective preferences, and enhance search with features such as questionnaires and algorithms that help a match-seeker better focus their search on the "right" type of candidates. For instance, in the business-to-business or industrial space, matching platforms such as Catalant and Powerlinx[1] help companies determine the type of partners they should seek, and then connect them to high-quality business partners of that type across the World. Even in the consumer context, matching platforms such as eharmony and match.com help match-seekers better *position* their search for partners.

When the matching process is conducted online, there is an additional *authentication* problem that becomes significant due to the informational asymmetry inherent in an anonymous online environment. The virtual nature of the online environment makes it difficult to ensure that participating entities are being truthful in their online representation. So it is easy for companies or individuals to misrepresent their attributes and credentials. The significance of this problem has been widely recognized, and a variety of online authentication mechanisms have been described in the literature (Kalakota and Whinston, 1996; Basu and Muylle, 2003; Basu et al., 2019). Online matching platforms can help match-seekers find the "best" candidates by offering authentication services that ensure that the information provided by participants on the platform is genuine and reliable. A

[1] Powerlinx provides a feature to help customers identify their business needs (https://www.powerlinx.com/pricing).

number of matching platforms in both business and consumer markets offer authentication services as an option. For instance, Upwork lets job-seekers authenticate their credentials from a list of approved certifications and display a 'verified badge'.[2] Upwork also verifies employers, by validating them against third party databases, checking government or legal documents.[3] In the consumer space, the eharmony platform offers match-seekers a premium service called relyID[4] to authenticate their features and credentials. It places a badge next to authenticated profiles, to increase their perceived value to potential matches.

When match-seekers can differ along both objective and subjective dimensions, an online matching platform can enhance its search capabilities with a positioning feature to determine the compatibility of potential matches, as well as offering an authentication service. However, it is not always obvious whether the platform should support each or even both. For instance, the matching platforms eharmony and match.com are both used by individuals to search for life partners. However, they have made different choices regarding support for positioning and authentication. Match.com provides enhanced search that includes positioning but offers very little authentication, while eharmony provides a proprietary algorithm to help match-seekers position their search, as well as an optional authentication service (relyID) to verify credentials.

In this paper, we model and analyze an online matching platform's decisions regarding positioning and authentication, in a setting where match-seekers' preferences vary across both objective and subjective dimensions. In addition, we examine how the optimal fees charged by the platform are impacted by key market parameters and the platform's positioning capability. We show that when the platform's positioning capability is modest, an increase in this capability increases the value of authentication (i.e., positioning *complements* authentication). However, when the platform's positioning capability is high, the value of authentication becomes lower as the positioning capability increases (i.e., positioning and authentication become *substitutes*); that is, narrowing the search to only high-value candidates through authentication can lead to a restricted choice set that excludes some compatible candidates. Furthermore, this interaction between authentication and positioning is also affected by the quality of the market. Thus, despite the orthogonal dimensions of positioning and authentication, we find that both are not always beneficial for the platform.

[2] Upwork also has a list of verifiable certificates that it encourages job-seekers to pursue.(https://support.upwork.com/hc/en-us/articles/215650138-Add-Certifications)

[3] https://community.upwork.com/t5/Freelancers/pointless-re-verify/m-p/660859

[4] https://www.eharmony.com/dating-advice/using-eharmony/new-feature-relyid/

Our work contributes to the field of online matching in important ways. To begin with, our analytical models and game-theoretic analysis of online matching platforms in the presence of both vertically and horizontally differentiated preferences are novel in the field. We show how online matching can be represented by a parsimonious model that captures the key features driving the behavior of match-seekers, as well as key decisions of platform operators with regard to positioning and authentication. The results of our analysis provide new insights into the interactions of two orthogonal dimensions of match-seeker differentiation, and into profit-maximizing strategies of platform operators under various market conditions. And in terms of managerial impact, our results provide valuable practical insights for owners and operators of online match-making platforms such as Powerlinx and eharmony, by helping them understand the interplay between positioning and authentication, and by providing guidelines for pricing decisions.

The paper is organized as follows. We start by reviewing the relevant literature in section 2. In section 3, we describe our setup and explain the equilibrium concept and approach. We then lay out our models for each setting in Sections 4 and 5, and analyze the behavior of match-seekers in each of them. Then in Section 6, we identify and characterize the optimal strategies of an online matching platform. Finally, in Section 7 we conclude with key observations and directions for future work.

2 Relevant Literature

Much of the past research in bilateral matching markets has been on two key areas: 1. the design of matching mechanisms, including issues related to pricing and market coverage and 2. the structural features of matching platforms (Roth and Sotomayor, 1992; Adachi, 2003). Our research builds upon this literature by studying the interaction of two services offered by an online platform – positioning and authentication of prospective matches.

The literature on matching mechanisms (Gale and Shapley, 1962; Shapley and Shubik, 1971; Roth, 1989) focuses on designing efficient algorithms to provide stable matches. Match-seekers' behavior have been explored in markets where consumers have strong class (quality) preferences and such preferences are similar across all match-seekers (McAfee, 2002; Damiano and Li, 2007; Damiano and Hao, 2008). A number of papers have also examined how class preferences among match-seekers can lead to different equilibria (McNamara and Collins, 1990; Chade, 2001; Smith, 2006).

The traditional matching literature (e.g.,Burdett and Coles 1997; Smith 2006) assumes that in the absence of a match-making intermediary, match-seekers find compatible matches through a random process. Boudreau and Knoblauch (2010) impose an additional restriction on the timing of match-seekers' choices and use simulation to study markets where only certain match-seekers can make proposals. By allowing match-seekers to create a sub-group of potential matches, Jacquet and Tan (2007) show that assortative matching can be an equilibrium outcome. A related stream of literature in "club theory" that followed the work of Buchanan (1965) focuses narrowly on the role of externalites on the formation of such sub-groups. Platforms that facilitate efficient search and matching take into consideration match-seekers' class preferences (Cosimano, 1996; Burdett and Coles, 1997; Damiano and Li, 2007), in their pricing decisions. The advent of Internet has allowed for increased search efficiency as a result of which online matching platforms can facilitate match-seekers to target their types and narrow their search space (Xu and Yang, 2019).

While the Internet enables a faster matching service, it also creates authentication challenges where a low-class match-seeker may imitate its high-class counterpart. By investing in authentication services (Ghani et al., 2014), a firm can mitigate such problems. Basu et al. (2019) show that an online matching platform may offer an authentication service as a loss leader when optimally pricing both search and authentication, and a superior authentication service may not justify higher authentication fees. Match-seekers may have not only commonly accepted preferences along the vertical dimension of attributes but also taste preferences along a horizontal dimension that allow them to value potential matches differently (Buss and Barnes, 1986; Boyd et al., 2003; Figueredo et al., 2006). In such settings, (Halaburda et al., 2018) show that restricting the choice set of match-seekers can be beneficial for a platform.

Several other papers have also considered the impact of subjective preferences on matching outcomes. Hitsch et al. (2010) empirically show that the impact of certain attributes (e.g., income, physical attributes) of match-seekers in an online dating market differs by gender while being similar across other dimensions including race. Such similarities in match-seeker preferences could also be motivated by strong societal biases (Banerjee et al., 2013). Other researchers have shown that match-seekers sequentially searching across different attributes can use the same attribute in both horizontal and vertical dimensions (Bruch et al., 2016). Gomes and Pavan (2015) find that a matching platform may find it optimal to induce negatively assortative matches in the vertical dimension as long as such matches are compatible along the horizontal dimension.Similar issues have also been considered in a number of studies on information disclosure and information sharing in settings that do not involve bilateral matching (Gu and Xie, 2013; Hao and Tan, 2019; Sun and Tyagi, 2020). As shown by Arora et al. (2020), such advisory activities by a service provider are valuable in non-profit settings as well.

In this paper, we explore a matching market where match-seekers have preferences over both vertical and horizontal dimensions. We consider the possibility that match-seekers have uncertainty about their subjective preferences which may result in matches with incompatible candidates. Such uncertainties may be alleviated if the matching platform includes a positioning feature that helps match-seekers understand their preferences in the matching process. Further, since a higher quality candidate is strictly preferred by all match-seekers, authentication also becomes an important issue in such settings. However, there is little research on online platforms that offer both positioning and authentication, even though many existing platforms (e.g., eharmony, Powerlinx, etc.) provide them. Our formal analysis of pricing and features offered by such platforms addresses this gap in the field.

3 Model and Analysis

Our modeling approach is consistent with and builds upon the approach in Basu et al. (2019), which examines online matching among match-seekers differentiated only in terms of a vertical quality dimension. As in that paper, we consider a two-period setting in which match-seekers (firms or individuals) seek matches with other match-seekers within a given population, since the two-period setting allows simpler exposition and analytical tractability while being sufficient to capture the temporal element associated with the matching process.

We also adopt the approach of Basu et al. (2019) in modeling the search process undertaken during online matching. That is, for every active match-seeker, we assume that one potential candidate from the overall population (who is also an active match-seeker) becomes available in each period. If the two parties detect each other, they can consider a match. If the match is not accepted by either party, the match-seeker waits for a candidate that becomes available in the following period. The population from which candidates become available is assumed to be sufficiently large that a match-seeker does not encounter the same candidate in both periods. To account for the fact that each match-seeker would like to find a match as soon as possible, the value of a match achieved in the second period is discounted by a factor δ ($0 < \delta \leq 1$) for any match-seeker.[5] This discount factor δ can be interpreted as the level of patience of the match-seekers in the market. Also, δ is assumed to be sufficiently high to ensure that the second period is a viable option for all match-seekers.

Match-seekers can search in one of two modes - either directly (offline), or through an online matching platform.[6] As in Basu et al. (2019), we model the efficiency of search as the probability that a match-seeker detects the available candidate in any period and denote it using the parameter μ (where $\mu \leq 1$). In *direct search*, match-seekers seek out and propose to potential matches, who can then accept or reject the available candidates. However, direct search is inefficient, in that the match-seeker may not always detect the available candidate (i.e., $\mu < 1$). On the other hand, the online platform enables efficient search by ensuring that the match-seeker always detects the available candidate (i.e., $\mu = 1$). We assume that match-seekers who use the online platform have to pay the platform an access fee p (Angerer et al., 2018).

We allow match-seekers to have both subjective and objective preferences. As an example, consider a placement search setting in which firms are looking for new professional hires for a specific role, from a pool of two types of candidates, specialists and generalists. Furthermore, both candidates and roles can be of either high or low quality. While both firms and candidates would prefer high-class to low-class matches, both the candidates and the firms may be uncertain about the type of match they need. Thus, the matching process involves consideration of both dimensions, which we define as follows:

1. **Type:** This is a horizontally differentiated dimension in which there is no single ordering of preferences across all match-seekers. Match-seekers can be either compatible or incompatible with each other, and *a priori*, cannot assess this compatibility by themselves. In the context of the above example, if a firm is matched with a compatible candidate, they will obtain the value associated with the match. The same is true for the candidate with which they match. On the other hand, if a firm is matched with an incompatible candidate, they realize no value

[5] A detailed table of notations (Table 4) is provided in the appendix.

[6] We focus on online matching platforms because the dual problems of positioning and authentication are particularly significant in the online setting. Consistent with prior literature, we make the simplifying assumption that match-seekers do not simultaneously use direct search and the platform (Bloch and Ryder, 2000; Basu et al., 2019).

from that match. In our example, while each hiring firm would value a high-quality candidate more than a low quality one, it would not benefit from hiring a generalist for a role that needs a specialist, regardless of their quality. Similarly, while each candidate would like to accept a high-quality role, a generalist would not get any value from a specialist role, and vice versa.

We assume that for any match-seeker, the probability that an available candidate is compatible with them is 1/2. The compatibility of a match becomes evident to each match-seeker only after they have accepted a match and exited the platform. When a match-seeker matches with a compatible candidate, they would receive a value of either v_H or v_L depending on the class of the other match-seeker. However, if the match is incompatible, neither party would realize any value irrespective of their class.

2. **Class:** This is a vertically differentiated dimension in which there is a single ordering of preferences across all match-seekers. As in Basu et al. (2019), we simplify this dimension into two classes of match-seekers: *H*igh-class (H-class) match-seekers and *L*ow-class (L-class) match-seekers. In the context of the hiring example above, both candidates and positions can be objectively classified as being of high or low class. Consistent with the notion of a bilateral match, the value derived by a match-seeker from a match depends on the match-seeker they are matched with. Specifically, we assume that a match-seeker who matches with a compatible H-class (L-class) candidate receives a value of v_H (v_L), where $v_H > v_L$ (we use η to denote the ratio v_H/v_L). This implies that when an H-class match-seeker matches with a compatible H-class candidate both parties receive a value of v_H. In contrast, when the same H-class match-seeker is matched with a compatible L-class candidate, the value that the H-class[7] derives from the match is v_L while the L-class derives a value v_H. It follows that a match with a compatible H-class is preferred by all match-seekers. Let α ($0 \leq \alpha \leq 1$) be the proportion of H-class in the population. Thus the probability that any candidate that becomes available from the overall population is of H-class will be α, where α is an exogenous characteristic of the overall population.[8]

[7]For brevity, we use the term "H-class (L-class)" to refer to H-class (L-class) match-seekers and candidates.

[8]When both classes are in the population (i.e., $\alpha < 1$), a match-seeker will not be able to match with the available candidate if either the latter is not detected (e.g., in direct search), or if they choose a different search mode. For example, if only the H-class uses the platform, and if the available candidate in period 1 is an L-class, the H-class match-seeker would not be able to match with this L-class candidate since the candidate is not using the platform for their search.

Combining the two dimensions, we assume that a match-seeker would realize a value of v_H (v_L) from a match with a compatible H-class (L-class), and a value of 0 otherwise. Note that this value is realized only after the matched pair has exited the matching market. These utilities are assumed to be non-transferable, based on the notion that the value derived by each match-seeker is determined solely by the fixed value corresponding to their match.

We assume that irrespective of whether the search is direct or through the platform, each match-seeker is initially uncertain about their preferences, so the probability of finding a compatible match is 1/2 as stated earlier. To mitigate the risk of incompatible matches, the platform can enhance its search by providing a positioning feature. For example, positioning may be implemented using a questionnaire or algorithm that match-seekers can use (as in eharmony and Powerlinx). This feature provides the match-seeker with a signal on whether a match with the available candidate is compatible with them. The efficacy of the positioning feature is characterized by the parameter λ ($0 \leq \lambda \leq 1$), which we call the platform's *positioning capability*.[9] This capability is determined by available technologies, and the platform incurs a fixed cost c_P to develop it. The probability that the platform's signal of compatibility is indeed correct is $(1 + \lambda)/2$. Thus positioning enables the match-seekers to reduce their uncertainty about match compatibility.

Although the online platform overcomes the inefficiencies of direct search, authentication of match-seekers is harder in an online setting, where match-seekers can easily misrepresent their credentials and attributes. The matching platform can mitigate this problem by offering an authentication service. Since every match-seeker would want to be viewed as an H-class, the goal of this authentication process is to classify only true H-class match-seekers as being of that class. If the platform offers this service, match-seekers who want to authenticate themselves can purchase it for an additional fee q. It costs the platform c_A to authenticate each match-seeker. As mentioned earlier, we assume that this authentication problem does not arise in direct search.

The process by which candidates become available is a function of the population and is thus external to the platform. The platform does not increase the availability of candidates, but rather ensures that available candidates are detected by match-seekers. Positioning helps the match seeker determine whether the available candidate is compatible with them. And authentication enables the match-seeker to determine whether the available candidate in each period is what they claim

[9] We assume that λ represents the platform's capability to correctly identify both compatible and incompatible matches.

they are (H- or L-class).

As described previously, a match requires acceptance by both match-seekers. In the first period, match-seekers can choose to reject any L-class candidate (since both classes prefer H-class matches) or not.[10] If the match-seeker is willing to accept an L-class match in the first period, this reduces their expected value, but it increases the chances of finding an acceptable match. In our setup, the second period is the terminal period, so the match-seeker would accept any candidate that is available, as that is still no worse than exiting without a match. We assume that when match-seekers exit the market, they are replaced by identical match-seekers, thus keeping the distribution invariant over time.

Given the bilateral nature of the matching process, we use the equilibrium concept of fulfilled rational expectations to determine the equilibria in the participation of match-seekers to ensure that they do not regret their choices (Bloch and Ryder, 2000). Otherwise, there can be coordination failures due to unfulfilled expectations, and that might result in multiple equilibria. Specifically, for a given access fee p and authentication fee q, match-seekers' actions are based on rational expectations regarding the participation of other match-seekers (both H- and L-classes). In equilibrium, these rational expectations are fulfilled. Figure 1 shows the timing of events in the game. We also restrict our attention to pure strategy equilibria for the match-seekers, which implies that all match-seekers of a particular class would behave in the same way.

We start by considering the scenario in which the online matching platform provides positioning in Section 4, and analyze its impact on the behavior of match-seekers, as well as on the platform's strategic choices. Then in Section 5, we examine how adding authentication impacts the decisions of match-seekers and the platform. Finally, in Section 6, we analyze the optimal strategy of the platform and characterize the conditions under which it is optimal to provide either or both services. In particular, we seek to understand the interconnected roles of authentication and positioning in online matching platforms.

4 Online Matching with Positioning

Without the help of a matching platform, match-seekers conducting a direct search face two challenges. First, their search is potentially inefficient, and an available candidate would be detected

[10]Note that the L-class can match with an H-class only if that H-class is willing to accept a match from an L-class.

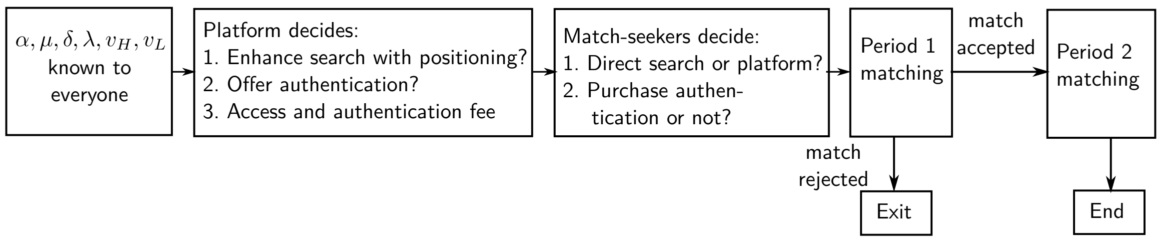


Figure 1: Timeline of events

with probability μ only, where $\mu < 1$. And second, they may not know the type of match they should be seeking, which can result in an ineffective search. In other words, there is a 50% likelihood that the candidate match ends up being incompatible.

We start by considering the setting in which the matching platform offers search enhanced with positioning, to address these challenges. First, as in every setting we consider, the platform enables efficient search, so that the match-seeker detects the available candidate in each period (whereas without the platform, the candidate would be detected with probability μ only). The candidate will be an H-class with probability α, and an L-class with probability $(1-\alpha)$. Second, the platform's positioning capability increases the probability of correctly identifying a compatible match to $(1+\lambda)/2$, where $0 \leq \lambda \leq 1$. Thus even when $\mu = 1$ (i.e. direct search is as efficient as the online platform), the online platform offers additional value due to its positioning capability.

4.1 Match-Seeker Behavior

In this setting, in order to determine the equilibrium match-seeker behavior, we have to consider several distinct scenarios, based on the choices made by match-seekers. First, both classes of match-seekers have to choose whether to use the online matching platform and pay the access fee p, or avoid the fee and conduct direct search. Second, match-seekers have to decide whether to accept the match in the first period. These scenarios are analyzed in the following subsections, followed by an analysis of the conditions under which each scenario occurs, and the optimal pricing strategy of the platform.

4.1.1 Both Classes Choose Direct Search

There are two possibilities to evaluate when both classes use direct search depending on whether match-seekers accept L-class candidates in the first period. Here, given α (the proportion of H-

class candidates in the population) and μ (the probability of detecting the available candidate), match-seekers face three types of uncertainty - first, they are uncertain about the compatibility of the available candidate (with probability 1/2); second, that candidate could be an L-class (with probability $1-\alpha$), and third, they may not detect the candidate (with probability $1-\mu$). In this setting, the expected value of a match-seeker from a compatible match is $\mu\left(\alpha v_H+(1-\alpha)\,v_L\right)$, and is 0 if the match turns out to be incompatible. If they do not find a match in the first period (because that candidate was not detected with probability $(1-\mu)$), they move to the second period, where they receive a value $\delta\mu\left(\alpha v_H+(1-\alpha)\,v_L\right)$ from a compatible match, and as before, 0 if the match turns out to be incompatible. Based on these, we can express the value function for match-seekers of both classes when they are willing to accept a match of any class in the first period as

$$
\begin{aligned}
V_{1H}=V_{1L} &= \frac{1}{2}\left[\mu\left(\alpha v_H+(1-\alpha)\,v_L\right)\right]+\frac{1}{2}(1-\mu)\left[\delta\mu\left(\alpha v_H+(1-\alpha)\,v_L\right)\right] \\
&= \frac{1}{2}\left(\delta(1-\mu)\,\mu+\mu\right)\left(\alpha v_H+(1-\alpha)\,v_L\right) \qquad (1)
\end{aligned}
$$

Now consider the case in which the H-class match-seeker accepts only H-class matches in the first period. In this case, their expected value is

$$
V_{2H} = \frac{1}{2}\left(\alpha\mu v_H+\delta\mu\,(1-\alpha\mu)\left(\alpha v_H+(1-\alpha)\,v_L\right)\right) \qquad (2)
$$

Comparing the equations 1 and 2 above, an H-class would reject L-class candidates in period 1 if $\eta={}^{v_H}\!/_{v_L}$ is sufficiently high i.e. $\eta>\eta_H=\frac{1-(1-\alpha)\delta\mu}{\alpha\delta\mu}$.

With respect to the expected value function of the L-class, if they choose to accept a match with L-class candidates in the first period, their value function will be

$$
V_{2La} = \frac{1}{2}\left((1-\alpha)\,\mu v_L+\delta\mu\alpha\,(1-\mu)\left(\alpha v_H+(1-\alpha)\,v_L\right)\right) \qquad (3)
$$

and if not, it will be

$$
V_{2Lb} = \frac{\delta\mu}{2}\left(\alpha v_H+(1-\alpha)\,v_L\right) \qquad (4)
$$

The above value functions incorporate the fact that an L-class is unable to match with the H-class in the first period because the H-class rejects all L-class candidates in that period. Comparing the equations 3 and 4, it can be shown that $V_{2La} < V_{2Lb}$ when $\eta > \eta_H$. So the optimal policy of the L-class is also to reject an L-class and wait for a second period match if the optimal policy of H-class was to reject an L-class candidate, consistent with Basu et al. (2019). So the value function of the L-class when the H-class match-seeker accepts only H-class matches in the first period will be

$$V_{2L} = \frac{\delta\mu}{2}\left(\alpha v_H + (1-\alpha)\, v_L\right) \tag{5}$$

Given this, the value function of a match-seeker of class $k \in [H, L]$ can be defined as follows:

$$V_k^{dd} = \begin{cases} V_{1k} & \text{when } \eta < \eta_H \\ V_{2k} & o.w. \end{cases} \tag{6}$$

where the superscripts refer to the fact that both classes are using direct search.

If either or both classes decide to use the platform, they have to decide whether to accept the available candidate in the first period. This depends upon whether both classes are using the online platform, and if so, whether the H-class match-seekers are willing to accept L-class matches or not in their search. It also depends on how well the match-seekers are positioned in their preferences.

4.1.2 Both Classes Use the Platform

Let us now consider the case in which both H and L-class match-seekers use the platform. Without any authentication, it would be optimal for the L-class to pretend to be H-class, so every candidate will present themselves as H-class. This implies that in this setting, a match-seeker can have no assurance of being successful in getting an H-class match.

However, the platform's positioning capability can help the match-seeker increase their chances of finding a compatible match. If the platform's positioning capability is λ, the platform's signal that a match with the available candidate is compatible will be correct with probability $(1+\lambda)/2$. For $\lambda > 0$, it is clearly optimal for the match-seeker to accept a match that the platform signals as compatible.

If however, the signal is that the match is incompatible, the match-seeker has to balance the decision to match with that candidate against the option of waiting until the second period and possibly obtaining a compatible match then. The optimal decision of the match-seeker in this situation thus depends on the comparison between the expected values from each choice, which we examine next.

With regard to the first option, with probability $(1-\lambda)/2$, a match that the platform signals as incompatible would turn out to be compatible with the match-seeker. Thus, if the match-seeker chooses to accept that match, their expected value is given by

$$v_{1PB} = \frac{(1-\lambda)}{2}\left(\alpha v_H + (1-\alpha)\, v_L\right) \tag{7}$$

As for the second option, it is optimal for the match-seeker to accept any candidate in period 2, since that is also the terminal period. Thus the expected value from this option is given by

$$v_{2PB} = \frac{\delta}{2}\left(\alpha v_H + (1-\alpha)\, v_L\right) \tag{8}$$

Comparing v_{1PB} and v_{2PB} in equations 7 and 8 respectively, gives us the optimal policy of a match-seeker as a function of λ, the positioning capability of the platform, as in the following proposition.

Proposition 1. *The optimal policy of the match-seeker is as follows:*
(i) If the candidate match is signaled as compatible, accept the match.
(ii) If the candidate match is signaled as incompatible, there exists a threshold λ_{PB} *such that if* $\lambda < \lambda_{PB}$*, accept the match. If not, wait until the second period to accept the match.*
Furthermore, λ_{PB} *is decreasing in* δ*.*

Proof. All proofs are provided in the Appendix. □

This proposition shows how the match-seeker's behavior is influenced by the platform's positioning capability. When λ is low, the match-seeker is better off accepting any match in the first period. However, when λ is sufficiently high, the match-seeker gains sufficient confidence in the platform's positioning capability to reject a match signaled as incompatible in the first period, in the hope of getting a better match in the second period. Furthermore, the willingness to wait till the second period increases with the value of δ.

Given the optimal policy described above in Proposition 1, the value functions of the match-seeker can be derived as follows:

In the first period, with probability 1/2 the available candidate will be compatible with the match-seeker. In the presence of positioning, with probability $(1+\lambda)/2$, the platform will correctly signal that this match is compatible. If the match-seeker wants to accept only matches that are signaled as compatible, the expected value from period 1 associated with this case is then $\frac{1}{2}\left[\left(\frac{1+\lambda}{2}\right)(\alpha v_H + (1-\alpha) v_L)\right]$. On the other hand, the match-seeker will reject the period 1 match and wait till the second period: first, if the available candidate in period 1 is compatible, but the platform signals that it is incompatible (with probability $(1-\lambda)/2$); and second, if the available candidate is indeed incompatible and this is correctly signaled by the platform (with probability $(1+\lambda)/2$). Combining these cases, the discounted expected value to the match-seeker from waiting till the second period is $\left(\frac{1}{2}\right)\left[\frac{\delta}{2}(\alpha v_H + (1-\alpha) v_L)\right]$. So the total expected value from both periods together of matching with only candidates signaled as compatible (in period 1) is given by

$$\begin{aligned} V_{1PB} &= \frac{1}{2}\left[\left(\frac{1+\lambda}{2}\right)(\alpha v_H + (1-\alpha) v_L)\right] + \frac{1}{2}\left[\frac{\delta}{2}(\alpha v_H + (1-\alpha) v_L)\right] \\ &= \frac{1}{4}(1+\lambda+\delta)(\alpha v_H + (1-\alpha) v_L) \end{aligned} \tag{9}$$

In contrast, if the match-seeker is willing to accept a match in period 1 regardless of the platform's positioning signal, their total expected value is given by

$$V_{2PB} = \frac{1}{2}(\alpha v_H + (1-\alpha) v_L) \tag{10}$$

It follows that the total expected value of the match-seeker of either class in this scenario can be stated as

$$V_{3H} = V_{3L} = \begin{cases} V_{1PB} & \text{if } \lambda \geq \lambda_{PB} \\ V_{2PB} & \text{o.w} \end{cases} \tag{11}$$

4.1.3 Only One Class Uses the Platform

We next consider the setting in which either the H-class or L-class match-seekers use the online platform, while the other class uses direct search. When only H-class match-seekers use the platform, they implicitly would match with only H-class candidates (because the only candidates that will become available in the online platform will be of H-class). As in the previous case, if an H-class candidate is available in the first period (with probability α),[11] the match-seeker has to decide whether or not to accept that candidate depending on the platform's positioning signal. On the other hand, the available candidate will be L-class with probability $(1-\alpha)$, and will therefore not appear in the online platform, so that the H-class match-seeker will have to wait till the second period.

Consider the case in which an H-class match-seeker accepts only matches from candidates signaled as compatible. As before, with probability 1/2, the available candidate in each period would be compatible, and with a probability $(1+\lambda)/2$, that candidate would be signaled as compatible. Since the probability of the candidate being an H class is α, the expected value from period 1 associated with this case is $\frac{1}{2}\left[\left(\frac{1+\lambda}{2}\right)(\alpha v_H)\right]$. Additionally, if the available candidate in period 1 is an H-class signaled as incompatible $\left(\text{the probability of which is } \left(1-\frac{\alpha}{2}\right)\right)$, then the match-seeker would wait until the second period and receive a discounted expected value of $\left(1-\frac{\alpha}{2}\right)\left[\frac{\delta}{2}(\alpha v_H)\right]$. So the total expected value from both periods of matching with only candidates signaled as compatible (in period 1) is

$$\begin{aligned} V_{4Ha} &= \frac{1}{2}\left[\left(\frac{1+\lambda}{2}\right)(\alpha v_H)\right] + \left(1-\frac{\alpha}{2}\right)\left[\frac{\delta}{2}(\alpha v_H)\right] \\ &= \frac{\alpha v_H}{2}\left(\frac{(1+\lambda)}{2} + \delta\left(1-\frac{\alpha}{2}\right)\right) \end{aligned} \tag{12}$$

If an H-class match-seeker, in contrast, also accepts matches in period 1 from candidates signaled as incompatible, they would accept a match with any H-class candidate that becomes available (with probability α). The expected value of this case is $\left[\frac{\alpha v_H}{2}\right]$. If the available candidate is of L-class (with probability $(1-\alpha)$), the H-class match-seeker would have to wait until the second period, and the expected value would be $\delta(1-\alpha)\left[\frac{\alpha v_H}{2}\right]$. Thus the total expected value from accepting a candidate of either type in the first period is

$$\begin{aligned} V_{4Hb} &= \left[\frac{\alpha v_H}{2}\right] + \delta(1-\alpha)\left[\frac{\alpha v_H}{2}\right] \\ &= \frac{\alpha v_H}{2}\left(1+\delta(1-\alpha)\right) \end{aligned} \tag{13}$$

[11] Note that the candidate who becomes available from the population in each period may be of either class, and will be H-class only with probability α. If the available candidate of L-class, that candidate will not appear in the online platform.

Since L-class match-seekers are the only ones using direct search, they too are limited to matches within their class. As before, a match-seeker using direct search would detect an available candidate with probability μ only. Since the available candidate in each period will be of L-class with probability $(1-\alpha)$, using the same reasoning as for the H-class above, the total expected value to an L-class is

$$V_{4L} = \frac{1}{2}(1-\alpha)\mu v_L (1+\delta(1-(1-\alpha)\mu)) \tag{14}$$

Next consider the case in which only L-class match-seekers use the online matching platform, while the H-class uses direct search. Using similar reasoning as before, the total expected value to an L-class from matching in period 1 with only candidates signaled as compatible is

$$V_{5La} = \frac{(1-\alpha)v_L}{4}((1+\lambda)+\delta(1+\alpha)) \tag{15}$$

and the total expected value to an L-class from matching with any available candidate on the platform in the first period is

$$V_{5Lb} = \frac{(1-\alpha)v_L}{2}(1+\delta\alpha) \tag{16}$$

Finally, the total expected value to an H-class (which would be using direct search in this setting) is

$$V_{5H} = \frac{1}{2}\alpha\mu v_H (1+\delta(1-\alpha\mu)) \tag{17}$$

Proposition 2. *When only one class uses the platform, the optimal policy of the online match-seeker is as follows:*

(i) If the candidate match is signaled as compatible, accept the match.

(ii) If the H-class (L-class) uses the platform and the candidate match is signaled as incompatible, there exists a threshold λ_{PH} (λ_{PL}) such that if $\lambda < \lambda_{PH}$ ($\lambda < \lambda_{PL}$), the match-seeker should accept a match. If not, they should wait until the second period to accept the match.

Furthermore, $\lambda_{PH}, \lambda_{PL}$ are decreasing in δ, and both are greater than λ_{PB}.

Proposition 2 illustrates that the behavior of match-seekers in the online platform is influenced by

			H-class	
			Direct (d)	Online (o)
L-class	Direct(d)		V_H^{dd}, V_L^{dd}	V_H^{do}, V_L^{do}
	Online(o)		V_H^{od}, V_L^{od}	V_H^{oo}, V_L^{oo}

Table 1: Payoffs for online matching with positioning

the platform's positioning capability in the same way as when both classes use the platform. When λ is sufficiently high, a match-seeker of either class would, in the first period, reject a candidate signaled as incompatible, though the threshold values of λ for H- and L- classes are different. It is interesting to note that when only one of the classes uses the platform, the threshold on positioning capability above which a match-seeker would reject a candidate signaled as incompatible is higher than when both classes use the platform. The reason for this is that when only one class uses the platform, there is a greater risk of not getting any match at all in the second period, which causes the match-seeker in the first period to be more willing to accept candidates signaled as incompatible.

4.2 Choices of Match-Seekers in Equilibrium

As mentioned earlier, both classes of match-seekers have to choose whether to use the online matching platform and pay the access fee p, or avoid the fee and conduct direct search. We next analyze the conditions under which each choice is optimal for the match-seeker in equilibrium.

The payoffs from these choices can be organized as in Table 1. Each of these value functions can be derived from the value functions that are defined earlier. For example, when $\lambda > \lambda_{PH}$,

$$
\begin{aligned}
V_H^{do} &= V_{4Ha} - p \\
V_L^{do} &= V_{4L}
\end{aligned}
$$

Lemma 1. *There exists thresholds on p_{1P} and p_{2P} on p, and λ_A and λ_B on λ such that:*

i) If $p > p_{2P}$, neither class uses the platform

ii) If $p_{1P} \leq p \leq p_{2P}$ and $\lambda \geq \lambda_A$, only H-class uses the platform, and in period 1 accepts matches with only candidates signaled as compatible.

iii) If $p_{1P} \leq p \leq p_{2P}$, and $0 < \lambda < \lambda_A$, only H-class uses the platform, and accepts any match in period 1.

iv) If $p < p_{1P}$ and $\lambda_B < \lambda \leq 1$, both classes use the platform and in period 1 accept matches with only candidates signaled as compatible.

v) If $p < p_{1P}$ and $\lambda \leq \lambda_B$, both classes use the platform and accept any match in period 1.

There are two important implications of Lemma 1. First, L-class match-seekers use the matching platform only when the H-class does so as well. In other words, L-class match-seekers would be willing to pay the access fee to use the online platform only if there is a possibility of matching with an H-class in that setting. Second, the platform could potentially discourage L-class match-seekers from using it, by charging a very high access fee. The higher quality of the resulting market may still motivate the H-class to pay the high fee as well in equilibrium.

It is also worth noting that the positioning capability of the platform also influences the match-seeker's equilibrium choices. When the capability is very high, a match-seeker using the platform would accept only a candidate signaled as compatible, whereas they would accept a match with any available candidate if the capability were low. In other words, there is a threshold value of λ above which match-seekers leverage the positioning capability of the platform. Interestingly, when the positioning capability is in the intermediate range (i.e., $\lambda_B \leq \lambda < \lambda_A$), the willingness of an H-class match-seeker to accept a match with a candidate signaled as incompatible depends on the access fee. When the fee is very low, both classes use the platform and in the first period a H-class match-seeker accepts only a candidate signaled as compatible. However, when the fee is higher and L-class match-seekers choose not to use the platform, the H-class match-seeker in the online platform accepts any available match in the first period. The reason for this is that with only H-class match-seekers in the online platform, these match-seekers perceive a higher risk of not finding a match in the second period, which motivates them to accept any match in the first period (which at least will be an H-class match).

4.3 Optimal pricing strategy of the platform

Having characterized the equilibrium search behavior of match-seekers, we now examine the optimal pricing strategy of the matching platform in this setting. The key question is whether the platform should charge a high access fee that attracts only H-class match-seekers, or instead charge a low fee that motivates both classes to use the platform.

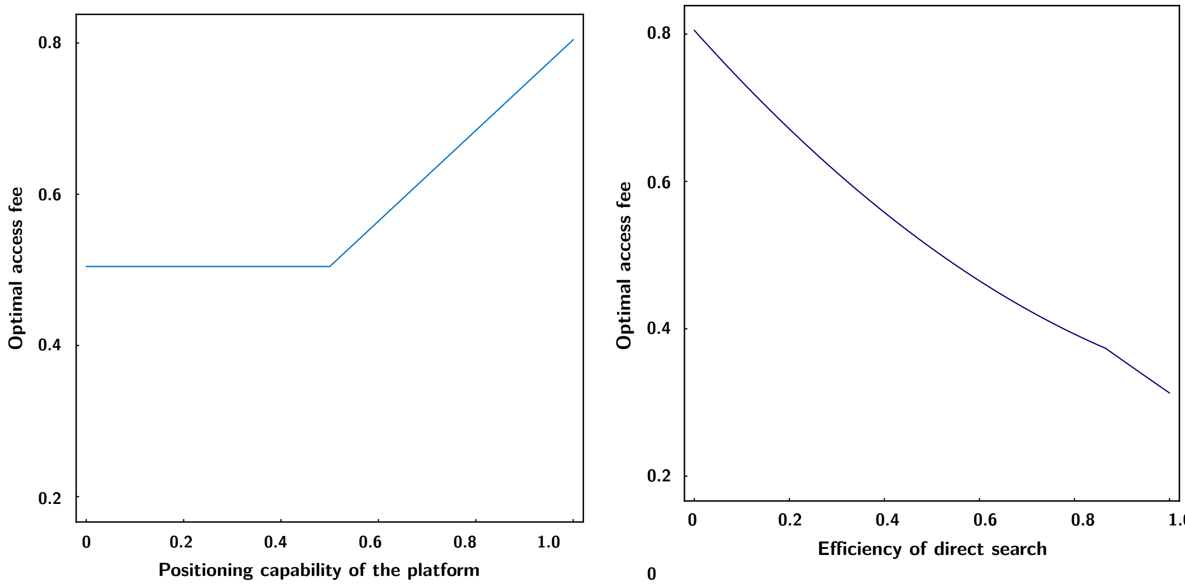
Proposition 3. *The optimal pricing strategy of the platform results in both H- and L-classes using the platform. In addition, the optimal access fee p^* is increasing in λ, decreasing in μ and non-monotonic in η, α.*

In a setting limited to vertical differentiation, Basu et al. (2019) show that without authentication, it is optimal for online matching platforms to set an access fee that attracts all classes of match-seekers. The above proposition shows that this result holds even when the platform supports positioning. In a high-quality market characterized by high values of α and η, the matching platform may be motivated to focus on H-class match-seekers. However, such a market is also attractive to L-class match-seekers, particularly since the online platform includes positioning. And without any authentication mechanism, L-class match-seekers will use the online platform pretending to be H-class match-seekers. Given this, the platform is better served by charging an access fee that attracts both classes.

As shown above, both classes have to be motivated to use the platform and hence the access fee that the platform can charge is determined by the minimum of the willingness to pay of both classes. Since better positioning capability λ increases the value of the platform to both classes, higher values of λ result in higher access fees. On the other hand, higher values of μ result in lower access fees due to the increasing threat of the direct search option (Fig. 2). However, the effect of η and α are different. When η is low, the L-class's progressively increasing willingness to pay to use the platform drives the platform's pricing decision. On the other hand, for high values of η, direct search becomes a more attractive alternative for H-class match-seekers resulting in decreasing access fee. For similar reasons, the market composition parameter α also has a non-monotonic effect on the optimal access fee (Fig. 3).

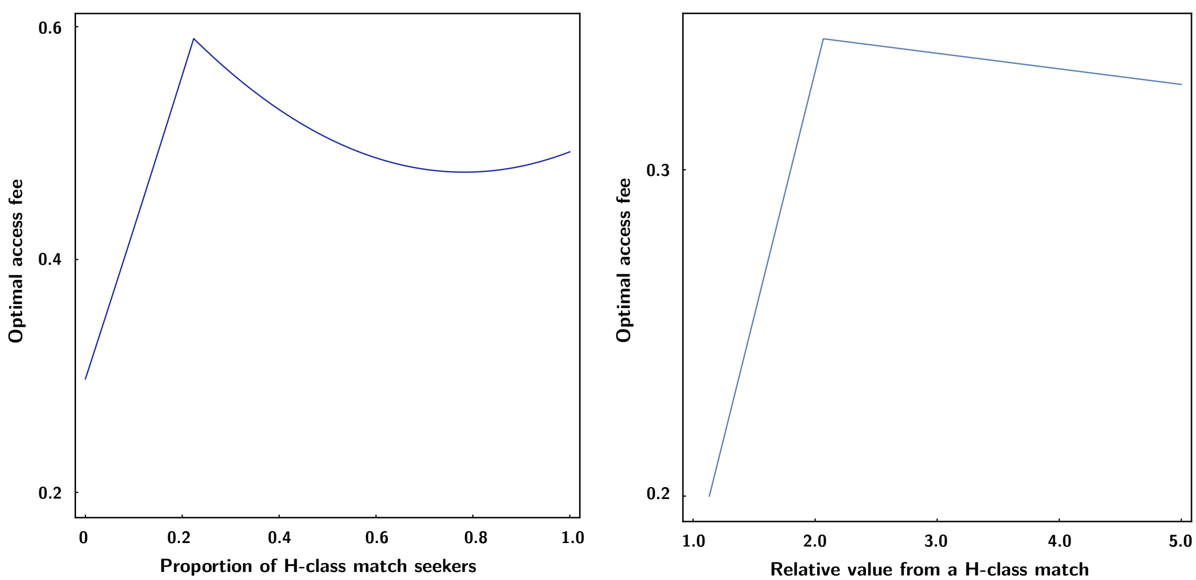
5 Online Matching with Positioning and Authentication

We next consider a setting in which the online matching platform offers both authentication and positioning. In this setting, the platform provides an authentication service that enables match-seekers to increase their ability to match with higher quality candidates. The platform continues to charge an access fee p, and in addition, charges an authentication fee q if a match-seeker purchases it.



$\mu = 0.5, \alpha = 0.7, \delta = 0.5, \eta = 3$ $\alpha = 0.2, \delta = 0.8, \eta = 3, \lambda = 0.5$

Figure 2: Optimal access fee



$\mu = 0.7, \delta = 0.5, \eta = 3, \lambda = 0.2$ $\mu = 0.7, \alpha = 0.3, \delta = 0.6, \lambda = 0.3$

Figure 3: Optimal access fee

5.1 Match-seeker Behavior

In the first period, if the match-seeker purchases authentication,[12] they encounter and have to evaluate a potential match from one of the following four possibilities:

1. An H-class, candidate signaled as compatible
2. An L-class, candidate signaled as compatible
3. An H-class, candidate signaled as incompatible
4. An L-class, candidate signaled as incompatible.

Of these, the optimal decision in case 1 would be to accept the match. Similarly, the optimal decision in case 4 would be to reject the match and wait till the second period if the match-seeker purchased authentication in order to match with an H-class. The optimal decision in the other two cases, however, depends on the positioning capability λ of the platform. The optimal decisions of an H-class who has purchased authentication can be characterized by Lemmas 2 and 3.

Lemma 2. *(i) There exists a threshold λ_{B1} such that if $\lambda < \lambda_{B1}$, an H-class who is matched with an H-class candidate signaled as incompatible, will accept the match.*
(ii) $\lambda_{B1} > \lambda_{PB}$

As before, if the capability of the platform is sufficiently high, the match-seeker would be confident that the candidate signaled as incompatible would indeed turn out to be incompatible. It is only when the platform's positioning capability is sufficiently low i.e. ($\lambda < \lambda_{B1}$) that the match-seeker would be motivated to accept any H-class match in period 1.

To understand the impact of authentication on this decision, we compare λ_{B1} to λ_{PB}, which was the threshold below which the match-seeker chose to accept a candidate match in Period 1 when the platform did not offer authentication. For instance, a candidate signaled as incompatible could be either H- or L-class. With authentication being available, the match-seeker can determine the class of the candidate. If the available candidate is an H-class (even though it is a candidate signaled as incompatible), without any assurance that they would encounter an H-class in the second period, the match-seeker would be motivated to accept the first-period match. In other words, they would

[12] Given that there are only two classes of participants, when at least one of the match-seeker classes purchases authentication, the classes of all participants are implicitly known.

be willing to reject the match with a candidate signaled as incompatible only if the platform's positioning capability is even higher, i.e. $\lambda_{B1} > \lambda_{PB}$.

Lemma 3. *(i) There exists a threshold λ_{B2} such that if $\lambda > \lambda_{B2}$, an H-class facing a match with an L-class candidate signaled as compatible will accept the match.*
(ii) $\lambda_{B2} > \lambda_{PB}$ iff α is sufficiently high.

Corollary 1. *There exists a threshold α_2 such that if $\alpha \geq \alpha_2$, $\lambda_{B2} > \lambda_{B1}$.*

We know that an H-class would always reject an L-class candidate signaled as incompatible. However, Proposition 3 shows that there are conditions under which that match-seeker would accept an L-class if that candidate is signaled as compatible. If the platform's positioning capability is sufficiently high, the match-seeker's higher level of confidence in the platform's positioning capability would prompt them to accept that match even though the resulting value is lower (i.e., v_L) rather than risk getting matched with an incompatible candidate in period 2.

Again, it is worth comparing the optimal policy of the match-seeker to the setting in which the platform does not offer authentication. When authentication is available and an H-class is faced with an L-class candidate signaled as compatible, the potential to be able to match with an H-class candidate signaled as compatible in the second period should encourage the match-seeker to reject the L-class candidate. However, this would result in a compatible H-class match only if the proportion of H-class (i.e., α) is sufficiently high. When α is low, delaying the decision only defers the same outcome i.e. of getting matched with an incompatible L-class.

It follows that the optimal policy of both classes will be as stated in Propositions 4 and 5.

Proposition 4. *A) When $0 < \alpha < \alpha_1$ then the optimal policy of an H-class is to accept any candidate. Otherwise*
B) When $\alpha_1 < \alpha \leq \alpha_2$, then the optimal policy of an H-class is as follows:
(i) If $0 \leq \lambda < \lambda_1$, accept any H-class.
(ii) If $\lambda_1 < \lambda < \lambda_2$, accept any candidate and class other than a L-class candidate signaled as incompatible
(iii) If $\lambda \geq \lambda_2$, accept only a candidate signaled as compatible.

C) When $\alpha_2 < \alpha \leq 1$, then the optimal policy of an H-class is as follows:

(i) If $0 \leq \lambda < \lambda_1$, *accept any* H*-class.*

(ii) If $\lambda_1 < \lambda < \lambda_2$, *accept only an* H*-class candidate signaled as compatible.*

(iii) If $\lambda \geq \lambda_2$, *accept only a candidate signaled as compatible.*

Proposition 4 above illustrates the interesting interaction between positioning and authentication as a function of the quality of the matching market (as represented by α) and the positioning capability of the platform (as represented by λ). When the positioning capability of the platform is either very high or very low, the optimal policy of H-class match-seekers is relatively straightforward. Specifically, when λ is very low ($\lambda < \lambda_1$), H-class accepts matches from only H-class candidate regardless of the platform's positioning signal. In other words, positioning is not valuable for the H-class. Similarly, when λ is high ($\lambda > \lambda_2$), the H-class accepts candidates of both classes but only candidates signaled as compatible; in other words, even though positioning offers value for the H-class, authentication does not.

However, when λ is in the intermediate range ($\lambda_1 < \lambda < \lambda_2$), the match-seeker's optimal policy is further moderated by the quality of the market. In this range of λ, when α is low ($\alpha_1 < \alpha \leq \alpha_2$), H-class match-seekers hedge their risks by accepting any match other than L-class candidates signaled as incompatible. However, when α is higher, the confidence of H-class about the market quality justifies a more selective policy. Thus we see an interesting progression in the preferences of H-class, based on market quality and the platform's positioning capability. In a low-quality market, as the positioning capability improves, H-class goes from accepting only H-class matches to also accepting L-class matches but still restricting themselves to only candidates signaled as compatible. However, in a high-quality market ($\alpha > \alpha_2$), the H-class goes from being selective on the class dimension to being selective on both dimensions to eventually being selective only about compatibility. The reason for this interesting final shift is that the H-class gains enough confidence in the platform's positioning capability to accept an L-class candidate signaled as compatible, rather than waiting for the second period where they may only meet an incompatible candidate.

Proposition 5. *The optimal policy of an L-class in period 1 can be characterized as below*

A) When $0 < \alpha < \alpha_1$ *, then accept any candidate.*

B) When $\alpha_1 \leq \alpha < \alpha_2$ *and* $\lambda_1 < \lambda < \lambda_2$, *accept any candidate other than a L-class candidate signaled as incompatible.*

C) When $\lambda > \lambda_2$, accept any candidate signaled as compatible.[13]

D) Otherwise, wait till second period.

The optimal policy of the L-class is dictated by the choices of the H-class. When α is very low ($\alpha < \alpha_1$), the H-class accepts any candidate and so the L-class also follows the same policy. Similarly, when the values of α and λ are such that the H-class finds it optimal to match with all candidates other than L-class candidates signaled as incompatible ($\alpha_1 \leq \alpha < \alpha_2$ and $\lambda_1 < \lambda < \lambda_2$), the L-class also accepts any candidate other than L-class candidates signaled as incompatible. This is also the case when $\lambda > \lambda_2$. If any of the above 3 conditions are not satisfied, the H-class finds it optimal to reject matches from any L-class. In the absence of the possibility of a match from H-class in period 1, the L-class now finds it optimal to wait until the second period.

These interactions have useful implications for platform operators. They indicate that when the quality of the matching market is low, the platform should focus on its positioning feature, since that is what will lead to more accepted matches. On the other hand, when the quality of the market is high, the platform should focus on authentication, since that is what will lead to better quality matches.

5.2 Choices of Match-Seekers in Equilibrium

As before, both classes of match-seekers have to choose whether to use direct search or use the online matching platform and pay the fee p. In addition, they can purchase authentication for an additional fee q. We now analyze the conditions under which each choice is optimal for the match-seeker in equilibrium.

The payoffs from these choices can be organized as in Table 2. When the H-class buys authentication and the L-class does not, the value functions of both classes would be determined by the optimal policies of both classes as characterized in Lemma 3, which in turn depend on the positioning capability of the platform. Again, we provide detailed expressions for these value functions in the Appendix. When neither class purchases authentication, the value functions are identical to the situation in which the platform does not offer any authentication (and are discussed in Section 4). The following lemma characterizes the equilibrium behavior of both classes when the platform supports both positioning and authentication.

[13] Note that when $\lambda > \lambda_2$, $\alpha_1 < 0$

		H-class		
		Direct Search	Online Search only	Search +Authentication
	Direct Search	V_H^{dd}, V_L^{dd}	V_H^{do}, V_L^{do}	V_H^{da}, V_L^{da}
***L*-class**	Online Search only	V_H^{od}, V_L^{od}	V_H^{oo}, V_L^{oo}	V_H^{oa}, V_L^{oa}
	Search+Authentication	V_H^{ad}, V_L^{ad}	V_H^{ao}, V_L^{ao}	V_H^{aa}, V_L^{aa}

Table 2: Payoffs for matching with positioning and authentication

Lemma 4. *There exist thresholds p_{1B}, p_{2B} and q_B such that for any access fee p and authentication fee q, the equilibrium behavior of the H- and L-classes will be as follows:*

i) If $p > p_{2B}$, both H- and L-classes engage in direct search.

ii) If $p_{1B} < p \leq p_{2B}$, only the H-class uses the platform but does not purchase authentication.

iii) If $p \leq p_{1B}$ and $q > q_B$, both H- and L-classes use the platform, but neither purchases authentication.

iv) If $p \leq p_{1B}$ and $q \leq q_B$, both H- and L-classes use the platform, and the H-class also purchases authentication.

As in the previous case, Lemma 4 indicates that there are only four pure-strategy equilibria in this setting. This is because the L-class does not purchase authentication, since that reveals their true class. Furthermore, the H-class finds authentication useful only when they want to match with only H-class candidates and both classes use the platform. Thus there is an equilibrium in which both classes use the platform and pay the access fee p, while the H-class also pays the additional fee q to purchase authentication.

5.3 Optimal Pricing Strategy

Having identified the equilibrium behavior of the match-seekers, we now turn our attention to the platform's pricing strategy when the platform offers authentication. In formulating this strategy, the platform has to consider the following: (i) the costs of positioning and authentication, (ii) the profits from offering authentication to the H-class, and (iii) the value from motivating the L-class to use the platform.

Specifically, we examine how the pricing strategy is impacted by the market composition and the platform's positioning capability. We restrict our attention to the setting in which both classes use the platform and the H-class purchases authentication. The profits of the platform under this case would be $\pi_B = p + \alpha (q - c_A) - c_P$ where $p \leq p_{1B}$ and $q \leq q_B$. Note that the platform's access fee decision is driven by its desire to motivate both H- and L-class match-seekers to use the platform. Thus, the access fee has to be lower than the willingness to pay of both classes. On the other hand, the authentication fee choice is determined by the additional value that the H-class can obtain by being able to search only for H-class matches, while the platform is also constrained by the total fees, t, that it can charge (where $t = p + q$).

Let p^* and q^* be the optimal access and authentication fees of the platform. As we saw earlier, the platform's positioning capability influences the behavior of the H-class match seeker (Proposition 5). Recognizing these patterns, the platform sets the access and authentication fees to both motivate participation and maximize profits, and these fees can be determined by examining the implications of the match-seeker choices on the profitability of the platform. We derive the optimal access and authentication fees for various ranges of the platform's positioning capability in the appendix as Lemma 5. Using a numerical analysis, we illustrate the relationship between these fees and two key factors, the platform's positioning capability and the quality of the matching market, as shown in Figure 4. Additionally, we conduct a comprehensive sensitivity analysis in Proposition 6 to better understand the relationship between these factors and the optimal fees.

Proposition 6. *(i) The optimal access fee p^* is non-decreasing in λ if $\lambda > \lambda_L$. Additionally, it is non-monotonic in α and η.*
(ii) The optimal authentication fee q^* is non-monotonic in λ, α and η.

Sensitivity analysis of the optimal access and authentication fees reveals interesting effects due to the interaction between the market quality and the platform's positioning capability. An increase in λ implies that the value provided by the platform is greater, and thus match-seekers of both classes would be motivated to use the platform, so the platform can charge a higher access fee. In contrast, an increase in the platform's positioning capability does not always lead to higher optimal authentication fees. When λ is low, positioning and authentication are complementary to each other, so that an increase in positioning capability leads to higher authentication fees. In this region, an increase in λ enables the match-seeker to increase their ability to recognize a compatible candidate, and authentication provides an added benefit.

However, beyond a point, the relationship between authentication and positioning switches so that they become substitutes, and an increase in λ may decrease the value of authentication and

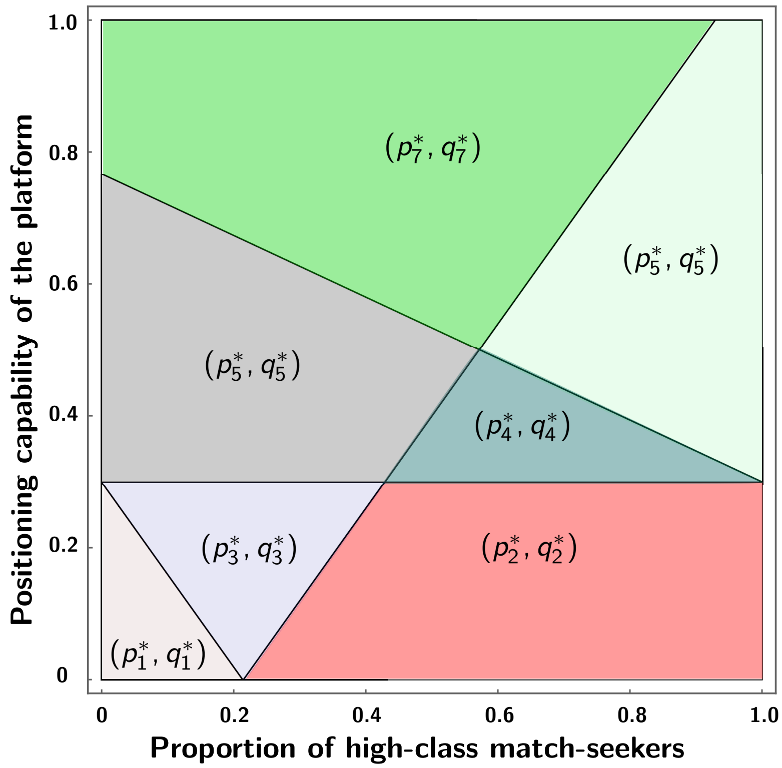
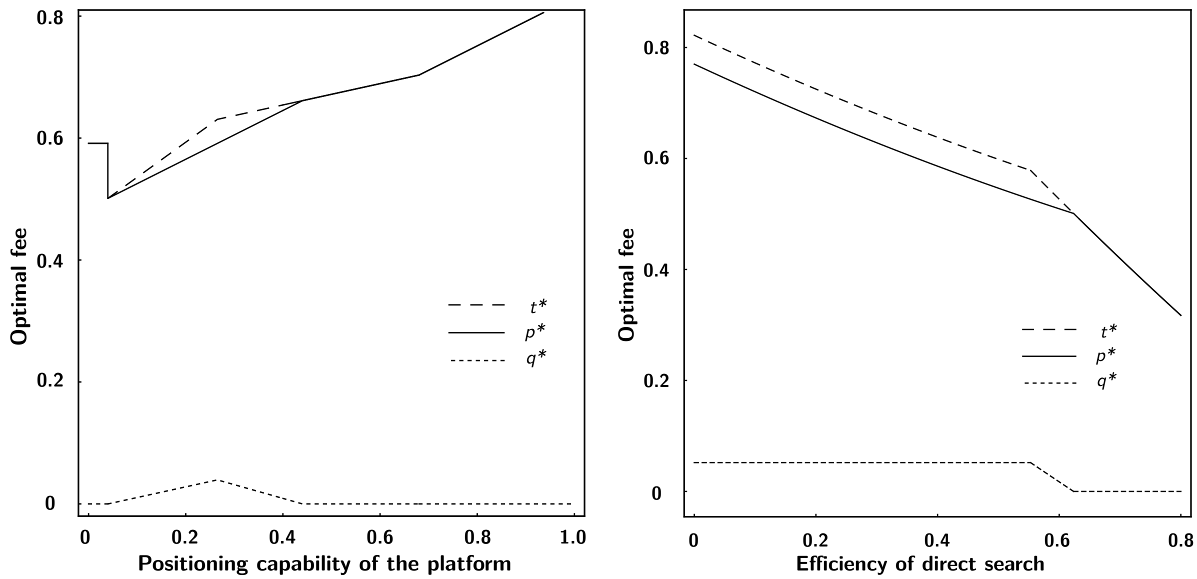


Figure 4: Optimal access and authentication fees

lead to lower optimal authentication fees. For high-values of λ, the H-class is already predisposed to reject a candidate signaled as incompatible even if that candidate is of H-class. Thus there is less value in detecting the true class of a candidate signaled as compatible as the match-seeker would rather match with any candidate signaled as compatible than a H-class candidate signaled as incompatible. As λ increases, this effect is even more pronounced and the value of authentication decreases. This non-monotonic relationship is illustrated in Figure 5. As for the effect of μ, the efficiency of direct search, the increasing threat of direct search when μ increases leads to the access fee being monotonically decreasing in μ, as shown in the right of Figure 5.

We also find a non-monotonic relationship between the optimal access and authentication fees, and the market composition parameters α and η. When α is low, an increase in α improves the market quality and makes it attractive to both classes of match-seekers. The platform can take advantage of this by increasing both the access and authentication fees. However, when α is very high, the chances of an H-class match is already high, reducing the additional value from authentication. But the threat of direct search still is a concern for the platform. So in order to prevent the H-class from moving to direct search, the platform has to offer authentication but for a lower



$\delta = 0.6, \eta = 3, \mu = 0.3, \alpha = 0.3$ $\delta = 0.6, \eta = 3, \mu = 0.3, \alpha = 0.3$

Figure 5: Optimal access and authentication fees

fee. Furthermore, the stronger threat from the direct search also reduces the platform's ability to increase its access fees. The effect of η on the access and authentication fees are similar, as shown in Figure 6.

6 Optimal Platform Strategy

Having developed the optimal fees of the platform, we now turn our attention to the optimal strategy of the platform i.e., what the platform should offer under various conditions. Thus, the four potential strategies that the platform can pursue are as follows:

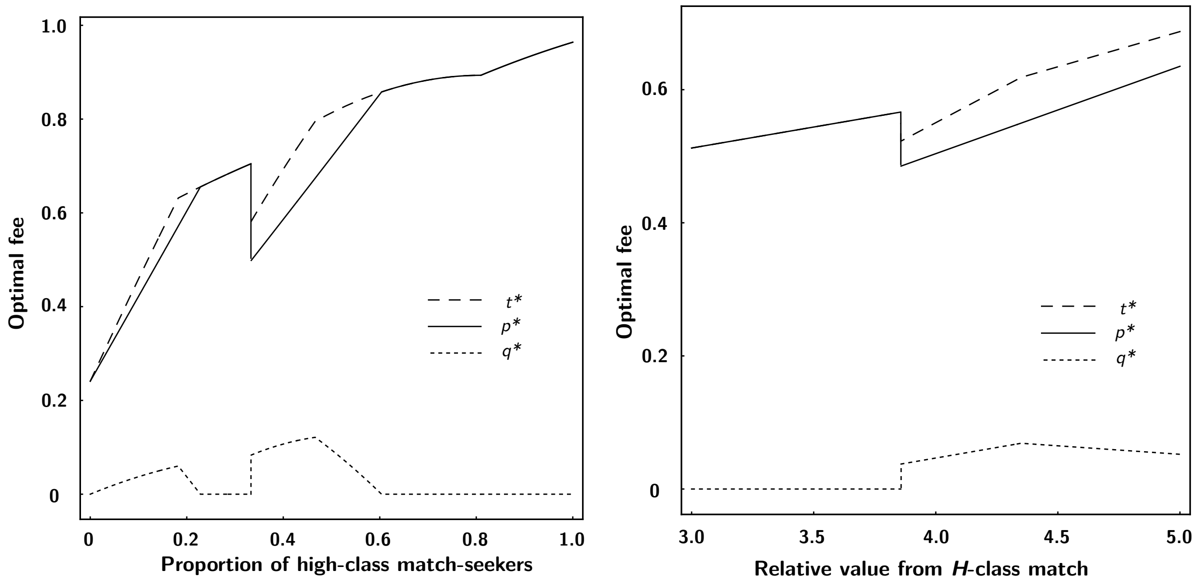
1. Online matching with no positioning or authentication.

2. Online matching with only positioning.

3. Online matching with only authentication.

4. Online matching with both authentication and positioning.

To analyze the situation in which the platform supports neither authentication nor positioning, we can set $\lambda = 0$ in the value functions derived in section 4 where the platform supports only positioning. The third option, in which the platform supports only authentication, was analyzed in Basu et al. (2019), in a simpler setting in which the dimension of subjective preference was not



$\delta = 0.7, \eta = 4, \mu = 0.4, \lambda = 0.4$ $\delta = 0.75, \mu = 0.55, \alpha = 0.35, \lambda = .5$

Figure 6: Optimal access and authentication fees

considered. We summarize this scenario here for the sake of completeness but relegate the details of the analysis to the Appendix.

Similar to the case when both positioning and authentication are supported by the platform, it is never in the interest of the L-class to purchase authentication. Thus, the optimal fees charged by the platform are influenced by two factors. First, the access fee should motivate both classes to use the platform rather than using direct search. Second, the authentication fee has to be sufficiently low to encourage the H-class to purchase authentication. These constraints ensure that neither the authentication fee nor the total fee (the sum of access and authentication fees) get too high. The optimal access and authentication fees are therefore as follows:

$$\begin{aligned} p_A^* &= \min\left[\frac{1}{2}\delta\left(\alpha v_H + (1-\alpha) v_L\right) - \frac{1}{2}(1-\alpha)\delta\mu v_L\left(1-(1-\alpha)\mu\right) - \frac{1}{2}(1-\alpha)\mu v_L,\right. \\ &\quad \left. -\frac{1}{2}\alpha\delta\mu v_H(1-\alpha\mu) - \frac{\alpha\mu v_H}{2} + \frac{1}{2}\left(\alpha v_H + (1-\alpha)\delta\left(\alpha v_H + (1-\alpha) v_L\right)\right)\right] \\ q_A^* &= \min\left[\frac{1}{2}\left(\alpha v_H + (1-\alpha)\delta\left(\alpha v_H + (1-\alpha) v_L\right)\right) - \frac{1}{2}\left((1-\alpha) v_L + \alpha v_H\right),\right. \\ &\quad \left. -\frac{1}{2}\alpha\delta\mu v_H(1-\alpha\mu) - \frac{\alpha\mu v_H}{2} + \frac{1}{2}\left(\alpha v_H + (1-\alpha)\delta\left(\alpha v_H + (1-\alpha) v_L\right)\right) - p_A^*\right] \end{aligned}$$

Finally, the optimal profit of the platform in this setting is $\pi_A^* = p_A^* + \alpha\left(q_A^* - c_A\right)$.

Having considered all four possible strategies, we now analyze the optimal strategy of the platform. We start by considering the situation in which $\mu = 0$, i.e. there is no threat for the platform from direct search. The following results characterize the optimal strategy of the platform in this scenario.

Proposition 7. *When $\mu = 0$, there exists thresholds λ_X and λ_Y such that the optimal policy of the platform is as follows:*
(i) If $\lambda < \lambda_X$, support neither authentication nor positioning.
(ii) If $\lambda_X \leq \lambda \leq \lambda_Y$, support both authentication and positioning.
(iii) If $\lambda > \lambda_Y$, support only positioning.

Corollary 2. *When $\mu = 0$ and $c_P, c_A \approx 0$, there exists a threshold α_A such that*
(i) If $\alpha \leq \alpha_A$, $\lambda_X \leq \lambda_{PB} \leq \lambda_Y$.
(ii) If $\alpha > \alpha_A$, $\lambda_X = \lambda_Y = \lambda_{PB}$.

The optimal strategies of the platform are illustrated in Fig.7. The first part of proposition 7 indicates that the platform supports positioning only if its positioning capability exceeds a minimal threshold. When the platform's positioning capability is low, match-seekers would ignore the platform's positioning signal and accept any match in period 1. Thus the positioning capability does not provide any value, and therefore the platform does not support it. It is interesting to note that under these conditions, it is optimal for the platform to not offer authentication either.

However, when the platform's positioning capability is above the minimum threshold, match-seekers follow the platform's positioning signal in the first period and reject a candidate signaled as incompatible, waiting until the second period in the hope of a better match. This value provided by positioning allows the platform to support it and charge a higher access fee.

The platform's optimal strategies with respect to authentication are more nuanced in this setting (*i.e.*, $\mu = 0$). First, it is optimal for the platform to offer authentication only if its positioning capability is in the intermediate range (*i.e.*, $\lambda_X \leq \lambda \leq \lambda_Y$). Second, it is not optimal for the platform to offer authentication unless it also supports positioning. To understand these results, it is useful to examine match-seeker behavior when the platform offers them.

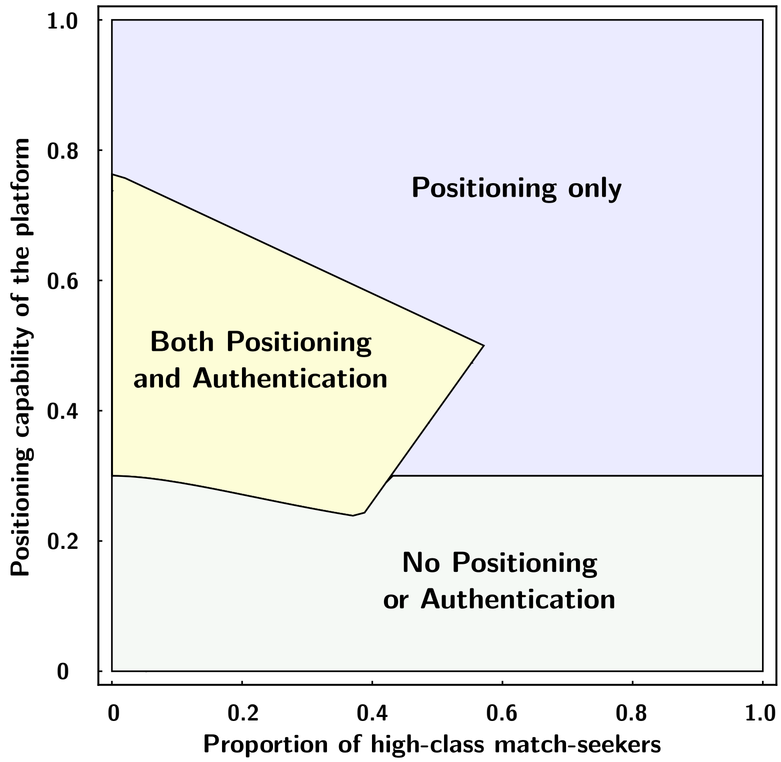
Recall that the benefits of authentication are realized only by H-class match-seekers. Additionally, without authentication, the H-class may choose to use direct search to be able to match only with H-class candidates. In the same vein, because the L-class will not be able to match with an

H-class in the presence of authentication, their willingness to pay the access fee to use the platform goes down when the platform enables H-class to recognize and reject L-class candidates. In general, the threat of direct search motivates the platform to offer authentication, and the higher total fee from the H-class compensates for the lower access fee from the L-class. However, in the absence of direct search ($\mu = 0$), this threat no-longer exists. When $\lambda < \lambda_X$,the platform finds it optimal to discard authentication as well and charge both classes more for search.

As λ increases, the positioning offered by the platform can improve the chances of finding a compatible match and thus its value increases, particularly for the H-class. In fact, H-class match-seekers find value in being able to reject incompatible L-class candidates, which is possible only if the platform offers both positioning and authentication. When the positioning capability of the platform is very high, the H-class prefers to reject even H-class candidates who are incompatible, which reduces the value of authentication. Thus the platform finds it optimal to charge a high access fee that is attractive to both classes, by supporting positioning without authentication. It is worth noting that the absence of authentication increases the value of the platform to L-class match-seekers as they now have the potential to match with H-class candidates as well. Thus the platform's optimal strategy is driven by the anticipated behavior of match-seekers in equilibrium as characterized in Propositions 4 and 5 (Section 5).

An interesting implication of the above results is that positioning and authentication can be complements or substitutes, depending on the platform's positioning capability and market quality. When the platform's positioning capability is low, positioning and authentication are complements in that an increase in the positioning capability increases the value of authentication. Additionally, as illustrated in Figure 7 and in Corollary 2, when α is sufficiently low, the availability of authentication can motivate the platform to support positioning at a level of λ that can be even below λ_{PB}. On the other hand, at very high levels of λ, positioning and authentication are substitutes, in the sense that an increase in λ reduces the value of authentication. Indeed, when λ is very high ($\lambda > \lambda_Y$), the platform ceases to offer authentication altogether and only supports positioning.

Characterizing the platform's optimal strategy when $\mu > 0$ is complex because we need to account for several variations of the match-seeker behaviors and different pricing responses from the platform for each of those cases (as illustrated in Propositions 4 and 5, and Lemma 5). Thus, analytical results are harder to derive for this setting. Instead, we conduct an extensive numerical



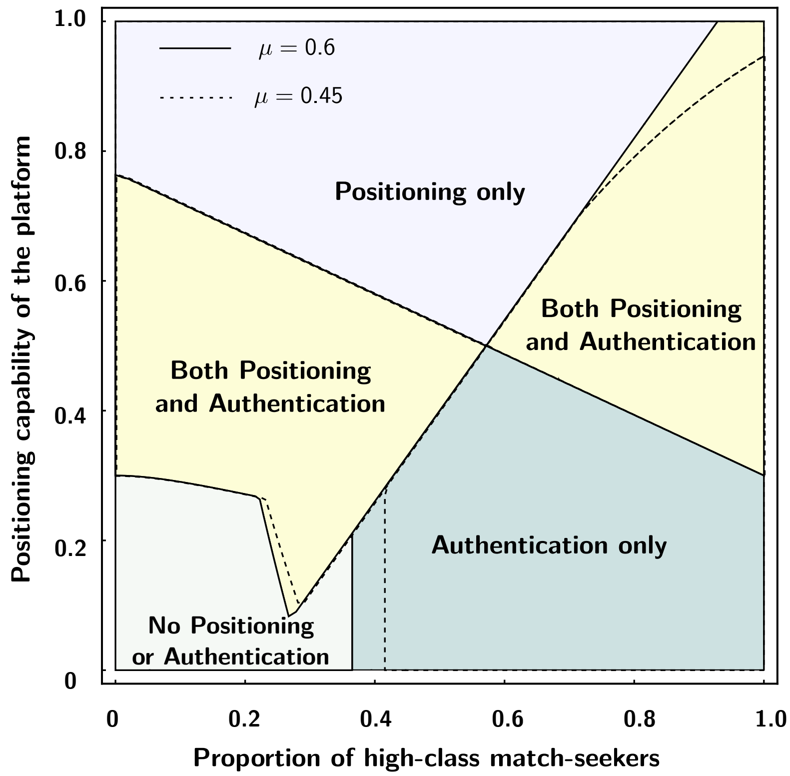
$c_A \approx 0, c_P \approx 0, \delta = 0.7, \eta = 3, \mu = 0$

Figure 7: Optimal Platform Strategy in the absence of direct search ($\mu = 0$)

analysis which shows that the patterns with respect to positioning and authentication strategy of the platform are consistent with the special case of $\mu = 0$. These results are summarized in Fig 8.

This numerical analysis illustrates two important aspects of the platform's optimal strategy. First, we see that when λ is sufficiently low and α is sufficiently high, offering authentication without support for positioning can be an optimal strategy for the platform. In this region, the real threat of direct search motivates the platform to offer authentication as a means to retain the H-class match-seekers. However, as before, when the platform is able to acquire only a low positioning capability is low, it is no longer attractive to enhance search with positioning.

Second, when α is sufficiently high, an increase in the positioning capability of the platform results in a shift from supporting only authentication to supporting both positioning and authentication. Eventually, when the platform's capability is very high, the platform's optimal strategy reverts back to supporting only positioning. Thus, the complement vs. substitute relationship between positioning and authentication that was analytically characterized in Proposition 7 is not peculiar to that special case when there is no threat from direct search, but also holds in a more general setting.



$c_A \approx 0, c_P \approx 0, \delta = 0.7, \eta = 3$

Figure 8: Optimal Platform Strategy when $\mu > 0$

Note that these strategic choices are similar to those analyzed in the special case of $\mu = 0$. Furthermore, Figure 8 illustrates the sensitivity of the platform's optimal strategy to the efficiency of direct search by comparing the strategy space for two values of μ. Here, as μ increases from 0.45 to 0.6, it can be seen that the two choices of (i) authentication only and (ii) both positioning and authentication become optimal under a broader range of conditions. In other words, the value of authentication increases when the threat of direct search as an outside option is greater.

To summarize, we have used the case of $\mu = 0$ to illustrate the fundamental trade-offs driving the platform's optimal strategy, and use the extensive numerical analysis to illustrate the robustness of these findings. The results of this analysis show that the fundamental trade-offs between positioning and authentication are consistent with what we analytically derive for the special case in which $\mu = 0$. In other words, the impact of positioning and authentication on the equilibrium match-seeker behavior is what drives the platform's optimal strategy (rather than the presence or absence of direct search).

7 Discussion and Conclusion

In this paper, we examine some key issues in the design of online matching platforms when match-seekers are looking for matches that are both compatible (i.e., "right") as well as of the "best" quality. Prior research has examined how online platforms can offer appropriate authentication services to address the quality problem. In this paper, we examine the first problem, namely the correct positioning of an online match-seeking process, and examine how an online matching platform's decision to support such positioning features interact with any authentication services it may want to offer. To the best of our knowledge, this is one of the first papers to use a game theoretic analysis to characterize match-seeker behavior, and derive optimal platform strategies in the presence of both objective and subjective dimensions of match-seeker preferences.

Our analytical models and game-theoretic analysis lead to some valuable insights, about both the choices of match-seekers as well as the online platform owner/operator. We consider a general setting in which match-seekers can choose to search broadly or narrowly along objective and subjective dimensions. First, we show that the value of authentication can be significantly impacted by match-seekers' ability to better position their search. And second, consistent with prior research, we show that both higher and lower quality match-seekers can play pivotal roles in the platform's decisions about offering and pricing its services.

Our analysis also yields key insights into the optimal strategies of the online matching platform with respect to offering authentication and positioning in addition to its basic search. To start with, we show the important role that the availability of an outside option such as direct search plays in determining match-seekers' behavior, and in turn, the online platform's optimal strategy. For instance, we show that without an outside option, it is never optimal for the platform to offer authentication unless it also supports positioning. Even in the presence of the outside option, we show that authentication and positioning have interesting interactions, so that under some conditions, the platform would view them as complements, while in other situations, they act as substitutes. Specifically, they complement each other only when both the market quality and the platform's positioning capability are either very high or low. For intermediate values of both factors, the services act as substitutes. This underscores the value of our models in providing online matching platform operators with valuable guidance in their strategic choices regarding such services. These insights from our analyses can be used by a matching platform in deciding whether to support authentication and positioning, as well as how the platform's prices are impacted by factors such as the quality of other options, the composition of the market served by the platform, and the positioning capability of the platform.

The findings from our models are applicable to both consumer matching platforms such as online dating and marriage platforms such as match.com and eharmony, as well as business matching platforms such as Powerlinx and Catalant. In the B-B setting, the Powerlinx platform provides clients with a dedicated advisor to help with positioning their searches, and also offers an authentication service (the Powerlinx Verified Badge). This is consistent with the strategy guidelines from our model (Figure 8) regarding the impact of increasing positioning capability of the platform.

In the consumer setting, while the casual dating platform Tinder does not offer any authentication service to its customers, Bumble (another dating app where women take the initiative) suggests customers take and upload a "selfie" in a pose randomly selected by the platform, which is manually reviewed by the platform. In the marriage-oriented matching market, the eHarmony platform uses a proprietary algorithm driven with input from an extensive survey (called the Personality Profile) that new match-seekers have to complete (Piskorski et al., 2008), to focus the matches provided by the platform to the priorities of each match-seeker. In addition, eharmony also offers match-seekers the ability to authenticate themselves from publicly available data and government issued documents, through an optional service (relyID). On the other hand, while match.com does not have any significant authentication mechanism, it does support a positioning feature that answers questions such as "What type of woman/man is truly right for me?".[14] A possible reason for this is that the positioning capability of match.com is relatively high for the dating market where match-seekers are looking for short-term commitments. In such a situation, our model suggests that the platform should support only positioning without any authentication, as reflected in match.com's approach.

While we believe that this work is an important step towards understanding the economic behavior of online matching platforms, there are several opportunities for additional work in this general area. For instance, we have considered positioning as an up-front feature that match-seekers use when initiating a search process. As mentioned in the introduction, this is effectively the practice in several platforms, such as eharmony and Powerlinx. However, we have not considered the additional process of experiential learning that match-seekers obtain through repeated match

[14] https://www.iac.com/media-room/press-releases/matchcom-launches-matchcom-advisors-only-expert-advice-network-be-offered

attempts. Such learning can complement both the authentication and positioning supported by the platform. However, it is different in that it does not require any investment from the matching platform, beyond the ability to repeat match attempts. Also, if the platform's positioning capability is not determined by available technologies but instead is a function of its investment, it might be useful to also determine the optimal level of investment. Another possible direction for further study is the restriction of positioning to a subset of match-seekers. And yet another direction for future work is empirical analysis of the performance of online matching platforms, comparing platforms that offer both positioning and authentication, with platforms that offer only search (and perhaps also authentication).

References

Adachi, H. (2003). A search model of two-sided matching under nontransferable utility. *Journal of Economic Theory 113*(2), 182–198.

Angerer, P., S. Zimmermann, and B. R. Nault (2018). Pricing in C2C sharing platforms. *Journal of the Association for Information Systems 19*(7), 671–687.

Arora, P., M. Rahmani, and K. Ramachandran (2020). Doing Less to Do More? Optimal Service Portfolio of Non-profits that Serve Distressed Individuals. *Manufacturing and Service Operations Management (forthcoming)*.

Banerjee, A., E. Duflo, M. Ghatak, and J. Lafortune (2013). Marry for what? caste and mate selection in modern india. *American Economic Journal: Microeconomics 5*(2), 33–72.

Basu, A., S. Bhaskaran, and R. Mukherjee (2019). An Analysis of Search and Authentication Strategies for Online Matching Platforms. *Management Science 65*(5), 2412–2431.

Basu, A. and S. Muylle (2003). Authentication in e-commerce. *Communications of the ACM 46*(12), 159–166.

Bloch, F. and H. Ryder (2000). Two-sided search, marriages, and matchmakers. *International Economic Review 41*(1), 93–116.

Boudreau, J. W. and V. Knoblauch (2010). Marriage matching and intercorrelation of preferences. *Journal of Public Economic Theory 12*(3), 587–602.

Boyd, D., H. Lankford, S. Loeb, and J. Wyckoff (2003). Analyzing the determinants of the matching public school teachers to jobs: Estimating compensating differentials in imperfect labor markets. Technical report, National Bureau of Economic Research.

Bruch, E., F. Feinberg, and K. Y. Lee (2016). Extracting multistage screening rules from online dating activity data. *Proceedings of the National Academy of Sciences 113*(38), 10530–10535.

Buchanan, J. M. (1965). An economic theory of clubs. *Economica 32*(125), 1–14.

Burdett, K. and M. G. Coles (1997). Marriage and class. *The Quarterly Journal of Economics*, 141–168.

Buss, D. M. and M. Barnes (1986). Preferences in human mate selection. *Journal of personality and social psychology 50*(3), 559.

Chade, H. (2001). Two-sided search and perfect segregation with fixed search costs. *Mathematical Social Sciences 42*(1), 31–51.

Cosimano, T. F. (1996). Intermediation. *Economica*, 131–143.

Damiano, E. and L. Hao (2008). Competing matchmaking. *Journal of the European Economic Association 6*(4), 789–818.

Damiano, E. and H. Li (2007). Price discrimination and efficient matching. *Economic Theory 30*(2), 243–263.

Figueredo, A. J., J. A. Sefcek, and D. N. Jones (2006). The ideal romantic partner personality. *Personality and Individual Differences 41*(3), 431–441.

Gale, D. and L. S. Shapley (1962). College admissions and the stability of marriage. *American Mathematical Monthly*, 9–15.

Ghani, E., W. R. Kerr, and C. Stanton (2014). Diasporas and outsourcing: Evidence from odesk and india. *Management Science 60*(7), 1677–1697.

Gomes, R. and A. Pavan (2015). Matching plans for agents with vertically and horizontally differentiated preferences. Technical report, Northwestern University working paper.

Gu, Z. and Y. Xie (2013). Facilitating fit revelation in the competitive market. *Management Science 59*(5), 1196–1212.

Halaburda, H., M. Jan Piskorski, and P. Yıldırım (2018). Competing by restricting choice: The case of matching platforms. *Management Science 64*(8), 3574–3594.

Hao, L. and Y. Tan (2019). Who wants consumers to be informed? facilitating information disclosure in a distribution channel. *Information Systems Research 30*(1), 34–49.

Hitsch, G. J., A. Hortacsu, and D. Ariely (2010). Matching and sorting in online dating. *American Economic Review 100*(1), 130–63.

Jacquet, N. L. and S. Tan (2007). On the segmentation of markets. *Journal of Political Economy 115*(4), 639–664.

Kalakota, R. and A. B. Whinston (1996). *Frontiers of Electronic Commerce*. USA: Addison Wesley Longman Publishing Co., Inc.

McAfee, R. P. (2002). Coarse matching. *Econometrica 70*(5), 2025–2034.

McNamara, J. M. and E. Collins (1990). The job search problem as an employer-candidate game. *Journal of Applied Probability*, 815–827.

Piskorski, M. J., H. Halaburda, and T. Smith (2008). Eharmony.

Roth, A. E. (1989). Two-sided matching with incomplete information about others' preferences. *Games and Economic Behavior 1*(2), 191–209.

Roth, A. E. and M. A. O. Sotomayor (1992). *Two-sided matching: A study in game-theoretic modeling and analysis*. Number 18. Cambridge University Press.

Shapley, L. S. and M. Shubik (1971). The assignment game i: The core. *International Journal of Game Theory 1*(1), 111–130.

Smith, L. (2006). The marriage model with search frictions. *Journal of Political Economy 114*(6), 1124–1144.

Sun, M. and R. K. Tyagi (2020). Product fit uncertainty and information provision in a distribution channel. *Production and Operations Management 29*(10), 2381–2402.

Xu, Y. and H. Yang (2019). Targeted search with horizontal differentiation in the marriage market. *Journal of Economic Behavior & Organization 164*, 31–62.

Appendix: Proofs

Proof of Proposition 1

i) and ii) To determine the threshold, we compare V_{3L} and V_{3H} from equations 7 and 8 respectively. This gives us that $v_{1PB} \geq v_{2PB}$ iff $\lambda \geq \lambda_{PB} = 1 - \delta$. Additionally, it is easy to see that λ is decreasing in δ.

Proof of Proposition 2

i) and ii) First consider the case in which only the H-class uses the platform. In this case, we can compare V_{4Ha} and V_{4Hb} from equations 12 and 13 respectively to determine the threshold above which $V_{4Ha} > V_{4Hb}$. It can be seen that this happens when $\lambda \geq \lambda_{PH} = 1 - \alpha\delta$.

For the case in which the L-class uses the platform, we compare V_{5La} and V_{5Lb} from equations 15 and 16. This gives us that $V_{5La} > V_{5Lb}$ iff $\lambda \geq \lambda_{PL} = 1 - (1-\alpha)\,\delta$.

Finally, it can be seen easily that both λ_{PH} and λ_{PL} are greater than λ_{PB}, and that these thresholds are decreasing in δ.

Proof of Lemma 1

First note that when only the L-class uses the platform while the H-class uses direct search, the value functions must satisfy the conditions $V_H^{od} > V_H^{oo}$ and $V_L^{od} > V_{iL}^{dd}$. Depending on the value of λ, we need to consider three cases viz., 1) $\lambda > \lambda_{PL}$ and 2) $\lambda_{PB} \leq \lambda \leq \lambda_{PL}$ and 3) $\lambda < \lambda_{PB}$. Let us consider these cases separately:

1. Case 1: $\lambda > \lambda_{PL}$. Here, for an equilibrium strategy, we will require that

$$\begin{aligned} p &\geq p_{1a} = \frac{1}{4}\left(\alpha v_H\left(2 - 2\delta\mu\left(1-\alpha\mu\right) + \delta - 2\mu\right) + \left(1-\alpha\right)\left(\delta+2\right)v_L\right) \\ p &\leq p_{2a} = \frac{1}{4}\left((1-\alpha)v_L\left(\delta\left(\alpha + 2\mu^2 - 2\mu + 1\right) + \lambda - 2\mu + 1\right) - 2\alpha\mu v_H\left(\delta\left(1-\mu\right)+1\right)\right) \end{aligned}$$

Comparing the upper and lower bounds p_1 and p_2, we have that

$$p_{1a} - p_{2a} = \frac{1}{4}\left(\alpha v_H\left(2(\alpha-1)\delta\mu^2+\delta+\lambda+1\right)+(\alpha-1)v_L(\alpha\delta+2\mu(\delta(\mu-1)-1))\right) > 0$$

when $0 < \alpha, \mu, \delta < 1$. It follows that this cannot be an equilibrium solution in this range.

2. Case 2: $\lambda_{PB} \leq \lambda \leq \lambda_{PL}$. Here the conditions simplifies to

$$\begin{aligned} p &\geq p_{1a} = \frac{1}{4}\left(\alpha v_H\left(2-2\delta\mu\left(1-\alpha\mu\right)+\delta-2\mu\right)+\left(1-\alpha\right)\left(\delta+2\right)v_L\right) \\ p &\leq p_{2b} = \frac{1}{2}\left(\left(1-\alpha\right)v_L\left(\alpha\delta+\left(\mu-1\right)\left(\delta\mu-1\right)\right)-\alpha\mu v_H\left(\delta\left(1-\mu\right)+1\right)\right) \end{aligned}$$

Again, comparing the upper and lower bounds, we have

$$p_{1a} - p_{2b} = \frac{1}{4}\left(\alpha v_H\left(2(\alpha-1)\delta\mu^2+\delta+\lambda+1\right)+(\alpha-1)v_L\left(\delta\left(2\alpha+2\mu^2-2\mu-1\right)-\lambda-2\mu+1\right)\right) > 0$$

when $0 < \alpha, \mu, \delta < 1$. It follows that this cannot be an equilibrium solution in this range.

Case 3: $\lambda < \lambda_{PB}$. Here, the condition simplifies to

$$\begin{aligned} p &\geq p_{1b} = \frac{1}{2}\left(\alpha v_H\left(\alpha\delta\mu^2-(\delta+1)\mu+1\right)-\alpha v_L+v_L\right) \\ p &\leq p_{2b} = \frac{1}{2}(\alpha\mu v_H(\delta(\mu-1)-1)-(\alpha-1)v_L(\alpha\delta+(\mu-1)(\delta\mu-1))) \end{aligned}$$

Similar to before, the comparison of upper and lower bounds gives us

$$p_{1b} - p_{2b} = \frac{1}{2}\left(v_H\left(\alpha-\left(1-\alpha\right)\alpha\delta\mu^2\right)+\left(1-\alpha\right)v_L\left(\mu\left(\delta\left(1-\mu\right)+1\right)-\alpha\delta\right)\right) > 0$$

when $0 < \alpha, \mu, \delta < 1$. It follows that this cannot be an equilibrium solution in this range. So the L-class using the platform and the H-class using direct search cannot be an equilibrium. So we focus on the remaining cases.

Both classes of match-seekers use direct search if $V_H^{dd} > V_H^{do}$ and $V_L^{dd} > V_L^{od}$. This results in the following condition:

$$V_H^{dd} \geq \max[V_{4Ha}, V_{4Hb}] - p$$
$$V_L^{dd} \geq \max[V_{5La}, V_{5Lb}] - p$$

Thus direct search is an equilibrium when $p \geq p_{1P}$ where

$$p_{1p} = \max\left[\max[V_{4Ha}, V_{4Hb}] - V_H^{dd}, \max[V_{5La}, V_{5Lb}] - V_L^{dd}\right]$$

The H-class uses the platform while the L-class uses direct search if $V_H^{do} > V_H^{dd}$ and $V_L^{do} > V_L^{oo}$. From equations 6, 7, 8, 12, 13 and 14, we have the following condition depending on the value of λ. Consider $\lambda > \lambda_{PH}$. Here the condition simplifies to

$$V_H^{do} = V_{4Ha} - p \geq V_H^{dd}$$
$$V_L^{do} = V_{4L} \geq V_{3H} - p$$

So this case is an equilibrium only if $V_{3H} - V_{4L} \leq p \leq V_{4Ha} - V_H^{dd}$. Additionally, since $\lambda > \lambda_{PH}$, an H-class will choose to match only with candidates signaled as compatible.

Similarly, when $\lambda < \lambda_{PH}$, we require

$$V_H^{do} = V_{4Hb} - p \geq V_H^{dd}$$
$$V_L^{do} = V_{4L} \geq \max[V_{3H}, V_{3L}] - p$$

which results in the condition $\max[V_{3H}, V_{3L}] - V_{4L} \leq p \leq V_{4Hb} - V_H^{dd}$. Here since $\lambda < \lambda_{PH}$, the H-class match-seeker will accept a match from all candidates.

When both types use the platform, their value functions must satisfy $V_H^{oo} > V_H^{od}$ and $V_L^{oo} > V_L^{do}$. Therefore the access fee p must satisfy the following conditions, depending on the value of λ. Suppose $\lambda < \lambda_{PB}$

$$V_H^{oo} = V_{3L} - p \geq V_H^{od}$$
$$V_L^{oo} = V_{3L} - p \geq V_L^{do}$$

which results in the condition $p \leq \max\left[V_{3L} - V_H^{od}, V_{3L} - V_L^{do}\right]$. Here too, since $\lambda < \lambda_{PB}$, both classes of match-seekers will accept matches from all candidates.

Finally, when $\lambda_{PB} \leq \lambda \leq \lambda_{PH}$, the access fee conditions reduce to

$$\begin{aligned} V_H^{oo} &= V_{3H} - p \geq V_H^{od} \\ V_L^{oo} &= V_{3H} - p \geq V_L^{do} \end{aligned}$$

This results in the condition $p \leq \max\left[V_{3H} - V_H^{od}, V_{3H} - V_L^{do}\right]$. Finally, since $\lambda \geq \lambda_{PB}$, both classes of match-seekers will accept matches only from candidates signaled as compatible.

This characterizes all the thresholds on λ and p for the equilibrium behavior of all match-seekers.

Proof of Proposition 3

As shown in Proposition 1, the participation behavior is such that either (i) only the H-class uses the platform or (ii) both classes use the platform. When the access fee is p, the profits under the case (i) in which only the H-class uses the platform is $\pi_{PH} = \alpha p - c_P$ while that in case (ii) in which both classes use the platform will be $\pi_{PB} = p - c_P$. Let us look at each of these cases in turn.

Case 1. $\lambda < \lambda_{PB}$:

Here, $\pi_{PH} = \alpha\left(V_{4Ha} - \max\left[V_{eH}^{dd}, V_{iH}^{dd}\right]\right) - c_P$ and $\pi_{PB} = V_{3L} - \max\left[V_H^{od}, V_L^{od}\right] - c_P$. First note that when $\eta > \eta_H$, $V_{eH}^{dd} > V_{iH}^{dd}$. We will compare profits when $\eta > \eta_H$.

In this case, there are two possibilities depending on the relative values of V_H^{od} and V_L^{od}. First, suppose $V_H^{od} < V_L^{od}$. The difference in profit for the two cases is

$$\begin{aligned} \pi_{PH} - \pi_{PB} &= \frac{1}{2}\left(\alpha v_H\left(\alpha^2\delta\left(\mu^2 - 1\right) - \alpha(\delta+1)(\mu-1) - 1\right)\right. \\ &\quad \left. + (1-\alpha)\, v_L\left(\left(\alpha^2 + \alpha - 1\right)\delta\mu^2 + \mu\left(1 - \alpha\delta + \delta\right) - 1\right)\right) \\ \frac{\partial\left(\pi_{PH} - \pi_{PB}\right)}{\partial v_H} &= \frac{\alpha}{2}\left(\alpha\left(1-\mu\right)\left(1 - \delta\left(\alpha\mu + \alpha - 1\right)\right) - 1\right) < 0 \end{aligned}$$

Since $\pi_{PH} - \pi_{PB}$ is decreasing in v_H, the highest value occurs when $v_H = v_L$ where

$$\pi_{PH} - \pi_{PB}\big|_{v_H = v_L} = v_L\left(-\alpha^3\delta + \alpha^2\left(1 + \delta - \mu\right) - \alpha\mu\left(1 + 2\delta\left(1-\mu\right)\right) - \left(1-\mu\right)\left(1 - \delta\mu\right)\right) < 0$$

Now suppose $V_H^{od} \geq V_L^{od}$. The difference in profit for the two cases is

$$\pi_{PH} - \pi_{PB} = \frac{(1-\alpha)}{2}\left(\alpha v_H\left(\delta\left(\alpha+\mu-\alpha\mu^2\right)+\mu-1\right) - v_L\left(1+\alpha\delta\mu\left(1-\alpha\mu\right)\right)\right)$$

First, note that the co-efficient for v_L is negative. So, the above expression is positive only if the co-efficient of v_H is also positive i.e. $\left(\delta\left(\alpha+\mu-\alpha\mu^2\right)+\mu-1\right) > 0$. i.e only if $\alpha > \frac{1-\delta\mu-\mu}{\delta(1-\mu^2)}$. Additionally, for high-class only using the platform to be feasible, we require that $p_{1P} < p < p_{2P}$, where

$$\begin{aligned} p_{1P} &= \frac{1}{2}\left(\alpha v_H + (1-\alpha)v_L\left(1+(1-\alpha)\delta\mu^2 - (1+\delta)\mu\right)\right) \\ p_{2P} &= \frac{1}{2}\left(\alpha v_H(1-\mu)\left(\delta(-\alpha\mu-\alpha+1)+1\right) - \delta\mu v_L(1-\alpha)(1-\alpha\mu)\right) \end{aligned}$$

Comparing p_{1P} and p_{2P}, we see that

$$\begin{aligned} p_{2P} - p_{1P} &= \frac{1}{2}\left(\alpha v_H\left(\delta(1-\mu)(1-\alpha\mu-\alpha)-\mu\right) - v_L(1-\alpha)\left(\mu-(1-2\alpha)\delta\mu^2-1\right)\right) \\ \frac{\partial\left(p_{2P}-p_{1P}\right)}{\partial v_H} &= \frac{\alpha}{2}\left(\delta(1-\mu)(1-\alpha\mu-\alpha)-\mu\right) \end{aligned}$$

Thus $\frac{\partial(p_{2P}-p_{1P})}{\partial v_H} < 0$ when $\alpha > \frac{\delta-\delta\mu-\mu}{\delta(1-\mu^2)}$. So the maximum value of $p_{2P}-p_{1P}$ will occur when $v_H = v_L$. In addition, the coefficient of v_H is negative when $\alpha > \frac{\delta-\delta\mu-\mu}{\delta(1-\mu^2)}$. Since $p_{2P}-p_{1P}$ can be positive only if v_H is sufficiently high, it is sufficient to verify only the sign of the coefficient of v_L at $\alpha = \frac{\delta-\delta\mu-\mu}{\delta(1-\mu^2)}$. It can be see that $\left(\mu-(1-2\alpha)\delta\mu^2-1\right)\Big|_{\alpha=\frac{\delta-\delta\mu-\mu}{\delta(1-\mu^2)}} = -\frac{\mu^2\left(\delta(\mu-1)^2-2\mu\right)}{\mu^2-1}+\mu-1<0$. Since $\frac{1-\delta\mu-\mu}{\delta(1-\mu^2)} > \frac{\delta-\delta\mu-\mu}{\delta(1-\mu^2)}$, it follows that when $V_H^{od} \geq V_L^{od}$, then $p_{2P} < p_{1P}$. It follows that offering the platform only to H-class is infeasible. This eliminates all cases in which only the H-class using the platform is optimal for the platform for $\lambda < \lambda_{PB}$ and $\eta > \eta_H$.

Finally, when $\eta < \eta_H$, the value of π_{PH} would only be lower; hence H-class only using the platform cannot be optimal even then.

2. $\lambda_{PB} < \lambda < \lambda_{PH}$: In this case, the fee when only the H-class uses the platform is the same as under the previous case, but the fee in the case in which both classes the platform is $V_{3H} - \max\left[V_H^{od}, V_L^{od}\right]$ which is greater than the fee in the previous case. It follows that $\pi_{PB} > \pi_{PH}$ in this range as well.

3. $\lambda > \lambda_{PH}$: In this case, the fee in the case in which only the H-class uses the platform is $V_{4Ha} - V_{iH}^{dd}$, while the fee in the case in which both classes use the platform is $V_{3H} - \max\left[V_H^{od}, V_L^{od}\right]$. Let us compare V_{4Ha} and V_{3H}. We have that

$$\begin{aligned} V_{3H} - V_{4Ha} &= \frac{1}{4}(1-\alpha)\left(v_L(1+\delta+\lambda) - \alpha\delta v_H\right) \\ \frac{\partial\left(V_{3H} - V_{4Ha}\right)}{\lambda} &= \frac{1}{4} v_L (1-\alpha) > 0 \end{aligned}$$

Thus the difference is increasing in λ and the difference is the least when $\lambda = \lambda_{PH}$. But when $\lambda = \lambda_{PH}$, $V_{4Ha} = V_{4Hb}$. From case 2, we already know that when V_{4Hb} is used in the H-class only fee, the profits when both H and L-class use the platform is higher. Thus, this should continue to be true when λ is even higher.

Thus it is never optimal for the platform to price such that only the H-class uses the platform.

Sensitivity analysis of optimal access fee

1. Effect of λ

It is optimal for the platform to set its access fee at p^* so that both classes can use the platform as described above where $p^* = \min\left[V_{3L} - \max\left[V_H^{od}, V_L^{od}\right], V_{3H} - \max\left[V_H^{od}, V_L^{od}\right]\right]$. Both V_H^{od} and V_L^{od} are independent of λ and V_{3H} and V_{3L} are increasing in λ. Additionally, $V_{3H} > V_{3L}$ when λ sufficiently high. So the optimal fee is increasing in λ.

2. Effect of α:

Let $p_{1PB} = V_{3H} - \max\left[V_H^{od}, V_L^{od}\right]$ and $p_{2PB} = V_{3L} - \max\left[V_H^{od}, V_L^{od}\right]$.

First consider the case when $\lambda < \lambda_{PB}$. Here, the optimal fee is p_{2B}. In this region,

$$\frac{\partial\left(V_H^{od} - V_L^{od}\right)}{\partial\alpha} = \frac{1}{2}\mu\left((1+\delta)(v_H + v_L) - 2\delta\mu\left((1-\alpha)v_L + \alpha v_H\right)\right) > 0$$

This implies that when α is sufficiently high, the optimal fee $p^* = V_{3L} - V_H^{od}$; otherwise it is $p^* = V_{3L} - V_L^{od}$. Let α_k be the threshold on α above which $p^* = V_{3L} - V_H^{od}$.

When $\alpha < \alpha_k$, we have that

$$\frac{\partial p^*}{\partial \alpha} = \frac{1}{2}(v_H + v_L(\mu(2(\alpha-1)\delta\mu + \delta + 1) - 1)) > 0$$

Further, when $\alpha > \alpha_k$, $p^* = V_{3L} - V_L^{od}$, and

$$\frac{\partial^2 p^*}{\partial \alpha^2} = v_H \delta \mu^2 > 0$$

which implies that p^* is convex in α in this range. Additionally, we have that

$$\frac{\partial p^*}{\partial \alpha} = \frac{1}{2}\left(2\alpha\delta\mu^2 v_H - (\delta+1)\mu v_H + v_H - v_L\right)$$

Note that $\frac{\partial p^*}{\partial \alpha} > 0$ if $\alpha > \alpha_m = \frac{v_H(\delta\mu+\mu-1)+v_L}{2\delta\mu^2 v_H}$ and $\frac{\partial p^*}{\partial \alpha} > 0$ if $\alpha < \alpha_m$. If follows that when $\alpha < \alpha_k$, p^* is increasing in α. When $\alpha_k < \alpha < \alpha_m$, p^* is decreasing in α and when $\alpha > \alpha_m$, p^* is increasing in α. This establishes the non-monotonicity of the optimal fee w.r.t α.

3. Effect of η:

To determine the sensitivity with respect to η, note that

$$\frac{\partial\left(V_H^{od} - V_L^{od}\right)}{\partial \eta} = \frac{\alpha\mu}{2}(1 + \delta - \alpha\delta\mu) > 0$$

In addition

$$\frac{\partial\left(V_{3L} - V_L^{od}\right)}{\partial \eta} = \frac{\alpha}{2} > 0$$
$$\frac{\partial\left(V_{3L} - V_H^{od}\right)}{\partial \eta} = \frac{1}{2}\alpha\left(\alpha\delta\mu^2 - (\delta+1)\mu + 1\right)$$

Note that $\frac{\partial\left(V_{3L} - V_H^{od}\right)}{\partial \eta} < 0$ if δ is sufficiently high. So $p^* = \min\left[V_{3L} - V_H^{od}, V_{3L} - V_L^{od}\right]$ is non-monotonic in η.

4. Effect of μ

For the sensitivity of p^* with respect to μ, note that V_{3H} and V_{3L} are independent of μ. so we can restrict our attention to V_H^{od} and V_L^{od}

$$\begin{aligned}\frac{\partial V_L^{od}}{\partial \mu} &= \frac{1}{2}v_L\left(1-\alpha\right)\left(1+\delta\left(1-2\left(1-\alpha\right)\mu+1\right)\right)>0\\ \frac{\partial V_H^{od}}{\partial \mu} &= \frac{1}{2}v_H\alpha\left(1+\delta\left(1-2\alpha\mu\right)\right)>0\end{aligned}$$

So $p^*=\min\left[V_{3L}-V_H^{od},V_{3L}-V_L^{od}\right]$ should be decreasing in μ.

Proof of Lemma 2

When an H-class meets a candidate signaled as incompatible in period 1, the probability that the candidate is indeed incompatible is $\left(\frac{1+\lambda}{2}\right)$ and the probability that the candidate is compatible is $1-\left(\frac{1+\lambda}{2}\right)$. So the value from accepting the match would be

$$V_a = \left(1-\frac{1+\lambda}{2}\right)v_H$$

The value from waiting until the second period in the hope for a better match is

$$V_b = \delta\left(\frac{1}{2}\left(1-\frac{1}{2}\left(1+\lambda\right)\right)+\frac{\lambda+1}{4}\right)\left(\alpha v_H+\left(1-\alpha\right)v_L\right)$$

Comparing V_a and V_b, we have that $V_a>V_b$ *iff* $\lambda<\lambda_{B1}$ where

$$\lambda_{B1} = 1-\alpha\delta-\frac{\left(1-\alpha\right)\delta}{\eta}$$

In addition, we have that

$$\lambda_{B1}-\lambda_{PB} = \frac{\left(1-\alpha\right)\delta(\eta-1)}{\eta}>0$$

Proof of Lemma 3

When an H-class meets an L-class candidate signaled as compatible in period 1, the probability that the candidate is indeed compatible is $\left(\frac{1+\lambda}{2}\right)$ and the probability that the candidate is incompatible is $1-\left(\frac{1+\lambda}{2}\right)$. So the expected value from accepting the match would be

$$V_{a1} = \left(\frac{1+\lambda}{2}\right) v_L$$

Alternatively, the expected value from rejecting the match and waiting till second period is

$$V_{b1} = \delta\left(\frac{1}{2}\left(1-\frac{1}{2}(1+\lambda)\right)+\frac{1+\lambda}{4}\right)(\alpha v_H+(1-\alpha)v_L)$$

Comparing them, we see that $V_{b1} > V_{b2}$ *iff* $\lambda > \lambda_{B2}$ where

$$\lambda_{B2} = \delta + \alpha\delta(\eta-1) - 1$$

In addition,

$$\lambda_{B2} - \lambda_{PB} = \delta(\alpha(\eta-1)+2) - 2$$

which is positive only if α is sufficiently high.

Proof of Corollary 1

The proof follows directly from the fact that $\lambda_{B1} > \lambda_{PB}$ and that λ_{B2} is greater than λ_{PB} only if α is sufficiently high.

Proof of Proposition 4

Consider an H-class match-seeker who meets a candidate in period 1. Given the different possibilities, there are only 5 possibilities with respect to a match in period 1:

1. Accept the match only if the candidate is an H-class signaled as compatible. Let the value from this case be V_1
2. Accept any H-class candidate. Let the value from this case be V_2
3. Accept any candidate signaled as compatible. Let the value from this case be V_3

4. Accept any candidate as long as that candidate is not an L-class signaled as incompatible. Let the value from this case be V_4

5. Accept any candidate. Let the value from this case be V_5

These value functions can be written as below:

$$
\begin{aligned}
V_1 &= \frac{1}{4}\alpha(\lambda+1)v_H + \frac{1}{2}\left(1-\frac{\alpha}{2}\right)\delta(\alpha v_H + (1-\alpha)v_L) \\
V_2 &= \frac{\alpha v_H}{2} + \frac{1}{2}(1-\alpha)\delta(\alpha v_H + (1-\alpha)v_L) \\
V_3 &= \frac{1}{4}\delta(\alpha v_H + (1-\alpha)v_L) + \frac{1}{4}(\lambda+1)(\alpha v_H + (1-\alpha)v_L) \\
V_4 &= \frac{\alpha v_H}{2} + \frac{1}{4}(1-\alpha)\delta(\alpha v_H + (1-\alpha)v_L) + \frac{1}{4}(1-\alpha)(\lambda+1)v_L \\
V_5 &= \frac{1}{2}\left(\alpha v_H + (1-\alpha)\, v_L\right)
\end{aligned}
$$

First let us examine when V_5 is the highest of the values which will ensure that accepting any candidate match is the optimal policy of an H-class match-seeker. Comparing V_4 and V_5, it can be seen that $V_5 > V_4$ when $\alpha < \alpha_1$, where

$$\alpha_1 = \frac{(1-\delta-\lambda)}{\delta(\eta-1)}$$

In addition, the following are also true:

$$V_5 > V_3 \quad \text{if} \quad \lambda < \lambda_{C0} < \lambda_{B1}$$

$$V_4 > V_3 \quad \text{if} \quad \lambda < \lambda_{B1}$$

So, this implies that $V_5 > V_3$. Also,$V_5 > V_2$ if $\alpha < \alpha_1$. Finally, $V_2 > V_1$ when $\alpha < \alpha_1$ implying that $V_5 > V_1$ when $\alpha < \alpha_1$. It follows that when $\alpha < \alpha_1$, the optimal policy of the H-class match seeker is to accept all matches.

Now let us focus on the range where $\alpha > \alpha_1$. First, note that 1) $V_1 > V_2$ and $V_3 > V_4$ if $\lambda > \lambda_{B1}$ and 2) $V_1 > V_3$ and $V_2 > V_4$ if $\lambda < \lambda_{B2}$. This implies that when $\lambda < \lambda_1 = \min[\lambda_{B1}, \lambda_{B2}]$, $V_2 > \max[V_1, V_3, V_4]$. So in this range, the optimal policy is to match with only H-class candidates.

Similarly, when $\lambda > \lambda_2 = \max[\lambda_{B1}, \lambda_{B2}]$, $V_3 > \max[V_1, V_2, V_4]$. Thus when $\lambda > \lambda_2$, the optimal policy is to match only with candidates signaled as compatible. Also, $\lambda_2 > \lambda_1$ by definition.

Finally comparing λ_{B1} and λ_{B2}, we see that, we see $\lambda_{B1} > \lambda_{B2}$ when $\alpha < \alpha_2$ where

$$\alpha_2 = \frac{((2-\delta)\,\eta - \delta)}{\delta\,(\eta^2 - 1)} > \alpha_1$$

So when $\alpha < \alpha_2$, $\lambda_1 < \lambda < \lambda_2$ is equivalent to $\lambda_{B2} < \lambda < \lambda_{B1}$ and $V_4 > \max[V_1, V_2, V_3]$. Thus, the optimal policy in this range would be to match with all candidates other than L-class candidates signaled as incompatible.

Similarly, when $\alpha > \alpha_2$, $\lambda_1 < \lambda < \lambda_2$ is equivalent to $\lambda_{B1} < \lambda < \lambda_{B2}$ and $V_1 > \max[V_2, V_3, V_4]$. It follows that the optimal policy in this range would be to match with only H-class candidates signaled as compatible.

Proof of Proposition 5

The value functions of the L-class depends on the optimal policy of the H-class.

Case 1: First consider the case when the H-class accepts only matches from an H-class candidate signaled as compatible i.e. $\alpha > \alpha_2$and $\lambda_{B1} < \lambda < \lambda_{B2}$. Here, the L-class match-seeker can pursue three options:

a. Accept matches from everyone other than L-class candidates signaled as incompatible: The value from this policy would be

$$V_{1La} = \frac{1}{2}\left(1 - \frac{1-\alpha}{2}\right)\delta\,(\alpha v_H + (1-\alpha)v_L) + \frac{1}{4}\,(1-\alpha)\,(\lambda+1)\,v_L$$

b. Accept matches from all L-class candidates: The value from this policy would be

$$V_{1Lb} = \frac{1}{2}\alpha\delta\,(\alpha v_H + (1-\alpha)v_L) + \frac{1}{2}(1-\alpha)v_L$$

c. Reject all matches in period 1 for a better match period 2: The value from this policy would be

$$V_{1Lc} = \frac{\delta}{2}\left(\alpha v_H + (1-\alpha)\, v_L\right)$$

Comparing these values, it can be seen than $V_{1La} > V_{1Lb}$ if $\alpha > \alpha_1$. In addition, $V_{1Lc} > V_{1La}$ if $\lambda < \lambda_{B2}$. So the optimal policy of the L-class in this case would be to reject all matches and wait for better one in Period 2.

Case 2: Suppose an H-class match-seeker accepts matches only from any other H-class candidate i.e. $\lambda < \min\left[\lambda_{B1}, \lambda_{B2}\right]$. Here again, the L-class match-seeker has the same three options as in the previous case. The value functions from these cases are also identical. Since $\lambda < \lambda_{B2}$ in this case, the optimal policy of the L-class remains wait until second period for a match.

Case 3: Suppose the H-class accepts only matches from candidates who are signalled as compatible i.e., $\lambda > \max\left[\lambda_{B1}, \lambda_{B2}\right]$. Here, in addition to previous options, the L-class can also choose to accept all candidates signaled as compatible, the value of which is

$$V_{3La} = V_{3H} > V_{1La}$$

Since $\lambda > \lambda_{B2}$, the optimal policy of the L-class would be to accept a match from all candidates signaled as compatible.

Case 4: Suppose the H-class will accept any candidate as long as that candidate is not an L-class signaled as incompatible i.e. $\lambda_{B2} < \lambda < \lambda_{B1}$ and $\alpha_1 < \alpha < \alpha_2$. The options for the L-class are the same as in case 1. However, the value function associated with accepting all matches other than L-class candidates signaled as incompatible is

$$V_{4La} = \frac{1}{4}\delta\left(\alpha v_H + (1-\alpha) v_L\right) + \frac{1}{4}(\lambda+1)\left(\alpha v_H + (1-\alpha) v_L\right) > V_{1La}$$

Since $\lambda > \lambda_{B2}$,the optimal policy of the L-class would be to accept all matches other than with L-class candidates signaled as incompatible.

Case 5: Suppose the H-class accepts all candidate matches, i.e. $\alpha < \alpha_1$. In this case. it is optimal for the L-class also to accept all matches and the value from that match is V_{3L}.

Proof of Lemma 4

Several cases in the table can be eliminated as an equilibrium strategy as described below. First, the L-class purchasing authentication is a dominated strategy. Second, since, $V_H^{da} > V_H^{do}$, so the case in which the H-class purchases authentication when the L-class uses direct search is also a dominated strategy. Finally, from Proposition 1, we know that the case where the L-class uses the platform while the H-class uses direct search is also not an equilibrium. That leaves only the 4 cases which are given in the proposition to be the possible equilibria. Let us examine these cases one by one.

1. Both classes of match-seekers use direct search if $\max\left[V_H^{do}, V_H^{da}\right] = V_H^{do}$ and $V_L^{dd} > V_L^{od}$. This again results in the following condition:

$$\begin{aligned} V_H^{dd} &\geq \max\left[V_{4Ha}, V_{4Hb}\right] - p \\ V_L^{dd} &\geq \max\left[V_{5La}, V_{5Lb}\right] - p \end{aligned}$$

Thus direct search is an equilibrium when $p \geq p_{2B}$ where

$$p_{2B} = \max\left[\max\left[V_{4Ha}, V_{4Hb}\right] - V_H^{dd}, \max\left[V_{5La}, V_{5Lb}\right] - V_L^{dd}\right]$$

2. The H-class uses the platform while the L-class uses direct search if $V_H^{do} > V_H^{dd}$ and $V_L^{do} > V_L^{oo}$. In the proof of Proposition 1, we already showed that this case is an equilibrium only if $V_{3H} - V_{4L} \leq p \leq V_{4Ha} - V_H^{dd}$ when $\lambda > \lambda_{PH}$, and if $\max\left[V_{3H}, V_{3L}\right] - V_{4L} \leq p \leq V_{4Hb} - V_H^{dd}$ when $\lambda < \lambda_{PH}$.

3. Now consider the case when both classes the platform and neither purchase authentication. Let V_{BH} be the value obtained by the H-class when they follow the optimal policy as described in Proposition 4 and V_{BL} be the value derived by the L-class when they follow the optimal policy in Proposition 5.

As before, the value functions of both classes must satisfy $V_H^{oo} > \max\left[V_H^{oa}, V_H^{od}\right]$ and $V_L^{oo} > \max\left[V_L^{do}, V_L^{oa}\right]$. Again as described in the proof of Proposition 1, it results in the condition $p \leq \max\left[\max\left[V_{3H}, V_{3L}\right] - V_H^{od}, \max\left[V_{3H}, V_{3L}\right] - V_L^{do}, V_{BL} - V_{4L}\right]$.

In addition, the comparison of V_H^{oo} and V_H^{oa} results in the condition on authentication fee. Since $V_H^{oa} = V_{BH} - p - a$ this results in the condition that $q \geq V_{BH} - \max\left[V_{3H}, V_{3L}\right]$

4. Finally, when both classes use the platform and only the H-class purchases the authentication service, $V_H^{oa} > \max\left[V_H^{oo}, V_H^{od}\right]$and $V_L^{oa} > V_L^{da} = V_{4L}$. This results in the following condition

$$
\begin{aligned}
p &< \min\left[\max\left[V_{3H}, V_{3L}\right] - V_H^{od}, \max\left[V_{3H}, V_{3L}\right] - V_L^{do}, V_{BL} - V_{4L}\right] \\
q &< \min\left[V_{BH} - \max\left[V_{3H}, V_{3L}\right], V_{BH} - V_{4L}\right]
\end{aligned}
$$

Lemma 5. *Let $\lambda_L = 1 - \delta\left(1 + \alpha\left(\eta - 1\right)\right)$. Then the optimal access fee p^* and authentication fee q^* can be characterized as a function of the firm's positioning capability as represented in Table 3 below.*

Proof of Lemma 5

The optimal access and authentication fee needs to be determined only for the case in which both classes use the platform, but only the H-class purchases authentication. In this case, let p be the access fee, q be the authentication fee and t be the total fee (the sum of access and authentication fees).

From the proof of Proposition 4, we know that for this case to be an equilibrium, we require $V_H^{oa} > V_H^{oo}$, $V_H^{oa} > V_H^{od}$ and $V_L^{oa} > V_L^{da}$. This results in the following condition

$$
\begin{aligned}
p &\leq p_m = V_{BL} - V_{4L} \\
q &\leq q_m = V_{BH} - \max\left[V_{3H}, V_{3L}\right] \\
t &\leq t_m = V_{BH} - V_{5H}
\end{aligned}
$$

Case	Access fee (p^*)	Authentication fee (q^*)
$\lambda < \lambda_L$	$p_1^* = \min\left[\frac{1}{2}\left(\alpha v_H - (1-\alpha)\, v_L \left((1+\delta)\,\mu - (1-\alpha)\,\delta\mu^2 - 1\right)\right),\right.$ $\left.\frac{1}{2}\left(\alpha v_H \left(\alpha\delta\mu^2 - (\delta+1)\mu + 1\right) + (1-\alpha)\, v_L\right)\right]$	0
$\max\left[\lambda_L, \lambda_{B2}\right] < \lambda < \lambda_{PB}$	$p_3^* = \min\left[\frac{1}{4}\left(\alpha v_H \left(\alpha\delta\left(2\mu^2-1\right) - 2\delta\mu + \delta - 2\mu + 2\right)\right.\right.$ $\left. + (1-\alpha)\, v_L\, (1 + \lambda - (1-\alpha)\,\delta)\right),$ $\frac{1}{4}\left(\alpha v_H\, (\delta + \lambda + 1) + \right.$ $\left.\left.(1-\alpha)\, v_L \left(\delta\left(2\,(1-\alpha)\,\mu^2 - 2\mu + 1\right) + \lambda - 2\mu + 1\right)\right)\right]$	$\min\left[\frac{1}{4}\,(1-\alpha)\,(\alpha\delta v_H + (1-\alpha)\,\delta v_L + \lambda v_L - v_L),\right.$ $\frac{1}{4}\left(\alpha v_H \left(\alpha\delta\left(2\mu^2-1\right) - 2\delta\mu + \delta - 2\mu + 2\right) + \right]$ $(1-\alpha)\, v_L\, ((1-\alpha)\,\delta + \lambda + 1)) - p_3^*]$
$\lambda < \min\left[\lambda_{B2}, \lambda_{PB}\right]$	$p_2^* = \min\left[\frac{1}{2}\left(\alpha\,(1-\mu)\, v_H\, (1 + \delta\,(1 - \alpha\mu - \alpha)) + (1-\alpha)^2\,\delta v_L\right),\right.$ $\left.\frac{1}{2}\left(\alpha\delta v_H - (1-\alpha)\, v_L \left(\mu - \delta\left(1 + \mu - (1-\alpha)\,\mu^2\right)\right)\right)\right]$	$\min\left[\frac{1}{2}\,(1-\alpha)\,(\alpha\delta v_H + (1-\alpha)\,\delta v_L - v_L),\right.$ $\left.\frac{1}{2}\left(\alpha(1-\mu) v_H\, (1 - \delta\,(1 - \alpha\mu - \alpha)) + (1-\alpha)^2\,\delta v_L\right) - p_2^*\right]$
$\lambda_{PB} < \lambda < \min\left[\lambda_{B2}, \lambda_{B1}\right]$	$p_4^* = \min\left[\frac{1}{2}\left(\alpha\,(1-\mu)\, v_H\, (1 + \delta\,(1 - \alpha\mu - \alpha)) + (1-\alpha)^2\,\delta v_L\right),\right.$ $\left.\frac{1}{2}\left(\alpha\delta v_H + (1-\alpha)\, v_L \left(1 - \mu + \delta\left((1-\alpha))\,\mu^2\right) - \mu\right)\right)\right]$	$\min\left[\frac{1}{4}\,(\alpha v_H\, (1 - 2\alpha\delta + \delta - \lambda) - (1-\alpha)\, v_L\, (1 + \lambda - (1-2\alpha)\,\delta)),\right.$ $\left.\frac{1}{2}\left(\alpha\,(1-\mu)\, v_H\, (1 + \delta\,(1 - \alpha\mu - \alpha)) + (1-\alpha)^2\,\delta v_L\right) - p_4^*\right]$
$\max\left[\lambda_{B2}, \lambda_{PB}\right] < \lambda < \lambda_{B1}$	$p_5^* = \min\left[\frac{1}{4}\left(\alpha v_H \left(\alpha\delta\left(2\mu^2-1\right) - 2\delta\mu + \delta - 2\mu + 2\right)\right.\right.$ $\left. + (1-\alpha)\, v_L\, ((1-\alpha)\,\delta + \lambda + 1)\right),$ $\frac{1}{4}\left(\alpha v_H\, (\delta + \lambda + 1) + \right.$ $\left.\left.(1-\alpha)\, v_L \left(\delta\left(2\,(1-\alpha)\,\mu^2 - 2\mu + 1\right) + \lambda - 2\mu + 1\right)\right)\right]$	$\min\left[\frac{1}{4}\alpha\,(v_H\, (1 - \alpha\delta - \lambda) - (1-\alpha)\,\delta v_L),\right.$ $\frac{1}{4}\left(\alpha v_H \left(\alpha\delta\left(2\mu^2-1\right) - 2\delta\mu + \delta - 2\mu + 2\right) + \right]$ $(1-\alpha)\, v_L\, ((1-\alpha)\,\delta + \lambda + 1)) - p_5^*]$
$\lambda_{B1} < \lambda < \lambda_{B2}$	$p_6^* = \min\left[\frac{1}{2}\left(\alpha\delta v_H + (1-\alpha) v_L \left(1 - \mu + \delta\left((1-\alpha)\mu^2\right) - \mu\right)\right),\right.$ $\frac{1}{4}\left(\alpha v_H \left(\delta\left(2\alpha\mu^2 - \alpha - 2\mu + 2\right) + \lambda - 2\mu + 1\right)\right.$ $\left.\left. + \left(\alpha^2 - 3\alpha + 2\right)\delta v_L\right)\right]$	$\min\left[\frac{1}{4}\,(1-\alpha)\,(\alpha\delta v_H - v_L\, (1 + \lambda - (1-\alpha)\delta)),\right.$ $\frac{1}{4}\left(\alpha v_H \left(\delta\left(2\alpha\mu^2 - \alpha - 2\mu + 2\right) + \lambda - 2\mu + 1\right)\right.$ $\left.\left. + \left(\alpha^2 - 3\alpha + 2\right)\delta v_L\right) - p_6^*\right]$
$\lambda > \max\left[\lambda_{B2}, \lambda_{B1}\right]$	$p_7^* = \min\left[\frac{1}{4}\left(\alpha v_H \left(2\alpha\delta\mu^2 - 2\delta\mu + \delta + \lambda - 2\mu + 1\right)\right.\right.$ $\left. + (1-\alpha)\, v_L\, (\delta + \lambda + 1)\right),$ $\frac{1}{4}\left(\alpha v_H\, (\delta + \lambda + 1) + \right.$ $\left.\left.(1-\alpha)\, v_L \left(\delta\left(2\,(1-\alpha)\,\mu^2 - 2\mu + 1\right) + \lambda - 2\mu + 1\right)\right)\right]$	0

Table 3: Optimal access and authentication fees

In addition, the profits of the platform are $\pi = p + \alpha (q - c_A) - c_P$ where $p, q \geq 0$, where p and q satisfies the above constraints. Given that the profit function is increasing in p, q and that $0 < \alpha < 1$, the optimal fees has to be one of the following:

- If $q_m \leq 0$, then $p^* = p_m$ and $q^* = \epsilon \approx 0$
- If $p_m + q_m < t_m$, then $p^* = p_m$ and $q^* = q_m$.
- If $p_m < t_m < p_m + q_m$, then $p^* = p_m$ and $q^* = t_m - p_m$.
- If $t_m < p_m$, then $p^* = t_m$ and $q^* = \epsilon \approx 0$.

Since these values depend on the optimal policy of the match-seekers characterized in Proposition 4 and 5, the optimal fees will depend on the policy of the match-seekers.

Let us examine the values for each of these regions.

Case 1: H-class accepts all matches: This is optimal when $\alpha < \alpha_1$. When this condition is satisfied, $\lambda < \lambda_L = 1 - \delta (1 + \alpha (\eta - 1))$. Also, the optimal policy of the L-class is also to accept all matches. The expected value of both classes are $V_H^{oa} = V_5 - p - q$ and $V_L^{oa} = V_5 - p$. This implies that $p_m = V_5 - V_{4L}$, $q_m = V_5 - V_{3L} = 0$; $t_m = V_5 - V_{5H}$. This implies that the prices have to be as follows:

$$
\begin{aligned}
p^* &= \min \left[V_{3L} - V_{4L}, V_{3L} - V_{5H} \right] \\
&= \min \left[\frac{1}{2} \left(\alpha v_H - (1-\alpha) v_L \left((1+\delta) \mu - (1-\alpha) \delta \mu^2 - 1 \right) \right), \right. \\
&\qquad \left. \frac{1}{2} \left(\alpha v_H \left(\alpha \delta \mu^2 - (\delta + 1) \mu + 1 \right) + (1-\alpha) v_L \right) \right] \\
q^* &= 0
\end{aligned}
$$

Case 2: H-class rejects only L-class candidates signaled as incompatible: This is optimal when $\alpha_1 < \alpha < \alpha_2$ and $\lambda_{B2} < \lambda < \lambda_{B1}$, which can be rewritten as $\max [\lambda_L, \lambda_{B2}] < \lambda < \lambda_{B1}$ and $\alpha < \alpha_2$. The optimal policy of the L-class is also to reject all L-class candidates signaled as incompatible. So, the expected value of both classes in this region are $V_H^{oa} = V_4 - p - q$ and $V_L^{oa} = V_{4La} - p$. This implies that $p_m = V_{4La} - V_{4L}$, $q_m = V_4 - \max [V_{3H}, V_{3L}]$ and $t_m = V_4 - V_{5H}$.

Since $\lambda_{B1} > \lambda_{PB}$, (and recall that $V_{3H} > V_{3L}$ when $\lambda > \lambda_{PB}$) the optimal access and authentication fees will be as follows, depending on λ. If $\max[\lambda_L, \lambda_{B2}] < \lambda < \lambda_{PB}$, the optimal fees will be

$$\begin{aligned}
p^* &= \min[V_{4La} - V_{4L}, V_4 - V_{5H}] \\
&= \min\Big[\frac{1}{4}\left(\alpha v_H\left(\alpha\delta\left(2\mu^2-1\right) - 2\delta\mu + \delta - 2\mu + 2\right) + (1-\alpha)v_L(1+\lambda-(1-\alpha)\delta)\right), \\
&\quad \frac{1}{4}\left(\alpha v_H(\delta+\lambda+1) + (1-\alpha)v_L\left(\delta\left(2(1-\alpha)\mu^2 - 2\mu + 1\right) + \lambda - 2\mu + 1\right)\right)\Big] \\
q^* &= \min[V_4 - V_{3L}, V_4 - V_{5H} - p^*] \\
&= \min\Big[\frac{1}{4}(1-\alpha)(\alpha\delta v_H + (1-\alpha)\delta v_L + \lambda v_L - v_L), \\
&\quad \frac{1}{4}\left(\alpha v_H\left(\alpha\delta\left(2\mu^2-1\right) - 2\delta\mu + \delta - 2\mu + 2\right) + (1-\alpha)v_L((1-\alpha)\delta + \lambda + 1)\right) - p^*\Big]
\end{aligned}$$

If $\lambda_{PB} < \lambda < \lambda_{B1}$, the optimal fees will be

$$\begin{aligned}
p^* &= \min[V_{4La} - V_{4L}, V_4 - V_{5H}] \\
&= \min\Big[\frac{1}{4}\left(\alpha v_H\left(\alpha\delta\left(2\mu^2-1\right) - 2\delta\mu + \delta - 2\mu + 2\right) + (1-\alpha)v_L(1+\lambda-(1-\alpha)\delta)\right), \\
&\quad \frac{1}{4}\left(\alpha v_H(\delta+\lambda+1) + (1-\alpha)v_L\left(\delta\left(2(1-\alpha)\mu^2 - 2\mu + 1\right) + \lambda - 2\mu + 1\right)\right)\Big] \\
q^* &= \min[V_4 - V_{3H}, V_4 - V_{5H} - p^*] \\
&= \min\Big[\frac{1}{4}\alpha(v_H(1-\alpha\delta-\lambda) - (1-\alpha)\delta v_L), \\
&\quad \frac{1}{4}\left(\alpha v_H\left(\alpha\delta\left(2\mu^2-1\right) - 2\delta\mu + \delta - 2\mu + 2\right) + (1-\alpha)v_L((1-\alpha)\delta + \lambda + 1)\right) - p^*\Big]
\end{aligned}$$

Case 3: H-class accepts only candidates signaled as compatible:

This is optimal when $\lambda \geq \max[\lambda_{B1}, \lambda_{B2}]$. The expected value of both classes in this region are $V_H^{oa} = V_3 - p - q$ and $V_L^{oa} = V_{3La} - p$. Here, $\lambda > \lambda_{PB}$ as well. This implies that $p_m = V_{3La} - V_{4L}$, $q_m = V_3 - V_{3H} = 0$ and $t_m = V_3 - V_{5H}$. So the optimal access and authentication fees will be

$$\begin{aligned}
p^* &= \min[V_{3La} - V_{4L}, V_3 - V_{5H}] \\
&= \min\Big[\frac{1}{4}\left(\alpha v_H\left(2\alpha\delta\mu^2 - 2\delta\mu + \delta + \lambda - 2\mu + 1\right) + (1-\alpha)v_L(\delta+\lambda+1)\right), \\
&\quad \frac{1}{4}\left(\alpha v_H(\delta+\lambda+1) + (1-\alpha)v_L\left(\delta\left(2(1-\alpha)\mu^2 - 2\mu + 1\right) + \lambda - 2\mu + 1\right)\right)\Big] \\
q^* &= 0
\end{aligned}$$

Case 4: H-class accepts only another H-class

This is optimal when $\lambda \leq \min[\lambda_{B1}, \lambda_{B2}]$. The optimal policy of the L-class is to wait until second period in this case. So the expected value of both classes in this region are $V_H^{oa} = V_2 - p - q$ and $V_L^{oa} = V_{1Lc} - p$. This implies that $p_m = V_{1Lc} - V_{4L}$, $q_m = V_2 - \max[V_{3H}, V_{3L}]$ and $t_m = V_2 - V_{5H}$. If $\lambda_{B2} < \lambda < \lambda_{PB}$, then $V_{3L} > V_{3H}$ and so the optimal fees will be

$$\begin{aligned}
p^* &= \min[V_{1Lc} - V_{4L}, V_2 - V_{5H}] \\
&= \min\left[\frac{1}{2}\left(\alpha(1-\mu)v_H(1+\delta(1-\alpha\mu-\alpha)) + (1-\alpha)^2\delta v_L\right),\right. \\
&\quad \left.\frac{1}{2}\left(\alpha\delta v_H - (1-\alpha)v_L\left(\mu - \delta\left(1+\mu-(1-\alpha)\mu^2\right)\right)\right)\right] \\
q^* &= \min[V_2 - V_{3L}, V_2 - V_{5H} - p^*] \\
&= \min\left[\frac{1}{2}(1-\alpha)(\alpha\delta v_H + (1-\alpha)\delta v_L - v_L),\right. \\
&\quad \left.\frac{1}{2}\left(\alpha(1-\mu)v_H(1-\delta(1-\alpha\mu-\alpha)) + (1-\alpha)^2\delta v_L\right) - p^*\right]
\end{aligned}$$

If $\lambda_{PB} < \lambda < \min[\lambda_{B1}, \lambda_{B2}]$, then $V_{3H} > V_{3L}$, and the optimal fees will be

$$\begin{aligned}
p^* &= \min[V_{1Lc} - V_{4L}, V_2 - V_{5H}] \\
&= \min\left[\frac{1}{2}\left(\alpha(1-\mu)v_H(1+\delta(1-\alpha\mu-\alpha)) + (1-\alpha)^2\delta v_L\right),\right. \\
&\quad \left.\frac{1}{2}\left(\alpha\delta v_H - (1-\alpha)v_L\left(\mu - \delta\left(1+\mu-(1-\alpha)\mu^2\right)\right)\right)\right] \\
q^* &= \min[V_2 - V_{3H}, V_2 - V_{5H} - p^*] \\
&= \min\left[\frac{1}{4}(\alpha v_H(1-2\alpha\delta+\delta-\lambda) - (1-\alpha)v_L(1+\lambda-(1-2\alpha)\delta)),\right. \\
&\quad \left.\frac{1}{2}\left(\alpha(1-\mu)v_H(1-\delta(1-\alpha\mu-\alpha)) + (1-\alpha)^2\delta v_L\right) - p^*\right]
\end{aligned}$$

Case 5: H-class accepts only H-class candidates signaled as compatible: This is optimal when $\alpha > \alpha_2$ and $\lambda_{B1} < \lambda < \lambda_{B2}$. The optimal policy of the L-class is to wait until second period. So the expected value of both classes in this case are $V_H^{oa} = V_1 - p - q$ and $V_L^{oa} = V_{1Lc} - p$. This implies that $p_m = V_{1Lc} - V_{4L}$, $q_m = V_1 - V_{3H}$ and $t_m = V_1 - V_{5H}$. The optimal fees will be

$$\begin{aligned} p^* &= \min\left[V_{1Lc} - V_{4L}, V_1 - V_{5H}\right] \\ &= \min\left[\frac{1}{2}\left(\alpha\delta v_H + (1-\alpha)v_L\left(1-\mu+\delta\left((1-\alpha)\mu^2\right)-\mu\right)\right),\right. \\ &\quad \left.\frac{1}{4}\left(\alpha v_H\left(\delta\left(2\alpha\mu^2-\alpha-2\mu+2\right)+\lambda-2\mu+1\right)+\left(\alpha^2-3\alpha+2\right)\delta v_L\right)\right] \\ q^* &= \min\left[V_1 - V_{3H}, V_1 - V_{5H} - p^*\right] \\ &= \min\left[\frac{1}{4}(1-\alpha)\left(\alpha\delta v_H - v_L\left(1+\lambda-(1-\alpha)\delta\right)\right),\right. \\ &\quad \left.\frac{1}{4}\left(\alpha v_H\left(\delta\left(2\alpha\mu^2-\alpha-2\mu+2\right)+\lambda-2\mu+1\right)+\left(\alpha^2-3\alpha+2\right)\delta v_L\right)-p^*\right] \end{aligned}$$

Proof of Proposition 6

Sensitivity analysis of access fee with respect to λ: First consider the range $\alpha > \alpha_2$. Here, the ordering of the λ thresholds will be as follows: $0 < \lambda_{PB} < \lambda_{B1} < \lambda_{B2} < 1$. Let us look at each of those regions separately.

1. $0 < \lambda < \lambda_{B1}$: Here the optimal access fee is $p_2^* = p_4^*$ where $p_2^* = \min\left[V_{1Lc} - V_{4L}, V_2 - V_{5H}\right]$ and is independent of λ.

2. $\lambda_{B1} < \lambda < \lambda_{B2}$: Here the optimal access fee is p_5^* where $p_5^* = \min\left[V_{1Lc} - V_{4L}, V_1 - V_{5H}\right]$. Also, $V_{1Lc} - V_{4L}$ is independent of λ and

$$\frac{\partial\left(V_1\right)}{\partial\lambda} = \frac{\alpha v_H}{4} > 0$$

So p_5^* is increasing in λ

3. $\lambda_{B2} < \lambda < 1$: Here optimal fees are $p_7^*, q_7^*$where $p_7^* = \min\left[V_{3La} - V_{4L}, V_3 - V_{5H}\right]$. Also,

$$\begin{aligned} \frac{\partial\left(V_{3La}\right)}{\partial\lambda} &= \frac{\left(\alpha v_H + (1-\alpha)v_L\right)}{4} > 0 \\ \frac{\partial\left(V_3\right)}{\partial\lambda} &= \frac{\left(\alpha v_H + (1-\alpha)v_L\right)}{4} > 0 \end{aligned}$$

So p_7^* is increasing in λ.

Finally, note that $V_{3La} > V_{1Lc}$ and $V_3 > V_1$ when $\lambda > \lambda_2$. As a result, optimal access fee can only increase when going from one threshold region to another threshold region when λ increases.

It follows that p^* is non-decreasing in λ when $\alpha > \alpha_2$.

Let α_{th} be the threshold on α above which $\lambda_{B2} > \lambda_{PB}$. Since $\lambda_{PB} < \lambda_{B1}$, we will have $\alpha_{th} < \alpha_2$. Now consider the range $\alpha_{th} < \alpha < \alpha_2$. The ordering of λ thresholds in this region would be $0 < \lambda_{PB} < \lambda_{B2} < \lambda_{B1}$. As before, we can examine these regions separately.

1. $0 < \lambda < \lambda_{B2}$: Here the optimal access fee is $p_2^* = p_4^*$ where $p_2^* = \min\left[V_{1Lc} - V_{4L}, V_2 - V_{5H}\right]$ and is independent of λ.

2. $\lambda_{B2} < \lambda < \lambda_{B1}$: Here the optimal access fee is p_5^* where $p_5^* = \min\left[V_{4La} - V_{4L}, V_4 - V_{5H}\right]$.

$$
\begin{aligned}
\frac{\partial\left(V_{4La}\right)}{\partial\lambda} &= \frac{\left(\alpha v_H + (1-\alpha)\, v_L\right)}{4} > 0 \\
\frac{\partial\left(V_4\right)}{\partial\lambda} &= \frac{(1-\alpha)\, v_L}{4} > 0
\end{aligned}
$$

Thus p_5^* is increasing in λ.

3. $\lambda_{B1} < \lambda < 1$: Here the optimal fees are p_7^*, q_7^*, where $p_7^* = \min\left[V_{3La} - V_{4L}, V_3 - V_{5H}\right]$. Also,

$$
\begin{aligned}
\frac{\partial\left(V_{3La}\right)}{\partial\lambda} &= \frac{\left(\alpha v_H + (1-\alpha)\, v_L\right)}{4} > 0 \\
\frac{\partial\left(V_3\right)}{\partial\lambda} &= \frac{\left(\alpha v_H + (1-\alpha)\, v_L\right)}{4} > 0
\end{aligned}
$$

So p_7^* is increasing in λ.

Finally, note that $V_{3La} > V_{4La}$ and $V_3 > V_4$ when $\lambda \geq \lambda_{B1}$; in addition, $V_{4La} > V_{1Lc}$ and $V_4 > V_2$ when $\lambda_{B2} < \lambda < \lambda_{B1}$. Thus, as before, the optimal access fee can only increase when going from one threshold region to another threshold region due to an increase in λ. It follows that p^* is non-decreasing in λ when $\alpha_{th} < \alpha < \alpha_2$.

Now let us consider the region in which $\alpha < \alpha_{th}$.

1. $\lambda_{B2} < \lambda < \lambda_{PB}$: Here the optimal access fee is p_3^* where $p_3^* = \min\left[V_{4La} - V_{4L}, V_4 - V_{5H}\right]$.

$$\begin{aligned}\frac{\partial\left(V_{4La}\right)}{\partial\lambda} &= \frac{\left(\alpha v_H + \left(1-\alpha\right)v_L\right)}{4} > 0\\ \frac{\partial\left(V_4\right)}{\partial\lambda} &= \frac{\left(1-\alpha\right)v_L}{4} > 0\end{aligned}$$

Thus p_3^* is increasing in λ.

2. $\lambda_{PB} < \lambda < \lambda_{B1}$: Here the optimal access fee is $p_5^* = p_3^*$ which we have already shown earlier to be increasing in λ.

3. $\lambda_{B1} < \lambda < 1$: Here the optimal fees are p_7^*, q_7^*, which are also increasing in λ.

Again, due to the ordering of the value functions, the optimal fee only increases as it moves from one region of λ to a higher one. Thus, p^* is increasing in λ when $\alpha < \alpha_{th}$.

This establishes that p^* is non-decreasing in λ when $\lambda > \lambda_T$.

Sensitivity analysis of authentication fee w.r.t to λ: To realize the non-monotonicity of q^* with respect to λ, we can examine

$$\begin{aligned}q_3^* &= \min\left[V_4 - V_{3H}, V_4 - V_{5H} - p^*\right]\\ q_5^* &= \min\left[V_4 - V_{3L}, V_4 - V_{5H} - p^*\right]\end{aligned}$$

Differentiating each of these with respect to λ gives us

$$\begin{aligned}\frac{\partial\left(V_4 - V_{3H}\right)}{\partial\lambda} &= \frac{\left(1-\alpha\right)v_L}{4} > 0\\ \frac{\partial\left(V_4\right)}{\partial\lambda} &= \frac{\left(1-\alpha\right)v_L}{4} > 0\\ \frac{\partial\left(V_4 - V_3\right)}{\partial\lambda} &= -\frac{\alpha v_H}{4} < 0\end{aligned}$$

It follows that q^* is non-monotonic in λ.

The non-monotonicity of the optimal access and authentication fees are illustrated through the Figure 6.

Proof of Proposition 7

When $\mu = 0$, there is no direct search. Under this condition, we examine the profits of the platform as function of different strategies

1. Neither Positioning nor Authentication: The expected profits are:

$$\pi_N = V_{3L}$$

2. Positioning only: The expected profits are

$$\pi_P = \begin{cases} V_{3H} - c_P & \text{if } \lambda > \lambda_{PB} \\ V_{3L} - c_P & \text{o.w.} \end{cases}$$

3. Authentication only: The access fee has to motivate the L-class to participate and the authentication fee should be such that the H-class should purchase it. This means that:

$$\begin{aligned} p^* &= \frac{1}{2}\delta\left(\alpha v_H + (1-\alpha)\, v_L\right) \\ q^* &= \frac{1}{4}(1-\alpha)\left(\alpha\delta v_H + v_L\left((1-\alpha)\,\delta - \lambda - 1\right)\right) \end{aligned}$$

The expected profits are:

$$\begin{aligned} \pi_A &= p^* + \alpha\left(q^* - c_A\right) \\ &= \frac{1}{2}\left(\alpha\left(1+\alpha-\alpha^2\right)\delta v_H - (1-\alpha)\left(\alpha^2\delta - \alpha\delta + \alpha - \delta\right) v_L\right) - \alpha c_A \end{aligned}$$

4. Both Authentication and Positioning: When both positioning and search are offered, the profits can be characterized as follows:

1. If $\lambda > \lambda_2$, the profits are

$$\pi_{B1} = \begin{cases} V_{3H} - c_P - c_A & \text{if } \lambda > \lambda_{PB} \\ V_{3L} - c_P - c_A & \text{o.w.} \end{cases}$$

2. If $\lambda < \lambda_1$, the profits are

$$\pi_{B2} = \pi_A - c_P$$

3. If $\lambda_1 < \lambda < \lambda_2$ and $\alpha > \alpha_2$, the optimal access and authentication fees are

$$\begin{aligned} p^* &= \frac{1}{2}\delta\left(\alpha v_H + (1-\alpha)\, v_L\right) \\ q^* &= \frac{1}{4}(1-\alpha)\left(\alpha\delta v_H + v_L\left((1-\alpha)\,\delta - \lambda - 1\right)\right) \end{aligned}$$

So the expected profits are

$$\begin{aligned} \pi_{B3} &= p^* + \alpha\left(q^* - c_A\right) - c_P \\ &= \frac{1}{4}\left(\alpha\left(-\alpha^2 + \alpha + 2\right)\delta v_H + (\alpha-1)v_L\left(\alpha^2\delta + \alpha(-\delta + \lambda + 1) - 2\delta\right)\right) - \alpha c_A - c_P \end{aligned}$$

4. If $\lambda_{PB} < \lambda < \lambda_2$ and $\alpha_1 < \alpha < \alpha_2$, the optimal access and authentication fees are

$$\begin{aligned} p^* &= \frac{1}{4}(1 + \delta + \lambda)\left(\alpha v_H + (1-\alpha)\, v_L\right) \\ q^* &= \frac{1}{4}(1-\alpha)\left(\alpha\delta v_H + v_L\left((1-\alpha)\,\delta - \lambda - 1\right)\right) \end{aligned}$$

The expected profits are

$$\begin{aligned} \pi_{B4} &= p^* + \alpha\left(q^* - c_A\right) - c_P \\ &= \frac{1}{4}\left(\alpha v_H\left(\left(-\alpha^2 + \alpha + 1\right)\delta + \lambda + 1\right) + v_L\left(\alpha^3\delta - \alpha^2(2\delta + \lambda - 1) - 2\alpha + \delta + \lambda + 1\right)\right) - \alpha c_A - c_P \end{aligned}$$

5. If $\lambda < \lambda_{PB}$ and $\alpha_1 < \alpha < \alpha_2$, the optimal access and authentication fees are

$$\begin{aligned} p^* &= \frac{1}{4}(1+\delta+\lambda)(\alpha v_H + (1-\alpha) v_L) \\ q^* &= \frac{1}{4}(1-\alpha)(\alpha\delta v_H + v_L((1-\alpha)\delta - \lambda - 1)) \end{aligned}$$

The expected profits are

$$\begin{aligned} \pi_{B5} &= p^* + \alpha (q^* - c_A) - c_P \\ &= \frac{1}{4}\left(\alpha v_H \left(\left(-\alpha^2+\alpha+1\right)\delta + \lambda + 1\right) + v_L\left(\alpha^3\delta - \alpha^2(2\delta+\lambda-1) - 2\alpha + \delta + \lambda + 1\right)\right) - \alpha c_A - c_P \end{aligned}$$

6. If $\alpha < \alpha_1$, the expected profits are

$$\pi_{B6} = V_{3L} - \alpha c_A - c_P$$

We now compare the different profit functions. For the purpose of comparisons, we assume that $c_A = c_P = \epsilon \approx 0$. This simplifies the functions but does not affect results. These comparisons show the following:

1. When $\lambda > \lambda_1$, π_P is greater than all other profit functions.

2. When $\lambda_{PB} < \lambda < \lambda_2$ and $\alpha > \alpha_2$, π_P is greater than all other profit functions.

3. $\pi_N > \pi_A > \pi_{B2}$

4. When $\lambda_{PB} < \lambda < \lambda_2$ and $\alpha_1 < \alpha < \alpha_2$, π_{B4} is greater than all other profit functions.

5. When $\lambda < \lambda_{PB}$ and $\alpha_1 < \alpha < \alpha_2$, there exists a threshold λ' such that if $\lambda' < \lambda < \lambda_{PB}$, $\pi_{B5} > \pi_N$ where

$$\lambda' = \frac{\alpha v_H\left(\left(\alpha^2-\alpha-1\right)\delta+1\right) - v_L\left(\alpha^3\delta + \alpha^2(1-2\delta) + \delta - 1\right)}{\alpha v_H - \alpha^2 v_L + v_L}$$

Proof of Corollary 2

The proof of the corollary follows directly from the thresholds derived in proof of proposition above.

Parameter	**Definition**
α	Proportion of H-class match-seekers in the population
δ	Discount factor
η	Relative valuation of the H-class match-seekers
λ	Platform's positioning capability
μ	Efficiency of direct search
c_A	Cost of authentication
c_P	Cost of positioning
p	Access fee
q	Authentication fee
t	Sum of access and authentication fees
v_H	Value from a compatible H-class match
v_L	Value from a compatible L-class match

Table 4: Table of Notations