Heterogeneous Applications and Platform Competition: Mobile Apps *

Timothy Bresnahan, Joe Orsini, and Pai-Ling Yin

Stanford University, Nuna, and University of Southern California

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Abstract

~~Motivated by tipping and stability in mobile applications platform markets, we estimate a new model of application demand and supply under platform competition and calculate stability bounds.~~ In the US and a few other large, rich countries, there has been a longstanding approximately symmetric Android/Apple duopoly. At the same time, smaller US platforms (e.g., Windows Mobile) have tipped out of the market, and most countries have tipped to Android. We incorporate heterogeneity across applications in attractiveness to users into the classical model of equilibrium stability and tipping in platform markets to explain both ~~the observed tippiness and the observed stability of mobile applications platform markets.~~

We estimate ~~an empirical version of our~~ model using data on the US mobile phone application platforms and undertake a number of platform market stability analyses. For modern consumer-oriented platforms, where star applications coexist with less popular ones, the degree to which the most attractive applications are inframarginally supplied explains stability in markets where all platforms have large user installed bases and tippiness in markets where at least one platform has a small installed base.

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I Introduction

Platform markets often tip to a dominant platform, with applications developers tending to supply the platform with the most users and users choosing the platform with the most applications.[1] ~~Platform industries have been around for many decades, and have~~ recentl~~y~~ exten~~ded~~ to consumer mass markets~~. This~~ has led to a surprising mix of market structures. The smartphone market in the United States, the largest applications development platform of all time, has persisted in an equilibrium approximately equally divided between iOS and Android.[2] While some other large countries also exhibit that novel pattern, platform market equilibri~~um~~ in smaller national markets ha~~s~~ tipped to Android or, in a few cases, to iOS. Even in the large US market, platforms with a smaller installed base, like Windows Mobile, have tipped out. These 21st century consumer-oriented development platform markets present as both tippy and not~~. S~~tability appears to arise where all platforms have large installed bases, tippiness where one or more platforms have a smaller installed base.

In this paper, we provide an economic explanation of these phenomena with both theoretical and empirical analysis. The core theoretical idea is that applications heterogeneity in attractiveness to users changes platform stability analysis. ~~We start with the obvious fact of~~ our industry ~~that~~ applications are heterogeneous in how attractive they are to users: some are stars, demanded by many or even most users, while other applications are mundane, attracting only a modest amount of user demand. Supply of a star application to a platform increases the value of that platform to users more than supply of a mundane application. At the same time, the developer of a star application earns more economic return per customer than does a mundane application, making the star application's threshold installed base for profitability on a platform lower than that of the mundane application. This means that the supply of the most attractive applications is inframarginal for a platform with a large installed base of users. By definition, inframarginally supplied applications do not change behavior as the size of the installed base changes, dampening the strength of the platform positive feedback loop. The elasticity of supply of what users care about in a platform, not just many applications but attractive ones, is lowered by inframarginal star-application platform supply behavior.

1. See Farrell and Klemperer (2007) and Rysman (2009) for reviews of the platform literature, and see section II below for further citations.

2. See industry structure discussion at Figure 4 and below.

The shape of the distribution of attractiveness across applications determines whether the inframarginal supply behavior of star applications leads to inelastic aggregate application supply to a platform and thus to stability. Intuitively, aggregate application supply will be elastic when the bulk of the attractiveness of a platform comes from its star applications, not from the (potentially more numerous) mundane applications that are only marginally profitable on the platform. The intuition comes from the dual role of applications attractiveness. In the supply of applications to the platform, it is the mundane applications that are marginal. In the demand for platforms, it is the star applications that produce much of the value. ~~When applications supply increases at~~ platform installed base grows create little additional value to the platform. In Section III, we formalize the idea of a star-dominant distribution of applications attractiveness and show a richer result than just the intuition. In particular the intuition is right only when the platform installed base is large, for only then will star applications be inframarginally supplied. At a smaller installed base, supply will be more elastic under star-dominance, as more valuable applications are at the margin of supply. While new in platform economics, star-dominant product heterogeneity is familiar in media and entertainment mass markets,[3] not just consumer-oriented smartphone application markets.

Inelastic application supply to all platforms at a candidate equilibrium pushes it toward stability. Only if consumer platform choice is highly elastic with respect to applications availability will the equilibrium be unstable. This, plus star-dominance, is the key to our theoretical results about platform market stability. First, in large markets, an evenly divided platform market equilibrium can be stable with unstable equilibria at less evenly divided platform market shares. This result reverses the "folk theorem" of platform models with indirect network effects and traditional assumptions about representative applications, in which it is the divided equilibrium which is unstable and dominant-platform equilibria are stable. Applications heterogeneity changes the result *without* dropping the positive feedback elements of platform markets. Second, with the same supply of applications and demand for applications and platforms, divided platform market equilibrium will be more stable in a large market and more unstable in a small one. The reverse result holds under traditional assumptions, Finally, a platform market equilibrium with two (or more) approximately equally-sized platforms and another smaller one can be unstable even when a divided platform market equilibrium would be stable. All these surprising results require star-dominance in the application heterogeneity and that user platform demand not be too elastic with respect to available applications. In our empirical results, we shall measure the shape of the smartphone application heterogeneity and the elasticity of supply of applications to platforms and bound the elasticity of user demand for platforms.

We empirically estimate the distribution of heterogeneity in applications attractiveness to users and a

3. See review articles by Sorensen (2017) and Waldfogel (2017) as well as further discussion of the literature in Section II.

linked platform supply model for applications developers in the US smartphone market. We assemble a dataset in which an observation is an application. The cross-section dataset contains information on all the economically significant iOS and Android applications in the US market. Section IV has details on the sources of our data: a commercial product ~~attempting to~~ measure~~, user~~ applications~~ demand and~~ our own collection of information about applications and developers.

~~To measure~~ the distribution across applications of attractiveness to users is conceptually simple but econometrically complex. An applications' attractiveness to consumers on a platform is only observed if it is 1) supplied for that platform and 2) included in our baseline commercial sample.[4] Accordingly, we estimate a joint model of observed application attractiveness (conditional on being observed on a platform), ~~of~~ supply to platforms, and ~~of~~ presence in the sample. The distribution of attractiveness underlies all these as a primitive. We also gather additional data on supply in order to make the selection model sharper.

The economics of developer supply are like those of market entry, with access to groups of customers driving the profitability of supplying one or more platforms.[5] With heterogeneity, a developer's profits on a platform depend on their application's attractiveness as well as on the installed base of users. Our model allows the attractiveness of a given application to differ across each of the major platforms, iOS and Android, and estimates the degree to which they are dependent. Our market, like many consumer-facing media and entertainment markets, has uncertainty about product success *ex ante*.[6] We model this uncertainty explicitly. A related modeling issue concerns the set of potential suppliers to each platform. Like many entry studies starting with Berry (1992), we only observe a potential entrant if it is an actual entrant in the market of interest or in an adjacent market~~, in our case,~~ the other platform. This leads us to solve a longstanding problem in entry models.

To move toward a quantitative realization of the stability analysis, our empirical model adds a number of elements. Some of these elements are general to mass market consumer goods industries, including discovery of applications by users and, relatedly, gaps between a developer's forecast of an application's attractiveness at the time of entry and its ultimate market importance. These forces are important for quantification because they loosen the connection between applications heterogeneity in demand and in supply. Other elements of our empirical model are related to platform industries with competing platforms, including the possibilities that the distribution of application attractiveness is different on different platforms and

4. It is not practical to avoid selection of a sample of applications in our industry. While there are millions of applications on each platform, the bulk of user demand is for a modest number of applications (Bresnahan, Davis and Yin (2015)). We use a sample of applications that are economically important on either iOS and Android.

5. Profit is an economic concept here, not an accounting one. In our industry, some applications earn profit literally by being sold or, more typically, through "in app purchases." Others are advertising supported. Others are complements to products sold by the firm, such as air travel and an airline's mobile app.

6. See Aguiar and Waldfogel (2018) and Hendricks and Sorensen (2009) for empirical examinations of particular industries, and Sorensen (2017) and Waldfogel (2017) for broader reviews of the literature.

that application profitability varies across platforms. These forces are important for quantification because they weaken the sense in which an applications development platform is, from a star developer perspective, just another market with some installed base of customers to enter. Finally, we account for the highly heterogeneous nature of mobile applications and developers. For example, entrepreneurial Rovio's "Angry Birds" games and long-established Citibank's mobile banking application are both attractive to users, but we do not want to assume *ex ante* that their costs or the value they place on additional customers on a platform are the same.

Our estimates show, as is also clear in an examination of the raw data, that the assumption of undifferentiated applications is not tenable. Application heterogeneity is a first order phenomenon in both demand and supply. The quantification in our estimates, together with our update of the theory, leads to a series of results about platform market equilibrium. The distinct supply behavior of star applications and more mundane ones is central both to the stability analysis of the US market and to examination of how stability would change if there were a smaller platform (like Windows Mobile) or if the analysis were applied to a smaller economy.

Our estimates (Section VI) show that the density function of application attractiveness for each of the iPhone and Android platforms are similar, and each has a star-dominant shape. When we calculate the elasticity of the total value of applications available on the platform with respect to the installed base of users, the critical supply elasticity, we get quite low numbers, around 0.02 for either iOS or Android. As a result, we calculate a bound in which the elasticity of iPhone users' demand for iPhones with respect to available applications would need to be over 25 for the observed US duopoly. In contrast, if we examine supply at smaller installed bases for both platforms (a within-sample analysis), we get much larger elasticities. The implication is that a market about 1/6 the size of the US would have a stability index about an order of magnitude larger than the US, i.e. be much closer to tippyness. Similarly, a three-platform equilibrium, with the third ~~about~~ the largest size ever achieved by Microsoft smartphone operating systems, would be much closer to tippyness than the historical duopoly. ~~We conclude that o~~ur model can at once rationalize stability of the US duopoly and tippiness of an equilibrium with at least one platform having a significantly smaller installed base. Whether those results apply to WinMo's tipping out or to the tip to Android in most countries and iOS in some requires, of course, a further assumption that the relevant heterogeneity distribution and supply conditions would be similar to those we have estimated for the two US platforms. Our finding that the two US platforms are very similar in this regard is encouraging evidence for our extrapolation's validity. ~~Consideration~~ of applications heterogeneity along the lines suggested by modern consumer mass markets ~~provides an explanation of~~ stability of divided equilibrium in the largest application development platform market seen thus far without ruling out the possibility of tippiness observed ~~at~~ other platform market structures.

II Prior Work

We draw on two previously unlinked literatures, platform stability analysis and mass media markets.

The economic impact of network effects is a well-studied problem theoretically with papers dating to Rohlfs (1974). The competition between platforms is another rich theoretical literature, with seminal contributions from Katz and Shapiro (1985) and Farrell and Saloner (1985). Farrell and Klemperer (2007) provide a deep review of the literature on network effects, with more emphasis on the theory. The theoretical literature has analysis of the sources and implications of network effects (where we make our contribution) and of their normative implications. Rysman (2004) provides an overview of both empirical and theoretical work on platform markets. We do not draw out the normative implications of our work, focusing instead on the positive economics of stability.

A number of papers consider the possibility that platform competition could be dulled by forces that offset network effects and positive feedback, emphasizing the economic relationships between users and developers. One structure offsets the positive externalities of indirect network effects by adding negative externalities among users, such as congestion, or among developers, such as competition to sell similar applications (e.g. Ellison and Fudenberg (2003)). Another structure, closer to our approach, assumes that platforms themselves are differentiated products either to developers or to users or to both (e.g., Church and Gandal (1992), Cantillon and Yin (2008)). One form of differentiation is related to user preference heterogeneity; if some users value the number of applications on the platform more than others, platform markets can be vertically differentiated with a many-application and a few-application platform in equilibrium (e.g.Gabszewicz and Wauthy (2014)). None of these treatments, however, embody our model of heterogeneity in attractiveness.

There are a number of empirical papers which examine platform industries with a focus on tipping and tippiness. If the market has tipped to a dominant platform, it is hard to find variation in the installed base or in the number of applications in-sample. Rysman (2004) is one of the few empirical studies of platform industries which observes variation in industry structure, in this case variation across local yellow pages markets. More papers examine the process of moving toward an equilibrium (e.g., Church and Gandal (1992), Gandal, Kende and Rob (2000)). In this vein, Augereau, Greenstein and Rysman (2006) distinguish between a process of “coordination” on a common standard versus moving to a divided equilibrium with “differentiation” in standards, with one side of the market preferring the differentiation to impose switching costs on the other side. Another approach is to look at performance indicia of the platform to infer network effects. If an incumbent platform performs worse than an entrant, for example, but nonetheless maintains a high market share, one might infer that network effects are holding back the entrant platform. This approach leads to a finding of "endogenous platform differentiation" rather than a clear advantage for the incumbent platform in Hendel, Nevo and Ortalo-Magne (2009) and to a finding of network advantages in Brown and Morgan (2009). The other foundational literature for us studies consumer media markets.

Hendricks and Sorensen (2009) study music sales, noting that product demand is distributed so that most of industry profit is earned by only a few products. Further, they conclude that ~~the~~ the tendency for much of sales to come from only a few products is heightened by consumers' difficult~~ies~~ learning about products. Aguiar and Waldfogel (2018) note that uncertainty by producers about a product's success at the time of investment plays a similar role and that better *ex ante* signals of demand, which they call "quality predictability," lead to changes in supply that expand demand. Similar findings about the shape of demand and the role of information have been found for books (Chevalier and Mayzlin (2006), Sorensen (2007)), motion pictures, and other consumer media industries. Sorensen (2017), reviewing studies of a number of industries, notes what we call a star-dominant distribution of success for a wide range of consumer product industries~~,~~ and discusses how incomplete consumer information plus search leads, in some circumstances ~~(which may well be applicable to the mobile apps industry)~~ to even greater spread in demand heterogeneity at the product level.[7] Waldfogel (2017) notes that the return to digitization in many media markets often arises because lower costs create more chances for the creation of very high quality products, i.e., star products at the top of the distribution of applications heterogeneity. Whether either of these ideas – the importance of a small number of products in aggregated demand and the importance of information to consumers and producers about products – applies to our industry is an empirical question.

The kind of supply behavior we study is familiar in empirical models of entry.[8] Finally, while we know of no other paper studying inframarginal supply of applications to a platform, in our industry it takes the form of inframarginal multihoming. ~~There is a rich literature on multihoming.~~[9]

III Application Heterogeneity and Platform Market Tipping

In this section, we show how the classical indirect network effects model changes when applications supply is changed from platform selection by undifferentiated application developers to heterogeneous applications' use of platforms to gain access to customers. In both versions of the model, we maintain the two elements

~~7. Briefly revisit this discussion when we introduce λ.~~

8. See Berry and Reiss (2007).

9. Corts and Lederman (2009) study developer supply in the game console industy, emphasizing the role of multihoming in limiting tipping. Venkataraman, Ceccagnoli and Forman (2017) analyze multihoming in a very different context where strong links between platform provider and complementors flow through shared human capital. Grajek and Kretschmer (2012) examine "critical mass" in mobile telephony itself.

that lead to positive feedback: applications developers are more profitable on platforms with more users while users value a platform more if it gains applications. We are interested in two features of the model. What happens in larger markets? What forces tend to make divided equilibrium less stable than equilibria with a dominant platform?

III.A The Folk Theorem

We start with the familiar version in which undifferentiated applications select between two platforms. Strong enough positive feedback leads to tippiness, with an unstable divided equilibrium and stable equilibria with a dominant platform ~~are stable~~. (Proofs of the propositions in this section are in the Appendix.) This result underlies much thinking about platform markets and has become a folk theorem. This version of the model is also always less stable in larger market.

The profit for application a on platform p depends on the installed base of users, U_p:

$$\Pi_{ap} = \mu U_p - C_p - \epsilon_{ap} \tag{1}$$

Each application's profit increases with the installed base at rate $\mu \geq 0$. Developers are heterogeneous only in the fixed costs of writing an application, $C_p + \epsilon_{ap}$. This familiar version of the classical model captures an essential feature of application supply to platforms. Software has high fixed costs and low marginal costs, so developers tend to prefer a platform with a larger installed base.[10] Each developer chooses ~~that~~ platform, 1 or 2, for which Π_{ap} is larger. Let $F_a()$ be the strongly unimodal and symmetric cdf of $\epsilon_{a1} - \epsilon_{a2}$, and let $\Delta_\pi \equiv \mu(U_1 - U_2) - C_1 + C_2$ be the difference in mean profitability between platforms. The developer platform supply equation is

$$N(U) = [F_a(\Delta_\pi(U)) \quad 1 - F_a(\Delta_\pi(U))] \tag{2}$$

The second positive feedback element is that users value platforms in part for the applications available on them. In this version applications are undifferentiated and user platform demand depends on the number of available applications on each platform. Let $\gamma \geq 0$ be users' valuation of applications, and γ_p be the mean intrinsic value of platform p to users, so the utility of choosing platform (p) for user (u) is[11]

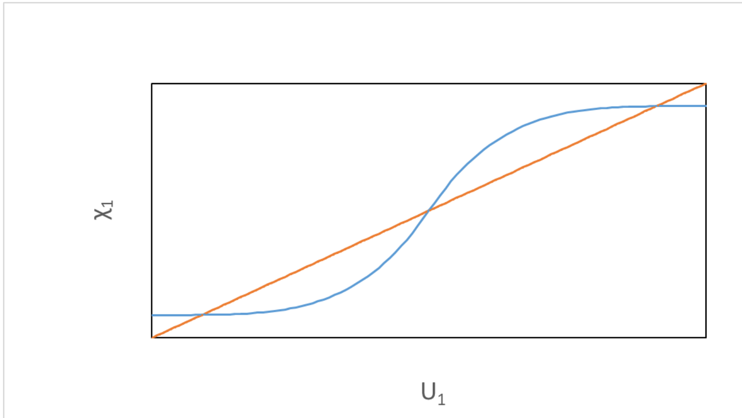
$$V_{up} = \gamma N_p + \gamma_p + \epsilon_{up} \tag{3}$$

We normalize the mass of applications to 1, but to examine market-size effects we let the mass of users, U_M,

10. The pricing of apps is pushed into the background here, but most models have a positive equilibrium (after pricing) marginal profit as the size of the installed base increases.

11. In a familiar notation from the work of S. Berry, $\delta_p = \gamma_p + \gamma N_p$.

Figure 1: The Geometry of the Folk Theorem



vary. Let $\Delta_V(N) = \gamma(N_1 - N_2) + \gamma_1 - \gamma_2$ and let $F_u()$ be the cdf of $\epsilon_{u1} - \epsilon_{u2}$ (heterogeneity across individual users) which is smooth, strongly unimodal, and has infinite support. User platform demand is

$$U(N) = U_M \times [F_u(\Delta_V(N)) \quad 1 - F_u(\Delta_V(N))] \tag{4}$$

Finally, an equilibrium U^e of the system defined by (2) and (4) is a fixed point of the mapping $\chi(U) \equiv U(N(U))$. An equilibrium is stable if the real part of all eigenvalues of $J_\chi(U^e)$, the Jacobian of χ, are less than 1 in absolute value. In this two-platform case with single-homing users, an equilibrium is also a fixed point of $U_1 = \chi_1(U_1)$ where χ_1 is defined in two steps. We calculate $\chi(U)$, set $U_2 = U_M - U_1$, and keep $\chi_1(U_1)$ as the first element of $\chi(U_1, U_M - U_1)$. ~~Also,~~ the largest eigenvalue of the Jacobian of χ and the slope of χ_1 are ~~the same~~ in the two platform case. Accordingly, we define the slope of χ_1 ~~(equivalently, the largest eigenvalue)~~ as the stability index at a point (lower is more stable). That index is

$$2 * \gamma * f_u(\Delta_V(N)) * U_M * \mu * f_a(\Delta_\pi(U)) \tag{5}$$

The index, plus the geometry of χ_1, a function from a closed interval on $\mathbb{R}^1$ to itself, permits simple demonstration of a number of points (more details in Appendix). An equilibrium is a point where χ_1 cuts the 45° line. There will always be at least one equilibrium, and there will always be one stable equilibrium, at which χ_1 cuts the 45° line from above. If demand and supply are symmetric, i.e. $\delta_1 = \delta_2$ and $C_1 = C_2$, there will always be a divided equilibrium with $U_1 = U_2$ and $N_1 = N_2$. Assuming symmetry, the equilibrium correspondence can take on only two forms: one with a unique, divided equilibrium, and one with three equilibria, as in Figure 1, i.e. an unstable divided equilibrium and two, stable dominant-platform equilibria. In one of these, with high U_1 and N_1 but low U_2 and N_2 – users choose platform 1 because developers do, and developers choose platform 1 because users do. The other stable equilibrium is the opposite, with high U_2 and N_2 because of positive feedback. The three-equilibria scenario is the core of the folk theorem.

It is also simple to see the condition determining whether there is a unique, stable, divided equilibrium ~~vs. when there are~~ three equilibria. Expression (5) will be greater than 1 at the divided equilibrium if $\gamma * \mu * U_M > 1/(2 * f_a(0) * f_u(0))$. In that case, there are three equilibria, following the pattern of Figure 1. If the opposite inequality holds, there is a unique, stable divided equilibrium. The economics have two elements. If $\gamma * \mu$ is sufficiently small, network effects are modest and platform market equilibrium is unique. With larger network effects there are multiple equilibria. Perhaps less well known, U_M plays exactly the same role as γ or μ; modeling developers as having fixed plus constant marginal costs makes their response to larger installed base be ~~just like~~ an increase in per-customer profit μ.

Tipping to a dominant platform, arbitrarily either 1 or 2, and unstable divided equilibrium~~,~~ are the core positive economics results of the familiar version. These results require not only the assumption of indirect network effects but also the representative application assumption that application supply and user platform demand can both be ~~well~~ modeled ~~with~~ N_p, the number of applications on a platform.

III.B Application Heterogeneity

We consider a form of applications heterogeneity prevalent in many modern consumer-oriented platforms. Some applications are stars, ~~with great~~ attractive~~ness~~ to consumers and higher profit for a given user installed base. Other applications are less attractive and profitable. The supply behavior of applications to platforms changes with this assumption~~, with~~ more attractive applications earn~~ing~~ more per-customer profit and profitably supply~~ing~~ all platforms with substantial user installed bases. We call this assumption "applications heterogeneity in attractiveness," sometimes abbreviated as "applications heterogeneity" in what follows. In this section, we examine how this general phenomenon changes platform market stability, partially reversing the folk theorem.

In our industry, heterogeneity shows up through important differences in profitability, supply behavior, and value creation for users across applications. All of these differ markedly between stars like "Angry Birds" with tens of millions of users, and "Bird Sounds Ringtones" with hundreds of thousands.

Each application, a, has an index of attractiveness, r_a, with $0 \leq r_a \leq 1$. If application a is available on platform p, the number of users of the platform demanding it is $r_a U_p$. Denoting the application's per-customer profit once again by μ, and setting the fixed costs of writing an app for platform p to C_p the total return to supplying an app of attractiveness r_a to a platform with U_p users is

(6) $$\Pi_{ap} = \mu r_a U_p - C_p$$

Since $\partial \Pi_{ap} / \partial r_a > 0$, demand heterogeneity means different applications have different supply behavior. The applications supplied to a platform are those with r_a above a breakeven threshold, which we call $\widehat{r}_p$, where

$$\widehat{r}_p = C_p / (\mu U_p) \tag{7}$$

Note that an application with high r_a can have $r_a > \widehat{r}_p$ for several p and thus choose to supply them all, gaining access to different customers on each platform.[12]

A higher-r application has higher user demand in the application market, so we make the conforming assumption that higher r applications make a larger contribution to the attractiveness to users in the *platform* market. Specifically, the index of application attractiveness on platform p, called v_p, is the total, across all applications available on p, of their attractiveness:

$$v_p = \int_{\widehat{r}_p}^{1} t f_r(t) dt, \tag{8}$$

where $f_r()$ is the density function of the distribution of r_a across apps. We continue to assume user platform demand equation (4) holds, but change the definition of Δ_V to $\Delta_V = \gamma(v_1 - v_2) + \gamma_1 - \gamma_2$, so that instead of $U(N)$ we have $U(v)$ given by

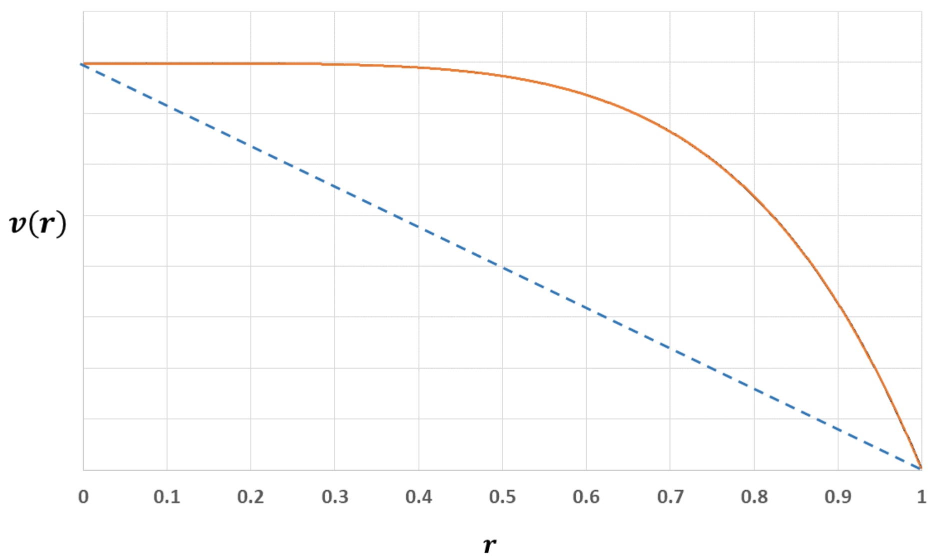
$$U(v) = U_M \times [F_u(\Delta_V(v)) \;\; 1 - F_u(\Delta_V(v))] \tag{9}$$

An equilibrium is a fixed point of $U = \chi(U) = U(v(\widehat{r}(U)))$.

The function $v(\widehat{r})$ encapsulates the role of applications heterogeneity at the platform demand level and determines whether stability analysis changes when applications heterogeneity is introduced. In Figure 2, we show two different versions of v corresponding to different $f(r)$. Both have the same value for $v(0)$, i.e., the same total available value to users if *all* apps are supplied to a platform. The approximately linear dotted blue curve corresponds to a density function $f(r)$ proportional to $1/r$, so that the marginal contribution of a reduction in $\widehat{r}$, $\widehat{r}f(\widehat{r})$, is a constant. In this easy-to-understand special case, the lower attractiveness of each lower-r app is just offset by ~~there being~~ more of them. The solid orange curve corresponds to a distribution $f(r)$ in which higher-r apps make a larger contribution to total attractiveness. We define a star-dominant f_r as one that has a decreasing and con[illegible] $r \cancel{*} f_r(r)$. This corresponds to v ~~being steeper at higher r, and~~ ~~also to the aggregate contribution to~~ user value v coming ~~more~~ from higher-r ~~than lower-r~~ applications. Applications heterogeneity does not remove the positive feedback loop from platform economics, but it ~~changes~~ applications supply ~~behavior to a platform measured by platform attractiveness to users, v_p~~. We can see the implications for stability analysis by examining the functions $\widehat{r}_p(U_p)$ and $v_p(\widehat{r}_p)$ defined in Equations (7) and (8) ~~which determine whether the supply response tends to be explosive or dampened~~.

12. Our empirical model does not impose the assumption that r_a is the same across platforms. ~~There is~~ positive dependence: applications with high r on Android tend also to have high r on iOS. There is no pricing equation for the app. If there is pricing in applications, r_a is the equilibrium attractiveness and, as in the prior model, μ is the equilibrium marginal profit.

Figure 2: Star-Dominant Valuation Distribution



There are two forces which ~~tend to~~ lead to a dampen~~ing of~~ supply response~~s~~ in large markets ~~and~~ with divided platform shares. First, as U_p grows, individually less attractive applications are supplied to the platform, but the margin $\widehat{r}_p$ falls at a decreasing rate:

$$\partial \widehat{r}_p / \partial U_p = -C_p / \left(\mu U_p^2 \right)$$

The second force depends very much on the shape of $f_r()$, in particular, on the economic importance of star versus mundane applications. As $\widehat{r}_p$ falls, the ~~aggregate contribution to~~ user ~~welfare~~ will be larger or smaller depending on ~~how many~~ applications cross the threshold:

$$\partial v_p / \partial \widehat{r}_p = -f_r(\widehat{r}_p) \widehat{r}_p$$

A star-dominant shape for $f_r()$ leads to dampen~~ing of the~~ supply response~~, measured in user value, of applications to a platform that has a large U_p.~~

(10) $$\partial v_p(\widehat{r}_p(U_p)) / \partial U_p = -f_r(\widehat{r}_p)\widehat{r}_p \ \times \ -C_p / \left(\mu U_p^2 \right) = (\mu / C_p) * \widehat{r}_p^2 * f_r(\widehat{r}_p)\widehat{r}_p$$

The last equality uses $1/U_p = (\mu/C_p)\widehat{r}_p$. Evaluating this expression at larger U_p is the same as evaluating it at a smaller $\widehat{r}_p$. ~~Looking at the expression in Equation (10) that depends only on $\widehat{r}_p$ shows immediately that,~~ if the distribution of applications heterogeneity is star-dominant, $\partial v_p(\widehat{r}_p(U_p))/\partial U_p$ is smaller at a smaller $\widehat{r}_p$ i.e. a larger U_p. The economics of this result follow from the shape of Figure 2. In a small platform, an increase in U_p draws in attractive applications at the margin. If $f_r()$ is star-dominant, that adds a great deal of user value to the platform. In a larger platform, a similar increase draws in less attractive applications, and, if $f_r()$ is star-dominant, adds less user value to the platform. The shape of $rf_r()$ in our market will become a focus of our empirical work.

A star-dominant shape for $f_r()$ cannot lead to dampen~~ing the~~ supply response at all U_p. The arc elasticity of supply from any U_p down to zero or, correspondingly, from any $\widehat{r}$ up to 1, is unity.[13] ~~The economics are that it is~~ *inframarginal* supply ~~that~~ dampens the supply response in large markets~~. In sufficiently small markets,~~ star applications are marginally supplied. As a result, the supply of applications switches from inelastic to elastic at some point as market size falls.

~~The stability of divided equilibrium in~~ large markets with applications heterogeneity ~~has a specific supply behavior for~~ the most attractive applications~~, i.e., they~~ are *inframarginal multihomers*, far from the margin of not supplying either platform. The math of equilibrium ~~tells us to~~ emphasize the *inframarginal* supply behavior, not the multihoming. Consider the example in which there are distinct groups of applications, with no overlap~~,~~ that might supply each platform. Then, of course, there is no multihoming. But if the distributions of app attractiveness within each group are the same, i.e. $f_{rp}(r) = f_{rp'}(r)$ for any two platforms p and p' at all r, the stability index is exactly the same as in the model we just considered where all apps consider all platforms. The central economic role of a star-dominant $f_r()$ is to make the bulk of value to users of applications on a platform *inframarginal* to changes in installed base, whether this occurs by multihoming or not.

This economic logic underlies a number of stability results~~,~~ laid out in the Appendix. Consider the case in which the economic fundamentals are symmetric across platforms and $f_r()$ is star-dominant. First, there is always a divided equilibrium (as in the traditional version) when the economic fundamentals are symmetric. Second, departing from the traditional version, as U_M grows, the stability index at the divided equilibrium falls.[14] Indeed, for sufficiently large U_M, the divided equilibrium is stable. These results follow because the larger market will have larger U_p for all platforms in a divided equilibrium; equation (10) tells us that the slope of the supply of applications to each platform, measured in v_p, will fall with larger U_p. In smaller markets, however, the supply behavior is much more reactive to changes in U because the marginal app gets a much bigger profit boost from an increase in installed base: $\partial^2\Pi/\partial r\partial U > 0$ (and the marginal r is proportional to $1/U$). The relationship of stability to market size is the opposite with heterogeneity in app attractiveness than it is with app/platform cost heterogeneity.

That amounts to a partial reversal of the folk theorem with regard to the stability of the divided equi-

13. There is no supply at $U_p = 0$ or at $\widehat{r} > 1$, illustrated in Figure 2 if we reinterpret the dotted line as the arc.

14. Of course, it would be possible to get this result in the traditional model by limiting positive feedback and positive network effects, perhaps by having more competition among applications on larger platforms (a source of negative feedback) or by adding diminishing returns to applications attractiveness so that the diminishing returns offset the social increasing returns of the platform. With applications heterogeneity, the result arises without removing positive feedback and positive externalities.

librium. We say partial because the role of user platform demand is unchanged between the traditional version and the applications heterogeneity version. The different roles of platform supply (measured in user attractiveness) and user platform demand can be seen in the stability index:

$$f_u(\Delta_V) * U_M * \gamma * \mu \left[\frac{\widehat{r}_1^2 * f_r(\widehat{r}_1)\widehat{r}_1}{C_1} + \frac{\widehat{r}_2^2 * f_r(\widehat{r}_2)\widehat{r}_2}{C_2} \right]. \tag{11}$$

If $f_u(0)$ is large enough, the stability index will be greater than 1, i.e. explosive, at a divided equilibrium. If the distribution of user platform tastes, F_u, is near degenerate, the user platform demand responds to changes in platform application attractiveness v with elasticities near ∞. That makes $\chi(U)$ explosive even if developer supply has only a modest response to user installed base. That demand behavior leads to multiple equilibria: one with all on platform 1, one with all on platform 2, and an unstable divided one.[15] In light of this, we report empirical stability results through a bound, the smallest elasticity of user platform demand with respect to v consistent with instability.

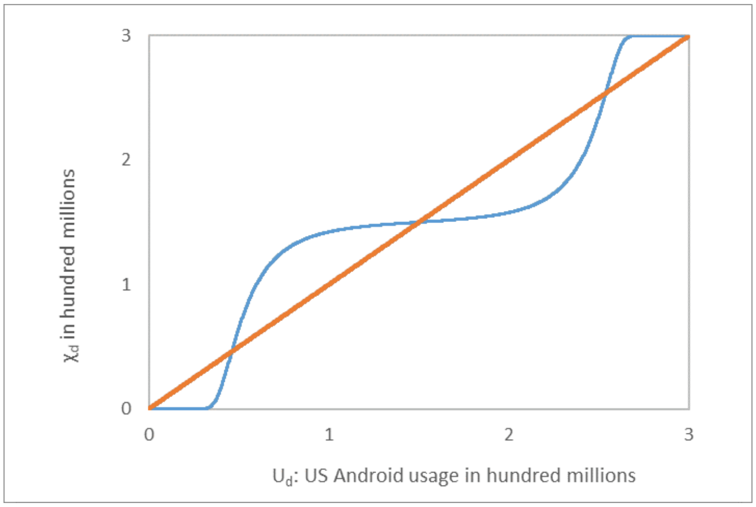
In the traditional version, a stable divided equilibrium will be unique, while an unstable divided equilibrium has stable, dominant-platform equilibria next to it. (See Figure 1.) Under applications heterogeneity, there are two opposing forces, which can be seen in (11). One is the same as in the traditional model. Start from a divided stable equilibrium, and increase U_1 but decrease U_2 while holding U_M fixed. As we move away from $U_1 = U_2$, $f_u(\Delta_V)$ falls, tending toward more stability. But in the applications heterogeneity version of the model, as we increase U_1 and decrease U_2 by the same amount, the term in square brackets in (11) *rises*, tending away from stability. (See Appendix for demonstration.) Indeed, while $f_u()$ is maximized at the divided equilibrium the term in square brackets is *minimized* there. As U_1 and U_2 grow farther apart with $U_2 < U_1$, the fact that $v_p(U_p))$ is convex means that the supply of $v_2 - v_1$ grows more responsive to $U_2 - U_1$. The smaller platform 2 is, at the margin, losing more valuable applications ($\widehat{r}_2$ is larger than $\widehat{r}_1$) which with a star-dominant f_r that 2 losing more aggregate applications attractiveness to users at the margin. So this is another partial reversal of the folk theorem.

"Two opposing forces" theoretical results are more interesting if there is a context in which the unfamiliar result is true. Figure 3, shows χ_1 when the supply of v_p to platforms and the distribution of app heterogeneity are based on our model estimates from Table 2 and we assume user platform demand is a two-parameter logit calibrated to predict shares at the divided equilibrium and to have an elasticity of demand for iPhones with respect to v_{iOS} of 10.[16] The figure shows a stable divided equilibrium, even with that explosive response by users to available applications. However, this is not because the model rules out tippiness. The figure

15. As in the traditional variant, existence of the divided unstable equilibrium requires sufficient symmetry to solve the relevant equations at an interior point. See Appendix.

16. The divided equilibrium is not exactly symmetric since our estimates vary by platform, and there are other gaps, such as allowing for observable application heterogeneity, between the empirical model and the theory. See Appendix VIII.C for a complete description.

Figure 3: Multiple Equilibria with Applications Heterogeneity



also shows two adjacent equilibria, one with mostly Android usage and more applications for Android, the other with mostly iPhone usage and ~~with~~ more applications for iPhone. Critically, these divided equilibria are unstable. Each shows a powerful tendency for the smaller platform to tip out. Once again, ~~what is going on is that~~ the marginal developer ~~at~~ a low-installed-base platform~~, here the smaller platform of a dominant platform equilibrium, has powerful incentives at the margin.~~

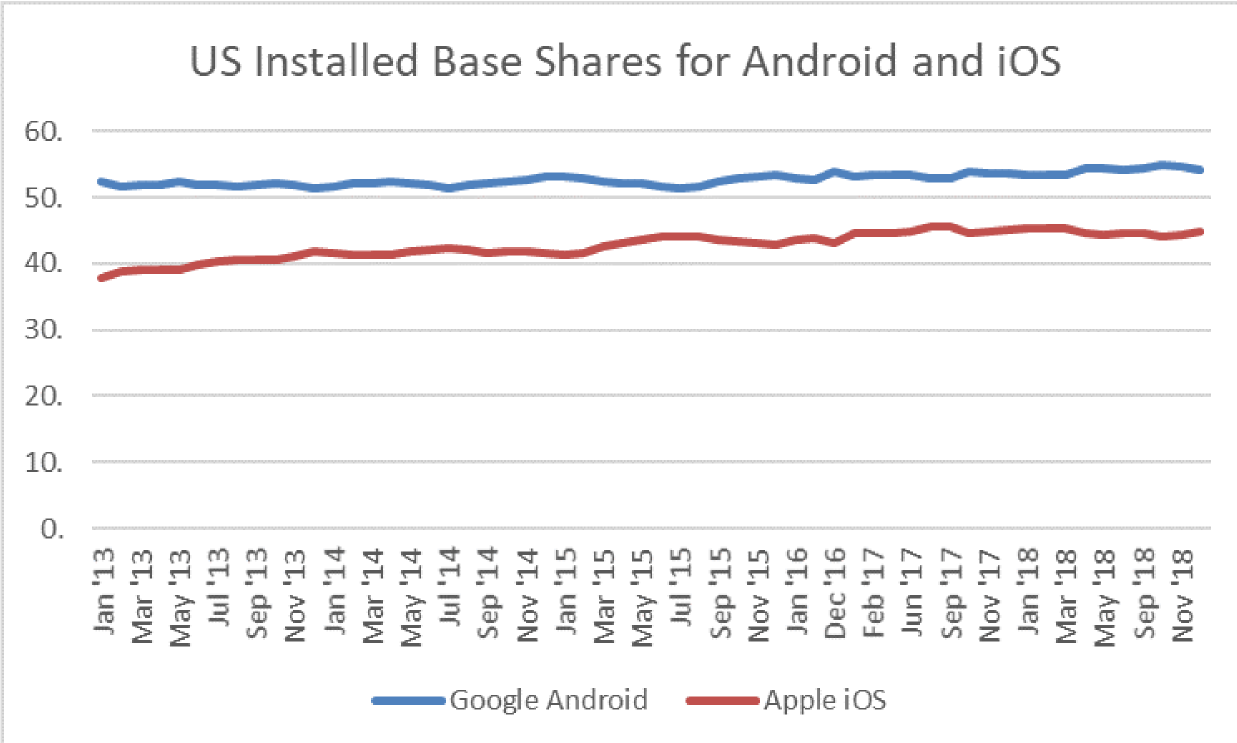
~~The f~~undamental economic assumptions ~~of~~ users valuing applications and applications valuing users ~~lead to the possibility of~~ multiple equilibria and ~~the possibility that some are~~ unstable. However, the ~~pattern that divided equilibria tend to be the unstable ones depends on~~ economic assumptions about applications heterogeneity. The shape of that heterogeneity and its implications for stability can be studied empirically, the topic to which we now turn.

IV Industry and Data

Today's mass market consumer smartphone industry began with the 2007 introduction of the iPhone.[17] The iPhone came with a more consumer-friendly design than any previous smartphone. Smartphones~~,~~ fill~~ing~~ the market gap for a competent consumer computer~~,~~ became the fastest growing~~,~~ and ~~soon the~~ largest~~,~~ development platform market ever. Although Google's Android was introduced 16 months after the iPhone, Google's open systems strategy allowed it to quickly catch up to Apple's user base and app supply. Platform shares were volatile for a period, but since early 2013 there has been little movement away from an approx-

17. Before that, RIM dominated a much smaller market with its Blackberry devices for business communicators, and Nokia offered a "smart" phone that was not an important development platform. See discussion in Bresnahan and Greenstein (2014) about the loss of dominance ~~at~~ those firms in the new consumer-oriented platform market.

Figure 4: Divided Platform Equilibrium in US



Source: comScore "Subscriber Share held by smartphone operating systems in the Unted States." (https://www.statista.com/statistics/266572/market-share-held-by-smartphone-platforms-in-the-united-states). Shares sum to < 100% as only top two platforms shown. The figure shows a three-month moving average stock of phones, labeled by the first of the three months, thus it is smoother than comparable figures based on new phone sales, which shows a seasonal saw-tooth from the annual release cycle of new iPhone variants. Note that both Android and iOS are used in devices other than smartphones, such as tablets.

imately equally divided US platform market. Android's user installed base has been about 5/4 of iOS on smartphones (see Figure 4).

~~N~~o comparable installed base dataset covers a wide variety of countries~~.~~ ~~I~~t is clear~~, however,~~ from data on the new~~-~~phone market and from data based on website access from smartphones~~,~~ that the divided market structure of the US occurs ~~only~~ in a few countries.[18] ~~The divided platform market equilibrium seen in the US is not the norm.~~ Worldwide iOS share is around 14%, though ~~it is likely~~ demand for iOS with its expensive handsets is ~~small~~ in poor countries. ~~Looking at~~ rich countries, we see a wide range of equilibria: Japan and Denmark are about two-thirds iOS and Germany and France are under 30% iOS. ~~Industry sources point to iPhone countries and Android phone countries.~~

Two smaller smartphone platforms tipped out of the US market. Blackberry had been dominant in

18. New phones sales are difficult to convert into national installed base via a perpetual inventory method because there is a wide variety in the useful life of phones – most "burner" phones are Android and a lively international market in used phones. Web access data, such as those from DeviceAtlas or statcounter somewhat overcount iOS because of iOS users' tendency to be richer and to use commercial services more. The figures quoted in text are from statcounter.

smartphones in an earlier, business-user era. The existing applications running on Blackberry phones were broadly irrelevant to consumers, and Blackberry's business user installed base was soon much smaller than the mass-market consumers served by the iPhone. Blackberry, after difficult technical and management decisions, switched to a more consumer-friendly strategy, but found itself in a downward tip with too few users to attract apps and too few consumer-oriented apps to attract users.[19] Microsoft, a late but well-funded entrant into consumer-oriented mobile phone platforms, found itself ~~with tipping forces pushing "Windows Mobile"~~ out. On the developer side of the platform, Microsoft paid small bounties to each developer who submitted an app~~,~~ and ~~very large bounties,~~ reportedly up to \$100,000~~,~~ to selected developers.[20] Unsurprisingly, the ~~negative prices~~ did not draw developers' best work. On the user side, Microsoft bought Nokia, an important smartphone platform firm in Finland, and switched its phones to Windows Mobile. With limited availability of quality applications[21] Windows Mobile tipped downward. Microsoft took a \$7.6 billion charge, laid off more than 3,000 Finns, and then sold Nokia. While the Android/iOS duopoly appears stable, there are important tippyness elements in the US market. We cannot estimate app supply to these two smaller platforms, as industry sources do not find it worthwhile to collect suitable data on them. We shall, however, consider whether our model can explain their tipping out under the assumption that app supply to them and app heterogeneity on them resembled the larger two platforms.

Almost all apps are supplied by third party developers, and relationships between developers and platform providers are arms-length and market-like. A few apps are officially sponsored. They are the main software candidates for divided technical leadership, e.g., browser, mail, maps. Exclusive contracts with outside developers are rare at the two largest platforms.[22]

We measure the attractiveness of a mobile app on a platform ~~through its per-user quantity demanded on the platform~~. If app prices or revenues were systematically observable, this measure could likely be improved. The wide variety of app "monetization" strategies, including earning the economic return entirely through selling complements to the app, render this impractical.[23]

In the US platform market, there are millions of economically unimportant applications on each platform. We build our empirical analysis around a commercial data set that reports information on applications used

19. See Bresnahan and Greenstein (2014) for more complete analysis of this incident and for cites to industry sources.

20. See Bass and Satariano (2010). Microsoft also created programming frameworks that ~~let~~ applications be shared between Windows PCs and mobile devices. The ease-of-porting strategy has worked well for Microsoft tablets but was ineffective for smartphones.

21. ~~Industry observers noted this.~~ William Stofega, IDC program director for mobile phones~~:~~ "The quality of the apps in the store was inferior – not every one of them – but a lot compared to Apple or Android." ~~Quoted in~~ Wheelwright (2017)~~.~~

22. A few predictably popular games were exclusive to a platform for a short period of time at initial launch but not long-term. Also, both iOS and Android have used Yahoo apps to fill gaps before supplying officially sponsored apps (see Figure 5).

23. See Bresnahan, Davis and Yin (2015) and Miric, Boudreau and Jeppesen (2019) for discussion of and statistics on monetization strategies. Some apps have comparatively price-like monetization strategies, such as an initial price or a recurring subscription price. These, however, are not as common as other forms, such as advertising-supported apps, apps with delayed pricing (either "freemium" or for enhancements), and corporate apps that support consumer product and services companies in their main lines of business outside of mobile (e.g., an airline) which typically do not monetize directly through the app.

by at least 0.0012 of users on either iOS or Android phones in a large sample of users. Specifically, we use the January 2013 Mobile Metrix dataset from comScore.[24] comScore has two samples, one of 5,000 adult Android smartphone users in the US and a parallel sample of 5,000 iPhone users. comScore only reports data on an app on a platform if there are more than 5 users in their sample for that platform.[25] Finally, we keep only applications that come from "independent software vendors," as it is the supply behavior of these and the demand for their applications that lie at the heart of our economic enquiry.[26] This yields our final sample of $1,044$ apps. We will address the sample selection issues shortly.

First, based only on the comScore data, we define four variables for each app. We use the * notation to denote that a variable comes from comScore and $p = d$ for Android and $p = i$ for iOS. S^*_{pa} is a dummy variable for the event ~~"~~app a is observed on platform p in the comScore data". At least one of S^*_{da} or S^*_{ia} is 1 for every sample app. comScore reports~~, for each platform,~~ a projection of the fraction of the US population who used the app during the month. This fraction, denoted r^*_{pa}, is called "reach" in the industry. Obviously, r^*_{pa} is truncated from below at 0.0012 for all apps that are actually supplied to platform p ~~and~~ both S^* and r^* are selected by comScore's rules.

To improve our models of selection, truncation, and ~~our model of~~ app supply to platforms, we ~~also~~ go outside comScore and define S_{pa} as a dummy for whether the app was in fact supplied to the platform. For each sample app that appears in the comScore sample on only one platform, we undertook an extensive search to determine whether the app was also supplied to the other platform.[27]

Obviously, $S_{pa} \geq S^*_{pa}$ since an app can be available on a platform but not used by more than 5 people in the comScore sample.

We employ r^*_{pa} as our index of app a's attractiveness on platform p.[28] The dependent variables in our model are r^*_{ap}, S^*_{ap}, and S_{ap}.

24. This dataset is available for subscription at academic rates. We will provide our programs for processing it to anyone seeking to replicate this paper, but you will need to buy your own copy of the underlying data.

25. comScore also reports a few apps with less than this level of usage if a client has requested tracking. We drop these. Modeling those requests in order to gain a few data points seems likely to lower, not raise, statistical reliability.

26. That is, we exclude apps produced by Apple, Google, carriers (e.g., Verizon), and OEMs (e.g., Samsung). These apps are often pre-loaded and thus may not reflect user demand – certainly not user download~~ing~~ and often not user usage of the app. Carrier and OEM apps are not very important in total demand. Apple and Google, as we noted above, each provide a set of core applications that we think of as part of the platform itself, not the indirect network effects positive feedback loop.

27. Our main sources were the two platforms' app stores, developers' websites, app data sources Distimo and AppAnnie, and ~~finally~~ Crunchbase. For apps that appear on only one platform in ~~C~~omScore, we first looked in AppAnnie and Distimo. If we can find the app, we are finished ~~for~~ those sources report when and if the app was first available on the other platform. ~~However, l~~inking to those sources is not always possible, since there are no developer ~~and~~ app unique identifiers common to ~~the app stores~~ and ~~them or to C~~omScore. Our next source was the the developer's website, if it ~~is~~ listed on the app store ~~(as it usually is)~~. That often led us to App Annie, or to a direct statement of whether or not the developer supplies to the other platform. ~~Next, we found~~ some developers with an uninformative website or no link to it on the primary platform ~~in~~ Crunchbase, which frequently lists available products by platform. If none of those links can be found, we look on the other platform's app store for an app with the same name~~, and attempt to follow its link to a developer website. That may be either the same or a different developer, in either case resolving the question.~~ If no such link is available, we had to verify whether it is a version of the same app as on the first platform.

28. The other candidate variable, the time spent in the application, appears to be badly measured and varies remarkably little conditional on r^*_{pa}. A plot of their joint distribution appears in Bresnahan, Davis and Yin (2015).

~~In Table 1, we also report~~ characteristics associated with app a or its developer ~~which we will employ as type regressors~~, i.e. observable heterogeneity measures~~, called X~~. The first is a feature of the app itself: *Game* (abbreviated G) indicates whether the app is in the game category.[29] We also use measures about the firm that supplies the app. We designate three mutually exclusive firm types based on the firm's technological era: offline, online, and mobile. If the firm was founded ~~making~~ mobile app~~s~~ (e.g. Rovio), *Mobile Era* (abbreviated O) equals 1. *Offline Era* (abbreviated F) equals 1 if the developer had an offline business before having an online business or mobile app (e.g., Delta Airlines, CVS pharmacy). Finally, *Online Era* (abbreviated L) equals 1 if the firm has an online business, and was, at the time of its founding, an online-only firm (e.g. Facebook). Obviously $O + L + F = 1$. We also define *Publicly Traded* (abbreviated T) $= 1$ if the developer is a publicly traded firm.

Central to any platform industry is all the technologies and services an application developer need *not* provide. For both the iOS and Android platforms, these include computer hardware (the phone) networking services (via cell networks and wi-fi) and system software running on the phone, on the web, and so on. Platform app stores also provide distribution services.

While all those technologies and services are available on both platforms, there are important differences in how they are structured and managed. Apple is vertically integrated, while Android smartphones come from a wide variety of sellers. Typically there are only a few iPhone models, and typically they are expensive.[30] Android phones vary widely in price and features. The large business advice literature for app developers, and our many dozens of interviews with developers, suggests that per-customer profits will be higher on iOS than on Android.

Developer costs vary between the platforms. Apple mandates app distribution only through its app store, with an approval process that imposes costs of consumer protection and security review. Apple restricts app access to many phone and operating system features. Comparatively permissive Google lets developers distribute through third party stores, and has a less-intrusive security policy.[31] On the other hand, the wide variety of Android phone screen shapes and sizes means that developers bear additional user-interface development costs on that platform. UI costs are typically a large fraction of the fixed costs of a mass-market app, second only to marketing costs.[32] Our empirical model will measure, in the notation of (6), C_p/μ_p but

29. Other app category variables are available from comScore, from the platforms' app stores, or from other commercial sources. However, the game/non-game distinction is the only categorization which is measured reliably. See discussion in Davis ~~et al.~~ (2014).

30. Bundling the smartphone with a cell subscription provides a source of financing for consumers. See Sinkinson (2020) for a very interesting strategic discussion of this practice in the context of the short-lived exclusive availability of the iPhone on AT&T.

31. Effective competition from Android led Apple to relax some of its developer restrictions. Similarly, Android has overcome some of its early shortcomings as a development platform for commercial apps, such as weak payment systems, and its security model has moved towards Apple's over time.

32. The largest costs of entry onto a platform are the marketing costs to "gain visibility", i.e., to make a new population of users on the platform aware of the app's existence. According to our discussions with industry participants (Li, Bresnahan

Table 1: Descriptive Statistics (1,044 apps)

Variable	**Mean**	Variable	**Mean**
S_{ia}	0.765	S^*_{ia}	0.574
S_{da}	0.820	S^*_{da}	0.657
S_{ba}	0.647	S^*_{ba}	0.231
Mobile Era (O)	0.420		
Online Era (L)	0.290		
Offline Era (F)	0.290		
Publicly Traded (T)	0.300		
Game (G)	0.313		
Variable	**Mean (St Dev)**		
r^*_{ia}	0.021 (0.060)		
r^*_{da}	0.018 (0.050)		

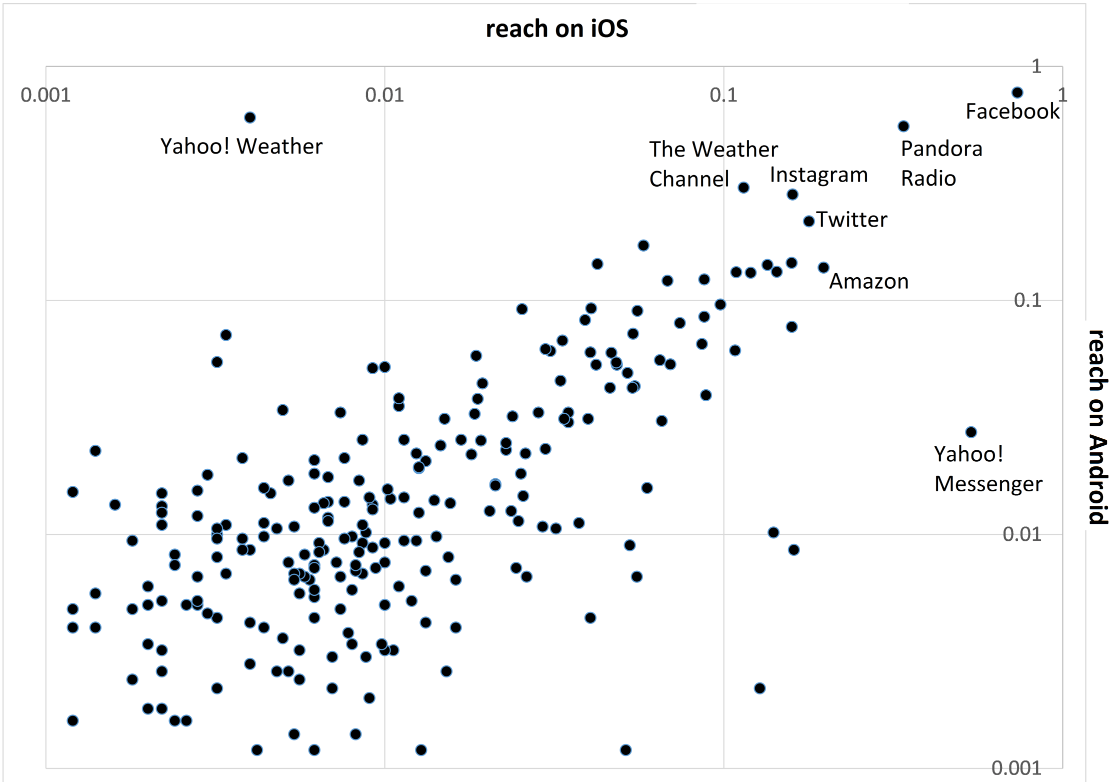
S_{ia}, S_{da}, S_{ba} are indicator variables for whether the app was supplied to iOS, Android, or both. S^*_{ia}, S^*_{da}, S^*_{ba} are indicator variables for whether the app was observed in comScore on iOS, Android, or both. r^*_{ia} and r^*_{da} are the reach of apps observed in comScore on each platform. *Mobile Era, Online Era, Offline Era, Publicly Traded,* and *Game* are indicator variables for characteristics of the developer or app.

we cannot measure the two elements separately. We have a strong conjecture that μ_p is higher for iOS but there are good reasons why C_p could go either way.

We show descriptive statistics in Table 1 and Figures 5, 7, and 6. The first three lines of the Table show application supply to platforms as S_{pa} – the application is available on the platform's app store – next to S^*_{pa} – the app appears in comScore's sample for the platform more than 5 times. An app is about 20 percentage points more likely to appear on the iTunes app store than to be observed in comScore's iOS sample (.77 vs .57) and about 16 percentage points more likely on the Android side. We also constructed the multihoming or ~~”~~both” supply row. Multihoming is common in our sample, but not universal, as about 2/3 of apps multihome. Far fewer multihome and achieve enough success on both platforms to appear in both comScore samples, about 23%. Developers tell us that some apps simply fail to be discovered by the customers on a platform and end up with very low demand. We model this, and also model dependence in user app demand across platforms for the same app. ~~T~~he degree of dependence ~~appears, from looking at Figure 5,~~ likely ~~to~~ be ~~estimated as~~ high. In fact, $\text{corr}(r^*_{ia}, r^*_{da} \,|\, S^*_{ba} = 1) = 0.691$.

and Yin (2016)), launch campaign costs average approximately \$0.5 million. Entrepreneurial app developers tell us that they buy ads displayed in other app developers' apps and pay for "incentivized downloads" in an effort to gain visibility in a mass market. For this reason, our model allows costs to differ by platform and does not restrict the joint costs of multihoming to be less. By contrast, corporate apps can directly access their established firms' existing customers. We thus also permit costs to vary by developer type.

Figure 5: Joint distribution of r^* on Android and iOS for $S_b^* = 1$ (logscale)



~~T~~he minimum of r^*_{pa} is (by construction~~)~~ of the sample, .0012 and the maximum is near 1 as can be seen in Figure 5. The means are both around .02, meaning that star applications are less frequent than mundane applications, as one would expect.

Figures 6 and 7 graph an empirical version of $v(\hat{r})$ (Equation (8)) for each platform. We take the set of observed r^*_{pa} and calculate the step function $v^*_p(\hat{r}_p) \equiv \sum_{r^*_{pa} > \hat{r}_p} r^*_{pa}$. Both clearly have the star-dominant shape shown by the orange curve in Figure 2. Our estimates will deal with the selection behind using the observed set of r_{pa} and will convert the empirical v into quantitative form so that we can perform stability analysis~~. Yet~~ these simple descriptives suggested by the theory demonstrate that application heterogeneity is a first significant digit force.

Finally, the similarity of Figures 6 and 7 to one another, the visible symmetry of Figure 5, and the tendency of all the iOS and Android data in Table 1 to be similar to one another suggest that applications supply and demand are approximately symmetric across platforms in the divided US platform market. While there are somewhat more Android than iOS apps, there are not proportionately as many more as the~~ir~~ are US Android users than iOS users.

The distribution of all the variables, including the Xs, in Table 1 reflect selection on the criterion that the app is economically important. The bulk of the millions of apps that are on the app stores or even of the

Figure 6: Empirical v for iOS $v_i^*(\hat{r}_i)$

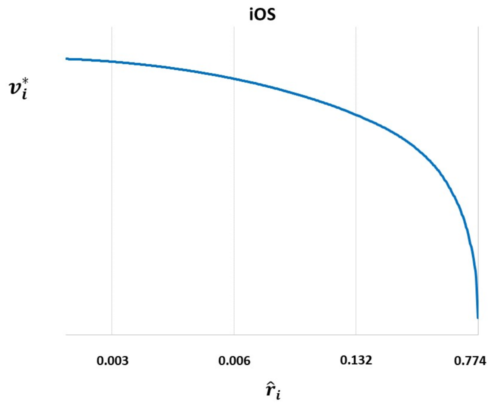
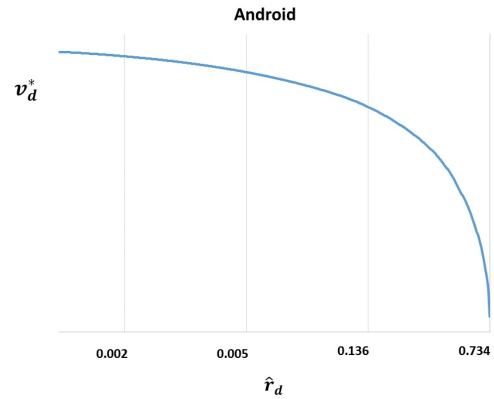


Figure 7: Empirical v for Android $v_d^*(\hat{r}_d)$



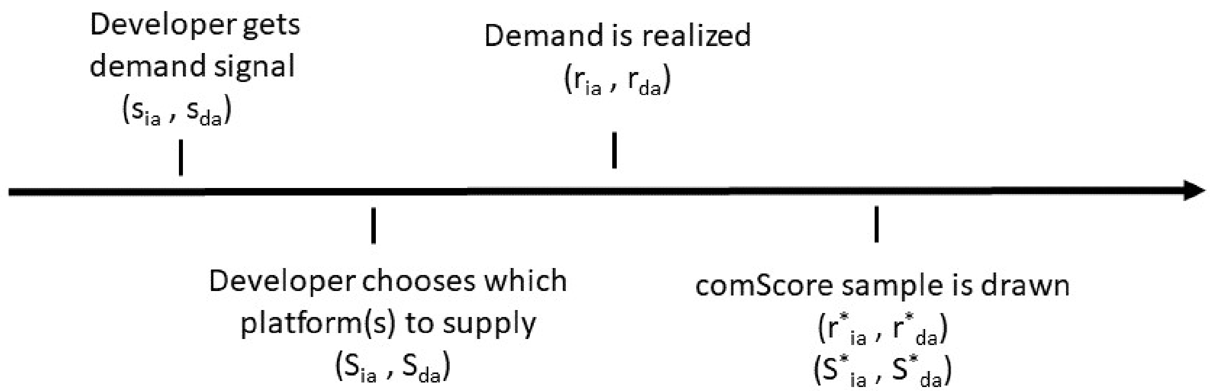
tens of thousands of apps that have been on a top 500 list are from new entrepreneurial firms, but our sample includes only 42% Mobile Era firms (new) and 30% Publicly Traded firms (not newly entrepreneurial) as the supplier. Similarly, about half the apps on the top free list are games, but only 31% of our sample are games. We shall examine whether this tendency for certain kinds of apps to be more valuable economically is still visible after we account for selection and see how selection impacts the inclusion of apps of different types in our estimation sample.

V Developer Supply to Platforms and User App Demand

In this section, we specify an econometric model of user app demand and developer supply with applications heterogeneity. It has the basic structure of the model in section III.B, above, but also includes a number of elements related to our industry. We seek to estimate the supply function of apps to a platform and the demand distribution of apps' attractiveness to users. Our model is parametric, though we also examine the raw data to see what pins down the key elasticities identified in the theory.

Figure 8 shows the model timeline. Developers may have incomplete knowledge of how well their app will be received by the consumers on a given platform. Some applications fail to gain much visibility with consumers on a platform. We treat these together in the two events above the timeline. A developer first gets a signal, $s_a = (s_{ia}, s_{da})$ of its app's demand on each platform. If the app is supplied to a platform p, it either gains visibility with a consumer or it does not, and the resulting reach is r_{pa}. The random variable r_{pa} has support $(0, 1)$, so we will model both reach and signals of reach as having *beta* distributions. We first model the distribution of the signal, then demand conditional on the signal. In our simplified theoretical model, r_a is the same on all platforms. In our empirical model, the developer gets the same signal of profitability

Figure 8: Timeline of Empirical Model



(and thus has the same entry behavior) with probability ω_X. ~~The "X" introduces a notation we shall use throughout.~~ Let X_a be observable features of app a and of the firm supplying the app. We frequently write a deep parameter θ as depending on X_a and denote that by θ_X. If the parameter also varies across platforms, we write θ_{Xp}. Finally, we use the notation θ to mean the vector of all θ_X or of all θ_{Xp}.

We denote the common, cross-platform signal as q_b ("b" for both) and assume that it has a *beta* distribution. To permit many economic and econometric calculations in closed form within the model, we assume that the signal is a mixture of *beta* distributions.[33] Symmetry across platforms is not assumed but the distribution of q_{ba} is restricted to be an average of the two platform distributions

$$q_{ia} \sim beta(\alpha_{Xi}, \beta_{Xi}), \quad q_{da} \sim beta(\alpha_{Xd}, \beta_{Xd}),$$
$$q_{ba} \sim beta((\alpha_{Xi} + \alpha_{Xd})/2, (\beta_{Xi} + \beta_{Xd})/2).$$

$$s_a = \begin{cases} (q_{ia}, q_{da}) \text{ with probability } \omega_X, \\ (q_{ba}, q_{ba}) \text{ otherwise.} \end{cases} \tag{12}$$

To specify the relationship between the signal of reach and reach we introduce λ, the probability that all users learn about the app. If the app is not visible to users, then realized demand for the app, $\overline{r}_{pa}$, is drawn as a new random variable. It is a "shrunk" version of reach:

$$\overline{r}_p \sim beta(\delta_{Xp} * \alpha_{Xp}, \beta_{Xp}) \tag{13}$$

33. There are several other ways to model dependent *beta* distributions. The Sarmonov method fits very badly in our application, since it limits the correlation to be near zero. Another method is to build up the distributions from ratios of *gamma* distributions. This, however, does not lead to both marginal and conditional *beta* distributions and thus would leave a number of the economic as well as econometric calculations below much more difficult.

with $0 < \delta_{Xp} < 1$ parameters to be estimated. Then

$$r_{pa} = \begin{cases} s_{pa} \text{ with probability } \lambda_X, \\ \bar{r}_{pa} \text{ otherwise.} \end{cases} \tag{14}$$

Using (14), we can easily calculate $f(r_a \,|\, s_a, X, \lambda, \delta, \alpha, \beta))$. Using (14) together with (12), we can easily calculate the joint $f(s_a, r_a \,|\, X, \lambda, \delta, \alpha, \beta, \omega)$.

While our demand distribution is parametric and built of *beta* distributions, it is not highly restrictive. First, ~~there is considerably more generality than a *beta* distribution in~~ our model of the size distribution of app demand~~. There are~~ two mixtures, with parameters ω and λ, of *beta* draws. Second, there is observable heterogenity through the dependence on X. The model also permits us easily to deal with the selection in the distribution of r that arises ~~because we~~ only observe an app on a platform if a developer chooses to supply it there and thus to calculate the expectation of developer returns (which depend on the realization of r) conditional on s and thereby impose the same model of heterogeneity on both demand and supply of apps.

In our model, developer supply to platforms is like entry into markets, and shares elements with models reviewed in Berry and Reiss (2007). If app a is published on platform p, it earns $\pi_{pa} = \mu_{pa} \times U_p \times r_{pa} - C_{pa}$, where μ_{pa} is the marginal profit per customer of the app to the developer, U_p is the number of users on platform p, r_{pa} is the ~~por~~tion of users who demand app a on platform p (reach), and C_{pa} is the fixed costs of supplying app a on platform p. In our timeline, the realization of r_a is not known to the developer at the time of the entry decision, so conceptually the condition for supplying a platform to be profitable is

$$\mu_{pa} \times U_p \times \mathbb{E}[\, r_{pa} \,|\, s_a \,] \geq C_{pa} \tag{15}$$

The supply condition is, of course, an entry threshold but $\widehat{r}_{Xp}$ is now the smallest value of the *signal* for which (15) is satisfied. Given our parametric assumptions, we can solve for that in closed form in two steps. First, letting θ be an abbreviation for all parameters,

$$\mathbb{E}[\, r_{pa} \,|\, s_a, X_a, \theta \,] = \lambda_{Xp} s_{pa} \;+\; (1 - \lambda_{Xp}) \mathbb{E}[\, \bar{r}_{pa} \,|\, X_a, \theta \,]$$

We define supply parameters, κ_{Xp}, as the ratio of fixed costs to variable profit, $\kappa_{Xp} = C_{Xp}/\mu_{Xp}$. (We cannot in principle separately identify μ and C.) Now $\widehat{r}_{Xp}$ solves

$$\lambda_{Xp} \widehat{r}_{Xp} \;+\; (1 - \lambda_{Xp}) \mathbb{E}[\, \bar{r}_{Xp} \,|\, X, \theta \,] \;= \kappa_{Xp}/U_p. \tag{16}$$

This leads us to normalize κ for estimation as κ_{Xp}/U_p, putting it in the same units as developer signals and

~~as~~ reach. Thus an app is supplied to a platform, i.e. $S_{pa} = 1$, if it has $s_{pa} > \widehat{r}_{Xp}$ where

$$\widehat{r}_{Xp} = \kappa_{Xp}/\lambda_{Xp}/U_p - (1 - \lambda_{Xp})/\lambda_{Xp} \times \mathbb{E}[\overline{r}_{Xp} \,|\, X, \theta] \tag{17}$$

This lets us make the first of several steps in calculating the likelihood. The supply to each platform of apps of observable type X is the set of signals above the value for $\widehat{r}_{Xp}$ given by (17). The distribution function for those signals comes from (12). We calculate $Pr(S \,|\, X, \theta)$, the probability of any of the four possible values of S_a as a function of X_a and parameters. This is not yet corrected for selection (the event $S = (0, 0)$ cannot occur).

Above, we showed the calculation of $f(r_a \,|\, s_a, X, \lambda, \delta, \alpha, \beta)$. Since S_a is just a coarsening of s_a, this makes calculating $f(r_a \,|\, S_a, X, \lambda, \delta, \alpha, \beta)$ simple in principle, and our parametric assumptions mean that we can make this calculation in closed form. Next, we deal with the truncation problem that, conditional on $S_p = 1$, we will only observe r_p^* if it is at least 6/5000. Then we deal with the selection problem that we only observe an app at all if it satisfies that condition on at least one platform.

Conditional on $S_{pa} = 1$, the probability that an app is observed in the comScore sample and the observed sampling distribution of r_{pa}^* follow from the distribution of r_p which is, conditional on $S_{pa} = 1$, a *beta* mixture. Let g_p be the number of platform p users that have the app in the comScore sample. The distribution of g_p conditional on r_p is binomial and involves no new parameters as the sample size of 5000 is known; unconditional on r_p, g_p has a $beta - binomial$ mixture. The app is in the comScore sample if $g_p \geq 6$. Thus $Pr(S_p^* = 1 \,|\, S_p = 1, X, \theta) = Pr(g_p \geq 6 \,|\, S_p = 1, X, \theta)$.

Next, we calculate $f(r_p^* \,|\, S_p^* = 1, S_p = 1, X, \theta)$ taking into account the realization of r_p^* is always a value of the form $k/5000$, where k is an integer, using $Pr(g_p = k \,|\, g_p \geq 6, X_a, \theta)$ for any k from 6 to 5000.

We can now write the joint distribution of r^*, S^*, and S given X and parameters θ, denoted $f_Y(S, S^*, r^* \,|\, X, \theta) = Pr(S \,|\, X, \theta) * Pr(S^* \,|\, S, X, \theta) * f(r^* \,|\, S, X, \theta)$. We do not observe an app unless it meets comScore's sampling criterion on at least one platform, i.e., $S_{ia}^* + S_{da}^* > 0$. The structure of the joint distribution of r^*, S^*, and S and our parametric assumptions permit us to calculate the probability of this event, denoted $\Pr(S_{ia}^* + S_{da}^* > 0 \,|X_a, \theta)$ in closed form. We correct for sample selection by dividing by this probability and maximizing the conditional likelihood:

$$L_C(S, S^*, r^* \,|\, X, \theta) = \sum_a \log\left(\frac{f_Y(S_a, S_a^*, r_a^* \,|\, X_a, \theta)}{\Pr(S_{ia}^* + S_{da}^* > 0 \,|\, X_a, \theta)}\right).$$

This solves a long-standing "potential entrants" problem in entry models. The problem arises in analyses which, like Berry (1992), identify a list of potential entrants into one market as the actual entrants into other markets.[34] If a firm's profit in one market is not independent of its profit in another, the list of potential

34. Some entry models like those of Bresnahan and Reiss (1990) and Seim (2006) identify a set of market niches rather than

entrants is not exogenous. This problem certainly applies to our application: we use actual entrants into platform p as the potential entrants into platform p'. Our model adds no additional "selection" parameters to be estimated because we build economic and econometric models of entry into both markets. This creates not only a model of supply but also a model of selection, as entry into at least one of the markets is how a firm qualifies for the list of potential entrants into the other.

VI Results

We first estimated our model without restrictions. That means letting all the deep parameters vary with X and, where ~~it is~~ possible, with p. That specification led to many poorly-estimated parameters. Thus we report the more restricted specification where the free parameters are those listed in Table 2.[35]

VI.A App Demand on iOS and on Android

We begin with the demand estimates, reported at the top of Table 2. We write α_{Xp} as a regression on the supply firm characteristics F, O, and T and the app characteristic G. Thus the baseline constant α_p applies to the values Online Era (L), Non-Game (Y), Privately Held (not publicly traded) (H).

Table 2: Parameter Estimates

	Estimate	Standard Error		Estimate	Standard Error
α_d Constant	0.3124*	(0.1035)*	α_i Constant	0.3003**	(0.0984)
α_d Offline Era (F)	-0.0083	(0.0710)	α_i Offline Era (F)	-0.0000	(0.0795)
α_d Mobile Era (O)	-0.1195	(0.1175)	α_i Mobile Era (O)	-0.2005*	(0.1088)
α_d Game (G)	-0.0781**	(0.0328)	α_i Game (G)	-0.0224	(0.0252)
α_d Publ. Traded (T)	0.0670	(0.0550)	α_i Publ. Traded (T)	0.1485**	(0.0672)
β	22.5658**	(0.4536)			
ω	0.3482**	(0.0631)			
κ_d/U_d	0.0011**	(0.0004)	κ_i/U_i	0.0011**	(0.0004)
λ	0.6480**	(0.0728)	λ (O)	-0.0983	(0.1158)

Note: Bootstrapped standard errors are presented (250 draws). **significant at 5% *significant at 10%. The parameters β, ω, and λ do not vary by platform. The parameters α and κ/U do vary by platform.

The first row of Table 2 indicates that $\alpha_i \approx \alpha_d$ for baseline apps/firms. Equality across platforms is less clear in the point estimates of the rest of the α parameters, but we cannot reject the hypotheses that

a set of potential entrants and thereby avoid this problem.

35. We shut down variation over X in the dependency parameter ω, and all but one X coefficient in the uncertainty parameters λ. We set $\delta = 0.02$, consistent with findings in Li ~~et al.~~ (2016) regarding the difficulty of getting visibility on a top list in the app stores, which uses time series data on the apps' progress in app store top lists. While in this specification only the α_{Xp} vary across observable type X and platform p, we also have a more richly parameterized specification in which β_{Xp} also vary. Little differs in that specification, so we do not present those results here. κ/U varies with p but not X.

$\alpha_d = \alpha_i$, either when we test one row at a time or for the entire α_p vector. There appears to be little statistical difference between app demand on the two platforms. Now turning to the question of α varying with X, we reject the hypothesis that X does not matter for α statistically. The larger effects are for Mobile Era (O) firms (negative on both platforms, significantly so on iOS), Publicly Traded ~~firms~~ (T) (positive on both, significant on iOS) and Game apps (negative on both, significant on Android). We cannot reject the hypothesis that β and ω do not vary with X or p and have imposed this restriction in the results shown. The estimate of ω of a little over a third suggests substantial but not overwhelming dependence across platforms in the unobserved portion of app demand. The estimate of β is quite large, and the regressors in α are all dummy variables, so we can immediately see that $\beta \gg \alpha$ for any X. This implies the unsurprising result that the mean predicted demands are small~~,~~

We can see the structure of observed heterogeneity in app demand across firm and app characteristics and across platforms in Figure 9. For each of the 12 unique values of X we plot the point estimate of the mean of expected reach on iOS and on Android if consumers learned about all apps.[36] Each point is labeled by its X values; for example, the leftmost point is OHG, i.e. Game apps from Mobile Era, Privately Held firms. The notation for all the other points is explained in the table footnote.

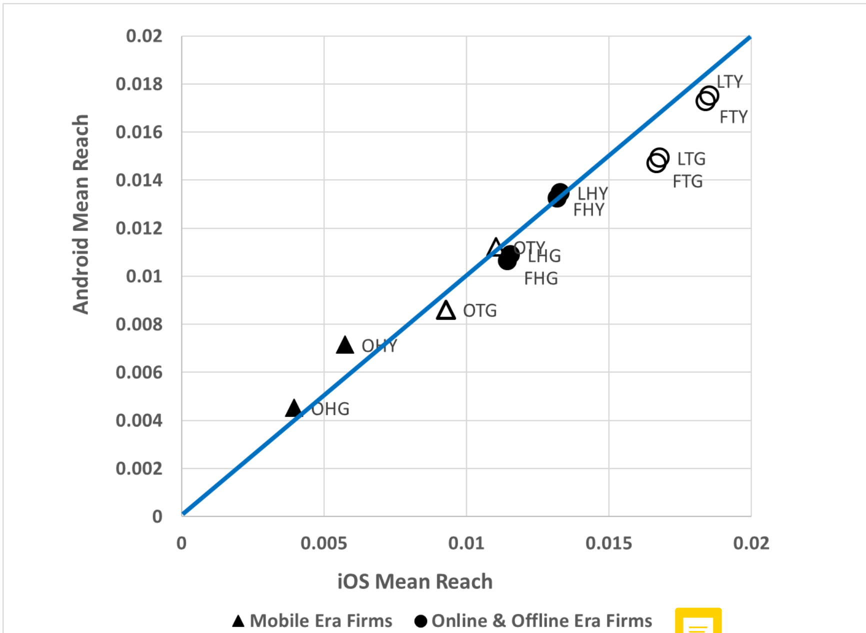
Our estimates reveal different roles for observable variation across apps in (fully informed) demand as X and p vary. First, ~~A~~s shown in Figure 9, the mean of fully-informed demand conditional on X not vary much with p. In light of the symmetry across p of the joint distribution of observed r seen in Figure 5, this is unsurprising.

As shown in Figure 9, the mean of fully-informed demand does vary with X. This variation, however, is not a large portion of the total variation in fully informed demand across apps. We calculate two model-based quanta. First, the population-weighted variance of mean demand as X varies is $var_X(E[Y|X,p])$. This is the population weighted variance of the means in Figure 9~~)~~. Second, we calculate the variance of an individual app's demand, if consumers were fully informed, around that mean, $E_X[var(Y|X,p)]$. We calculate this variance for each X, and then take the weighted average over X. The first of these quanta is the observable variation in fully informed demand, the second, the unobservable variation. The unobserved variation is 13x as large for iOS, 18x as large for Android. In light of the very wide spread in realized app demand seen in Figure 5, this too is unsurprising.

There is a clear, unexpected, pattern in the variation of mean demand across X. To see it, we have used different symbols in the figure for mobile era firms vs firms founded in earlier eras, and different shading for publicly traded firms. The established firms – both ones from the older technological eras and publicly

36. That is, for iOS it calculates $\omega * \alpha_{Xi}/(\alpha_{Xi}+\beta)+(1-\omega)(\alpha_{Xi}+\alpha_{Xd})/(\alpha_{Xi}+\beta+\alpha_{Xd}+\beta)$ and symmetrically for Android. These follow from our assumption that developers' signals are the reach of their app if users could learn about the apps without search. In another notation, the figure shows $(E[s_i|X], E[s_d|X])$.

Figure 9: Mean Reach by Platform and X if all apps were visible to users



Points are labeled by their X in the notation ABC, where C=G (Game) or Y (Non-Game), B=T (Publicly Traded) or H (Privately Held) and A=O (Mobile era), L (Online Era), or F (Offline Era). The L and F points are marked with a different symbol than are the O points.

traded ones – tend to have higher demand. Entrepreneurial firms from the mobile era tend to have lower demand. This contradicts an expectation held by many industry observers that mobile app development would be an entirely new, entrepreneurial industry. It certainly is very entrepreneurial if you count firms, but not if you count market success.

That pattern in demand if consumers were fully informed about all apps is reinforced by the point estimates of λ, the probability that consumers are fully informed. The point estimates ~~are~~ for ~~a~~ λ ~~of about~~ two-thirds, smaller for Mobile Era apps (λ (O) < 0). Though imprecisely estimated, this suggests Mobile Era apps h[illegible] a lower probability of being visible and achieving their fully-informed consumer reach. In short, a version of Figure 9 displaying the point estimates of the means of realized reach r_{Xp} using equation (14) would reveal an even greater relative demand disadvantage for Mobile Era firms' apps.

While we can reject the model of our theory section in which app has the same attractiveness to users of both platforms, demand for apps is quantitatively similar on iOS and Android. When we turn to the analysis of stability based on our estimates, we will use these quantitatively similar but not identical demand estimates.

VI.B Supply for iOS vs. Android.

The supply parameters (κ) are presented in Table 2. We estimate them as κ_p/U_p ~~as~~ those are the units of reach and ~~of~~ developer's signal. The table shows that estimated $\kappa_i/U_i = \kappa_d/U_d$ to three significant digits (they also cannot be distinguished statistically). This result has two economic implications.

First, these estimates imply $\mu_d/C_d = 0.75\mu_i/C_i$.[37] That is, [per customer return]/[fixed costs] is 75% as large on Android as on iOS. This is consistent with the developer opinion we discussed above, in which per-customer profits are higher on iOS, since iOS users tend to be richer. Second, the higher μ_i/C_i is just offset by the lower U_i and therefore $\mu_p/C_p/U_p$ is almost exactly equal on the two platforms. This leads to very nearly symmetric supply behavior between the two platforms.[38] The source of this symmetry in the data is not surprising. First, the demand estimates are very close to symmetric across platforms. Second, as we saw in Table 1, approximately the same fraction of sample apps were written for iOS (77 percent) as for Android (82 percent).

Our sample-selection calculation varies with X, since both α and λ do. Apps with lower realized demand are more likely to fall below the threshold for inclusion in the sample. For example, $\Pr(S^*_{ia} + S^*_{da} > 0 \,|\, X, \theta)$ is about half as large for Mobile Era applications as for others. See details in Table 4.

~~Taking it all in,~~ the distribution of applications heterogeneity, the degree of developer uncertainty about applications success, and developer profitability post~~-~~entry (adjusted for slightly different installed bases) are similar across the two platforms. While we do not estimate a user platform demand equation, the 5/4 split in favor of Android is consistent with somewhat higher prices for iPhones and with a slightly lower v, total value of applications, on iOS (~~shown~~ in Table 3). The divided US equilibrium is close enough to the theoretical construct of symmetry to allow investigation of the question of stablity.

VI.C Platform Market Equilibrium Stability

We now examine the equilibrium stability index based on our empirical estimates, providing a bound for stability at the historical US duopoly and in a smaller country with the same per-capita demand as the US and in an asymmetric equilibrium with a much smaller platform, like the historical Windows mobile.

An equilibrium is a fixed point of the function of U to itself, $\chi(U) = D(v(\widehat{r}(U)))$ and its Jacobian is given by $J_D J_v J_{\widehat{r}}(U)$. We have estimated $J_v(v \,|\, X, \theta)$ and $J_{\widehat{r}}(U \,|\, X, \theta)$ and will calculate various bounds on J_D.

One change from the theoretical model is that the estimates of application heterogeneity, ~~of~~ the quality

37. Since $U_i/U_d = 0.39/0.52$ in January-March 2013, the period including our estimation month.

38. Since the thresholds to supply each platform depend on expected demand and on λ as well as on κ_p/U_p (See Equation (17)) there are small differences in $\widehat{r}_{Xp}$ across p and larger differences across X. The absolute value of $\widehat{r}_{Xd} - \widehat{r}_{Xi}$ is less than 0.0015 for 10 out of 12 values of X. The largest difference arises for Publicly Traded, Offline Era firms supplying a Game app, where $\widehat{r}_{Xd} = 0.0090$ and, $\widehat{r}_{Xi} = 0.0115$.

of developer signals, and ~~of~~ supply costs vary with X and p. As we show in Appendix VIII.D.1, $J_{\hat{r}}(U)$ is a $P * dim(X)$ by P matrix where P is the number of platforms under consideration and $dim(X)$ is the number of unique values of X with typical element:

$$\frac{\partial \hat{r}_{Xp}}{\partial U_p} = -\frac{C_{Xp}}{\mu_{Xp}\ \lambda_{Xp}\ U_p^2}. \tag{18}$$

The same changes are relevant to our empirical J_v. As we show in appendix VIII.D.2, the typical element of the P by $P * dim(X)$ matrix J_v is

$$\frac{\partial v_p}{\partial \hat{r}_{Xp}} = N_X \left[-\lambda_{Xp}\hat{r}_{Xp} f_{Xp}(\hat{r}) + (1 - \lambda_{Xp}) E[\overline{r}_{Xp}] f_{Xp}(\hat{r}) \right] \tag{19}$$

where N_X is the population size of potential developers of apps of type X[39] and $f_{Xp}()$ is the density function of the signal seen by the developer. We note that the P by P matrix $J_v J_{\hat{r}}(U)$ depends both on estimated parameters and on the point at which we are evaluating it, U.

Without an estimate of $D(v)$, we proceed to bound it. In Figure 3 we plot $\chi_1(U_d)$ using our estimated $\hat{r}_{Xp}(U)$ and our estimated $v(\hat{r}_{Xp})$ and an assumed $D(v)$, a logit demand system with a γ (coefficient of v) picked to make the own-v elasticity of demand for the iPhone be 10 and $\gamma_i - \gamma_d$ picked to predict the observed shares. That figure shows a quite stable divided equilibrium.

In the first row of Table 3, we report calculations associated with the Observed US Duopoly, all calculated at the divided equilibrium point. The final column reports our bound. We increase γ in the assumed logit $D(v)$ until the divided equilibrium is just unstable. We report this bound in terms of the own-app-availability elasticity for iOS, $\eta_i^i \equiv \partial ln(U_i)/\partial ln(v_i)$. We get an extraordinarily large bound, over 25. Such a large elasticity is implausible,[40] so we conclude that our model is consistent with the multiyear persistence of the US observed divided duopoly. How our model produces this result is easily understood. Our estimates let us calculate (not bound) $J_v J_{\hat{r}}(U)$ in elasticity form. The table reports this in elasticity form, $\partial \ln(v_p)/\partial \ln(U_p)$. For both Android and iOS is small, around 0.02. The behavior of developers in supplying applications to platforms, *measured in terms of the applications' attractiveness to users*, is very damped. Accordingly, the response of users to the availability of applications would need to be explosive for the divided equilibrium to be unstable.

39. Our procedure for estimating N is laid out in the Appendix VIII.D.4.

40. For our bound on η_i^i to be violated requires a strength of user reaction such that, for example, if the eBay app were unavailable on iPhones, over a quarter of iPhone users would switch to Android phones. Such a large platform demand elasticity is inconsistent with the widely held view among industry participants that iPhones have been successfully differentiated from (most) Android phones, all but the most expensive ones – see Bresnahan and Greenstein (2014). The available estimate of the *price* elasticity of demand for iPhones, from Sinkinson (2020), is less than 1. However, it is likely not reasonable to assume that the price elasticity is a good quantitative estimate of other demand elasticities. Users may respond less to handset prices than to other elements of the platform surrounding the handset. Handset prices are often not charged directly to the user, instead ~~being~~ paid as a part of a cell services subscription. Handsets are a durable good, and financed in opaque ways. Still, the best quantitative evidence about demand elasticities does not suggest an explosive user response.

Table 3: Supply Elasticities and Stability Bounds

Evaluated at:	v_d	v_i	v_3 [c]	$\frac{\partial \ln(v_d)}{\partial \ln(U_d)}$	$\frac{\partial \ln(v_i)}{\partial \ln(U_i)}$	$\frac{\partial \ln(v_3)}{\partial \ln(U_3)}$	Bound on η_i^i
Observed US Duopoly [a]	14.90	13.51		0.022	0.020		25.39
Small-Market Duopoly[b]	13.03	11.92		0.179	0.168		3.100
Triopoly; smaller platform added to observed duopoly[c]	14.90	13.51	11.26	0.022	0.020	0.236	3.104

[a] The "Observed Duopoly" row is evaluated at historical U_d, U_i. Supply and v_p are based on our estimates. The assumed logit has consumers choosing only between iOS and Android. Shares are 0.559 and 0.441.

[b] The "Small-Market Duopoly" has two platforms with the same shares as the historical duopoly, but with U_d and U_i 1/6 as large. We hold all the economic parameters fixed.

[c] The "Triopoly; smaller platform added to observed duopoly" row leaves U_i and U_d at observed levels and assigns all other observed smartphones (Symbian, Windows Phone, Blackberry) in use in January 2013 to U_3. The resulting shares are 0.530, 0.418, and 0.052. The artificial Platform 3 occurs only in triopoly situations and is endowed with iOS economic parameter values.

Of course, the Observed US Duopoly bound is high in part because because both platforms' installed bases are so large. To understand the implications of our estimates for stability if there were a third, smaller platform or if the market size were much smaller, we calculate two further bounds.

One of these bounds is for a smaller market size, 1/6 that of the US. We evaluate the Jacobian and calculate the bounds in the Small-Market Duopoly row using the same platform market shares as in the observed historical duopoly. We use all of the same parameter estimates. What changes are the $\widehat{r}$ values which rise to reflect the smaller installed base and, of course, the possibility that v_p is more responsive to U_p in a smaller market. The values for $f_{Xp}(r)$ at those higher thresholds are in-sample. We have plenty of observations of applications with r above and below these higher thresholds. If we tried a bound moving the other direction, to a market even larger than the US duopoly, it would be out of sample, so we do not attempt this.

The differences between the Observed US Duopoly and the Smaller Market Duopoly results illustrate the logic of our model given the applications heterogeneity observed in the US market. The bound on η_i^i is much lower, just over 3, making instability of the divided equilibrium in a smaller market imaginable. The changed bound arises because the estimated quantities $\partial \ln(v_p)/\partial \ln(U_p)$ are significantly larger evaluated at this point, over 8x as large as in the observed duopoly row for each platform. Another advantage of computing equilibrium quanta for a very different market size is that we can see the supply elasticities over a wide arc. Much of the supply of applications, measured in terms of attractiveness to users, is inframarginal. If we reduced the market size by 6/6, v would fall to zero. Reducing market size by 5/6, and thus reducing each platform's installed base by that proportion, reduces v by about 12%. Over a wide range, the distribution of application heterogeneity is star-dominant and the supply behavior of the star applications is inframarginal. Of course, the inframarginal supply in large markets must become marginal supply at some smaller one – the supply of applications switches from inelastic to elastic at some point as market size falls.

We can learn more about the logic of our economic model, evaluated at our estimates, by looking at the Triopoly row of ~~the~~ Table. Here we consider stability when a third, smaller, platform is added to the Observed Duopoly. We leave U_d and U_i at their observed levels, and give this third platform an installed base (U_3) 1/8 the size of the iOS installed base at the Observed Duopoly. We give this third platform the same parameters as iOS. Once again, we evaluate the stability index with the same parameters but at a different point. Here, too, the stability index bound is much smaller, about 3.1, so once again, tipping seems plausible. Here the bound is based on the eigenvalues of a 3*3 matrix because there are three platforms. Note that only one of the $\partial \ln(v_p)/\partial \ln(U_p)$ differs from the Observed Duopoly row of the table, the one for $\partial \ln(v_3)/\partial \ln(U_3)$. This is by construction, but it also informs our interpretation of the Triopoly results. The small platform is likely to tip out. Like the Small-Market Duopoly calculation, the Triopoly calculation draws our attention to the possibility that applications heterogeneity makes platform supply behavior, measured in terms of the value of the applications to consumers, much more explosive with a smaller installed base for any platform.

~~To say that these calculations explain why the smartphone platform market appears to have a stable equilibrium in the US but not in many smaller countries and why the stable duopoly does not rule out a smaller platform, like Windows Mobile, tipping out, involves further unverified assumptions.~~

Interpreting the smaller-market results as applying to a real-world market with 1/6 the users of the US requires the assumption that applications heterogeneity, applications demand by consumers, and applications supply are like the US, just smaller. That is, however, a bit too strict. If we thought of a market as both smaller than the US and poorer than the US, ~~and~~ thus having both lower U_M and μ, this analysis could still apply. The developer supply model has μU_p wherever it has U_p. With this change, however, we are interpreting the results as applying to a smaller market, not necessarily 1/6 as large. The key assumption is about application heterogeneity. A smaller economy with a similar distribution of application attractiveness to users as in the USA, i.e., a star-dominant one, could have an unstable, rather than a stable, divided equilibrium.

Interpreting the Triopoly results as applying to the experience of Windows Mobile also involves additional unverified assumptions. Here, we are still looking at the USA. However, when we assign the iOS parameters to our third, hypothetical, platform, we are clearly assuming a very capable third platform. So that is an extrapolation. At the minimum, our analysis suggests that a third US platform might need to grow significantly larger than WinMo ever did, i.e., to create an approximately evenly divided Triopoly, for equilibrium to be stable rather than for the smallest platform to face a powerful tendency to tip out. That possibility may have fallen within Microsoft's aspirations for WinMo, but ~~very far~~ exceeded the resources the firm put into the mobile effort.

VII Conclusion

The market structure in smartphone applications development platforms in the US, the largest application development platform market (as yet) ever seen, is surprising. Rather than tipping to a dominant platform, the market has stayed at approximately evenly divided duopoly. This is not because there are no positive feedback forces benefiting larger platforms and penalizing smaller ones. Smaller platforms have tipped out of the US, including an effort by a firm expert in platform supply which lost billions on the effort. The platform market in most other countries has tipped to Android, and in a few to iOS. These apparently contradictory outcomes can be easily explained with a single change to traditional platform market stability analysis, adding heterogeneity in application attractiveness to users. Heterogeneity of a star-dominant form is particularly important. It renders ~~inelastic~~ the supply of applications to a large platform~~, measured by the attractiveness of all the applications available on the platform.~~ The mechanism is simple. The most attractive applications supply even a moderately-sized platform; their supply behavior is inframarginal if a platform has many users. With one side of the positive feedback loop damped, the entire loop can be explosive only if users respond very elastically to changes in available applications on a platform.

We have estimated the distribution of applications attractiveness to users across both iOS and Android apps, resolving the selection and truncation issues associated with our data supplier's sampling frame and with endogenous supply of applications to platforms. We have also estimated a application supply to platforms model, taking into account the problem that demand is imperfectly forecastable by developers *ex ante* entry. The estimated attractiveness heterogeneity distribution, like the empirical ~~CDF~~ of the raw data (with its selection and truncation problems) is star-dominant. As a result, supply of applications to platforms~~, measured in attractiveness to users,~~ is quite inelastic. Even a substantial reduction in installed base size from the very high US levels leads only to modest declines in aggregate applications attractiveness on platforms. Since the arc elasticity of supply from any installed base to zero must be one (none of the applications would be supplied to a platform with no users) the low elasticity at high installed bases implies a high elasticity at low installed bases. Both follow from the star-dominant shape. Our estimates thus show explosive applications supply (at low installed bases) and very damped applications supply at US-duopoly level installed bases.

These results resolve the empirical puzzle at the heart of this paper. With further assumptions about the comparability of app demand and supply across platforms and countries, they resolve the puzzle of stability in the US duopoly and tippiness in other countries and for smaller platforms. More importantly, our work merges ideas from the literature on consumer entertainment and media products with classical platform stability analysis to create models suitable for modern consumer-oriented platforms.

VIII Appendix

In this appendix we state more carefully and prove the theoretical results of Section III. The traditional version is covered in Section VIII.A, and the applications heterogeneity version in Section VIII.B. Finally, this appendix lays out the formulae for stability indexes in our empirical model, the assumptions behind our bounds, and provides some background tables in Section VIII.D.

VIII.A The Folk Theorem

We begin with the model defined by (1), (2), (3), and (4). The main economic assumptions are noted in the text. Here we state regularity conditions and show the results.

We assume that both F_u and F_a are continuous in all their derivatives, are symmetric around zero, and that F_u has infinite support. Each is strongly unimodal in the sense that $f_a(x) < f_a(y)$ and $f_u(x) < f_u(y)$ whenever $|x| > |y|$. They are, in short, error terms.

Recalling $\Delta_V(N) \equiv \gamma(N_1 - N_2) + \gamma_1 - \gamma_2$, the Jacobian of $U(N)$ for this model is

$$f_u(\Delta_V(N)) * U_M * \gamma * \begin{Bmatrix} 1 & -1 \\ -1 & 1 \end{Bmatrix} \tag{A20}$$

Recalling $\Delta_\pi \equiv \mu(U_1 - U_2) - C_1 + C_2$, the Jacobian of $N(U)$ for this model is

$$f_a(\Delta_\pi(U)) * \mu * \begin{Bmatrix} 1 & -1 \\ -1 & 1 \end{Bmatrix}. \tag{A21}$$

The Jacobian of χ is the product of these two matrices. It has two eigenvalues, 0 and

$$2 * \gamma * f_u(\Delta_V(N)) * U_M * \mu * f_a(\Delta_\pi(U)) \tag{A22}$$

So that (A22) is the stability index for this version.

As we increase either γ or μ, the stability index given by (A22) increases at the divided equilibrium. Obviously, the index is 0 for $\gamma = \mu = 0$, so there is a hyperbola in γ, μ space ($\gamma * \mu = 1/(2 * f_u(0) * U_M * f_a(0))$) above which the divided equilibrium is unstable and below which it is stable.

Note that $\chi_1(0) > 0$ and $\chi_1(U_M) < U_M$ because of the infinite support assumption. Also, χ_1, formed from the continuous $U(N)$ and $N(U)$, is continuous. The geometry leads to a number of propositions.

- There is a stable equilibrium. The continuous χ_1 must pass from above the 45° line to below it.
- If $\gamma * \mu$ is low enough, (A22) will be less than 1 for all U_1 and equilibrium will be unique and stable. Under this condition, all crossings of the 45° line will be from above, so there can be only one crossing.
- When there are multiple equilibria, the equilibria with the largest and smallest U_1 are always stable.
  - Proof: We have $\chi_1(0) > 0$ and χ_1 continuous, so the lowest crossing must come from above. Symmetrically for the highest crossing. See Figure 1.
  - This result will always hold if there are two platforms, one side of the market single-homes, and that side has an infinite-support error term.
- If there is a strictly unstable equilibrium, there are at least three equilibria. This follows because the largest and smallest equilibria are stable.
  1. In the symmetric case, if the divided equilibrium is unstable, there are exactly three equilibria. Proof: The divided stable equilibrium occurs at $U_1 = 0.5U_M$. (A22) is maximized at that point, declining monotonically as U_1 increases above that point and declining monotonically as U_1 decreases below it. This follows because, as U_1 increases above that point, Δ_Π increases monotonically (by arithmetic) as does Δ_V (because N_1 is increasing, and N_2 decreasing, in $U_1 - U_2$.) The monotone (A22) cannot change from above 1 to below 1 to above.
  2. In the symmetric case, the equilibrium correspondence is everywhere continuous except at the point where one equilibrium becomes three, i.e. at $\gamma\mu = x_0$.
  3. Outside the symmetric case, there can be two equilibria, where one is a point of tangency between χ_1 and the 45 degree line. Increases in $\gamma * \mu * U_M$ at that point add a third equilibrium. Except at such points, the equilibrium correspondence is everywhere continuous.
  4. In either the symmetric or assymetric case, for low values of $\gamma * \mu * U_M$ there is a unique stable equilibrium. Increases in γ, μ, or U_M can change the equilibrium correspondence from unique to multiple.
- Since the equilibrium correspondence is continuous almost everywhere, for economic primitives close to symmetry and close to a symmetric case with three equilibria, there are also three equilibria with the divided equilibrium not at the symmetric point.

An extreme version of the folk theorem arises when both F_a and F_u are degenerate distributions at 0. There is then no continuity and either two or three equilibria:

1. All developers and users choose 1.
2. All developers and users choose 2.

3. $N_1 - N_2 = (\gamma_2 - \gamma_1)/\gamma$ and $U_1 - U_2 = (C_1 - C_2)/\mu$ so that all developers and users are indifferent between 1 and 2.

That third, divided equilibrium exists only if the solutions to the two equations fall in $0 < N_1 < 1$ and $0 < U_1 < U_M$. It is a "knife edge," i.e. unstable.

VIII.B Heterogeneous Application Variant

We examine the model defined by (6), (7), (8), and (9) with $\Delta_V = \gamma(v_1 - v_2) + \gamma_1 - \gamma_2$. We assume that F_u is continuous in all its derivatives and has infinite support, is symmetric around zero, and is strongly unimodal in the sense defined above. We assume that F_r has support on $[0, 1]$, consistent with our interpretation of r_a as the fraction of users who demand app a.

For $U_P < U_p^L = C_p/\mu$, no app finds it profitable to supply platform p. Reprinting (6)

$$\Pi_{ap} = \mu r_a U_p - C_p,$$

Taking this into account, application supply to platform p is

$$\text{(A23)} \qquad \begin{aligned} v_p(U_p) &= 0 \quad \text{for } U_p < U_p^L \\ v_p(U_p) &= \int_{\widehat{r}_p}^{1} t f_r(t) dt \quad \text{for } U_p \geq U_p^L \end{aligned}$$

The supply equations can be written one platform at a time, and thus apply to any number of platforms. For the case of two platforms, we have user demand for platforms defined by

$$U(v) = U_M \times [F_u(\Delta_V(v)) \;\; 1 - F_u(\Delta_V(v))]$$

This, (A23), and $U_2 = U_M - U_1$ let us once again define the function $\chi_1(U_1)$. χ_1 is continuous, $\chi_1(0) > 0$ and, symmetrically, $\chi_1(U_M) < U_M$, since with infinite support for F_u, some users always choose each platform. Thus there is always a stable equilibrium. To avoid repeated uninformative case-checking, we examine the case of two platform symmetric demand and supply only at sufficiently large market size so that there is some app supply to both platforms in divided equilibrium, i.e. $v_p(U_M/2) > 0$.

Turning now to the stability analysis, we note that χ_1 is smooth except at $U_1 = U_1^L$ and $U_1 = U_M - U_1^L$, where its derivative is not continuous. The three-case structure of (A23) means that there are three cases for the Jacobian of χ. We start with the interior case $U_M - U_1^L > U_1 > U_1^L$ where both $\widehat{r}_1$ and $\widehat{r}_2$ are relevant. Also, it is easy to calculate everything from U, including $\widehat{r}_p$, and therefore v_p, and therefore

$\Delta_V = \gamma(v_1 - v_2) + \gamma_1 - \gamma_2$. The Jacobian of χ at an arbitrary point in the interior range is

$$f_u(\Delta_V) * U_M * \gamma * \begin{Bmatrix} 1 & -1 \\ -1 & 1 \end{Bmatrix} * \begin{Bmatrix} -\widehat{r}_1 f_r(\widehat{r}_1) & 0 \\ 0 & -\widehat{r}_2 f_r(\widehat{r}_2) \end{Bmatrix} * \begin{Bmatrix} -C_1/\mu U_1^2 & 0 \\ 0 & -C_2/\mu U_2^2 \end{Bmatrix}.$$

This Jacobian matrix has two eigenvalues, 0 and

$$\text{(A24)} \qquad \frac{f_u(\Delta_V) * U_M * \gamma}{\mu} \left[\frac{\widehat{r}_1 f_r(\widehat{r}_1) C_1}{U_1^2} + \frac{\widehat{r}_2 f_r(\widehat{r}_2) C_2}{U_2^2} \right].$$

In the other cases, $U_1 > U_M - U_1^L$ or $U_1 < U_1^L$, one of the terms in square brackets is zero. Thus the stability index is discontinuous at U_1^L and $U_M - U_1^L$, taking on smaller values just outside the interior interval.

We now assume symmetry, i.e. $C_1 = C_2$ and $\gamma_1 = \gamma_2$. It is immediate that there is a symmetric equilibrium with $U_1 = U_2$, $\widehat{r}_1 = \widehat{r}_2 = U_2/(C/\mu)$, $v_1 = v_2$, and $\Delta_V = 0$. Symmetry also lets us simplify the expression for the stability index at the divided equilibrium. We have

$$\text{(A25)} \qquad \frac{2\gamma C f_u(0)}{\mu} \left[\frac{\widehat{r}_1 f_r(\widehat{r}_1)}{U_1} + \frac{\widehat{r}_2 f_r(\widehat{r}_2)}{U_2} \right] = \frac{4\gamma C f_u(0)}{\mu} \left[\frac{\widehat{r}_2 f_r(\widehat{r}_2)}{U_2} \right] = 4\gamma f_u(0) \widehat{r}_2 \widehat{r}_2 f_r(\widehat{r}_2)$$

where the last equality uses $1/U_2 = (\mu/C)\widehat{r}_2$.

We use this expression to get results about the role of user heterogeneity and of market size.

- If F_u is nearly degenerate,[41] then the divided equilibrium is unstable.
  - As we change F_u, the divided equilibrium has the same $\widehat{r}_2$. Thus it is unstable for all $f_u(0) > 1/(4\gamma \widehat{r}_2^2 f_r(\widehat{r}_2))$.

For any finite $f_u(0)$, there is a large enough U_M so that the equilibrium is stable. However, for any fixed U_M, there is a large enough $f_u(0)$ so that divided equilibrium is stable.

- Under symmetry with star-dominance, at the divided equilibrium the stability index is decreasing in market size.
  - If f_r is star-dominant, then $r * (r * f_r(r))$ is an increasing function of r (product of positive increasing functions). In defining (A25), we imposed $U_1 = U_2$ and $=U_M = 2 * U_2$. Thus, large U_2 and small $\widehat{r}^2$ are, in (A25), the same as large U_M.
- Under symmetry with star-dominance, there is a U_M^* such that for all $U_M > U_M^*$, the symmetric equilibrium is stable.

41. Under our assumptions that F_u is symmetric, smooth, and has $f_u(x) > f_u(y)$ whenever $|x| > |y|$, the distribution is close to degenerate when $f_u(0)$ is large.

– If f_r is star-dominant, then $r^2 * f_r(r) < \epsilon$ for any $\epsilon > 0$ for some $r > 0$. In particular, there is an $\widehat{r}_2 > 0$ such that $4\gamma f_u(0)\widehat{r}_2\widehat{r}_2 f_r(\widehat{r}_2) < 1$. Star-dominance means that the supply of applications to a platform, measured in terms of the incremental impact on v_p, is decreasing in $\widehat{r}$, i.e. increasing in U_p, and that the supply elasticity becomes arbitrarily small for large enough U_p.

– There will also be a non-empty range of $U_M < U_M^*$ where the divided equilibrium in unstable if, for some r, $r^2 * f(r) > 1/(4\gamma f_u(0))$. This condition does not contradict the assumption that F_r is star-dominant, but is not implied by it.

This reverses, partially, the folk theorem. Larger markets are more stable here, and large enough markets will have a stable, not unstable, divided equilibrium under symmetry.

The geometry tells us that there are two different possible outcomes when there is a stable divided equilibrium. The essential geometric facts here are that the largest and smallest equilibrium must be stable (this includes the case of unique equilibrium trivially) and that, in the case of equilibrium not being unique, the neighbor(s) of a stable (unstable) equilibrium must be unstable (stable). Figure 3 shows an example of the smallest possible equilibrium set under symmetry with a stable divided equilibrium that is not unique – five equilibria.

This is another partial reversal of the results of the traditional version. Instead of Stable/Unstable/Stable around the divided equilibrium, it has Unstable/Stable/Unstable. The platform market is still tippy, but only in the sense that a platform with a smaller installed base tends to tip out. If there are two platforms with substantial installed bases, neither tends to tip out. We first show how the stability condition demonstrates this intuition and then examine the conditions for existence of unstable equilibria adjacent to the stable divided one (assuming it exists).

If the stability index is less than 1 at the divided equilibrium, can it be greater than 1 elsewhere? This depends on two opposing forces, which can be seen clearly in (A26). This evaluates the index at an arbitrary point (i.e., U_1 may differ from U_2) under the assumption that the economic fundamentals are symmetric and at a fixed U_M.

(A26) $$\nu * f_u(\Delta_V) * [\widehat{r}_1^3 f_r(\widehat{r}_1) + \widehat{r}_2^3 f_r(\widehat{r}_2)]$$

As U_1 increases and U_2 decreases away from $U_1 = U_2$, $f_u(\Delta_V)$ falls. This force is the same as in the traditional variant, and tends to make the stability index smaller, i.e. more stable, at a more divided point. However, the other part of the stability index increases, i.e., makes it less stable. This follows because the function $r^3 f(r)$ is convex. Which of these forces dominates depends on the relative strength of demand forces, which grow less responsive as the candidate point is farther from the center of the taste distribution, and supply forces, which grow more responsive as the candidate point is farther from evenly divided between platforms.

VIII.C Empirical Model Less restrictive than theory – two quantitatively important and two unimportant differences

The theoretical model gives developers a perfect pre-entry forecast of r_a. The empirical model dampens the supply response by giving developers an imperfect signal. This is quantitatively significant. λ is typically about .64, meaning that while the signal is more often than not good, about a third of the time the signal is uninformative.

The theoretical model has a single $f_r()$. The empirical model conditions $f_r()$ on observables about the developer and the application. Conditional on the distribution of X, this is a distinction without a difference within-platform.

The theoretical model endows each application with the same r_a on all platforms. In the empirical model estimates, r_{ai} and r_{ad} are not identical. However, because the parameters that condition $f_r()$ on observables are very similar between platforms, even the independent component of r_a has very similar distribution on the two platforms.

Finally, while U_d is about 5/4 the size of U_i so the divided equilibrium is not symmetric, this difference is somewhat offset by a higher μ/C on the iOS platform. However, $v_i < v_d$ by about 1.3 at the divided equilibrium, so the slope of $v_i - v_d$ as $U_i - U_D$ changes is not zero there as it is in the symmetric divided equilibrium.

VIII.D Empirical Jacobian Details

VIII.D.1 $J_{\widehat{r}}$

In the empirical model, X takes on multiple values. This means that the supply threshold function $\widehat{r}(U)$ is from $\mathbb{R}^P$ to $\mathbb{R}^{P*dim(X)}$ where P is the number of platforms and $dim(X)$ is the number of values of X. Thus, we are going to define a value of $\frac{\partial \widehat{r}_{Xp}}{\partial U_p}$ for each X and p. $\frac{\partial \widehat{r}_{Xp'}}{\partial U_p} = 0$ for $p' \neq p$.

(A27) $$\widehat{r}_{pa} = \frac{\kappa_{Xp}}{\lambda_{Xp}\, U_p} - (1-\lambda_{Xp})/\lambda_{Xp} \times \mathbb{E}[\,\overline{r}_{pa}\,|\, X, \theta\,],$$

which implies

(A28) $$\frac{\partial \widehat{r}_{Xp}}{\partial U_p} = -\frac{\kappa_{Xp}}{\lambda_{Xp} U_p^2},$$

VIII.D.2 J_v

First, we need to calculate the portion of v_p that comes from a given X and take into account the gap between developer supply to a platform (based on their signal) and how much they contributed to user app demand (based on realized r). This means that the contribution from developers of type X to platform p is

$$v_{Xp}(\widehat{r}_{Xp}) = N_X \left[\lambda_{Xp} \int_{s=\widehat{r}_{Xp}}^{1} s f_{Xp}(s) ds \; + \; (1-\lambda_{Xp}) Pr(s > \widehat{r}_{Xp}) E[\overline{r}_{Xp}]\right], \tag{A29}$$

where N_X is the number of developers who might write an attractive app of type X, $f_{Xp}(s)$ is as defined above, and the two terms in the large brackets represent the contributions of applications that do and do not realize their signaled reach, respectively. We sum these over all the values of X to get $v_p(\widehat{r}_p)$.

Accordingly, the element of J_v for platform p and app type X is

$$N_X \left[-\lambda_{Xp} * \widehat{r}_{Xp} * f_{Xp}(\widehat{r}_{Xp}) \; + \; (1-\lambda_{Xp}) E[\overline{r}_{Xp}] f_{Xp}(\widehat{r}_{Xp})\right]. \tag{A30}$$

VIII.D.3 J_D

For empirical stability bounds, we assume a one-parameter logit model and put a bound on that parameter. We write:

$$\bar{v}_p + \gamma v_p,$$

where $\bar{v}_p$ captures all the factors that lead users to pick platforms other than app availability, such as the price and quality of the platform's devices. We will always be evaluating the elasticities at fixed shares, so the $\bar{v}_p$ disappear into the shares, leaving only γ. Using σ for shares, we have

$$J_D = \gamma U_M \begin{pmatrix} \sigma_1(1-\sigma_1) & \sigma_1\sigma_2 & \dots \\ \sigma_2\sigma_1 & \ddots & \\ \vdots & & \ddots \end{pmatrix}.$$

We report η_i^i, the elasticity of demand for iOS with respect to v_i, where $\eta_i^i = \gamma v_i/(1-\sigma_i)$.

The mechanics of the bounding calculation itself is simple. We calculate J_D under the assumption that $\gamma = 1$, then find the largest eigenvalue of $J_D J_v J_{\widehat{r}}$. 1/that eigenvalue is the bound on γ.

VIII.D.4 Population of Capable Developers

We estimate the number of app developers of each observable type X, N_X, as

$$N_X = n_X / Pr(S_i > 0 \;\vee\; S_D > 0 \,|\, X),$$

where the numerator n_X is the number of apps of type X observed in our sample and the denominator is the probability that an app is observed. The sum of n_X is 1,044 (our sample size) and the sum of N_X is just under 2,646. We think of this as an estimate of the population of developers who had an app potentially suitable for mass-market distribution and sufficient resources to market it, not of all developers.

Table 4: Sample Selection By X

X^a	n	N	Pr()
LHG	48	89.8	0.5343
OHG	177	824.1	.2148
FHG	22	41.6	0.5287
LTG	24	37.0	0.6493
OTG	3	7.6	0.3945
FTG	53	82.1	0.6454
LHY	151	254.0	0.5944
OHY	257	841.7	0.3053
FHY	76	128.8	0.5898
LTY	80	115.8	0.6910
OTY	1	2.2	0.4609
FTY	152	221.0	0.6878
All L[b]	303	496.6	0.610
All O	438	1675.6	0.261
All F	303	473.6	0.640

[a] X is the value of X in the notation of Figure 9. n is the number of sample points for that value of X. $Pr()$ is the probability that an application of type X will be observed $\Pr(S^*_{ia} + S^*_{da} > 0 \mid X, \theta)$ N is the number of unselected applications of type X predicted by $n/Pr()$. For example, the first row points to X=LHG, Game apps from Online-Era firms that are Privately Held; there are 48 of these in our sample, about 90 of these in the population of capable developers, and a 53% chance of an application of this type appearing in our sample.

[b] The rows All L, All O, and All F aggregate the rows above them. For example, the All L row shows that we have 303 apps from Online Era firms, and that an Online-Era firm has a 61% chance of being observed. This clarifies the definition of "potential entrant" in our model. There are many millions of developers who have written an app. Economically, a potential entrant is one with the resources 1) to make an app which appeals to a meaningful number of users and 2) to market the app and bring it to those users' attention.

The striking thing about Table 4 is that the probability of selection for a Mobile Era (O) firm's app is much lower at 26% than for an app from firms of the other two, "established" eras.

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