Optimal discoverability on platforms*

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April 2023

Abstract

A key issue for the design of platforms is how much discoverability to enable. Some platforms are primarily aimed at providing tools for suppliers to serve their existing buyers, and offer no or limited ability for buyers to discover new sellers or content (e.g. Shopify, Substack, Teachable). Others are buyer-focused, offering search tools for buyers to discover the most suitable suppliers or content (Amazon, Medium, Udemy). We study what drives a platform's choice between these two extremes.

1 Introduction

A key issue for the design of platforms is how much discoverability to enable. Some platforms are primarily aimed at providing tools for sellers to serve their existing buyers, and offer no or limited ability for buyers to discover new sellers or content providers they did not know about (e.g. Shopify, Substack, Teachable). Other platforms, in addition to seller tools, offer search tools and recommendations that make it easier for buyers to discover new sellers or content providers (Amazon, Medium, Udemy).

We study a platform's optimal choice of how much it wants to enable such discoverability. Enabling more discoverability generates a fundamental tradeoff for platforms. On the one hand, it creates more transactions by inducing buyers to purchase from new sellers, thereby increasing the platform's revenue for any given transaction fee it sets. This can also potentially increase the transaction fees sellers are willing to pay because it allows them to be found by new buyers. On the other hand, it also commoditizes sellers, by making it easier for a seller's a priori captive buyers to find and purchase from other sellers. This means some sellers may be reluctant to participate on platforms that enable too much discoverability,

*We would like to thank Chiara Farronato, Shane Greenstein, and Sam Ransbotham as well as other seminar participants at Harvard Business School for valuable comments and suggestions, as well as David Ronayne for helpful discussions.

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potentially decreasing the transaction fees sellers are willing to pay. Indeed, it is this fear of commoditization at the hands of large platforms that has created an opportunity for new platforms to emerge that promise to only enable limited discoverability in order to attract sellers.

To analyze this tradeoff, we build a model in which a platform attracts sellers, each of whom brings some initially captive buyers. By its design choices (e.g. how easy it is for buyers to search and compare across the listed sellers), the platform determines what fraction of these buyers see other sellers that are also participating on the platform. We find that the optimal extent of discoverability is higher when (i) sellers' products are less substitutable, (ii) the tools the platform offers to sellers are more valuable, (iii) the number of potential sellers that can be brought onto the platform is higher, (iv) sellers are less asymmetric in terms of the number of buyers they bring to the platform, and (v) the platform has a larger installed base of buyers.

To help fix ideas, it is useful to briefly describe a few examples:

- Shopify competes with Amazon.com for attracting third-party e-commerce sellers. On Amazon.com, sellers are exposed to intense competition with a large number of other sellers, which the buyers can easily compare against each other and sort by price. By contrast, Shopify has been very deliberate in not creating a similar marketplace that enables discovery for buyers, and emphasizing to sellers that they can maintain full control over their buyers and that they would not be "shopped around" to other sellers.

- Both Medium and Substack are platforms connecting independent writers with readers. Medium is explicitly focused on making it easier for readers to discover posts and writers. By contrast, Substack is primarily focused on providing writers the tools they need to create and manage newsletters: each author must build their own reader audience, without much (if any) help from Substack. Recently, Substack has created a centralized website where readers can bookmark posts and authors they like, and potentially discover new ones. However, most authors still obtain the majority of their audience through their own efforts (e.g. social media, etc.).

- Both Teachable and Udemy are platforms connecting instructors that offer courses on a wide range of topics with learners. Udemy is essentially a version of the Amazon marketplace for online courses, where learners can browse and discover instructors and courses, complete with a recommendation system based on the learners' interests and courses they have previously taken. By contrast, Teachable is much closer to the Shopify model: it started off by solely providing instructors the tools they needed to offer their courses online, without any marketplace enabling discovery of courses for learners. In 2019, Teachable launched Discover, a new dedicated sub-domain, created for students to browse, preview or enroll in courses from Teachable instructors. Teachable instructors opt in to appear on the discovery site. Importantly, Teachable goes out of its way to make it clear to instructors that Discover is not a full-fledged marketplace (like Udemy and others) which commoditizes instructors.[1]

- Doordash, Grubhub and Uber Eats offer online food delivery marketplaces where consumers can search and discover listed restaurants to order from. By contrast, Olo and Toast only offer software tools for restaurants, which allow them to accept and fulfill online orders from their own websites.

Motivated by these contrasting examples, after fully analyzing the choice of discoverability by a monopoly platform, we **also** explore what happens when there are competing platforms, showing that competition between symmetric platforms reduces the level of discoverability platforms choose. Moreover, we illustrate how discoverability can be an endogenous way for otherwise identical platforms to differentiate, with one platform offering maximum discoverability and attracting smaller sellers, and the rival platform choosing no discoverability to attract larger sellers.

1.1 Related literature

The paper fits within the burgenoning literature on platform design, with other authors exploring how platforms optimally design consumer search (Hagiu and Jullien, 2011; White, 2013; Dukes and Liu, 2016; Jiang and Zou, 2020; and Zhong, 2023), how many or which sellers are allowed to participate (Casner, 2020), their product recommendations (Barach et al, 2020; Zhou and Zou, 2023), and their reputation system (Shi et al, 2023). Teh (2022) explores how such optimal design choices vary with different platform business models and how this can lead to misalignment with welfare objectives (see also Choi and Jeon, 2023, who consider how platform design is biased in ad-funded platforms). Hagiu and Wright (2023) explore how different design choices can be used by platforms to limit leakage or disintermediation. In terms of this literature, our paper is closest to those works focused on search design, and in particular papers that show that the platform may want to add frictions to consumer search in order to relax seller competition, thereby allowing the platform to extract more revenue from sellers. Our focus is on how the extent of discoverability affects sellers' participation incentives given sellers can always sell to their buyers directly without the platform. This contrasts with other papers in this literature, which typically take seller

[1] See https://teachable.com/blog/discover-by-teachable

participation as exogenously given. Another distinction is that in our setting, each seller brings its own buyers, so discovery arises from one seller's buyers discovering another seller, rather than the platform's buyers discovering the sellers. As a result, and in constrast to the existing literature, the platform in our setting may optimally choose no discoverability at all. This is despite the fact we focus on the platform charging per-transaction fees, which in standard settings imply the platform will choose its design to maximize the volume of transactions, for example, by making search as easy as possible (see Teh, 2022).

Our paper is also related to the four key strategies that can be used to turn product firms into platforms (Hagiu and Altman, 2017). Previous work has only analyzed one of these strategies formally. Specifically, Hagiu et al. 2020, explore the possibility of a multiproduct firm becoming a platform by inviting rivals to sell products or services on top of its core product. The current paper can be seen as a first attempt at considering one of the other key strategies proposed in Hagiu and Altman, which is reaching out to customers' customers. Here the original customers of the "product" firm are sellers, which it sells software tools to, and their customers are the initially captive buyers they bring. By offering discovery for these buyers, the firm creates network effects and turns itself into a proper platform that helps buyers discover new sellers.

Finally, our paper is part of an emerging literature that focuses on the downsides of participating on platforms from the perspective of sellers. In our paper, the downside is a form of commoditization: sellers bring their buyers to the platform, which then allows those buyers to discover other, possibly competing, sellers. In a sense, by participating on platforms, sellers can lose control over their relationship with their own buyers, something which has been widely discussed in the popular press (see, Hagiu and Wright, 2021 for a discussion). Other related work exploring the downsides of participating on platforms include recent work on possible imitation and self-preferencing by hybrid marketplaces like those offered by Amazon and Apple (Anderson and Bedre-Defolie, 2022; Hagiu et al., 2022; and Madsen and Vellodi, 2022). In a similar vein, Mayya and Li (2022) show empirically how participating on food-delivery platforms may commoditize restaurants.

2 Baseline model with two sellers

We start with a simple baseline model, which we will later extend in various directions. Suppose there are two symmetric sellers, each of which sells a product that buyers value at v. Both sellers have marginal cost $c < v$. Each seller $i = 1, 2$ starts with a measure λ of captive buyers (buyers who only know about the seller). So all buyers are initially captive, half belonging to each seller.

The platform offers participating sellers B2B tools (which we will call "tools" for brevity) which is captured by a reduction in sellers' marginal costs by b. This could be software and other infrastructure that more efficiently handles the building of a website, payments, delivery, customer service, record-keeping and receipts, and so on. If this is the only thing the platform does the platform can be thought of as just a SaaS (software-as-a-service) company. The platform charges each seller a non-negative per transaction fee of f, so the effective marginal cost for a seller on the platform is $c + f - b$. If both sellers participate, the platform can choose to make a fraction x of buyers aware of both sellers (i.e. make them discover the seller they were not initially aware of). With probability θ a buyer who is aware of both sellers views their products as perfect substitutes, while with probability $1 - \theta$ a buyer views them as independent (and therefore buys from both if they are aware of both).

The timing is natural. In period 1 the platform chooses its level of discoverability x and transaction fee f.[2] In period 2, each seller decides whether to join the platform. Then in period 3, each seller sets its price, and demands and payoffs are realized.

For certain choices of f and x, it is possible that there are multiple equilibria in sellers' decisions in period 2, one in which all sellers join given they expect the other sellers to join, and one where no sellers join given they expect the other sellers not to join. In such cases, we select the equilibrium in which all sellers join, which is sometimes referred to as "favorable beliefs" on the part of sellers.[3]

Some comments about our modelling assumptions are in order. First, to interpret x, one could think of a more elaborate setting in which buyers are heterogeneous in their search cost. There is some cutoff level such that all buyers with search cost below the cutoff discover the rival seller (this is the fraction x) and all those with search cost above the cutoff (i.e. $1 - x$ of buyers) do not search, i.e. they just know their original seller.[4] In this context, one can interpret the platform's choice of x as representing its ability to shift everyone's search cost up or down by its design of the search process. Examples of a platform's design choices that affect x include how prominent they make buyer search, the ability to search based on price or to do side-by-side comparisons, and whether the platform recommends a particular seller to buyers based on price and other factors (e.g. Amazon's buybox).

Second, there are two features of the model that are necessary to obtain an interest-

[2]For a monopoly platform, it is irrelevant whether discoverability x is set before or after (or the same time as) its transaction fee f. The timing of these decisions will matter when we consider competing platforms.

[3]In Section A.1 of the Online Appendix we redo the baseline analysis in case sellers hold "unfavorable beliefs," so that they coordinate on the equilibrium in which none of them join whenever that equilibrium exists. As we show there, while the platform's profit is lower in the face of unfavorable beliefs, the baseline characterization of optimal discoverability remains unchanged.

[4]When there are more than two sellers, this same interpretation works if we consider buyers engaging in simultaneous search, so they search all sellers provided their search cost is below some cutoff.

ing tradeoff when choosing discoverability. Specifically, discoverability must result in more transactions in total, but it must also make some transactions contested. Our stark formulation of buyer demand has these features: with some probability, buyers are interested in both products, so view them as independent, and with the complementary probability they are just interested in one product, so they view competing products as identical.A more realistic but less tractable setting might have buyers always interested in both products so that when they are exposed to both, they buy more in total than if they are just exposed to one, and how much more depends on the degree of substitutability they perceive between the products. We will show the robustness of our main results to this alternative formulation.

Third, a key assumption implicit in our timing is that the platform commits to its choice of x. This captures that it is harder for the platform to change its design choices (e.g. due to technologicial commitments in designing its search, as well as possibly brand or reputation concerns) than it is for sellers to list (or delist). Without commitment to x, since the sellers' listing decisions would be treated as if they are fixed, the platform will always choose maximum discoverability ($x = 1$) given that doing so expands demand (and so transactions) as much as possible.[5]

Finally, sellers are assumed to each set a single price, so we rule out price discrimination across their different buyers (e.g. assuming some discoverability, seller 1 would sell to some buyers who are initially captive to seller 1 and some buyers who are initially captive to seller 2). This reflects that sellers may find it difficult to distinguish between buyers. Indeed, buyers may be able to disguise their identity to obtain the more attractive offer in case sellers try to set differential prices based on whether buyers come via a given seller's own channel, or from the rival seller.[6]

3 Analysis and results

If neither seller joins the platform, then each makes profits

$$\lambda (v - c) . \tag{1}$$

If only one seller joins the platform, that seller's marginal cost is $c - (b - f)$, instead of c for the non-joining seller. Each seller just faces its captive buyers and prices at v. Thus, the

[5] We have redone our analysis of the baseline setting without commitment in Section A.2 of the Online Appendix.

[6] In Online Appendix A.3, we extend the baseline analysis to the case sellers can effectively price discriminate between their "own" captive buyers and those coming from the rival seller after discovery via the platform. This does not change the optimal level of discoverability in a systematic way.

joining seller's profit is

$$\lambda\left(v+b-f-c\right)$$

while the profit of the non-joining seller is still $\lambda\left(v-c\right)$.

Finally, consider the case both sellers join the platform. Given the sellers are symmetric and face equal fees, to determine each seller's expected profit, we just need to determine the measure of captive buyers each seller has after the platform's choice of x. This reflects that given sellers have some fraction of captives and some fraction of buyers who compare the two identical sellers, prices are determined by a mixed strategy pricing equilibrium. In such an equilibrium, each seller's expected profit will equal the profit it can guarantee if it just sells to its captives.[7]

The captive buyers for seller i are now made up of seller i's initial captives that did not discover seller j (measure $\lambda\left(1-x\right)$), seller i's captives that discovered seller j but view the two sellers' products as independent (measure $\lambda x\left(1-\theta\right)$) and seller j's initial captives that discovered seller i but view the two sellers' products as independent (measure $\lambda x\left(1-\theta\right)$). Thus, each seller's expected profit is

$$\left(v+b-f-c\right)\left(\lambda\left(1-x\right)+2\lambda x\left(1-\theta\right)\right). \tag{2}$$

This is increasing in the extent of discovery x if and only if $\theta<\frac{1}{2}$, i.e. if and only if the two sellers' products are not too substitutable. This makes sense: sellers want to join a platform that induces discovery only if the other participating sellers are not too close substitutes.

The condition for both sellers joining the platform to be an equilibrium is that (2) is no less than (1), or equivalently

$$f\leq b+\left(v-c\right)\frac{x\left(1-2\theta\right)}{1+x\left(1-2\theta\right)}. \tag{3}$$

When $x=0$, this constraint reduces to $f\leq b$. Without discoverability, there are no interactions between the sellers and no network effects, so the platform just provides tools: each seller adopts it if and only if it offers more value than it charges, i.e. $b\geq f$.

When $x>0$, if the two sellers' products are not too substitutable ($\theta<\frac{1}{2}$), then the platform can charge $f>b$ and still get both sellers to join given we have assumed sellers coordinate on the equilibrium in which they both join (i.e. they hold "favorable beliefs"). In this case, the maximum fee the platform can charge to get both sellers to join is increasing

[7] This is just a special case of the more general result from Lemma 2 in Myatt and Ronayne (2019) which characterizes the expected profits of the sellers in the mixed strategy equilibrium when there are any number of sellers and allowing for the fact that these sellers can be asymmetric (either in their costs or in their measure of captives). We provide this more general characterization in Online Appendix A.4.

in the amount of discoverability x. On the other hand, when $x > 0$ and the products are sufficiently substitutable ($\theta \geq 1/2$), the platform must charge $f < b$ if it wants both sellers to join. Furthermore, more discoverability now decreases the maximum fee the platform can charge to get both sellers to join.

The platform's demand when it attracts both sellers consists of the $2\lambda(1-x)$ buyers who are informed of only one product (and who buy only that product only), the $2\lambda x(1-\theta)$ buyers who are informed of both products and view them as independent (they buy both), and the $2\lambda x\theta$ buyers who are informed of both products and view them as substitutes (they buy one product only). Thus, the platform's profit when both sellers join is

$$\Pi(f,x) = f\left(2\lambda(1-x) + 4\lambda x(1-\theta) + 2\lambda x\theta\right) = 2\lambda f\left(1 + x(1-\theta)\right). \tag{4}$$

Clearly, the platform's profit is always increasing in the extent of discovery, holding f and the participation of both sellers fixed. This is natural: discovery expands the number of transactions on the platform.

Substituting in the maximum fee the platform can charge while ensuring the sellers still participate from (3) and defining

$$\mu = \frac{b}{v-c}$$

as the ratio of the value provided by tools to the value provided by the underlying product being sold, we obtain the platform's maximum profit as a function of x only[8]:

$$\Pi(x) = 2\lambda\left(\mu + \frac{x(1-2\theta)}{1+x(1-2\theta)}\right)\left(1+x(1-\theta)\right)(v-c). \tag{5}$$

When products are not too substitutable ($\theta < \frac{1}{2}$), since both the platform and the participating sellers benefit from discovery, Π is increasing in x and the platform will naturally set $x^* = 1$, the maximum amount of discovery. However, when products are more substitutable ($\theta > \frac{1}{2}$), the platform faces a trade-off when choosing the amount of discovery x: on the one hand, a higher x increases the number of transactions, but on the other hand it lowers the participating sellers' profits, so it also lowers the maximum transaction fee f that the platform can extract from the sellers. This can lead to the optimal level of discovery to be set less than one. Relegating the remaining analysis to the appendix, we obtain the following proposition.

Proposition 1. *Suppose each seller starts with a measure λ of captive buyers. The platform*

[8] It is straightforward to confirm that the platform always prefers to have both sellers join. Indeed, the platform's profit with one seller joining is half of what it could get with both sellers joining and setting $x = 0$, which is always an option it could choose when it induces both sellers to join.

always finds it optimal to induce both sellers to join and its optimal level of discovery is given by

$$x^* = \begin{cases} 1 & if \quad 0 < \theta \leq \theta_1(\mu) \\ \frac{1-\sqrt{\frac{\theta}{(\mu+1)(1-\theta)}}}{2\theta-1} & if \quad \theta_1(\mu) \leq \theta \leq \frac{\mu+1}{\mu+2} \\ 0 & if \quad \theta \geq \frac{\mu+1}{\mu+2} \end{cases}, \tag{6}$$

where $\theta_1(\mu) \in \left(\frac{1}{2}, \frac{\mu+1}{\mu+2}\right)$ is the unique solution in θ to

$$\frac{\theta}{(1-\theta)^3} = 4(\mu+1).$$

The optimal level of discoverability x^* is decreasing in θ and increasing in μ.

The proposition fully characterizes the platform's optimal choice of discoverability, which is just a function of the underlying parameters θ and μ. A greater level of substitutability between products (i.e. higher θ) induces the platform to choose a lower level of discoverability x^*, because discoverability leads to more intense competition between the sellers, and so makes it harder to attract the two sellers to join the platform.

Meanwhile, an increase in the value offered by the platform's tools for sellers (i.e. higher μ) means the platform can charge a higher fee per transaction while keeping sellers willing to participate. This in turn make it more profitable to increase the number of transactions enabled, which the platform does by increasing discoverability. This is why x^* is increasing in μ. In short, the platform's investment in tools and provision of discoverability are strategic complements. Note, however, that even if $\mu = 0$, the platform will set $x = 1$ and derive positive profits if and only if $\theta < \frac{1}{2}$. In other words, provided the sellers' products are not too substitutable, the platform can create positive value for sellers via discoverability and extract positive profits.[9]

In Figure 1, we have mapped out the optimal x^* when θ is on the horizontal axis and μ is on the vertical axis. The figure show levels of x^* from $x^* = 0$ (lightest colour) to $x^* = 1$ (darkest colour). The upward sloping relationship seen in the figure reflects that with higher θ, one would require a higher μ to leave the level of x^* unchanged.

One feature of the unit demand setting we used is that the lower prices resulting from seller competition do not lead to an increase in overall demand. In Online Appendix A.6 we use a less tractable elastic demand setting in which this effect is accounted for, and show that the main comparative static results are very similar.

[9] More generally, in Online Appendix A.5 we show that the platform can derive positive profits when its tools are worth less than seller tools that are available competitively in the outside market.

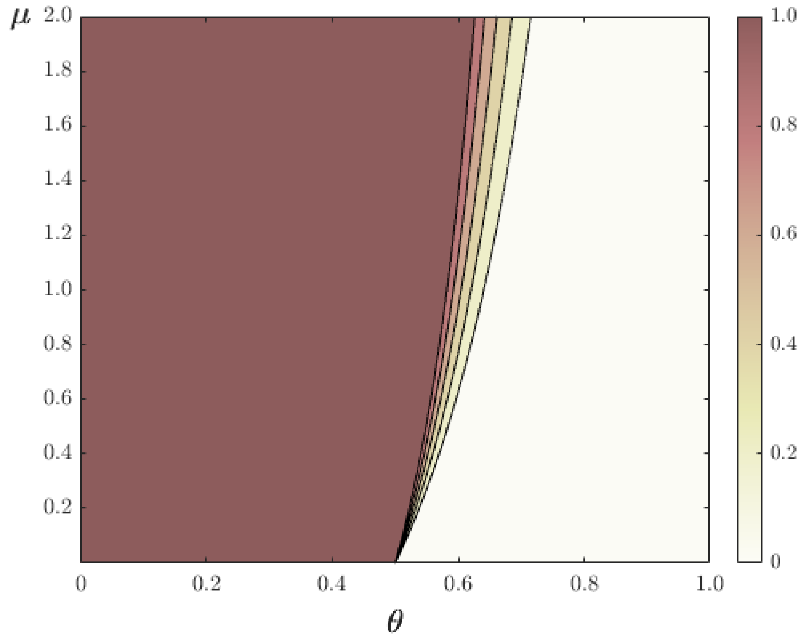

Figure 1: The platform's optimal level of discoverability x^*

4 Extensions

In this section we explore several interesting extensions of the baseline model.

4.1 Platform brings in its own buyers

Often platforms may have some of their own buyers: these could be buyers obtained directly via the platform's own marketing efforts or through selling to buyers itself (i.e. being a reseller) as Amazon did before it opened up to third-party sellers. It is therefore interesting to explore how optimal discoverability changes when the platform has its own buyers.

Suppose the platform starts with its own measure $\eta\lambda$ of buyers, where $\eta > 0$ is the ratio of the platform's buyers to each seller's captive buyers. We assume the platform's buyers are initally uninformed of the two sellers. By choosing x, the platform also determines the fraction x of the platform's buyers that discover the two sellers.

The effect of these platform buyers is to increase each seller's captives by $\eta\lambda x (1-\theta)$, since a fraction $1-\theta$ of the platform's buyers that become informed of both sellers will buy from both sellers. Thus, modifying (3), the condition for each seller to join given that the other does becomes

$$f \leq \left(b + (v - c)\,\frac{x\,(1 - 2\theta + \eta\,(1 - \theta))}{1 + x\,(1 - 2\theta + \eta\,(1 - \theta))}\right). \tag{7}$$

The platform's demand when it attracts both sellers is the same as before, plus an additional $\eta\lambda\,(\theta + 2\,(1 - \theta))\,x$ buyers that come directly from the platform. Thus, the platform's profit when both sellers join is now

$$\Pi\,(f, x) = (2\lambda\,(1 + x\,(1 - \theta)) + \lambda\eta x\,(2 - \theta))\,f. \tag{8}$$

Combining (7) and (8), and using the definition of μ, implies the platform's problem is to choose x to maximize

$$\Pi\,(x) = \lambda\,(2\,(1 + x\,(1 - \theta)) + x\eta\,(2 - \theta))\left(\mu + \frac{x\,((1 - 2\theta) + \eta\,(1 - \theta))}{1 + x\,((1 - 2\theta) + \eta\,(1 - \theta))}\right)(v - c)\,. \tag{9}$$

Relegating the maximization problem to the appendix, we obtain the following results.

Proposition 2. *Suppose each seller starts with a measure λ of captive buyers and the platform starts with $\eta\lambda$ buyers of its own. The platform always finds it optimal to induce both sellers to join and its optimal level of discovery is given by*

$$x^* = \begin{cases} 1 & if \quad 0 < \theta \leq \theta_1\,(\eta, \mu) \\ \frac{1 - \sqrt{\frac{(2+\eta)\theta}{(2(1-\theta)+\eta(2-\theta))(\mu+1)}}}{2\theta - 1 - \eta(1-\theta)} & if \quad \theta_1\,(\eta, \mu) \leq \theta \leq \theta_2\,(\eta, \mu) \\ 0 & if \quad \theta \geq \theta_2\,(\eta, \mu) \end{cases}$$

where

$$\theta_2\,(\eta, \mu) = \frac{2\,(1 + \eta)\,(\mu + 1)}{(2 + \eta)\,(\mu + 2)} > \frac{1 + \eta}{2 + \eta}$$

and $\theta_1\,(\eta, \mu) \in \left(\frac{1+\eta}{2+\eta}, \theta_2\,(\eta, \mu)\right)$ is the unique solution in θ to

$$\frac{\theta}{(1 - \theta)^2\,(2\,(1 - \theta) + \eta\,(2 - \theta))} = (2 + \eta)\,(\mu + 1)\,.$$

The optimal level of discoverability x^* is decreasing in θ, increasing in μ and increasing in η.

Proposition 2 shows that the bigger the installed base of buyers the platform starts with (relative to the measure of the sellers' initial captives), the higher level of discoverability it will choose. This can be seen from the fact that when $\eta = 0$, we are back to the solution from the baseline (Proposition 1), and that $\theta_1(\eta,\mu)$, $\theta_2(\eta,\mu)$ and x^* are all increasing in η. Put differently, the darkest area in Figure 1 (with highest x^*) expands to the right as η increases.[10] The rationale for this result is that informed platform buyers are a net benefit for the sellers, and they create more transactions for the platform. This increases the willingness of sellers to pay to participate (i.e. the fee that the platform can charge), thereby increasing the value of further expanding demand, which the platform does by increasing discoverability. Thus, we expect platforms that already have a lot of buyers coming to them directly, will be more likely to offer maximum discoverability. This also suggests that, over time, as sellers' initially captive buyers keep coming back to the platform to discover potentially new sellers, the platform will want to increase discoverability.

4.2 Two-part tariffs

We are interested in studying whether the platform would be better off charging sellers a fixed fee as opposed to, or in addition to, variable transaction fees. To model this, suppose that in addition to the transaction fee f, the platform can also charge each seller a fixed fee F. Everything else is as in the baseline model.

The sellers' payoffs are as in the baseline model except if they join the platform they also pay the fixed fee F. Thus, for both sellers to join the platform to be an equilibrium, we therefore must have

$$F+\lambda(f-b)(1+x(1-2\theta))\leq\lambda(v-c)x(1-2\theta) \tag{10}$$

and the platform's profit with both sellers joining is

$$2F+2f\lambda(1+(1-\theta)x) \tag{11}$$

from (4). The platform maximizes (11) over (F,f,x) subject to the constraint (10) above. It is easily seen that the constraint must be binding. Using that to write F as a function of f and x, we obtain that the platform maximizes

$$2\lambda((v-c)x(1-2\theta)+b(1+x(1-2\theta))+f\theta x)$$

with respect to f and x. Clearly, the last expression is increasing in f, so the platform will set

$$f^*=v-c+b,$$

[10] Apart from this feature, the figure remains qualitatively the same. This assumes η is not too large. When η is high enough, $\theta_2(\eta,\mu)>1$, and the platform always chooses a positive level of discoverability.

which then leads to

$$x^* = 1.$$

This implies

$$\begin{aligned} F^* &= \lambda\left(\left(v-c\right)x^*\left(1-2\theta\right)-\left(f^*-b\right)\left(1+x^*\left(1-2\theta\right)\right)\right) \\ &= -\lambda\left(v-c\right). \end{aligned}$$

With unrestricted two-part tariffs, the result shows that the platform chooses maximum discoverability and charges the maximum transaction fee. It extracts the entire margin of each seller's product, and subsidizes the participation of sellers by paying each seller the value of their outside option, which is equal to $\lambda\left(v-c\right)$.

At first glance, one may think that with inelastic demand both transaction fees and fixed fees work like transfers so it does not matter which is used by the platform. However, this is not the case, because an increase in f is just passed through by the sellers to the extent they compete, so doesn't impact their profit as much as an equivalent increase in a fixed fee that generates the same revenue for the platform. Specifically, in our model, each seller's net profit only reflects transactions with buyers for whom it doesn't compete (captive buyers and buyers who view the two sellers' products as independent), whereas the platform derives the transaction fee f from all transactions. Thus, the two sellers do not internalize all transactions they generate on the platform when making their participation decisions, which is why, provided there is some discovery, it always makes sense for the platform to load up on the per transaction fee and offset it with a fixed subsidy to the maximum extent possible.

A problem with the solution above is that it involves the platform paying each seller their outside option as a fixed subsidy upfront. The sellers then derive zero net revenues from their participation. In practice, this is unrealistic since it would lead to moral hazard problems (e.g. sellers participate just to collect the subsidy but then not doing anything to serve buyers) and the platform may also face a budget constraint. So it is reasonable to assume that the subsidy the platform can offer to sellers is limited by some exogenously given amount $K \geq 0$, which means we have the additional constraint

$$F \geq -K. \tag{12}$$

The platform maximizes (11) over (F, f, x) subject to the constraint (10) above and the additional constraint (12). Relegating the rest of the analysis to the appendix, we obtain the following result.

Proposition 3. *Suppose each seller starts with a measure λ of captive buyers, and the platform can charge a two-part tariff: a transaction fee f and a fixed fee F, subject to $F \geq -K$. The platform always finds it optimal to induce both sellers to join. If $K \geq \lambda(v-c)$, then the platform fully subsidizes the participation of the sellers by setting $F^* = -\lambda(v-c)$, extracts the entire margin by setting $f^* = v - c + b$, and maximizes discoverability by setting $x^* = 1$. If instead $0 \leq K < \lambda(v-c)$, then $F^* = -K$, $f^* < v - c + b$ and the optimal level of discovery is given by*

$$x^* = \begin{cases} 1 & if \quad 0 < \theta \leq \theta_1(\mu, \lambda, K) \\ \frac{1-\sqrt{\frac{\theta}{(\mu+1)(1-\theta)}\left(1-\frac{K}{\lambda(v-c)}\right)}}{2\theta-1} & if \quad \theta_1(\mu, \lambda, K) \leq \theta \leq \theta_2(\mu, \lambda, K) \\ 0 & if \quad \theta \geq \theta_2(\mu, \lambda, K) \end{cases}$$

where

$$\theta_2(\mu, \lambda, K) = \frac{\mu+1}{\mu+2-\frac{K}{\lambda(v-c)}} \in \left[\frac{1}{2}, 1\right]$$

and $\theta_1(\mu, \lambda, K) \in \left(\frac{1}{2}, \theta_2(\mu, \lambda, K)\right)$ is the unique solution to

$$\frac{\theta}{(1-\theta)^3} = \frac{4(\mu+1)}{1-\frac{K}{\lambda(v-c)}}$$

The optimal level of discoverability x^* is decreasing in θ and in λ and increasing in μ and in K.

It is easily verified that setting $K = 0$ leads to the results in Proposition 1. The platform wants to offer a subsidy, but if it cannot, it optimally chooses no fixed fee. This means our baseline results still apply even if the platform can use two-part tariffs provided moral hazard (or some other constraint) prevents the platform offering sellers fixed subsidies.

The reason the platform always chooses a subsidy here, if it can, reflects that the platform wants to push the final price charged by the two sellers up to the monopoly price v, thereby maximizing joint profit. The platform then extracts this profit subject to leaving each seller only with its outside option. Since in this Bertrand setting, the only way to achieve the monopoly price is to charge a transaction fee equal to the monopoly price, this leaves sellers with zero profit, which given their positive outside option, implies sellers must receive a subsidy to keep them willing to participate.[11]

[11] In more general models with imperfect competition, the platform may be able to induce the monopoly price while still leaving sellers with positive profits. Then whether the fixed fee is positive or negative depends on how these profits compare to the outside option, which is positive here. This is in contrast to traditional vertical relationship models, where it is never necessary to subsidize via the fixed fee given the option option is typically assumed to give zero profits.

The larger K, i.e. the more the platform can subsidize seller participation via a negative fixed fee, the higher the optimal level of discoverability (until $K \geq \lambda (v - c)$, at which point full discoverability is optimal). This makes sense: fixed subsidies are a way to compensate sellers for the individual downside of discoverability, and maximum discoverability is better from a joint profit perspective. Another way to understand the result is in terms of the usual tradeoff in setting discoverability: a higher fixed subsidy allows the platform to charge a higher transaction fee, which increases its profits from expanding the number of transactions, and shifts the tradeoff towards a higher level of discoverability.

4.3 Differential fees

The logic of two-part tariffs, in which the platform increases the transaction fee so as to increase seller prices, but then offers a fixed subsidy to sellers to make them willing to join, suggests that the platform can also do better if it can raise the fee it charges for transactions on which sellers compete and lower the fee it charges for transactions on which sellers do not compete. Indeed, in our model, charging $f_1 = b + v - c$ for transactions generated by buyers who are aware of both sellers but choose only one, and $f_2 = b + (v - c) \frac{1-2\theta}{2-2\theta} < f_1$ for all other transactions (such that the sellers are just willing to participate), would replicate the same outcome as with the optimal two-part tariff.

The problem with such a mechanism is that it requires the platform to distinguish between buyers for whom the sellers must compete more intensely from buyers for whom the sellers compete less (either because such buyers are only aware of one seller or because they view the products as independent rather than substitutes). But even if the platform can implement such a mechanism, each seller has no way to verify which of its own initial captive buyers become aware of the rival seller, and so is subject to manipulation by the platform which could overstate the fraction of transactions for which it earns a higher fee.

Taking into account these practical limitations, we focus on a more realistic second-best mechanism which is only based on what the seller can also verify. We allow the platform to charge each seller different transaction fees for selling to the buyers they brought to the platform (their initially captive buyers) vs. for selling to buyers that discovered the seller through the platform (the other seller's initially captive buyers). Since each seller knows which buyers it brings onto the platform, it can monitor the fees it pays are correct. This is indeed a practice that has been used. For example, Teachable, an online platform for instructors to sell courses to students charges instructors a lower fee for students that come via their own Teachable-powered sites[12] and a higher fee for students that come via

[12] At the time accessed, these fees varied between 5-10% of the revenue each instructor generated. See https://teachable.com/pricing.

Teachable's discovery page[13].

Intuitively, charging each seller a lower fee for transactions with buyers they brought to the platform vs. buyers that discovered them through the platform should make sellers more willing to participate and therefore allow the platform to increase the level of discoverability in order to increase the number of transactions enabled. As we will see, this intuition does not always hold.

To proceed, we allow the platform to charge each seller a transaction fee f_0 for transactions with the seller's initial captive buyers and a potentially different transaction fee f_1 for transactions with buyers that are not part of the seller's initial captive base.

Suppose both sellers join the platform (the payoffs from not joining are the same as in the baseline). The set of captive buyers for a seller is made up of three components:

- $\lambda(1-x)$ buyers on whom the seller incurs a marginal cost of $c+f_0-b$ and who only consider that seller
- $\lambda x(1-\theta)$ buyers on whom the seller incurs a marginal cost of $c+f_0-b$ and who consider both sellers
- $\lambda x(1-\theta)$ buyers on whom the seller incurs a marginal cost of $c+f_1-b$ and who consider both sellers.

Each seller's profit is then

$$(v+b-f_0-c)(\lambda(1-x)+\lambda x(1-\theta))+(v+b-f_1-c)\lambda x(1-\theta).$$

Indeed, the two sellers are symmetric, so each seller's expected profit from setting any other price in the support of its mixed strategy would have to be the same as the seller can obtain simply by serving its captive buyers (which here incur different marginal costs).

The sellers' profits are increasing in x for given fees if and only if

$$\theta<\frac{v+b-c-f_1}{(v+b-c-f_0)+(v+b-c-f_1)}.$$

In the baseline, sellers' profit are increasing in x if and only if $\theta<\frac{1}{2}$. Thus, the platform's ability to charge different fees makes it less likely for discoverability to be good for sellers' profits whenever $f_0<f_1$. The reason is that when $f_0<f_1$, each seller makes a higher margin on its own initially captive buyers (for whom it prefers less discoverability) vs. on buyers

[13] Teachable takes 30% of the instructors' revenue when they sell courses via its discovery page which is available at https://www.spotlightapp.io/.

that discovered it through the platform (for whom it prefers more discoverability), so overall each seller prefers less discoverability.

To determine the platform's revenue, consider the λ buyers that are initially captive to seller i. Out of these buyers, $\lambda(1-x)$ remain captive to seller i and buy from that seller only, so the platform makes $f_0\lambda(1-x)$ on them. Another fraction $\lambda x(1-\theta)$ are informed of both products and view them as independent, so they buy both and the platform makes $(f_0+f_1)\lambda x(1-\theta)$ on them. And the remaining fraction $\lambda x\theta$ are informed of both products and view them as substitutes, so they buy one product only. Given that the sellers are symmetric and therefore have the same price distributions in equilibrium, half of these buyers will buy from seller i and half will buy from seller j, so the platform makes $\frac{(f_0+f_1)\lambda x\theta}{2}$ on these buyers. Thus, in total, the platform's profit when both sellers join is

$$\begin{aligned}&\lambda\left(2f_0(1-x)+2(f_0+f_1)x(1-\theta)+(f_0+f_1)x\theta\right)\\=\;&\lambda\left(f_0(2-x\theta)+f_1x(2-\theta)\right).\end{aligned}$$

The platform's problem is to set x, f_0 and f_1 to maximize the above expression, subject to the following three constraints:

$$\begin{aligned}0&\leq f_0\leq v+b-c\\0&\leq f_1\leq v+b-c\end{aligned}$$

$$(v+b-f_0-c)(1-x+x(1-\theta))+(v+b-f_1-c)x(1-\theta)\geq v-c.$$

The first two constraints rule out negative transaction fees[14] and ensure that buyers want to participate at the competitive price. The third constraint ensures that each seller wants to participate on the platform.

Relegating the calculations to the appendix, we obtain the following proposition.

Proposition 4. *Suppose the platform can charge each seller a fee f_0 for transactions with its initially captive buyers and f_1 for transactions with buyers it gains through discovery on the platform. Then the platform always finds it optimal to induce both sellers to join and to*

[14] Indeed, negative transaction fees are seldom used in practice because they create arbitrage-type problems (e.g. some sellers might join just to buy from themselves and thereby collect the subsidy).

set $f_1 > f_0$. The optimal level of discovery is given by

$$x^* = \begin{cases} 1 & \textit{if} \quad 0 < \theta \leq \frac{1}{2} \\ \frac{\mu}{(\mu+1)\theta} & \textit{if} \quad \frac{1}{2} \leq \theta \leq \frac{2}{3+\mu} \\ \frac{1-\sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta} & \textit{if} \quad \frac{2}{3+\mu} \leq \theta \leq \frac{2(\mu+1)}{2(\mu+1)+1} \\ 0 & \textit{if} \quad \theta \geq \frac{2(\mu+1)}{2(\mu+1)+1} \end{cases} \tag{13}$$

when $\mu \leq 1$, and by

$$x^* = \begin{cases} 1 & \textit{if} \quad 0 < \theta \leq \theta_0(\mu) \\ \frac{1-\sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta} & \textit{if} \quad \theta_0(\mu) \leq \theta \leq \frac{2(\mu+1)}{2(\mu+1)+1} \\ 0 & \textit{if} \quad \theta \geq \frac{2(\mu+1)}{2(\mu+1)+1} \end{cases} \tag{14}$$

when $\mu \geq 1$, where $\theta_0(\mu)$ is the unique solution to

$$\frac{\theta}{(1-\theta)^3} = 2(\mu+1).$$

Again, the optimal level of discoverability is decreasing in the degree of substitutability θ between the sellers' products. In terms of fees, the key result is that the platform always finds it optimal to charge each seller a higher fee for transactions with buyers that are not part of the seller's initial captive base (f_1) than for transactions with the seller's initial captive buyers (f_0). The reason for this is that provided $0 < x < 1$, a larger share of a seller's transactions that come from the rival seller's buyers involve head-to-head competition (and therefore which do not contribute to the seller's expected profit), relative to the seller's transactions that come from its own initially captive buyers. Indeed, the share of "discovery transactions" (i.e. transactions generated by the rival seller's buyers) that result in head-to-head competition is $\frac{x\theta}{x-\frac{x\theta}{2}}$, whereas the share of transactions with a seller's own buyers that result in head-to head competition is $\frac{x\theta}{1-\frac{x\theta}{2}} < \frac{x\theta}{x-\frac{x\theta}{2}}$.

It is important to emphasize that this differential fee strategy only works if $0 < x < 1$.[15] For this reason, the range over which partial discoverability is optimal (i.e. $0 < x^* < 1$) is now larger than in the baseline model where the platform could only charge a single fee. The platform prefers partial discoverability because it allows it to exploit this profitable differential fee strategy. This is illustrated in Figure 2, which is constructed with $\mu = 1$: the

[15]If $x = 0$ or $x = 1$, then the platform does not gain anything from charging differential fees. Indeed, if $x = 0$, then there is no discovery so f_1 is irrelevant, whereas if $x = 1$, then all buyers are equivalent, so only $f_0 + f_1$ matters.

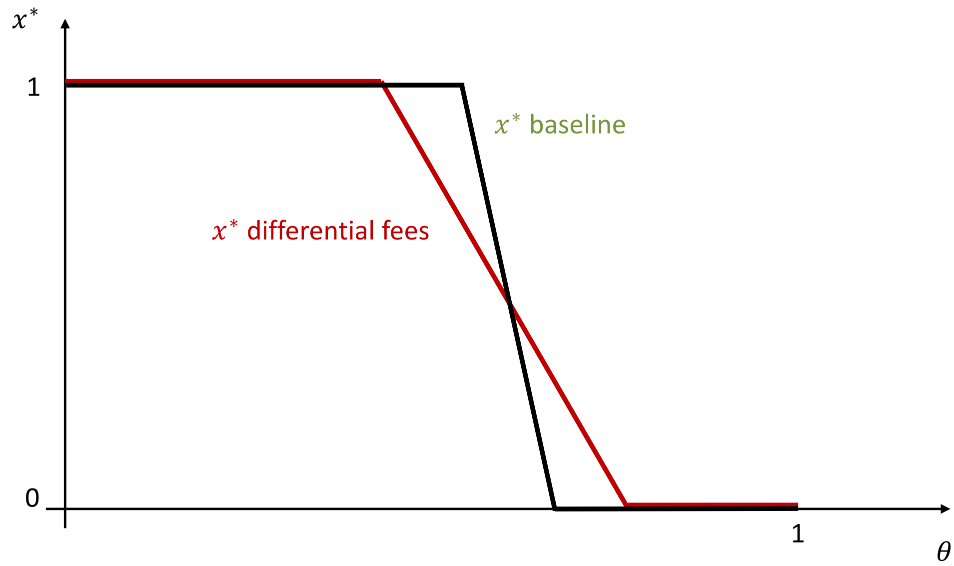

Figure 2: The platform's optimal level of discoverability x^*

black line represents $x^*(\theta)$ in the baseline and the red line represents $x^*(\theta)$ with differential fees.

This also leads to the following corollary, which compares the optimal level of discoverability here to the one from the baseline.

Corollary 1. *Denote by x_b^* the optimal level of discoverability from the baseline, given by (6), and by x_{df}^* the optimal level of discoverability with differential fees, given by (13) when $\mu < 1$ and by (14) when $\mu > 1$. For every $\mu > 0$, there exists a unique $\theta_3 \in \left[\theta_1(\mu), \frac{\mu+1}{\mu+2}\right]$ such that $x_{df}^* \leq x_b^*$ if $\theta \leq \theta_3$ and $x_{df}^* \geq x_b^*$ if $\theta \geq \theta_3$.*

Corollary 1 implies that being able to set different fees leads to less discoverability when θ is less than some threshold (denoted θ_3 in the Corollary) and leads to more discoverability when θ is more than that threshold. Moreover, the threshold always arises in the range where there is partial discoverability in the baseline, as can be seen from Figure 2.

4.4 More than two sellers

So far we have focused on the case with only two sellers. Suppose now there are $n \geq 2$ sellers: each seller brings a measure λ of buyers who are informed of the particular seller but not any of the other sellers.

Compared to before, the only difference that arises to a seller's payoff is the payoff it gets from joining when it expects more than one other seller to also join. Specifically, if $m-1 \geq 1$

other sellers join, then each seller's expected profit from joining is

$$((1-x)\lambda + m\lambda x(1-\theta))(v-c+b-f). \tag{15}$$

As before, $1-x$ of a seller's λ initial captives do not discover other sellers, so remain captive. The remaining fraction λx discover all other $m-1$ sellers, with a fraction $1-\theta$ of these viewing all sellers's products as independent, so $\lambda x(1-\theta)$ also remain captive from each seller's perspective. Likewise, each seller sells to the $\lambda x(1-\theta)$ buyers it gets exposed to from each of the other $m-1$ sellers' initial captives. So each seller ends up with

$$(1-x)\lambda + \lambda x(1-\theta) + (m-1)\lambda x(1-\theta)$$

captives, compared to λ original captives, thus leading to the result in (15).

Comparing (15) with the payoff $\lambda(v-c)$ when not joining, a seller will want to join when it expects $m-1$ other sellers to do so iff

$$f \leq b + \frac{x(m(1-\theta)-1)}{x(m(1-\theta)-1)+1}(v-c). \tag{16}$$

As is clear from (15), there are positive network effects across sellers. The more sellers join, the higher the payoff from joining for each seller (and therefore the higher f the platform can charge). Note that we continue to adopt favorable beliefs, in that sellers always coordinate on the highest number of available sellers joining that is an equilibrium given the fee f charged.[16]

Suppose the platform attracts m sellers in total. Each of these sellers has $\lambda(1-x)$ captive buyers who are only informed of one product and buy that product only, so the platform demand generated by these buyers is $m\lambda(1-x)$. Meanwhile, there are a total of $m\lambda x$ buyers who discover all sellers on the platform. Out of these, a fraction $1-\theta$ view all products as independent so buy all of them, while the remaining fraction only buy one product. The platform demand generated by these informed buyers is $m\lambda x((1-\theta)m+\theta)$. Total demand for the platform when m sellers join is thus

$$m\lambda(1+x(m-1)(1-\theta)). \tag{17}$$

Since (16) and (17) are both increasing in m, the platform obtains its maximum payoff by inducing all sellers to join ($m=n$) and setting f so (16) is binding when $m=n$. The

[16] In Online Appendix A.6, we explore less favorable beliefs. These lower the platform's profit, as it has to set a lower fee to attract all sellers to join, but as was the case for the baseline setting with two sellers, they have no effect on the platform's optimal choice of x^*.

resulting platform profit is

$$\Pi(x) = \left(b + \frac{x(n(1-\theta)-1)}{x(n(1-\theta)-1)+1}(v-c)\right)(n\lambda(1+x(n-1)(1-\theta))). \qquad (18)$$

Relegating the optimization problem over x to the appendix, we obtain the following proposition.

Proposition 5. *Suppose each of n sellers starts with a measure λ of captive buyers. The platform always finds it optimal to induce all sellers to join and its optimal level of discovery is given by*

$$x^* = \begin{cases} 1 & if \quad 0 < \theta \leq \theta_1(\mu, n) \\ \frac{1-\sqrt{\frac{\theta}{(\mu+1)(1-\theta)(n-1)}}}{1-n(1-\theta)} & if \quad \theta_1(\mu, n) \leq \theta \leq \theta_2(\mu, n) \\ 0 & if \quad \theta \geq \theta_2(\mu, n) \end{cases}, \qquad (19)$$

where $\theta_1(\mu, n) \in \left(1 - \frac{1}{n}, \theta_2(\mu, n)\right)$ is the unique solution in θ to

$$\frac{\theta}{(1-\theta)^3} = n^2(n-1)(\mu+1)$$

and $\theta_2(\mu, n) = \frac{(\mu+1)(n-1)}{(\mu+1)(n-1)+1}$.

It is straightforward to confirm that the comparative statics with respect to θ and μ remain the same as in the baseline. The extent of discoverability x^* is decreasing in θ and increasing in μ.

More interesting is that θ_1 and θ_2 are increasing in n, and so is x^* for the interior solution. More sellers always increase the amount of discoverability the platform will choose. In part this reflects that our model over-emphasizes the positive network effect across sellers due to discoverability and de-emphasizes the negative substitution effect that can arise as more and more sellers are added. Indeed, the only thing that matters for a seller's expected profit is the profit from captives, which always increases when more sellers join the platform. Meanwhile, the number of sellers that compete for contested buyers turns out not to affect a given seller's equilibrium profit. With a more general demand function, each seller's profits from contested buyers would be decreasing in the number of participating sellers, so that adding more sellers can lower each seller's equilibrium profit. In Online Appendix A.7 we use a less tractable setting with elastic demand, and show that adding more sellers always reduces the optimal level of discoverability.

One way to interpret these different results is that our baseline demand specification captures that each additional seller serves a unique product category, with buyers sometimes only wanting to buy from one such product category (and viewing them as perfect substitutes), and other times wanting to buy from all of them. The result says the platform should increase the level of discoverability as it adds more product categories. In contrast, the alternative elastic-demand specification captures the idea of adding more sellers within a given product category. In that case, we find the platform should decrease the level of discoverability as it adds more sellers within a given product category.

Moreover, it is important to recall that in our model discoverability involves buyers seeing all listed sellers. An alternative setting, would be that discoverability involves buyers seeing a fixed number of sellers, say $j \geq 2$ out of a total n participating sellers, where $j < n$. In this case, $1 - x$ of a seller's λ initially captive buyers do not get to see any other seller, and x of them get to observe $j - 1$ other randomly selected sellers. In Online Appendix A.8 we show that the optimal level of discoverability x^* in this case is the same as above in the case there are j sellers on the platform to start with. Thus, for instance, if buyers only look at most at two sellers, then the optimal level of discoverability is the same as in the baseline no matter how many sellers join the platform.

4.5 Heterogeneous sellers

So far we have assumed all sellers are identical, each starting with the same measure λ of captive buyers. In this section we analyze two different cases where the sellers are not symmetric: in the first case we explore how asymmetry changes the optimal level of discoverability, and in the second case we illustrate the possibility that a platform may choose to only attract smaller sellers and leave larger sellers out by setting a high level of discoverability.

4.5.1 Two asymmetric sellers

Consider first the case with two sellers but seller i has measure λ_i of initially captive buyers, and assume $\lambda_1 \geq \lambda_2$. This captures asymmetric sellers, with seller 1 having a larger initial base of captive buyers than seller 2.

If seller i does not join the platform, then its profit is $\lambda_i (v - c)$. If only seller i joins the platform, its profit is $\lambda_i (v + b - f - c)$, while the profit of the non-joining seller is still $\lambda_j (v - c)$. If both sellers join the platform, then seller i and seller j will compete with different measures of captive buyers. The analysis in this case turns out to be more complicated, given that the seller with fewer captives will act more aggressively and its profit will be higher than what it can obtain by just charging the monopoly price on its captives. Despite this, as we show in the proof of Proposition 6 below, it is still the seller with more captives (seller 1) that turns out to constrain the fee the platform can set to induce the two sellers to participate in case $x > 0$. This is intuitive: that seller has a better outside option, and discoverability brings more of its buyers to the other seller, than vice-versa.

The captive buyers for seller 1 are now made up of seller 1's initial captives that did not discover seller 2 (measure $\lambda_1 (1-x)$), seller 1's captives that discovered seller 2 but view the two sellers' products as independent (measure $\lambda_1 x (1-\theta)$) and seller 2's initial captives that discovered seller 1 but view the two sellers' products as independent (measure $\lambda_2 x (1-\theta)$). Thus, seller 1's profit is

$$\begin{aligned} & (v-c+b-f)\left(\lambda_1 (1-x) + (\lambda_1+\lambda_2) x (1-\theta)\right) \\ = \ & (v-c+b-f)(\lambda_1+\lambda_2)\left(\beta_1 (1-x) + x(1-\theta)\right), \end{aligned}$$

where

$$\beta_1 = \frac{\lambda_1}{\lambda_1+\lambda_2} \geq \frac{1}{2}$$

is seller 1's relative market share of initial captives. Comparing this with seller 1's profit if it doesn't join, the platform must set

$$f \leq b + (v-c)\left(1 - \frac{\beta_1}{\beta_1 (1-x) + x(1-\theta)}\right), \tag{20}$$

to ensure seller 1 participates, which also ensures seller 2 participates.[17]

The platform's demand when it attracts both sellers consists of the $(\lambda_i+\lambda_j)(1-x)$ buyers who are informed of only one product (and who buy only that product only), the $(\lambda_i+\lambda_j) x (1-\theta)$ buyers who are informed of both products and view them as independent (they buy both), and the $(\lambda_i+\lambda_j) x\theta$ buyers who are informed of both products and view them as substitutes (they buy one product only). Thus, the platform's profit when both sellers join is

$$\begin{aligned} & f\left((\lambda_1+\lambda_2)(1-x) + 2(\lambda_1+\lambda_2) x (1-\theta) + (\lambda_1+\lambda_2) x\theta\right) \\ = \ & f\left(1 + x(1-\theta)\right)(\lambda_1+\lambda_2). \end{aligned}$$

Note that the platform can set $x = 0$ and $f = b$ to obtain $b(\lambda_1+\lambda_2)$, which is strictly higher than $b\lambda_1$, the maximum profit it can achieve by attracting one seller only. Thus, it is optimal

[17] To see this, note that the right-hand side of (20) is decreasing in β_1, and the participation constraint for seller 2 is identical other than β_1 is replaced by $\beta_2 = \frac{\lambda_2}{\lambda_1+\lambda_2} < \beta_1$.

for the platform to attract both sellers.

The platform will therefore set f and x to maximize the last expression above subject to (20), which ensures both sellers participate.

Relegating the rest of the analysis to the appendix, we obtain the following proposition.

Proposition 6. *Suppose seller i starts with a measure λ_i of captive buyers, where $\lambda_1 \geq \lambda_2$. The platform always finds it optimal to induce both sellers to join and the optimal level of discovery is given by*

$$x^* = \begin{cases} 1 & if \quad 0 < \theta \leq \theta_1(\mu, \beta_1) \\ \frac{1-\sqrt{\frac{2-\theta-\frac{1-\theta}{\beta_1}}{(\mu+1)(1-\theta)}}}{1-\frac{1-\theta}{\beta_1}} & if \quad \theta_1(\mu, \beta_1) \leq \theta \leq \theta_2(\mu, \beta_1) \\ 0 & if \quad \theta \geq \theta_2(\mu, \beta_1) \end{cases}$$

where

$$\theta_2(\mu, \beta_1) = \frac{\mu + \frac{1}{\beta_1} - 1}{\mu + \frac{1}{\beta_1}} \in [1 - \beta_1, 1]$$

and $\theta_1(\mu, \beta_1) \in (1 - \beta_1, \theta_2(\mu, \beta_1))$ is the unique solution to

$$\frac{2 - \frac{1}{\beta_1} + \left(\frac{1}{\beta_1} - 1\right)\theta}{(1-\theta)^3} = \frac{\mu + 1}{\beta_1^2}.$$

The comparative statics of x^* with respect to μ and θ remain the same as before. More interesting is that x^* is decreasing in β_1, the larger seller's market share of initial captives. Thus, the bigger the difference in initial market shares of captives, the less discoverability the platform will provide. The reason is that the binding participation constraint that the platform's fee and level of discoverability must respect is that of the larger seller. And the larger seller necessarily prefers less discoverability since it brings more buyers to the platform than it stands to gain from discoverability. It is indeed easily verified that seller 1's profits are decreasing in discoverability whenever $\beta_1 > 1 - \theta$, which must be the case for the interior solution x^* to hold.

4.5.2 Why a large seller may not participate on the platform

As shown in Proposition 6, with two sellers, even if asymmetric, it is always profitable for the platform to attract both of them. With more than two sellers, if they are symmetric, the platform also wants to attract all of them (as we saw in Section 4.4). However, with more than two sellers, if they are heterogeneous, the platform may be better off setting its transaction fee and level of discoverability such that not all sellers participate.

In particular, the previous analysis with asymmetric sellers shows that it is the larger seller (in terms of their initial captives) that constrains the platform's transaction fee because it benefits less from joining the platform. This is consistent with real world observations: larger and more established brands are the ones least likely to participate on large market-places (e.g. Amazon.com), preferring to sell through their own channels instead.

In what follows we confirm that in a setting with three sellers, such that $\lambda_1 > \lambda_2 = \lambda_3$, it may be optimal for the platform to set its fee and level of discoverability such that the larger seller 1 does not participate in equilibrium. Denote

$$\beta = \frac{\lambda_1}{\lambda_1 + 2\lambda_2}.$$

First, it can never be optimal for the platform to induce only one seller to join because that implies no discovery, so the most the platform could obtain is $b\lambda_1$. The platform could do strictly better setting $x = 0$ and the same $f = b$, so all sellers are willing to join, yielding $b(\lambda_1 + 2\lambda_2)$ for the platform. Second, it can never be optimal to induce the large seller to join together with only one small seller. We prove this result as part of Proposition 7 below. The reason is essentially the same as above. The large seller is the least likely to wish to participate on the platform when other sellers are present, and given Bertrand competition for buyers who view the products as substitutes, having two small sellers join is actually better for the large seller than having just one small seller due to the possibility of discovery. So if the large seller participates, then the second small seller is even more willing to participate, and the platform certainly benefits from having three rather than two sellers via an increased number of transactions.

Taking these two results into account, the platform's optimal strategy is either to induce all three sellers to join, or only induce the two small sellers to join. If the platform induces all three sellers to join, it is easily verified that the binding constraint on the platform's optimal fee is once again the participation of the large seller (we show this in the proof of Proposition 7 below), so the platform's profits in this case are

$$\begin{aligned} & \max_x \left\{ f\left(\lambda_1 + 2\lambda_2\right)\left(1 + x\left(1-\theta\right)\right)\right\} \\ \text{subject to } \lambda_1 (v-c) \quad \leq & \quad (v - c + b - f)\left(\lambda_1(1-x) + (\lambda_1 + 2\lambda_2)\, x\,(1-\theta)\right), \end{aligned}$$

which is equal to

$$\max_x \left\{ (\lambda_1 + 2\lambda_2)(v-c)\left(\mu + 1 - \frac{1}{1 - x + \frac{x(1-\theta)}{\beta}}\right)(1 + x(1-\theta)) \right\}.$$

Meanwhile, if the platform only induces the two small sellers to join, then the analysis is the same as in the baseline model, so the platform's profits in this case are

$$\max_x \left\{ 2\lambda_2 (v-c)\left(\mu + 1 - \frac{1}{1 + x(1-2\theta)}\right)(1 + x(1-\theta)) \right\}.$$

Consider the tradeoff between these two options. The total number of buyers is larger when attracting all three sellers ($\lambda_1 + 2\lambda_2$ instead of $2\lambda_2$), but the transaction fee can be higher when attracting just the two small sellers:

$$(v-c)\left(\mu + 1 - \frac{1}{1 + x(1-2\theta)}\right) > (v-c)\left(\mu + 1 - \frac{\lambda_1}{1 - x + \frac{x(1-\theta)}{\beta}}\right)$$

which is true if and only if

$$\beta > \frac{1}{2}.$$

Thus, the large seller has to be at least as large as the two small sellers combined in order for the maximum transaction fee that can be charged with two small sellers to be higher than that charged with all three sellers. In this case, the size disparity between the large seller and the two small sellers is so big that for the same level of discoverability, the platform must charge a lower transaction fee if it wants to attract all three sellers than when it wants to attract only the two small sellers.

By contrast, if the large seller is close in size to each of the small sellers ($\frac{1}{2} < \beta \leq 1$), then there is no tradeoff and the platform always prefers to induce all three sellers to join, consistent with the results from Section 4.4 with multiple sellers: profits are increasing in the number of (equal) sellers that join.

The following proposition confirms this by focusing on the specific case when tools have no value ($b = 0$), so the only valuable service that the platform can provide is discovery.

Proposition 7. *Suppose there are three sellers, one large of size λ_1 and two identical smaller sellers, each of size $\lambda_2 < \lambda_1$. Suppose also $b = 0$. When $\theta \geq \frac{1}{2}$, the platform strictly prefers to induce all three sellers to join if $\beta < 1 - \theta$ and is indifferent between two or three sellers joining (with zero resulting profits) when $\beta \geq 1 - \theta$. When $\theta < \frac{1}{2}$, the platform prefers to induce all three sellers to join if $\beta \leq \frac{1}{1+2\theta}$ and prefers to induce only the two small sellers* *to join if $\beta > \frac{1}{1+2\theta}$. If it is optimal to induce only the two small sellers to join, the optimal level of discoverability is as in the baseline. If it is optimal to induce all three sellers to join, the optimal level of discoverability is higher than in the baseline, strictly so if $\frac{1}{2} \leq \theta < 1 - \beta$.*

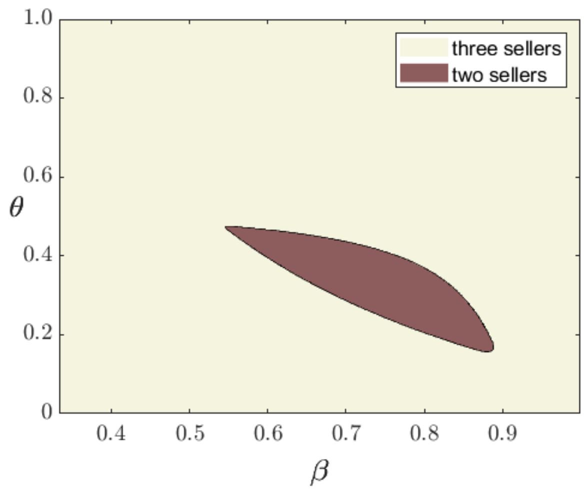

Figure 3: Parameter region where two sellers join and where three sellers join

This case is represented in Figure 3 which shows the combination of (θ, β) where the platform prefers to host the two small sellers only and where the platform prefers to host all three sellers when $\mu = 0$. (In case, the platform is indifferent, we assume it chooses the option which would also be implied by the limit as $\mu \to 0$ from above). In particular, note that for any θ, there exists a threshold such that the platform prefers to induce all three sellers to join when β is below that threshold and prefers (strictly only if $\theta < \frac{1}{2}$) to induce only the two small sellers to join when β is above that threshold. This is an artifact of the assumption that $\mu = 0$, so the platform has no valuable tools to offer aside from discovery. Indeed, this implies that when the large seller becomes sufficiently large relative to the two small sellers (i.e. β becomes large), the platform prefers to drop the large seller because attracting it means choosing almost no discovery and therefore vanishingly small profits in the absence of valuable tools.

In general however, with $\mu > 0$ so the platform offers valuable tools, if the large seller becomes sufficiently big relative to the small sellers, then the platform once again strictly prefers inducing all three sellers to join (which it can always do by setting x equal or close to zero), for the simple reason that the large seller is too big to leave out and it can be served profitably with tools. For the platform to prefer inducing only the two small sellers to join, λ_1 has to be in some intermediate range relative to λ_2 (given θ). This is confirmed in Figure 3, which also shows the platform's optimal choice of sellers as a function of θ and β when

$\mu = 0.1$. As μ increases, the region in Figure 3 where the platform prefers to only induce the two sellers to join shrinks, and we note that for any $\mu \geq 0.2$, there is no θ and β for which the platform ever prefers only selling to the two small sellers.

4.6 Competing platforms

So far we have assumed there is a single monopoly platform. In this subsection we consider two extensions to handle competing platforms, the first to provide the simplest and more direct extension of our baseline setting to competing platforms, and the second to illustrate how it is possible to sustain an equilibrium where the platforms endogenously differentiate themselves by offering different levels of discoverability, and thereby attracting different sellers. In each case, the model will have two identical platforms, with the idea the platforms first determine if they want to invest in offering some level of discoverability (which incurs some arbitrarily small fixed costs to provide). Then after observing each other's choice of x, they simultaneously set their fees f_1 and f_2. The sellers, then decide which platform to join, if any. Note if platform i chooses not to invest in any discoverability, then by default it has $x_i = 0$.

4.6.1 Symmetric platform competition

Suppose there are two platforms 1 and 2, and two symmetric sellers with $\lambda_1 = \lambda_2 = \lambda$.

Proposition 8. *If $\theta < \frac{1}{2}$, then the only possible equilibrium is that both sellers join the same platform i, and in this equilibrium we have $x_i = 1$, $x_j = 0$, $f_i = (v + b - c)\frac{1-2\theta}{2-2\theta}$, $f_j = 0$. If $\theta \geq \frac{1}{2}$, then in equilibrium we have $f_1 = f_2 = 0$, $x_1 = x_2 = 0$, and each seller joins either platform.*

Comparing this to the baseline with a monopoly platform and two symmetric sellers, we have that the equilibrium level of discoverability is lower under platform competition. The reason is that when platforms compete, they focus on maximizing the payoff to sellers in order to attract them, and sellers generally prefer less discoverability than the platforms.

4.6.2 Endogenous platform differentiation

Consider again the setting from Section 4.5.2 and Proposition 7, but now allow for our setting with competing platforms. Then we claim the following holds.

Proposition 9. *Suppose there are three sellers, one large of size λ_1 and two identical smaller sellers, each of size $\lambda_2 < \lambda_1$. Suppose also $b = 0$. When $\theta < \frac{1}{2}$ and provided*

$\lambda_1 > \frac{2(1-\theta)\lambda_2}{\theta}$, there is an equilibrium where platform 1 attracts the two small sellers, setting $f_1^* = (v-c)\left(1 - \frac{1}{2(1-\theta)}\right)$ and $x_1^* = 1$, and platform 2 attracts the large seller, setting $f_2^* = 0$ and $x_2^* = 0$. (There is another equivalent equilibrium with the roles of the two platforms reversed).

This result shows the possibility for the co-existence of two competing platforms, one that attracts the larger seller by not offering discoverability, and one that attracts the multiple smaller sellers by offering maximum discoverability.

5 Conclusion

We have provided a framework for analyzing to what extent platforms will want to allow buyers who are brought in by participating sellers to discover rival sellers. While we explored many different extensions of the simple baseline setting in the paper, there remain many more avenues to explore in future work.

Further analysis of competing platforms seems warranted, although this remains challenging. For instance, it would be interesting to explore other types of heterogeneity between sellers, to understand how different seller characteristics drive their preferences over platforms that offer different levels of discoverability. In our analysis of competing platforms, we assumed a seller would only go to one platform or the other, bringing all its buyers onto the chosen platform. Another possibility would be to allow the seller to determine the portion of its initially captive buyers it brings onto each platform, or possibly to both. Extending our analysis to a dynamic setting where the sellers' initial capitives become loyal to the platform after some time would possibly provide a rationale for platforms to increase the extent of discoverability they offer over time.

6 Appendix

We provide the remaining details for the proofs of each proposition.

6.1 Proof of Propositions 1 and 2

We prove directly Proposition 2, which is more general. The proof of Proposition 1 follows automatically simply by setting $\eta = 0$ (i.e. the platform starts with no buyers of its own).

Factoring out the constant term $\lambda(v-c)$, the derivative of (9) with respect to x is

$$(2(1-\theta)+\eta(2-\theta))(\mu+1)-\frac{(2+\eta)\theta}{(1+x((1-2\theta)+\eta(1-\theta)))^{2}}. \tag{21}$$

If $\theta \leq \frac{1+\eta}{2+\eta}$, then (21) is increasing in x and is non-negative when evaluated at $x=0$, so we must have $x^{*}=1$. If $\theta > \frac{1+\eta}{2+\eta}$, then (21) is decreasing in x, so the SOC holds. Setting (21) equal to zero and solving for x implies the unconstrained solution

$$x(\theta)=\frac{1-\sqrt{\frac{(2+\eta)\theta}{(2(1-\theta)+\eta(2-\theta))(\mu+1)}}}{2\theta-1-\eta(1-\theta)}.$$

Given $x(\theta)$ is decreasing in θ for $\theta > \frac{1+\eta}{2+\eta}$, and given $x\left(\frac{1+\eta}{2+\eta}\right) > 1$ and $x(\theta) < 0$ for θ sufficiently high, the constrained solution is given by x^{*} in Proposition 2 where $\theta_1(\eta,\mu)$ is the unique solution to $x(\theta)=1$ and where $\theta_2(\eta,\mu)=\frac{2(1+\eta)(\mu+1)}{(2+\eta)(\mu+2)} > \frac{1+\eta}{2+\eta}$ is the unique solution to $x(\theta)=0$. It is easily verified that $\frac{1+\eta}{2+\eta} < \theta_1(\eta,\mu) < \theta_2(\eta,\mu)$.

Setting $\eta=0$, we obtain the results in Proposition 1. Note that $\theta_2(0,\mu)=\frac{\mu+1}{\mu+2} < 1$, but with $\eta > 0$, we can have $\theta_2(\eta,\mu) > 1$.

6.2 Proof of Proposition 3

Recall the problem is to maximize (11) over (F,f,x) subject to the constraints (10) and (**??**). Since platform profits are increasing in F and f, we must have

$$F+\lambda(f-b)(1+x(1-2\theta))=\lambda(v-c)x(1-2\theta).$$

Using this to replace F in the platform's profits and (12), the problem becomes to choose f and x to maximize

$$2\lambda((v-c+b)x(1-2\theta)+b+f\theta x).$$

subject to

$$\lambda(v-c)x(1-2\theta)-\lambda(f-b)(1+x(1-2\theta)) \geq -K.$$

If the constraint is not binding, then $f^{*}=v-c+b$ and $x^{*}=1$, which is valid iff $\lambda(v-c) \leq K$. So assume $0 \leq K < \lambda(v-c)$, and the constraint is binding. Solving the binding constraint for f implies

$$f=(v-c)\frac{x(1-2\theta)}{(1+x(1-2\theta))}+\frac{K}{\lambda(1+x(1-2\theta))}+b.$$

Substituting this into the platform's profit, after factoring out the constant $2\lambda(v-c)$, the problem is to choose x to maximize

$$\frac{x(1-2\theta)(1+x(1-\theta))}{1+x(1-2\theta)}+\frac{\theta x K}{\lambda(v-c)(1+x(1-2\theta))}+\mu(1+x(1-\theta)).$$

It is easily verified that if $\theta\leq\frac{1}{2}$, this is increasing in x, so the platform sets $x^*=1$ regardless of K. Assume therefore $\theta>\frac{1}{2}$. The derivative in x is

$$\frac{(1-2\theta)\left(1+2x(1-\theta)+x^2(1-\theta)(1-2\theta)\right)+\mu(1-\theta)(1+x(1-2\theta))^2+\frac{\theta K}{\lambda(v-c)}}{(1+x(1-2\theta))^2},$$

which is decreasing in x for $\theta>\frac{1}{2}$, so the SOC holds. This derivative is zero when the numerator equals zero, which gives the unconstrained solution

$$x(\theta)=\frac{1-\sqrt{\frac{\theta}{(\mu+1)(1-\theta)}\left(1-\frac{K}{\lambda(v-c)}\right)}}{2\theta-1}.$$

This is the same as $x(\theta)$ in the baseline except $\mu+1>1$ is replaced by $\frac{\mu+1}{1-\frac{K}{\lambda(v-c)}}>1$, with the expressions for the cutoffs adjusted accordingly.

6.3 Proof of Proposition 4

The sellers' participation constraint can be rewritten as

$$f_1x(1-\theta)+f_0(1-x\theta)\leq(v-c)x(1-2\theta)+b(1+x(1-2\theta)).$$

Suppose the sellers' participation constraint is not binding at the optimum. Then we must have

$$f_0=f_1=v+b-c,$$

otherwise the platform could profitably increase either f_0 or f_1. But then the sellers' participation constraint is equivalent to

$$v-c\leq 0,$$

which is not possible.

So the sellers' participation constraint must be binding at the optimum, i.e. we must have

$$f_1x(1-\theta)+f_0(1-x\theta)=(v-c)x(1-2\theta)+b(1+x(1-2\theta)).$$

We can use this to express f_1 as a function of f_0. After factoring out the constant λ, the platform's profits can then be written as

$$-f_0 \frac{\theta(1-x)}{1-\theta} + \frac{(2-\theta)}{1-\theta}\left((v-c+b)\,x(1-2\theta)+b\right),$$

which the platform maximizes over (f_0, x) subject to

$$0 \leq f_0 \leq v+b-c$$

and

$$0 \leq (v-c)\frac{1-2\theta}{1-\theta} + b\frac{1+x(1-2\theta)}{x(1-\theta)} - f_0\frac{(1-x\theta)}{x(1-\theta)} \leq v+b-c.$$

Since the last expression of platform profits is decreasing in f_0, we must either have

$$f_0 = 0$$

or

$$(v-c)\frac{1-2\theta}{1-\theta} + b\frac{1+x(1-2\theta)}{x(1-\theta)} - f_0\frac{(1-x\theta)}{x(1-\theta)} = v+b-c.$$

Suppose first $f_0 = 0$. Then the platform is maximizing profit

$$\frac{(2-\theta)}{1-\theta}\left((v-c+b)\,x(1-2\theta)+b\right)$$

over x subject to

$$0 \leq \frac{(v-c)\,x(1-2\theta)+b\,(1+x(1-2\theta))}{x(1-\theta)} \leq v+b-c.$$

Clearly, the platform will set x such that $(v-c+b)\,x(1-2\theta)+b > 0$. So the only relevant constraint is

$$\frac{(v-c)\,x(1-2\theta)+b\,(1+x(1-2\theta))}{x(1-\theta)} \leq v+b-c,$$

which is equivalent to

$$\frac{\mu}{1+\mu} \leq x\theta$$

where $b = \mu(v-c)$. There are three cases:

1. If

$$\theta < \frac{\mu}{\mu+1},$$

then the constraint cannot be satisfied, so we can't have $f_0 = 0$.

2. If $\frac{\mu}{\mu+1} \leq \theta \leq \frac{1}{2}$, then $x^* = 1$ and the platform's maximum profits conditional on $f_0 = 0$ are

$$\begin{aligned} & \frac{(2-\theta)}{1-\theta}\left((v-c+b)(1-2\theta)+b\right) \\ = & \frac{(2-\theta)(1-2\theta)}{1-\theta}(v-c)+2(2-\theta)b. \end{aligned}$$

3. If $\theta \geq \max\left\{\frac{\mu}{\mu+1}, \frac{1}{2}\right\}$, then $x^* = \frac{\mu}{(\mu+1)\theta}$ and the platform's maximum profits conditional on $f_0 = 0$ are

$$\begin{aligned} & \frac{(2-\theta)}{1-\theta}\left((v-c+b)\frac{\mu(1-2\theta)}{(\mu+1)\theta}+b\right) \\ = & b\frac{2-\theta}{\theta}. \end{aligned}$$

Now suppose $f_0 > 0$, so we must have

$$f_1 = (v-c)\frac{1-2\theta}{1-\theta} + b\frac{1+x(1-2\theta)}{x(1-\theta)} - f_0\frac{(1-x\theta)}{x(1-\theta)} = v+b-c,$$

which is equivalent to

$$f_0 = b-(v-c)\frac{x\theta}{1-x\theta} < v-c+b.$$

The platform's profits as a function of x are then

$$\begin{aligned} & -\left(b-(v-c)\frac{x\theta}{1-x\theta}\right)\frac{\theta(1-x)}{1-\theta} + \frac{(2-\theta)}{1-\theta}\left((v-c+b)x(1-2\theta)+b\right) \\ = & (v-c)\left(-\left(\mu-\frac{x\theta}{1-x\theta}\right)\frac{\theta(1-x)}{1-\theta} + \frac{(2-\theta)}{1-\theta}\left((1+\mu)x(1-2\theta)+\mu\right)\right) \\ = & (v-c)\left(1+2\mu+2(1+\mu)x(1-\theta)-\frac{1}{1-x\theta}\right). \end{aligned}$$

The platform maximizes these profits subject to $f_0 \geq 0$ (all other constraints are satisfied), which is equivalent to

$$x \leq \frac{\mu}{\theta(1+\mu)}.$$

The derivative of the last expression of platform profits above with respect to x is

$$(v-c)\left(2(1+\mu)(1-\theta)-\frac{\theta}{(1-x\theta)^2}\right),$$

so the second derivative is clearly negative, which means the SOC holds. The unconstrained optimal x is then

$$x^* = \frac{1 - \sqrt{\frac{\theta}{2(1+\mu)(1-\theta)}}}{\theta}.$$

There are three cases.

1. If $\frac{\theta}{2(1+\mu)(1-\theta)} \geq 1$, which is equivalent to

$$\theta \geq \frac{2(\mu+1)}{2(\mu+1)+1},$$

then the optimal solution conditional on $f_0 > 0$ is $x^* = 0$, which implies $f_0 = b$ and the platform's profits are $2b$.

2. If

$$0 \leq \frac{1 - \sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta} \leq \min\left\{1, \frac{\mu}{\theta(1+\mu)}\right\},$$

which is equivalent to

$$\frac{\theta}{(1-\theta)^3} \geq 2(\mu+1) \text{ and } \frac{2}{3+\mu} \leq \theta \leq \frac{2(\mu+1)}{2(\mu+1)+1},$$

then the optimal solution conditional on $f_0 > 0$ is

$$x^* = \frac{1 - \sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta},$$

which implies $f_0 = b - (v-c)\frac{x^*\theta}{1-x^*\theta}$ and the platform's profits are

$$(v-c)\left(1 + 2\mu + 2(1+\mu)x^*(1-\theta) - \frac{1}{1-x^*\theta}\right).$$

3. If

$$\frac{1 - \sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta} \geq \min\left\{1, \frac{\mu}{\theta(1+\mu)}\right\},$$

which is equivalent to

$$\frac{\theta}{(1-\theta)^3} \leq 2(\mu+1) \text{ or } \theta \leq \frac{2}{3+\mu},$$

then the optimal solution conditional on $f_0 > 0$ is

$$x^* = \min\left\{1, \frac{\mu}{\theta(1+\mu)}\right\},$$

which implies $f_0 = b - (v-c)\frac{x^*\theta}{1-x^*\theta}$ and the platform's profits are

$$(v-c)\left(1 + 2\mu + 2(1+\mu)x^*(1-\theta) - \frac{1}{1-x^*\theta}\right).$$

The platform compares the best solution conditional on $f_0 = 0$ to the best solution conditional on $f_0 > 0$. We distinguish two cases: $\mu \leq 1$ and $\mu \geq 1$. Let $\theta_0(\mu)$ denote the unique solution to

$$\frac{\theta}{(1-\theta)^3} = 2(\mu+1).$$

Suppose first $\mu \leq 1$. Then we have

$$\frac{\mu}{\mu+1} \leq \theta_0(\mu) \leq \frac{1}{2} \leq \frac{2}{3+\mu} \leq \frac{2(\mu+1)}{2(\mu+1)+1}.$$

So:

- if $\theta \leq \frac{\mu}{1+\mu}$, then there is no solution with $f_0 = 0$, so the optimal solution is

$$\begin{aligned} x^* &= 1 \\ f_0 &= b - (v-c)\frac{\theta}{1-\theta} \\ f_1 &= v + b - c \end{aligned}$$

  and yields platform profits

$$(v-c)(2-\theta)\left(2(1+\mu) - \frac{1}{1-\theta}\right).$$

- if $\frac{\mu}{\mu+1} \leq \theta \leq \frac{1}{2}$, then the solution with $f_0 > 0$ has $x^* = \frac{\mu}{\theta(1+\mu)}$, which implies $f_0 = 0$. So this is weakly dominated by the solution conditional on $f_0 = 0$, which has

$$\begin{aligned} x^* &= 1 \\ f_1 &= \frac{(v-c)(1-2\theta)}{1-\theta} + 2b \end{aligned}$$

and yields platform profits

$$(v-c)(2-\theta)\left(2(1+\mu)-\frac{1}{1-\theta}\right)$$

- If $\frac{1}{2}\le\theta\le\frac{2}{3+\mu}$, then the solution with $f_0>0$ has $x^*=\frac{\mu}{\theta(1+\mu)}$, which implies $f_0=0$. So this is weakly dominated by the solution conditional on $f_0=0$, which has

$$\begin{aligned} x^* &= \frac{\mu}{(\mu+1)\theta} \\ f_1 &= \frac{b(\mu+1)}{\mu} \end{aligned}$$

and yields platform profits

$$(v-c)\,\mu\frac{2-\theta}{\theta}.$$

- if $\frac{2}{3+\mu}\le\theta\le\frac{2(\mu+1)}{2(\mu+1)+1}$, then the solution with $f_0>0$ has

$$\begin{aligned} x^* &= \frac{1-\sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta} \\ f_0 &= b-(v-c)\frac{x^*\theta}{1-x^*\theta} \\ f_1 &= v-c+b \end{aligned}$$

and yields platform profits

$$(v-c)\left(1+2\mu+2(1+\mu)x^*(1-\theta)-\frac{1}{1-x^*\theta}\right).$$

We know that $x^*=\frac{1-\sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta}$ maximizes this last expression, so it must be higher than when it is evaluated at $x^*=\frac{\mu}{(\mu+1)\theta}$, where it is equal to $(v-c)\frac{(2-\theta)\mu}{\theta}$. The latter is the optimal platform profit that can be obtained conditional on $f_0=0$ (because $\theta\ge\frac{2}{3+\mu}>\frac{1}{2}$). So the optimal solution is the one above, with $f_0>0$.

- if $\theta\ge\frac{2(\mu+1)}{2(\mu+1)+1}$, then the solution conditional on $f_0>0$ is

$$\begin{aligned} x^* &= 0 \\ f_0 &= b, \end{aligned}$$

with indeterminate f_1 and yielding platform profits

$$2b.$$

This dominates the solution with f_0, which yields $\frac{(2-\theta)b}{\theta}$, because $\theta \geq \frac{2(\mu+1)}{2(\mu+1)+1} > \frac{2}{3}$.

Now suppose $\mu \geq 1$. Then we have

$$\frac{2}{3+\mu} \leq \frac{1}{2} \leq \theta_0(\mu) \leq \frac{\mu}{\mu+1} < \frac{2(\mu+1)}{2(\mu+1)+1}.$$

So:

- if $\theta \leq \theta_0(\mu)$, then there is no solution with $f_0 = 0$, so the optimal solution has

$$\begin{aligned} x^* &= 1 \\ f_0 &= b - (v-c)\frac{\theta}{1-\theta} \\ f_1 &= v + b - c \end{aligned}$$

  yielding platform profits

$$(v-c)(2-\theta)\left(2(\mu+1) - \frac{1}{1-\theta}\right).$$

- if $\theta_0(\mu) \leq \theta \leq \frac{\mu}{\mu+1}$, then there is no solution with $f_0 = 0$, so the optimal solution has

$$\begin{aligned} x^* &= \frac{1 - \sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta} \\ f_0 &= b - (v-c)\frac{x^*\theta}{1-x^*\theta} \\ f_1 &= v + b - c \end{aligned}$$

  yielding platform profits

$$(v-c)\left(1 + 2\mu + 2(1+\mu)x^*(1-\theta) - \frac{1}{1-x^*\theta}\right).$$

- if $\frac{\mu}{\mu+1} \leq \theta \leq \frac{2(\mu+1)}{2(\mu+1)+1}$, then the optimal solution with $f_0 > 0$ is

$$\begin{aligned} x^* &= \frac{1 - \sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta} \\ f_0 &= b - (v-c)\,\frac{x^*\theta}{1 - x^*\theta} \\ f_1 &= v + b - c, \end{aligned}$$

  yielding platform profits

$$(v-c)\left(1 + 2\mu + 2\,(1+\mu)\,x^*\,(1-\theta) - \frac{1}{1 - x^*\theta}\right).$$

  We know that $x^* = \frac{1-\sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta}$ maximizes this expression, so it must be higher than when it is evaluated at $x = \frac{\mu}{(\mu+1)\theta}$, where it is equal to $(v-c)\,\mu\frac{(2-\theta)}{\theta}$. The latter is the optimal profit that can be obtained conditional on $f_0 = 0$ (because $\theta \geq \frac{\mu}{\mu+1} \geq \frac{1}{2}$). So the optimal solution is the one above, with $f_0 > 0$.

- if $\theta \geq \frac{2(\mu+1)}{2(\mu+1)+1}$, then the solution conditional on $f_0 > 0$ is

$$\begin{aligned} x^* &= 0 \\ f_0 &= b, \end{aligned}$$

  with indeterminate f_1 and yielding profits

$$2b.$$

  This dominates the solution with f_0, which yields $\frac{(2-\theta)b}{\theta}$, because $\theta \geq \frac{2(\mu+1)}{2(\mu+1)+1} > \frac{2}{3}$.

6.4 Proof of Corollary 1

Recall the optimal level of discoverability in the baseline is

$$x_b^* = \begin{cases} 1 & \text{if} \quad 0 < \theta \leq \theta_1(\mu) \\ \frac{1-\sqrt{\frac{\theta}{(\mu+1)(1-\theta)}}}{2\theta-1} & \text{if} \quad \theta_1(\mu) \leq \theta \leq \frac{\mu+1}{\mu+2} \\ 0 & \text{if} \quad \theta \geq \frac{\mu+1}{\mu+2} \end{cases},$$

with $\theta_1(\mu) \in \left(\frac{1}{2}, \frac{\mu+1}{\mu+2}\right)$ the unique solution in θ to

$$\frac{\theta}{(1-\theta)^3} = 4(\mu+1).$$

Consider first the case $\mu \geq 1$, so the optimal level of discoverability with differential fees is

$$x^*_{df} = \begin{cases} 1 & \text{if} \quad 0 < \theta \leq \theta_0(\mu) \\ \frac{1-\sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta} & \text{if} \quad \theta_0(\mu) \leq \theta \leq \frac{2(\mu+1)}{2(\mu+1)+1} \\ 0 & \text{if} \quad \theta \geq \frac{2(\mu+1)}{2(\mu+1)+1} \end{cases},$$

where $\theta_0(\mu)$ is the unique solution to

$$\frac{\theta}{(1-\theta)^3} = 2(\mu+1),$$

so

$$\theta_0(\mu) < \theta_1(\mu)$$

Note that

$$\frac{1-\sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta} > \frac{1-\sqrt{\frac{\theta}{(\mu+1)(1-\theta)}}}{2\theta-1}$$

is equivalent to

$$\frac{1-\left(2-\sqrt{2}\right)\theta}{1-\theta}\sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}} > 1.$$

The LHS is increasing in θ. Furthemore, it is easily verified that the inequality holds for $\theta = \frac{\mu+1}{\mu+2} < \frac{2(\mu+1)}{2(\mu+1)+1}$ and does not hold when $\theta = \theta_1(\mu)$. Thus, there exists $\theta_3 \in \left[\theta_1(\mu), \frac{\mu+1}{\mu+2}\right]$, such that the inequality holds for $\theta > \theta_3$ and does not hold for $\theta \leq \theta_3$. This implies the result for this case.

Now consider the case $\mu \leq 1$, so the optimal level of discoverability with differential fees is

$$x^*_{df} = \begin{cases} 1 & \text{if} \quad 0 < \theta \leq \frac{1}{2} \\ \frac{\mu}{(\mu+1)\theta} & \text{if} \quad \frac{1}{2} \leq \theta \leq \frac{2}{3+\mu} \\ \frac{1-\sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta} & \text{if} \quad \frac{2}{3+\mu} \leq \theta \leq \frac{2(\mu+1)}{2(\mu+1)+1} \\ 0 & \text{if} \quad \theta \geq \frac{2(\mu+1)}{2(\mu+1)+1} \end{cases},$$

Note that

$$\frac{\mu}{(\mu+1)\theta} > \frac{1-\sqrt{\frac{\theta}{(\mu+1)(1-\theta)}}}{2\theta-1}$$

is equivalent to

$$\frac{\theta\left(1-\mu\right)+\mu}{\theta\sqrt{\frac{\theta}{(\mu+1)(1-\theta)}}}<\left(\mu+1\right),$$

and the LHS of the last inequality is decreasing in θ. Furthermore, we still have

$$\frac{1-\sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta}>\frac{1-\sqrt{\frac{\theta}{(\mu+1)(1-\theta)}}}{2\theta-1}$$

iff

$$\frac{1-\left(2-\sqrt{2}\right)\theta}{1-\theta}\sqrt{\frac{\theta}{2\left(\mu+1\right)\left(1-\theta\right)}}>1$$

and the LHS of the last inequality is increasing in θ. And

$$\frac{\mu}{\left(\mu+1\right)\theta}=\frac{1-\sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta}$$

when $\theta=\frac{2}{3+\mu}$. Define

$$f\left(\theta\right)=\begin{cases}\frac{\mu}{(\mu+1)\theta} & \text{if} \quad \theta\leq\frac{2}{3+\mu}\\ \frac{1-\sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta} & \text{if} \quad \theta\geq\frac{2}{3+\mu}\end{cases}.$$

We have

$$f\left(\theta\right)<\frac{1-\sqrt{\frac{\theta}{(\mu+1)(1-\theta)}}}{2\theta-1}=1$$

when $\theta=\theta_1\left(\mu\right)$ and

$$f\left(\theta\right)>\frac{1-\sqrt{\frac{\theta}{(\mu+1)(1-\theta)}}}{2\theta-1}$$

when $\theta=\frac{\mu+1}{\mu+2}$.

So we can conclude there exists $\theta_3\in\left[\theta_1\left(\mu\right),\frac{\mu+1}{\mu+2}\right]$ such that $f\left(\theta\right)>\frac{1-\sqrt{\frac{\theta}{(\mu+1)(1-\theta)}}}{2\theta-1}$ iff $\theta>\theta_3$, which implies the result for this case as well.

6.5 Proof of n-firm case

To show the result, we can define the function $X\left(\theta\right)=n^2\left(n-1\right)\left(1-\theta\right)^3\left(1+\mu\right)$, which is strictly decreasing in θ with $X\left(1-\frac{1}{n}\right)>1-\frac{1}{n}$. This implies $1-\frac{1}{n}<\theta_1$ and $X\left(\theta_2\right)<\theta_2$, implying $\theta_2>\theta_1$.As a result for $\theta\leq\theta_1$, $x^*=1$ and for $\theta>\theta_2$, $x^*=0$.

6.6 Proof of Proposition 6

Let the measure of captives that seller i obtains be denoted λ_i'. To handle this case we use the result in Lemma 2 of Myatt and Ronayne (2019) to determine each seller's expected profit.[18] Their result covers the case of two sellers i and j with $\lambda_i' > \lambda_j'$ captives and the same marginal costs c. Seller i is the less aggressive seller as it has more captives, meaning $p_i^+ > p_j^+$ in their notation. Then seller j's expected profit is

$$\left(\lambda_j' + \phi\right)\left(p_i^+ - c\right) = \frac{\lambda_j' + \phi}{\lambda_i' + \phi}\lambda_i'\left(v - c\right) > \lambda_j'\left(v - c\right),$$

while seller i's expected profit is $\lambda_i'(v-c)$, where ϕ is the measure of buyers informed of both sellers and view them as substitutes.

Following the same logic for the mesaure of captives of seller 1 in the main text, the captive buyers for seller i in general are

$$\lambda_i' = \lambda_i\left(1 - x\right) + \lambda_i x\left(1 - \theta\right) + \lambda_j x\left(1 - \theta\right).$$

Given $\lambda_1 > \lambda_2$, we have $\lambda_1' > \lambda_2'$. Moreover, $\phi = \left(\lambda_i + \lambda_j\right)x\theta$.

Thus, seller 1's profit is

$$\begin{aligned} & \left(v - c + b - f\right)\left(\lambda_1\left(1 - x\right) + \left(\lambda_1 + \lambda_2\right)x\left(1 - \theta\right)\right) \\ = \; & \left(v - c + b - f\right)\left(\lambda_1 + \lambda_2\right)\left(\beta_1\left(1 - x\right) + x\left(1 - \theta\right)\right) \end{aligned}$$

and seller 2's profit is

$$\begin{aligned} & \left(v - c + b - f\right)\frac{\lambda_2\left(1 - x\right) + \left(\lambda_1 + \lambda_2\right)x}{\lambda_1\left(1 - x\right) + \left(\lambda_1 + \lambda_2\right)x}\left(\lambda_1\left(1 - x\right) + \left(\lambda_1 + \lambda_2\right)x\left(1 - \theta\right)\right) \\ = \; & \left(v - c + b - f\right)\left(\lambda_1 + \lambda_2\right)\frac{\left(1 - \beta_1\right)\left(1 - x\right) + x}{\beta_1\left(1 - x\right) + x}\left(\beta_1\left(1 - x\right) + x\left(1 - \theta\right)\right), \end{aligned}$$

where $\beta_1 \in \left[\frac{1}{2}, 1\right]$ is defined in the main text.

Seller 1 participates iff (20) and seller 2 participates iff

$$f \leq b + \left(v - c\right)\left(1 - \frac{\left(1 - \beta_1\right)\left(\beta_1\left(1 - x\right) + x\right)}{\left(\left(1 - \beta_1\right)\left(1 - x\right) + x\right)\left(\beta_1\left(1 - x\right) + x\left(1 - \theta\right)\right)}\right).$$

[18] For completeness, we've restated the relevant part of Lemma 2 of Myatt and Ronayne in the Online Appendix A.4, which is much more general than the result stated here.

Since $\beta_1 \geq \frac{1}{2}$, we have

$$\frac{\beta_1}{\beta_1(1-x)+x(1-\theta)} \geq \frac{(1-\beta_1)(\beta_1(1-x)+x)}{((1-\beta_1)(1-x)+x)(\beta_1(1-x)+x(1-\theta))},$$

so the binding constraint is (20) of seller 1. Clearly f will be set at the maximum value allowed by the constraint, so the platform maximizes

$$(\lambda_1+\lambda_2)(v-c)\left(\mu+1-\frac{1}{1-x+\frac{x(1-\theta)}{\beta_1}}\right)(1+x(1-\theta)).$$

over x.

If $\theta \leq 1-\beta_1$, then $1-x+\frac{x(1-\theta)}{\beta_1}$ is increasing in x, so the profit expression above is increasing in x, which means $x^*=1$. Now suppose $\theta > 1-\beta_1$. The derivative of the profit expression above in x is

$$(\lambda_1+\lambda_2)(v-c)\left((\mu+1)(1-\theta)-\frac{2-\theta-\frac{1-\theta}{\beta_1}}{\left(1-x+\frac{x(1-\theta)}{\beta_1}\right)^2}\right).$$

Since $2-\theta-\frac{1-\theta}{\beta_1}\geq 0$ and we have assumed $\theta > 1-\beta_1$, the last expression above is decreasing in x, so the second-order condition holds. From this, we directly conclude:

- If
$$\theta \geq \frac{\mu+\frac{1}{\beta_1}-1}{\mu+\frac{1}{\beta_1}},$$
then $x^*=0$.

- If
$$\frac{\mu+1}{\beta_1^2} \geq \frac{2-\frac{1}{\beta_1}+\left(\frac{1}{\beta_1}-1\right)\theta}{(1-\theta)^3}$$
then $x^*=1$.

- Otherwise,
$$x^* = \frac{1-\sqrt{\frac{2-\theta-\frac{1-\theta}{\beta_1}}{(\mu+1)(1-\theta)}}}{1-\frac{1-\theta}{\beta_1}}$$

6.7 Proof of Proposition 7

As argued in the main text, it can never be optimal for the platform to induce only one seller to join. Furthermore, it can never be optimal for the platform to induce the large seller to join together with only one small seller. Indeed, if this was the case, the large seller must prefer joining together with a small seller than its outside option, i.e. we would have

$$(v - c + b - f)\left((1-x)\lambda_1 + (\lambda_2 + \lambda_1)x(1-\theta)\right) \geq (v-c)\lambda_1$$

Meanwhile, the condition for the second small seller to prefer not joining when the other two sellers have joined is

$$(v - c + b - f)\left((1-x)\lambda_2 + (\lambda_1 + 2\lambda_2)x(1-\theta)\right) < (v-c)\lambda_2.$$

It can be easily verified that these two conditions are incompatible, so there cannot be an equilibrium with one large seller and one small seller joining for any (f, x). Nor would the platform want to force the outcome in which only one large seller and one small seller join. Indeed, from the analysis above, the maximum transaction fee it could charge would be

$$f = b + (v-c)\left(1 - \frac{1}{(1-x) + x(1-\theta)\frac{\lambda_2+\lambda_1}{\lambda_1}}\right).$$

At this fee, we know that the second small seller would also be willing to join. The platform's profits with one large seller and one small seller are

$$f(\lambda_1 + \lambda_2)(1 + x(1-\theta)),$$

whereas with all three sellers participating, the platform would make

$$f(\lambda_1 + 2\lambda_2)(1 + x(1-\theta)),$$

which is strictly larger.

Thus, there are only two possibilities for the platform's optimal strategy: either all three sellers join the platform or only the two small sellers join.

In the case where only the two small sellers join, the platform sets x as in the baseline, except here we have assumed $b = \mu = 0$, so

$$x_2^* = \begin{cases} 1 & if \quad 0 < \theta \leq \frac{1}{2} \\ 0 & if \quad \theta \geq \frac{1}{2} \end{cases},$$

The platform's optimal fee and resulting profits for this case are

$$
\begin{aligned}
f_2^* &= \begin{cases} (v-c)\,\frac{1-2\theta}{2(1-\theta)} & if \quad 0<\theta\leq\frac{1}{2} \\ 0 & if \quad \theta\geq\frac{1}{2} \end{cases} \\
\Pi_2^* &= \begin{cases} \lambda_2\,(v-c)\,\frac{(1-2\theta)(2-\theta)}{1-\theta} & if \quad 0<\theta\leq\frac{1}{2} \\ 0 & if \quad \theta\geq\frac{1}{2} \end{cases} .
\end{aligned}
$$

In the case where all three sellers join the platform, the large seller's profit is

$$(v-c-f)\,(\lambda_1\,(1-x)+(\lambda_1+2\lambda_2)\,x\,(1-\theta)),$$

while the two small sellers each make a profit equal to

$$(v-c-f)\,(\lambda_2\,(1-x)+(\lambda_1+2\lambda_2)\,x\,(1-\theta)).$$

For the large seller to participate we must have

$$f\leq(v-c)\left(1-\frac{\lambda_1}{\lambda_1\,(1-x)+(\lambda_1+2\lambda_2)\,x\,(1-\theta)}\right)$$

For the small sellers to participate we must have

$$f\leq(v-c)\left(1-\frac{\lambda_2}{\lambda_2\,(1-x)+(\lambda_1+2\lambda_2)\,x\,(1-\theta)}\right).$$

Since $\lambda_1>\lambda_2$, the binding constraint must be that of the large seller, so for f to be optimal, it must be that

$$f=(v-c)\left(1-\frac{1}{1-x+\frac{x(1-\theta)}{\beta}}\right).$$

Platform profits are then

$$
\begin{aligned}
& f\,(\lambda_1+2\lambda_2)\,(1+x\,(1-\theta)) \\
= \;& (\lambda_1+2\lambda_2)\,(v-c)\left(1-\frac{1}{1-x+\frac{x(1-\theta)}{\beta}}\right)(1+x\,(1-\theta))
\end{aligned}
$$

In this case, the optimal level of discoverability is

$$x_3^*=\begin{cases} 1 & \text{if} \quad 0<\theta\leq1-\beta \\ 0 & \text{if} \quad \theta\geq1-\beta \end{cases} .$$

And the platform's profits are

$$\Pi_3^* = \begin{cases} (\lambda_1 + 2\lambda_2)(v-c)\frac{(1-\beta-\theta)(2-\theta)}{1-\theta} & \text{if} \quad 0 < \theta \le 1-\beta \\ 0 & \text{if} \quad \theta \ge 1-\beta \end{cases}.$$

Thus, when $\theta \ge \max\left\{\frac{1}{2}, 1-\beta\right\}$, we have $\Pi_3^* = \Pi_2^* = 0$, and when $\theta \le \min\left\{\frac{1}{2}, 1-\beta\right\}$, we have $\Pi_3^* \ge \Pi_2^*$ iff $\lambda_2 \ge \theta\lambda_1$. If $1-\beta \le \theta < \frac{1}{2}$ (which can only happen when $\beta > \frac{1}{2}$), then $\Pi_2^* > \Pi_3^*$. And if $\frac{1}{2} \le \theta < 1-\beta$ (which can only happen when $\beta > \frac{1}{2}$), then $\Pi_3^* > \Pi_2^*$. From this we can conclude:

- If $\theta \ge \frac{1}{2}$, then $\Pi_3^* > \Pi_2^*$ for all $\beta < 1-\theta$ and $\Pi_3^* = \Pi_2^* = 0$ for all $\beta \ge 1-\theta$.
- If $\theta < \frac{1}{2}$, then $\Pi_2^* > \Pi_3^*$ iff $\beta > \frac{1}{1+2\theta}$.

6.8 Proof of Proposition 8

Let's first look for an equilibrium in which both sellers join the same platform, say platform 1 (the same analysis applies with the roles of the two platforms reversed). The payoff to each seller when they both join platform i is

$$(v + b - f_i - c)(\lambda(1-x_i) + 2\lambda x_i(1-\theta)).$$

This payoff is increasing in x_i if $\theta \le \frac{1}{2}$ and decreasing in x_i if $\theta > \frac{1}{2}$. Thus, for platform 1 to attract the two sellers in the fee-setting stage, we must have $x_1 \ge x_2$ if $\theta \le \frac{1}{2}$ and $x_1 \le x_2$ if $\theta > \frac{1}{2}$. And working backwards to the discovery-setting stage, if $\theta \le \frac{1}{2}$, then we must have $x_1 = 1$ (otherwise platform 2 could profitably deviate to $x_2 = 1$ and attract the two sellers in the second stage) and $x_2 = 0$ (otherwise platform 2 would make negative profits). And if $\theta > \frac{1}{2}$, then by a similar logic we must have $x_1 = x_2 = 0$.

So the equilibrium with both sellers joining platform 1 always exists. It entails

$$(x_1, x_2) = \begin{cases} (1,0) & \text{if} \quad \theta \le \frac{1}{2} \\ (0,0) & \text{if} \quad \theta > \frac{1}{2} \end{cases}$$

and

$$(f_1, f_2) = \begin{cases} \left((v+b-c)\left(\frac{1-2\theta}{2-2\theta}\right), 0\right) & \text{if} \quad \theta \le \frac{1}{2} \\ (0,0) & \text{if} \quad \theta > \frac{1}{2} \end{cases}$$

Now let's look for an equilibrium in which the two sellers are split between the two platforms, say seller 1 is on platform 1 and seller 2 is on platform 2. Seller i's profits are then

$$\lambda\left(v+b-f_i-c\right)$$

and platform i's profit is λf_i.

Suppose first $\theta \geq \frac{1}{2}$. Then for platform i, setting $x_i > 0$ is a dominated strategy because the payoff to each seller when both join platform i is $\left(v+b-f_i-c\right)\left(1-x_i+2x_i\left(1-\theta\right)\right)$, which is decreasing in x_i. So there is no advantage to set $x_i > 0$, and indeed there is a disadvantage given the fixed cost ε involved in doing so. This implies in equilibrium we must have $x_1 = x_2 = 0$ in the first stage. Which in turn implies that in the second stage the only possible equilibrium is $f_1 = f_2 = 0$.

Now suppose $\theta < \frac{1}{2}$. In this case, the payoff to each seller when both join platform i is $\left(v+b-f_i-c\right)\left(1-x_i+2x_i\left(1-\theta\right)\right)$, which is increasing in x_i. Thus, if $x_i \geq x_j$, the only possible equilibrium in the fee-setting stage is that both sellers join platform i. This means there is no possible equilibrium in which the two sellers split across the two platforms in this case.

So the equilibrium with the two sellers splitting between the two platforms exists iff $\theta \geq \frac{1}{2}$. If it exists, it involves $x_1 = x_2 = 0$ and $f_1 = f_2 = 0$.

6.9 Proof of Proposition 9

Consider the proposed equilibrium in Proposition 9. The first thing to note is if the large seller ever joins the same platform as the other two sellers, the highest expected profit it can get on a platform charging f with discoverability x is $\left(\lambda_1\left(1-x\right)+\left(\lambda_1+2\lambda_2\right)x\left(1-\theta\right)\right)\left(v-c-f\right)$. When the condition in the proposition holds (i.e. $\lambda_1 > \frac{2(1-\theta)\lambda_2}{\theta}$), the amount the large seller gets is decreasing in x, and so is strictly less than what it gets with $x = 0$, i.e. $\lambda_1\left(v-c-f\right)$, which is less than $\lambda_1\left(v-c\right)$, the amount the larger seller gets if it doesn't join either platform. Thus, there is no way for either platform to attract the large seller together with the two small sellers other than to set $x = 0$ and $f = 0$. This is what platform 2 does in the proposed equilibrium.

The payoff for a small seller on platform 1 when the other small seller also joins is $\left(\lambda_2\left(1-x_1\right)+2\lambda_2 x_1\left(1-\theta\right)\right)\left(v-c-f\right)$. Note this is increasing in x_1 provided $\theta < \frac{1}{2}$, so platform 1 can charge the most when it sets $x_1 = 1$. In this case, each small seller gets expected profit of $2\lambda_2\left(v-c-f_1\right)\left(1-\theta\right)$. This compares to its next best alternative which is $\left(v-c\right)\lambda_2$ if it goes to platform 2 or does not join either platform. So we require

$$2\lambda_2\left(v-c-f_1\right)\left(1-\theta\right) \geq \left(v-c\right)\lambda_2$$

or

$$f_1 \leq \left(1 - \frac{1}{2\,(1-\theta)}\right)(v-c) = f_1^*.$$

We require $\theta \leq \frac{1}{2}$ so that $f_1^* \geq 0$. Under this condition, platform 2 cannot attract the small sellers even though it sets $f_2 = 0$ given it sets $x_2 = 0$. Thus, given $x_1 = 1$ and $x_2 = 0$ in the first stage, the equilibrium in the second stage is indeed $f_1 = f_1^*$ and $f_2 = 0$, with the two small sellers going to platform 1 and the large seller going to platform 2.

If platform 1 were to set some lower (still positive) x_1 in the first stage, it would earn strictly lower profit given the amount the small sellers are willing to pay to join is increasing in x and the large seller does not join unless $x_1 = 0$ and $f_1 = 0$.

Lastly, we need to rule out that platform 2 could make a positive profit by incurring the small fixed cost ε in order to set some $0 < x_2 \leq 1$, and competing. As discussed above, with $0 < x_2 \leq 1$, platform 2 cannot attract all three sellers. Also, because $x_2 \leq 1$ and $\theta < \frac{1}{2}$, it cannot attract the two sellers and make positive profits to cover ε. And it also can not attract the larger seller only and make positive profits because it offers no tools so cannot charge $f_2 > 0$.

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