

How Does Best Seller Recommendation Shape the Ecosystem of an Online Marketplace?

Farzad Fathi, Yi Xu, Bo Zhou

Smith School of Business, University of Maryland, College Park, Maryland

This paper studies the impact of the best seller recommendation, a widely used popularity-based system, on consumers, sellers, and the online marketplace. In a two-period model, sellers decide whether to join the marketplace and consumers decide whether to make a purchase. Upon entry, competing sellers decide their prices, while consumers form their consideration set and search products within that set before making the purchase decision. Without a recommendation system, consumers' consideration set will consist of sellers randomly selected in both periods. With the best seller recommendation, consumers still randomly search in the first period, but they will definitely include the best seller of the first period in their consideration set in the second period. We show that the best seller recommendation intensifies competition among sellers, resulting in a lower equilibrium price and decreased seller participation. Both the reduced price and the decreased seller participation hurt the marketplace's commission revenue. When sellers' pricing competition induced by the best seller recommendation is sufficiently intense, adopting the best seller recommendation system is not beneficial to the marketplace. We also identify conditions under which the best seller recommendation simultaneously benefits everyone in the ecosystem. In addition, we find that the main results are robust even if consumers engage in costly search and endogenously determine the size of their consideration set, or when the marketplace optimizes its commission rate. These results highlight the importance of accounting for the strategic response of the sellers before an online marketplace implements the best seller recommendation system.

Key words: Best Seller Recommendation, Online Marketplace, Consideration Set, Search, Price Competition

1. Introduction

Online marketplaces that connect sellers and consumers are playing a growing role in today's economy. According to Digital Commerce 360, the top online marketplaces in the world sold \$3.23 trillion in goods in 2021. Sales through marketplaces like those operated by Amazon, eBay, Alibaba, and others accounted for two-thirds of global e-commerce sales in that year.¹ In the U.S., Amazon generated over \$610 billion in its total gross merchandise volume (GMV), accounting for almost half of all retail sales in 2021.² With millions of items on major online marketplaces (e.g., Amazon currently has over 350 million items on its site³), recommendation systems become their important

¹<https://www.digitalcommerce360.com/article/infographic-top-online-marketplaces/>

²<https://www.marketplacepulse.com/marketplaces-year-in-review-2021>; <https://www.drip.com/blog/e-commerce-statistics>

³<https://www.sunkenstone.com/blog/amazon-sku/>

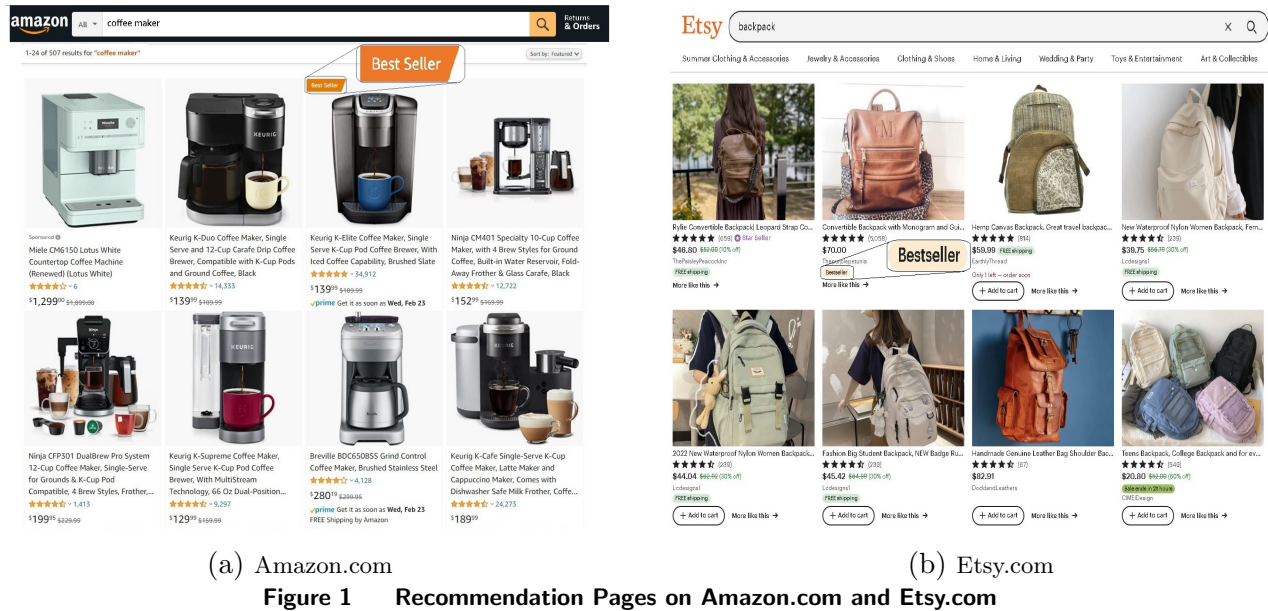
instrument. Effective recommendation systems that connect relevant and desirable products with consumers can prevent them from information overload, reduce search friction, and ultimately facilitate transactions and increase engagement. For example, 30% of Amazon’s page view results from its recommendation system (Sharma et al. 2015), 40% of App installs on Google Play are from Google’s recommendations, and e-commerce shoppers that clicked on recommendations are 4.5 times more likely to add items to cart, and 4.5 times more likely to complete their purchase (Salesforce).⁴

Due to their ease of implementation, popularity-based recommendation systems are widely used in practice. This type of recommendation system uses the sales volume of products in a particular category and recommends products with the highest sales as “Best Seller”, “Popular Product” or “Customer Favorite”. Amongst different labels, “Best Seller” is observed most frequently, as it sends the simplest message, lends credibility to the featured products, and adds a social proof element to the recommendation (Myers and Sar 2013, Sorensen 2016). Furthermore, statistically, products previously chosen by most consumers are very likely to be appealing for a large portion of the new consumers. In short, the best seller recommendation can help increase brand visibility, facilitate consumer search, and help cope with the cold-start problems. This type of system is particularly suited to environments in which there are many sellers and most consumers are anonymous and vastly heterogeneous (Garcin et al. 2014, Cremonesi et al. 2010).

For example, both Amazon and Etsy are currently using the best seller recommendation. To illustrate the specifics, Figure 1 shows a recommendation page of these two online marketplaces. In Figure 1a, one coffee-maker is labeled as “Best Seller” in the list displayed by Amazon’s ranking system. In general, if a search term belongs to a specific category, the best seller for that category will show up in a prominent spot with the label “Best Seller” clearly displayed. Similarly, Figure 1b illustrates a recommendation page of Etsy for backpacks. On this webpage, the one at the top of the page is tagged as “Bestseller”. Specifically, Etsy displays the “Bestseller” tag for the product that had the highest sales volume over the past six months.

There are quite a few advantages of the popularity-based recommendation system compared to other recommendation systems. First, it is much easier to implement and adjust. The only metric that the marketplace and the sellers need to measure and track is a given time period’s sales. In addition, the marketplace can adjust the time window of the sales measurement easily (e.g., switching from best seller of the month to best seller of the quarter). As a result, there is no need for the marketplace to invest in sophisticated algorithms that are both computationally and financially demanding. Second, it is straightforward to understand the mechanism of the popularity-based

⁴<https://developers.google.com/machine-learning/recommendation/overview>; <https://netcorecloud.com/blog/12-e-commerce-personalized-non-personalized-product-recommendation-tactics-to-set-your-sales-soaring/>



system and thus easy to communicate to the sellers. In other words, the transparency of this recommendation system saves the sellers from wasting resources to speculate over what factors they should strategize to win over recommendations. Third and perhaps most importantly, growing concerns over individual consumers' data and their privacy have led to the California Consumer Privacy Act (CCPA) and General Data Protection Regulation (GDPR), and more recently, the new law of Digital Markets Act in Europe in 2022.⁵ Collection and utilization of individual consumers' information by major online marketplaces has been under greater scrutiny than ever before. Compared to any personalized system which inevitably collects and analyzes individual-level data to make its recommendation, the popularity-based system only tracks the transaction outcome – the sales level. As a result, the popularity-based system can completely circumvent the public's concerns over marketplaces' potential violation of individual consumers' privacy.

Despite the advantages of the popularity-based recommendation system and its wide adoption in practice, relatively little is known on its impact from the extant literature. Instead, prior work has largely focused on personalized recommendation systems which is another widely adopted recommendation system in the field. Different from a popularity-based system, a personalized recommendation system collects individual consumers' behavioral data and tries to match each consumer with personally relevant products that can lead to the highest purchase likelihood or generate the greatest profits (Choudhary and Zhang 2019, Zhou and Zou 2022). Field experiments have also shown that personalized recommendation systems can be effective in raising firms' profits across different product categories (Dias et al. 2008, Jannach and Hegelich 2009, Kumar and

⁵https://competition-policy.ec.europa.eu/sectors/ict/dma_en

Hosanagar 2019). In reality, it is likely that many online marketplaces are using a combination of personalized and popularity-based recommendation to improve the system’s overall performance (Amatriain and Basilico 2012, Zhang et al. 2020, Tam and Ho 2006). For instance, Netflix admitted that its personalized recommendation uses “a pretty healthy dose of (non-personalized) popularity information in their ranking method.” (Gomez-Uribe and Hunt 2015). To better understand the impact of the “hybrid” recommendation system on the entire ecosystem, it is helpful to first delineate the effects of the personalized factor and the non-personalized (popularity-based) factor.⁶

Given the importance of the popularity-based system as well as sellers’ price competition in this ecosystem and the gap in the current literature, our paper seeks to address the following research questions.⁷ First, how does the best seller recommendation influence sellers’ competition on the marketplace? Second, how does the best seller recommendation affect the marketplace and the sellers’ profits as well as consumer surplus? Third, when should the marketplace adopt the best seller recommendation?

To answer these questions, we build an analytical model that consists of an online marketplace, multiple sellers and multiple consumers. In particular, sellers offer horizontally differentiated products to compete for consumers over two periods. Sellers will only join the marketplace if their total expected profits across two periods are greater than their reservation profits. In each period, new consumers with unit demand and different outside options enter the marketplace. They search and compare two products in their consideration set, and will purchase the product that generates the highest utility. The marketplace charges sellers a commission on each transaction and may or may not use the best seller recommendation system, which in turn affects the formation of consumers’ consideration set. Specifically, the best seller from the first period automatically enters consumers’ consideration set when the best seller recommendation system is adopted. Otherwise, consumers just randomly search two options. Note that this setup captures the key features of the online marketplace whose recommendation determines a product’s prominence and in turn the competition outcome amongst the sellers.

Comparison between the equilibrium outcome under the best seller recommendation with the outcome without recommendation (equivalent to a random recommendation) leads to several novel insights. First, compared to the benchmark case without any recommendation, the best seller

⁶We focus on the impact of the popularity-based (best seller) recommendation in this paper, and leave the impact of the hybrid recommendation system for future research.

⁷The primary objective of this paper is to develop a tractable model to understand the impact of the best seller recommendation on sellers’ pricing competition and all participants’ payoffs. We do not intend to analyze the optimal algorithm of the recommendation system given a specific objective, which was the focus of a stream of prior literature (e.g., Ricci et al. (2011), Ren et al. (2012), Zuva et al. (2012), Elahi et al. (2016), Yoganarasimhan (2020), Feldman et al. (2021), and Ferreira et al. (2021)).

recommendation intensifies the price competition among sellers and leads to a lower equilibrium price. The intuition is that the best seller recommendation guarantees the seller a spot in consumers' consideration set and thus increases consumers' purchase likelihood (compared to the case where the seller enters consumers' consideration set randomly) in the second period. As a result, competing sellers have incentives to reduce their prices in the first period to boost sales to increase their chance to be the best seller which will be recommended by the marketplace in the second period.

Second, as equilibrium price decreases, adopting the best seller recommendation increases the number of consumers making a purchase on the marketplace and consumer surplus. By contrast, the best seller recommendation has two opposing effects on sellers and the marketplace. On the one hand, a lower equilibrium price hurts sellers' and marketplace's profit margin. On the other hand, the recommendation system induces more customers to make a purchase so sellers have a greater demand, and the marketplace enjoys commission from more transactions. When the best seller recommendation intensifies seller competition so much that the equilibrium price is reduced to below a threshold, using this system decreases each seller's expected profit, the number of active sellers on the marketplace (thus product variety), and the marketplace's expected profits. Managerially, this result means that using the best seller recommendation may backfire for the marketplace compared to the case with no recommendation.

Third, when the price reduction induced by the best seller recommendation is moderate (not too large), each seller's expected profit will be higher, and there will be more active sellers on the marketplace. As a result, the marketplace will also be more profitable. Recall that consumers are overall better off. In other words, utilizing the best seller recommendation can create a win-win-win situation for all three parties. This desirable outcome is more likely to happen in a product category with a low base utility, a high unit cost, a high commission rate, and greater heterogeneity in consumer tastes. In such a product category, sellers would have enough pricing cushion to absorb a more intense price competition induced by the best seller recommendation.

Finally, we consider two extensions where consumers engage in costly search and endogenously determine the size of their consideration set, and the marketplace optimizes its commission rate. We show that the equilibrium prices under the best seller recommendation are still lower than the ones without the recommendation in these situations. In addition, whether the best seller recommendation benefits everyone in the ecosystem still depends on the extent of the price reduction. These results demonstrate the robustness of the main finding on the impact of the best seller recommendation on the entire ecosystem.

The rest of the paper is organized as follows. We first review the relevant literature in Section 2. Then we introduce the model in Section 3. Next, we analyze the equilibrium outcome without the best seller recommendation in Section 4. Section 5 presents the equilibrium under the best

seller recommendation. Section 6 discusses the differences between the cases with and without recommendation. Section 7 presents several extensions. Finally, Section 8 concludes and discusses future research directions. All the proofs are presented in the Appendix.

2. Literature Review

This paper contributes to the literature that studies the impact of personalized product recommendations on the marketplaces (Tam and Ho 2005; Hosanagar et al. 2008; Hagiu and Jullien 2011; Hagiu and Jullien 2014; Yang 2013; Yang and Gao 2017; Ke et al. 2019; Choudhary and Zhang 2019). This stream of work typically abstracts away from the issue of price competition between sellers and its impact on the marketplace’s recommendation outcomes, with the following few exceptions. Teh and Wright (2022) find that if competing sellers can choose prices and the commission paid to the marketplace, recommendations will be steered towards products with higher commissions, which increases the equilibrium commissions and prices. Zhong (2022) study how the marketplace’s search design interacts with its revenue models and show that the effect of product-buyer precision on price is determined by the interplay between competition and incentives to search. Zhou and Zou (2022) find that sellers’ incentive to compete for the recommendation spot pushes prices to the medium levels and under certain conditions the marketplace can be more profitable to exclude products’ pricing information in its recommendation system. While accounting for sellers’ strategic pricing decisions and their incentive to win over the recommendation spot, our work also differs from this stream of prior research in that we focus on the popularity-based recommendation instead of personalized recommendation, which does not raise any privacy concerns.

Our work is related to research on product salience and consumers’ consideration set formation. A stream of empirical work has documented that in-store displays and feature ads in the offline setting can significantly increase a brand’s sales even without price promotions (Gupta 1988, Grover and Srinivasan 1992, Chintagunta 1992, Papatla 1996). Importantly, the rationale behind these empirical patterns is that in-store displays and feature ads can make a brand more prominent, and are used by consumers to form their consideration set (e.g., Hauser and Wernerfelt 1990, Roberts and Lattin 1991, Fader and McAlister 1990, Allenby and Ginter 1995, Andrews and Srinivasan 1995, Bronnenberg and Vanhonerack 1996, Mehta et al. 2003). For example, Zhang et al. (2009) find that the gaze duration on a feature advertisement has a positive and significant effect on the sales of the featured product beyond the mere presence of an ad. In addition, using ACNielsen scanner panel data on single-wrap cheese slices, Zhang (2006) show that over half of the consumers use feature and in-store ad for consideration set formation.

Building on the empirical findings, a stream of theoretical work studies the impact of product prominence/salience on pricing competition. Armstrong et al. (2009) show that a prominent firm

that will be searched first by consumers will charge a lower price than its non-prominent rivals. Armstrong and Zhou (2011) analyze how firms can pay to make their products prominent and how the equilibrium retail pricing is higher than the case of random search. Relatedly, Haan and Moraga-González (2011) show that when consumers first visit the firm whose advertising is most salient, the equilibrium advertising in competition increases as search costs rise. Amaldoss and He (2013) find that prototypicality shapes the composition of consumers' consideration set, and it may or may not lead to a higher price compared to a non-prototypical competitor. Liu and Dukes (2013) show that within-firm evaluation costs and across-firm evaluation costs are different in consumers' consideration set formation process, and both costs affect firms' product line design. Our work contributes to this line of literature by analyzing the impact of a marketplace's decision on the sellers' prominence (recommendation) on their pricing competition. In addition to the new layer of the marketplace, we also note that the best seller recommendation in our framework is organically determined by previous sales and there is no direct payment exchanged between sellers and the marketplace.

Our paper also contributes to the literature on consumer search in online marketplaces. Shulman et al. (2015) show that firms can provide prepurchase information to reduce consumer uncertainty and aid their purchase decision. Dukes and Liu (2016) show that an online shopping intermediary will design its search environment by raising search costs to prevent consumers from evaluating too many sellers while ensuring them to search each product in depth. Wang and Sahin (2018) study the impact of search costs on assortment planning and pricing when customers have uncertainties in their valuation. Derakhshan et al. (2022) show that when consumers sequentially search multiple products, ranking products in decreasing order of their preferences is not optimal for the marketplace. Aouad et al. (2019) assume that customers only consider the items they have clicked on before they proceed to compare their random utilities, and show that this estimation approach outperforms the traditional Multinomial Logit choice model. Chu et al. (2020) show that consumers' sequential search leads to the optimal ranking algorithm in which the objective is a linear combination of consumer surplus and sellers' and the marketplace's revenue. Jiang and Zou (2020) find that the effects of filtering and those of a decrease in search cost are qualitatively different on a retail marketplace. Shi and Raghu (2020) show that when consumers search in the presence of both vertical and horizontal dimensions of product heterogeneity, recommending the high quality product can be sub-optimal. Zou and Zhou (2022) show that search neutrality, which bans the marketplace's self-preferencing when it competes with third-party sellers, may lead to higher prices and thus harm consumers. This stream of papers mostly considers sequential search (e.g., Wolinsky 1986, Anderson and Renault 1999, Gretzel and Fesenmaier 2006, Greminger 2021,

Gao et al. 2022). By contrast, building on Cachon et al. (2008), our paper uses a parallel (simultaneous) search model. Our framework is more applicable to settings where consumers are not familiar with the product category and may need to go back and forth to compare two (or more) options. Our setting also captures a key feature of the online search environment where visiting two sellers simultaneously is feasible.

Finally, our paper is related to the growing literature on the impact of recommendation systems on consumers’ purchase behavior. Fleder and Hosanagar (2009) show that recommendation systems can push customers toward the same products and can create a rich-get-richer effect for popular products. Prawesh and Padmanabhan (2014) show that recommending the N most popular articles is easily susceptible to manipulation but a probabilistic variant is more robust to common manipulation strategies. l’Ecuyer et al. (2017) considers the ranking problem in response to organic search where the ranking tries to balance long-term and short-term revenue. Adomavicius et al. (2018) experimentally show that recommendation can change consumers’ willingness to pay. Gallego et al. (2020) propose a product framing model where consumer choice is influenced by how products are displayed. Zhang et al. (2020) use an agent-based model to study the impact of various factors on the temporal dynamics of recommendation systems. Li et al. (2018) study the impact of a recommendation system that is a weighted combination of expected customer utility and retailer revenue. Furthermore, Li et al. (2020) find that recommendation increases competition and results in lower advertisement by sellers. Our work differs from this stream of literature in two important dimensions. First, unlike most prior studies that consider a static setting, we focus on a dynamic setting where the previous sales endogenously affect the current recommendation outcome. Second, instead of designing the optimal recommendation system based on specific objectives, we take the best seller recommendation strategy (commonly observed in practice) as given, and analyze how it influences sellers’ competition, the marketplace’s revenue, and consumer surplus.

3. Model

In this section, we present our model of the online ecosystem, which consists of the marketplace, sellers, and consumers. On the online retail marketplace, multiple competing sellers in the same product category interact with consumers over two periods, $t = 1, 2$. Below, we detail the specific assumptions and settings, starting with the sellers.

3.1. Sellers

There are $M > 1$ potential sellers who offer differentiated products in the same product category. Each seller has one product under his brand (To facilitate exposition, we use he/him to denote a seller and she/her to denote a consumer.). The per unit production cost is assumed to be $c > 0$, and it is homogeneous across all products from all sellers. Seller $j \in \{1, \dots, M\}$ has a reservation

profit $\pi_{0j} \geq 0$, which is uniformly distributed over $[0, \pi_0]$. If a seller decides to join the marketplace, he will remain on the marketplace in both periods. Therefore, seller j will join the marketplace if and only if his expected total profit from selling on the marketplace across the two periods is higher than his reservation profit π_{0j} .⁸ Let $m \leq M$ denote the actual number of sellers joining the marketplace. Once seller $j \in \{1, \dots, M\}$ joins the marketplace, he sets the price of his product, $p_j > c$, which remains unchanged throughout the two periods. After transactions with consumers complete, each seller needs to pay the marketplace a fixed percentage of commission fee (more on this later). Sellers are risk neutral and they maximize their total expected profits over the two periods.

3.2. Consumers

At the beginning of each period $t = 1, 2$, $N_t > 1$ new consumers enter the marketplace and consider making a purchase in the focal product category. These N_1 and N_2 consumers are two different groups of consumers, each of them will make at most one purchase in period 1 and period 2, respectively.⁹ To simplify exposition, we consider the case where $N_1 = N_2 = N$. If consumer i decides not to make a purchase in this category on the marketplace, she will take the outside option with a utility of U_{i0} which is uniformly distributed over $[0, u_0]$. Therefore, consumer i will make a purchase if and only if her expected utility from buying a product is higher than U_{i0} .

If a consumer decides to make a purchase in the focal product category, we assume that she will only search and compare two products listed by two sellers on the marketplace. Then she will buy the product that generates the highest utility. This assumption of searching and comparing only two products is consistent with the notion that the size of consumers' consideration set is limited due to costly information processing (e.g., Chakravarti and Janiszewski 2003, Chen and Riordan 2007, Lleras et al. 2017). Furthermore, we will relax this assumption in Section 7.2 and show that our main results remain unchanged.

The utility consumer i obtains from purchasing the product sold by seller j is given by

$$U_{ij} = u - p_j + \epsilon_{ij},$$

where u represents the baseline utility from any product in this product category, p_j is the price for product j , and ϵ_{ij} captures the (random) fit or match value that consumer i draws for product j . Note that ϵ_{ij} for consumer i is realized after she finishes searching product j in her consideration

⁸An alternative interpretation of π_{0j} is that this is seller j 's fixed costs of entry. As a result, seller j only joins the marketplace if his expected total profit exceeds the entry cost, and upon entry, he is incentivized to stay on the marketplace across both periods of the game.

⁹In theory, our model also applies to the situation where some consumers are the same across the two periods, as long as their preferences are uncorrelated over time. We focus on "new consumers" in each period to ease exposition.

set. To simplify the analysis, we assume that ϵ_{ij} is a random variable that follows an i.i.d. zero-mean Gumbel distribution (McFadden et al. 1973) with the following CDF:

$$F(x) = \exp[-\exp(-(x/\mu + \gamma))],$$

where μ is a scale parameter and γ is Euler’s constant. The scale parameter μ can be viewed as a measure of consumer heterogeneity in terms of their preferences over the products under consideration. The higher the value of μ is, the more heterogeneous consumers’ preferences are. Because the baseline utility u is identical across all products, if all the products are identically priced, they are equally likely to be preferred by a consumer, i.e., they are ex ante symmetric.

When consumer i searches and compares two products j and k ($j \neq k$) listed on the marketplace, the probability that this consumer purchases product j , i.e., product j generates a higher utility than product k , $q_j(p_j, p_k)$ is given by the well-known multi-nomial logit (MNL) formula,

$$q_j(p_j, p_k) = \mathbb{P}(U_{ij} \geq U_{ik}) = \frac{\exp((u - p_j)/\mu)}{\exp((u - p_j)/\mu) + \exp((u - p_k)/\mu)}.$$

Furthermore, according to the properties of the Gumbel distribution and the MNL model (p. 60, Anderson et al. 1992), the expected utility for consumer i from considering products j and k is

$$U_i = \mathbb{E}[\max\{U_{ij}, U_{ik}\}] = \mu \ln [\exp((u - p_j)/\mu) + \exp((u - p_k)/\mu)]. \quad (1)$$

3.3. The Marketplace

To focus on the sellers’ competition and the impact of the best seller recommendation, we assume that the online marketplace charges sellers a commission rate $0 < \ell < 1$ (exogenously given) on the revenue from each transaction between sellers and consumers (In an extension in Section 7.1, we will analyze the case when the marketplace endogenously sets its commission rate). The marketplace may or may not recommend products to consumers to influence what products they would search. After observing the marketplace’s decision on whether to use a recommendation system, sellers and consumers make their entry and purchase decisions conditional on their rational expectations on the equilibrium outcome on the marketplace. For sellers who decide to join the marketplace, they choose the prices to maximize their expected profits. For consumers with a relatively undesirable outside option, they will search and make a purchase decision to maximize their expected utilities. Finally, given that sellers are ex ante symmetric, we focus on the symmetric interior equilibrium throughout the paper.

4. The Case Without Recommendation

In this section, we analyze the benchmark case where the marketplace does not use any recommendation system. As a result, each consumer who decides to purchase will randomly select two listed products to search and compare. The sequence of events are illustrated in Figure 2. Note that to focus on the impact of the best seller recommendation (or lack thereof), we assume that consumers engage in parallel/simultaneous search on the two alternatives within their consideration sets. We use superscript “ o ” to denote equilibrium in the benchmark case.

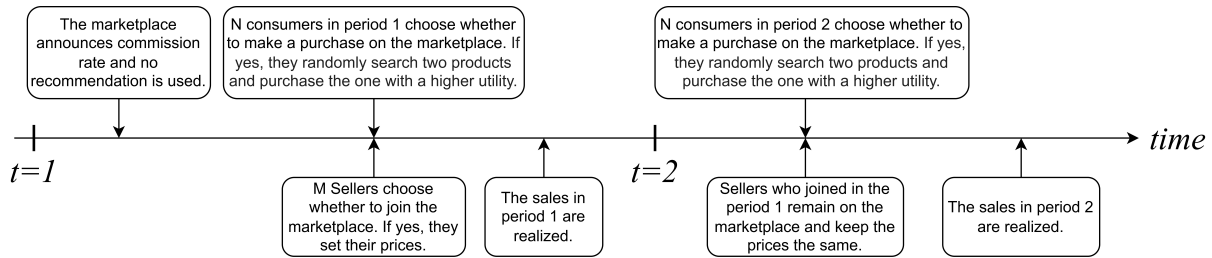


Figure 2 Timeline of the benchmark model without recommendation

We first consider the consumers' decisions. All consumers in each period expect a symmetric equilibrium on the marketplace where all products are priced at p^o . Therefore, substituting p^o into Equation (1), we obtain the expected utility for consumer i to purchase a product in her consideration set in period 1 as follows

$$U_i = u + \mu \ln 2 - p^o. \quad (2)$$

Comparing her expected utility from the purchase with her outside option, consumer i will make a purchase if and only if $U_i \geq U_{i0}$. Further, consumers in period 2 expect the same symmetric equilibrium outcome because the same set of sellers on the marketplace from period 1 remain and they will charge the same prices. Therefore, the expected number of customers who will make a purchase in period 2 remains the same as that in period 1, i.e., $n_1^o = n_2^o = n^o$. Essentially, the outcomes across the two periods are identical in this benchmark case without recommendations. In aggregate, the expected number of consumers who will make a purchase (i.e., the expected demand) in each period is given by

$$n^o = N \frac{u + \mu \ln 2 - p^o}{u_0}. \quad (3)$$

Next, we consider the sellers' decisions. We will take seller k 's perspective. Seller k expects a symmetric equilibrium where m^o sellers who join the marketplace including seller k himself compete for the n^o active consumers in each period, and all the other sellers charge the symmetric equilibrium price p^o . To ensure the existence of the symmetric equilibrium, we next examine if

seller k can do better by deviating from this symmetric equilibrium and setting a price $p_k \neq p^o$. The expected number of consumers who will purchase his product (i.e., his expected demand) in period $t = 1, 2$, $\mathbb{E}[D_{k,t}(p_k, p^o, n^o, m^o)]$, is given by

$$\mathbb{E}[D_{k,t}(p_k, p^o, n^o, m^o)] = n^o \left(\frac{2}{m^o} \right) \frac{\exp((u - p_k)/\mu)}{\exp((u - p_k)/\mu) + \exp((u - p^o)/\mu)}. \quad (4)$$

The first term n^o in this demand function is the equilibrium number of consumers making a purchase on the marketplace. The second term, $\frac{2}{m^o}$, is the probability that a consumer includes seller k in her search. This probability is calculated as $1/m^o + (1 - 1/m^o)(1/(m^o - 1)) = \frac{2}{m^o}$, where $1/m^o$ is the probability that seller k is selected first, and $(1 - 1/m^o)(1/(m^o - 1))$ is the probability that he is selected second, because the consumer randomly selects two out of m^o products. The last term, $\exp((u - p_k)/\mu)/[\exp((u - p_k)/\mu) + \exp((u - p^o)/\mu)]$, is the probability that a consumer purchases seller k 's product conditional on her searching seller k . Similar to the discussion on consumers, seller k 's expected demand across the two periods is also identical, i.e., $\mathbb{E}[D_{k,1}(p_k, p^o, n^o, m^o)] = \mathbb{E}[D_{k,2}(p_k, p^o, n^o, m^o)]$.

Note that seller k 's profit margin after accounting for the commission and the marginal cost is $(1 - \ell)p_k - c$. Thus, the expected profit for seller k , $\pi_k(p_k, p^o, n^o, m^o)$, can be written as

$$\pi_k(p_k, p^o, n^o, m^o) = ((1 - \ell)p_k - c) (\mathbb{E}[D_{k,1}|p_k, p^o, n^o, m^o] + \mathbb{E}[D_{k,2}|p_k, p^o, n^o, m^o]).$$

Seller k 's optimal price, $p_k^o(p^o, m^o)$, solves the following optimization problem,

$$p_k^o(p^o, n^o, m^o) = \arg \max_{p_k \geq c} \pi_k(p_k, p^o, n^o, m^o).$$

In a symmetric equilibrium, we have

$$p_k^o(p^o, n^o, m^o) = p^o. \quad (5)$$

In other words, the above equation ensures that the symmetric equilibrium indeed arises in this setting. Note that in the symmetric equilibrium, we observe $\mathbb{E}[D_{k,1}(p^o, n^o, m^o)] = \mathbb{E}[D_{k,2}(p^o, n^o, m^o)] = n^o/m^o$. The total expected profit for each seller in the symmetric equilibrium is thus

$$\pi^o(p^o, n^o, m^o) = 2 \left((1 - \ell)p^o - c \right) \frac{n^o}{m^o}.$$

Seller k would join the marketplace if his total expected equilibrium profit is higher than his reservation profit, i.e., if $\pi^o(p^o, n^o, m^o) \geq \pi_{0k}$. Therefore, the equilibrium number of sellers who will join the marketplace is given by

$$m^o = M \frac{\pi^o(p^o, n^o, m^o)}{\pi_0}. \quad (6)$$

The symmetric equilibrium in the benchmark case is a 3-tuple $\{p^o, n^o, m^o\}$ that satisfies Equations (3), (5) and (6) simultaneously. The following proposition characterizes this symmetric equilibrium.

Proposition 1. *Without any recommendation, there exists a unique (interior) symmetric equilibrium, $\{p^o, n^o, m^o\}$ that is defined as,*

$$p^o = \frac{c}{1-\ell} + 2\mu, \quad (7)$$

$$n^o = \frac{N}{u_0} \left(u - (2 - \ln 2)\mu - \frac{c}{1-\ell} \right), \quad (8)$$

$$m^o = \sqrt{\frac{4MN\mu(1-\ell) \left(u - (2 - \ln 2)\mu - \frac{c}{1-\ell} \right)}{\pi_0 u_0}}.$$

The expected equilibrium profits for each seller joining the marketplace and the marketplace itself are respectively

$$\pi^o = 4\mu(1-\ell)\frac{n^o}{m^o}, \quad \text{and} \quad \Pi^o = 2\ell p^o n^o.$$

The equilibrium price p^o follows a simple structure as the sum of the adjusted unit cost (by the commission rate), $c/(1-\ell)$, and a mark-up of 2μ . Essentially, the sellers would pass part of the commission charged by the marketplace to consumers. The mark-up is proportional to the extent of consumers' heterogeneity in their preferences over all competing sellers' products, μ . In other words, sellers enjoy a higher mark-up in equilibrium when consumers view their products to be more differentiated.

Interestingly, the equilibrium price p^o is independent of the number of active sellers on the marketplace, m^o , and the number of purchasing customers, n^o . This is because regardless of the total number of sellers on the marketplace, competition is limited between the only two sellers in any given consumer's consideration set (Recall that each consumer only searches two products in her consideration set). Similarly, the total consumer demand n^o is decreasing in the price of the product p^o , but is independent of the number of active sellers m^o . As a result, in the benchmark case without any recommendation, the marketplace's expected profit, Π^o , is independent of the participating sellers m^o too. Finally and intuitively, more consumers will attract more sellers onto the marketplace, i.e., m^o is increasing in n^o (see Equation (6)). Later on, we will see how some of these properties in this equilibrium would change if the best seller recommendation is implemented by the marketplace.

5. The Best Seller Recommendation

In this section, we consider the case in which the marketplace implements the best seller recommendation system. In our context, the best seller recommendation is based on the unit sales in the first period. Specifically, once the first period is over, the marketplace will identify the “Best Seller” which had the highest number of sales in this period. At the beginning of the second period, the marketplace will then recommend the “Best Seller” product to all consumers. Given the salience

of the “Best Seller” badge on a web page (as well as the ex ante symmetry amongst all competing sellers), it is much easier for the product with that badge to enter consumers’ consideration set. Translating the power of the “Best Seller” label into our framework, we assume that all consumers on the marketplace in the second period will automatically search the “Best Seller,” along with another randomly selected product. From a seller’s perspective, being the best seller in the first period increases his probability of being considered and searched by consumers in the second period compared to that of the previous period.

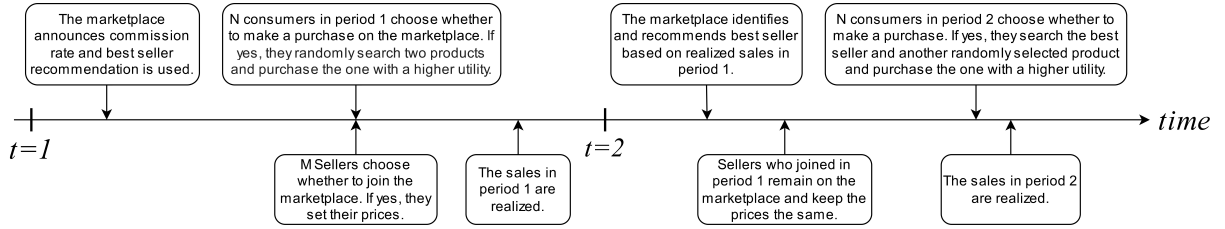


Figure 3 Timeline of the model with the best seller recommendation

To delineate the impact of the best seller recommendation, we consider the case with recommendation while maintaining most of the assumptions in the benchmark case. In particular, we continue to assume that once a seller joins the marketplace and sets a price, he will remain on the marketplace with the same price throughout the two periods. The sequence of events with the best seller recommendation is illustrated in Figure 3. It is easy to see that the first period problem is identical to the one in the benchmark case, whereas the second period problem fundamentally changes as one seller is identified and recommended as the “Best Seller.” We next derive the symmetric equilibrium of the model with the best seller recommendation which is again a 3-tuple $\{p^*, n^*, m^*\}$.

All consumers expect a symmetric equilibrium on the marketplace where all competing products are priced at p^* . The first period consumers’ decisions in this case with the best seller recommendation exhibit the identical structure to the one in the benchmark model as the best seller recommendation only affects the second period outcomes. In the second period, consumers who decide to make a purchase will search the best seller product along with another randomly selected product. However, because the same set of sellers remain on the marketplace and keep their prices unchanged from period 1, the second period consumers’ decisions also display the same structure as that in the benchmark case. Therefore, following a similar process of deriving Equation (3) in the benchmark model, the expected number of consumers who will make a purchase (i.e., consumer demand) in each period in the presence of the best seller recommendation, n^* , is given below

$$n^* = N \frac{u + \mu \ln 2 - p^*}{u_0}. \quad (9)$$

We now consider how the sellers' decisions change when the best seller recommendation is utilized. As before, we take the perspective of seller k who expects a symmetric equilibrium where m^* sellers who joined the marketplace including seller k himself compete for n^* active consumers. In addition, all the other sellers charge the symmetric equilibrium price p^* . To ensure the existence of the symmetric equilibrium, we investigate if seller k has incentives to deviate from the symmetric equilibrium pricing. Suppose that seller k deviates from the symmetric equilibrium price by setting a price $p_k \neq p^*$, his expected demand in period 1 will be:

$$\mathbb{E}[D_{k,1}(p_k, p^*, n^*, m^*)] = n^* \left(\frac{2}{m^*} \right) \frac{\exp((u - p_k)/\mu)}{\exp((u - p_k)/\mu) + \exp((u - p^*)/\mu)}.$$

This demand function has the same structure as his expected demand in period 1 in the benchmark model given in Equation (4).

Once the first period concludes, the marketplace observes the realized sales of each seller, and then identifies and recommends the best seller in the second period. Therefore, a seller's expected profit in the second period critically depends on whether he is the best seller or not. Subsequently, we analyze the probability that seller k is the best seller, which is calculated based on the realized sales of all sellers in period 1.

Recall that in period 1, each of the n^* consumers considers and searches two randomly selected products, and then purchases one of them. The probability that a consumer purchases product k (with price p_k) is

$$g_k = \left(\frac{2}{m^*} \right) \frac{\exp((u - p_k)/\mu)}{\exp((u - p_k)/\mu) + \exp((u - p^*)/\mu)}, \quad (10)$$

and the probability that the consumer purchases one of the $(m^* - 1)$ products (with price p^*) other than product k is

$$g = \frac{1 - g_k}{m^* - 1}.$$

As a result, the sales of the m^* sellers in period 1, $D_{j,1}^*$, $j \in \{1, \dots, m^*\}$, are random variables that follow a multinomial distribution with the parameters (n^*, g_k, g, \dots, g) . Their probability distribution function is given by

$$\mathbb{P}(D_{1,1}^* = y_1, \dots, D_{k,1}^* = y_k, \dots, D_{m^*,1}^* = y_{m^*}) = \frac{n^*!}{y_1! \dots y_k! \dots y_{m^*}!} g_k^{y_k} g^{n^* - y_k}.$$

Let $Q_k(p_k, p^*, n^*, m^*) = \mathbb{P}(D_{k,1}^* \geq \max_{1 \leq j \leq m^*} D_{j,1}^*)$ denote the probability that seller k is the best seller, i.e., the realized sale of seller k 's product in period 1 is the highest among all m^* sellers. Based on the above distribution function of the sales, we derive $Q_k(p_k, p^*, n^*, m^*)$ in the appendix.

To facilitate exposition, we abbreviate $Q_k(p_k, p^*, n^*, m^*)$ to Q_k . Then, the expected demand for seller k in period 2 is given by

$$\mathbb{E}[D_{k,2}(p_k, p^*, n^*, m^*)] = n^* \left(Q_k + (1 - Q_k) \left(\frac{1}{m^* - 1} \right) \right) \frac{\exp((u - p_k)/\mu)}{\exp((u - p_k)/\mu) + \exp((u - p^*)/\mu)},$$

where the term $Q_k + (1 - Q_k)(1/(m^* - 1))$ is the probability that seller k will be searched by a consumer in period 2. With probability Q_k , seller k is the best seller in period 1, and he will be searched in period 2 with probability 1. With probability $(1 - Q_k)$, seller k fails to be the best seller in period 1, and he will be searched in period 2 by a consumer with probability $1/(m^* - 1)$.

The total expected profit of seller k over the two periods is

$$\pi_k((p_k, p^*, n^*, m^*)) = ((1 - \ell)p_k - c) (\mathbb{E}[D_{k,1}(p_k, p^*, n^*, m^*)] + \mathbb{E}[D_{k,2}(p_k, p^*, n^*, m^*)]).$$

In this case, seller k chooses its price $p_k^*(p^*, n^*, m^*)$ to maximize his expected profit:

$$p_k^*(p^*, n^*, m^*) = \arg \max_{p_k \geq c} \pi_k(p_k, p^*, n^*, m^*).$$

In a symmetric equilibrium, the following equation holds

$$p_k^*(p^*, n^*, m^*) = p^*. \quad (11)$$

Substituting the above equation into the expected demand in both periods, we have $\mathbb{E}[D_{k,1}(p^*, p^*, n^*, m^*)] = \mathbb{E}[D_{k,2}(p^*, p^*, n^*, m^*)] = n^*/m^*$ in the symmetric equilibrium. Consequently, the total expected profit for each seller in equilibrium is given by

$$\pi^*(p^*, n^*, m^*) = 2((1 - \ell)p^* - c) \frac{n^*}{m^*}. \quad (12)$$

Although the above equilibrium profit for a seller $\pi^*(p^*, n^*, m^*)$ seems to have a similar structure as the one in the benchmark case, we will show later that the equilibrium prices are fundamentally different across the two cases, which directly reflects the impact of the best seller recommendation.

Knowing his expected profits upon entry, seller k would only join the marketplace if and only if his expected equilibrium profit is higher than his reservation profit, i.e., when $\pi^*(p^*, n^*, m^*) \geq \pi_{0k}$. Therefore, the equilibrium number of sellers who will join the marketplace is given by

$$m^* = M \frac{\pi^*(p^*, n^*, m^*)}{\pi_0}. \quad (13)$$

To summarize, the symmetric equilibrium in the case with the best seller recommendation, $\{p^*, n^*, m^*\}$, satisfy Equations (9), (11) and (13) simultaneously. The following proposition characterizes this symmetric equilibrium.

Proposition 2. *With the best seller recommendation, there exists a (interior) symmetric equilibrium, $\{p^*, n^*, m^*\}$, which are defined by the following equations:*

$$p^* = \frac{c}{1 - \ell} + \frac{2\mu}{1 + T(n^*, m^*)}, \quad (14)$$

$$n^* = N \frac{u + \mu \ln 2 - p^*}{u_0},$$

$$m^* = \sqrt{\frac{2M(1-\ell)}{\pi_0} \left(p^* - \frac{c}{1-\ell} \right) n^*},$$

where $T(n^*, m^*) > 0$ is defined in the appendix. The expected equilibrium profits for each seller joining the marketplace and the marketplace itself are respectively

$$\pi^* = 4\mu(1-\ell) \frac{n^*}{m^*} \left(\frac{1}{1+T(n^*, m^*)} \right), \quad \text{and} \quad \Pi^* = 2\ell p^* n^*.$$

The equilibrium price under the best seller recommendation, p^* , still follows the same simple structure as the sum of the commission adjusted unit cost $c/(1-\ell)$, and a mark-up of $2\mu/(1+T(n^*, m^*))$. However, unlike the equilibrium price p^o in the benchmark case, p^* is not independent of the numbers of consumers and sellers on the marketplace, n^* and m^* , anymore because the mark-up depends on the function $T(n^*, m^*)$. In the presence of the best seller recommendation, the competition is no longer limited within the two sellers in her consideration set a consumer searches. Instead, all sellers on the marketplace will compete in the first period to become the best seller to gain an advantage in the second period. The function $T(n^*, m^*)$ in the mark-up of p^* reflects this intensified competition among sellers caused by the best seller recommendation. This subtle change under the best seller recommendation will cascade into every part of the ecosystem as we can see from the equilibrium characterized in Proposition 2.

6. The Impact of the Best Seller Recommendation

In Sections 4 and 5, we have analyzed the equilibrium outcomes with and without the best seller recommendation, respectively. In this section, we compare the equilibrium outcomes between the two cases to reveal the impact of the best seller recommendation on the entire ecosystem. First, the following proposition compares the equilibrium prices in the two cases.

Proposition 3. *Compared to the benchmark without recommendation, the best seller recommendation intensifies the price competition among sellers and leads to a lower equilibrium price, i.e., $p^* < p^o$.*

Comparing Equations (7) and (14), we can clearly see that $p^* < p^o$ because the extra term $T(n^*, m^*)$ in the denominator of the mark-up of p^* is strictly positive. As we discussed before, $T(n^*, m^*)$ reflects the strengthened competition among sellers under the best seller recommendation. To increase the chances to be the best seller and thus guarantee its presence in consumers' consideration set in period 2, sellers have to reduce their prices to try to increase unit sales in

period 1. Thus, this incentive induced by the best seller recommendation intensifies sellers' price competition, which leads to a lower equilibrium price. A natural question will be how the reduced equilibrium price affects consumers, sellers, and the marketplace itself. The following proposition addresses this question.

Proposition 4. *Compared to the benchmark without recommendation, in the equilibrium with the best seller recommendation,*

- (i) *the number of consumers making a purchase on the marketplace is higher: $n^* > n^o$.*
- (ii) *if $p^o \leq u + \mu \left(\ln 2 - \frac{2}{1+T(n^*, m^*)} \right)$, each seller's expected profit and the number of sellers on the marketplace are lower: $\pi^* \leq \pi^o$ and $m^* \leq m^o$. Otherwise, they are higher.*
- (iii) *if $p^o < u + \mu \left(\ln 2 - \frac{2}{1+T(n^*, m^*)} \right) - \frac{c}{1-\ell}$, the best seller recommendation reduces the marketplace's expected profit, $\Pi^* < \Pi^o$. Otherwise, it increases the marketplace's expected profit.*

Consumers clearly benefit from the lower equilibrium price under the best seller recommendation. As a result, part (i) of the Proposition 4 states that more consumers will make a purchase on the marketplace when the best seller recommendation is implemented. In other words, the best seller recommendation enhances the overall demand on the marketplace, i.e., $n^* > n^o$.

By contrast, the story for the sellers is more ambiguous. On the one hand, the demand-enhancing effect of the best seller recommendation makes the marketplace more attractive to sellers. On the other hand, all sales will have to be made at a lower price due to the intensified price competition. The net effect of the best seller recommendation for the sellers is not clear-cut and it depends on which one of the two effects is stronger.

According to part (ii) of Proposition 4, if the original equilibrium price in the benchmark case, p^o is already sufficiently low (e.g., possibly due to a low level of differentiation in the focal product category μ), to further intensify the price competition by introducing the best seller recommendation will hurt the equilibrium profit for sellers and discourage them from joining the marketplace. With a low enough equilibrium price, many consumers would have already made a purchase on the marketplace. In other words, relatively few consumers are inactive and will choose the no-buy outside option. This implies that the demand-enhancing effect of the best seller recommendation is limited. In this case, the best seller recommendation will reduce the already low equilibrium price to an even lower level where the competition-intensifying effect would dominate its demand-enhancing effect. As a result, sellers' profits drop and the number of sellers entering the marketplace decreases too. However, if the original equilibrium price in the benchmark case, p^o is relatively high, there exists a significant number of inactive consumers who can potentially be attracted to make a purchase if the equilibrium price becomes lower. The demand-enhancing effect of the best seller recommendation is strong. In this case, there is room for sellers to cut prices without hurting

their profits so that introducing the best seller recommendation can improve seller profits and thus attract more sellers to join the marketplace.

Note that the marketplace collects a commission on the revenue of each unit of sales. Hence, the marketplace faces the same trade-off: a lower price and a higher volume under the best seller recommendation, and a higher price and a lower volume without recommendation. In aggregate, the marketplace may not benefit from implementing the best seller recommendation. Part (iii) of Proposition 4 indeed confirms that blindly implementing the widely used best seller recommendation can backfire for the marketplace, especially when the equilibrium price in the focal product category is already sufficiently low. Finally, a comparison between the condition in part (iii) with that in part (ii) leads to the following Corollary.

Corollary 1. *Compared to the sellers, the marketplace is more likely to benefit from the best seller recommendation system.*

Corollary 1 states that the marketplace is more likely to be more profitable with the recommendation than without the recommendation compared to the sellers. In other words, the marketplace can sustain the pressure from a lower equilibrium price better than the sellers as the former does not incur a marginal cost on each unit of sales whereas the latter do. Managerially, this result implies that the incentives of the sellers and the marketplace are not always perfectly aligned. When the latter actively promotes a new recommendation system, it does not necessarily benefit the former group.

Given its popularity, one important question is whether the best seller recommendation can benefit all participants on the marketplace. The following proposition answers this question and reveals when a win-win-win outcome can occur.

Proposition 5. *Compared to the benchmark,*

(i) *only when $p^o > u + \mu \left(\ln 2 - \frac{2}{1+T(n^*, m^*)} \right)$, the best seller recommendation benefits everyone in the ecosystem: $n^* > n^o$, $\pi^* > \pi^o$, $m^* > m^o$, and $\Pi^* > \Pi^o$.*

(ii) *the best seller recommendation is more likely to benefit the whole ecosystem in a product category with a low base utility u , a high unit cost c , a high commission rate ℓ , and/or greater heterogeneity in consumer tastes μ .*

Recall from Proposition 4 that the sellers are the most vulnerable party on the marketplace who are most likely to be hurt by the best seller recommendation. Part (i) of Proposition 5 states that only when the sellers are better off, implementing the best seller recommendation can achieve a win-win-win outcome to make all parties in the ecosystem better off. Surprisingly, in this case, more intense pricing competition amongst sellers can improve their payoffs. This result echoes the finding

of Jain and Qian (2021) where more intense competition can lead to improved profits for sellers on a marketplace. Part (ii) of Proposition 5 suggests a win-win-win is more likely to happen in a product category with a low base utility, or a high unit cost, a high commission rate, and greater heterogeneity in consumer tastes. In such a product category, sellers would have enough pricing cushion to absorb the price competition induced by the best seller recommendation. For example, hand-made fashion items such as designer bags and jewels fit with this criterion, which is consistent with the context at Etsy; on the other hand, a wide selection of products sold on WalMart.com probably falls into the other end of the spectrum. Similarly, consumer package goods/staples such as toothpastes and toothbrushes offer relatively high base utilities and low unit production costs. As a result, a win-win-win is less likely to occur there.

Existing literature on recommendation systems has mostly focused on how they attract more consumers to marketplaces without paying much attention to sellers (e.g., Abdollahpouri et al. 2019). Propositions 4 and 5 collectively show that sellers’ payoffs are significantly influenced by the marketplace’s recommendation decision. Although we caution that our results apply to the best seller recommendation, and they may not be readily generalized to other recommendation systems, our results indeed indicate that sellers are actually a key part of the equation in determining the overall effect of a recommendation system on the whole ecosystem of a marketplace. It is crucial for a marketplace to make sure the sellers’ interests and their strategic responses are taken into account when considering a recommendation system.

7. Extensions

In the main model, we made a few simplifying assumptions to focus on the impact of the best seller recommendation on sellers’ competition. Specifically, we assumed that the marketplace’s commission rate was exogenously given, and the size of consumers’ consideration set was two (so consumers only searched two alternatives before purchase). In this section, we relax these assumptions, and analyze the cases of endogenized commission rate and endogenized consideration set formation respectively.

7.1. Endogenous Commission Rate

We now consider the case where the marketplace can optimize its commission rate. Given the complexities of the choice probabilities, especially in the case with the best seller recommendation, analytical tractability is limited. Therefore, we rely on numerical studies to investigate the equilibrium outcomes of the models under endogenous commission rates.

To improve computational efficiency, we approximate the $T(n^*, m^*)$ function in the model with the best seller recommendation by a continuous function specified in Appendix B. In the numerical studies, we limit the commission rate that the marketplace can charge to be no more than 50% to

be consistent with practice in reality (e.g., the average commission rate is about 30% on Apple App Store, 15% on Amazon, and 10% on eBay). We observe some interesting patterns in the numerical analysis, and present a representative example in Figure 4.

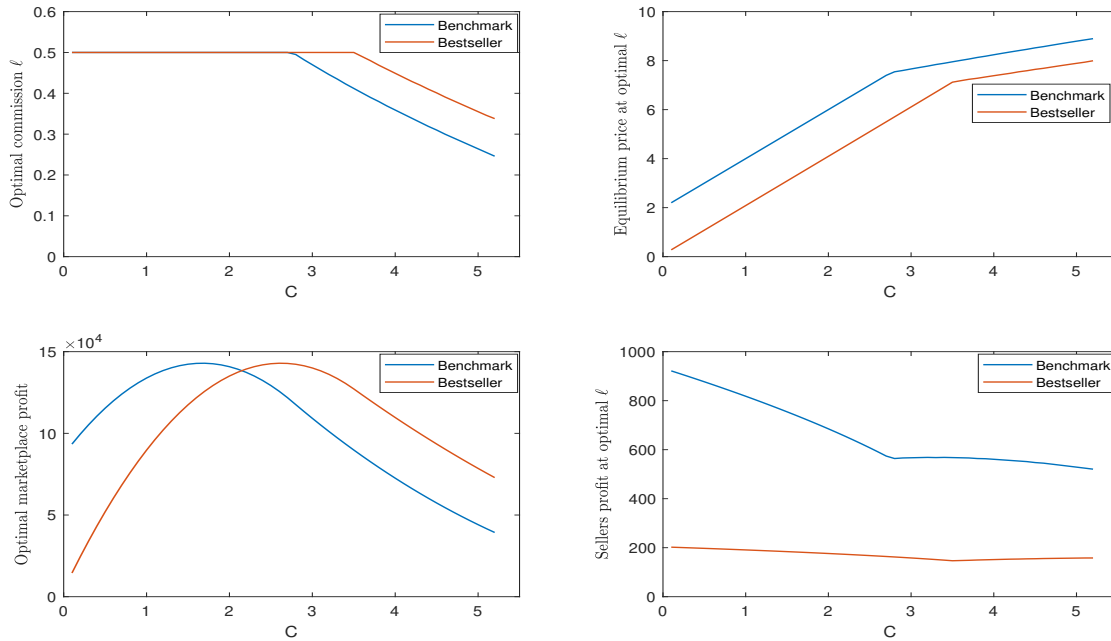


Figure 4 Comparisons of the optimal commission rates, equilibrium prices, marketplace profits and seller profits without recommendation and with the best seller recommendation ($u = 10, \mu = 1, N = 10^5, M = 200, u_0 = 20$, and $\pi_0 = 2000$).

Figure 4 shows how the marketplace’s optimal commission rate and expected profit, and the equilibrium price and seller profit change with the unit production cost, c , for a given set of parameters, and compare them between the cases with and without the best seller recommendation.

Regarding the marketplace’s optimal commission rate, we have two observations. First, the marketplace’s optimal commission rates are decreasing in the unit production cost of the product c in both cases. Second, the marketplace’s optimal commission rate is higher in the case with the best seller recommendation. As we discussed before, implementing the best seller recommendation intensifies the price competition, resulting in lower equilibrium prices which could hurt the marketplace’s profit. To at least partially neutralize this effect, the marketplace would raise the commission rate to incentivize the sellers not to drop their prices too much.

However, as we can see from the figure, even with a higher commission rate, the equilibrium price in the case with the best seller recommendation is still lower than the one in the benchmark without recommendation. Therefore, the competition-intensifying effect of the best seller recommendation

is robust. Optimizing the commission rate might weaken this effect, but does not eliminate it completely. As a result, the sellers' equilibrium profit is lower when the best seller recommendation is implemented in the example shown in Figure 4.¹⁰ Furthermore, implementing the best seller recommendation can hurt the marketplace's profit, especially when the unit production cost c is relatively low, even when the marketplace can optimize its commission rate. All these observations are consistent with the analytical results obtained in the main model with an exogenous commission rate.

7.2. Endogenous Consideration Set Formation for Consumers

In this subsection, we relax the assumption that consumers' consideration set only consists of two products. We analyze the case where consumers' consideration set is endogenously formed to maximize their expected utilities. In other words, consumers will endogenously choose the optimal number of sellers to include in their consideration sets given their expectations on the equilibrium.

We develop a consideration set formation model similar to the one used by Cachon et al. (2008) and Liu and Dukes (2013). A consumer incurs a search cost of τ for each seller that is included in her consideration set. If a consumer decides to make a purchase, she chooses the number of sellers to include in her consideration set, r , to maximize her expected utility. The total search cost for the consumer with r sellers in her consideration set will be $r\tau$. The consumer will purchase the product from the seller in the consideration set that generates the highest utility to her. According to the properties of the MNL model, the expected utility of including r sellers who price their products at p is given by

$$U(r) = u - p + \mu \ln r - r\tau.$$

The consumer chooses r to maximize the above expected utility given her expectation about the prices set by the sellers on the marketplace. Anticipating the optimal size of consumers' consideration set, sellers strategically set their prices. The following proposition characterizes the optimal size of consumers' consideration set, the equilibrium prices in the cases with and without the best seller recommendation, and the condition under which the best seller recommendation creates a win-win-win situation for all three parties.

Proposition 6. *(i) Regardless of whether the best seller recommendation is implemented, the optimal number of sellers to include in a consumer's consideration set is*

$$r^* = \frac{\mu}{\tau},$$

which is independent of the equilibrium price set by the sellers.

¹⁰There exist cases where sellers are better off as well when the best seller recommendation is implemented, for example, with $u = 10$, $\mu = 4$, $N = 10^5$, $M = 200$, $u_0 = 20$, $\pi_0 = 4000$, and $c = 4$.

(ii) The equilibrium prices in the benchmark without recommendation and the case with the best seller recommendation are

$$p^o = \frac{c}{1-\ell} + \frac{r^*\mu}{r^*-1}, \quad (15)$$

and

$$p^* = \frac{c}{1-\ell} + \frac{r^*\mu}{(r^*-1)(1+T(n^*, m^*, r^*))}, \quad (16)$$

respectively, where $T(n^*, m^*, r^*) > 0$, $T(n^*, m^*, m^*) = 0$ and $T(n^*, m^*, 2) = T(n^*, m^*)$.

(iii) Only when $p^o > u + \mu \left(\ln r^* - \frac{r^*}{(r^*-1)(1+T(n^*, m^*, r^*))} \right)$, the best seller recommendation benefits everyone in the ecosystem: $n^* > n^o$, $\pi^* > \pi^o$, $m^* > m^o$, and $\Pi^* > \Pi^o$.

According to part (i) of Proposition 6, consumers will form a consideration set with a fixed number of μ/τ sellers, regardless of whether the best seller recommendation is implemented, as long as they expect a symmetric equilibrium emerging on the marketplace. The size of consumers' consideration set is increasing in the heterogeneity in consumer tastes, μ , and decreasing in the search cost, τ . Intuitively, consumers want to consider and search more products when they have more heterogeneous preferences, because they value the best match within their consideration set more (technically, the expected maximum utility of the best-matched product within the consideration set is $\mu \ln r^*$). In addition, they want to search more alternatives when the cost of search is lower.

Note that when $r^* = 2$, $T(n^*, m^*, 2) = T(n^*, m^*)$, so the equilibrium prices given in part (ii) of the above proposition is the same as the prices given in Equations (7) and (14) when consumers only search two sellers. Furthermore, we can clearly see from Equations (15) and (16) that $p^* < p^o$ continues to hold as $T(n^*, m^*, r^*) > 0$. In other words, the competition-intensifying effect of the best seller recommendation still exists, even when consumers endogenously optimize the size of their consideration set. As a result, all the other results we presented when consumers consider only two firms in the previous sections continue to hold qualitatively when they optimize the size of their consideration set. In particular, the best seller recommendation still does not necessarily benefit the marketplace as well as the sellers.

The new condition for the best seller recommendation to create a win-win-win situation for all parties on the marketplace is given in part (iii) of Proposition 6.¹¹ Due to the complexity of the function $T(n^*, m^*, r^*)$, it is challenging to analytically compare this condition with that in Proposition 5. Numerical studies show that the condition of a win-win-win in part (iii) of Proposition 6 is less likely to hold as r^* increases because of two reasons. First, as consumers form

¹¹For example, the condition for a win-win-win situation holds with $u = 4$, $\mu = 4$, $N = 10^5$, $M = 200$, $u_0 = 100$, $\pi_0 = 4000$, $c = 4$, $\ell = 0.05$, and $r^* = 5$.

a larger consideration set to search more sellers, the competition among these sellers becomes more intense, leading to a lower equilibrium price, p^o , in the benchmark case without recommendation. As we discussed in the previous section, a lower p^o leaves even less room for the sellers and the marketplace to benefit from the best seller recommendation. This result stands in stark contrast with Li et al. (2019) where a reduction in consumer search (deliberation) costs leads to greater profits for a seller in a traditional channel without competition. Second, as consumers search more sellers, each seller has a higher chance to be included in a consumer’s consideration set in the second period regardless of whether he was the best seller in the previous period. In other words, the advantage to be the best seller is weaker for the sellers, which makes them less likely to benefit from the best seller recommendation. This diminished advantage for sellers from the recommendation also reduces the likelihood for the win-win-win situation to occur.

8. Conclusion

This paper examines how using the best seller recommendation system influences the pricing competition amongst sellers, and the payoffs of sellers, the marketplace as well as consumers. To this end, we develop a two-period analytical model in which consumers’ and sellers’ participation is determined based on their expectations about equilibrium price and whether the best seller recommendation system is utilized. Upon joining the marketplace, sellers choose their prices at the beginning of the first period and keep their prices unchanged in the second period. In the first period, the marketplace displays products randomly to customers. A consumer randomly selects a subset of displayed products to form her consideration set, and purchases the product with the highest realized utility in this set according to the multinomial logit model (MNL). In the second period, the marketplace displays products using the best seller recommendation system. This recommendation system shows the product with the highest sales in the first period at the top of the webpage, and randomly fills the other spots on the webpage with other sellers. The best seller product automatically enters consumers’ consideration set in the second period, and they compare this product with another randomly selected product in their consideration set before making a purchase. Finally, the marketplace collects a commission over each transaction on its site. Analysis of the equilibrium outcome with and without the best seller recommendation offers important managerial insights on the following questions.

First, how does the best seller recommendation affect sellers’ competition on the marketplace? We find that compared to the case without any recommendation, the best seller recommendation intensifies the price competition among sellers and leads to a lower equilibrium price. The reason is that the best seller recommendation automatically secures the seller a spot in consumers’ consideration set and in turn increases consumers’ purchase likelihood (compared to the case of randomly

entering consumers' consideration set) in the second period. As a result, competing sellers have incentives to reduce their prices in the first period to improve their chances of being recommended by the marketplace.

Second, how does the best seller recommendation affect the payoffs of the marketplace, sellers and consumers? We show that adopting the best seller recommendation increases the number of consumers making a purchase on the marketplace due to a lower equilibrium price. At the same time, consumers may suffer from a lower level of product variety because fewer sellers will choose to join the marketplace. Overall, the adoption of the best seller recommendation increases consumer surplus. By contrast, the best seller recommendation system has two opposing effects on sellers and the marketplace. On the one hand, a lower equilibrium price hurts sellers' and marketplace's profit margin. On the other hand, the recommendation system invites more customers to the marketplace so sellers have access to a greater demand, and the marketplace enjoys commission from more transactions. When the best seller recommendation intensifies seller competition so much that the equilibrium price is reduced to a very low level, using this system decreases each seller's expected profit, the number of active sellers on the marketplace, and the marketplace's expected profits. Managerially, this result suggests that using the best seller recommendation may backfire for the marketplace even compared to the case of no recommendation at all.

Third, when should the marketplace adopt the best seller recommendation? Our analysis suggests that when the price reduction induced by the best seller recommendation is moderate, each seller's expected profit will be higher, and there will be more active sellers on the marketplace. As a result, the marketplace will also be more profitable. Recall that consumers always benefit from such a recommendation system. In other words, utilizing the best seller recommendation can create a win win situation. This desirable outcome is more likely to happen in a product category with a low base utility, a high unit cost, a high commission rate, and greater heterogeneity in consumer tastes. In such a product category, sellers would have enough pricing cushion to absorb the price competition induced by the best seller recommendation.

Finally, what happens to the impact of the best seller recommendation when consumers optimize over their consideration set, or when the marketplace optimizes its commission rate? We find that in these cases, the equilibrium price with the best seller recommendation is still lower than the case without the recommendation. More importantly, when the pricing competition caused by the best seller recommendation is not too intense, this recommendation system benefits everyone in the ecosystem: consumers, sellers, and the marketplace. On the other hand, when price reduction by the recommendation system is severe, sellers and the marketplace will be worse off. These results highlight that the main finding of more intense price competition resulting from the best seller recommendation is robust.

We conclude this paper by discussing several directions for future research. First, our model has focused on the impact of the best seller recommendation on sellers' competition and the marketplace's payoff. Future research could analyze the effect of other types of recommendation systems. In particular, it will be useful to understand the influence of hybrid recommendation systems where both personalized and popularity-based factors are accounted for. In that case, we expect that the popularity-based factor in the system will continue to put a downward pressure on competing sellers' prices. Furthermore, we conjecture that as data privacy concerns and the regulatory pressure increase, online marketplaces may put a bigger weight on the popularity-based factor. Second, future research can investigate the interaction between organic recommendations and sponsored brands/products where sellers pay to obtain prominence. Marketplaces need to understand the trade-off between commission and direct payment from sponsorship, and sellers also need to optimize their strategies over different instruments. Third, to simplify the analysis, our model assumed that the best seller recommendation guarantees its position in consumers' consideration set. Future research can study a more general situation with a probabilistic (higher probability than other products) entry into consumers' consideration set by the best seller. Finally, more empirical research is needed to quantify the effect of the best seller recommendation. In particular, it will be helpful to measure the change in consumer surplus with and without such recommendation systems.

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Appendix A: Proofs

Proof of proposition 1: Suppose that all sellers expect n^o consumers and m^o joined the marketplace and all sellers on the marketplace charge price p^o . Given the expectations, we now consider seller k 's optimal price p_k . Substituting expected demand from (4) into seller k 's expected profit, we have

$$\pi_k(p_k, p^o) = 2((1-\ell)p_k - c)n^o \left(\frac{2}{m^o} \right) \frac{\exp((u - p_k)/\mu)}{\exp((u - p_k)/\mu) + \exp((u - p^o)/\mu)}. \quad (17)$$

We will use the following abbreviation in the proof:

$$q_k = \frac{\exp((u - p_k)/\mu)}{\exp((u - p_k)/\mu) + \exp((u - p^o)/\mu)}.$$

Note that q_k is decreasing in p_k because $\partial q_k / \partial p_k = -q_k(1 - q_k)/\mu < 0$. Differentiate $\pi_k(p_k, p^o)$ with respect to p_k and simplifying, we obtain the first order condition as

$$(1-\ell)n^o \left(\frac{2}{m^o} \right) q_k \left(1 - \left(p_k - \frac{c}{1-\ell} \right) \frac{1 - q_k}{\mu} \right) = 0,$$

which can be rewritten as

$$p_k - \frac{c}{1-\ell} = \frac{\mu}{1 - q_k}. \quad (18)$$

Observe that both sides of equation (18) are positive, and the left hand side of the equation is increasing in p_k while the right hand side is decreasing in p_k . Therefore, there exists only one solution to equation (18).

Now we show that the optimal price resulted from the first order condition is a maximum. Differentiating π_k twice, we have

$$\begin{aligned} \frac{\partial^2 \pi_k}{\partial p_k^2} &= 2(1-\ell)n^o \frac{2}{m^o} \left[-q_k(1 - q_k) \frac{1}{\mu} - q_k(1 - q_k) \frac{1}{\mu} + \left(p_k - \frac{c}{1-\ell} \right) (1 - 2q_k) q_k (1 - q_k) \frac{1}{\mu^2} \right] \\ &= 2(1-\ell)n^o \frac{2q_k(1 - q_k)}{\mu m^o} \left[-2 + \left(p_k - \frac{c}{1-\ell} \right) \frac{1 - 2q_k}{\mu} \right]. \end{aligned}$$

Evaluating the above second order derivative at first-order condition by substituting $p_k - c/(1-\ell)$ from equation (18), we have

$$\left. \frac{\partial^2 \pi_k}{\partial p_k^2} \right|_{FOC} = 2(1-\ell)n^o \frac{2q_k(1 - q_k)}{\mu m^o} \left(-2 + \frac{1 - 2q_k}{1 - q_k} \right) = -2(1-\ell)n^o \frac{2q_k}{\mu m^o} < 0,$$

which implies that π_k is quasiconcave in p_k . Thus, an equilibrium exists.

At the symmetric equilibrium, we have $p_k = p^o$. Plugging $p_k = p^o$ and using $q_k(p_k = p^o) = 1/2$, the first order condition can be simplified to

$$p^o - \frac{c}{1-\ell} = 2\mu,$$

which gives us the equilibrium price presented in the proposition.

Plugging p^o into (3) gives the number of consumers on the marketplace at the equilibrium as

$$n^o = N \frac{u - (2 - \ln 2)\mu - \frac{c}{1-\ell}}{u_0}. \quad (19)$$

Substituting p^o in equation (17), we have the equilibrium seller profit as

$$\pi^o = 2((1-\ell)p^o - c) \frac{n^o}{m^o} = 4\mu(1-\ell) \frac{n^o}{m^o}. \quad (20)$$

Substituting (20) into (6), the equilibrium number of sellers m^o solves the following equation:

$$m^o = \frac{2M((1-\ell)p^o - c)}{\pi_0} \frac{n^o}{m^o} = M \frac{4\mu(1-\ell)}{\pi_0} \frac{n^o}{m^o}, \quad (21)$$

which yields that solution

$$m^o = \sqrt{M \frac{4\mu(1-\ell)}{\pi_0} n^o} = \sqrt{\frac{4MN\mu(1-\ell)}{\pi_0 u_0} \left(u - (2 - \ln 2)\mu - \frac{c}{1-\ell} \right)},$$

where the last equality is obtained by substituting n^o using (19).

Given the equilibrium $\{p^o, n^o, m^o\}$, the marketplace's expected profit is given as $\Pi^o = 2\ell p^o n^o$.

□

Proof of proposition 2: We first derive $Q_k(p_k, p^*, n^*, m^*) = \mathbb{P}(D_{k,1}^* \geq \max_{1 \leq j \leq m^*} D_{j,1}^*)$, the probability that seller k is the best seller, i.e., the realized sale of seller k 's product in period 1 is the highest among all sellers. For ease of exposition, we will abbreviate notation by omitting the arguments, e.g., $Q_k(p_k, p^*, n^*, m^*)$ to Q_k , whenever possible.

We consider seller k 's pricing problem when he expects n^* consumers and m^* sellers join the marketplace and all other sellers charge p^* . Recall that with n^* consumers, the sales distribution over m^* sellers in the first period follows a multinomial distribution whose probability distribution function is given as

$$\mathbb{P}(D_{1,1}^* = y_1, \dots, D_{k,1}^* = y_k, \dots, D_{m^*,1}^* = y_{m^*}) = \frac{n^*!}{y_1! \dots y_k! \dots y_{m^*}!} g_k^{y_k} g^{n^* - y_k},$$

where $D_{k,1}^*$ is demand for product k in the first period, g_k is defined in (10). Therefore, Q_k can be written as

$$Q_k = \mathbb{P}(D_{k,1}^* \geq \max_{1 \leq j \leq m^*} D_{j,1}^*) = \sum_{(y_1, \dots, y_{m^*}) \in S} \binom{n^*}{y_1, \dots, y_{m^*}} g_k^{y_k} g^{n^* - y_k}, \quad (22)$$

where set S contains all sales vectors that sum up to n^* in which seller k 's sales is the highest, i.e., the best seller,

$$S = \left\{ (y_1, y_2, \dots, y_{m^*}) \mid \sum_{j=1}^{m^*} y_j = n^*, \text{ and } y_i \leq y_k \ \forall i \right\}.$$

Given Q_k , seller k 's expected sales in the second period is

$$\begin{aligned} \mathbb{E}[D_{k,2}] &= n^* \left(Q_k + (1 - Q_k) \left(\frac{1}{m^* - 1} \right) \right) \frac{\exp((u - p_k)/\mu)}{\exp((u - p_k)/\mu) + \exp((u - p^*)/\mu)} \\ &= n^* \left(\frac{1 + (m^* - 2)Q_k}{m^* - 1} \right) q_k \\ &= n^* H(p_k, p^*) q_k, \end{aligned} \quad (23)$$

where $H(p_k, p^*) = (1 + (m^* - 2)Q_k)/(m^* - 1)$. Note that seller k 's expected sales in the first period is

$$\mathbb{E}[D_{k,1}] = n^* \left(\frac{2}{m^*} \right) q_k.$$

Thus, seller k 's price optimization problem is

$$\begin{aligned} \max_{p_k \geq c} \pi_k(p_k) &= \max_{p_k \geq c} ((1 - \ell)p_k - c) \{ \mathbb{E}[D_{k,1}] + \mathbb{E}[D_{k,2}] \} \\ &= n^* \max_{p_k \geq c} ((1 - \ell)p_k - c) q_k \left(\frac{2}{m^*} + H(p_k, p^*) \right). \end{aligned}$$

Differentiating $\pi_k(p_k)$ with respect to p_k , the first order condition is given as

$$(1 - \ell)n^*q_k \left(\frac{2}{m^*} + H(p_k, p^*) \right) + n^* [(1 - \ell)p_k - c] \left[\left(\frac{2}{m^*} + H(p_k, p^*) \right) \frac{\partial q_k}{\partial p_k} + q_k \frac{\partial H(p_k, p^*)}{\partial p_k} \right] = 0.$$

Using

$$\frac{\partial q_k}{\partial p_k} = -\frac{1}{\mu} q_k (1 - q_k),$$

and

$$\frac{\partial H(p_k, p)}{\partial p_k} = \frac{\partial}{\partial p_k} \frac{(m^* - 2)Q_k}{m^* - 1} = \frac{m^* - 2}{m^* - 1} \frac{\partial Q_k}{\partial p_k},$$

we can rewrite the first order condition as

$$\frac{2}{m^*} + \frac{1 + (m^* - 2)Q_k}{m^* - 1} + \left(p_k - \frac{c}{1 - \ell} \right) \left[-\frac{(1 - q_k)}{\mu} \left(\frac{2}{m^*} + \frac{1 + (m^* - 2)Q_k}{m^* - 1} \right) + \frac{m^* - 2}{m^* - 1} Q'_k \right] = 0, \quad (24)$$

where $Q'_k = \partial Q_k / \partial p_k$ which is given as

$$\begin{aligned} \frac{\partial Q_k}{\partial p_k} &= \sum_{(y_1, \dots, y_{m^*}) \in S} \binom{n^*}{y_1, \dots, y_{m^*}} \left\{ y_k g_k^{y_k - 1} \left(\frac{1 - g_k}{m^* - 1} \right)^{n^* - y_k} - (n^* - y_k) g_k^{y_k} \left(\frac{1 - g_k}{m^* - 1} \right)^{n^* - y_k - 1} \frac{1}{m^* - 1} \right\} \frac{\partial \bar{q}_k}{\partial p_k} \\ &= \sum_{(y_1, \dots, y_{m^*}) \in S} \binom{n^*}{y_1, \dots, y_{m^*}} g_k^{y_k} \left(\frac{1 - g_k}{m^* - 1} \right)^{n^* - y_k} \left\{ y_k - \frac{n^* - y_k}{1 - g_k} \right\} \frac{2}{m^*} \frac{\partial q_k}{\partial p_k}. \end{aligned}$$

Now we evaluate first order condition at the symmetric equilibrium, $p_k = p^*$. At the symmetric equilibrium, $p_k = p^*$, we have

$$q(p_k = p^*) = \frac{1}{2},$$

$$Q_k(p_k = p^*) = \sum_{(y_1, \dots, y_{m^*}) \in S} \binom{n^*}{y_1, \dots, y_{m^*}} \left(\frac{1}{m^*} \right)^{n^* - y_k} \left(\frac{1}{m^*} \right)^{y_k} = \left(\frac{1}{m^*} \right)^{n^*} \sum_{(y_1, \dots, y_{m^*}) \in S} \binom{n^*}{y_1, \dots, y_{m^*}} = \frac{1}{m^*},$$

and

$$H(p^*, p^*) = \frac{2}{m^*}. \quad (25)$$

The last equality follows from the fact that when all sellers pick the same price, they have an equal chance to have the highest sales, i.e., be the best seller in the first period.

Then, evaluating the derivatives at $p_k = p^*$, we have

$$\left. \frac{\partial q_k}{\partial p_k} \right|_{p_k = p^*} = -\frac{1}{4\mu},$$

and

$$\begin{aligned} \left. \frac{\partial Q_k}{\partial p_k} \right|_{p_k = p^*} &= \sum_{(y_1, \dots, y_{m^*}) \in S} \binom{n^*}{y_1, \dots, y_{m^*}} \left(\frac{1}{m^*} \right)^{n^*} \left\{ m^* y_k - \frac{m^*}{m^* - 1} (n^* - y_k) \right\} \frac{-1}{2\mu m^*} \\ &= -\frac{1}{2\mu} \left(\frac{m^*}{m^* - 1} \right) \sum_{(y_1, \dots, y_{m^*}) \in S} \binom{n^*}{y_1, \dots, y_{m^*}} \left(\frac{1}{m^*} \right)^{n^*} \left\{ y_k - \frac{n^*}{m^*} \right\}. \end{aligned}$$

Using the above equations, the first order condition (24) calculated at symmetric equilibrium, $p_k = p^*$, can be simplified to

$$\frac{4}{m^*} + \left(p^* - \frac{c}{1 - \ell} \right) \left\{ -\frac{2}{\mu m^*} - \frac{2}{\mu m^*} \frac{m^* - 2}{4} \left(\frac{m^*}{m^* - 1} \right)^2 \sum_{(y_1, \dots, y_{m^*}) \in S} \binom{n^*}{y_1, \dots, y_{m^*}} \left(\frac{1}{m^*} \right)^{n^*} \left\{ y_k - \frac{n^*}{m^*} \right\} \right\} = 0.$$

Solving the above equation, we have p^* as

$$p^* = \frac{c}{1-\ell} + \frac{2\mu}{1 + \frac{m^*-2}{4} \left(\frac{m^*}{m^*-1}\right)^2 \left(\frac{1}{m^*}\right)^{n^*} \sum_{(y_1, \dots, y_{m^*}) \in S} \binom{n^*}{y_1, \dots, y_{m^*}} \left\{y_k - \frac{n^*}{m^*}\right\}}.$$

Let's define

$$T(n^*, m^*) := \frac{m^*-2}{4} \left(\frac{m^*}{m^*-1}\right)^2 \left(\frac{1}{m^*}\right)^{n^*} \sum_{(y_1, \dots, y_{m^*}) \in S} \binom{n^*}{y_1, \dots, y_{m^*}} \left\{y_k - \frac{n^*}{m^*}\right\}.$$

We can rewrite the symmetric equilibrium price as

$$p^* = \frac{c}{1-\ell} + \frac{2\mu}{1 + T(n^*, m^*)}.$$

Observe that summation in $T(n^*, m^*)$ is over all combinations $y_k \geq y_i$ for all i , resulting $y_k \geq \frac{n^*}{m^*}$. Therefore, $y_k - \frac{n^*}{m^*} > 0$. Additionally, all other terms in $T(n^*, m^*)$ are positive. Therefore, we have $T(n^*, m^*) > 0$.

We next derive a condition to ensure that $\pi_k(p_k)$ is quasiconcave so that p^* is a maximum. Differentiating $\pi_k(p_k)$ twice and evaluating it at the first order condition (24), we have

$$\begin{aligned} \frac{1}{n^*(1-\ell)q_k} \frac{\partial^2 \pi_k}{\partial p_k^2} \Big|_{FOC} &= 2 \frac{m^*-2}{m^*-1} Q'_k - \left(\frac{2}{m^*} + \frac{1}{m^*-1} + \frac{m^*-2}{m^*-1} Q_k \right) \frac{1-q_k}{\mu} + \\ &\quad \left(p_k - \frac{c}{1-\ell} \right) \left[\frac{m^*-2}{m^*-1} Q''_k - Q'_k \frac{m^*-2}{m^*-1} \frac{1-q_k}{\mu} - \left(\frac{2}{m^*} + \frac{1}{m^*-1} + \frac{m^*-2}{m^*-1} Q_k \right) \frac{q_k(1-q_k)}{\mu^2} \right]. \end{aligned}$$

Substituting $p_k - \frac{c}{1-\ell}$ at the first order condition from (24) again, we can simplify the above second order derivative to

$$\frac{1}{n^*(1-\ell)q_k} \frac{\partial^2 \pi_k}{\partial p_k^2} \Big|_{FOC} = -2Q'_k \left(Q'_k - (\alpha(m^*) + Q_k) \frac{1-q_k}{\mu} \right) - (\alpha(m^*) + Q_k) \left[(\alpha(m^*) + Q_k) \frac{1-q_k}{\mu^2} - Q''_k \right],$$

where $\alpha(m^*) := 2(m^*-1)/(m^*(m^*-2)) + 1/(m^*-2)$. The first term of the above expression is negative because Q is decreasing in p_k , and the second term can be positive. The condition to ensure the quasiconcavity of $\pi_k(p_k)$ is

$$-2Q'_k \left(Q'_k - (\alpha(m^*) + Q_k) \frac{1-q_k}{\mu} \right) - (\alpha(m^*) + Q_k) \left[(\alpha(m^*) + Q_k) \frac{1-q_k}{\mu^2} - Q''_k \right] < 0.$$

According to (9), the equilibrium number of consumers on the marketplace is

$$n^* = N \frac{u + \mu \ln 2 - p^*}{u_0}.$$

Using (23) and (25), the symmetric equilibrium expected sales for each seller on the marketplace over the two periods are

$$\mathbb{E}[D_{k,1}^*] = \mathbb{E}[D_{k,2}^*] = \frac{n^*}{m^*}.$$

Thus, the expected profit of each seller on the marketplace at the symmetric equilibrium is given as

$$\pi^* = ((1-\ell)p^* - c) \left(\mathbb{E}[D_{k,1}^*] + \mathbb{E}[D_{k,2}^*] \right) = \frac{4\mu(1-\ell)}{1 + T(n^*, m^*)} \frac{n^*}{m^*}. \quad (26)$$

Note that seller k would join the marketplace if the equilibrium expected profit exceeds his reservation profit. The equilibrium number of sellers that join the marketplace is given as

$$m^* = M \frac{\pi^*}{\pi_0}. \quad (27)$$

Substituting π^* from (26) into (27), and solving for m^* , we have

$$m^* = \sqrt{\frac{2M(1-\ell)}{\pi_0} \left(p^* - \frac{c}{1-\ell} \right) n^*}. \quad (28)$$

Given the equilibrium $\{p^*, n^*, m^*\}$, the marketplace's expected total profit is $\Pi^* = 2\ell p^* n^*$.

□

Proof of proposition 3: In the proof of proposition 2, we proved that $T(n^*, m^*) > 0$, thus

$$p^* = \frac{c}{1-\ell} + \frac{2\mu}{1+T(n^*, m^*)} < \frac{c}{1-\ell} + 2\mu = p^o.$$

□

Proof of proposition 4: To prove Part (i), from (9) and (3), we have,

$$n^* - n^o = N \frac{u + \mu \ln 2 - p^*}{u_0} - N \frac{u + \mu \ln 2 - p^o}{u_0} = N \frac{p^o - p^*}{u_0} > 0,$$

where the last inequality follows from $p^* < p^o$. Thus, it is true that $n^* > n^o$.

To show Part (ii), it is sufficient to show that $m^* - m^o < 0$ or $(m^*)^2 - (m^o)^2 < 0$, if and only if the condition in the proposition is satisfied. Using (21) and (28), substituting n^* and n^o , and collecting terms, we have

$$\begin{aligned} (m^*)^2 - (m^o)^2 &= \frac{2M}{\pi_0} [(1-\ell)p^* - c] n^* - \frac{2M}{\pi_0} [(1-\ell)p^o - c] n^o \\ &= \frac{2M}{\pi_0} [(1-\ell)p^* - c] \frac{N}{u_0} (u + \mu \ln 2 - p^*) - \frac{2M}{\pi_0} [(1-\ell)p^o - c] \frac{N}{u_0} (u + \mu \ln 2 - p^o) \\ &= \frac{2MN}{\pi_0 u_0} (p^o - p^*) [(1-\ell)(p^o + p^*) - (u + \mu \ln 2)(1-\ell) - c]. \end{aligned}$$

Because $p^o - p^* > 0$, the terms outside of square bracket in the last equation are positive. Thus, $(m^*)^2 - (m^o)^2 < 0$, if and only if the term inside the square bracket is negative, or,

$$p^o + p^* < u + \mu \ln 2 + \frac{c}{1-\ell}. \quad (29)$$

Substituting p^* from (14) into the above inequality, we have

$$p^o \leq u + \mu \left(\ln 2 - \frac{2}{1+T(n^*, m^*)} \right).$$

Hence, $(m^*)^2 - (m^o)^2 < 0$, or equivalently $m^* \leq m^o$ if and only if the above condition holds. Note that $m = M \frac{\pi(p)}{\pi_0}$, and M and π_0 are constant and the same for both models. Therefore, $\pi^* \leq \pi^o$ if and only if $m^* \leq m^o$, or the same above condition holds.

From propositions 1 and 2, we have

$$\begin{aligned} \Pi^* - \Pi^o &= 2\ell (n^* p^* - n^o p^o) = \frac{2\ell N}{u_0} (p^* (u + \mu \ln 2 - p^*) - p^o (u + \mu \ln 2 - p^o)) \\ &= \frac{2\ell N}{u_0} (p^o - p^*) [p^* + p^o - (u + \mu \ln 2)]. \end{aligned}$$

The terms outside of the square bracket in the last equation are all positive because $p^* < p^o$. Hence, $\Pi^* < \Pi^o$ if and only if the term inside the square bracket in the last equation is negative, or equivalently

$$p^o < u + \mu \ln 2 - p^*. \quad (30)$$

Substituting p^* from (14) into the above inequality, we have $\Pi^* < \Pi^o$ if and only if

$$p^o < u + \mu \left(\ln 2 - \frac{2}{1 + T(n^*, m^*)} \right) - \frac{c}{1 - \ell},$$

which proves Part (iii). \square

Proof of proposition 5 Part (i) directly follows from proposition 4.

From proof of proposition 4 and equation (30), we can conclude that best seller recommendation hurts sellers, and the marketplace if

$$p^o < u + \mu \ln 2 - p^*$$

which surely holds if the following condition holds

$$p^o < u + \mu \ln 2 - p^o,$$

because $p^* < p^o$. Substituting p^o from (7) into the above condition and collecting terms, the best seller recommendation hurts all parties on the marketplace if

$$\frac{2c}{1 - \ell} + \mu(4 - \ln 2) < u,$$

in other words, the best seller recommendation would be more likely to benefit all parties if the above condition does not hold, which is more likely to happen for low base utility, u , and high unit cost, c , commission rate, ℓ and heterogeneity of consumer taste, μ . \square

Proof of proposition 6: To show Part (i), we solve the consumer's consideration set problem given as

$$\max_{0 < r \leq m} U(r) = \max_{0 < r \leq m} u - p + \mu \ln r - r\tau$$

to obtain

$$r^* = \frac{\mu}{\tau}.$$

The derivations of the equilibrium prices follow the same processes in proofs of Proposition 1 and Proposition 2. Thus, we only present the abbreviated proofs for part (ii). We start by deriving the equilibrium price in the model without recommendation. We still take seller k 's perspective. Now, with r^* , each seller can be searched by a consumer with probability $\frac{r^*}{m^o}$. Given seller k is searched by the consumer, the probability that the consumer purchases product k becomes

$$q_k = \frac{\exp((u - p_k)/\mu)}{\exp((u - p_k)/\mu) + (r^* - 1) \exp((u - p^o)/\mu)}.$$

Note that $q_k(p_k = p^o) = 1/r^*$. The expected profit of seller k is

$$\pi_k(p_k, p^o) = 2((1 - \ell)p_k - c) n^o \left(\frac{r^*}{m^o} \right) q_k.$$

Differentiate $\pi_k(p_k, p^o)$ with respect to p_k yields the first order condition as

$$\frac{\partial \pi_k}{\partial p_k} = 2(1-\ell)n^o \left(\frac{r^*}{m^o} \right) q_k \left(1 - \left(p_k - \frac{c}{1-\ell} \right) (1-q_k) \frac{1}{\mu} \right) = 0,$$

which implies

$$p_k - \frac{c}{1-\ell} = \frac{\mu}{1-q_k}. \quad (31)$$

Evaluate the second order condition at the first order condition, we have

$$\frac{\partial^2 \pi_k}{\partial p_k^2} \Big|_{FOC} = -2(1-\ell)n^o \left(\frac{r^*}{m^o} \right) \frac{q_k}{\mu} < 0.$$

Solving the first order condition (31) for the symmetric equilibrium ($p_k = p^o$) yields the equilibrium price,

$$p^o = \frac{c}{1-\ell} + \frac{r^* \mu}{r^* - 1}.$$

Now we derive the equilibrium price with the best seller recommendation. All derivations of the equilibrium in Proposition 2 will remain unchanged with q_k and g_k redefined as

$$q_k = \frac{\exp(-p_k/\mu)}{\exp(-p_k/\mu) + (r^* - 1) \exp(-p^*/\mu)}$$

and

$$g_k = \left(\frac{r^*}{m^*} \right) \frac{\exp(-p_k/\mu)}{\exp(-p_k/\mu) + (r^* - 1) \exp(-p^*/\mu)}.$$

Then, the probability that seller k is the best seller in the first period is

$$Q_k = \mathbb{P}(D_{k,1}^* \geq \max_{1 \leq j \leq m^*} D_{j,1}^*) = \sum_{(y_1, \dots, y_{m^*}) \in S} \binom{n^*}{y_1, \dots, y_{m^*}} g_k^{y_k} g^{n^* - y_k}.$$

Seller k 's expected sales in each period are

$$\mathbb{E}[D_{k,1}] = n^* \left(\frac{r^*}{m^*} \right) q_k,$$

$$\mathbb{E}[D_{k,2}] = n^* H(p_k, p^*) q_k,$$

respectively, where $H(p_k, p^*) = (r^* - 1 + (m^* - r^*)Q_k)/(m^* - 1)$ is the probability that a consumer searches product k in period 2.

Seller k 's price optimization problem is

$$\max_{p_k \geq c} \pi_k(p_k) = \max_{p_k \geq c} ((1-\ell)p_k - c) \{ \mathbb{E}[D_{k,1}] + \mathbb{E}[D_{k,2}] \}.$$

Differentiating $\pi_k(p_k)$, the first order condition is given as

$$(1-\ell)q_k \left(\frac{r^*}{m^*} + H(p_k, p^*) \right) + [(1-\ell)p_k - c] \left[\left(\frac{r^*}{m^*} + H(p_k, p^*) \right) \frac{\partial q_k}{\partial p_k} + q_k \frac{\partial H(p_k, p^*)}{\partial p_k} \right] = 0. \quad (32)$$

At the symmetric equilibrium $p_k = p^*$, we have

$$Q_k = \frac{1}{m^*},$$

$$H(p^*, p^*) = \frac{r^*}{m^*},$$

$$\left. \frac{\partial q_k}{\partial p_k} \right|_{p_k=p^*} = \frac{-(r^*-1)}{r^{*2}\mu},$$

$$\begin{aligned} \frac{\partial Q_k}{\partial p_k} &= \sum_{(y_1, \dots, y_{m^*}) \in S} \binom{n^*}{y_1, \dots, y_{m^*}} g_k^{y_k} \left(\frac{1-g_k}{m^*-1} \right)^{n-y_k} \left\{ y_k - \frac{n^*-y_k}{1-g_k} \right\} \frac{\partial g_k}{\partial p_k}, \\ \left. \frac{\partial Q_k}{\partial p_k} \right|_{p_k=p^*} &= \frac{-(r^*-1)}{r^*\mu} \left(\frac{m^*}{m^*-1} \right) \sum_{(y_1, \dots, y_{m^*}) \in S} \binom{n^*}{y_1, \dots, y_{m^*}} \left(\frac{1}{m^*} \right)^{n^*} \left\{ y_k - \frac{n^*}{m^*} \right\}, \end{aligned}$$

and

$$\frac{\partial H(p_k, p^*)}{\partial p_k} = \frac{\partial}{\partial p_k} \frac{(m^*-r^*)Q_k}{m^*-1} = \frac{m^*-r^*}{m^*-1} \frac{\partial Q_k}{\partial p_k}.$$

The first order condition calculated at symmetric equilibrium ($p_k = p^*$) is

$$1 + \left[p^* - \frac{c}{1-\ell} \right] \left\{ \frac{-r^*-1}{r^*\mu} - \frac{(r^*-1)(m^*-r^*)}{2r^{*2}\mu} \left(\frac{m^*}{m^*-1} \right)^2 \sum_{(y_1, \dots, y_{m^*}) \in S} \binom{n^*}{y_1, \dots, y_{m^*}} \left(\frac{1}{m^*} \right)^{n^*} \left\{ y_k - \frac{n^*}{m^*} \right\} \right\} = 0,$$

which yields

$$p^* = \frac{c}{1-\ell} + \frac{r^*\mu}{r^*-1} \frac{1}{1 + \frac{m^*-r^*}{2r^*} \left(\frac{m^*}{m^*-1} \right)^2 \left(\frac{1}{m^*} \right)^{n^*} \sum_{(y_1, \dots, y_{m^*}) \in S} \binom{n^*}{y_1, \dots, y_{m^*}} \left\{ y_k - \frac{n^*}{m^*} \right\}}.$$

Let's define

$$T(n^*, m^*, r^*) := \frac{m^*-r^*}{2r^*} \left(\frac{m^*}{m^*-1} \right)^2 \left(\frac{1}{m^*} \right)^{n^*} \sum_{(y_1, \dots, y_{m^*}) \in S} \binom{n^*}{y_1, \dots, y_{m^*}} \left\{ y_k - \frac{n^*}{m^*} \right\}.$$

Note that summation in the above equation is over all combinations $y_k \geq y_i, \forall i$, which implies $y_k \geq \frac{n^*}{m^*}$.

Therefore, the term in the curly bracket, $y_k - n^*/m^*$, is always positive. Moreover, all other terms in the above equation are positive. Therefore, $T(n, m, r^*) > 0$. The symmetric equilibrium price can be rewritten as

$$p^* = \frac{c}{1-\ell} + \frac{r^*\mu}{(r^*-1)(1+T(n^*, m^*, r^*))}. \quad (33)$$

Evaluating the second order condition at the first order condition, we have the condition to ensure $\pi_k(p_k)$ to be quasiconcave as

$$-2Q'_k \left(Q'_k - (\alpha(m^*, r^*) + Q_k) \frac{1-q_k}{\mu} \right) - (\alpha(m^*, r^*) + Q_k) \left[(\alpha(m^*, r^*) + Q_k) \frac{1-q_k}{\mu^2} - Q_k'' \right] < 0,$$

where

$$\alpha(m^*, r^*) = \frac{r^*(m^*-1)}{m^*(m^*-r^*)} + \frac{r^*-1}{m^*-r^*}.$$

This completes the proof for part (ii). We now proof part (iii). Using the equilibrium prices derived above, and following similar processes in proofs of Proposition 1 and Proposition 2, we can derive $n^o, n^*, m^o, m^*, \pi^o, \pi^*, \Pi^o$ and Π^* . Then, we can obtain $\Pi^* > \Pi^o$ if and only if

$$p^o > u + \mu \ln r^* - p^*,$$

and $m^* > m^o$ if and only if

$$p^o > u + \mu \ln r^* + \frac{c}{1-\ell} - p^*.$$

Therefore, $m^* > m^o$ and $\Pi^* > \Pi^o$ can occur simultaneously when $p^o > u + \mu \ln r^* + \frac{c}{1-\ell} - p^*$. Substituting p^* from (33) into the above inequality, we have the condition as

$$p^o > u + \mu \left(\ln r^* - \frac{r^*}{(r^*-1)(1+T(n^*, m^*, r^*))} \right).$$

□

Appendix B: A Continuous Approximation of $T(n, m, r)$

To improve computational efficiency in numerical studies, we develop a continuous approximation of $T(n, m, r)$ which is a discrete function. First, we rewrite $T(n, m, r)$ in terms of the expectation of the first order statistic of the equal probability multinomial distribution. Recall that

$$\begin{aligned} T(n, m, r) &:= \frac{m-r}{2r} \left(\frac{m}{m-1} \right)^2 \left(\frac{1}{m} \right)^n \sum_{(y_1, \dots, y_m) \in S} \binom{n}{y_1, \dots, y_m} \left\{ y_k - \frac{n}{m} \right\} \\ &= \frac{m-r}{2r} \left(\frac{m}{m-1} \right)^2 \left(\mathbb{E} [y_k; y_k \geq y_i \forall i \in \{1, 2, \dots, m\}] - \frac{n}{m^2} \right). \end{aligned}$$

We next develop a continuous approximation of $\mathbb{E} [y_k; y_k \geq y_i \forall i \in \{1, 2, \dots, m\}]$ because other terms are straightforward to compute. Note that

$$\begin{aligned} \mathbb{E} [y_k; y_k \geq y_i \forall i \in \{1, 2, \dots, m\}] &= \sum_{(y_1, \dots, y_m) \in S} \binom{n}{y_1, \dots, y_m} \left(\frac{1}{m} \right)^n y_k \\ &= \mathbb{E} \left[\max_i \{y_1, \dots, y_m\} \right] \mathbb{P}(y_k = \max_i \{y_1, \dots, y_m\}) \\ &= \frac{1}{m} \mathbb{E} \left[\max_i \{y_1, \dots, y_m\} \right]. \end{aligned} \quad (34)$$

In the first equation, the summation is over all combination of (y_1, \dots, y_m) such that y_k is largest element in (y_1, \dots, y_m) . Thus, it is the expectation of y_k only when y_k is the largest element in (y_1, \dots, y_m) as shown in the second equation. The last equation follows from the fact that all element of (y_1, \dots, y_m) have equal probability, $\frac{1}{m}$, to be the largest element as they are ex ante symmetric. We now only need to approximate $\mathbb{E} [\max_i \{y_1, \dots, y_m\}]$. We do so by using a result from Kolchin (1969) showing that the distribution of order statistics of a multinomial distribution can be approximated by a Gumbel distribution asymptotically.

Lemma B.1. Suppose $(a_1, \dots, a_m) \sim \text{Multinomial}(n, 1/m)$, and for $n \rightarrow \infty$ and $\frac{n}{m \ln m} \rightarrow \infty$ then,

$$\mathbb{E} \left[\max_{1 \leq i \leq m} a_i \right] \approx \frac{n}{m} + \sqrt{\frac{n}{m}} \left(\sqrt{2 \left(\ln m - \frac{1}{2} \ln \ln m \right)} + \frac{\gamma - \frac{1}{2} \ln 4\pi}{\sqrt{2 \ln m}} \right).$$

Proof: To prove the lemma we utilize following theorem presented in Kolchin (1969). For $n \rightarrow \infty$ and $\frac{n}{m \ln m} \rightarrow \infty$ then,

$$\mathbb{P} \left\{ \frac{\max_{1 \leq i \leq m} a_i - \frac{n}{m} - \frac{n}{m} f \left(\left(\ln m - \frac{1}{2} \ln \ln m \right) / \frac{n}{m} \right)}{\sqrt{\frac{n}{m} / 2 \ln m}} + \frac{1}{2} \ln 4\pi \leq z \right\} \rightarrow e^{-e^{-z}}, \quad (35)$$

where $f(w)$ is a function defined in the interval $0 \leq w < \infty$ by the following equation

$$-f(w) + (1 + f(w)) \ln(1 + f(w)) = w.$$

Rearranging terms in (35) gives us the approximate distribution of $\max_{1 \leq i \leq m} a_i$ which is a Gumbel distribution with following CDF:

$$F(x) = \exp[-\exp(-(\frac{x - \delta}{\omega}))]$$

where

$$\delta = \frac{n}{m} + \frac{n}{m} f \left(\left(\ln m - \frac{1}{2} \ln \ln m \right) / \frac{n}{m} \right) - \frac{1}{2} \ln(4\pi) \sqrt{\frac{n}{m} / 2 \ln m}$$

and

$$\omega = \sqrt{\frac{n}{m}/2 \ln m}.$$

Therefore, according to the property of Gumbel distribution, we have

$$\mathbb{E} \left[\max_{1 \leq i \leq m} a_i \right] \approx \delta + \omega \gamma, \quad (36)$$

where γ is Euler's constant. We can further approximate $f(\cdot)$ to first order using its expansion:

$$\frac{n}{m} f \left(\frac{\ln m - \frac{1}{2} \ln \ln m}{n/m} \right) \approx \sqrt{2 \frac{n}{m} \left(\ln m - \frac{1}{2} \ln \ln m \right)}.$$

Using the above approximation and substituting it into (36), we can show that

$$\mathbb{E} \left[\max_{1 \leq i \leq m} a_i \right] \approx \frac{n}{m} + \sqrt{\frac{n}{m}} \left(\sqrt{2 \left(\ln m - \frac{1}{2} \ln \ln m \right)} + \frac{\gamma - \frac{1}{2} \ln 4\pi}{\sqrt{2 \ln m}} \right).$$

□

The above approximation is proper for our model since it is reasonable that the number of consumers on the marketplace is larger than the number of sellers. Combining the approximation presented in Lemma B.1. with equation (34), we have the following approximation of $T(n, m, r)$ as

$$T(n, m, r) \approx \sqrt{\frac{n}{m}} \frac{(m-r)m}{2(m-1)^2 r} \left(\sqrt{2 \left(\ln m - \frac{1}{2} \ln \ln m \right)} + \frac{\gamma - \frac{1}{2} \ln 4\pi}{\sqrt{2 \ln m}} \right).$$