Platform Transformation Risk and the Role of Hosting Rivals*

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April 19, 2023

Abstract

This paper studies the decision of traditional firms to transform into digital platforms. While digital transformation enhances value through externalizing value creation, it also entails investment risk. We show that when investments entail high risk and the value of network effects is low, firms should avoid transforming into a platform and retain their traditional form. By contrast, low transformation risk or high value of network effects make digital transformation profitable. Interestingly, when firms choose to transform, we show that inviting rivals onto the platform can raise profits. Indeed, the platform may even pay rivals to join its platform in certain cases. The benefit of enhancing network effects through demand aggregation can be more profitable than competing as separate platforms. Further, inviting rivals onto a proprietary platform lowers the rival's competitive aggressiveness. This is a novel strategic rationale for inviting rivals on to the platform elicited in this paper. Yet, when the value of network effects is very high and investments are near certain, the platform chooses not to invite rivals. We provide clear managerial and policy implications from these results and use real world examples to illustrate our theory.

Keywords: Digital transformation, platforms, hosting rivals, investment risks, external value creation, developers, cross-sided network effects.

JEL Codes: L22, L23, M11, O32.

*We thank Ann Majchrzak, Ricky Tan, Sunil Mithas for their valuable comments. We also thank the participants at the University of Houston seminars, University of South Florida (Moma School of Business), Workshop on Information Technology and Systems (WITS) 2022, Conference on Information Systems and Technology (CIST) 2022, CESifo Area Conference on Economics of Digitization 2022 for their insights.

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1 Introduction

Relatively young digital platform firms which were once considered scrappy, underdog startups have turned the tables and dominated the stock market leaving analog incumbents behind (Report 2020).[1] This relative stagnation of traditional firms led executives to recognize that digital transformation would drive survival. The opportunities presented by digital transformation of traditional businesses into platforms can seem unlimited, nudging these executives to rapidly invest and transform (Bonnet 2016).[2] Transformation augments internal value creation by encouraging third parties to add value and shifting managerial focus from inside to outside. This transformation, "inverting the firm," moves from internal production to external orchestration of value creation (Van Alstyne & Geoffrey G. 2021, Sandberg et al. 2019, Parker et al. 2016).[3] This is because such a strategy harnesses third party resources. However, platformization of traditional firms is no panacea. It brings forth new risks and new problems and a high fraction of such project fail (Fuller et al. 2019, Bonnet & Westerman 2021).

Although digital transformation into a platform may be appealing as it enables external value creation through third party developers, transformation is also a source of multiple risks such as business risks, strategic risks, security risks and investments risks (Duc & Chirumamilla 2019, Jurisch et al. 2016).[4] As a result, compared to more straightforward product enhancement investments, digital transformation diverts resources to uncertain business-model as the scope of challenges broaden. This handicaps a firm's efficiency compared to traditional and experienced rivals.[5] More certain investors can also be more aggressive in their investments. Apart from the usual increased risk, this suggests that firms contemplating risky digital transformation must also anticipate the response of their rivals and its impact on their profitability (Markides & Sosa 2013,

[1]See House Judiciary Report (2020). In 2018, 7 of the 10 firms ranked by market capitalization were digital firms, a stark contrast to the rankings in 2008 (*The Platform Economy* 2019).

[2]A Gartner survey found that 80% of the responding business leaders expect their firm to transform into a digital enterprise. See WEF (2016) report on digital transformation for more details.

[3]There are multiple instances of traditional firms creating platforms to augment their product value through improved data collections or by encouraging third parties to create additional value. For instance, Becton Dickinson, a medical equipment manufacturer, has developed a platform that offers expert advice to Hospitals by leveraging on data-driven insights (Correani et al. 2020). Similarly, LEGO, a plastic toy manufacturer, successfully transformed into a platform by allowing fans and partners to experiment with "micro-businesses" and enhance their value proposition See link for more details. Finally, CNH Industrial, an agriculture equipment producer, has recently launched its AGXTEND platform where startups and other partners can offer technologies to customers. See link for more details. For more examples of traditional firms making the digital transformation leap, See WEF report on digital transformation (WEF 2016) and more recently Srai et al. (2022).

[4]A study found that between 66-84% of digital transformations fail (Jacquemont et al. 2015, Libert et al. 2016). For instance, in 2011, Johnson control introduced "Panoptix," a cloud-based platform that sought to consolidate data from disparate building systems (e.g., building automation systems (BAS), smart grids, security systems) and enable applications to use that data to monitor and manage building operations. The platform was intended to be agnostic to the equipment manufacturers and be open to third party developers as well as including community elements for building managers and experts to interact and share knowledge. In 2016, the platform was shutdown and removed from the company's product portfolio (Schultz 2016). Other notable failures include GE Predix and IBM Watson.

[5]Michael Nilles, Chief Digital and Information Officer at Henkel, points out that innovation around new business models is risky (Hinterhuber & Nilles 2021). This may be due to low digital literacy among managers and employees which may diminish the real value of investments (Engler 2020). Further, he stresses the importance of co-innovation where both internal and external value creation are important.

Karhu & Ritala 2021). Therefore, digital transformation is a complex organizational change where firms must carefully balance increased market opportunities with the associated vulnerabilities.

In this paper, we consider the decision of a traditional firm to proceed with digital transformation into a multi-sided platform in a competitive setting while facing digital transformation risk. If it transforms into a platform, then it also decides whether to invite its rival on its platform or not. We interpret digital transformation as traditional firms evolving their business towards a platform business model enabling external value creation and adding complementary value to their core product, for instance, through cross-network interactions between consumers and developers. At the same time, digital transformation of a firm into a platform presents increased investment risk vis à vis traditional business models. This increased risk lowers its investment in internal value creation. It further downgrades market expectations on the external value generation through cross-sided network interactions on the platform. We find that when the risks associated with digital transformation are high, and the per-unit value of cross-network interactions (external value creation) is low, firms may be better off avoiding a digital transformation strategy and remain as traditional firms. Under these circumstances, the benefits of digital transformation are outweighed by the risks of transformation. Specifically, in these circumstances, the appeal of (deterministic) investments by remaining a traditional firm is higher than undergoing digital transformation where investment risks are high. Further, when the value of per-unit cross-sided network interactions is high, the benefits of being a platform dominate the risks. Interestingly, we find that, in most cases, the platform firm finds it profitable to transform into a platform even to the point of hosting its rival. The intuition for this result is as follows. Participation of the rival firm on the platform favorably enhances market expectation on the volume of the cross-sided network interactions and as a result increases platform participation of both consumers and developers. There is also an additional more nuanced rationale for the platform firms to invite rivals. By inviting rivals on to its platform, the investments of rivals also become risky (stochastic) due to migration from product to platform complement. This lowers their investment aggressiveness and thus enhances the platform's flow profit by reducing competition. Further, we find that when investment uncertainty is high, and the value of cross-network interactions is moderately low, the platform firm finds it profitable to even pay its rival to join the platform. Hosting occurs because the rival firm's flow profits are higher when it is a traditional firm competing with a platform facing very risky investments. In these circumstances, the platform must compensate the rival for taking on investment risk when affiliating with the platform. Yet, when the value of cross-network interactions is very high and investments are very certain, the platform firm decides not to invite its rival, seeking to foreclose the market. Refusal to host occurs because the platform already dominates the market and the (negative) competition effect of inviting the rival when investments are certain dominates the increase in volume of cross-network interactions.

Results extend further to the case of two competing platforms. We build upon yet contrast with now classic results of Katz & Shapiro (1985). They find that firms with larger networks choose incompatibility, even when total welfare increases, while firms with smaller networks choose compatibility, even in some cases where social costs outweigh benefits. Their model considers only rival firms. By contrast, we consider a case where the platform firm, by hosting a rival, can tax a competitor's gains from compatibility in a manner that is Pareto improving. In certain cases, the host platform can increase profit even at the cost of cannibalizing its own products. Market expansion effects dominate competition effects. Demand aggregation strategies become far more important to aid transformation, raise profits, and weaken competition. These new results imply that product managers' insights regarding competitors must change as firms move to platform strategies. They also imply that regulators' views regarding breakup and collaboration among competitors must change lest they retard investment or reduce welfare.

Our model provides novel insights for managers of traditional firms who contemplate digital transformation. First, our results suggest that when the risks associated with digital transformation into a platform are high, and the perceived value of cross-network interactions is low, it may be advisable for firms to remain traditional and focus on less risky core value proposition. Second, when the risks associated with digital transformation are intermediate or low, it may be profitable for a firm to transform into a platform and to host its rivals. Inviting rivals on the platform signals greater volume of cross-sided network interactions thereby increasing market participation on both sides of the market. A more nuanced rationale for inviting rivals is also to lower a rival's investment aggressiveness while increasing the probability of successful innovation on the platform. Further, the profit from taxing rivals can outweigh the cannibalization costs. Together, these three factors lower competitive intensity for the platform firm while increasing the expected platform size. Finally, platform firms may find it profitable to even pay their rivals to join the platform when the value of cross-network interactions is not high. Inviting the rival enhances the volume of cross-network interactions which increases the industry profits.

The rest of this paper is organized as follows. Section (2) discusses the related literature and our contribution. Section (3) lays down the basic model. Section (4) characterizes the equilibrium outcomes and the welfare implications. Then, in Section 5, we discuss some extensions to our model such as price charged to developers and differentiated price competition. Section (6) discusses managerial implications and Section (7) concludes. Appendix contains two sections. Section (A) extends the basic model to the case when platform charges a fee to the developers. All proofs are in Section (B).

2 Related Literature

Our research is linked to two main strands of literature on digital transformation. First, we contribute to the literature on digital transformation of traditional firms with a focus on transformation of incumbent firms and its strategic value (Bharadwaj et al. (2013), Kane et al. (2015), Pagani & Pardo (2017), Parker et al. (2016), Matt et al. (2016), Sebastian et al. (2020), Clayton M. et al. (2018)).[6] The decision to pursue a digital transformation brings forth multiple strategic and tech-

[6]See Hanelt et al. (2021), Vial (2019), Verhoef et al. (2021), Rêgo et al. (2021), Reis et al. (2018) among others.

nological risks that must be considered and firms should exercise caution while doing so (Clayton M. et al. (2018), Markides & Sosa (2013), Karhu & Ritala (2021), Tekic & Koroteev (2019), Hinterhuber & Nilles (2021)). Differently from the previous literature, we game-theoretically formalize the digital transformation decision of a traditional firm when such a decision entails additional risk. We are able to elicit novel insights on strategic concerns that firms must bear in mind when pursuing digital transformation into platform. Additionally, our paper is also linked to the strand of literature that focuses on firms transforming into platforms and opening access of their platform to third party developers/complementors (Boudreau (2010), Mantovani & Ruiz-Aliseda (2016), Parker & Van Alstyne (2018), Tan et al. (2020)). Opening up a platform implies that third party developers can add value to the products on the platform. This shifts value creation away from the traditional firm to the complementors. However, transforming into a platform makes internal value creation more risky which lowers investment in value creation in the first place. Riskier investments (after digital transformation) make it harder for a platform to convince the market participants of its value proposition and thus also unable to attract developers and consumers. This is evidence of the importance of co-innovation by the platform firm along with external value creators.[7] Therefore, platforms must balance the risks associated with digital transformation (and lower internal value creation) with the benefits arising from external value creation by developers.

Our paper also contributes to the nascent yet growing literature on incentives of platform firms to invite rivals and encourage coopetition. Hagiu et al. (2020) considers the incentive of a multi-product firm to host specialist non-core product rivals on its platform. They find that it can be profitable to host a rival in the non-core product market, while earning in the core product market. Differently from them, we consider the incentive of a platform to host direct core-product rivals. The presence of a rival on the platform drives platform demand which expands the network of developers which feeds back into higher consumer value. Considering network effects and the incentives to invite direct rivals our work is related to Economides (1996) and Niculescu et al. (2018). We confirm the main result in Economides (1996) in a two-sided market setting when the platform is not a quantity leader. Further, we extend the model where platform firms incur risky value-enhancing investments and show that internal value creation and external value creation complement each other. Niculescu et al. (2018) also consider platform incentives to invite rivals on their exclusive intellectual property (IP) platform in presence of same-sided network effects when the entrant invests. First, we confirm the results of Niculescu et al. (2018) that when the investments are certain and value of network effects are very high, it is preferable for the platform to not invite its rival on its platform. In contrast to their result, we find that the platform does not invite its rival to its platform despite the rival being an active player in the market. Moreover, our paper differs from their paper along multiple lines. The two main differences are as follows. First, investments of firms on a platform (after digital transformation) are stochastic. Secondly, both the incumbent and the entrant invest. These differences elicit a novel strategic mechanism

[7] Michael Nilles, Chief Digital and Information Officer at Henkel, stresses the importance of co-innovation where both internal and external value creation are crucial (Hinterhuber & Nilles 2021).

through which inviting rivals may be beneficial under investment uncertainty. Inviting rivals on the platform makes their investments also stochastic and as a result lower their aggressiveness in investment. In addition to this, when the rival joins the platform, there is an ecosystem expansion effect as market participants expect a greater volume of cross-network interactions. Interestingly, we show that in some cases (when the platform's investment uncertainty is large), the platform firm may actually find it profitable to pay its rival to affiliate with its platform and participate in the external value creation. Finally, our work is also related to Huang et al. (2020). As in our paper, this work considers stochastic investment by firms and the incentive of firms to share their innovation with rivals. Differently from them, firm investments are stochastic only if firms digitally transform and are active on the same platform which enables cross-side network interaction. We find that inviting rivals to a platform can be a strategic move to lower their investment aggressiveness as investments are riskier when the rival affiliates with the platform. Nevertheless, consumers benefit when platform firms invite their rivals to participate in the ecosystem when the value of cross-sided network interactions is sufficiently high.

Our paper is also closely related to the literature on investment in platform markets. Relevant literature on this includes Anderson Jr et al. (2014), Parker & Van Alstyne (2018), Tan et al. (2020) among others. The closest paper to our work is Anderson Jr et al. (2014) where competing platforms choose value enhancing investment. In our setting, investments are also made to enhance product value by firms on the same platform, but they are stochastic as digital transformation makes investments risky while enabling external value creation.[8] In contrast to their competitive setting, we find that an increase in network effects also increases investment. This difference arises because in their setting two platforms compete while in our setting competition is between firms within a platform whose investments can (indirectly) positively impact their rival by enhancing the external value creation on the platform. Another paper closely related to ours is Tan et al. (2020). In their paper, platform investments are a co-innovation strategy which directly increase participation of third party developers as they reduce entry cost of developers. In our setting as well, platform investments also enhance external value creation through increased demand as a result it is also a co-innovation process.

3 The baseline model

Players and environment. Consider a market with two firms R_1 and R_2 that compete by setting quantities x_1 and x_2 respectively. To drive down the point on how traditional firm transformation into platforms, which orchestrate value creation from third parties, can lead to inversion of value generation, we model consumer demand in three market structures. First, when the two firms are traditional firms that have not opened up their products to third party developers. Second, we consider the case when R_2 remains a traditional firm while R_1 transforms into a two-sided platform and invites complementors (developers) to exploit application programming interfaces

[8] Xin & Choudhary (2019) is a recent work where platform investments are stochastic.

(API) that access functionality of its products and create value for consumers on the platform. Digital transformation into a platform presents business risks which are modeled as risks in the outcome of investment in product enhancement.[9] Finally, we consider the case when R_1 hosts R_2 on its platform and study the incentives to invite a rival on to the platform. Inviting a rival has two opposing effects. First, by inviting rivals on to its platform, R_1's advantage relative to its rivals diminishes, which discourages opening the platform. Second, by inviting its rival, consumer participation rises and thus cross-side interactions on the platform also rise, which makes it profitable for a larger mass of developers to be active on the platform. This encourages opening the platform. It is unclear ex-ante which of the two effects will dominate.

The augmented inverse demand. To incorporate the above features, we consider three market structures. (i) R_1 and R_2 are traditional ($g = T$), (ii) R_1 is a platform and R_2 is a traditional firm ($g = P$), and (iii) R_1 is a platform and it hosts R_2 on its platform ($g = H$).[10] The inverse demand function of two firms are as follows.[11]

$$P_1^g(v_1, \Psi_1^g, X) = 1 + v_1 + \Psi_1^g - 3X, \qquad P_2^g(v_2, \Psi_2^g, X) = 1 + v_2 + \Psi_2^g - 3X,$$

where Ψ_i^g is the degree of external value creation in the market structure $g \in \{T, P, H\}$ and v_i is the internal value investment made my firms. The inverse demand function behaves as follows. $\frac{\partial P_i^g(\cdot)}{\partial X} < 0$ for $i \in \{1, 2\}$ and $\frac{\partial P_i^g(\cdot)}{\partial \Psi_i^g} \geq 0$.

In case $g = T$, the variable representing external value creation is equal to the value 0 — $\Psi_1^T = \Psi_2^T = 0$. In case $g = P$, we extend the Katz & Shapiro (1985) framework to a two-sided framework where R_1 is a two-sided platform with developers on one side that create value for consumers and on the other side, the presence of consumers is valuable to developers.[12] To be more specific, we assume that the consumers value the presence of developers which is a proxy for external value creation denoted by $\Psi_1^P = \gamma Dev^P$ where γ is the marginal consumer value from interacting with an additional developer on platform, and Dev^P is the total mass of developers on the platform.[13] On the contrary, since R_2 remains as a traditional firm, there is no external value creation at firm R_2 — $\Psi_2^P = 0$. In essence, consumers that buy from platform R_1 benefit from value enhancing network interactions while consumers that buy from R_2 are unable to do so. Finally in case $g = H$, R_2's product also benefits from the presence of developers as consumers benefit from the external value creation from developers hosted on the platform. As a result, the additional value for the products offered by R_1 and R_2 is symmetric as external value creation arising from cross-network

[9] Digital transformation (into a platform) may divert resources of a firm towards investment in platform related products that may be risky (Hinterhuber & Nilles 2021).

[10] We discuss the scenario where both firms form separate platforms in Subsection 4.6. We demonstrate that it is never optimal for firm 2 to create its own separate platform when firm 1 has already done so.

[11] The microfoundations for these demands are presented in the Appendix E.

[12] A discussion on this is also available in Padilla et al. (2021).

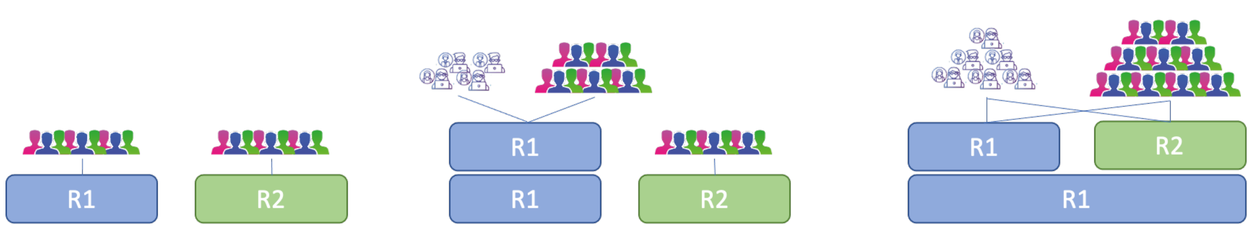
[13] For simplicity and to focus on our main point, we assume that consumers interact and derive value from the presence of all developers active on the platform. This is a common assumption in the literature on platform economics. See Armstrong (2006), Belleflamme & Peitz (2019), Parker & Van Alstyne (2005), Parker et al. (2016) among others.

interactions is the same on the platforms. Specifically, $\Psi_1^H = \Psi_2^H = \gamma Dev^H > 0$ where Dev^H is the total mass of active developers on the platform under market structure H.

Developer market. In our modelling set-up, the developer side is active only after R_1 transforms into a platform — i.e., case $g = P$ or $g = H$. Developers derive value ϕ per unit of consumer when interacting with consumers that buy products affiliated with the platform. We assume developers are heterogeneous in their investment cost of developing applications. Let k be the cost which follows the distribution $\Lambda(\,)$. For simplicity we assume that Λ is a uniform distribution over 0 and 1 i.e. $k \sim U[0,1]$. Developers' participation in the platform market in the two market cases ($g = P$ and $g = H$) is described below.[14]

Market structure $g = P$. In this market structure, the utility of a type-k developer, is $\pi^P(k) = \phi x_1^e - k$, where x_1^e is developers' expectation on the total mass of consumers participating on the platform R_1. Developers affiliate with the platform only if they obtain positive value from the participating in the platform — i.e., for any $k < \phi x_1^e$. Thus, the mass of developers participating in the platform is $Dev^P = \phi x_1^e$.

Market structure $g = H$. When platform R_1 invites its rival R_2 to participate in the platform, the developers now are able to interact with all the consumers in the market. The utility of a developer of type k is $\pi^H(k) = \phi X^e - k$ where $X^e = \sum_{i=1}^2 x_i^e$ is the total market demand that developers expect in the market. As before, developers affiliate with a platform only if they gain positive value from the participating in the platform — i.e., for any $k < \phi X^e$. Thus, the mass of developers participating in the platform is $Dev^H = \phi X^e$. Note that the benefit of higher consumer demand on the platform is that now even the high cost developers find it profitable to affiliate with the platform which was not possible with lower levels of consumer demand. We discuss the key differences in the three market-structures in Figure (1).



(a) Both rivals are traditional pipeline firms $g = T$

(b) Digital Transformation, only R1 is platform $g = P$

(c) Digital Transformation, R1 as platform hosts R2 $g = H$

Figure 1: Industry Structure: In Panel (a), both rival firms remain traditional ($g = T$). In Panel (b), R_1 risks platform transition to attract developers and consumers, while R_2 remains traditional ($g = P$). In Panel (c), R_1 risks platform transition and hosts R_2, further increasing network effects ($g = H$).

[14]Here we assume that developers' participation is free. We extend this model to the case where platform orchestrates value creation through developers along with charging a fixed fee for their participation. Our results hold in this general model as well. Details are available in the Appendix, Section A.

The following table summarizes inverse demand in different market-structures.

Market Structure	Both rivals are traditional pipeline firms ($g = T$)	Digital transformation, only R_1 is platform ($g = P$)	Digital transformation, R_1 as platform hosts R_2 ($g = H$)
External value creation (Ψ_i)	$\Psi_1^T = 0$; $\Psi_2^T = 0$	$\Psi_1^P = \gamma Dev^P$; $\Psi_2^P = 0$	$\Psi_1^H = \Psi_2^H = \Psi^H = \gamma Dev^H$
Inverse Demand	$P_1 = 1 + v_1 - 3X$	$P_1 = 1 + v_1 + \Psi_1^P - 3X$	$P_1 = 1 + v_1 + \Psi^H - 3X$
	$P_2 = 1 + v_2 - 3X$	$P_2 = 1 + v_2 - 3X$	$P_2 = 1 + v_2 + \Psi^H - 3X$

Table 1: Inverse demand in different market structures

Firm profits. Firm profits are just the market revenue given market structure $g \in \{T, P, H\}$ minus the investment costs. Specifically, the profit of firm i in the market structure g is given as

$$P_i^g(v_i, \Psi_i^g, X)x_i - I(v_i).$$

where x_i is the output of R_i and Ψ_i^g is the external value creation given market structure g.[15] We assume the investment cost is given as $I(v) = \frac{v^2}{2}$ which is strictly increasing, strictly convex and symmetric for both firms — $I'(v) > 0$ and $I''(v) > 0$.
For a quick perusal of the important notations, a table of important notations and their associated interpretations is available after the Conclusion in Table (2).

Risks associated with digital transformation. To model risks associated with digital transformation, we assume that investments become risky when firms are part of a platform while their investments are secure and certain if they remain traditional. This difference in investment certainty between the two business models can arise because of firms' long term experience in traditional business models and thus being able to better gauge the value creation from investment. Instead, when firms migrate their business to a new business model, they face multiple uncertainties including investment uncertainties. To capture this, if R_1 transforms into a platform and then invests in value additions v_1, the success of this investment is stochastic while investment costs are certain. In particular, R_1's investment is successful with a probability of $\Omega \in [0, 1]$. On the contrary, R_1's investment is unsuccessful with a probability of $(1 - \Omega)$ and in this case $v_1 = 0$ implying consumers do not obtain any positive value. Further, if R_1 does not invite R_2 ($g = P$), R_2's investment is successful with certainty, yet it must take into account the rival's investment outcome. When R_1 invites R_2 to join its platform ($g = H$), the investments of both firms become uncertain, and the probability of its success is Ω.

Timing, contracts and equilibrium concept. The timing of the game is as follows:

[15] In favor of brevity, we have normalized the marginal cost of production to zero. Introducing a positive per unit cost would not (qualitatively) affect our results.

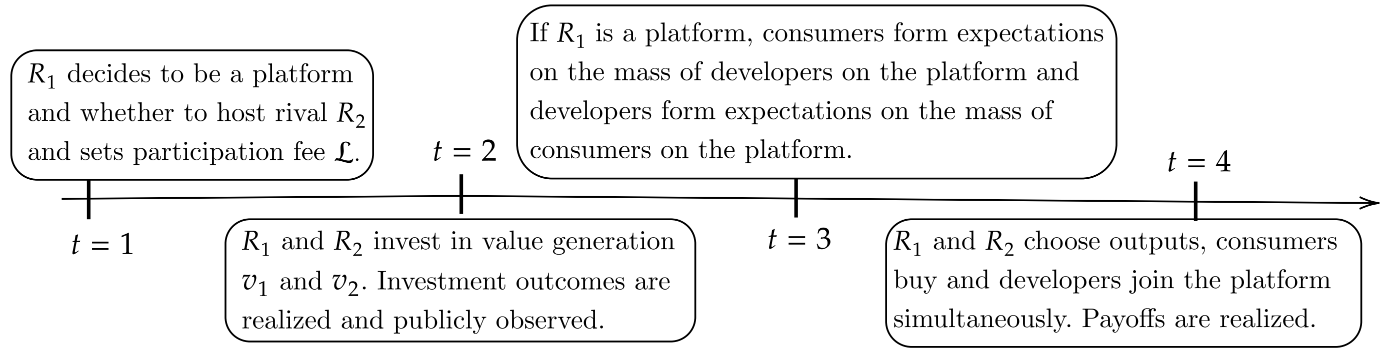


Figure 2: Timeline of the game

As Katz & Shapiro (1985), our solution concept in the retail competition game is Fulfilled Expectations Cournot Equilibrium, where each firm chooses its output level under the assumption that consumers' expectations are consistent with the equilibrium outcome — i.e., rational expectations.

Assumption 1 *For brevity, we will employ a variable transformation where* $\theta = \phi\gamma$ *with* $0 < \theta < \frac{(15-\sqrt{105})}{4}$.

This variable reflects the intensity of cross-network benefit from interactions at the platform. The bounds of this variable ensure that the value of cross-network interactions is not too high that digital transformation creates a monopoly. In this paper, we focus on the case where digital transformation is risky along with the strategic risks arising from the response of traditional rivals.

4 Analysis

In this section, we discuss each of the market outcomes $g \in \{T, P, H\}$ and present a detailed analysis for any given transformation risk $\Omega \in [0, 1]$.

4.1 R_1 and R_2 are traditional firms ($g = T$)

In this market structure, there is no active market for developers (complementors) who add complementary value to the product— $\Psi_i^T = 0$ for each $i \in \{1, 2\}$. The two firms compete in a traditional manner without any external value creation. Panel (a) of Figure 1 presents the market structure when $g = T$. Given investment in stage 2, firms set outputs to maximize their profits

$$\max_{x_i} \Pi_i^T(v_i, 0, X) - I(v_i) = P_i^T(v_i, 0, X)x_i - I(v_i).$$

Differentiating the profit expression of firm R_i with respect to its output and solving the system of first order conditions yields the individual and total equilibrium outputs as a function of investments of each individual firm $i \in \{1, 2\}$ given by $x_i^T(v_i, v_{-i}) = \frac{1+2v_i-v_{-i}}{9}$, $X^T(v_1, v_2) = \frac{2+v_1+v_2}{9}$.
In stage 2, each firm R_i invests to maximize profits. Before we proceed further to the investment outcomes, it is worthwhile to discuss the best responses of the two firms. The best response of

each firm is $v_i^{T,BR}(v_{-i}) = \frac{4(1-v_{-i})}{19}$ whose slope is negative with respect to the rival's investment — $\frac{\partial v_i^{T,BR}(v_{-i})}{\partial v_{-i}} = -\frac{4}{19} < 0$. Thus, we conclude that investments are strategic substitutes in our model. The equilibrium investment outcome is symmetric and given as $v_1^T = v_2^T = v^T = \frac{4}{23}$. As a consequence, the associated outputs and profits of each firm are also symmetric and given as $x_i^{T\star} = \frac{3}{23}$ and $\Pi_i^T = \frac{19}{529}$.

4.2 R_1 is a platform and R_2 is a traditional firm ($g = P$)

In this subsection, we consider the case when R_1 transforms into a platform and is a closed platform i.e. R_1 does not invite R_2 over its platform and hence, R_2 remains a traditional firm. After transformation, R_1 invites developers (complementors) to create additional complementary value for its product in the market. In this case, investment uncertainty affects $R_1's$ investment while $R_2's$ investment is certain. Panel (b) in Figure 1 presents the market structure when $g = P$.

Let $\hat{v}_1$ and $\hat{v}_2$ be the values of realized investment outcomes after stage (2.1). We employ a different notation of $\hat{v}_i$ representing the realization of uncertain investment rather than using actual investment v_i levels. This is to emphasize that the equilibrium outputs and profits in stage 4 are functions of realized value of investment rather than the actual investment. When $g = P$, if actual investment levels are v_1 and v_2 then $\hat{v}_1 \in \{0, v_1\}$ and $\hat{v}_2 = v_2$. The asymmetry between the two firms due to the market structure $g = P$ is modeled by $\Psi_1^P = \gamma Dev^{P,e}$ while $\Psi_2^P = 0$ which is reflected in the inverse demands.

$$P_1^P(\hat{v}_1, \Psi_1^P, X) = 1 + \hat{v}_1 + \Psi_1^P - 3X, \qquad P_2^P(\hat{v}_2, 0, X) = 1 + \hat{v}_2 - 3X. \tag{1}$$

Output setting stage. Given consumer and developer expectations in stage 3 and investment levels in stage 2, in stage 4 firms set outputs to maximize their profits

$$\max_{x_i} \Pi_i^P(\hat{v}_i, \Psi_i^P, X) - I(v_i) = P_i^P(\hat{v}_i, \Psi_i^P, X)x_i - I(v_i).$$

Note that the investment cost is not a function of $\hat{v}_i$ and instead a function of the actual investment in Stage 2. This reflects our transformation risk which accounts for the fact that while investment costs are certain, the value generated by such investments may not always be realized. Differentiating the profit expression of firm R_i with respect to its output and imposing rational expectations — $x_1^P(v_1, v_2) = x_1^e$, $\Psi_1^P(v_1, v_2) = \gamma Dev^P(v_1, v_2) = \gamma\phi x_1^P(v_1, v_2)$, yields the output of R_1 and R_2 as a function of realized investments. From here on, we will employ the variable transformation $\theta = \gamma\phi$. The outputs as a function of realized investments are given as $x_1^P(\hat{v}_1, \hat{v}_2) = \frac{1+2\hat{v}_1-\hat{v}_2}{9-2\theta}$, $x_2^P(\hat{v}_2, \hat{v}_1) = \frac{3(1-\hat{v}_1)-\theta+\hat{v}_2(6-\theta)}{27-6\theta}$. The total output as a function of investment realization and firm profits are given as $X^P(\hat{v}_1, \hat{v}_2) = x_1^P(\hat{v}_1, \hat{v}_2) + x_2^P(\hat{v}_2, \hat{v}_1) = \frac{3(2+\hat{v}_1+\hat{v}_2)-\theta(1+\hat{v}_2)}{27-6\theta}$ and $\Pi_i^{P\star}(\hat{v}_i, \hat{v}_{-i}) = P_i^P(\hat{v}_i, \Psi_i^P(\hat{v}_i, \hat{v}_{-i}), X^P(\hat{v}_1, \hat{v}_2))x_i^P(\hat{v}_i, \hat{v}_{-i})$ for $i \in \{1, 2\}$.

Innovation stage. In stage 2, firms unilaterally invest in the value for the product v_i to maximize expected profits as expressed below. The expected profits of platform R_1 and firm R_2 are given as

$$\begin{aligned} \mathsf{E}\Pi_1^{P?}(v_1, v_2) \quad I(v_1) &= \Omega\Pi_1^{P?}(v_1, v_2) + (1 \quad \Omega)\Pi_1^{P?}(0, v_2) \quad I(v_1), && (2) \\ \mathsf{E}\Pi_2^{P?}(v_2, v_1) \quad I(v_2) &= \Omega\Pi_2^{P?}(v_2, v_1) + (1 \quad \Omega)\Pi_2^{P?}(v_2, 0) \quad I(v_2). && (3) \end{aligned}$$

Notice that investment decision of R_1 relies totally on the revenues arising in the case when investment is successful.

Differentiating the profit of platform firm R_1 as expressed in equation (2) with respect to v_1 and employing the envelope theorem yields

$$\Omega \left[x_1^P(\,) \left(\underbrace{\frac{\partial P_1^P(\,)}{\partial \hat{v}_1}}_{\substack{\text{Internal} \\ \text{value e ect}}(+)} + \underbrace{\frac{\partial P_1^P(\,)}{\partial \Psi_1^P} \frac{\partial \Psi_1^P}{\partial x_1^e} \frac{\partial x_1^P(\,)}{\partial \hat{v}_1}}_{\text{External value e ect }(+)} + \underbrace{\frac{\partial P_1^P(\,)}{\partial X} \frac{\partial x_2^P(\,)}{\partial \hat{v}_1}}_{\text{Strategic e ect}(+)} \right) \right] \Big|_{\hat{v}_1 = v_1} \quad \frac{\partial I(v_1)}{\partial v_1} = 0. \qquad (4)$$

The incentive to innovate by R_1 can be broken down into multiple effects. First, there is effect of risk multiplied to marginal revenue term. As Ω increases, transformation gets less risky which encourages firm 1 to invest more. Second, there is the direct positive effect on prices that increases margins of R_1 for every unit of output sold through increased consumers' willingness to pay. Third, there is a positive effect on margins through increase in developer participation which enhances consumers' willingness to pay. Notice that an increase in internal value also positively impacts external value. This is evidence of how the internal and external value creation are intertwined.[16] Fourth, there is the positive effect on R_1's price through reduction in R_2^0s output. Instead, the cost of investment v_1, which is increasing and convex, represents a negative effect on investment incentives. R_1 trades-off these positive and negative effects of an increase in v_i on profitability and on equilibrium these marginal gains are exactly equal to the marginal cost of investment.

Analogously, differentiating the profits of firm R_2 with respect to v_2 and employing the envelope theorem yields

$$\mathsf{E}_{\hat{v}_1} \left[x_2^P(\,) \left(\underbrace{\frac{\partial P_2^P(\,)}{\partial v_2}}_{\substack{\text{Internal} \\ \text{value e ect}}(+)} + \underbrace{\frac{\partial P_2^P(\,)}{\partial X} \frac{\partial x_1^P(\,)}{\partial v_2}}_{\text{Strategic e ect}(+)} \right) \right] \quad \frac{\partial I(v_2)}{\partial v_2} = 0. \qquad (5)$$

The innovation incentives of R_2 also can be decomposed into two effects. First, there is the direct positive effect on prices that increases margins of R_2 for every unit of output sold. Second, there is the positive effect on price through reduction in the R_1^0s output. Solving the above first order conditions respectively for v_1 and v_2, we compute the best responses and observe that the slope of the best response is negative for both R_1 and R_2.[17] This gives us the important result that

[16] See Hinterhuber & Nilles (2021) where Henkel's Chief Digital Officer stresses the importance of co-innovation.

[17] For further details, see equation (42) and (43) presented in Appendix (B.1).

investments in our setting are strategic substitutes.

By solving for the investment best responses of both firms simultaneously, we obtained the optimal investment levels, which are

$$v_1^P = \frac{12\Omega(9-2\theta)(15-4\theta)}{((9-2\theta)^2-24\Omega)(171-2\theta(42-5\theta))-72\Omega^2(6-\theta)} \text{ and}$$

$$v_2^P = \frac{2(6-\theta)\left(((9-2\theta)^2-24\Omega)(3-\theta)-36\Omega^2\right)}{((9-2\theta)^2-24\Omega)(171-2\theta(42-5\theta))-72\Omega^2(6-\theta)}.$$

These investment levels by R_1 (R_2) rises (falls) as investments gets more certain — $\frac{\partial v_1^P}{\partial \Omega} > 0$ ($\frac{\partial v_2^P}{\partial \Omega} < 0$). As the uncertainty regarding investment in a platform rises (as Ω falls), the platform firm is more cautious in its investment strategy and finds it profitable to invest lesser. This lower investment levels by R_1 encourages R_2 (the traditional firm) to invest more as investments are strategic substitutes.

The expected optimal profit of platform firm R_1 and its rival R_2 in case $g = P$ are given as

$$\Pi_1^{P**} = \mathsf{E}\Pi_1^{P*}(v_1^P, v_2^P) - I(v_1^P) \text{ and } \Pi_2^{P**} = \mathsf{E}\Pi_2^{P*}(v_2^P, v_1^P) - I(v_2^P).$$[18]

Performing comparative statics on the expected profits of R_1 and R_2, we present the result in the following Lemma.

Lemma 1 (Case $g = P$: Expected profit) *The expected profit of R_1 (R_2) rises (falls) with the certainty of investment* | $\frac{d\Pi_1^{P**}}{d\Omega} > 0$ ($\frac{d\Pi_2^{P**}}{d\Omega} < 0$). *Moreover, the expected profit of platform R_1 is higher than the expected profit of R_2 when $\theta > \tilde{\theta}^P(\Omega)$.*

As investments at the platform R_1 become more certain (as Ω increases), the platform finds it profitable to invest more and benefits from the external value creation. On the contrary, as investments become more certain (as Ω gets larger), firm R_2 lowers its investment as it expects the rival to invest on average more. This lowers its market price and output and thus also its profitability falls. We compare the profit of platform R_1 with the profit of traditional firm R_2 and illustrate the two cases when platform R_1 earns more than its rival R_2 and vice-versa present our results in Figure (3). This figure emphasizes the fact that being a platform may not always lead to a market-winning situation for R_1 even if it may gain from external value creation. This is because transforming into a platform can be risky and can negatively impact incentives to invest. This investment cautiousness of the platform R_1 when investment is risky is internalized by its rival who then finds it profitable to enhance its investment levels and increase its profits. As a result, transformation of R_1 into a platform may make it worse-off relative to its rival. Intuitively, this happens when transformation is quite risky, and the value of cross-sided network effects is sufficiently low (grey shaded region in Figure (3)).[19]

[18] The explicit expressions can be found in the appendix in the proof of Lemma 1.

[19] On the contrary when network effects are high and investment is sufficiently certain, R_1's profit from being a platform are higher than its rival's profit (brown shaded region in Figure (3)).

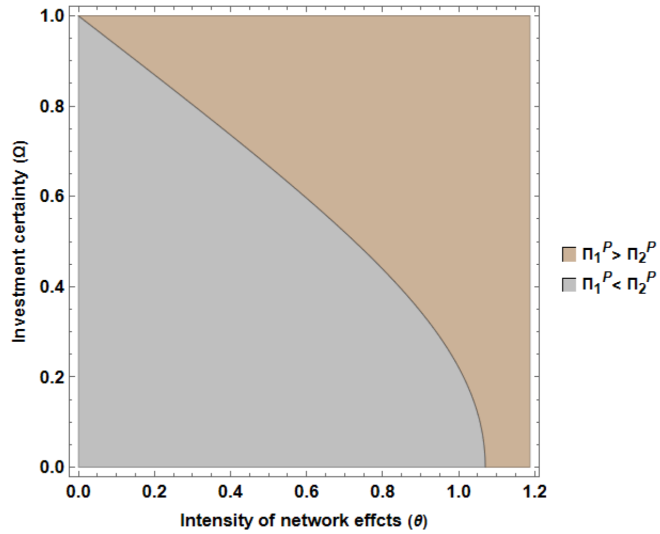


Figure 3: Comparing of platform R_1's profit vs. its rival R_2.

4.3 Platform R_1 hosts the rival firm R_2 $(g = H)$

In this subsection, we consider the case where R_1 transforms into an open platform and invites rival R_2 to participate on its platform.[20] The platform R_1 when inviting its rival on the platform must consider the following trade-off. On the one hand, by inviting the rival R_2 onto its platform, R_1 gives R_2 access to developers on its platform and lowers its platform advantage which negatively impacts R_1's profitability. On the other hand, inviting the rival implies increased external value creation and a more vibrant platform ecosystem. In addition, there is a nuanced benefit of R_1 for inviting its rival on the platform. By affiliating with the platform, R_2's investments also become stochastic. This makes R_2 more cautious in its investment strategy as the investment uncertainty of migrating to a new business model rises. This increased investment uncertainty results in dampening of investment externality exerted by R_2 upon R_1. Panel (c) of Figure (1) presents the market structure when $g = H$.

Let $\hat{v}_1$ and $\hat{v}_2$ be the values of realized investment outcomes.[21] Then, we present the inverse demand expression of R_1 and R_2 as

$$P_1^H(\hat{v}_1, \Psi^H, X) = 1 + \hat{v}_1 + \Psi^H \quad 3X, \qquad P_2^H(\hat{v}_2, \Psi^H, X) = 1 + \hat{v}_2 + \Psi^H \quad 3X. \tag{6}$$

Recall that the mass of developers is given as $Dev^H(X^e)$, where developers join expecting the total market demand.

[20] For instance, Klockner, a steel distributor, developed a platform and invited its rivals to participate.

[21] In the case $g = H$, if actual investment levels are v_1 and v_2, then $\hat{v}_1$ 2 f0; v_1g and $\hat{v}_2$ 2 f0; v_2g.

Output setting stage. Each firm sets outputs to maximize profits which are given as

$$\max_{x_1} P_1^H(\hat{v}_1, \Psi^H, X)x_1 - I(v_1), \ \max_{x_2} P_2^H(\hat{v}_2, \Psi^H, X)x_1 - I(v_2).$$

Differentiating the profit expression of firm R_i for $i \in \{1, 2\}$ with respect to its output and imposing rational expectations — $X^H(\hat{v}_1, \hat{v}_2) = x_1^e + x_2^e$, $\Psi^H(\hat{v}_1, \hat{v}_2) = \theta X^H(\hat{v}_1, \hat{v}_2)$, yields the output of R_1 and R_2 as a function of realized investments. The output of firm R_i as a function of realized investments are given as $x_i^H(\hat{v}_i, \hat{v}_{-i}) = \frac{3+\hat{v}_i(6-\theta)-\hat{v}_{-i}(3-\theta)}{27-6\theta}$. The total output and gross profit as a function of realized investments are given as $X^H(\hat{v}_1, \hat{v}_2) = x_1^H(\hat{v}_1, \hat{v}_2) + x_2^H(\hat{v}_2, \hat{v}_1) = \frac{2+\hat{v}_1+\hat{v}_2}{9-2\theta}$ and

$$\Pi_i^{H\star}(\hat{v}_i, \hat{v}_{-i}) = P_i^H((\hat{v}_i, \Psi^H(\hat{v}_1, \hat{v}_2), X(\hat{v}_1, \hat{v}_2))x_1^H(\hat{v}_i, \hat{v}_{-i}). \tag{7}$$

We employ the above expression to construct the expected revenue of the two firms on the platform.

$$\mathbb{E}\Pi_i^{H\star}(v_i, v_{-i}) = \Omega^2\Pi_i^{H\star}(v_i, v_{-i}) + \Omega(1-\Omega)\Pi_1^{H\star}(0, v_{-i}) + \Omega(1-\Omega)\Pi_1^{H\star}(v_i, 0) + (1-\Omega)^2\Pi_1^{H\star}(0, 0). \tag{8}$$

Innovation stage: In stage 2, given fixed fee L, firms unilaterally invest in the value for the product v_i to maximize expected profits given as

$$\mathbb{E}\Pi_1^{H\star}(v_1, v_2) - I(v_1) + L, \quad \text{and} \quad \mathbb{E}\Pi_2^{H\star}(v_2, v_1) - I(v_2) - L.$$

Differentiating the profits of firm R_i with respect to v_i for $i \in \{1, 2\}$ and employing the envelope theorem yields

$$\mathbb{E}_{\hat{v}_{-i}}\left[x_i^H(\cdot)\left(\underbrace{\frac{\partial P_i^H(\cdot)}{\partial \hat{v}_i}}_{\text{Internal value effect}(+)} + \underbrace{\frac{\partial P_i^H(\cdot)}{\partial \Psi^H}\frac{\partial \Psi^H}{\partial X^e}\frac{\partial X^H(\cdot)}{\partial \hat{v}_i}}_{\text{External value effect }(+)} + \underbrace{\frac{\partial P_i^H(\cdot)}{\partial X}\frac{\partial x^H{}_{-i}(\cdot)}{\partial \hat{v}_i}}_{\text{Strategic effect}(+)}\right)\Bigg|_{\hat{v}_i = v_i}\right] - \frac{\partial I(v_i)}{\partial v_i} = 0, \text{ for } i \in \{1, 2\}. \tag{9}$$

As before, we can decompose down the incentives to invest into internal value effect, external value effect and the strategic effect. All of these effects positively impact the incentives to invest and are countervailed by the increased investment costs. A cursory look at the best responses, we find that they are downward sloping.[22] Further, to understand how the slopes of best responses differ in the two market structures $g = P$ and $g = H$, we compare slopes of best responses of the two firms across market structures. We note that, for firm R_1, the difference between the slope of best response in case $g = H$ and $g = P$ is positive i.e. — $\frac{\partial v_1^{H,BR}(v_2)}{\partial v_2} - \frac{\partial v_1^{P,BR}(v_2)}{\partial v_2} > 0$.[23] Similarly for firm R_2 the same relation holds $\frac{\partial v_2^{H,BR}(v_1)}{\partial v_1} - \frac{\partial v_2^{P,BR}(v_1)}{\partial v_1} > 0$.[24] Notice that by inviting its rival R_2,

[22] For further details, see equation (46) presented in Appendix (B.1).

[23] In particular, note that $\frac{\partial v_1^{H,BR}(v_2)}{\partial v_2} - \frac{\partial v_1^{P,BR}(v_2)}{\partial v_2} = 2\Omega\left(\frac{6}{(9-2\theta)^2-24} - \frac{(6-\theta)(3-\theta)}{3(9-2\theta)^2-2(6-\theta)^2}\right) > 0$.

[24] Further, observe that $\frac{\partial v_2^{H,BR}(v_1)}{\partial v_1} - \frac{\partial v_2^{P,BR}(v_1)}{\partial v_1} = \frac{2(6-\theta)\left(-(5\theta(2(\theta-12)\theta+99)-729)+9(9-2\theta)^2\right)}{(2\theta(5\theta-42)+171)\left(3(9-2\theta)^2-2(6-\theta)^2\right)} > 0.$

platform R_1's negative response to a unit increase in R_2's investment is dampened. This implies that the investment externality affects R_1 to a lower degree in case $g = H$ than in case $g = P$. Likewise, by inviting the rival on the platform, the rivals' response to increased investment is also dampened in comparison to case $g = P$. This dampening of the best responses arises from the fact that an increase in the rivals' investment positively impacts the profitability of R_i through increased developer participation on the platform.

Solving simultaneously the best responses of the two firms, we obtain the optimal symmetric investments $v^H = \frac{6 \ (6 \ )}{2 \ ^2(6 \ )(3 \ ) \ 2 \ (6 \ )^2+3(9 \ 2 \ )^2}$. This equilibrium investment level rises as investments get more certain — $\frac{@v^H}{@} > 0$.[25] This relation is quite intuitive. An increase in Ω has two opposing effects. Firstly, an increase in Ω has a direct effect of enhancing the incentive to invest. Second, an indirect effect through increased investment incentive of the rival, which reduces investment incentive. The former direct effect dominates the latter and thus, we have the result that as Ω increases, investments rise as well.

Optimal contract L. The optimal fixed fee is given as

$$L^{?} = \mathsf{E}\Pi_2^{H?}(v^H, v^H) \quad I(v^H) \quad \left(\mathsf{E}\Pi_2^{P?}(v_2^P, v_1^P) \quad I(v_2^P)\right).$$

The optimal contract is set such that R_2 is indifferent between accepting or rejecting R_1^{0}s optimal contract offer.

Lemma 2 *The optimal contract is unambiguously increasing as investments become more certain (as Ω rises). There exists a threshold $\tilde{\Omega}^L(\theta)$ below which the optimal fee is negative. In other words, Platform R_1 pays its rival to join the platform when $\Omega < \tilde{\Omega}^L(\theta)$.*

The intuition for main result that the optimal contract is increasing in Ω is nuanced. There are two forces whose total gives us the result that L is increasing with Ω. First, as investment certainty increases, both firms find it profitable to increase investments which may increase their profitability as external value creation also rises. Second, as investment certainty at the platform increases, the profit of R_2 in case $g = P$ falls due to increased investment by R_1. This fall in the outside option of R_2 further increases $L^{?}$. This negative effect which increases the optimal fee outweighs any opposing effect that might arise from changes in the flow profit. The intuition for the negative fees is straightforward. When Ω is small, there is large uncertainty in platform investments and as a result R_2^{0}s investment is also low when it accepts the offer of the platform. In this case, R_2 earns higher profits by rejecting any free offer ($L = 0$) by platform R_1 to join its platform. This is because as Ω falls (investments are more uncertain), the profit of R_2 in case $g = P$ rises and can be above what it gains by being in the platform. Therefore, the platform must compensate its rival for this loss in profit when inviting R_2. In the following, we discuss and present the circumstances where such a strategy of compensating the rival to join its platform can be an equilibrium outcome.

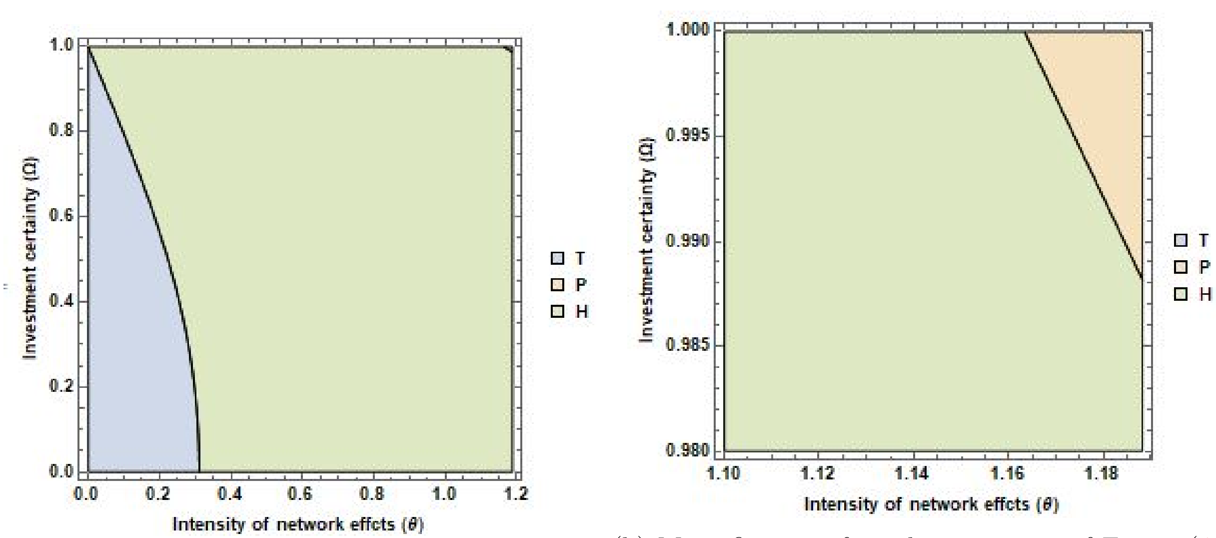
[25] The expression is given as $\frac{\partial v^H}{\partial} = \frac{6(6 \ \theta)(3(9 \ 2\theta)^2+2 \ ^2(6 \ \theta)(3 \ \theta))}{(2 \ ^2(6 \ \theta)(3 \ \theta) \ 2 \ (6 \ \theta)^2+3(9 \ 2\theta)^2)^2} > 0$:

4.4 To be a platform or not and the incentives to host rival R_2

Comparing the profit of R_1 in the three market structures g 2 fT, P, Hg, we discuss the results in the following Proposition.

Proposition 1 *It is preferable for R_1 to stay a traditional rm when $\theta < \theta^T = 0.311$ and $\Omega < \Omega^T(\theta)$ with $\frac{@\ ^T(\ )}{@} < 0$. R_1 chooses to be a platform and hosts its rival R_2 on its platform when (i) $\theta < \theta^T$ and $\Omega > \Omega^T(\theta)$, (ii) $\theta^T < \theta < \tilde{\theta} = 1.16$ and $0 < \Omega < 1$, (iii) $\tilde{\theta} < \theta$ and $0 < \Omega < \Omega^H(\theta)$. R_1 chooses to be a platform and does not host its rival when $\theta > 1.16$ and $\Omega > \Omega^H(\theta)$.*

R_1 prefers to stay a traditional firm and not transform when the investment risks are high, and the value of cross-network interactions is sufficiently low. The intuition for the results in the above proposition is as follows. Firm R_1 must trade-off the benefits of being a platform with the potential risks. Figure (4a) illustrates the regions in the relevant (Ω, θ) space where the different market structures occur.



(a) Equilibrium market structures

(b) Magnification of north-east corner of Figure (4a) to show the case $g = P$.

Figure 4: Market structure and magnification of area where market structure is $g = P$

When investment uncertainty is high (Ω is low) and value of cross-side network interactions (θ) is low, R_1 avoids digital transformation and the equilibrium outcome is $g = T$ (blue shaded region in Figure (4a)). In this parameter constellation, the potential negative impact on R_1's profits by being a platform dominates any positive benefits from being a platform vis-á-vis being a traditional firm. When the value of cross-network interactions is sufficiently high, it is always profitable for R_1 to transform into a platform based business model for any level of investment uncertainty. This is because now external value creation is high enough to justify any losses from uncertain investment in relation to deterministic profits in the traditional business model case. Given that R_1 transforms into a platform, for intermediate values of cross-network interactions, the platform always finds it profitable to invite its rival on its platform. This is because by inviting its rival R_2 on its platform,

R_1 dampens R_2's aggressiveness in investment while at the same time enhancing external value creation through increased platform demand due to R_2^{0}s participation on the platform. Such a strategy is profitable in most cases except when the value of cross-network interactions is very high and investments are close to certain. In this case, the platform R_1 chooses not to invite its rival to participate in the platform and marginalizing its rival R_2 by dominating the market (yellow shaded region in Figure (4a) and in Figure (4b)).

Interestingly, we find that under some parameter constellations, it is profitable for R_1 to pay its rival to participate in its platform.

Corollary 1 *Platform R_1 nds it pro table to pay its rival R_2 to participate in its platform when network e ects are moderate and certainty in investment is low.*[26]

The intuition for this interesting but counter-intuitive result is as follows. Being a platform has its drawbacks arising from uncertainty of investments. By inviting a rival onto its platform, R_1 makes R_2 less aggressive in its investments strategy. Figure 5 illustrates the regions in the $(\Omega,\ \theta)$ space where R_1 hosts its rival and when the optimal contracts are negative.

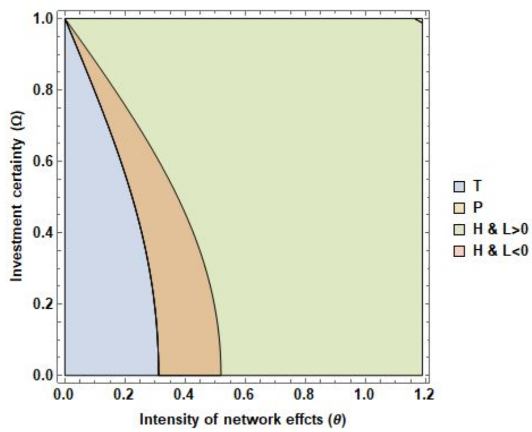


Figure 5: Negative fees charges to rivals

The orange shaded region is the case when R_1 chooses to be a platform than being a traditional firm and compensates R_2 ($L^? < 0$) to encourage its participation in the value creation at the platform. When the value of cross-sided network interactions is not high (for low θ) and investments are not certain, R_2's losses from participating in the platform through uncertain investments may be higher than forgoing third party value creation. In such circumstances, R_1 still prefers a platform structure to a traditional firm structure but must compensate R_2 to encourage its participation on the platform. This resembles developer subsidies (Parker & Van Alstyne 2005, Rochet & Tirole 2006) but operates via a different mechanism on competitors. In the green region, R_2^{0}s value from participating in the platform is larger than being a traditional firm and R_1 extracts this surplus by setting $L^? > 0$.

[26] Specifically, the platform finds it profitable to hosts its rival and charge a negative fee (i) when $< 0{:}311$, $\Omega^k < \Omega < \Omega^l$, (ii) when $0{:}311 < \ < 0{:}519$, $0 < \Omega < \Omega^l$. The expressions for Ω^l and Ω^k are involved and are available upon request. We provide a graphical proof in the paper.

4.5 Consumer surplus and welfare implications

Consumer surplus in the traditional market structure $g = T$ is as before.

Under uncertainty of investment, we consider the expected consumer surplus in the market structures $g = P$ is given as

$$\mathsf{E}CS^{P?} \quad = \quad \frac{(\Omega(X^P(v_1^P, v_2^P))^2 + (1 \quad \Omega)(X^P(0, v_2^P))^2)}{6}. \tag{10}$$

Similarly, the expected consumer surplus in the market structure $g = H$ is given as

$$\mathsf{E}CS^{H?} \quad = \quad \frac{(\Omega^2(X^H(v^H, v^H))^2 + 2\Omega(1 \quad \Omega)(X^H(0, v^H))^2) + (1 \quad \Omega)^2(X^H(0,0))^2))}{6}. \tag{11}$$

Comparing the consumer surplus in the three cases, we present the results in the following Proposition.

Proposition 2 *Consumer surplus under hosting rival R_1 is highest when $\theta > \theta^{CS;H}$ $|$ $\mathsf{E}CS^{H?} >$ Max$f\mathsf{E}CS^{P?}, CS^T g$. Else, when $\theta < \theta^{CS;H}$, the market structure $g = T$ when R_1 and R_2 are traditional rms leads to the highest consumer surplus.*

It is straightforward that consumers benefit from both firms being hosted on the platform. This is because of increased expected value creation on the platform and also fiercer competition. The case $g = P$ can never be an outcome where consumer surplus is maximized. However, it is unclear whether adopting a platform business model and hosting rivals is better for consumers than when firms compete under a traditional business model. The following plot illustrates our results for the relevant parameter space.

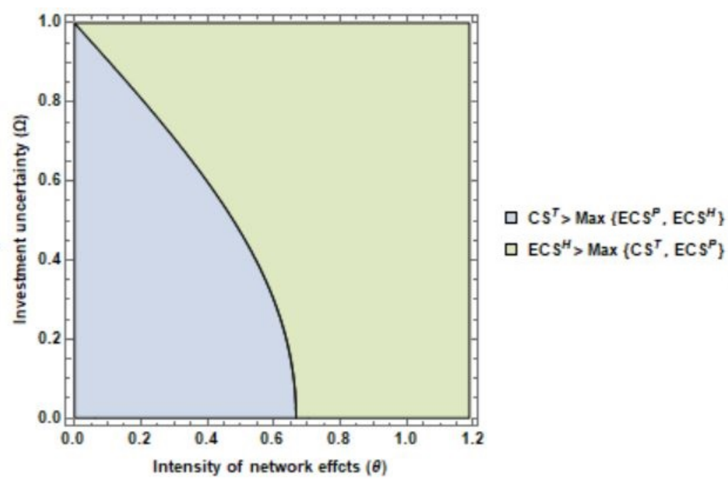


Figure 6: Regions where CS in a regime is the highest.

When investment uncertainty levels are high, investments on platform are much lower than when firms compete in a traditional market setting. Consumers care about total value of the products and when the external value creation on platform is unable to compensate for reduced investment intensity of firms, it is better for consumers when firms do not pursue a digital transformation.

Now, we study the cases when the firm decision to pursue digital transformation is aligned with consumer surplus maximizing market structure. Comparing the results in Proposition (2) and in Proposition (3), we present our findings in the following Proposition.

Proposition 3 *The decision of R_1 to pursue digital transformation is aligned with the consumer surplus maximizing market structure in most cases except when the value of cross-sided network interactions is moderately low or when the value of cross-sided network interactions is very large and investments are certain.*

The above proposition states that there are two regions when the choice of R_1 is not aligned with the consumer surplus maximizing market structure. We illustrate these results in the following plot.

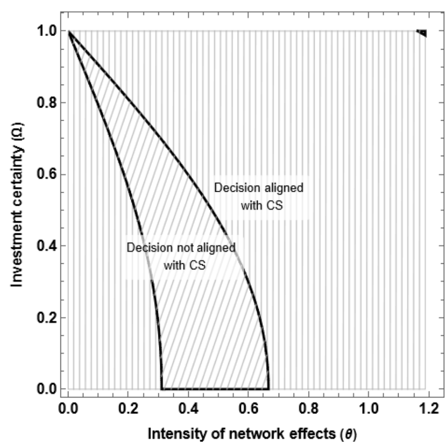


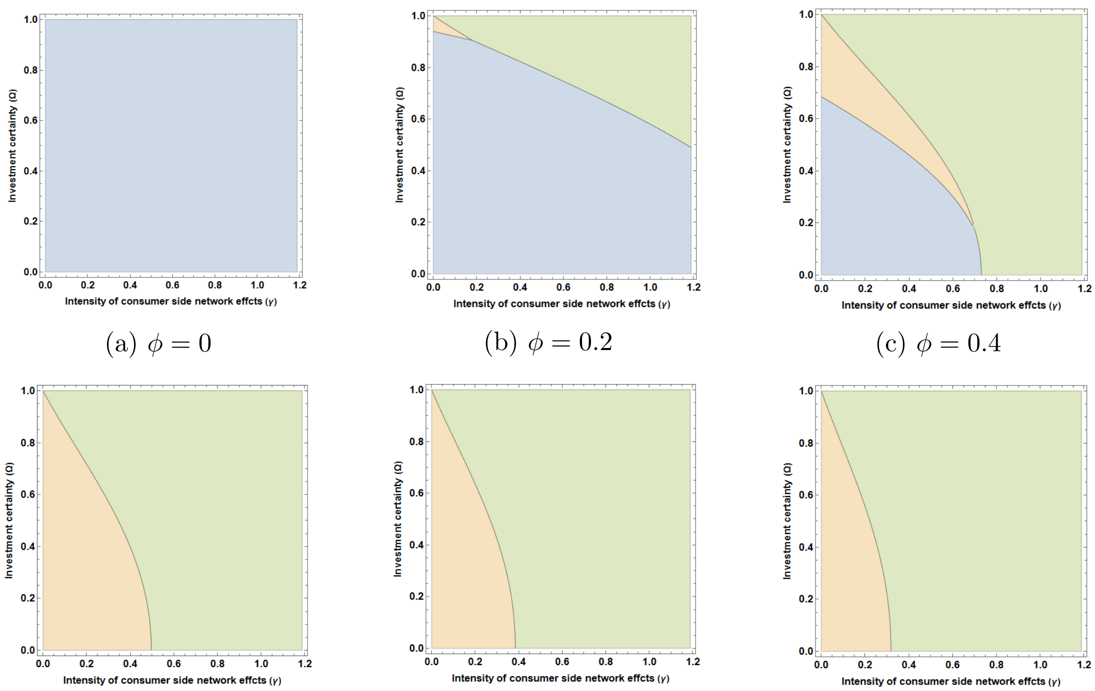
Figure 7: Regions where CS and market structure choice of R_1 are aligned and when misaligned.

In Figure 7, the region shaded with diagonal lines is where the digital transformation decision of R_1 is misaligned with the consumer surplus maximizing market structure and the region shaded with vertical lines depict when the decision of R_1 is aligned. Specifically, when investments are close to certain (after digital transformation) and the value of cross-sided network interactions is high, platforms find it profitable not to host their rival. However, we know that when R_1 decides to be a platform, consumers are always better off when it hosts its rival. As a result, in this parameter constellation, there is a misalignment of R_1's choice and consumer surplus maximizing market structure. Interestingly, in a region where the value of cross-sided network interactions is moderate, we find that consumers may be better off when R_1 chooses not to digitally transform and focus on certain investments than transforming into a platform and hosting its rival. This is because digital transformation leads to investment uncertainty which lowers investment incentives. When the value of cross-network interactions is low and uncertainty sufficiently high, increased value creation by third party developers is not sufficient to compensate for the fall in investment incentives by R_1 and R_2. This results in a misalignment of firm choice and consumer surplus maximizing decision.[27]

Total welfare. Total welfare in the market is computed as the sum of firm profits, consumer surplus and developer surplus.[28] Specifically,

$$TW^{g?} = \mathsf{E}CS^{g?} + \mathsf{E}\Pi_1^{g?} + \mathsf{E}\Pi_2^{g?} + \mathsf{E}DS^{g?}.$$

Note that $DS^{T?} = 0$ as there is no developer market. Comparing the total welfare in the three regimes, we present our results in the following graphs. The Figure 8 simulates the regions when



(a) $\phi = 0$ (b) $\phi = 0.2$ (c) $\phi = 0.4$

(d) $\phi = 0.6$ (e) $\phi = 0.8$ (f) $\phi = 1.0$

Figure 8: Total welfare ranking. The blue region indicates that case $g = T$ has the highest welfare, the orange region indicates that case $g = P$ has the highest welfare and the green region indicates that case $g = H$ has the highest welfare.

total welfare is highest. When $\phi = 0$, developers do not value the presence of consumers and as a result there is no cross-sided network effects and hence also no external value creation. In this parameter constellation, transforming into a platform is a fool's errand as it only creates risk which not adding any value. A direct consequence of this is that innovation levels are highest under no transformation and therefore total welfare is also highest in this regime. This is clearly indicated in the figure (8a). Notice that as the developers' value from the presence of consumers increases

[27]Our main results continue to hold when platform sets a fixed developer participation fee. See Appendix A for more details.

[28]The expression for developers' surplus can be found in equations (30) —(30) in the Proof of Proposition (2).

(as ϕ increases), case $g = T$ is less likely to result in the highest total welfare. For ϕ large enough, it is always better in terms of total welfare to transform into a platform than not (see figures (8d) - (8f)). Interestingly, we find that inviting rivals (case $g = H$) onto a platform is not always total welfare maximizing and there are regions where total welfare is maximized for case $g = P$. This is because when developers' value for consumers (ϕ) is high enough but consumers do not value external value creation (γ is low enough), consumers are more sensitive to investments by firms than external value creation. In this case, we find that $v_2^P > v^H$ implying welfare enhancement effect of increased innovation by firm 2 in case $g = P$ dominates the welfare enhancement from external value creation. This gives us the result that total welfare is higher under case $g = P$. In all other cases, total welfare under case $g = H$ (green shaded region) is higher and the intuitions are quite straightforward.

4.6 Profitability of R_2 building its own platform

In this subsection, we discuss the profitability of the case where R_2 also builds its own platform.[29] The following figure illustrates the market structure.

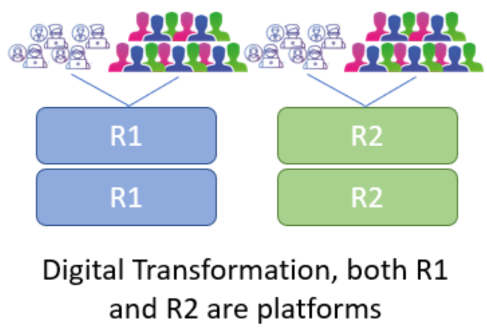


Figure 9: Industry Structure when R_1 and R_2 form separate competing platforms.

We find that the total expected industry profits of the two firms is higher when R_1 hosts its rival than when R_2 creates a competing standalone platform. As before, let's denote this case as $g = C$ where both R_1 and R_2 create their own platforms and consumers expect the value of interactions with the developer network on each platform i as Ψ_i^C.

Following, the same steps as in the main paper, we express the inverse demand expression at the two firms as

$$P_1^C(v_1, \Psi_1^C, X) = 1 + v_1 + \Psi_1^C \quad 3X, \qquad P_2^C(v_2, \Psi_2^C, X) = 1 + v_2 + \Psi_2^C \quad 3X.$$

Developer market. The utility of a type-k developer of platform i, is $\pi_i^C(k) = \phi x_i^e \quad k$, where x_i^e is developers' expectation on the total mass of consumers participating on the platform R_i. Developers affiliate with a platform only if they obtain positive value from the participating in the

[29] A detailed discussion on this case is available in Appendix (C).

platform — i.e., for any $k < \tilde{k}_i^C = \phi x_i^e$. Thus, the mass of developers participating on a platform i is $Dev^C(x_1^e) = \tilde{k}^C$. We delegate the detailed computations of this model to the Appendix and instead focus on the incentives to create separate platforms vis-á-vis being hosted.

Proposition 4 *It is always jointly pro table for the two rms to be in case $g = H$ than in case $g = C$ | $\sum_{i=1}^{2} \left[\left(\mathsf{E}\Pi_i^{H?}(v^H, v^H) \quad I(v^H) \right) \quad \left(\mathsf{E}\Pi_i^{C?}(v^C, v^C) \quad I(v^C) \right) \right] > 0$.*

The intuition for the above is straightforward. By creating its own platform in case $g = C$, R_2 competes more aggressively with R_1 while fragmenting demand and hence also network effect. This implies that the additional network value creation arising purely from aggregation is lost. On the contrary, in case $g = H$, by aggregating consumer demand, the two firms increase the mass of active developers in the market. This enhances the value creation and thus the consumers' willingness to pay. This increased value creation in case $g = H$ enhances firm margins while lowering competitive intensity as was discussed earlier because the two firms transform into co-opetitors from being fierce competitors.

Corollary 2 *There is a range of transfers t 2 $(\underline{t}, \bar{t})$ where both R_1 and R_2 are better o in case $g = H$ than each rm forming a separate platform, $g = C$.*

The above result suggests that R_1 can always find a fee such that R_2 accepts the offer of joining its platform and both firms are better-off compared to $g = C$. Thus, we can state that there will never be a situation where R_2 develops its own standalone platform.

Consumer surplus in the case when R_2 also forms a separate platform is given as

$$\mathsf{E}CS^{C?} = \frac{(\Omega^2(X^S(v^C, v^C))^2 + 2\Omega(1 \quad \Omega)(X^C(0, v^C))^2) + (1 \quad \Omega)^2(X^C(0,0))^2))}{6}. \tag{12}$$

Comparing the consumer surplus above with the consumer surplus under case $g = H$, we present the results in the following Proposition.

Proposition 5 *Consumer surplus under hosting rival is unambiguously greater than the consumer surplus when R_2 forms a separate platform | $\mathsf{E}CS^{H?} > \mathsf{E}CS^{C?}$.*

The above result is quite intuitive. Transforming into a platform is risky. By forming a separate platform, R_2 is faced with (the same) transformation risks as in the case when it was hosted by R_1 (case $g = H$), while fragmenting the value of network benefits. Thus, losing out on the benefits of aggregation and risk sharing while facing the same level of transformation risk. This discourages investments and the total output in the market. A direct consequence of this is that consumers are worse-off when R_2 forms its own platform.

5 Robustness of our results

We discuss in the following subsection an extension to our benchmark model where the platform can charge external value creators a price.

5.1 Participation fee charged to developers

In our benchmark model, we assumed that the platform manages cross-sided externalities through value enhancing investments. These investments serve as the basis for expectation formation on the mass of valuable cross-network interactions and affect actual demand. In reality, platforms have a wider set of strategic tools at their disposal to manage market expectations on cross-sided interactions. One tool commonly employed is a network participation fee charged to developers. In the case when platforms charge a network participation fee, our results qualitatively hold and in some cases our result on digital transformation may be more pronounced than in the benchmark model without any participation fee. This is because the platform now has two strategic instruments to better organize the market, which increases the likelihood of digital transformation. For more details, see Appendix (Section A).

6 Managerial and Policy Implications

Policy implications. Digital firms once considered scrappy, underdog startups have come to dominate the digital sphere (Report 2020). Policy makers now scramble to avoid market dominance and cultivate healthy competition. An important step towards contestability in digital markets is to encourage incumbent firms to digitally transform and carve out their own digital stronghold to weather competition from these dominant digital firms. Our results show digital transformation has several policy relevant effects. First, a policy that encourages digital transformation of firms into open platforms that allow rivals to co-create network effects under fair and reasonable terms increases consumer surplus. Welfare improves because demand aggregation motivates external value creation by third party developers who increase their market participation. Second, digital transformation into platforms is not always optimal. Firms can prefer to retain a traditional product focus and avoid a risky transformation process when network benefits are small. This may actually enable them to compete more aggressively with platforms and maintain their market power by offering significantly superior products.

A third policy insight is that legislation to increase competition can reduce the benefits of these aggregation and market expansion strategies. Consumer surplus can inadvertently ***fall***. Standard regulatory policies focus on traditional products where competition among firms and breakup of dominant firms increases welfare by lowering prices (Areeda & Hovenkamp 2011). Rivalry among smaller firms can also boost innovation when competition is not too intense (Aghion et al. 2005). These results, however, overlook the market expansion effects that arise when developers can co-ordinate on the same market. Expansion effects can overturn both sets of traditional results. Platform firms that host rivals, rather than compete with them, invite competition with their own products which lowers prices. The host recovers the substitution effect by taxing the rival and by the market expansion effect. Both properties benefit consumers. Similarly, the divided market resulting from competing platforms reduces developer and firm investment incentives, which both reduce innovation. The consequence of competition is less innovation not more. Policy rules must therefore change to incorporate the increased investment incentives and market expansion effects of third party investment. Network effects necessitate policy reform.

Managerial implications. Our findings provide novel managerial insights for firms considering the transformation from a traditional business model into a platform-based business model. First, it is profitable to transform a traditional firm into a platform either when value enhancing investments are successful with a high probability or network effect gains are sufficiently high. Communicating value of a multi-sided platform to the platform participants is crucial specially when a traditional firm is transforming into a platform based business. Certainty of platform investments encourages platforms to invest more in value enhancement and as a result this makes their value proposition more salient to the market participants. This increased investment in product value signals greater value of participating in the platform to the consumers and developers and benefiting the cross-sided network interactions. As a consequence, more developers affiliate with the platform which results in increased value for consumers thus, kick-starting a virtual cycle of value generation. This leads to a win-win situation for developers, consumers and the platform if investment risks are managed well. Lego represents a leading example. Its digital transformation is a platform called *Lego Ideas* that allows creators to submit ideas for Lego products that may be commercially available, with the creator receiving a 1% royalty rate.[30] Since creators use Lego's building blocks to create new products, there is little or no risk to Lego. Instead, Lego benefits from external value creation which further makes it profitable to invest more in value. As expected, Lego has not invited other toy rivals to join its platform as the fall in profits due to increased competition dominate any network expansion effect.

Second, our results suggest that when platform transformation entails risky product investments that offer little value in terms of network effects, it may be best for managers to avoid transforming their traditional firm into a platform. It's important to recognize that not every firm is well-suited to becoming a platform, and in situations where the risks outweigh the potential benefits, it may be better to stick with a traditional business model. Although, transformation can bring forth network benefits, it is well documented that transforming into a platform entails multiple risks and not all firms necessarily find it profitable (Bonnet & Westerman 2021).[31] Our results suggest that when the uncertainty of platform investments is high and the value of cross-sided interactions is low, platform firms do not find it profitable to invest heavily in the platform. Demand growth and aggregation benefits are not high enough to convince other market participants to join the platform. Thus, traditional markets may be preferable because of certainty of their investments. A classic example of this is perhaps the market for toothpaste in the consumer goods. For instance, competitive manufacturers such as *Crest* and *Colgate* may be worse off seeking to digitally transform toothpaste, solicit developers, or invite interactions on a product that is not intrinsically shareable. In such a situation, it may be better for these firms to avoid transforming into a platform to benefit from external value creation.

[30] See Gurcaylilar-Yenidogan & Gul (2021) for a detailed discussion on Lego's digital transformation.

[31] Roughly 70% of all digital transformation fall short of their stated goals (Bucy et al. 2019).

Third, our results suggest that, it can be quite profitable for a platform to host their competitors. Managers can gain a valuable new perspective from our research, which suggests that when uncertainty of platform product investments is intermediate and cross-sided interactions are present but not high, it can be profitable for firms to spur cross-sided interactions by even paying its rival to participate on the platform. This insight suggests that inviting rivals on its platform is a signaling device to the market participants regarding the volume of cross-sided interactions that may compensate for lower product investments and/or low value of interactions. For instance, Kloeckner, a German steel and metal distributor, invited its competitors to join the platform *XOM Materials* to distribute their products as well.[32] Specifically, Kloeckner helped a direct competitor, *Outokumpu* — the biggest Finnish Steel manufacturer and distributor, to join its platform.[33] As pointed out by Smilen Hazhikostov, XOM's Head of Growth, XOM exerted a lot of effort to help competing suppliers to digitize their files during the onboarding process.[34] As elicited in our paper, Klöckner's CEO also points out that despite competition shrinking margins, volume increases as the common platform has the potential to reach 100 percent of customers rather than just that segment already familiar with a company's individual proprietary platform.[35]

7 Conclusions

The risk of digital transformation is a hot debate topic inside and outside corporate boardrooms.[36] This paper provides insights on when should firms choose to digitally transform and if they do so, whether hosting rivals is profitable. We focus on the trade-offs a firm encounters when it decides to digitally transform itself from a traditional business to a platform based business model. Transforming into a platform creates opportunities for external value creation, expanding demand while at the same time also manifesting investment risks. A firm contemplating digital transformation must balance these risks and benefits of digital transformation. We find that it is not always beneficial for firms to pursue a digital transformation strategy, particularly when the investment risks are too high and network externalities are too low. Interestingly, we show that when firms decide to transform into platform, it may be profitable for them to invite rivals on their platform even to the point of subsidizing them. This is because inviting rivals on the platform lowers their rival's market aggressiveness through two channels. First, invited rivals become less aggressive in their own investments. Second, the invited rival internalizes the value generated by the platform firm as the two firms become competitive complements and thus compete less aggressively. Very interestingly, when the value of network externalities is moderately low, platforms may even find it profitable to pay their rivals to join their platform. Yet, when network externalities are quite high,

[32] See Bonnet & Westerman (2021), Jacobides et al. (2019), and WEF (2016).

[33] See link for more details

[34] A case study on Kloeckner and its digital transformation sheds more light on this (see Duke Kominers & Knoop (2020)).

[35] See link for detailed review of the case study.

[36] See WEF (2016) report on digital transformation that states 80% of managers surveyed contemplated digital transformation.

platforms find it profitable to exclude rivals.

Our findings bear valuable insights for managers of firms considering digital transformation. First, positive externalities make each unit of investment more productive. This increases incentives to invest relative to the traditional no externalities case. Second, such externalities encourage market expansion. Third parties, who value demand, represent an external source of value addition, meaning that platforms can seek to pull in demand to win these third parties even via competitors. The demand benefit softens the cost of competition. Competition is more likely to transform to coopetition. The consequence is that managers can and should deploy more demand aggregation strategies than traditional firms.

List of variables

Variable	Interpretation
Case $g = T$	Both firms R_1 and R_2 are traditional.
Case $g = P$	R_1 is a platform and R_2 is a traditional firm.
Case $g = H$	R_1 is a platform and it hosts R_2 on its platform.
r	Consumers' basic valuation for the products sold.
k	Developers' cost of production or the value of outside option.
ϕ	Per unit consumer benefit to developers that buy products affiliated with the platform.
γ	Per unit developer benefit to consumers that are affiliated with the platform.
θ	Variable transformation, $\theta = \phi\gamma$.
Ω	Probability of successful investment.
$u_i^g(\ )$	Consumers' utility from buying from firm i in case $g \in \{T, P, H\}$.
$P_i^g(\ )$	Inverse demand function of firm i in case $g \in \{T, P, H\}$.
$\Psi_i^g(\ )$	The degree of external value creation in the market structure $g \in \{T, P, H\}$.
$\Pi_i^g(\ )$	Profit of each firm i in the market structure $g \in \{T, P, H\}$.
$I(\ )$	Investment cost of developing consumer value.
$\pi^g(k)$	Profit of developer of type k in the market structure $g \in \{P, H\}$.
Dev^g	Mass of developers active on the platform in case $g \in \{P, H\}$.
	Decision variables.
x_i^g	output choice by firm i in case $g \in \{T, P, H\}$.
X^g	Total industry output in case $g \in \{T, P, H\}$.
v_i^g	Internal value creation by firm i in case $g \in \{T, P, H\}$.
L	Participation fee charged by platform R_1 to R_2 in case $g = H$.

Table 2: Table of notation.

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Online Appendix

A Platform sets a price to developers

In this section, we confirm that our main insights hold when platforms can orchestrate third party value creation also through a participation fee set to developers. We highlight the changes from our benchmark model below.

A.1 Developers

In our modelling set-up, the developer side is active only after R_1 transforms into a platform. Specifically, developers are active in the market structure $g = P$ and $g = H$. Developers derive value ϕ when interacting with consumers through the platform. We assume developers are heterogeneous in their investment cost of developing applications. Let k be the cost which follows the distribution $\Lambda(\cdot)$. For simplicity we assume that Λ is a uniformly distribution over 0 and 1 i.e. $k \sim U[0,1]$. Developer participation in the platform market in the two market cases ($g = P$ and $g = H$) is described below.

Market structure $g = P$. In this market structure, R_1 is a platform and R_2 is a traditional firm, developers interact with consumers active only on R_1. Thus, the utility of a type-k developer, is $\pi^P(k) = \phi x_1^e - l_1 - k$ where developers affiliate with the platform only if they obtain positive value from the participating in the platform — i.e., for any $k < \tilde{k}^P = \phi x_1^e - l_1$. Thus, the mass of developers participating in the platform is $Dev^P(x_1^e, l_1) = \tilde{k}^P$.

Market structure $g = H$. When platform R_1 invites its rival R_2 to participate in the platform, the developers now are able to interact with all the consumers in the market. The utility of a developer of type k is $\pi^H(k) = \phi(X^e) - l_1 - k$ where l_1 is the participation fee being charged to developers, $X^e = \sum_{i=1,2} x_i^e$ and developers affiliate with a platform only if they gain positive value from the participating in the platform — i.e., for any $k < \tilde{k}^H = \phi X^e - l_1$. Thus, the mass of developers participating in the platform is $Dev^H(X^e, l_1) = \tilde{k}^H$.

Firm profits. The profit of firm 1 and 2 in the market structure $g \in \{T, P, H\}$ is given as

$$P_1^g(v_1, \Psi_1^g, X)x_1 + l_1 Dev^g - I(v_1), \quad P_2^g(v_2, \Psi_2^g, X)x_2 - I(v_2),$$

where l_1 is the participation fee charged to developers.

Timing, contracts and equilibrium concept. The timing of the game is as follows:

$t = 1$ R_1 decides whether to be a platform or not. If it decides to be a platform then it also decides whether to host rival R_2 on its platform. If it hosts its rival then it charges the participation fee L.

$t = 2$ R_1 and R_2 invest v_1 and v_2 and R_1 sets a participation fee l_1 for developers.

$t = 2.1$ Investment outcomes are realized and publicly observable.

$t = 3$ If R_1 is not a platform, move to stage 4. When R_1 is a platform, its hosting decisions are observed. Based on that consumers and developers respectively form expectations on the mass of developers and consumers on the platform.

$t = 4$ R_1 and R_2 choose their outputs, consumers buy and developers join the platform simultaneously. Payoffs are realized accordingly.

Assumption 2 *We make the following assumptions:* $\gamma = 1$ *and* $0 < \phi < \frac{(3\sqrt{41} - 15)}{4}$.

The above parameter restriction are sufficient conditions that ensure the second order conditions are satisfied and we are in an interior solution case for the full spectrum of uncertainty — $\Omega \in [0, 1]$.[37] This allows us to simplify the number of cases while confirming our results in this extension while providing new insights.

The case of traditional firms is as in the baseline model. There is no change in the analysis. Therefore, we proceed directly to the market structures $g = P$ and $g = H$ to understand how investment uncertainty of new business models impacts the incentive to host rivals on a platform.

R_1 is a platform and R_2 is a traditional firm ($g = P$)

In this case, investment uncertainty affects $R_1's$ investment while $R_2's$ investment is certain. Despite this, R_2 when investing must consider the outcome of the investment of the platform when deciding on its level of investment.

Given investment outcome and investment levels, the output setting stage is solved as in our benchmark case. Let $\hat{v}_1$ and $\hat{v}_2$ be the values of realized investment outcomes.[38] The individual outputs of each firm R_i, total output and mass of active developers as a function of investments and investment outcome are given as

$$\begin{aligned} x_1^P(\hat{v}_1, \hat{v}_2, l_1) &= \frac{1 - 2l_1 + 2\hat{v}_1 - \hat{v}_2}{9 - 2\phi}, \quad x_2^P(\hat{v}_2, \hat{v}_1, l_1) = \frac{3(1 - \hat{v}_1 + l_1) - \phi + \hat{v}_2(6 - \phi)}{27 - 6\phi}, \\ X^P(\hat{v}_1, \hat{v}_2, l_1) &= \frac{3(2 + \hat{v}_1 + \hat{v}_2 - l_1) - \phi(1 + \hat{v}_2)}{27 - 6\phi}, \quad Dev^P(\hat{v}_1, \hat{v}_2, l_1) = \phi x_1^P(\cdot) - l_1. \end{aligned}$$

Again, in the following, we define profits of R_1 and R_2 as a function of realized investment levels.

$$\Pi_1^{P*}(\hat{v}_1, \hat{v}_2, l_1) = P_1^P(\hat{v}_1, \Psi_1^P(\hat{v}_1, \hat{v}_2, l_1), X^P(\hat{v}_1, \hat{v}_2, l_1))x_1^P(\hat{v}_1, \hat{v}_2, l_1) + l_1 Dev^P(\cdot), \tag{13}$$

$$\Pi_2^{P*}(\hat{v}_2, \hat{v}_1, l_1) = P_2^P(\hat{v}_2, 0, X^P(\hat{v}_1, \hat{v}_2, l_1))x_2^P(\hat{v}_2, \hat{v}_1, l_1). \tag{14}$$

[37] Technically, we need $= \; < \frac{(3\sqrt{41} - 15)}{4}$ but for brevity and ease of presentation we fix $= 1$. Our results will hold qualitatively for all other values of and as long as their product is below the threshold.

[38] As previously, in the case $g = P$, if actual investment levels are v_1 and v_2 then $\hat{v}_1 \in \{0, v_1\}$ and $\hat{v}_2 = v_2$.

Innovation stage and fee setting. In stage 2, firms unilaterally invest in the value for the product v_i to maximize expected profits as expressed below. The expected profits of platform R_1 and firm R_2 are given as

$$\mathsf{E}\Pi_1^{P?}(v_1, v_2, l_1) \quad I(v_1) = \Omega\Pi_1^{P?}(v_1, v_2, l_1) + (1 \quad \Omega)\Pi_1^{P?}(0, v_2, l_1) \quad I(v_1), \tag{15}$$

$$\mathsf{E}\Pi_2^{P?}(v_2, v_1, l_1) \quad I(v_2) = \Omega\Pi_2^{P?}(v_2, v_1, l_1) + (1 \quad \Omega)\Pi_2^{P?}(v_2, 0, l_1) \quad I(v_2). \tag{16}$$

Notice that investment decision of R_1 relies totally on the revenues arising in the case when investment is successful. In contrast to our benchmark, this revenue is weighted by the probability of success Ω.

Differentiating the expected profits of firm R_1 with respect to its investment v_1 and the participation fee l_1 and the expected profit of R_2 with respect to v_2 and solving simultaneously yields the optimal investment levels and participation fee in case $g = P$ as

$$v_1^P = \frac{\Omega(9 \quad 2\phi)^2(15 \quad 4\phi)\left(\phi^2+12\right)}{B}, \; l_1^P = \frac{(4\phi \quad 15)(\phi(2\phi \quad 9)+12)\left(24(\Omega \quad 1)\Omega + (9 \quad 2\phi)^2\right)}{B},$$

$$v_2^P = \frac{(\phi \quad 6)\left(\Omega^2(2\phi(\phi(4(\phi \quad 9)\phi+129) \quad 252)+936)+72\Omega(14 \quad 5\phi)+3(5\phi \quad 14)(9 \quad 2\phi)^2\right)}{B},$$

where $B = 2\Omega^2(\phi(\phi(\phi(4\phi(5\phi \quad 66)+1401) \quad 3825)+5796) \quad 4968) \quad 72\Omega(\phi(29\phi \quad 226)+429)+3(\phi(29\phi \quad 226)+429)(9 \quad 2\phi)^2$. The comparative statics qualitatively hold as in Lemma (**??**). It is worth noting that $l_1^P < 0$ as consumer valuation per unit of external value creation is very high — $\gamma = 1$.[39] The intuition for this is straightforward. The platform finds it profitable to subsidize entry of developers as consumers value them highly and charge a higher price to consumers. The expected optimal profit of platform firm R_1 and its rival R_2 in case $g = P$ are given as

$$\Pi_1^{P??} = \mathsf{E}\Pi_1^{P?}(v_1^P, v_2^P, l_1^P) \quad I(v_1^P) = \frac{\begin{array}{c}(2\Omega^2(\phi(2\phi \quad 9)+12)^2+(23 \quad 6\phi)(72\Omega \quad 3(9 \quad 2\phi)^2)) \\ (15 \quad 4\phi)^2\left(\phi^2+12\right)\left(24(1 \quad \Omega)\Omega \quad (9 \quad 2\phi)^2\right)\end{array}}{4B^2},$$

and

$$\Pi_2^{P??} = \mathsf{E}\Pi_2^{P?}(v_2^P, v_1^P) \quad I(v_2^P) = \frac{G}{4B^2}$$

where $G = 8\Omega^4(\phi(\phi(\phi(\phi(\phi(\phi(8\phi(2\phi(\phi(5\phi \quad 132)+1563) \quad 22167)+854379) \quad 3013524)+8085636) \quad 16488036)+24691392) \quad 25516944)+14789952)+12\Omega^3(\phi(\phi(\phi(\phi(4\phi(\phi(8\phi(2\phi \quad 183)+26253) \quad 219195)+4204593) \quad 12884616)+27360936) \quad 39857184)+29513808)+12\Omega^2(5\phi \quad 14)(2\phi(5\phi \quad 42)+171)(\phi(\phi(8\phi(\phi(2(\phi \quad 18)\phi+267) \quad 1071)+21393) \quad 35100)+31860) \quad 432\Omega(9 \quad 2\phi)^2(14 \quad 5\phi)^2(2\phi(5\phi \quad 42)+171)+9(9 \quad 2\phi)^4(14 \quad 5\phi)^2(2\phi(5\phi \quad 42)+171)$.

The comparative statics on the expected profits of R_1 and R_2 are qualitatively similar to the

[39] This is a common result in the two-sided market literature. See Armstrong (2006), Rasch & Wenzel (2013) among others.

results in Lemma 1. The following Figure (10) is the counterpart illustration for Figure (3) in the benchmark.

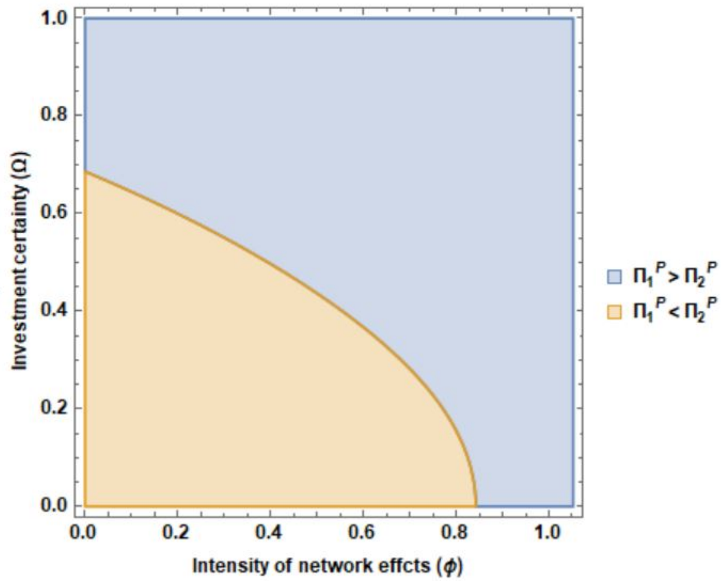


Figure 10: Comparing platform R_1's profit vs. its rival R_2.

The intuition for the Figure (10) is qualitatively similar as in the benchmark model. Interestingly, the mass of active developers is always — $Dev^P(v_1^P, v_2^P, l_1^P) > 0$, even when $\phi \to 0$. This is because in this extension where $\gamma = 1$ the platform subsidizes entry of developers.

A.2 Platform R_1 hosts the rival firm R_2 $(g = H)$

In this case, platform R_1 decides to host its rival R_2 as well on its platform. By inviting its rival R_2, R_1 is able to expand the consumer demand affiliated with the platform which attracts more developers (external value creators) and hence also the total ecosystem size. By doing so, R_1 also shares it's platform advantage with R_2.

Let $\hat{v}_1$ and $\hat{v}_2$ be the values of realized investment outcomes.[40] Given investment levels, participation fees and investment outcomes, solving as in the benchmark case, the output of firm R_i as a function in investments and the participation fees is given as

$$x_i^H(\hat{v}_i, \hat{v}_{-i}, l_1) = \frac{3(1-l_1) + \hat{v}_i(6-\phi) - \hat{v}_{-i}(3-\phi)}{27-6\phi},$$

[40] As previously, in the case $g = H$, if actual investment levels are v_1 and v_2 then $\hat{v}_1 \in \{0, v_1\}$ and $\hat{v}_2 \in \{0, v_2\}$.

The total output as a function of investments and revenue of R_i are given as

$$
\begin{aligned}
X^H(\hat{v}_1, \hat{v}_2, l_1) &= x_1^H(\hat{v}_1, \hat{v}_2) + x_2^H(\hat{v}_2, \hat{v}_1) = \frac{2(1 \quad l_1) + \hat{v}_1 + \hat{v}_2}{9 \quad 2\phi}, && (17)\\
Dev^H(\hat{v}_1, \hat{v}_2, l_1) &= \phi X^H(\hat{v}_1, \hat{v}_2, l_1) \quad l_1, && (18)\\
\Pi_1^{H?}(\hat{v}_1, \hat{v}_2, l_1) &= P_1^H((\hat{v}_1, \Psi^H(\hat{v}_1, \hat{v}_2, l_1), X(\hat{v}_1, \hat{v}_2, l_1))x_1^H(\hat{v}_1, \hat{v}_2, l_1) + l_1 Dev^H(\hat{v}_1, \hat{v}_2, l_1), && (19)\\
\Pi_2^{H?}(\hat{v}_2, \hat{v}_1, l_1) &= P_2^H((\hat{v}_2, \Psi^H(\hat{v}_1, \hat{v}_2, l_1), X(\hat{v}_1, \hat{v}_2, l_1))x_1^H(\hat{v}_2, \hat{v}_1, l_1). && (20)
\end{aligned}
$$

We employ the above expression to construct the expected revenue of the two firms in the platform and present them in the following equations.

$$
\mathsf{E}\Pi_1^{H?}(v_1, v_2, l_1) = \Omega^2 \Pi_1^{H?}(v_1, v_2, l_1) + \Omega(1 \quad \Omega)\Pi_1^{H?}(0, v_2, l_1) + \Omega(1 \quad \Omega)\Pi_1^{H?}(v_1, 0, l_1) + (1 \quad \Omega)^2 \Pi_1^{H?}(0, 0, l_1),
$$
$$
\mathsf{E}\Pi_2^{H?}(v_2, v_1, l_1) = \Omega^2 \Pi_2^{H?}(v_2, v_1, l_1) + \Omega(1 \quad \Omega)\Pi_2^{H?}(v_2, 0, l_1) + \Omega(1 \quad \Omega)\Pi_2^{H?}(0, v_1, l_1) + (1 \quad \Omega)^2 \Pi_2^{H?}(0, 0, l_1).
$$

Innovation and fee setting stage. In stage 2, given fixed fee L, firms unilaterally invest in the value for the product v_i to maximize expected profits given as

$$
\mathsf{E}\Pi_1^{H?}(v_1, v_2, l_1) \quad I(v_1) + L^?, \quad \text{and} \quad \mathsf{E}\Pi_2^{H?}(v_2, v_1, l_1) \quad I(v_2) \quad L^?.
$$

In contrast to case $g = P$, investment of both firms is now stochastic. This impacts their incentives to innovate and also their aggressiveness.

Differentiating the expected profits of firm R_1 with respect to its investment v_i and the participation fee l_i and the expected profit of R_2 with respect to v_2 and solving simultaneously yields the optimal investment levels and participation fee in case $g = H$ as

$$
v_1^H = \frac{2\Omega(9 \quad 2\phi)^2 \left(3(12 \quad (1 \quad \phi)\phi)(9 \quad 2\phi)^2 \quad \Omega^2(6 \quad \phi)(72 \quad \phi(\phi + 30)) \quad 2\Omega(\phi \quad 6)^2(12 \quad (1 \quad \phi)\phi)\right)}{H},
$$

$$
l_1^H = \frac{2(\phi(2\phi \quad 9) + 3)\left(2\Omega^2(\phi \quad 6)^2 \quad 2\Omega(\phi \quad 6)^2 + 3(9 \quad 2\phi)^2\right)\left(2\Omega^2(6 \quad \phi)(3 \quad \phi) + 2\Omega(6 \quad \phi)^2 \quad 3(9 \quad 2\phi)^2\right)}{3H},
$$

$$
v_2^H = \frac{2\Omega(6 \quad \phi)(9 \quad 2\phi)\left(6(9 \quad \phi)(9 \quad 2\phi)^2 \quad \Omega^2(\phi(\phi(4(\phi \quad 10)\phi + 165) \quad 450) + 648) \quad 4\Omega(9 \quad \phi)(\phi \quad 6)^2\right)}{H},
$$

where $H = 2\delta^4(\phi \quad 6)(\phi \quad 3)(\phi(\phi(4(\phi \quad 4)\phi \quad 185) + 1194) \quad 1800) + 2\Omega^3(\phi \quad 6)^2(\phi(\phi(4(\phi \quad 12)\phi + 207) \quad 336) + 72) + \Omega^2(252072 \quad \phi(\phi(4\phi(4\phi(3(\phi \quad 20)\phi + 455) \quad 5673) + 1053) + 152496)) + 48\Omega(\phi \quad 6)^2(3\phi \quad 13)(9 \quad 2\phi)^2 \quad 36(3\phi \quad 13)(9 \quad 2\phi)^4$. The comparative statics qualitatively hold as in Lemma (2).

In contrast to the benchmark case, one can notice that $v_1^H \neq v_2^H$. This is because now platform R_1 has a richer set of tools to maximize profits and orchestrate value creation. As a result, its optimization problem is different from the one in the benchmark. Comparing the two investment levels, we observe that

$$\text{when } \phi > \hat{\phi} = \frac{1}{4}\left(9 - \sqrt{57}\right), \; v_1^H > v_2^H \text{ and } l_1^H > 0.$$

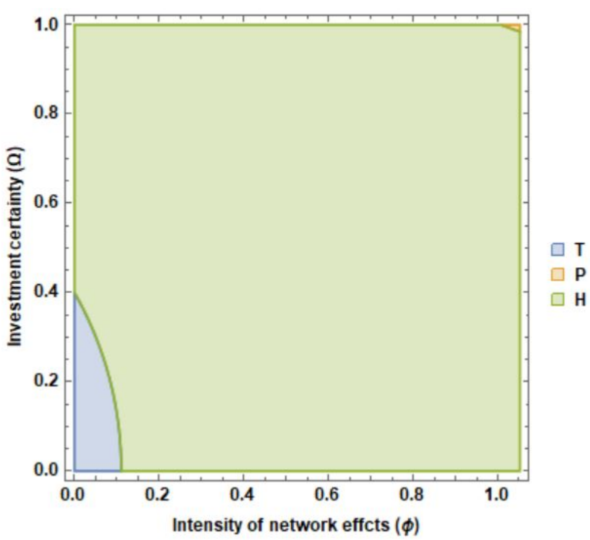
and vice-versa otherwise.
The comparative statics of investments with respect to Ω are qualitatively similar to the results as in Lemma (2). Notice that, in comparison to case $g = P$, in our example of $\gamma = 1$, the participation fee to developers can be positive. This is the case when developers place a high value on the participation of consumers on the platform — $\phi > \hat{\phi}$. This is because now developers are able to interact with a larger mass of consumers and when this value is high, the platform finds it profitable to charge a positive fee for their participation.

Optimal contract L. As in the benchmark case, the optimal contract is set such that R_2 is indifferent between accepting or rejecting R_1's optimal contract offer. Specifically, the optimal fixed fee is given as

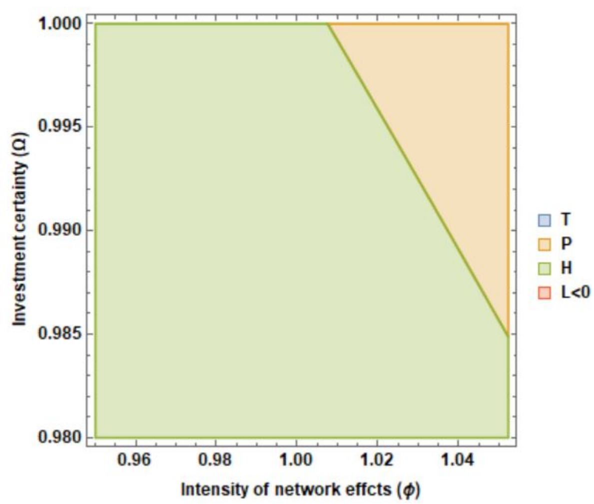
$$L^{*} = \mathsf{E}\Pi_2^{H*}(v^H, v^H) - I(v^H) - \left(\mathsf{E}\Pi_2^{P*}(v_2^P, v_1^P) - I(v_2^P)\right).$$

The comparative statics of profits and the optimal fees are qualitatively similar to Lemma (2).

The decision of R_1 to be a platform and the incentives to host rival R_2. Comparing the net profit of R_1 in the three market structures $g \in \{T, P, H\}$, we observe that the results are qualitatively similar as in Proposition (2). We present a region-plot in the Figure 11a, that confirms that our results hold qualitatively.



(a) Equilibrium market structures



(b) Magnification of north-east corner of Figure (11a) to show the case $g = P$.

Figure 11: Market structure and magnification of area where market structure is $g = P$

As expected, when ϕ is of a high magnitude and the investments are quite certain, we find that the platform R_1 chooses not to host its rivals. In all other cases, when R_1 chooses to be a platform, it hosts its rival.

Interestingly, we find that under some parameter constellations, it is profitable for R_1 to be a platform and pay its rival to participate in the platform and the results here are qualitatively similar as in Corollary (1). The Figure 12 confirms it. As in the benchmark, the orange shaded

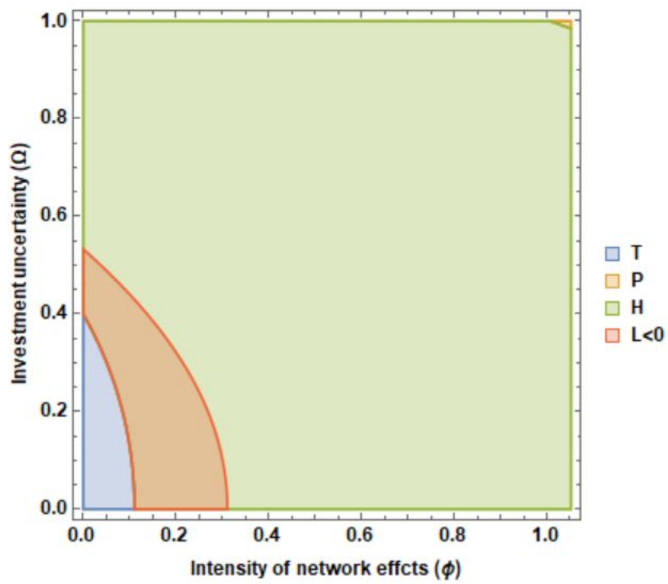


Figure 12: Equilibrium market structures and the region with negative fees for $\gamma = 1$.

region is the case when R_1 chooses to be a platform than being a traditional firm and compensates R_2 ($L^? < 0$) to encourage its participation in the value creation at the platform.

Interestingly, it is noteworthy to consider the case when the fees offered to developers is negative as well. In the following region-plot (Figure 13), we include another region when R_1 is a platform and offers negative fees to developers for participation on the platform. In the above figure, in addition to paying the rival to participate, we also consider when the platform chooses to be a platform and when it may want to subsidize entry of developers in case it invites its rival as well. The violet region in the above plot depicts the region where the platform subsidizes entry. Surprisingly, subsidizing entry of developers is profitable for the platform even when developers do not have any positive value of interactions with consumers — i.e., $\phi = 0$. This is because it is cheaper for the platform to subsidize entry and expand consumer value than invest further. Further, observe that the region when $L^? < 0$ is within the region when $l_1^H < 0$ implying that there is a region where the platform finds it profitable to subsidize entry of rival R_2 and third party developers.

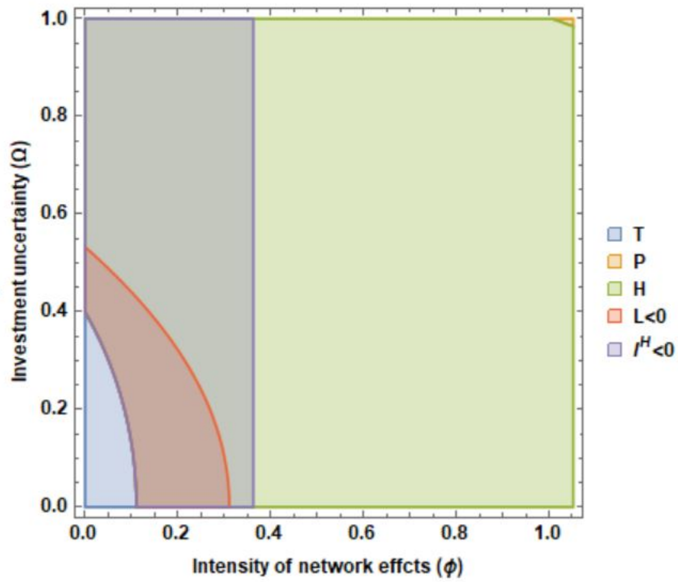


Figure 13: Equilibrium market structures and the region with negative fees for $\gamma = 1$.

B Proofs of results in the paper

Proof of Lemma 1. The equilibrium profits of platform R_1 and firm R_2 are given by substituting the equilibrium investment levels in equation (2) and equation (3) as

$$\Pi_1^{P??} = \mathsf{E}\Pi_1^{P?}(v_1^P, v_2^P) \quad I(v_1^P) = \frac{3(15 \quad 4\theta)^2 \left((9 \quad 2\theta)^2 \quad 24\Omega\right)\left((9 \quad 2\theta)^2 \quad 24\Omega(1 \quad \Omega)\right)}{\left(((9 \quad 2\theta)^2 \quad 24\Omega)(171 \quad 2\theta(42 \quad 5\theta)) \quad 72\Omega^2(6 \quad \theta)\right)^2},$$

and

$$\Pi_2^{P??} = \mathsf{E}\Pi_2^{P?}(v_2^P, v_1^P) \quad I(v_2^P) = \frac{B^P}{\left(((9 \quad 2\theta)^2 \quad 24\Omega)(171 \quad 2\theta(42 \quad 5\theta)) \quad 72\Omega^2(6 \quad \theta)\right)^2}$$

where $B^P = 864\Omega^4(6 \quad \theta)(24 \quad 7\theta) + 432\Omega^3(33 \quad 10\theta)\left(69 + 4\theta^2 \quad 34\theta\right) \quad 72\Omega^2(3 \quad \theta)(57 \quad 4\theta(7 \quad \theta))(171 \quad 2\theta(42 \quad 5\theta)) + (9 \quad 2\theta)^2(3 \quad \theta)^2(171 \quad 2\theta(42 \quad 5\theta))((9 \quad 2\theta)^2 \quad 48\Omega)$.

Differentiating the equilibrium profit with respect to Ω yields

$$\begin{aligned}
\frac{\partial \Pi_1^{P??}}{\partial \Omega} &= \frac{\Omega^b}{\left(((9 \quad 2\theta)^2 \quad 24\Omega)(171 \quad 2\theta(42 \quad 5\theta)) \quad 72\Omega^2(6 \quad \theta)\right)^3} > 0, \\
\frac{\partial \Pi_2^{P??}}{\partial \Omega} &= \frac{\Omega^c}{\left(((9 \quad 2\theta)^2 \quad 24\Omega)(171 \quad 2\theta(42 \quad 5\theta)) \quad 72\Omega^2(6 \quad \theta)\right)^3} < 0.
\end{aligned}$$

where $\Omega^b = 144\Omega(15 \quad 4\theta)^2\left((9 \quad 2\theta)^2 \quad 12\Omega\right)\left(((9 \quad 2\theta)^2 \quad 24\Omega)(207 \quad 10(9 \quad \theta)\theta) + 72\Omega^2(6 \quad \theta)\right) > 0$ and $\Omega^c = 144\Omega(15 \quad 4\theta)(72\Omega^3(6 \quad \theta)(639 \quad 4\theta(81 \quad 10\theta)) \quad 24\Omega^2(171 \quad 2\theta(42 \quad 5\theta))(117 \quad \theta(105 \quad 4(9 \quad \theta)\theta)) \quad 9\Omega(123 \quad 8(8 \quad \theta)\theta)(171 \quad 2\theta(42 \quad 5\theta))(9 \quad 2\theta)^2 + (3 \quad \theta)(171 \quad 2\theta(42 \quad 5\theta))(9 \quad 2\theta)^5) > 0$.

The above inequalities hold under Assumption 1.

Comparing $\Pi_1^{P??}$ and $\Pi_2^{P??}$, we observe that profit of R_1 is higher than profit of R_2 for any value of Ω if $\theta > \tilde{\theta}^P$.[41] ■

Proof of Lemma 2. The optimal contact is given as $L^? = \mathsf{E}\Pi_2^{H?}(v^H, v^H) \quad I(v^H) \quad \left(\mathsf{E}\Pi_2^{P?}(v_2^P, v_1^P) \quad I(v_2^P)\right)$. Differentiating the above with respect to Ω, we observe that $\frac{@L^\star}{@} > 0$.[42]

Solving for the inequality $L^? < 0$ yields the conditions $\theta < \tilde{\theta}^L(\Omega)$ for any Ω and otherwise $L^? > 0$.[43] ■

Proof of Proposition 1. In this proposition we compare the profits of R_1 under all three scenarios. Let us define D^T which is the collection of parameters θ and Ω where profits of R_1 is higher from being traditional than the other two forms i.e. $D^T = \left\{(\theta, \Omega) : \Pi_1^{T??} > \text{Max} f \Pi_1^{P??}, \Pi_1^{H??} g\right\}$. This parameter constellation can be expressed as follows.

$$D^T = 0 < \theta < 0.311 \text{ and } \Omega < \Omega^T(\theta). \tag{21}$$

The expression for $\Omega^T(\theta)$ is quite involved and is available upon request. The graphical proof is available in Figure (4a).

Similarly, let us define D^P which is the collection of parameters θ and Ω where profits of R_1 is higher from being a platform and not hosting its rival than the other two cases i.e. $D^P = \left\{(\theta, \Omega) : \Pi_1^{P??} > \text{Max} f \Pi_1^{T??}, \Pi_1^{H??} g\right\}$. This parameter constellation can be expressed as follows.

$$D^P = 1.16 < \theta \text{ and } \Omega^H(\theta) < \Omega. \tag{22}$$

The expression for $\Omega^H(\theta)$ is quite involved and is available upon request. The graphical proof is available in Figure (4a).

Similarly, let us define D^H which is the collection of parameters θ and Ω where profits of R_1 is higher from being a platform and hosting its rival than the other two cases i.e. $D^H = \left\{(\theta, \Omega) : \Pi_1^{H??} > \text{Max} f \Pi_1^{T??}, \Pi_1^{H??} g\right\}$. This parameter constellation can be expressed as follows.

$$D^H = \begin{cases} \text{when } 0 < \theta < 0.311 \text{ and } \Omega^T(\theta) < \Omega < 1, \\ \text{when } 0.311 < \theta < 1.16 \text{ and } 0 < \Omega < 1, \\ \text{when } 1.16 < \theta \text{ and } 0 < \Omega < \Omega^H(\theta). \end{cases} \tag{23}$$

The graphical proof is available in Figure (4a). ■

[41] The expression for ˜^{P} is available upon request. We provide a graphical proof in the paper.

[42] The expressions quite involved and in favor of brevity, we leave them out. A draft with the detailed expressions is available upon request.

[43] The expression for ˜$^L(\Omega)$ is quite involved and is available upon request.

Proof of Corollary 1. The intersection of the region when the platform charges a negative optimal fee and when the platform chooses to be a platform determines our relevant region. This region is expressed in terms of our parameters below. This parameter constellation is expressed as follows:

$$D^L = \left\{(\theta, \Omega) : \Pi^{H**} > \text{Max}\{\Pi_1^{P**}, \Pi_1^{T**}\} \text{ and } L^* < 0\right\}.$$

This parameter constellation can be expressed as follows.

$$D^L = \begin{cases} \text{when } 0 < \theta < 0.311 \text{ and } \Omega^k < \Omega < \Omega^l, \\ \text{when } 0.311 < \theta < 0.519 \text{ and } 0 < \Omega < \Omega^l. \end{cases} \tag{24}$$

The expressions for Ω^k and Ω^l are quite involved and available upon request. The graphical proof is available in Figure (5).

In all other cases, when the platform hosts its rival, it sets positive optimal fees. ∎

Proof of Proposition 2. The consumer surplus in the different market structures $g \in \{T, P, H\}$ is given as

$$CS^T = \frac{(X^T)^2}{6}, \tag{25}$$

$$\mathsf{E}CS^{P*} = \frac{\Omega(X^P(v_1^P, v_2^P))^2 + (1 - \Omega)(X^P(0, v_2^P))^2}{6}, \tag{26}$$

$$\mathsf{E}CS^{H*} = \frac{\Omega^2(X^H(v^H, v^H))^2 + 2\Omega(1 - \Omega)(X^H(0, v^H))^2) + (1 - \Omega)^2(X^H(0,0))^2)}{6}. \tag{27}$$

Let us define $D^{CS,H}$ which is the collection of parameters θ and Ω where consumer surplus in case $g = H$ is higher from being a platform and hosting its rival than the other two cases i.e. $D^{CS,H} = \left\{(\theta, \Omega) : \mathsf{E}CS^{H*} > \text{Max}\{CS^T, \mathsf{E}CS^{P*}\right\}$. This parameter constellation can be expressed as follows.

$$D^{CS,H} = \theta^{CS,H} < \theta, \forall \Omega. \tag{28}$$

The expression for $\theta^{CS,H}$ is quite involved and is available upon request.

Similarly, defining $D^{CS,T}$ which is the collection of parameters θ and Ω where Consumer surplus in case $g = T$ is higher from being a traditional firm than the other two cases i.e. $D^{CS,T} = \left\{(\theta, \Omega) : CS^T > \text{Max}\{\mathsf{E}CS^{H*}, \mathsf{E}CS^{P*}\right\}$. This parameter constellation can be expressed as follows.

$$D^{CS,T} = \theta < \theta^{CS,H}, \forall \Omega. \tag{29}$$

Finally, from the above it is obvious that the analogous set $D^{CS,P}$ is a null set. This implies that market structure $g = P$ never leads to the highest consumer surplus. ∎

Proof of Proposition 3. In this proposition, we consider when choice of R_1 is aligned with consumer surplus maximization choice. For this, we consider the union of intersection of the following regions,

$$D^A = (D^T \cap D^{CS;T}) \cup (D^H \cap D^{CS;H}).$$

This parameter constellation can be expressed as follows.

$$D^A = \begin{cases} 0 < \theta < 0.311 \text{ and } (\Omega < \Omega^{A\star} \cup \Omega^{A\star\star} < \Omega), \\ 0.311 < \theta < \frac{2}{3} \text{ and } \Omega^{A\star\star} < \Omega < 1, \\ \frac{2}{3} < \theta < 1.16 \text{ and } 0 < \Omega < 1, \\ 1.16 < \theta < 1.16 \text{ and } 0 < \Omega < \Omega^{A\star\star\star}. \end{cases}$$

The expression for $\Omega^{A\star}$, $\Omega^{A\star\star}$ and $\Omega^{A\star\star\star}$ are quite involved and are available upon request. The graphical proof is available in Figure (7).

In all other cases, R_1's decision is misaligned with the consumer surplus maximizing choice.

Developer surplus in market structures $g = P$ and $g = H$ are given as

$$DS^{P\star} = \int_0^{\bar{k}^{P\star}} (\phi x_1^{P\star} - k)\lambda(k)dk = \frac{(\phi x_1^{P\star})^2}{2}, \quad DS^{H\star} = \int_0^{\bar{k}^{H\star}} (\phi X^{H\star} - k)\lambda(k)dk = \frac{(\phi X^{H\star})^2}{2}.$$

This allows us to compute the expected developer surplus as

$$DS^T = = 0, \tag{30}$$

$$\mathsf{E}DS^{P\star} = \frac{\Omega(\phi x_1^P(v_1^P, v_2^P))^2 + (1-\Omega)(\phi x_1^P(0, v_2^P))^2}{2}, \tag{31}$$

$$\mathsf{E}DS^{H\star} = \frac{\Omega^2(\phi X^H(v^H, v^H))^2 + 2\Omega(1-\Omega)(\phi X^H(0, v^H))^2) + (1-\Omega)^2(\phi X^H(0,0))^2)}{2}. \tag{32}$$

■

Proof of Proposition 4. Given the realization of investment outcome, outputs as a function of investments realization is given as

$$x_i^C(\hat{v}_i, \hat{v}_{-i}) = \frac{3 - \theta - 3\hat{v}_{-i} + (6-\theta)\hat{v}_i}{(9-\theta)(3-\theta)}.$$

The total output and firm revenue as a function of investments are respectively given as

$$X^C(\hat{v}_1, \hat{v}_2) = x_1^S(\hat{v}_1, \hat{v}_2) + x_2^S(\hat{v}_2, \hat{v}_1) = \frac{(2 + \hat{v}_1 + v_2)}{9 - \theta}, \tag{33}$$

$$\Pi_i^{C\star}(\hat{v}_i, \hat{v}_{-i}) = P_i^C(\hat{v}_i, \Psi_i^C(\hat{v}_i, \hat{v}_{-i}), X^C(\hat{v}_1, \hat{v}_2))x_i^C(\hat{v}_i, \hat{v}_{-i}) \text{ for } i \in \{1, 2\}. \tag{34}$$

Innovation stage. In stage 2, firms unilaterally invest in the value for the product v_i to maximize expected profits given as

$$\mathsf{E}\Pi_1^{C?}(v_1,v_2)\quad I(v_1)=\Omega^2\Pi_1^{C?}(v_1,v_2)+\Omega(1\quad\Omega)\Pi_1^{C?}(0,v_2)+\Omega(1\quad\Omega)\Pi_1^{C?}(v_1,0)+(1\quad\Omega)^2\Pi_1^{C?}(0,0)\quad I(v_1),$$
$$\mathsf{E}\Pi_2^{C?}(v_2,v_1)\quad I(v_2)=\Omega^2\Pi_2^{C?}(v_2,v_1)+\Omega(1\quad\Omega)\Pi_2^{C?}(v_2,0)+\Omega(1\quad\Omega)\Pi_2^{C?}(0,v_1)+(1\quad\Omega)^2\Pi_2^{C?}(0,0)\quad I(v_2).$$

Differentiating the profits of platform firm R_1 and firm R_2 with respect to v_1 and v_2 and solving simultaneously for v_1 and v_2 yields the optimal investment levels in case $g=C$ by

$$v^C=\frac{6\Omega(6\quad\theta)(3\quad\theta)}{(3\quad\theta)^2(9\quad\theta)^2+6\Omega(6\quad\theta)(3\Omega\quad(6\quad\theta))}.$$

This investment is rising θ and Ω.

The expected optimal profit of platform firm R_1 and its rival R_2 in case $g=P$ are given as

$$\mathsf{E}\Pi_i^{C?}(v^C,v^C)\quad I(v^C).$$

Comparing the joint firm profit in case $g=H$ vs. case $g=C$. The joint firms profits in case $g=C$ is given as $2\Big(\mathsf{E}\Pi_1^{H?}(v^H,v^H)\quad I(v^H)\Big)$ and the joint firm profits in case $g=C$ is given as $2\Big(\mathsf{E}\Pi_1^{C?}(v^C,v^C)\quad I(v^C)\Big)$. Taking the difference between these two profit levels yields

$$2\Big(\mathsf{E}\Pi_1^{H?}(v^H,v^H)\quad I(v^H)\quad\big[\mathsf{E}\Pi_1^{C?}(v^C,v^C)\quad I(v^C)\big]\Big)>0.$$

The above inequality is always true under Assumption 1.

The above suggests that R_1 can always find a fee such that R_2 accepts the offer and R_1 is also better off. Thus, we can state that there will never be a situation where R_2 develops its standalone platform.

■

Proof of Proposition 5. The expression for Consumer Surplus is given as

$$\mathsf{E}CS^{C?}\quad=\quad\frac{(\Omega^2(X^S(v^C,v^C))^2+2\Omega(1\quad\Omega)(X^C(0,v^C))^2)+(1\quad\Omega)^2(X^C(0,0))^2))}{6}.\tag{35}$$

Comparing the consumer surplus in case $g=H$ presented in equation (27) with the expression for $\mathsf{E}CS^{C?}$ yields

$$\mathsf{E}CS^{H?}\quad\mathsf{E}CS^{C?}>0.$$

This is always true under Assumption 1. ■

Proof of Lemma 5. It is straightforward to calculate that for $\theta=0$ we have $v_1^P=v^T=v_2^P=\frac{4}{23}$.

Solving simultaneously the expressions in equation (52) and (53) for v_1 and v_2 yields

$$v_1^P = \frac{180 \quad 48\theta}{(69 \quad 10\theta)\big(15 \quad 2\theta(6 \quad \theta)\big)}, \qquad v_2^P = \frac{2(6 \quad \theta)\big(15 \quad \theta(15 \quad 2\theta)\big)}{(69 \quad 10\theta)\big(15 \quad 2\theta(6 \quad \theta)\big)}.$$

Taking the derivative of v_1^P and v_2^P with respect to θ, we get

$$\frac{\partial v_1^P}{\partial \theta} = \frac{126360 \quad 48\theta(1935 \quad \theta(483 \quad 40\theta))}{(69 \quad 10\theta)^2(15 \quad 2(6 \quad \theta)\theta)^2} > 0, \qquad \frac{\partial v_2^P}{\partial \theta} = \frac{6(6885 \quad 2\theta(1575 \quad \theta(21+4(12 \quad \theta)\theta)))}{(69 \quad 10\theta)^2(15 \quad 2(6 \quad \theta)\theta)^2} < 0.$$

The above inequalities hold under Assumption 1.

It is then immediate that that following inequality holds $v_1^P > v^T > v_2^P$ as we established before that $\frac{@v_1^P}{@} > 0$ and $\frac{@v_2^P}{@} < 0$. Hence, proved. ■

Proof of Lemma 6. Substituting the optimal investments in the outputs as expressed in equation (50), we get the individual outputs and the total outputs respectively as

$$\begin{aligned}
x_1^{P?} &= x_1^P(v_1^P, v_2^P) = \frac{135 \quad 66\theta + 8\theta^2}{(69 \quad 10\theta)(15 \quad 2\theta(6 \quad \theta))}, \\
x_2^{P?} &= x_2^P(v_2^P, v_1^P) = \frac{(9 \quad 2\theta)(15 \quad \theta(15 \quad 2\theta))}{(69 \quad 10\theta)(15 \quad 2\theta(6 \quad \theta))}, \\
X^{P?} &= \sum_{i=1;2} x_i^{P?} = \frac{(2 \quad \theta)(15 \quad 2\theta)(9 \quad 2\theta)}{(69 \quad 10\theta)(15 \quad 2\theta(6 \quad \theta))}.
\end{aligned}$$

Taking the difference of individual outputs in case $g = P$ with the outputs in case $g = H$ yields

$$\begin{aligned}
x_1^{P?} \quad x^T &= \frac{2\theta(708 \quad 5\theta(59 \quad 6\theta))}{23(69 \quad 10\theta)(15 \quad 2\theta(6 \quad \theta))} > 0, \\
x_2^{P?} \quad x^T &= \frac{\theta(861 \quad 330\theta + 32\theta^2)}{23(69 \quad 10\theta)(15 \quad 2\theta(6 \quad \theta))} < 0, \\
X^{P?} \quad 2x^T &= \sum_{i=1;2} x_i^{P?} = \frac{\theta(555 \quad 4\theta(65 \quad 7\theta))}{23(69 \quad 10\theta)(15 \quad 2\theta(6 \quad \theta))} > 0.
\end{aligned}$$

The above relations always hold under Assumption 1.

Differentiating $x_1^{P?}$, $x_2^{P?}$ and $X^{P?}$ with respect to θ yields

$$\begin{aligned}
\frac{\partial x_1^{P?}}{\partial \theta} &= \frac{63720 \quad 4\theta(13275 \quad 2\theta(2163 \quad 10\theta(33 \quad 2\theta)))}{(69 \quad 10\theta)^2(15 \quad 2(6 \quad \theta)\theta)^2} > 0, \\
\frac{\partial x_1^{P?}}{\partial \theta} &= \frac{9(4305 \quad 2\theta(1650 \quad \theta(483 \quad 4(17 \quad \theta)\theta)))}{(69 \quad 10\theta)^2(15 \quad 2(6 \quad \theta)\theta)^2} < 0, \\
\frac{\partial X^{P?}}{\partial \theta} &= \frac{24975 \quad 2\theta\left(11700 \quad \theta\left(4305 \quad 708\theta + 44\theta^2\right)\right)}{(69 \quad 10\theta)^2(15 \quad 2(6 \quad \theta)\theta)^2} > 0.
\end{aligned}$$

The above inequalities hold under assumption 1.

■

Proof of Lemma 7. Substituting the equilibrium outputs and investment levels in the expression for prices as expressed in equation (1) yields the equilibrium price at the two firms as

$$P_1^{P?} = \frac{3(9 \quad 2\theta)(15 \quad 4\theta)}{(69 \quad 10\theta)(15 \quad 2(6 \quad \theta)\theta)}, \qquad P_2^{P?} = \frac{3(9 \quad 2\theta)(15+\theta(\quad 15+2\theta))}{(69 \quad 10\theta)(15 \quad 2(6 \quad \theta)\theta)}.$$

Differentiating these price with respect to θ, yields

$$\frac{dP_1^{P?}}{d\theta} = \frac{12(15930+\theta(\quad 13275+2\theta(2163+10\theta(\quad 33+2\theta))))}{(69 \quad 10\theta)^2(15+2(\quad 6+\theta)\theta)^2} > 0 \text{ and}$$
$$\frac{dP_2^{P?}}{d\theta} = \frac{27(4305+2\theta(\quad 1650+\theta(483+4(\quad 17+\theta)\theta)))}{(69 \quad 10\theta)^2(15+2(\quad 6+\theta)\theta)^2} < 0.$$

The above inequalities hold under Assumption 1.

The equilibrium profits of platform R_1 and firm R_2 are given by substituting the equilibrium investment levels in equation (51) as

$$\Pi_1^{P?}(v_1^P, v_2^P) \quad I(v_1^P) = \frac{3(15 \quad 4\theta)^2(57 \quad 4(9 \quad \theta)\theta)}{(69 \quad 10\theta)^2(15 \quad 2(6 \quad \theta)\theta)^2},$$
$$\Pi_2^{P?}(v_2^P, v_1^P) \quad I(v_2^P) = \frac{(15 \quad \theta(15 \quad 2\theta))^2(171 \quad 2\theta(42 \quad 5\theta))}{(69 \quad 10\theta)^2(15 \quad 2(6 \quad \theta)\theta)^2}.$$

Differentiating the equilibrium profit with respect to θ yields

$$\frac{\partial(\Pi_1^{P?}(\,) \quad I(\,))}{\partial\theta} = \frac{24(15 \quad 4\theta)(80190 \quad \theta(104895 \quad 4\theta(13473 \quad 4\theta(852 \quad 5\theta(21 \quad \theta)))))}{(69 \quad 10\theta)^3(15 \quad 2(6 \quad \theta)\theta)^3} > 0,$$
$$\frac{\partial(\Pi_2^{P?}(\,) \quad I(\,))}{\partial\theta} = \frac{6(15 \quad \theta(15 \quad 2\theta))(266085 \quad 2\theta(146610 \quad \theta(65421 \quad 4\theta(3684 \quad \theta(423 \quad 20\theta)))))}{(69 \quad 10\theta)^3(15 \quad 2(6 \quad \theta)\theta)^3} < 0.$$

The above inequalities hold under Assumption 1. ■

Proof of Lemma 8. Solving the system of first order conditions as expressed in equation (9) for v_1 and v_2 yields the symmetric investment levels

$$v_1^H = v_2^H = \frac{2(6 \quad \theta)}{69 \quad 34\theta + 4\theta^2}.$$

Differentiating the investments levels with respect to θ gives

$$\frac{\partial v_1^H}{\partial\theta} = \frac{270 \quad 8\theta(12 \quad \theta)}{(69 \quad 34\theta+4\theta^2)^2} > 0.$$

The inequality is true under Assumption 1.

Comparing the investment levels of individual firms in case $g = H$ with the investment levels in case $g = P$, we have

$$
\begin{aligned}
v^H \quad v_1^P &= \frac{2\theta(9 \quad 2\theta)\left(243 + 10\theta^2 \quad 96\theta\right)}{(69 \quad 10\theta)\,(15 + 2\theta^2 \quad 12\theta)\,(69 + 4\theta^2 \quad 34\theta)} < 0, \\
v^H \quad v_2^P &= \frac{2\theta(6 \quad \theta)(9 \quad 2\theta)\left(63 + 4\theta^2 \quad 36\theta\right)}{(69 \quad 10\theta)\,(15 + 2\theta^2 \quad 12\theta)\,(69 + 4\theta^2 \quad 34\theta)} > 0.
\end{aligned}
$$

The above inequalities hold under Assumption 1. ∎

Proof of Lemma 9. Substituting the optimal investments in the outputs as expressed in equation (55), we get the individual outputs and the total outputs respectively as

$$
\begin{aligned}
x_i^{H?} &= x_i^H(v^H, v^H) = \frac{9 \quad 2\theta}{(69 \quad 34\theta + 4\theta^2)}, \\
X^{H?} &= \sum_{i=1;2} x_i^{H?} = \frac{2(9 \quad 2\theta)}{(69 \quad 34\theta + 4\theta^2)}.
\end{aligned}
$$

Taking the difference of individual outputs in case $g = P$ with the outputs in case $g = H$ yields

$$
x_1^{P?} \quad x_1^H = \frac{2\theta(9 \quad 2\theta)\,(96 \quad \theta(31 \quad 2\theta))}{(69 \quad 10\theta)\,(15 + 2\theta^2 \quad 12\theta)\,(69 + 4\theta^2 \quad 34\theta)} > 0, \tag{36}
$$

$$
x_2^{P?} \quad x_2^H = \frac{\theta(9 \quad 2\theta)^2\,(63 \quad 4\theta(9 \quad \theta))}{(69 \quad 10\theta)\,(15 + 2\theta^2 \quad 12\theta)\,(69 + 4\theta^2 \quad 34\theta)} < 0, \tag{37}
$$

$$
X^{P?} \quad 2x^H = \sum_{i=1;2} x_i^{P?} = \frac{\theta(9 \quad 2\theta)(3 \quad 2\theta)\left(125 \quad 46\theta + 4\theta^2\right)}{(69 \quad 10\theta)\,(15 + 2\theta^2 \quad 12\theta)\,(69 + 4\theta^2 \quad 34\theta)} < 0. \tag{38}
$$

The above relations always hold under Assumption 1.

Differentiating $x_1^{H?}$ and $X^{H?}$ with respect to θ yields

$$
\frac{\partial x_1^{H?}}{\partial \theta} = \frac{8(21 \quad \theta(9 \quad \theta))}{(69 \quad 34\theta + 4\theta^2)^2} > 0, \tag{39}
$$

$$
\frac{\partial X^{H?}}{\partial \theta} = 2\frac{\partial x_1^{H?}}{\partial \theta} > 0. \tag{40}
$$

The above inequalities hold under assumption 1.

∎

Proof of Lemma 10. For the case $g = H$, the firms are symmetric and as a result we also have a symmetric equilibrium.

Substituting the equilibrium outputs and investment levels in the expression for prices as expressed in equation (54) yields the equilibrium price at the two firms as

$$
P_1^{H?} = \frac{27 \quad 6\theta}{4\theta^2 \quad 34\theta + 69}. \tag{41}
$$

Differentiating this symmetric price level with respect to θ, yields

$$\frac{\partial P_1^{H?}}{\partial \theta} = \frac{24(21 \quad \theta(9 \quad \theta))}{(69 \quad 34\theta + 4\theta^2)^2} > 0.$$

The above inequalities hold under Assumption 1.

The equilibrium profits of platform R_1 and R_2 is given by substituting the equilibrium investment levels in equation (51). The expression for equilibrium profit of R_1 is

$$\begin{aligned}
\Pi_1^{H?}(v^H, v^H) \quad I(v^H) + L^? &= \Pi_1^{H?}(v^H, v^H) \quad I(v^H) + \Pi_2^{H?}(v^H, v^H) \quad I(v^H) \quad (\Pi_2^{P?}(v_2^P, v_1^P) \quad I(v_2^P)), \\
&= 2(\Pi_1^{H?}(v^H, v^H) \quad I(v^H)) \quad (\Pi_2^{P?}(v_2^P, v_1^P) \quad I(v_2^P)). \\
&= 2\underbrace{\left(\frac{2\theta(5\theta \quad 42) + 171}{(4\theta^2 \quad 34\theta + 69)^2}\right)}_{{}_1^{H\star}(v^H;v^H) \quad I(v^H)} \quad \frac{(15 \quad \theta(15 \quad 2\theta))^2(171 \quad 2\theta(42 \quad 5\theta))}{(69 \quad 10\theta)^2(15 \quad 2(6 \quad \theta)\theta)^2}.
\end{aligned}$$

Before proceeding, it is useful to consider how $\Pi_1^{H?}(v^H, v^H) \quad I(v^H)$ changes with respect to θ. Taking the derivative of $\Pi_1^{H?}(v^H, v^H) \quad I(v^H)$ with respect to θ yields

$$\frac{\partial(\Pi_1^{H?}(v^H, v^H) \quad I(v^H))}{\partial \theta} = \frac{4(9 \quad 2\theta)^2(18 \quad 5\theta)}{(4\theta^2 \quad 34\theta + 69)^3} > 0.$$

The above inequality always holds under Assumption 1. Keeping this result in mind, we take the derivative of R_1's profit

$$\frac{\partial(\Pi_1^{H?}(v^H, v^H) \quad I(v^H) + L^?)}{\partial \theta} = 2\underbrace{\frac{\partial(\Pi_1^{H?}(v^H, v^H) \quad I(v^H))}{\partial \theta}}_{(+)} \quad \underbrace{\frac{\partial(\Pi_2^{P?}(v_2^P, v_1^P) \quad I(v_2^P))}{\partial \theta}}_{(\;)}.$$

The above is positive as the firm term is always positive, while the second term, which enters the above expression negatively is the outside option of R_2 while always fall in θ as shown in the proof of Lemma (7).

Further, we know that the net profit of R_2 is

$$\Pi_2^{H?}(v^H, v^H) \quad I(v^H) \quad L^? = \Pi_2^{P?}(v_2^P, v_1^P) \quad I(v_2^P).$$

It is immediate from the previous results in Lemma (7) that the profit of R_2 falls in θ.

Finally, we have established before that X^H is increasing in θ and as a result it is immediate that the mass of developers is also rising in θ as $Dev^{H?} = \phi X^{H?}$. ■

Proof of Proposition 6. From Lemma 7 and Lemma 10, we know that $\frac{@(\ {}_1^{P\star}(\ ) \ I(v_1^P))}{@} > 0$, $\frac{@(\ {}_1^{H\star}(\ ) \ I(v^H))}{@} > 0$ and $\frac{@(\ {}_2^{P\star}(\ ) \ I(v_2^P))}{@} < 0$.

Further, it is immediate that for $\theta = 0$, we have $\Pi_i^{T?} \quad I(v^T) = \Pi_i^{P?} \quad I(v_i^P) = \Pi_i^{H?} \quad I(v^H) = \frac{19}{529}$

for all $i \in \{1, 2\}$. Therefore, for any θ in the feasible range, the following result is immediate. $\Pi_1^{T*}(v^T, v^T) - I(v^T) < \min\{\Pi_1^{P*}(v_1^P, v_2^P) - I(v_1^P), \Pi_1^{H*}(v^H, v^H) - I(v^H)\}$ and $\Pi_2^{T*} - I(v^T) > \Pi_2^{P*} - I(v_2^P)$. ■

Proof of Proposition 7. Firm R_1 hosts R_2 only if it is profitable to do so.

$$\Pi_1^{H*} - I(v^H) + L^* > \Pi_1^{P*} - I(v_1^P) \implies \Pi_1^{H*} + \Pi_2^{H*} - 2I(v^H) > \Pi_1^{P*} + \Pi_2^{P*} - I(v_1^P) - I(v_2^P).$$

From the above it is obvious that R_1's profit from hosting is higher than not hosting only if the joint profit from hosting is higher. Comparing the aggregate profits under the case $g = H$ with the profits in case $g = P$, we obtain that for the range $\theta \in (0, \tilde{\theta})$, with $\tilde{\theta} = 1.16$, profits under H are higher than P. For $\theta = \tilde{\theta}$ they are equal and for $\theta \in (\tilde{\theta}, \frac{15-\sqrt{105}}{4})$ the profits under latter are higher than former. ■

B.1 Comparative statics of the best responses

We first perform the comparative statics for the best responses in case $g = P$ and then proceed with case $g = H$.

Case $g = P$ The best responses are given as

$$v_1^{P;BR}(v_2) = \frac{12\Omega(1-v_2)}{(9-2\theta)^2-24\Omega},\ v_2^{P;BR}(v_1) = \frac{2(6-\theta)(3-\theta-3\Omega v_1)}{171-2\theta(42-5\theta)}. \tag{42}$$

The first derivative of the best response of $v_1^{P;BR}(\cdot)$ ($v_2^{P;BR}(\cdot)$) with respect to v_2 (v_1) is written as

$$\frac{\partial v_1^{P;BR}(v_2)}{\partial v_2} = -\frac{12\Omega}{(9-2\theta)^2-24\Omega} < 0,\quad \frac{\partial v_2^{P;BR}(v_1)}{\partial v_1} = -\frac{6\Omega(6-\theta)}{171-2\theta(42-5\theta)} < 0 \tag{43}$$

Differentiating the above expressions with respect to θ and Ω yields

$$\frac{\partial^2 v_1^{P;BR}(v_2)}{\partial v_2 \partial\Omega} = -\frac{12(9-2\theta)^2}{((9-2\theta)^2-24\Omega)^2} < 0,\quad \frac{\partial^2 v_2^{P;BR}(v_1)}{\partial v_1 \partial\Omega} = -\frac{6(6-\theta)}{171-2\theta(42-5\theta)} < 0, \tag{44}$$

$$\frac{\partial^2 v_1^{P;BR}(v_2)}{\partial v_2 \partial\theta} = -\frac{48\Omega(9-2\theta)^2}{((9-2\theta)^2-24\Omega)^2} < 0,\quad \frac{\partial^2 v_2^{P;BR}(v_1)}{\partial v_1 \partial\theta} = -\frac{6\Omega(333-10\theta(12-\theta))}{(2\theta(5\theta-42)+171)^2} < 0 \tag{45}$$

Case $g = H$ The best response of each firm i and its slope is given as

$$v_i^{H;BR}(v_{-i}) = \frac{2\Omega(6-\theta)(3-\Omega(3-\theta)v_{-i})}{3(9-2\theta)^2-2\Omega(6-\theta)^2},\ \frac{\partial v_i^{H;BR}(v_{-i})}{\partial v_{-i}} = -\frac{2\Omega^2(6-\theta)(3-\theta)}{3(9-2\theta)^2-2\Omega(6-\theta)^2} < 0. \tag{46}$$

C Profitability of forming separate platforms

Given the realization of investment outcome, outputs as a function of investments realization is given as

$$x_i^C(\hat{v}_i, \hat{v}_{-i}) = \frac{3 - \theta - 3\hat{v}_{-i} + (6 - \theta)\hat{v}_i}{(9 - \theta)(3 - \theta)}.$$

The total output and firm revenue as a function of investments are respectively given as

$$X^C(\hat{v}_1, \hat{v}_2) = x_1^C(\hat{v}_1, \hat{v}_2) + x_2^C(\hat{v}_2, \hat{v}_1) = \frac{(2 + \hat{v}_1 + \hat{v}_2)}{9 - \theta}, \tag{47}$$

$$\Pi_i^{C\star}(\hat{v}_i, \hat{v}_{-i}) = P_i^C(\hat{v}_i, \Psi_i^C(\hat{v}_i, \hat{v}_{-i}), X^C(\hat{v}_1, \hat{v}_2))x_i^C(\hat{v}_i, \hat{v}_{-i}) \text{ for } i \in \{1, 2\}. \tag{48}$$

Innovation stage. In stage 2, firms unilaterally invest in the value for the product v_i to maximize expected profits given as

$$\mathsf{E}\Pi_1^{C\star}(v_1, v_2) - I(v_1) = \Omega^2\Pi_1^{C\star}(v_1, v_2) + \Omega(1 - \Omega)\Pi_1^{C\star}(0, v_2) + \Omega(1 - \Omega)\Pi_1^{C\star}(v_1, 0) + (1 - \Omega)^2\Pi_1^{C\star}(0, 0) - I(v_1),$$
$$\mathsf{E}\Pi_2^{C\star}(v_2, v_1) - I(v_2) = \Omega^2\Pi_2^{C\star}(v_2, v_1) + \Omega(1 - \Omega)\Pi_2^{C\star}(v_2, 0) + \Omega(1 - \Omega)\Pi_2^{C\star}(0, v_1) + (1 - \Omega)^2\Pi_2^{C\star}(0, 0) - I(v_2).$$

Differentiating the profits of platform firm R_1 and firm R_2 with respect to v_1 and v_2 and solving simultaneously for v_1 and v_2 yields the optimal investment levels in case $g = C$. We perform some comparative statics and present the results in the following Lemma.

Lemma 3 (Case $g = C$: Investments) *The equilibrium investment level on each platform is symmetric and given by* $v^C = \frac{6\ (6\ )(3\ )}{(3\ )^2(9\ )^2+6\ (6\ )(3\ (6\ ))}$. *This investment is rising θ and Ω.*

The expected optimal profit of platform firm R_1 and its rival platform firm R_2 in case $g = C$ are given as

$$\mathsf{E}\Pi_i^{C\star}(v^C, v^C) - I(v^C).$$

Performing comparative statics on the expected profits of R_1 and R_2, we present the result in the following Lemma.

Lemma 4 (Case $g = C$: Expected profit) *The equilibrium profit of each platform is symmetric and this profit rises with an increase in the value of network interactions θ and in the certainty of investment Ω.*

The above results are quite intuitive.

Comparing the joint profit of firms in case $g = H$ vs. case $g = C$. The joint profit of firms in case $g = H$ is given as $2\big[\mathsf{E}\Pi_1^{H\star}(v^H, v^H) - I(v^H)\big]$ and the joint firm profits in case $g = C$ is given as $2\big[\mathsf{E}\Pi_1^{C\star}(v^C, v^C) - I(v^C)\big]$. Taking the difference between these two profit levels yields

$$2\Big[\big(\mathsf{E}\Pi_1^{H\star}(v^H, v^H) - I(v^H)\big) - \big(\mathsf{E}\Pi_1^{C\star}(v^C, v^C) - I(v^C)\big)\Big] > 0.$$

The above inequality is always true under Assumption 1.

The above result suggests that R_1 can always find a payment scheme under case $g = H$ such that R_2 accepts the offer and both firms are better-off in $g = H$ compared to $g = C$. Thus, we can state that there will never be a situation where R_2 transforms into a separate standalone platform.

D Platform transformation is certain

In this section, we focus on the case where there are no material risks associated with digital transformation — $\Omega = 1$. As a result, we ignore stage 2.1.

D.1 R_1 is a platform and R_2 is a traditional firm ($g = P$)

In this market structure, we consider the case when R_1 transforms into a platform and is a closed platform i.e. R_1 does not invite R_2 over its platform and hence, R_2 remains a traditional firm. After transforming itself into a platform, R_1 invites developers (complementors) to create additional complementary value for its product in the market. In this case, R_2 is at a disadvantage as it is unable to participate in the external value creation at the platform and thus offers no additional value to consumers while R_1 offers consumers this utility mark-up.[44] Figure 1, Panel (b) presents the market structure when $g = P$.

The asymmetry between the two firms due to the market structure $g = P$ is modeled by $\Psi_1^P = \gamma Dev^{P;e}$ while $\Psi_2^P = 0$. This is reflected in expression for market price of R_1 and R_2 which are respectively given as

$$P_1^P(v_1, \Psi_1^P, X) = 1 + v_1 + \Psi_1^P \quad 3X, \qquad P_2^P(v_2, 0, X) = 1 + v_2 \quad 3X. \tag{49}$$

R_1 being a platform offers developers access to consumers buying its products and consumers benefit from interacting with developers. Given investments in stage 2, developers join the platform expecting the total mass of consumers on platform R_1, x_1^e, and consumers join the platform expecting the total mass of developers on the platform. Specifically, $\Psi_1^P = \gamma Dev^{P;e}$ where $Dev^{P;e}$ is consumers' expectation on the mass of developers on the platform. Similarly, the mass of active developers (third party creators), $Dev^P = \tilde{k}^P = \phi x_1^e$ with x_1^e being developers' expectation on the demand for the product offered by platform R_1.

Output setting stage. Given consumer and developer expectations in stage 3 and investment levels in stage 2, in stage 4 of the game firms set outputs to maximize their profits

$$\max_{x_i} \Pi_i^P(v_i, \Psi_i^P, X) \quad I(v_i) = P_i^P(v_i, \Psi_i^P, X)x_i \quad I(v_i).$$

[44] Note that this is the case when innovation after digital transformation is certain. Instead after introducing investment uncertainty, we can show that in some cases R_2 is actually better off with R_1 transforming into a platform.

Differentiating the profit expression of firm R_i with respect to its output and imposing rational expectations — $x_1^P(v_1, v_2) = x_1^e$, $\Psi_1^P(v_1, v_2) = \gamma Dev^P(v_1, v_2) = \gamma\phi x_1^P(v_1, v_2) = \theta x_1^P(v_1, v_2)$, yields the output of R_1 and R_2 as a function of investments. From here on, we will employ the variable transformation $\theta = \gamma\phi$. The outputs as a function of investments are given as

$$x_1^P(v_1, v_2) = \frac{1 + 2v_1 - v_2}{9 - 2\theta}, \; x_2^P(v_2, v_1) = \frac{3(1 - v_1) - \theta + v_2(6 - \theta)}{27 - 6\theta}. \tag{50}$$

The total output as a function of investments and firm profits are given as $X^P(v_1, v_2) = x_1^P(v_1, v_2) + x_2^P(v_2, v_1) = \frac{3(2+v_1+v_2) - \theta(1+v_2)}{27 - 6\theta}$ and the profits are

$$\Pi_i^{P*}(v_i, v_{-i}) - I(v_i) = P_i^P(v_i, \Psi_i^P(v_i, v_{-i}), X^P(v_1, v_2))x_i^P(v_i, v_{-i}) - I(v_i). \tag{51}$$

Innovation stage. In stage 2, firms unilaterally invest in the value for the product v_i to maximize profits as expressed in (51). Further, recall that in this case $\Psi_1^P(v_1, v_2) = \gamma Dev^P(v_1, v_2)$ and $\Psi_2^P(v_2, v_1) = 0$. Differentiating the profits of platform R_1 with respect to v_1 and employing the envelope theorem yields

$$x_1^P(\cdot)\left(\underbrace{\frac{\partial P_1^P(\cdot)}{\partial v_1}}_{\substack{\text{Internal}\\ \text{value effect}}(+)} + \underbrace{\frac{\partial P_1^P(\cdot)}{\partial \Psi_1^P}\frac{\partial \Psi_1^P}{\partial x_1^e}\frac{\partial x_1^P(\cdot)}{\partial v_1}}_{\text{External value effect }(+)} + \underbrace{\frac{\partial P_1^P(\cdot)}{\partial X}\frac{\partial x_2^P(\cdot)}{\partial v_1}}_{\text{Price effect}(+)}\right) - \frac{\partial I(v_1)}{\partial v_1} = 0. \tag{52}$$

The incentive to innovate by R_1 can be broken down into multiple effects. First, there is the direct positive effect on prices that increases margins of R_1 for every unit of output sold through increased consumers' willingness to pay. Second, there is a positive effect on margins through increase in developer participation which enhances consumers' willingness to pay. Notice that an increase in internal value also positively impacts external value. This is evidence of how the internal and external value creation are intertwined.[45] Third, there is the positive effect on R_1's price through reduction in R_2's output. Instead, the cost of investment v_1, which is increasing and convex, represents a negative effect on investment incentives. R_1 trades-off these positive and negative effects of an increase in v_i on profitability and on equilibrium these marginal gains are exactly equal to the marginal cost of investment.

Analogously, differentiating the profits of firm R_2 with respect to v_2 and employing the envelope theorem yields

$$x_2^P(\cdot)\left(\underbrace{\frac{\partial P_2^P(\cdot)}{\partial v_2}}_{\substack{\text{Internal}\\ \text{value effect}}(+)} + \underbrace{\frac{\partial P_1^P(\cdot)}{\partial X}\frac{\partial x_1^P(\cdot)}{\partial v_2}}_{\text{Price effect}(+)}\right) - \frac{\partial I(v_2)}{\partial v_1} = 0. \tag{53}$$

The innovation incentives of R_2 also can be broken down into multiple effects. First, there is

[45] See Hinterhuber & Nilles (2021) where Henkel's Chief Digital Officer stresses the importance of co-innovation.

the direct positive effect on prices that increases margins of R_2 for every unit of output sold. Second, there is the positive effect on price through reduction in the R_1's output. One can observe when comparing the two first order conditions is the absence of the positive external value effect arising from increased developer expansion for R_2. Thus, it is straightforward that the equilibrium innovation levels at the platform R_1 are higher than at the rival R_2. Let's denote the equilibrium investment levels chosen by R_1 and R_2 as v_1^P and v_2^P respectively.

Solving the expression for first order conditions in equations (52) and (53), we present the equilibrium investment levels in the following Lemma.

Lemma 5 (Case $g = P$: Investments) *The equilibrium investment of the platform R_1 is higher and rival R_2 is lower than in the traditional firm case,*

$$v_1^P = \frac{180 - 48\theta}{(69 - 10\theta)\big(15 - 2\theta(6 - \theta)\big)} > v^T > v_2^P = \frac{2(6 - \theta)\big(15 - \theta(15 - 2\theta)\big)}{(69 - 10\theta)\big(15 - 2\theta(6 - \theta)\big)}.$$

Investment by R_1 (R_2) rises (falls) in the cross-network benefit at the platform — $\frac{\partial v_1^P}{\partial \theta} > 0$ and $\frac{\partial v_2^P}{\partial \theta} < 0$.

The transformation of R_1 into a platform gives it additional incentives to innovate through the positive feedback loop of developer participation than in the traditional market case, while R_2 does not have this incentive.[46] To be more specific, an increase in v_1 increases developers' expectation on the output of the platform R_1 which increases their participation on the platform. This, in turn, increases consumer valuation for the platform product which makes it profitable for R_1 to expand output. As the value of cross-network interactions, represented by θ, increase these innovation disparities at the two firms increase as well. Substituting these equilibrium external investment levels in the outputs and performing some comparative statics, we present the insights below.

Lemma 6 (Case $g = P$: Outputs) *The equilibrium output of R_1 (R_2) is higher (lower) than in the traditional firm case —*

$$x_1^{P*} = \frac{135 - 66\theta + 8\theta^2}{(69 - 10\theta)(15 - 2\theta(6 - \theta))} > x^T(v^T, v^T) > x_2^{P*} = \frac{(9 - 2\theta)(15 - \theta(15 - 2\theta))}{(69 - 10\theta)(15 - 2\theta(6 - \theta))}.$$

The output set by R_1 (R_2) rises (falls) in the cross-network benefit at the platform. Total market output, $X^{P*} = x_1^{P*} + x_2^{P*}$, is higher than in case $g = T$ and rises in intensity of cross-network benefits.

The intuition for R_1's equilibrium output being higher when it is a platform than when it was a traditional firm is that now it offers consumers additional value through two reinforcing channels. First, the presence of external value creators on its platform which increases margins. Second, higher (internal) investment by R_1 to enhance consumer value relative to R_2. These two intertwined value

[46] As shown later, when uncertainty of investments (after digital transformation) is high, investment of the platform can be lower than the investment of its traditional rival R_2.

creation channels increase its margins which makes it profitable to expand output as well. The fall in R_2's output arises from the fact that it is at a disadvantage through its inability to attract consumers through external value creation as it is not active on the platform. This also lowers its innovation incentives. Lower investment by R_2 than R_1 and considering the fact that outputs are strategic substitutes, it is straightforward that the output of R_2 is lower than the output of R_1. Technically, output expansion by the rival (platform R_1) lowers its market price which encourages R_2 to lower its output as well. Similarly, an increase in γ increases the value consumers derive from developer participation on the platform as well as increases investment by R_1. This increases R_1's quality advantage and thus also its output while reducing R_2's output.

An increase in total output with an increase in intensity (value) of cross-network interaction comes directly from the fact that an increase in θ positively impacts x_1^P directly through an increase in consumer value. The negative impact on x_2^P is indirect through increased output by R_1 which is a consequence of strategic substitutability in Cournot Games. This negative indirect impact is always dominated by the direct effect and thus, we have the result that total outputs are higher.[47] Substituting equilibrium output into developer demand, we obtain the equilibrium mass of developers active on the platform as $Dev^{P?} = k^{P?} = \phi x_1^{P?}$.

Lemma 7 (Case $g = P$: Market price and profits) *The market price of R_1 (R_2) rises (falls) in the value of cross-network interactions, θ, at the platform.*
The pro t of R_1 (R_2) increases (decreases) in cross-network bene ts. An increase in cross-network bene ts, θ, increases the mass of developers participating on the platform | $\frac{@Dev^{P\star}}{@} > 0$.

An increase in cross-network benefits behaves similar to an increase in the value offered to consumers on the platform from participating in the network effect. An increase in θ (suppose through an increase in γ) impacts market price at R_1 through three channels and sum of these effects determine its impact on the price. In the following expression, we break down the effects for clarity.

$$\frac{\partial P_1^P(v_1^P, \Psi_1^{P?}, X)}{\partial \theta} = \underbrace{\frac{\partial P_1^P(\,)}{\partial v_1}\frac{\partial v_1^P}{\partial \theta}}_{\text{Interval value e ect (+)}} + \underbrace{\frac{\partial P_1^P(\,)}{\partial \Psi_1^P}\left(\frac{\partial \Psi_1^{P?}}{\partial \theta} + \frac{\partial \Psi_1^{P?}}{\partial Dev^P}\frac{\partial Dev^P}{\partial x_1^e}\frac{\partial x_1^{P?}}{\partial \theta}\right)}_{\text{External value e ect (+)}} + \underbrace{\frac{\partial P_1^P(\,)}{\partial X}\frac{\partial X^{P?}}{\partial \theta}}_{\text{Output e ect (\ )}} > 0.$$

First, there is a direct effect arising from the increase in value consumers place on interacting with developers. Secondly, consumer value also increases indirectly through a larger mass of developers (greater volume of interactions) active on the platform. Thirdly, there is a negative impact on market price of R_1 through increased total output in the market. The sum of these three effects is positive giving us the result that an increase in θ increases market price of R_1.

Similarly, we describe how a change in θ impacts market price of R_2. Recall that R_2 does not have access to the platform and cannot participate in the external value creation. As before, we break down the impact of an increase in θ on market price of R_2 into the different components.

[47]The rationale for the indirect effect being dominated by the direct effect is straightforward. To ensure that best responses intersect, the slope of best responses must lie in the interval (-1,0). This also implies any opposing indirect effect that relies of firms responding will always be dominated.

This is reflected in the following expression with the conspicuous absence of external value effect.

$$\frac{\partial P_2^P(v_2^P, 0, X^{P?})}{\partial \theta} = \underbrace{\frac{\partial P_2^P(\ )}{\partial v_2}\frac{\partial v_2^P}{\partial \theta}}_{\text{Value e ect ( )}} + \underbrace{\frac{\partial P_2^P(\ )}{\partial X}\frac{\partial X^{P?}}{d\theta}}_{\text{Output e ect ( )}} < 0.$$

There are two reinforcing effects that negatively impact the market price of R_2. First, an increase in θ negatively affects R_2^0s investments. Secondly, market price is negatively affected by the output effect arising from an increase in θ. These two effects together give the result that R_2's price is falling in θ. Consequently, an increase in θ through an increase in γ and/or ϕ increases the profit of the platform R_1 while the profit of R_2 falls. Finally, recalling from Lemma 6 that the output of R_1 rises in θ gives us the result that the mass of active developers on the platform also increases with θ.

D.2 Platform R_1 hosts the rival firm R_2 ($g = H$)

In this market structure, we consider the case where R_1 transforms into an open platform and invites rival R_2 to participate on its platform for a fee which we denote as L.[48] The platform R_1 when inviting its rival on the platform must consider the following trade-off.

First, by inviting the rival R_2 onto its platform, R_1 gives R_2 access to developers on its platform and lowers its platform advantage which negatively impacts R_1's profitability. This lowering of R_1's platform advantage arises from the fact that by inviting the rival R_2 on to its platform, consumers experience an increase in quality for R_2^0s product as consumers are able to experience external value creation by interacting with developers on the platform.[49] Figure 1, Panel (c) presents the market structure when $g = H$.

This market structure is reflected in expression for market price of R_1 and R_2 which are respectively given as

$$P_1^H(v_1, \Psi^H, X) = 1 + v_1 + \Psi^H \quad 3X, \qquad P_2^H(v_2, \Psi^H, X) = 1 + v_2 + \Psi^H \quad 3X. \tag{54}$$

Note, that the platform external value creation through developers Ψ^H at the two firms on the platform is symmetric while the interval value creation v_i differs.[50]

Second, R_1 offers developers access to a larger base of consumers which also expands their participation on the platform in turn increasing R_1's profitability. Specifically, the mass of developers in case $g = H$ is given as

$$Dev^H(X^e) = \phi X^e = \phi(x_1^e + x_2^e),$$

where developers join expecting the total demand from consumers in the market. This increased

[48] For instance, Klockner, a steel distributor, developed a platform and invited its rivals to participate.

[49] We do not allow platform R_1 to reduce the compatibility of the platform with R_2^0s product which implies that consumers buying R_2^0s product benefit from interacting with a smaller mass of active developers on the platform R_1.

[50] Later, when we introduce uncertainty of investments, Ω, we show that there is an additional strategic effect of inviting the rival.

mass of developers positively impacts its output and margins. It is unclear ex-ante which of the two effects will dominate.

Output setting stage. Given consumer and developer expectations in stage 3, each firm sets outputs to maximize profits. The profit of R_1 and R_2 are respectively given as

$$\max_{x_1} \Pi_1^H(v_1, \Psi^H, X) + L - I(v_1) = P_1^H(v_1, \Psi^H, X)x_1 + L - I(v_1), \text{ and}$$

$$\max_{x_2} \Pi_2^H(v_2, \Psi^H, X) - L - I(v_2) = P_2^H(v_2, \Psi^H, X)x_1 - L - I(v_2).$$

Differentiating the profit expression of firm R_i for $i \in \{1, 2\}$ with respect to its output and imposing rational expectations — $X^H(v_1, v_2) = x_1^e + x_2^e$, $\Psi^H(v_1, v_2) = \gamma\phi X^H(v_1, v_2) = \theta X^H(v_1, v_2)$, yields the output of R_1 and R_2 as a function of investments. From here on, we will employ the variable transformation $\theta = \gamma\phi$. The output of firm R_i as a function of investments are given as

$$x_i^H(v_i, v_{-i}) = \frac{3 + v_i(6 - \theta) - v_{-i}(3 - \theta)}{27 - 6\theta}. \tag{55}$$

The total output as a function of investments and firm profits are given as

$$X^H(v_1, v_2) = x_1^H(v_1, v_2) + x_2^H(v_2, v_1) = \frac{2 + v_1 + v_2}{9 - 2\theta}. \tag{56}$$

The associated profit of the platform R_1 and its rival R_2 is given as

$$\Pi_1^{H\star}(v_1, v_2) + L - I(v_1) = P_1^H((v_1, \Psi^H(v_1, v_2), X(v_1, v_2))x_1^H(v_1, v_2) - I(v_1) + L, \tag{57}$$

$$\Pi_2^{H\star}(v_2, v_1) - L - I(v_2) = P_2^H(v_2, \Psi^H(v_1, v_2), X(v_1, v_2))x_2^H(v_2, v_1) - I(v_2) - L. \tag{58}$$

Innovation stage: In stage 2, given fixed fee L, firms unilaterally invest in the value for the product v_i to maximize profits as expressed in equations (57) and (58). Differentiating the profits of firm R_i with respect to v_i for $i \in \{1, 2\}$ and employing the envelope theorem yields

$$x_i^H(\cdot)\left(\underbrace{\frac{\partial P_i^H(\cdot)}{\partial v_i}}_{\substack{\text{Internal}\\ \text{value effect}}(+)} + \underbrace{\frac{\partial P_i^H(\cdot)}{\partial \Psi^H}\frac{\partial \Psi^H}{\partial X^e}\frac{\partial X^H(\cdot)}{\partial v_i}}_{\text{External value effect }(+)} + \underbrace{\frac{\partial P_i^H(\cdot)}{\partial X}\frac{\partial x^H{}_i(\cdot)}{\partial v_i}}_{\text{Price effect}(+)}\right) - \frac{\partial I(v_i)}{\partial v_i} = 0, \text{ for } i \in \{1, 2\}. \tag{59}$$

As before, we can break down the incentives to invest into internal value effect, external value effect and the price effect. All of these effects positively impact the incentives to invest. Each firm R_i must trade-off marginal revenue increase from an additional unit of investment with the associated marginal cost increase.

In comparison to the first order conditions in case $g = P$ (equations (52) and (53)), one can notice a few conspicuous differences in the first order condition of R_1 and R_2 (equation (59)). Notice, that now both R_1 and R_2 benefit from the external value creation from developers. This term positively impacts the innovation incentive of R_2 relative to R_2's innovation incentive in case $g = P$. Instead for R_1 now the positive effect of external value creation is dampened. This can be observed by comparing how outputs impact external value creation in the two cases. In case $g = H$, observe that the external value creation is impacted by changes in total output with a change in v_i, while in case $g = P$ (see equation (52)), external value creation is impacted only by changes in x_1^P. Therefore, it is sufficient to compare the slope of total demand in case $g = H$ (which is the volume of consumer interactions faced by developers in case $g = H$ as expressed in equation (56)) with the slope of x_1^P (which is the volume of consumer interactions faced by developers in case $g = P$, as expressed in equation (50)) with respect to v_1. Specifically, we observe that $\frac{\partial x_1^P(\cdot)}{\partial v_1} > \frac{\partial X^H(\cdot)}{\partial v_1} > 0$. Thus, we can state that in comparison to the case $g = P$, R_1's investment incentives are dampened in $g = H$.

Lemma 8 (Case $g = H$: Investments) *The equilibrium investment levels are symmetric and given by $v^H = \frac{2(6-\theta)}{69-34\theta+4\theta^2}$. This equilibrium investment level rises in the intensity of cross-network benefits. In comparison to the case $g = P$, investment by R_1 (R_2) is lower (higher) with hosting — $v_1^P > v^H > v_2^P$.*

It is straightforward that R_2 has higher incentives to invest in internal value creation under hosting than in case $g = P$. By participating in the platform, the marginal gain from every additional unit of investment is higher. This is because now an increase in internal value creation also leads to an increase in external value creation for R_2 and this makes it profitable for R_2 to raise investment levels.

The reduced incentive to invest for R_1 relative to case $g = P$ arises from levelling of the playing field for the two firms as both of them participate in the benefits from external value creation at the platform. Any investment increase by a firm R_1 also benefits its rival through increased external value which makes every additional unit of investment less profitable in comparison to case $g = P$ where all benefits of investment are accrued by R_1 only.

Substituting these equilibrium investment levels into output of the firms on the platform and performing some comparative statics, we discuss the results in the following Lemma.

Lemma 9 (Case $g = H$: outputs) *The equilibrium output of firms on the platform is symmetric and given as*

$$x^{H\star} = x^H(v^H, v^H) = \frac{9-2\theta}{69-34\theta+4\theta^2}.$$

Firm R_1's $(R_2's)$ output is lower (higher) in case $g = H$ relative to its output in case $g = P$ — $x_2^{P\star} < x^{H\star} < x_1^{P\star}$. The output of each firm in the platform unambiguously rises in the intensity of cross-network benefit. Total market output is higher in case $g = H$ than case $g = P$ and always rises in intensity of cross-network benefits.

The increase in output of R_2 is straightforward and is a direct consequence of participation in the platform which enhances consumer's willingness to pay for its products and hence makes it profitable to expand output. On a similar vein, since inviting the rival to participate in the platform lowers R_1's incentive to innovate as some of the gains from investments are also shared with the rival, its output is lower vis-a-vis case $g = P$. Nevertheless, total output is higher in case $g = H$ than in case $g = P$ as the positive effect on R_2^{0}s output outweighs any negative effect on R_1^{0}s output. Interestingly but not surprisingly, the output of both firms rise with an increase in θ. The rational for this is straightforward. An increase in θ enhances the value of cross-sided network interactions at both the firms on the platform. This increases the consumers' willingness to pay and hence also the price for each product. The direct positive effect on the market price of the two firms outweighs the second order negative effect on prices and thus makes it profitable for each firm to increase output as well.

Optimal contract L. In the following, we discuss the optimal fixed-fee contract offered by the platform R_1 to rival R_2 for joining its platform. We assume that platform R_1 has all the bargaining power and makes take-it-or-leave-it offers to its rival R_2. Therefore, R_1 sets its fixed fee to just ensure that $R_2^{0}s$ participation constraint is satisfied. To be precise, the optimal contract is set such that R_2 is indifferent between accepting or rejecting R_1^{0}s optimal contract offer. Specifically, the optimal fixed fee is given as

$$L^{?} = \Pi_2^{H?}(v^H, v^H) \quad I(v^H) \quad (\Pi_2^{P?}(v_2^P, v_2^P) \quad I(v_2^P)).$$

The associated total profit of platform R_1 is given as[51]

$$\Pi_1^{H?}(v^H, v^H) \quad I(v^H) + \underbrace{\Pi_2^{H?}(v^H, v^H) \quad I(v^H) \quad \Pi_2^{P?}(v_2^P, v_1^P) \quad I(v_2^P)}_{L^\star}$$

In essence by setting contract $L^{?}$, platform R_1 is able to appropriate all the industry profit and leaves rival R_2 it's outside option. Therefore, the net profit of R_2 is

$$\Pi_2^{H?}(v^H, v^H) \quad L^{?} \quad I(v^H) = \Pi_2^{P?}(v_2^P, v_1^P) \quad I(v_2^P). \tag{60}$$

The following Lemma presents some comparative statics on the market price and profits.

Lemma 10 (Case $g = H$: Market price and profits) *The market price of R_1 and R_2 is symmetric and rises in the cross-network bene t at the platform.*
The net pro t of R_1 (R_2) increases (decreases) in cross-network bene ts. An increase in cross-network bene ts θ, increases the mass of developers participating on the platform | $\frac{dDev^{H\star}}{d} > 0$.

The intuition for an increase in market price with cross-network effect θ is as follows. The effect of a change in cross-network effect intensity, θ, on market price can be broken down again into the internal value effect, external value effect and the output effect. The first two effects impact prices

[51] The explicit expression can be found in the appendix in the proof of Lemma 10.

positively and the output effect reduces market price.

$$\frac{\partial P^H(v^H, \Psi^H(v^H, v^H), X^{H?})}{\partial \theta} = \underbrace{\frac{\partial P^H(\ )}{\partial v_1}\frac{\partial v_1^H}{\partial \theta}}_{\text{Interval value e ect (+)}} + \underbrace{\frac{\partial P^H(\ )}{\partial \Psi^H}\left(\frac{\partial \Psi^H}{\partial \theta} + \frac{\partial \Psi^H}{\partial X^e}\frac{\partial X^{H?}}{\partial \theta}\right)}_{\text{External value e ect (+)}} + \underbrace{\frac{\partial P^H(\ )}{\partial X}\frac{\partial X^{H?}}{\partial \theta}}_{\text{Output e ect (\ )}} > 0.$$

The price increasing effect of an increase in value dominates the indirect price decreasing effect through increased output. As a result, market price increases in cross-network effects, θ. The net profit of R_1 always increases in θ. There are two reinforcing effects. First, the flow profit is increasing with θ. Second, the optimal contract is also increasing in θ. These two effects together imply that the profit of R_1 is rising. On the contrary, the net profit of R_2 is falling in θ. Recall from the expression in equation (60) that the net profit of R_2 is just its profit in case $g = P$. Further, from Lemma (7), we know that this net profit is falling in θ. Thus, we can state that the net profit of R_2 is falling in θ. Finally, from Lemma (9), we know that total market output is increasing in θ. As a consequence of this, the total mass of developers active on the platform also rises with an increase in θ.

D.3 To be a platform or not and the incentives to host rival R_2

In this subsection, we study the decision of R_1 whether to be a platform or not when digital transformation presents no innovation risks. This deterministic case helps us understand the role of risk in the decision-making process whether to digitally transform or not. Toward this, we first compare the profit of R_1 and R_2 in the three market structures g 2 fT, P, Hg.

Comparing the profit of R_1 and R_2 when R_1 is a traditional firm ($g = T$) relative to its profit when it is a closed platform ($g = P$), we discuss the results in the following proposition.

Proposition 6 *It is always more pro table for R_1 to be a platform than being a traditional rm. R_2's pro t falls when R_1 is a closed platform relative to its pro t when R_1 was a traditional rm.*

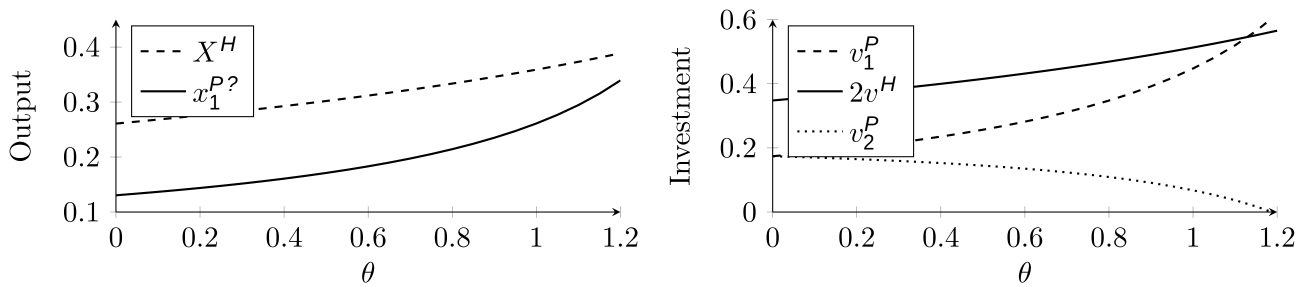
This is a straightforward result and arises directly from the fact that R_1 being a platform is able to externalize value creation and increase its quality advantage relative to R_2. As a consequence, R_1 gains from being a platform while R_2 loses. Next comparing R_1^0s profit when it hosts its rival than the case when it does not host its rival.

Proposition 7 *Platform R_1 prefers to host rival R_2 (case $g = H$) only when the cross-network e ects are su ciently low | i.e., $\theta < \tilde{\theta} = 1.16$. Otherwise, R_1 prefers not to host its rival stay a closed platform.*

Specifically, comparing R_1^0s profit with and without hosting, $R_1^0 s$ profit is greater with hosting R_2 than without hosting R_2 if and only if

$$\Pi_1^{H?} + L^? \quad I(v^H) > \Pi_1^{P?} \quad I(v_1^P) \implies 2(\Pi^{H?} \quad I(v^H)) > \Pi_1^{P?} \quad I(v_1^P) + \Pi_2^{P?} \quad I(v_2^P).$$

Interestingly, we observe that the industry profit is higher when $\theta < \tilde{\theta}$.[52] This is because an increase in cross-network benefit θ increases total outputs in both market structures. When cross-network benefits are large enough, the external value creation effect of hosting R_2 through increased platform output is not large enough to compensate for the increased competition effect on joint profits. We illustrate total output and investments in case $g = P$ with case $g = H$ in plots (14b) and (14a) respectively.



(a) Total output in case $g = H$ vs. output of R_1 in case $g = P$.

(b) Investment in case $g = H$ vs. investment by R_1 and R_2 in case $g = P$.

Figure 14: Comparing equilibrium output and investments in the cases $g = P$ and $g = H$.

Notice from the above that although the total output in case $g = H$ is greater than the output of R_1 in case $g = P$, the output of R_1 in case $g = P$ catches up with the total output in case $g = H$ as θ increases. Also, notice that the total investment in case $g = H$ is lower than the individual investment by R_1 in case $g = P$. This suggests that hosting the rival may not be profitable for R_1 when network effects are high as the fall in profitability from increased competition dominates any positive effect arising from hosting the rival.

Consumer Surplus and Welfare implications

Consumer surplus across the market structures g 2 fT, P, Hg is given as

$$CS^{g?} = 3(X^{g?})^2/2.$$

Thus, we can state the following regarding consumer surplus in the three market structures.

Proposition 8 (Consumer surplus) *Consumer surplus in the three market structures can be ordered as follows. $CS^{H?} > CS^{P?} > CS^{T?}$. Thus, when $\theta < \tilde{\theta}$, platform R_1's hosting choice is aligned with consumer surplus maximizing market structure and otherwise it is misaligned.*

This is a straightforward result which arises directly by comparing the market outputs in the three regimes.

[52] This result is qualitatively similar to Niculescu et al. (2018) who also find that a platform does not invite its rival to participate in the network effects when the value of cross-network effects is high.

Developer surplus in market structures $g = P$ and $g = H$ are given as

$$DS^{P?} = \int_0^{\tilde{k}^{P\star}} (\phi x_1^{P?} \quad k)\lambda(k)dk, \quad DS^{H?} = \int_0^{\tilde{k}^{H\star}} (\phi X^{H?} \quad k)\lambda(k)dk.$$

Since platform participation for developers solely depends on their development costs, it is straightforward that if there is a larger mass of developers participating on the platform, their surplus is also higher. Specifically, we know that $\tilde{k}^{H?} > \tilde{k}^{P?}$ which implies that a larger mass of developers is active in case $g = H$. We present our observations in the following

Corollary 3 (Developer surplus) *When platform R_1 hosts its rival R_2, developer surplus is higher than when it does not host the rival. Thus, when $\theta < \tilde{\theta}$, platform R_1's hosting choice is aligned with developer's surplus maximizing market structure.*

This is a direct implication of market outputs being higher in the case when R_1 hosts its rival than in case $g = P$ and the fact that R_1 chooses to host rivals only when $\theta < \tilde{\theta}$.

The total welfare in the market is computed as the sum of firm profits, consumer surplus and developer surplus. Specifically,

$$TW^{g?} = CS^{g?} + \Pi_1^{g?} + \Pi_2^{g?} + DS^{g;?}.$$

Note that $DS^{T?} = 0$ as there is no developer market. Comparing the total welfare in the three regimes, we present our results below.

Proposition 9 (Total welfare) *Total welfare in the three market structures can be ordered as follows. $TW^{H?} > TW^{P?} > TW^{T?}$. Thus, when $\theta < \tilde{\theta}$, platform R_1's hosting choice is aligned with the total welfare maximizing market structure and otherwise it is misaligned.*

The above result follows directly from the above discussion.

E Micro foundations of consumer demand

We present the micro foundations of the model in the three market scenarios below. Consumers have a basic valuation r with the support $[\quad 2, 1]$ which follows the Uniform distribution, i.e. r $U[\quad 2, 1]$.[53] The total valuation for consumers regarding the good also depends on the market structure denoted by the superscript g. Specifically, we consider three market structures. (i) R_1 and R_2 are traditional ($g = T$), (ii) R_1 is a platform and R_2 is a traditional firm ($g = P$), and (iii) R_1 is a platform and it hosts R_2 on its platform ($g = H$).

The utility of a consumer of type r that buys from R_1 or from R_2 given the market structure g is given as

$$u_1(r) = r + v_1 + \Psi_1^g \quad P_1, \qquad u_2(r) = r + v_2 + \Psi_2^g \quad P_2.$$

[53] As in Katz & Shapiro (1985), we assume that the lower bound of this support is sufficiently negative to ensure that the total market demand is within the bounds of $(0; 1)$.

where Ψ_i^g is the degree of external value creation in the market structure $g \in \{T, P, H\}$ and v_i is the internal value investment made my firms. Under the above specification, R_1 and R_2 have positive demand only if the following "no arbitrage" condition holds $\phi^g = P_1 - \Psi_1^g - v_1 = P_2 - \Psi_2^g - v_2$.[54] Consumers buy the good only if $r \geq r^g = \phi^g$.[55] Hence, total demand for the product in the market is $X = 1 - \frac{(r^g+2)}{3}$, where $X = \sum_{i=1}^{2} x_i$ being the total output in the market.

Rearranging and inverting the above expression yields the following inverse demand expression at the two firms as

$$P_1^g(v_1, \Psi_1^g, X) = 1 + v_1 + \Psi_1^g - 3X, \qquad P_2^g(v_2, \Psi_2^g, X) = 1 + v_2 + \Psi_2^g - 3X$$

with $\frac{\partial P_i^g(\cdot)}{\partial X} < 0$ for $i \in \{1, 2\}$ and $\frac{\partial P_i^g(\cdot)}{\partial \Psi_i^g} \geq 0$ and $\frac{\partial^2 P_i^g(\cdot)}{\partial (\Psi_i^g)^2} = 0$.

[54] Any consumer of type r should be indifferent between buying from R_1 or R_2 i.e. $U_1(r) = U_2(r)$. This gives us the desired no arbitrage condition.

[55] We obtain this condition from the inequality $U_i(r) \geq 0$ for both firms.