Third-degree price discrimination in two-sided markets*

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We investigate the welfare effects of third-degree price discrimination by a two-sided platform that facilitates interactions between buyers and sellers. Sellers are heterogenous with respect to their per-interaction benefit, and, under price discrimination, the platform can condition its fee on sellers' type. In a model with linear demand on each side, we show that price discrimination on the seller side: (i) increases participation on both sides; (ii) increases total welfare; (iii) may result in a Pareto improvement, with both seller types being better-off than under uniform pricing. These results, which are in stark contrast to the traditional analysis of price discrimination, are driven by the existence of cross-group network effects. By improving the firm's ability to monetize seller participation, price discrimination induces the platform to attract more buyers, which then increases seller participation. The Pareto improvement result means that even those sellers who pay a higher price under discrimination can be better-off, due to the increased buyer participation. These results provide clear and direct managerial and policy implications.

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1. Introduction

Online marketplaces often resort to price discrimination when dealing with a heterogenous population of sellers. For instance, Amazon and eBay charge different commission rates depending on the product category (electronics, clothes, etc.).[1] In their application stores, Apple and Google discriminate between large and small developers, by charging a higher commission rate (30% instead of 15%) for developers with more than $1m annual revenue (more examples can be found in Borck et al. (2020)).

What are the distributional and welfare consequences of such practices? While the effects of third-degree price discrimination have been widely studied (see our literature review below), an interesting feature of marketplaces is that they are two-sided markets, in which the presence of buyers and sellers generates cross-side (sometimes called indirect) network effects. To what extent do the lessons from the standard analysis of third-degree price discrimination apply to two-sided markets? How should a platform design its pricing policy in the presence of network effects? What are the managerial and policy lessons that can be learned?

To answer these questions, we study a simple model of monopoly price discrimination by a two-sided platform. There are two groups of agents, buyers and sellers. All buyers obtain the same per-seller benefit,[2] but sellers are heterogeneous with respect to their revenue: high-type sellers obtain a larger revenue for each buyer present on the platform than low-type sellers. The platform charges participation fees to buyers and sellers.[3] Agents also differ with respect to their exogenous participation cost (or outside option), which is distributed in such a way as to have linear demand on both sides of the market. We compare the situation where the platform charges the same participation fee to all sellers (uniform pricing) to one in which it can set different fees to high- and low-type sellers (third-degree price discrimination).

Our first result is that seller-side price discrimination leads to an increase in the participation of both buyers and sellers. Intuitively, allowing the monopolist to charge different seller fees allows it to better monetize buyer participation, thereby giving it an incentive to attract more buyers. This in turn attracts more sellers, resulting in overall larger participation on both sides. Second, we show that total welfare increases with price discrimination. Third, we show that price discrimination can constitute a *strict* Pareto improvement: because of increased buyer participation, high-type sellers may be better-off even if they end up paying a higher fee than

[1] See `https://sell.amazon.com/pricing` and `https://www.ebay.co.uk/help/selling/fees-credits-invoices/fees-business-sellers?id=4809`, accessed 30 December 2022.

[2] Our results are robust to the case where buyers obtain different per-seller benefits. See Subsection (5.1) for more details.

[3] Our results also hold when the platform employs an ad-valorem pricing structure on the seller side. See Subsection (5.2) for more details.

under uniform pricing.

These results stand in sharp contrast to the “traditional" analysis of third-degree price discrimination. Indeed with linear demands and no network effects, two cases can happen: either the weak market is not served under uniform pricing, in which case price discrimination allows to serve the weak market without changing the price in the strong market, resulting in a welfare increase and *weak* Pareto improvement; or, when the weak market is served under uniform pricing, price discrimination leaves total output unchanged but reduces total welfare. Throughout the paper we assume that the weak market (i.e. low-type sellers) is served under uniform pricing, to emphasize the importance of network effects.

Our analysis also delivers insights related to the platform's optimal pricing strategy. We identify several regimes, depending on parameter values. In the first, “typical" case, price discrimination leads to an increase in the fee paid by high-type sellers and to a decrease in the fee paid by low-type ones. In this regime, the fee paid by buyers decreases compared to uniform pricing if and only if buyers' network benefits are small enough: the platform then needs to lower buyers' fee to increase their participation. When buyers obtain large benefits from sellers' participation the platform can increase their fee without inducing a drop in participation. Other, more surprising patterns may also emerge in equilibrium. In the second regime, all seller fees increase under price discrimination, in which case buyers are subsidized (and more so under price discrimination). Alternatively, in the third regime all seller fees decrease while buyers pay a higher fee. Interestingly, whether a group (buyers or sellers) pay higher or lower fees does not change the result mentioned above that participation increases for both groups.

For analytical tractability, the baseline model relies on some simplifying assumptions, in partcular that buyer benefits are independent of sellers' types, and that the platform charges participation fees. As we show in Section 5, our main insights do not hinge on these assumptions. First, we consider the case in which buyer surplus depends on the seller type. There again, price discrimination increases participation on both sides and may constitute a Pareto improvement. Even though welfare no longer always increases, numerical results indicate that when it decreases the loss is very small (less than 1%), while welfare gains can be more substantial. In the second extension, the platform sets ad valorem instead of participation fees. This is more consistent with the business model of platforms such as Apple and Google. Analytical results are more difficult to obtain, but our main findings concerning the welfare-enhancing effect as well as the possibility of strict Pareto improvement under price discrimination continue to hold. In the third extension, we investigate the situation in which the platform cannot charge buyers. Welfare is no longer always higher under price discrimination, due to the fact that the platform has fewer instruments to attract buyers and sellers. We confirm, however, the existence of an parameter region in which price discrimination leads to a strict Pareto improvement.

2. Relevant literature

The analysis of third-degree price discrimination by a monopolist has a long tradition in economics (Pigou 1924, Robinson 1933, Schmalensee 1981, Varian 1985, Aguirre et al. 2010). Because it tends to lead to higher prices in some markets and to lower prices in others, its welfare effects are a priori ambiguous. As shown by Schmalensee (1981) and Varian (1985), a necessary condition for welfare to increase is that total output increases. Failing this, having different consumers face different prices leads to an inefficient "maldistribution of resources" (Robinson 1933). A case of particular interest thanks to its tractability is that of linear demands. There, Robinson (1933) shows that, provided that the firm made positive sales to each market under uniform pricing, output would remain the same under price discrimination, and therefore welfare would decrease. Our main contribution is to show that this result is overturned when the firm operates a two-sided market (the study of which was pioneered by Caillaud & Jullien 2003, Rochet & Tirole 2003, Parker & Van Alstyne 2005, Armstrong 2006, among others).

While Pareto improvement is possible in traditional (i.e. not two-sided) markets, in particular when not all markets are served under uniform pricing, we show that it can happen with linear demands even if all markets are served under uniform pricing, a result that cannot hold in traditional markets.

A few recent papers study price discrimination in two-sided markets, though of either the first or second-degree kind. Liu & Serfes (2013) show that first-degree price discrimination can soften competition in a setup where the opposite would happen absent cross-group network effects. In the context of second-degree price discrimination, Böhme (2016) shows that some properties of the optimal contract in traditional markets (e.g. no distortion at the top) no longer hold in two-sided markets. Jeon et al. (2022) provide condition for pooling to be optimal, and for second degree price discrimination to increase or decrease welfare. In a related setup, Lin (2020) shows that price discrimination is complementary across sides. Gomes & Pavan (2016) characterize the optimal many-to-many matching mechanism in the presence of two-sided asymmetric information.

Motivated by the app store controversies, Bhargava et al. (2022) study differential revenue sharing schemes, which bear some resemblance but are not equivalent to price discrimination. Indeed, they consider a platform returning to sellers a higher share for revenue-contributions up to a predetermined thereshold, and a smaller share above that. They find that the platform offering better terms to small developers may benefit large developers (a Pareto improvement), but do not consider the possibility for the platform to raise its commission to one group of developers. Also because of this constraint, the platform does not always gain from adopting a differential sharing scheme, and this represents another difference in comparison to our analysis.

Tremblay (2021) also considers a model of price discrimination by a monopolistic platform (in the absence of network externalities) who charges unit fees to merchants, and finds that perfect fee discrimination is likely to reduce welfare. This result, opposite from what we obtain, stems from a different set of modelling assumptions: we consider a model featuring network externalities, elastic participation on all sides, and a platform that is allowed to charge (or subsidize) buyers, while Tremblay (2021) views the platform as an upstream supplier who only charges merchants, and emphasizes the double marginalization problem.

Another related paper is Wang & Wright (2017), who argue that ad valorem fees are a way for platforms to efficiently price discriminate heterogenous merchants. Ding & Wright (2017) study price-discrimination by a payment card issuer, and find ambiguous welfare effects.

A few papers study third-degree price discrimination in one-sided platforms: Adachi (2005) considers a model where agents from each group enjoy the presence of agents ***from the same group***, and shows that welfare can increase with price discrimination even though total output remains the same. Belleflamme & Peitz (2020) analyze the monopoly provision of a network good where ***users care about the overall level of participation***; they show that, under particular circumstances, third-degree price discrimination is equivalent to versioning (second-degree price discrimination). Peitz & Reisinger (2022) demonstrate that operating multiple platforms allows to distinguish between single-homing and multi-homing sellers, which enables the platform owner to price discriminate between high-valuation and low-valuation sellers. Closer to us, Hashizume et al. (2021) consider third-degree price discrimination in a one-sided market in which the platform sells a network good in two separate markets. They provide conditions for price discrimination to constitute a Pareto improvement, but do not fully characterize its total welfare effects. Our model allows to investigate interaction between sellers and buyers that connect via the platform, and to consider the effect of increased participation on both sides.

3. Model

Consider a monopolist two-sided platform that orchestrates interactions between two groups, which we call buyers and sellers. The structure of the model is similar to Armstrong (2006): all pairs of participating agents interact, and the platform charges membership fees. While most of the examples of transaction platforms involve ad valorem fees, we focus on the case of membership fees for pedagogical and tractability reasons.[4] We show in Section ?? that our insights extend to the case of ad valorem fees.

[4] This approach is standard in the literature. See also Jullien et al. (2021), Belleflamme & Peitz (2020), Reisinger (2014), Shekhar (2021), Carroni et al. (2023) among others.

Sellers There are two categories of products, denoted L and H. Each category has a mass 1 of independent products, and each product is offered by a single seller.[5] We do not explicitly model sellers' pricing decisions. Instead, we assume that the seller of a product in category $j \quad \{L, H\}$ achieves a variable profit of θ_j for each buyer with which it interacts. We assume that $\theta_H > \theta_L > 0$, and refer to θ_j as the type of sellers in category j.

If we denote the number of buyers on the platform by N_B, the profit of a seller of type θ_j is $\theta_j N_B - f_j$, where f_j is the participation fee paid to the platform. We assume that sellers have an outside option whose value is uniformly distributed over $[0, 1]$, independently of their type. Thus, the demand from type j sellers is

$$D_j(N_B, f_j) = \theta_j N_B - f_j.$$

Total seller participation is

$$D_S(N_B, f_L, f_H) = (\theta_L + \theta_H)N_B - f_L - f_H.$$

We will compare two regimes: under uniform pricing, the fees must satisfy $f_L = f_H$, while no such constraint apply under price discrimination.

Buyers' payoffs. There is a mass 1 of buyers. Each buyer obtains a stand-alone value v from using the platform. In addition, they enjoy a benefit b for each seller who is present on the platform. For tractability reasons, in the baseline model we assume that the benefit b is independent of the type of the seller the buyers interacts with. While this assumption can be microfounded,[6] we relax it in Section ??.

If N_S sellers join the platform, a buyer obtains a utility $v + bN_S - p$ from joining the platform, where p is the participation fee set by the platform. Assuming that buyers also have an outside option whose value is uniformly distributed over $[0, 1]$, the participation level of buyers is

$$D_B(N_S, p) = v + bN_S - p. \qquad (1)$$

[5] In other words, there is no competition among sellers.

[6] For instance, suppose all marginal costs are zero. A buyer's willingness to pay for the product of a seller of type θ is equal to θ with probability x, and to $\theta + b/(1 - x)$ with probability $1 - x$, with $x > b/\theta_L$. The optimal price charged by a seller θ is equal to θ. In this case, per interaction profit is indeed equal to θ while consumers' expected per-interaction surplus is b.

Equilibrium demands and platform's profit. In an equilibrium with rational expectations, participation levels must satisfy

$$N_B = D_B(N_S, p) \quad \text{and} \quad N_S = D_S(N_B, f_L, f_H). \tag{2}$$

Instead of solving the above system to obtain participation levels as a function of fees, we use inverse demands and assume that the platform chooses participation levels and that fees adjust accordingly. In a monopoly setup the two approaches are equivalent, and the "quantity approach" allows us to convey the logic of our arguments more clearly.

Under uniform pricing, inverting the system (2) with the additional constraint that $f_L = f_H$ leads to the inverse demand system

$$P^U(N_B, N_S) = v + bN_S - N_B \quad \text{and} \quad F^U(N_B, N_S) = \frac{(\theta_L + \theta_H)N_B - N_S}{2}. \tag{3}$$

The platform then chooses N_B and N_S to maximize

$$\Pi^U(N_B, N_S) = N_B P^U(N_B, N_S) + N_S F^U(N_B, N_S). \tag{4}$$

Under price discrimination, on the other hand, inverting the system (2) leads to the following inverse demands, where N_L and N_H denote participation by sellers of type θ_L and θ_H respectively:

$$P^D(N_B, N_L, N_H) = v + b(N_L + N_H) - N_B, \quad F_H^D(N_B, N_H) = \theta_H N_B - N_H, \quad F_L^D(N_B, N_L) = \theta_L N_B - N_L. \tag{5}$$

Price discrimination enables the platform to choose the participation level of each type of seller independently, whereas under uniform pricing the platform can only choose the overall level of participation of sellers, without being able to change the composition of the set of sellers. The platform then chooses N_B, N_L and N_H to maximize

$$\Pi^D(N_B, N_L, N_H) = N_B P^D(N_B, N_L, N_H) + N_L F_L^D(N_B, N_L) + N_H F_H^D(N_B, N_H) \tag{6}$$

Interior solutions. Throughout the main text we focus on equilibria where all participation levels are strictly between 0 and 1. To ensure this, we make the following assumption:

Assumption 1. *Buyers' and sellers' valuation for participation on the other side as well as buyer intrinsic valuation are not too large, namely:* $0 < \theta_L < \sqrt{2}$, $\theta_H^2 + \theta_L^2 < 4$, $0 < v < \frac{4-\theta_H^2-\theta_L^2}{2}$ *and* $\max\{0, \frac{\theta_H - 3\theta_L}{2}\} < b < \bar{b} \equiv \frac{1}{2}(\sqrt{8 - 4v - (\theta_H - \theta_L)^2} - \theta_H - \theta_L)$.

In Subsection 4.4 we consider what happens when we relax some of these assumptions.

4. Analysis

4.1. Participation

Uniform pricing. Under uniform pricing, the platform chooses N_B and N_S to maximize $\Pi^U(N_B, N_S) = N_B P^U(N_B, N_S) + N_S F^U(N_B, N_S)$. Dropping the arguments to lighten notations, the first-order conditions are:

$$\frac{\partial \Pi^U}{\partial N_B} = 0 \Leftrightarrow P^U + N_B \frac{\partial P^U}{\partial N_B} + N_S \frac{\partial F^U}{\partial N_B} = 0, \tag{7}$$

$$\frac{\partial \Pi^U}{\partial N_S} = 0 \Leftrightarrow F^U + N_S \frac{\partial F^U}{\partial N_S} + N_B \frac{\partial P^U}{\partial N_S} = 0. \tag{8}$$

Beyond the standard marginal revenues, captured by the first two terms on the left-hand side of (7) and (8), the third terms in each equation capture the idea that attracting an extra agent on one side allows the platform to increase its revenue on the other side.

Price discrimination. Under price discrimination, the platform chooses N_B, N_L and N_H to maximize $\Pi^D(N_B, N_L, N_H) = N_B P^D(N_B, N_L, N_H) + N_L F_L^D(N_B, N_L) + N_H F_H^D(N_B, N_H)$. The first-order conditions are

$$\frac{\partial \Pi^D}{\partial N_B} = 0 \Leftrightarrow P^D + N_B \frac{\partial P^D}{\partial N_B} + N_L \frac{\partial F_L^D}{\partial N_B} + N_H \frac{\partial F_H^D}{\partial N_B} = 0, \tag{9}$$

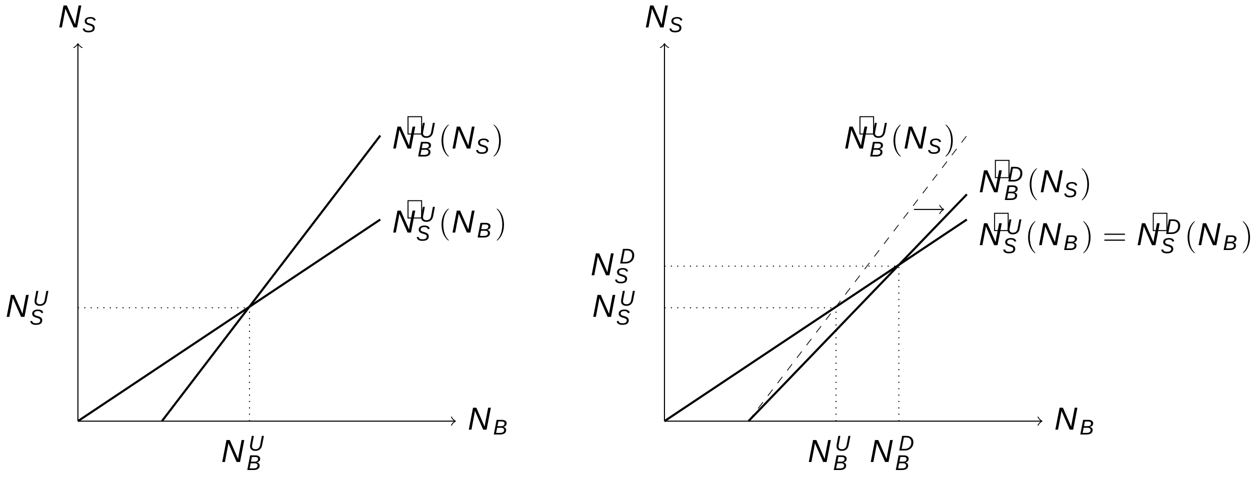
$$\frac{\partial \Pi^D}{\partial N_L} = 0 \Leftrightarrow F_L^D + N_L \frac{\partial F_L^D}{\partial N_L} + N_B \frac{\partial P^D}{\partial N_L} = 0, \tag{10}$$

and

$$\frac{\partial \Pi^D}{\partial N_H} = 0 \Leftrightarrow F_H^D + N_L \frac{\partial F_H^D}{\partial N_H} + N_B \frac{\partial P^D}{\partial N_H} = 0. \tag{11}$$

A first result. We are now ready to state our first main result:

Proposition 1. *Under price discrimination, the equilibrium number of both buyers and sellers increases compared to uniform pricing.*



(a) uniform pricing (b) price discrimination

Figure 1: Equilibrium participation

The proof of Proposition 1 can be found in Appendix A.1. Here we provide the intuition for it, illustrated in Figure 1. In the figure, $\hat{N}_B^U(N_S)$ is the profit-maximizing participation level for buyers, under uniform pricing, when N_S sellers participate (i.e. the solution to $\max_{N_B} \Pi^U(N_B, N_S)$). The other curves are defined similarly.

First, the participation level of buyers and sellers are complementary from the platform's point of view: increasing the participation level of sellers makes it more profitable for the platform to attract new buyers, and reciprocally. Indeed, as the number of sellers increase, not only can each buyer be charged a higher price (e.g. term $N_B \left(\partial P^U/\partial N_S\right)$ in (8), but attracting a new buyer allows the platform to increase its price to a larger base of sellers (e.g. term $N_S \left(\partial F^U/\partial N_B\right)$ in (7).[7] On the left panel of Figure 1 the equilibrium under uniform pricing is given by the intersection between the two increasing functions $\hat{N}_B^U(N_S)$ and $\hat{N}_S^U(N_B)$.

Second, *for a given level of buyer participation*, the profit-maximizing total number of sellers is the same under uniform pricing and discrimination. This result corresponds to the case of a traditional (one-sided) firm facing a linear demand: so long as both markets (here, both groups of sellers) are served under uniform pricing, discrimination does not affect total output (Robinson 1933). Formally, this follows from the fact that adding (10) and (11) gives (8). On the right panel of Figure 1, this observation means that $\hat{N}_S^U(N_B) = \hat{N}_S^D(N_B)$.

Third, *for a given level of seller participation* $N_S = N_H + N_L$, switching to the discrimination regime induces the platform to attract more buyers. Intuitively, being able to discriminate among sellers allows the firm to extract more of the value generated by each additional buyer:

[7] Formally, we have $\frac{\partial^2 \Pi^U}{\partial N_B \partial N_S} = \frac{\partial P}{\partial N_S} + \frac{\partial F^U}{\partial N_B} > 0$.

keeping the total number of sellers constant, an extra buyer generates additional revenues $\left(\frac{\theta_H+\theta_L}{2}\right)(N_H+N_L)$ under uniform pricing, against additional revenues of $\theta_H N_H+\theta_L N_L$ under discrimination. Because $N_H \geq N_L$ and $\theta_H > \theta_L$, the latter expression is larger. On the right panel of Figure 1, this corresponds to the shift from $\tilde{N}_B^U(N_S)$ to $\tilde{N}_B^D(N_S)$.

Put together, these observations imply that equilibrium participation of both sides is higher under price discrimination, driven by the extra incentive to attract buyers.

Discussion and generalization Two reasons motivate our choice of using linear demands, even though doing so is restrictive: First, we can obtain closed-form solutions, which allows us to provide a clean welfare analysis.[8] Second, they provide a nice benchmark when comparing our model to one without cross-side network effects: provided both markets are served under uniform pricing, output would remain the same and welfare would go down. Proposition 1 already shows that the output result is no longer true in two-sided markets. But actually the proposition holds under weaker assumptions. Indeed, the result holds if (i) N_B and N_S are complements, (ii) $\tilde{N}_B^D(N_S) > \tilde{N}_B^U(N_S)$, and (iii) $\tilde{N}_S^D(N_B)$ is not too much smaller than $\tilde{N}_S^U(N_B)$. Conditions (i) and (ii) are fairly natural: having more buyers tends to make attracting an extra seller more profitable (and reciprocally), and being able to extract more profit from sellers through price discrimination makes attracting extra buyers more profitable. Condition (iii) relates to a standard concern in the traditional analysis of third-degree price discrimination, namely the effect of discrimination on total output (see for instance Proposition 4 in Aguirre et al. 2010). Interestingly, the logic of Proposition 1 would hold even if output were to fall slightly in a traditional market, as illustrated in Figure 2.

4.2. Equilibrium

In order to provide welfare results, we need to explicitly compute the equilibria under uniform pricing and price discrimination.

Uniform pricing. Solving the system of first order conditions (7) and (8), we obtain:[9]

$$N_S^U = \frac{2v(2b+\theta_H+\theta_L)}{8-(2b+\theta_H+\theta_L)^2}, \quad N_B^U = \frac{4v}{8-(2b+\theta_H+\theta_L)^2}, \tag{12}$$

[8]Methods such as those used by Aguirre et al. (2010) to deal with more general demands do not work well with network effects.

[9]Participation of sellers of type $j \in \{L,H\}$ is $N_j^U = \frac{v(2b+3\theta_j-\theta_{-j})}{8-(2b+\theta_H+\theta_L)^2}$.

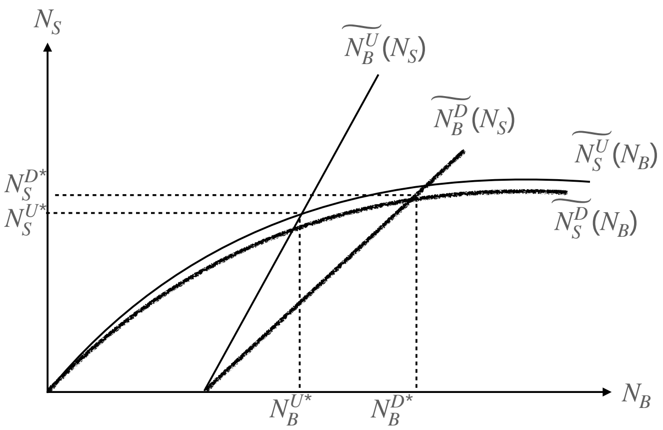


Figure 2: Example with $\widetilde{N_S^D}(N_B) < \widetilde{N_S^U}(N_B)$

which corresponds to equilibrium prices

$$p^U = \frac{v(4 - (\theta_H + \theta_L)(2b + \theta_H + \theta_L))}{8 - (2b + \theta_H + \theta_L)^2}, \quad f^U = \frac{v(\theta_H + \theta_L - 2b)}{8 - (2b + \theta_H + \theta_L)^2}, \tag{13}$$

and a profit for the platform equal to

$$\Pi^U = \frac{2v^2}{8 - (2b + \theta_H + \theta_L)^2}. \tag{14}$$

Price discrimination. Solving the system of first-order conditions (9), (11) and (10), we obtain

$$N_H^D = \frac{v(b + \theta_H)}{4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L)}, \quad N_L^D = \frac{v(b + \theta_L)}{4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L)},$$
$$N_B^D = \frac{2v}{4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L)},$$

which implies prices

$$f_H^D = \frac{v(\theta_H - b)}{4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L)}, \quad f_L^D = \frac{v(\theta_L - b)}{4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L)},$$
$$p^D = \frac{v(2 - \theta_H^2 - \theta_L^2 - b(\theta_H + \theta_L))}{4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L)}.$$

The platform's profit is then

$$\Pi^D = \frac{v^2}{4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L)}. \tag{15}$$

4.3. Comparison

Welfare analysis. Our main results concern the welfare effects of price discrimination. They are summarized in the proposition below, whose proof is in Appendix A.2

Proposition 2. *(i) The platform, buyers and low-type sellers are better-o under price discrimination.*

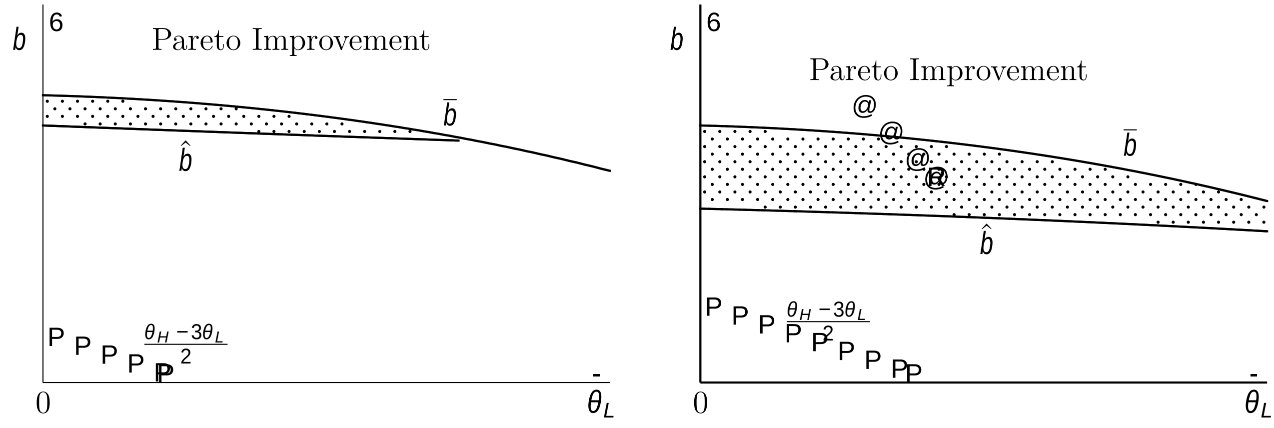
(ii) Total welfare is higher under price discrimination.

(iii) High-type sellers are better-o under price discrimination if and only if $b > \hat{b} \equiv \frac{\sqrt{32-7(\theta_H-\theta_L)^2}-3\theta_H-\theta_L}{4}$. *In this case, price discrimination constitutes a Pareto improvement over uniform pricing.*

Part (i) of Proposition 2 follows naturally from Proposition 1. That the platform is better-off follows from a revealed preference argument. Buyers are better-off, as revealed by their increased participation. Interestingly, this may happen even if the fee they pay increases (see next proposition for more details), as the increased seller participation compensates possible fee increases. The result that low-type sellers are better-off follows from inspection of their surplus.

Part (ii) stands in stark contrast with the traditional analysis of price discrimination. Recall that, when demands are linear and both markets are served under uniform pricing, third-degree price discrimination always lowers total welfare (Schmalensee 1981). The result is overturned in a two-sided context, thanks to the platform's incentive to increase participation on both sides, as we already explained. What is remarkable is that price discrimination always leads to a superior welfare than uniform pricing, meaning that the general benefits brought by higher participation are always bigger than the possible misallocation costs on the seller side (and, in our context, also marginally also on the buyer side) caused by different prices.

Part (iii) goes even further: when $b > \hat{b}$, the high-type sellers benefit from price discrimination, despite having to pay a higher fee than the low-type sellers. In the parametric region in which Pareto improvement holds, two cases can occur, explaining why high-type sellers can be better off. On the one hand, when their value for buyer participation is relatively low, they may receive a subsidy to join the platform, which becomes even bigger under price discrimination (see next proposition). On the other hand, when they highly value buyer participation, they end up paying more, but the additional benefit from increased buyer participation induced by price discrimination outweighs the higher participation fee. In this case, the platform can subsidize consumers to join the platform, and the amount of the subsidy increases under price discrimination.



Panel (a): $v = 0.1$ and $\theta_H = 0.5$. Panel (b): $v = 0.1$ and $\theta_H = 0.8$.

Figure 3: Regions with Pareto improvement

Figure 3 plots the region where price discrimination determines a Pareto improvement (dotted area), as indicated in part (iii) of Proposition 2. Following Assumption 1, we focus on the region where $\max\{0, \frac{\theta_H - 3\theta_L}{2}\} < b < \bar{b}$. We fix $v = 0.1$ and consider two possible values for θ_H to show that the area with Pareto improvement increases with θ_H (this is formally demonstrated in Appendix A.2).[10] This finding, together with condition $b > \hat{b}$ in part (iii), implies that we need a sufficiently high combination of network effects for the positive feedback loop induced by price discrimination (more buyer participation and therefore more value for sellers) to generate a Pareto improvement.

Prices. Having stated our main result, it is instructive to take a closer look at the platform's optimal pricing strategy. In Appendix A.3 we formally prove the following results.

Proposition 3. *(i) There exists $\tilde{b} > 0$ such that $p^D > p^U (> 0)$ if and only if $b > \tilde{b}$.*

(ii) $f_L^D < f_H^D$ for all parameter values. Depending on the parameter values, we can have: $f^U \leq f_L^D$, f^U (f_L^D, f_H^D), or $f^U \geq f_H^D$. When $f^U \leq f_L^D$, we necessarily have that $p^D < p^U < 0$; moreover, $f^U \geq f_H^D$ occurs only when $f^U < 0$.

Part (i) of of Proposition 3 reveals that, if buyers place a high value on seller participation ($b > \tilde{b}$), the platform increases its price to buyers. In spite of this, buyers are still better-off

[10] Also notice that the feasible region decreases in θ_H, as it can be easily derived from Assumption 1: apart from the evident conditions $\theta_H^2 + \theta_L^2 < 4$ and $0 < v < \frac{4-\theta_H^2-\theta_L^2}{2}$, it can be easily established that $\frac{\partial \bar{b}}{\partial \theta_H} < 0$.

because of the increased number of sellers under price discrimination. Such a strategy may require subsidizing seller participation, especially if sellers' value for buyer participation is relatively low. If b is smaller ($b < \tilde{b}$), the platform needs to lower its price to buyers in order to trigger the positive feedback loop leading to more participation on each side.

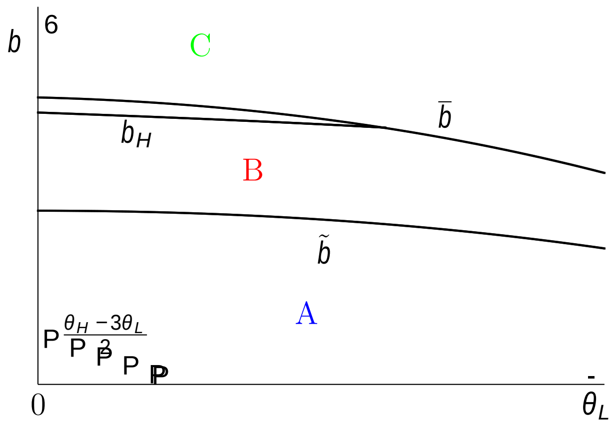
Part (ii) considers the price paid by sellers. Even though the typical case is such that $f_L^D < f^U < f_H^D$, there are regions in the parameter space such that both fees increase or decrease under price discrimination. On the one hand, when sellers scarcely value consumer interaction, the platform may decide to subsidize more (i.e. lowering their negative fees) under price discrimination in order to attract them. On the other hand, when sellers highly value consumer interaction, the platform may increase the fees for both of them when it can price discriminate. We provide more precise conditions in Appendix A.3.

In Figure 4 we illustrate the different cases. The standard results are obtained in Region A. In Region B, the platform's optimal strategy is to increase the price paid by buyers, while still moving f_L^D and f_H^D in opposite directions. In region C, sellers get a relatively low per-buyer benefit compared to buyers' per-seller benefit, and are subsidized under both regimes. Price discrimination induces the platform to increase the subsidy to both seller types and to charge a higher price to buyers. In region D, θ_H and θ_L are relatively high compared to b, and the platform increases both fees, while at the same time increasing the subsidy to buyers.

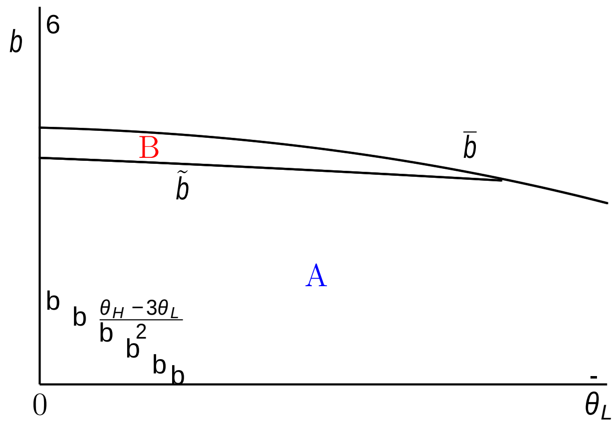
4.4. Corner solutions

The results above hold under Assumption 1, which ensures that equilibria under both uniform pricing and price discrimination are interior, meaning that all of N_B, N_L and N_H belong to $(0, 1)$. We now briefly discuss alternative scenarios.

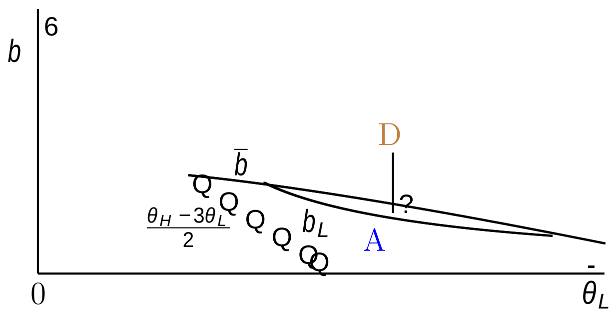
Exclusion of low-type sellers under uniform pricing. Suppose that θ_L is small enough compared to θ_H that, under uniform pricing, the platform finds it optimal to exclude the low-type sellers by setting a high fee: $N_L^U = 0$. Then price discrimination constitutes a Pareto improvement: the increase in low-type sellers induces the platform to attract more buyers, and in turn to also attract more high-type sellers. Even though the price to the latter may increase, their utility is always larger due to the increased buyer participation. Formally, when the platform opts to exclude the low-type sellers under uniform pricing, the optimal number of high-type sellers is $N_H^U(N_B) = \frac{\theta_H + b}{2} N_B$, which is equal to $N_H^D(N_B)$. Because N_B increases under discrimination, N_H must also do so, and high-type sellers are better off.



Panel (a): $v = 0.1$ and $\theta_H = 0.5$.



Panel (b): $v = 0.1$ and $\theta_H = 0.8$.



Panel (c): $v = 0.1$ and $\theta_H = 1.4$.

A: $f_L^D < f^U < f_H^D$ and $p^D < p^U$

B: $f_L^D < f^U < f_H^D$ and $p^D > p^U$

C: $f_L^D < f_H^D < f^U (< 0)$ and $p^D > p^U$

D: $f^U < f_L^D < f_H^D$ and $p^D < p^U (< 0)$

Figure 4: Pricing regimes

Note that this result is analogous to the case of discrimination in traditional markets when the weak market is not served under uniform pricing, so that network effects do not fundamentally change the analysis.

Full buyer participation Suppose that v is large enough so that all buyers participate under uniform pricing: $N_B^U = 1$. Then we must also have $N_B^D = 1$, so that price discrimination does not increase buyer participation. Because of the fixed participation on the buyer side, the analysis mirrors the traditional one: price discrimination leaves total participation on the seller side unchanged, but welfare goes down because of the misallocation due to sellers facing different prices. This case is illustrated in panel (a) of Figure 5.

Full seller participation Finally, suppose that parameters are such that the platform finds it optimal to serve all the low-type sellers under uniform pricing: $N_L^U = 1$.[11] Then price discrimination leads to an increase in buyer participation and in total welfare (since seller participation remains the same). This case is illustrated in panel (b) of Figure 5.

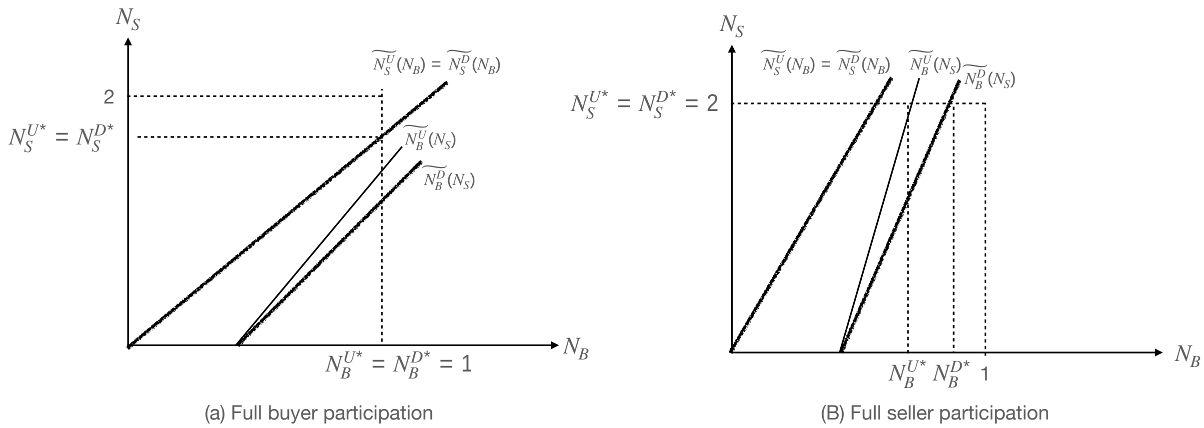


Figure 5: Cases with full participation on one side

5. Extensions

5.1. Heterogenous buyers' benefits

In order to obtain analytical results, we have assumed that buyers are indifferent with respect to the type of the sellers they interact with. While this assumption can be microfounded, a

[11] This happens when $(2b + \theta_L + \theta_H)(2b + \theta_L + \theta_H + v) > 8$.

more plausible assumption is that buyer surplus depends on the type of the seller, $b(\theta)$, and that buyers prefer to interact with high-type sellers: $b(\theta_H) > b(\theta_L)$.

One difference with the baseline model is that, under price discrimination, the platform may want to charge a higher participation fee to the low-type sellers. This is the case when $\theta_L - b(\theta_L) > \theta_H - b(\theta_H)$. Indeed, in that case, even though high-type sellers are willing to pay more, they also generate more benefits to buyers, so that the platform finds it optimal to offer them a lower price than to low-type sellers.

We have the following result, which is formally demonstrated in Appendix B.1:

Proposition 4. *Suppose that parameters are such that the equilibrium is interior. When $b(\theta_H) > b(\theta_L)$, participation on both sides increases under price discrimination.*

Proposition 4 is a generalization of Proposition 1. Recall that, in Proposition 1, part of the reasoning relied on seller's participation being constant across pricing regimes (for a given N_B). When buyers care about seller type, we need to take into account that, even though N_S is the same for a given N_B, the composition of the set of sellers is different so that buyers may be worse-off everything else being equal. The crux of the proof consists in showing that this composition effect is not enough to offset the platform's incentive to attract more buyers following the improvement of its ability to extract surplus from sellers.

Obtaining clean analytical results in this more general setup is difficult, but numerical simulations indicate that our main insights continue to hold. Even though total welfare may go down with price discrimination, we find that the magnitude of welfare losses is generally small (less than 1% compared of the welfare under uniform pricing), while the gains can be substantial (sometimes above 100%). There are also parameter regions such that price discrimination leads to a Pareto improvement.

5.2. Ad-valorem pricing

In this subsection, we show that our main welfare results are robust to a change in the pricing structure where the platform charges an ad-valorem fee to sellers. Specifically, the platform sets the percentage fee on the value of the transaction. This is commonly observed in B2C platforms such as app stores, marketplaces, gaming platforms etc.[12] Under such a pricing structure, we compare the uniform pricing regime where the platform charges the same ad-valorem fee to all sellers ($r_H = r_L = r$) to the one where it sets discriminatory prices $r_H = r_L$.

[12]For a detailed overview of different ad-valorem fee charged by platforms, see Borck et al. (2020).

Platform profit when employing uniform pricing and discriminatory pricing regimes are respectively given as

$$\max_{r,p} \Pi_U = (p + r(\theta_H N_H + \theta_L N_L))N_B, \quad \max_{r_H, r_L, p} \Pi_D = (p + r_H \theta_H N_H + r_L \theta_L N_L)N_B.$$

We find that our main welfare results are robust to the case where the platform charges ad-valorem fees to sellers. A detailed analysis is presented in Appendix B.2, where we also provide the relevant threshold value of b that appears below. In the following proposition, we present the ad-valorem fee counterpart to the results in Proposition 2.

Proposition 5. *(i) The platform, buyers and low-type sellers are better-o under price discrimination.*

(ii) Total welfare is higher under price discrimination.

(iii) High-type sellers are better-o under price discrimination if and only if $b > \hat{b}^{ad}$. In this case, price discrimination constitutes a Pareto improvement over uniform pricing.

The above proposition confirms that our welfare results hold when the platform employs an alternative pricing structure to sellers. The intuitions for these results are similar to the discussion in the benchmark after Proposition 2.

5.3. One-sided pricing

We now consider the case in which the platform does not charge buyers, whereas it still charges the participation fee f to sellers. The majority of apps available in Google Play and Apple's App Store are freely available for buyers.[13] This is also common in the lodging sector, in which Online Travel Agencies (OTAs) such as Booking.com and Expedia only charge hotels and lodging establishments.

As in the benchmark case, we compare uniform pricing with price discrimination. The formal analysis is reported in Appendix B.3, together with the expressions for the relevant threshold values of b which appear below. We present the main results in this proposition.

Proposition 6. *In comparison to uniform pricing, under price discrimination we find that the following holds.*

[13] According to Statista, as of November 2022, respectively 97 and 94 percent of apps in Google Play and Apple's app-store were freely available for buyers. For more information, visit https://www.statista.com/statistics/263797/number-of-applications-for-mobile-phones/

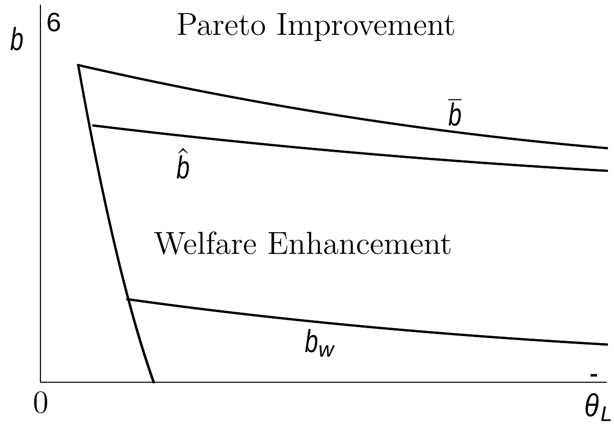


Figure 6: Pareto Improvement and Welfare Enhancement

(i) Total seller participation increases.

(ii) The platform, buyers and low-type sellers are better o .

(iii) Total welfare is higher under price discrimination if and only if $b > b_w$.

(iv) High-type sellers are better-o under price discrimination if and only $b > \hat{b}$, with $\hat{b} > b_w$. In this case, price discrimination constitutes a Pareto improvement over uniform pricing.

Figure 6 provides the interval regions of interest characterized by $\max\{0, \frac{2\theta_H}{\theta_L(\theta_H - \theta_L)}\} < b < \bar{b}$. It is plotted for $v = 0.1$ and $\theta_H = 0.5$. As one can easily notice, most of the main results of the benchmark case continue to hold, the most relevant exception being that price discrimination does not always enhance welfare. In fact, when consumer valuation for sellers' participation is not strong enough ($b < b_w$), total welfare is lower under price discrimination.

Consumers gain with price discrimination, given that sellers participation increases, following the same logic as Proposition 1. The platform obviously gains, and low-type sellers as well, given that $f_L^D < f^U < f_H^D$ always holds. This represents another difference with respect to the benchmark case, in which the standard result that price discrimination raises the price for the high type, while lowering that of the low type, did not always hold.

As per high-type sellers, they lose out under price discrimination when b is low, as they end up paying a higher fee without being compensated by a sufficiently strong increase in consumer participation. When $b < b_w$, the loss for high-type sellers overcomes the sum of the gains of the other market participants, and price discrimination reduces total welfare.

In comparison to the benchmark case, the platform has fewer instruments to attract buyers and sellers. Only for sufficiently high consumer valuation price discrimination improves social welfare, namely when $b > b_W$, as consumers value more the presence of sellers. Moreover, when $b > \hat{b}$, price discrimination is Pareto improving, as in the benchmark case. Consumers highly value the presence of sellers, and are therefore willing to join the platform; high-type sellers, who in turn value the presence of consumers, are more than compensated for the higher price they end up paying under price discrimination.

Finally, in terms of subsidization, another difference with respect with the benchmark case consists in the fact that we can only find parametric regions in which the low-type sellers are subsidized - especially when θ_L is relatively low - whereas the high-type sellers are always charged a positive fee.

6. Managerial and Policy Implications

In this section, we discuss how our results can be translated into clear managerial and policy implications.

Managerial implications. Our paper provides clear and intuitive managerial implications for managers of platforms, high-margin sellers and low-margin sellers.

Managerial Insight 1. *Third-degree price discrimination is a profitable pricing regime for platfortms which leads to higher total participation of sellers and consumers vis-à-vis uniform pricing.*

It is straightforward to follow that price discrimination is a profitable strategy for a platform and managers of platforms are aware of it. This is a well-known result. In platform markets, however, profitability is not the only metric of success of a platform. Another important metric is the number of transactions or active users. Managers of platforms who employ the well-known logic of one-sided traditional markets into two-sided markets might be concerned that third-degree price discrimination lowers participation on the platform. While this is a very serious concern, we find that third-degree price discrimination increases total participation on both sellers' and buyers' side. This suggests that the classical insights of traditional one-sided markets do not apply to multi-sided markets. Increased mass of active members on either sides, under price discrimination, fosters a healthy and vibrant platform ecosystem. Thus, apart from the standard increased profitability rationale in favor of price discrimination, managers of platforms should further welcome such a pricing scheme as it also enhances platform performance through other metrics employed that assess platform health and long-term viability.

Managerial Insight 2. *A (strict) zero pricing strategy on the consumers' side may actually lower the total surplus created on the ecosystem.*

Market analysts are often worried by consumer price of platforms being above or below zero. A zero price charged to consumers is seen as the sweet spot and such a simple pricing rule may be appealing to managers of platforms. However, such a pricing restriction by managers, apart from hurting profitability, may also hurt the ecosystem and result in lower value generation by the platform. Thus, managers must be careful in devising such simple yet destructive pricing rules which restrict surplus generation in the platform ecosystem.

Managerial Insight 3. *Third-degree price discrimination can be a simple yet powerful tool that fosters the whole platform ecosystem and can be surplus enhancing for every participant (Pareto improving) when the value of interactions is su ciently high.*

Above and beyond the obvious profitability and the total welfare increasing effect of third degree price discrimination, platform managers may be worried how third-degree price discrimination affects the participation of high-margin sellers. Specifically, platform managers may be concerned that after implementing price discrimination, these sellers may hesitate to affiliate with the platform. This may in turn hurt the brand value and the long-term health of the platform. In this paper, we find that, when the value of network interactions is sufficiently large, the participation of high-type sellers also rises. Therefore, our results may contribute to reducing the worries of platform managers regarding the platform brand value. These implications also inform managers that, when the value of interactions is big enough, price discrimination may make it profitable to enhance participation as the value of increased interactions will be larger than any perceived price inequality due to price discrimination.

Policy implications. Our paper brings forth many clear policy insights that inform policy makers and suggest they should be circumspect when applying results on the effect of price discrimination in traditional markets to multi-sided markets.

Policy Insight 1. *Third-degree price discrimination on the seller side is consumer surplus enhancing.*

In traditional markets, the focus is often only on the impact of price discrimination on the side where such a pricing scheme is employed. Instead, multi-sided markets are characterized by network effects where a change in the pricing structure and level impacts also the other side of the market. As shown in the paper, price discrimination enhances total participation on the sellers' side. A direct consequence of this is that, keeping consumer prices constant, the value derived by consumers by affiliating with the platform also rises. Since consumer demand is elastic, the monopolist platform is unable to extract all the surplus gain on the consumer side and thus consumers are also better off. This positive externality on the consumers' side due to a change in the pricing structure is a novel insight we elicit in this paper.

Policy Insight 2. *Third-degree price discrimination can be total welfare enhancing and benefit the whole ecosystem. Any regulatory restriction on consumer prices will lower the likelihood of total welfare increasing under price discrimination.*

The classical result in traditional one-sided markets is that the welfare effects of price discrimination are ambiguous. Despite this, it is accepted that, under linear demand systems, price discrimination lowers welfare in traditional one-sided setting as total participation/output remains constant. In a multi-sided setting where participation on each side presents cross-sided externalities on the other side, we find that total participation is increased on all sides under price discrimination. A direct consequence is that total welfare is higher under price discrimination and this is in contrast to the well-known results in two-sided markets (under linear demands). This finding is robust to a variety of extensions. As a consequence, policy makers interested in bolstering total welfare through regulating consumer price or seller fees may weaken our welfare result. This is because restricting the strategy space of the platform constrains its ability to enhance participation, thus hurting total welfare under certain cases.

Policy Insight 3. *Third-degree price discrimination can be Pareto improving and benefits the whole ecosystem when the value of network interactions is su ciently high.*

This result goes above and beyond the total welfare-enhancing effect of price discrimination discussed above. Pareto improvement arises because of multi-market interactions due to which even the group of sellers (seemingly) discriminated against benefit from price discrimination in place. Specifically, the value increase due to increased consumer participation dominates any perceived price inequality faced by high-type sellers. Thus, increasing their participation as well. This policy insight points out that, when the value of network interactions is high enough, policy makers may do best by letting platforms adopt third-degree price discriminate, which enables a bigger pie to be shared but also that each market participant is benefited from their portion in the pie vis-á-vis forcing them to set uniform prices.

7. Conclusion

In this paper, we argue that third-degree price discrimination in markets featuring network effects is not only welfare enhancing but can also be Pareto improving. This result arises only due to the presence of network externalities as in their absence our analysis would reproduce the well-known results from traditional markets. In particular, cross-sided network externalities render the multiple sides of a platform interdependent and changes in welfare on one side can have repercussions on the other side.

In the presence of two types of sellers, high-type and low-type, price discrimination enables a platform to profitably and more efficiently extract higher surplus from sellers. To do so, it chooses to enhance their value on the platform by boosting consumer participation. Since demands on the two sides are elastic, the platform only extracts a portion of this increased seller value which results in increased total participation of sellers. Ultimately, we find that price discrimination enhances platform profit, increases consumer surplus and the surplus of low-type sellers, and can even result in a Pareto improvement.

Our analysis is carried out in a simplified setting in which players on both sides pay participation fees to join, and buyers equally value the presence of sellers on the platform. However, we proved that our main results hold when more complex settings are taken into account, such as heterogenity in buyer valuation of sellers, ad valorem fees on the seller side, and buyers freely joining the platform. Finally, we used linear demands for tractability, but the mechanism underpinning our results, namely the increases in the participation of both buyers and sellers generated by price discrimination, holds more generally, as we explained at the end of Subsection 4.1.

Notwithstanding the limitations, the results that we obtain bear important managerial implications for executives of large platforms catering to a wide variety of demand segments. They also offer policy makers precious indications about the possible advantages that platforms can create for society at large when cross-sided network externalities are present. Indeed, the recently introduced Digital Market Act aims at fighting the dominant position of gatekeeper platforms by favoring direct intermediation between parties, banning contractual restrictions such as price parity clauses, and limiting self-preferencing. The intended effect is to curb the level of the commission fees charged especially to sellers. Regulatory bodies and scholars are also suggesting the possibility of capping commission fees (Gomes & Mantovani 2020, Bisceglia & Tirole 2022). Our paper shows that these provisions, particularly if too rigid, may accidentally reduce the benefits brought by price discrimination in the presence of network effects.

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A. Omitted proofs: baseline model

A.1. Proof of Proposition 1

In order to provide some intuition along with the proof, it is helpful to study the platform's dual problem of choosing the participation level on each side to maximize profit, while prices adjust accordingly. The idea of the proof is the following: writing $\hat{N}_j(N_{-j})$ to denote the profit-maximizing participation level of side $j \in \{B, S\}$ as a function of the total participation on the other side, we will first show that $\hat{N}_j$ is increasing in both pricing regimes. Then, we will show that, as in standard models of third-degree price discrimination in one-sided markets, $\hat{N}_S^U(N_B) = \hat{N}_S^D(N_B)$: taking buyers' participation as given, price discrimination leaves "output" on the seller side unchanged. Finally, we will show that, taking N_S as given, $\hat{N}_B^U(N_S) < \hat{N}_B^D(N_S)$: because the platform can extract more value from sellers under price discrimination, it has an incentive to attract more buyers. This in turn leads to more sellers joining the platform, and so on until the end of the feedback loop.

Uniform pricing Under uniform pricing, the platform chooses the quantity of buyers N_B and the total quantity of sellers N_S, and the participation fees adjust accordingly. By inverting system (**??**), we find that the inverse demands under uniform pricing are

$$P^U(N_B, N_S) = v + bN_S - N_B \quad \text{and} \quad F_U(N_B, N_S) = \frac{(\theta_L + \theta_H)N_B - N_S}{2}.$$

The platform's profit is $N_B P(N_B, N_S) + N_S F_U(N_B, N_S)$.

The first-order conditions are

$$\frac{\partial \pi^U}{\partial N_S} = 0 \Leftrightarrow N_S \frac{\partial F_U}{\partial N_S} + F_U + N_B \frac{\partial P}{\partial N_S} = 0 \Leftrightarrow \hat{N}_S^U(N_B) = \frac{(\theta_L + \theta_H + 2b)N_B}{2}, \tag{16}$$

$$\frac{\partial \pi^U}{\partial N_B} = 0 \Leftrightarrow N_B \frac{\partial P}{\partial N_B} + P + N_S \frac{\partial F_U}{\partial N_B} = 0 \Leftrightarrow \hat{N}_B^U(N_S) = \frac{v + bN_S}{2} + \frac{(\theta_H + \theta_L)N_S}{4}. \tag{17}$$

Price discrimination Under discrimination, the platform has an extra instrument, and can thus choose N_H and N_L independently. Inverting (**??**) leads to the following inverse demands:

$$P(N_B, N_H, N_L) = v + b(N_H + N_L) - N_B, \quad F_H(N_L, N_B) = \theta_H N_B - N_H, \quad \text{and} \quad F_L(N_L, N_B) = \theta_L N_B - N_L.$$

Profit is $N_B P(N_B, N_S) + N_H F_H(N_H, N_B) + N_L F_L(N_L, N_B)$.

The first-order conditions with respect to the number of sellers are:

$$\frac{\partial \pi^D}{\partial N_H} = 0 \Leftrightarrow N_H \frac{\partial F_H}{\partial N_H} + F_H + N_B \frac{\partial P}{\partial N_H} = 0 \Leftrightarrow N_H^{*D}(N_B) = \frac{(\theta_H + b)N_B}{2}, \tag{18}$$

$$\frac{\partial \pi^D}{\partial N_L} = 0 \Leftrightarrow N_L \frac{\partial F_L}{\partial N_L} + F_L + N_B \frac{\partial P}{\partial N_H} = 0 \Leftrightarrow N_L^{*D}(N_B) = \frac{(\theta_L + b)N_B}{2}. \tag{19}$$

Note that adding (18) and (19) gives (16), so that $N_S^{*D}(N_B) = N_S^{*U}(N_B)$.

The first-order condition with respect to the number of buyers is:

$$\frac{\partial \pi^D}{\partial N_B} = 0 \Leftrightarrow N_B \frac{\partial P}{\partial N_B} + P + N_H \frac{\partial F_H}{\partial N_B} + N_L \frac{\partial F_L}{\partial N_B} = 0 \Leftrightarrow N_B = \frac{v + bN_S}{2} + \frac{\theta_H N_H + \theta_L N_L}{2}. \tag{20}$$

In (20), N_B is obtained as a function of N_S, N_H and N_L. But from (18) and (19), we know that $N_H^{*D}(N_B) = \frac{\theta_H + b}{\theta_H + \theta_L + 2b} N_S^{*D}(N_B)$ and $N_L^{*D}(N_B) = \frac{\theta_L + b}{\theta_H + \theta_L + 2b} N_S^{*D}(N_B)$. Therefore we can rewrite (20) as

$$N_B^{*D}(N_S) = \frac{v + bN_S}{2} + \frac{\theta_H(\theta_H + b) + \theta_L(\theta_L + b)}{2(\theta_H + \theta_L + 2b)} N_S \tag{21}$$

Because $\theta_H > \theta_L$, simple algebra then reveals that $N_B^{*D}(N_S) > N_B^{*U}(N_S)$: for a given level of seller participation, the platform wants to serve more buyers in the discrimination regime.

Together, these observations imply that discrimination leads first to an increase in N_B, which leads to an increase in N_S, which further increases N_B, etc., until we converge to a point where both buyer and seller participation are higher than under uniform pricing.

A.2. Proof of Proposition 2

We first have to compute buyer surplus, sellers' surplus and total welfare in both scenarios. Platform profits are given by (14) and (15), respectively.

Uniform pricing. When the platform sets a unique fee, buyer surplus and type $j \in \{L, H\}$ sellers' surplus are respectively given by

$$CS^U = \int_0^{N_B^U(p^U, f^U)} (v + b(N_H^U(p^U, f^U) + N_L^U(p^U, f^U)) - p^U - k^B) dk^B = \frac{8v^2}{(8 - (2b + \theta_H + \theta_L)^2)^2},$$

$$DS_j^U = \int_0^{N_j^U(p^U, f^U)} (\theta_j N_B^U(p^U, f^U) - f^U - k^S) dk^S = \frac{v^2(2b + 3\theta_j - \theta_{-j})^2}{2(8 - (2b + \theta_H + \theta_L)^2)^2},$$

for a total welfare of

$$SW^U = CS^U + \Pi^U + \sum_{i=1,2} DS_j^U = \frac{v^2(24 - (2b + 3\theta_H - \theta_L)(2b - \theta_H + 3\theta_L))}{(8 - (2b + \theta_H + \theta_l)^2)^2}.$$

Price discrimination. When the platform charges two different fees, buyer surplus and type $j \in \{L, H\}$ sellers' surplus are respectively given by

$$\begin{aligned} CS^D &= \int_0^{N_B^D(p^D, f_H^D, f_L^D)} (v + b(N_H^D(p^D, f_H^D, f_L^D) + N_L^D(p^D, f_H^D, f_L^D)) - p^D - k^S)dk^S \\ &= \frac{2v^2}{(4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L))^2}. \end{aligned}$$

$$DS_j^D = \int_0^{N_j^D(p^D, f_H^D, f_L^D)} (\theta_j N_B^D(p^D, f_H^D, f_L^D) - f_j^D - k^B)dk^B = \frac{v^2(b + \theta_j)^2}{2(4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L))^2},$$

for a total welfare of

$$SW^D = CS^D + \Pi^D + \sum_{i=1,2} DS_j^D = \frac{v^2(12 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L)}{2(4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L))^2}.$$

We will now prove the three points of Proposition 2, taking into account the admissible parametric region defined by Assumption 1,

(i) By a revealed preference argument, the platform is necessarily better-off under price discrimination. Formally:

$$\Pi^D - \Pi^U = \frac{v^2(\theta_H - \theta_L)^2}{(4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H - \theta_L)(8 - (2b + \theta_H + \theta_L)^2)} > 0.$$

That buyers are also better-off is a corollary of Proposition 1. Regarding low-type sellers, one can check that, in the admissible parametric region:

$$DS_L^D - DS_L^U = \frac{1}{2}v^2 \left[\frac{(2b + 3\theta_L - \theta_H)^2}{(8 - (2b + \theta_H + \theta_L)^2)^2} - \frac{(b + \theta_L)^2}{(4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L))^2} \right] > 0.$$

(ii) Turning to total welfare, we obtain that:

$$SW^D - SW^U = \frac{v^2(\theta_H - \theta_L)^2\lambda}{2(4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L))^2(8 - (2b + \theta_H + \theta_l)^2)^2}$$

where $\lambda = 32 - 24b^4 - 7\theta_H^4 + 2\theta_H^3\theta_L + 28\theta_L^2 - 7\theta_L^4 - 48b^3(\theta_H + \theta_L) - 2\theta_L\theta_H(12 - \theta_L^2) + 14\theta_H^2(2 - \theta_L^2) + 2b(\theta_H + \theta_L)(16 - 13\theta_H^2 + 2\theta_H\theta_L - 13\theta_L^2) + 2b^2(16 - 25\theta_H^2 - 22\theta_L\theta_H - 25\theta_L^2)$.

Notice that sign of λ determines the sign of the difference in total welfare. Equating λ to 0 and solving for b, we get four solutions but only one is positive and given by:

$$b^{sol} = \frac{\sqrt{6}\sqrt{16 - 7(\theta_H - \theta_L)^2 + \sqrt{1024 + 256(\theta_H - \theta_L)^2 + (\theta_H - \theta_L)^4}} - 6(\theta_H + \theta_L)}{12}.$$

Further, this b^{sol} is greater than the upper bound of our feasible region $\bar{b}$.

Next differentiating the expression for λ with respect to b and computing it at $b = b^{sol}$, we get

$$\frac{\partial \lambda}{\partial b}\Big|_{b=b^{sol}} = -\sqrt{\frac{2}{3}}\sqrt{g(16 - 7(\theta_H - \theta_L)^2 + g)} < 0,$$

with $g = \sqrt{1024 + 256(\theta_H - \theta_L)^2 + (\theta_H - \theta_L)^4}$. Regardless of whether λ is convex or concave in b, for $b < \bar{b} < b^{sol}$, we must have $\lambda > 0$. We then confirm that the social welfare is higher under price discrimination than under uniform pricing.

(iii) This point follows from the comparison of DS_H^U and DS_H^D. We obtain that:

$$DS_H^D - DS_H^U = \frac{1}{2}v^2\left[\frac{(2b + 3\theta_H - \theta_L)^2}{(8 - (2b + \theta_H + \theta_L)^2)^2} - \frac{(b + \theta_H)^2}{(4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L))^2}\right] > 0$$

if and only if $b > \hat{b} = \frac{\sqrt{32 - 7(\theta_H - \theta_L)^2} - 3\theta_H - \theta_L}{4}$. Further notice that $\frac{\partial \hat{b}}{\partial \theta_H} < 0$, thus explaining why the parametric region with Pareto improvement enlarges when θ_H increases, as we can see in Figure 3 when comparing Panel (a) with Panel (b).

A.3. Proof of Proposition 3

Considering the conditions specified on Assumption 1, that define our feasible parametric region, we compare sellers' fees and buyers' participation prices across the two regimes.

Starting from buyers, we find that

$$p^D > p^U \qquad b > \tilde{b} = \frac{\sqrt{16 + (\theta_H + \theta_L)^2} - \theta_H - \theta_L}{4},$$

with $\tilde{b}$ admissible when θ_H is not very large. Hence, provided the high-type seller's valuation for buyer is not excessive, there exists a threshold value of b above which buyers pay a higher price under price discrimination. This represents another novel result of our analysis, as we prove that buyers may end up paying more under price discrimination. Remember that, by Proposition 1, participation of both sides increases under price discrimination. When b is low, that is when buyers do not highly value seller participation, attracting more of them requires lowering the price, and this could even be achieved through subsidization. On the contrary, when b is high, the increased seller participation is enough to attract more buyers, and the platform can also increase the price buyers have to pay. In other words, the platform and buyers share the increased gross surplus on the buyer side.

Turning to sellers, we first obtain that:

$$f_L^D < f_H^D < f^U \qquad b > b_H = \frac{\sqrt{32 + (\theta_H - \theta_L)(9\theta_H + 7\theta_L)} - 3\theta_H - \theta_L}{4},$$

with b_H admissible in the feasible region when both θ_H and θ_L are sufficiently low. Then,

$$f^U < f_L^D < f_H^D \qquad b > b_L = \frac{\sqrt{32 - 7\theta_H^2 - 2\theta_H\theta_L} - \theta_H - 3\theta_L}{4},$$

with b_L admissible in the feasible region when both θ_H and θ_l are sufficiently high.

Finally, $f_L^D < f^U < f_H^D$ for all remaining admissible parameter constellations, which reproduces a well-known result in the traditional one-sided market literature (Robinson 1933): price discrimination raises the price for the high-type, whereas it lowers that of the low type. This applies to the platform context that we consider, provided the sellers' values for buyer participation are neither too small nor too big.

Conversely, if sellers show more extreme attitudes towards the presence of buyers, the conventional result can be overturned. On the one hand, there is a region in which both sellers pay less under price discrimination. More precisely, when b_H is admissible, $f_L^D < f_H^D < f^U < 0$ if and only if $b > b_H$: both types of sellers are subsidized to join the platform, and such subsidy increases under price discrimination. On the other hand, when b_L is admissible, then both sellers pay a higher price under price discrimination if $b > b_L$: $0 < f^U < f_L^D < f_H^D$.

By considering together buyers and sellers, we summarize our main results on comparing prices across the two regimes as follows (see also Figure 4):

(i) When θ_H and θ_L are relatively low and $b > b_H$: $f_L^D < f_H^D < f^U < 0$ and $p^D > p^U > 0$. When $b < b_H$, $f_L^D < f^U < f_H^D$ (with subsidies for sellers when b is high enough, and $p^D > p^U$ when $b > \tilde{b}$).

(ii) For intermediate values of θ_H, we always have $f_L^D < f^U < f_H^D$, and $p^D > p^U$ when $b > \tilde{b}$.

(iii) When θ_H and θ_L are relatively high and $b > b_L$: $0 < f^U < f^D_L < f^D_H$ and $p^D < p^U < 0$. When $b < b_L$, $f^D_L < f^U < f^D_H$, with $p^D < p^U$ (with subsidies for buyers only when b is high enough).

Starting from point (i), price discrimination enables the platform to charge a high price to buyers, who highly value seller participation, in order to increase the subsidy for both sellers. The fact that the fees are negative implies that the platform can subsidize sellers more than under unique pricing in order to attract them, as their value for buyer participation is particularly low. When b is lower, we obtain the standard result that $f^D_L < f^U < f^D_H$.

Turning to point (ii), when the high-type sellers value for buyer participation is intermediate, we always obtain the standard result that price discrimination increases the price for the high type, while it lowers that of the low type. As per buyers, we can still find a region in which they end up paying more with price discrimination (when $b > \tilde{b}$), but this region shrinks in comparison to point (i).

Finally, when sellers' value for buyer participation is high, we find the interesting case in which price discrimination enables to subsidize buyers more than under uniform pricing, and this is possible as a higher fee is imposed on both sellers. This occurs when $b > b_L$. The fact that this scenario requires a sufficiently high value for b can be explained by the fact that consumers need to have a sufficiently high value for seller participation in order for the platform to decide to increase their subsidy at the expenses of sellers. When $b < b_L$, we obtain the standard result $f^D_L < f^U < f^D_H$, with $p^D < p^U$; buyers are subsidized only when b is high enough. In any case, they pay a lower participation fee (or obtain a higher subsidy) under price discrimination.

B. Omitted proofs: extensions to the baseline model

B.1. Proof of Proposition 4

Similarly to Appendix A.1, we study the platform's dual problem of choosing the participation level on each side to maximize. The aim is again to show that the platform can attract more buyers under price discrimination, and this in turn entices more sellers to join the platform.

Uniform pricing Under uniform pricing, one can view the platform's maximization program as choosing N_B and N_S to maximize profit, without being able to adjust N_L and N_H. For a given fee f, we have

$$N_H = \theta_H N_B - f, \quad \text{and} \quad N_L = \theta_L N_B - f. \tag{22}$$

Adding these two equations, one gets the market clearing uniform price

$$F^U(N_B, N_S) = \frac{\theta_H + \theta_L}{2} N_B - \frac{N_S}{2}. \tag{23}$$

On the buyer side, demand is given by

$$N_B = \theta_H N_H + \theta_L N_L - p. \tag{24}$$

The market clearing price thus depends on the allocation N_H and N_L, not only on the aggregate number of sellers N_S. However, using (22), we know that under uniform pricing N_H and N_L will necessarily satisfy $N_H = N_L + (\theta_H - \theta_L)N_B$. This implies that

$$N_L = \frac{N_S}{2} - (\theta_H - \theta_L)N_B, \quad N_H = \frac{N_S}{2} + (\theta_H - \theta_L)N_B. \tag{25}$$

Plugging this into (24), we obtain the market-clearing buyer price:

$$P^U(N_B, N_S) = \frac{b_H + b_L}{2} N_S - (1 + (b_H - b_L)(\theta_H - \theta_L))N_B. \tag{26}$$

The platform's profit is

$$\Pi^U(N_B, N_S) = N_S F^U(N_B, N_S) + N_B P^U(N_B, N_S). \tag{27}$$

It is straightforward to check that $\frac{\partial^2 \Pi^U(N_B,N_S)}{\partial N_B \partial N_S} > 0$, so that $\hat{N}_S^U(N_B)$ and $\hat{N}_B^U(N_S)$ are increasing. The first-order conditions are

$$\frac{\partial \Pi^U(N_B, N_S)}{\partial N_S} = 0 \Leftrightarrow \hat{N}_S^U(N_B) = \frac{(b_H + b_L + \theta_H + \theta_L)N_B}{2}, \tag{28}$$

$$\frac{\partial \Pi^U(N_B, N_S)}{\partial N_B} = 0 \Leftrightarrow 2\left(1 + (b_H - b_L)(\theta_H - \theta_L)\right) \hat{N}_B^U(N_S) = \frac{(b_H + b_L + \theta_H + \theta_L)N_S}{2}. \tag{29}$$

Price discrimination Under price discrimination the platform can choose N_B, N_L and N_H. Market-clearing prices are given by

$$F_L^D(N_B, N_L) = \theta_L N_B - N_L, \quad F_H^D(N_B, N_H) = \theta_H N_B - N_H, \tag{30}$$

$$P^D(N_B, N_L, N_H) = b_H N_H + b_L N_L - N_B. \tag{31}$$

The platform's profit is

$$\Pi^D(N_B, N_L, N_H) = N_L F_L^D(N_B, N_L) + F_H^D(N_B, N_H) + N_B P^D(N_B, N_L, N_H). \tag{32}$$

The first-order conditions are

$$\frac{\partial \Pi^D(N_B, N_L, N_H)}{\partial N_B} = 0 \Leftrightarrow 2\hat{N}_B^D(N_L, N_H) = (b_H + \theta_H)N_H + (b_L + \theta_L)N_L, \tag{33}$$

$$\frac{\partial \Pi^D(N_B, N_L, N_H)}{\partial N_H} = 0 \Leftrightarrow \hat{N}_H^D(N_B) = \frac{b_H + \theta_H}{2}N_B, \tag{34}$$

$$\frac{\partial \Pi^D(N_B, N_L, N_H)}{\partial N_L} = 0 \Leftrightarrow \hat{N}_L^D(N_B) = \frac{b_L + \theta_L}{2}N_B. \tag{35}$$

Note that adding (34) and (35) gives $\hat{N}_S^D(N_B) = \frac{b_H+\theta_H+b_L+\theta_L}{2}N_B = \hat{N}_S^U(N_B)$ (by (28)): for a given buyer participation level N_B, the optimal seller participation level is the same under the two pricing regimes.

Next, using (34) and (35), we obtain that:

$$\hat{N}_H^D(N_B) = \frac{b_H + \theta_H}{b_H + \theta_H + b_L + \theta_L}\big(\underbrace{\hat{N}_H^D(N_B) + \hat{N}_L^D(N_B)}_{=\hat{N}_S^D(N_B)}\big) \quad \text{and} \quad \hat{N}_L^D(N_B) = \frac{b_L + \theta_L}{b_H + \theta_H + b_L + \theta_L}\hat{N}_S^D(N_B). \tag{36}$$

Because the optimal ratios N_H/N_S and N_L/N_S are constant, we can rewrite (33) as a function of N_S:

$$2\hat{N}_B^D(N_S) = \frac{(b_H + \theta_H)^2 + (b_L + \theta_L)^2}{b_H + \theta_H + b_L + \theta_L}N_S. \tag{37}$$

Because $b_H + \theta_H > b_L + \theta_L$, the right-hand side of the previous equation is larger than $\frac{b_H+\theta_H+b_L+\theta_L}{2}N_S$, which, by (29), is equal to $2\left(1 + (b_H - b_L)(\theta_H - \theta_L)\right)\hat{N}_B^U(N_S)$. This implies that $\hat{N}_B^D(N_S) > \hat{N}_B^U(N_S)$.

Putting things together, the facts that (i) all the $\hat{N}_S$ functions are increasing, (ii) $\hat{N}_S^U(N_B) = \hat{N}_S^D(N_B)$, and (iii) $\hat{N}_B^D(N_S) > \hat{N}_B^U(N_S)$, imply that, in equilibrium, $N_S^D > N_S^U$ and $N_B^D > N_B^U$.

B.2. Proof of Proposition 5

In this extension, we consider the case where the monopolist platform charges sellers an ad-valorem fee. As in the benchmark, we compare the uniform pricing regime where the platform charges the same ad-valorem fee to all sellers ($r_H = r_L = r$) to the one where it sets $r_H = r_L$.

Sellers' payoffs. Suppose the platform charges ad-valorem fees r_j to sellers of type j. The payoff of a seller from group $j \in \{H, L\}$ with participation cost k^S from affiliating with the platform is

$$\tilde{\pi}_j(k^S) = (1 - r_j)\theta_j N_B^e - k^S,$$

where N_B^e is the sellers' expectations on the total mass of buyers affiliating with the platform.

Sellers affiliate with the platform if and only if they obtain positive utility from participating $\tilde{\pi}_j(k^S) \geq 0 \implies k^S \leq (1 - r_j)\theta_j N_B^e$ for $j \in \{H, L\}$. Thus, the mass of sellers of type j participating in the platform ecosystem are

$$\tilde{N}_j(N_B^e, f_j) = (1 - r_j)\theta_j N_B^e. \tag{38}$$

The total mass of sellers active on the platform under price discrimination is then

$$\tilde{N}_S(N_B^e, r_H, r_L) = ((1 - r_H)\theta_H + (1 - r_L)\theta_L)N_B^e. \tag{39}$$

Under a uniform pricing regime, the total mass of sellers active on the platform is instead

$$\tilde{N}_S(N_B^e, r, r) = (1 - r)(\theta_H + \theta_L)N_B^e. \tag{40}$$

Platform payoffs. Platform profit when employing uniform pricing and discriminatory pricing regimes are respectively given as

$$\max_{r,p} \Pi_U = (p + r(\theta_H N_H + \theta_L N_L))N_B, \quad \max_{r_H, r_L, p} \Pi_D = (p + r_H\theta_H N_H + r_L\theta_L N_L)N_B.$$

Timing and equilibrium concept are the same as in the baseline model, and to ensure an interior solution, we make the following assumption.

Assumption 2. *We assume that buyers' and sellers' valuation for participation on the other side as well as buyer intrinsic valuation are not too large, namely:* $0 < v < \frac{4 - 2b^2 - 2b\theta_H - \theta_H^2 - 2b\theta_L - \theta_L^2}{2}$, $b < \frac{\sqrt{8 - (\theta_H - \theta_L)^2} - (\theta_H + \theta_L)}{2} = \bar{b}^{ad}$, *and* $\theta_L^2 + \theta_H^2 < 4$ *and* $\theta_L < \sqrt{2}$.

Uniform pricing. In this pricing regime, recall the buyer and seller participation from equation (1) and (38). In a rational expectations equilibrium agents correctly anticipate participation by the other group, so that participation levels $\tilde{N}_B^U$ and $\tilde{N}_S^U$ satisfy

$$\tilde{N}_B^U = v + b\tilde{N}_S^U - p \quad \text{and} \quad \tilde{N}_S^U = (1 - r)(\theta_L + \theta_H)\tilde{N}_B^U. \tag{41}$$

Solving the above system of equations for N_B^U and N_S^U yields buyer participation and seller total participation as functions of prices. We present these demands below.

$$\tilde{N}_B^U(p,r) = \frac{v-p}{1-b(1-r)(\theta_H+\theta_L)}, \ \tilde{N}_S^U(p,r) = \frac{(\theta_H+\theta_L)(v-p)(1-r)}{1-b(1-r)(\theta_H+\theta_L)}. \tag{42}$$

Seller demand can be further decomposed into

$$\tilde{N}_H^U(p,r) = \frac{\theta_H(v-p)(1-r)}{1-b(1-r)(\theta_H+\theta_L)}, \ \tilde{N}_L^U(p,f) = \frac{\theta_L(v-p)(1-r)}{1-b(1-r)(\theta_H+\theta_L)}. \tag{43}$$

The platform sets prices to maximize profits

$$\max_{p,r}(p + r(\theta_H\tilde{N}_H(\cdot) + \theta_L\tilde{N}_L(\cdot)))\tilde{N}_B(\cdot).$$

Differentiating platform profits with respect to p and r and solving the system of first order conditions yields the following prices.

$$\begin{aligned}
\tilde{p}^U &= \frac{v(\theta_H^2+\theta_L^2)(2-\theta_H^2-\theta_L^2-b(\theta_H+\theta_L))}{2b\theta_H^3+\theta_H^4+2b\theta_H\theta_L(b+\theta_L)-\theta_L^2(4-(b+\theta_L)^2)-\theta_H^2(4-b^2-2b\theta_L-2\theta_L^2)}, \\
\tilde{r}^U &= \frac{\theta_H^2+\theta_L^2-b(\theta_H+\theta_L)}{2(\theta_H^2+\theta_L^2)}.
\end{aligned}$$

The associated equilibrium seller demands for type $j \in \{L,H\}$, buyer demand, and platform profit are respectively given by:

$$\begin{aligned}
\tilde{N}_j^U(\tilde{p}^U,\tilde{r}^U) &= \frac{\tilde{p}^U\theta_j(\theta_H^2+\theta_L^2+b(\theta_H+\theta_L))}{(\theta_H^2+\theta_L^2)(2-\theta_H^2-\theta_L^2-b(\theta_H+\theta_L))}, \\
\tilde{N}_B^U(\tilde{p}^U,\tilde{r}^U) &= \frac{2\tilde{p}^U}{(2-\theta_H^2-\theta_L^2-b(\theta_H+\theta_L))}, \\
\tilde{\Pi}^U &= \frac{v\tilde{p}^U}{(2-\theta_H^2-\theta_L^2-b(\theta_H+\theta_L))}.
\end{aligned}$$

Buyer surplus and type $j \in \{L,H\}$ sellers' surplus are respectively given by

$$\begin{aligned}
\tilde{CS}^U &= \int_0^{\tilde{N}_B^U(\tilde{p}^U,\tilde{r}^U)} (v + b(\tilde{N}_H^U(\tilde{p}^U,\tilde{r}^U) + \tilde{N}_L^U(\tilde{p}^U,\tilde{r}^U)) - \tilde{p}^U - k^B)dk^B \\
&= \frac{2(\tilde{p}^U)^2}{(2-\theta_H^2-\theta_L^2-b(\theta_H+\theta_L))^2},
\end{aligned}$$

$$\widetilde{DS}_j^U = \int_0^{\tilde{N}_j^U(\tilde{p}^U,\tilde{r}^U)} ((1-\tilde{r}^U)\theta_j \tilde{N}_B^U(\tilde{p}^U,\tilde{r}^U) - k^S)dk^S = \frac{(\tilde{N}_j^U(\tilde{p}^U,\tilde{r}^U))^2}{2},$$

for a total welfare of

$$\widetilde{SW}^U = \widetilde{CS}^U + \tilde{\Pi}^U + \sum_{i=1,2} \widetilde{DS}_j^U = (\tilde{p}^U)^2 X,$$

where

$$X = \frac{\theta_H^2(2(6-\theta_L^2) - b^2 - 2b\theta_L) + \theta_L^2(12-(b+\theta_L)^2) - \theta_H^4 - 2b\theta_H^3 - 2b\theta_H\theta_L(b+\theta_L)}{2(\theta_H^2+\theta_L^2)(2-\theta_H^2-\theta_L^2-b(\theta_H+\theta_L))^2}.$$

Price discrimination. Under price discrimination on the seller side, buyer participation is still given as in equation (??) and seller participation is given as in equation (40). Under rational expectations, equilibrium participation thus satisfies the following system:

$$\tilde{N}_B^D = v + b(N_L^D + N_H^D) - p, \quad \tilde{N}_H^D = (1-r_H)\theta_H N_B^D \quad \text{and} \quad \tilde{N}_L^D = (1-r_L)\theta_L N_B^D. \tag{44}$$

Solving the above system of equations for $\tilde{N}_B^D$, $\tilde{N}_H^D$ and $\tilde{N}_L^D$ yields buyer participation and seller participation as functions prices. We present these demands below. The solution is

$$\tilde{N}_B^D(p,r_H,r_L) = \frac{v-p}{1-b((1-r_H)\theta_H+(1-r_L)\theta_L)}, \tag{45}$$

$$\tilde{N}_H^D(p,r_H,r_L) = \frac{(v-p)(1-r_H)\theta_H}{1-b((1-r_H)\theta_H+(1-r_L)\theta_L)}, \tag{46}$$

$$\tilde{N}_L^D(p,r_H,r_L) = \frac{(v-p)(1-r_L)\theta_L}{1-b((1-r_H)\theta_H+(1-r_L)\theta_L)}. \tag{47}$$

The platform sets prices to maximize profits

$$\max_{p,r_H,r_L} (p + r_H\theta_H\tilde{N}_H^D(\cdot) + r_L\theta_L\tilde{N}_L^D(\cdot))\tilde{N}_B^D(\cdot).$$

Differentiating platform profits with respect to p and r_j, for $j \in \{L,H\}$ and solving the system of first order conditions yields the optimal prices as follows.

$$\tilde{p}^D = \frac{v(2-\theta_H^2-\theta_L^2-b(\theta_H+\theta_L))}{4-2b^2-\theta_H^2-\theta_L^2-2b(\theta_H+\theta_L)}, \quad \tilde{r}_j^D = \frac{\theta_j - b}{2\theta_j}, \text{ for } j \in \{H,L\},$$

where superscript D indicates the case with price discrimination.[14] The associated equilibrium

[14]The denominator is positive by Assumption 1.

seller demands for $j \in \{L, H\}$, buyer demand, and platform profit are respectively given as

$$\tilde{N}_j^D = \frac{v(b+\theta_j)}{4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L)},$$
$$\tilde{N}_B^D = \frac{2v}{4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L)},$$
$$\tilde{\Pi}^D = \frac{v^2}{4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L)}.$$

Before proceeding further, we make a few observations.

Observation 1. *The following equality holds true.*

- *Under price discrimination, the price charged to buyers remains unchanged regardless of the pricing structure incident on sellers — i.e., $p^D = \tilde{p}^D$.*
- *Under price discrimination, the total price charged to remains unchanged regardless of the pricing structure incident on sellers — i.e., $\tilde{r}_j^D \theta_H \tilde{N}_B^D(\tilde{p}^D, \tilde{r}^D) = f_j^D$.*

The above implies that the mass of buyers, sellers and platform profits are identical under price discrimination regime regardless of whether platforms charge sellers a fixed participation price or an ad-valorem fee. As a consequence, consumer surplus and welfare expressions are identical identical as well.

Price discrimination vs. uniform pricing. In the following, we show the robustness of the main result obtained in the baseline model.

Comparison of platform profit. Starting from platform profits, we obtain that:

$$\Pi^D - \tilde{\Pi}^U = \frac{b^2 v^2 (\theta_H - \theta_L)^2}{\begin{array}{l}(4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L))\\ \quad (\theta_H^2(4 - b^2 - 2b\theta_L - 2\theta_L^2) - \theta_L^2(4 - (b+\theta_L)^2) - 2b\theta_H\theta_L(b+\theta_L) - \theta_H^4 - 2b\theta_H^3)\end{array}}.$$

We observe that the sign of the difference in platform profit is determined by the sign of the expressions in the denominator. The two terms in the denominator of the difference in profits are positive as they are just the terms in the denominator of the platform profits in the two pricing regimes. Since Assumption 2 guarantees platform profits are positive, they must be positive as well because the numerator of the profits is always positive.

Comparison of consumer surplus. Comparing consumer surplus under price discrimination with the consumer surplus under uniform pricing yields

$$CS^D - \tilde{CS}^U = \frac{2}{(2 - \theta_H^2 - \theta_L^2 - b(\theta_H + \theta_L))^2}\left[(p_1^D)^2 - (\tilde{p}^U)^2\right].$$

Hence, the difference in buyer prices determines the sign of the difference in consumer surplus.

$$(p_1^D)^2 - (\tilde{p}^U)^2 = A((\theta_L^2 + \theta_H^2)(8 - 3b^2 - 4b\theta_L - 4\theta_L^2) - 2b\theta_H\theta_L(b + 2\theta_L) - 2\theta_H^3(\theta_H + 2b)),$$

where A is a composite term of squared expressions

$$A = \frac{2b^2v^2(\theta_H - \theta_L)^2}{(4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L))^2 \, (\theta_H^2(4 - b^2 - 2b\theta_L - 2\theta_L^2) - \theta_L^2(4 - (b + \theta_L)^2) - 2b\theta_H\theta_L(b + \theta_L) - \theta_H^4 - 2b\theta_H^3)^2} > 0.$$

Therefore, the sign of $(p_1^D)^2 - (\tilde{p}^U)^2$ is determined by the sign of

$$B = ((\theta_L^2 + \theta_H^2)(8 - 3b^2 - 4b\theta_L - 4\theta_L^2) - 2b\theta_H\theta_L(b + 2\theta_L) - 2\theta_H^3(\theta_H + 2b)).$$

Differentiating B with respect to b yields

$$\frac{\partial B}{\partial b} = -2(2(\theta_H + \theta_L)(\theta_H^2 + \theta_L^2) + b(3\theta_H^2 + 3\theta_L^2 + 2\theta_H\theta_L)) < 0.$$

Thus, it is sufficient to show that B at $b = \bar{b}^{ad}$ is positive.

$$B|_{b=\bar{b}^{ad}} = \frac{(\theta_H - \theta_L)^2(4 + 2\theta_H\theta_L - (\theta_H + \theta_L)\sqrt{(8 - (\theta_H - \theta_L)^2)})}{2}.$$

The second term in the numerator given by $(4 + 2\theta_H\theta_L - (\theta_H + \theta_L)\sqrt{(8 - (\theta_H - \theta_L)^2)})$ is always positive for $\theta_H > \theta_L > 0$. Thus, we show that consumer surplus is always higher under the price discrimination regime than under a uniform pricing regime.

Comparison of low-type seller surplus. Turning to the low-type sellers, a sufficient statistic for seller surplus is seller participation:

$$N_L^D - \tilde{N}_L^U = \frac{bv(\theta_H - \theta_L)Z_L}{(4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L)) \, (\theta_H^2(4 - b^2 - 2b\theta_L - 2\theta_L^2) - \theta_L^2(4 - (b + \theta_L)^2) - 2b\theta_H\theta_L(b + \theta_L) - \theta_H^4 - 2b\theta_H^3)}$$

where

$$Z_L = (\theta_H(4 - (b+\theta_H)^2) - (b+\theta_H)\theta_L(b+\theta_L)).$$

The sign of $N_L^D - \tilde{N}_L^U$ is determined by the sign of the term Z_L as all other terms are guaranteed to be positive under Assumption 2.

Differentiating Z_L with respect to b yields

$$\frac{\partial Z_L}{\partial b} = -(2\theta_H(b+\theta_H) + \theta_L(2b+\theta_H) + \theta_L^2) < 0.$$

Thus, it is sufficient to show that Z_L at $b = \bar{b}^{ad}$ is positive.

$$Z_L\big|_{b=\bar{b}^{ad}} = \frac{(\theta_H - \theta_L)(4 - \theta_L^2 - \theta_H(\sqrt{(8-(\theta_H-\theta_L)^2)} - \theta_L)}{2}.$$

The second term in the numerator given by $(4 - \theta_L^2 - \theta_H(\sqrt{(8-(\theta_H-\theta_L)^2)} - \theta_L)$ is always positive as Assumption 2 ensures $\theta_H > \theta_L > 0$ and $\theta_H^2 + \theta_L^2 < 4$. Thus, we show that the surplus of low-type sellers is always higher under price discrimination than under uniform pricing.

Comparison of total welfare. Comparing total welfare under price discrimination with the total welfare under uniform pricing yields

$$SW^D - \widetilde{SW}^U = \frac{A}{4}Y$$

where $Y \triangleq 4b\theta_H^5 + \theta_H^6 + 2b\theta_H\theta_L(2\theta_L^3 + 5b\theta_L^2 - \theta_L(24-5b^2) - 2b(6-b^2)) + \theta_L^2(80 - 2b^2(18-b^2) - 6b\theta_L(8-b^2) - \theta_L^2(24-7b^2) + 4b\theta_L^3 + \theta_L^4) + \theta_H^4(7b^2 + 4b\theta_L - 3(8-\theta_L^2)) + 2b\theta_H^3(3b^2 + 5b\theta_L - 4(6-\theta_L^2)) + \theta_H^2(80 + 2b^4 + 10b^3\theta_L - 48\theta_L^2 + 3\theta_L^4 - 8b\theta_L(6-\theta_L^2) - 2b^2(18-7\theta_L^2))$. Differentiating Y thrice with respect to b yields

$$\frac{\partial^3 Y}{\partial b^3} = 12(\theta_H + \theta_L)(3\theta_H^2 + 3\theta_L^2 + 2\theta_H\theta_L + 4b(\theta_H+\theta_L)) > 0.$$

Computing the second derivative of Y with respect to b at $b = \bar{b}^{ad}$ yields

$$\frac{\partial^2 Y}{\partial b^2}\Big|_{b=\bar{b}^{ad}} = -2(\theta_H - \theta_L)^2\left(12 + 2(\theta_H+\theta_L)^2 + 2\theta_H\theta_L - 3(\theta_H+\theta_L)\sqrt{8-(\theta_H-\theta_L)^2}\right) < 0.$$

Thus, we confirm that $\frac{\partial^2 Y}{\partial b^2}$ is always negative in the feasible region.

Evaluating the first derivative of Y with respect to b at $b = 0$ yields

$$\frac{\partial Y}{\partial b}\Big|_{b=0} = -4(\theta_H + \theta_L)(\theta_H^2 + \theta_L^2)(12 - \theta_H^2 - \theta_L^2) < 0.$$

The above is negative as Assumption 2 ensures that $\theta_H^2 + \theta_L^2 < 4$.

Finally, computing Y at $b = \bar{b}^{ad}$ yields

$$Y\big|_{b=\bar{b}^{ad}} = 4(\theta_H - \theta_L)^2 \left(4 + 2\theta_L - (\theta_L + \theta_H)\sqrt{8 - (\theta_H - \theta_L)^2}\right) > 0.$$

The above is always positive as Assumption 2 ensures that $\theta_H^2 + \theta_L^2 < 4$. Hence, we show that total welfare is always higher under price discrimination than under uniform pricing.

Comparison of high-type seller surplus. A sufficient statistic for seller surplus is seller participation, which yields:

$$N_H^D - \tilde{N}_H^U = \frac{bv(\theta_H - \theta_L)Z_H}{(4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L))\,(\theta_H^2(4 - b^2 - 2b\theta_L - 2\theta_L^2) - \theta_L^2(4 - (b + \theta_L)^2) - 2b\theta_H\theta_L(b + \theta_L) - \theta_H^4 - 2b\theta_H^3)}$$

where

$$Z_H = ((b + \theta_L)(\theta_H^2 + \theta_L^2 + b(\theta_H + \theta_L)) - 4\theta_L).$$

The sign of $N_L^D - \tilde{N}_L^U$ is determined by the sign of the term Z_H as all other terms are positive under Assumption 2.

Differentiating Z_H with respect to b yields

$$\frac{\partial Z_H}{\partial b} = \theta_H^2 + 2\theta_L^2 + \theta_H\theta_L + 2b(\theta_H + \theta_L) > 0.$$

Computing Z_H at $b = 0$, yields

$$Z_H\big|_{b=0} = \theta_L(4 - \theta_L^2 - \theta_H^2) > 0.$$

The above is positive as Assumption 2 ensures that $\theta_H^2 + \theta_L^2 < 4$.

Similarly, computing Z_H at $b = \bar{b}^{ad}$ yields

$$Z_H\big|_{b=\bar{b}^{ad}} = \frac{(\theta_H - \theta_L)(4 - \theta_H^2 + \theta_H\theta_L - \theta_L\sqrt{(8 - (\theta_H - \theta_L)^2)})}{2}.$$

The second term in the numerator given by $(4 - \theta_L^2 - \theta_H(\sqrt{(8 - (\theta_H - \theta_L)^2)} - \theta_L)$ is always positive as Assumption 2 ensures $\theta_H > \theta_L > 0$ and $\theta_H^2 + \theta_L^2 < 4$.

Thus, by intermediate value theorem, there must exist a critical level of b denoted by

$$\hat{b}^{ad} = \frac{1}{2}\left(\sqrt{\frac{\theta_H^4 + 2\theta_H\theta_L(8-\theta_H^2) + \theta_L^2(16+\theta_H^2) - 2\theta_L^2}{\theta_H + \theta_L}} - \theta_H\right)$$

where $N_H^D - N_H^U = 0$. For $b > \hat{b}^{ad}$, we must have $N_H^D - N_H^U > 0$ and for $b < \hat{b}^{ad}$, we must have $N_H^D - N_H^U < 0$.

Thus, we show that the surplus of high type sellers sellers can also increase giving us the result that price discrimination can result in Pareto improvement over uniform pricing.

B.3. Proof of Proposition 6

As in the benchmark case, in order to ensure that the maximization problem is concave, we impose the following conditions:

Assumption 3. *Provided buyer intrinsic valuation as well as sellers' valuations are sufficiently low, we consider the parametric region* $\max\{0, \frac{2\theta_H}{\theta_L(\theta_H-\theta_L)}\} < b < \bar{b} = \frac{\sqrt{17\theta_H^2 - 10\theta_H\theta_L + 9\theta_L^2} - 3\theta_H - \theta_L}{2(\theta_H-\theta_L)^2}$.[15]

Reproducing the analysis carried out in Section 4, we can easily see that Subsection 4.1 does not change, the only caveat being that we have to consider $p = 0$. As per the modification to Subsections 4.2 and 4.3, we obtain the following results.

Uniform pricing. The platform sets the uniform fee to maximize profits $fN_S^U(f)$, which yields the equilibrium fee:

$$f^U = \frac{v(\theta_H + \theta_L)}{4}.$$

The associated equilibrium seller demands for type $j \in \{L, H\}$, buyer demand, and platform profit are respectively given by:

$$N_j^U = \frac{v(\theta_j(3-b\theta_j) - \theta_{-j}(1-b\theta_{-j})}{4-4b(\theta_H+\theta_L)}, N_B^U = \frac{v(2-b(\theta_H+\theta_L))}{2-2b(\theta_H+\theta_L)}, \Pi^U = \frac{v^2(\theta_H+\theta_L)^2}{8-8b(\theta_H+\theta_L)}.$$

Total participation of the sellers is then $N_S^U = N_L^U + N_H^U = \frac{v(\theta_H+\theta_L)}{2-2b(\theta_H+\theta_L)}$.

Buyer surplus and type $j \in \{L, H\}$ sellers' surplus is respectively given by

[15] More precise conditions on v, θ_H and θ_L can be provided upon request.

$$CS^U = \frac{v^2(2 - b(\theta_H + \theta_L))^2}{8(1 - b(\theta_H + \theta_L))^2}, DS_j^U = \frac{v^2(\theta_j(3 - b\theta_j) - \theta_{-j}(1 - b\theta_{-j}))^2}{32(1 - b(\theta_H + \theta_L))^2}.$$

Total welfare amounts to:

$$SW^U = \frac{v^2(b^2(2 + (\theta_H - \theta_L)^2)(\theta_H + \theta_L)^2 - 2b(\theta_H + \theta_L)\Sigma)}{16(1 - b(\theta_H + \theta_L))^2}.$$

where $\Sigma = (4 + 3\theta_H^2 - 2\theta_H\theta_L + 3\theta_L^2) + 8 + 7(\theta_H^2 + \theta_L^2) - 2\theta_H\theta_L)$.

Price Discrimination. The platform sets two different fees in order to maximize $f_H N_H^D(f_H, f_L) + f_L N_L^D(f_H, f_L)$, which yields at equilibrium

$$f_j^D = \frac{v(2\theta_j(1 - b\theta_j) - b\theta_{-j}(\theta_j - \theta_{-j}))}{4 - 4b(\theta_H + \theta_L) - b^2(\theta_H - \theta_L)^2}, \text{ for } j \in \{H, L\}.$$

The associated equilibrium seller demands for $j \in \{L, H\}$, buyer demand, and platform profit are respectively given as

$$N_j^D = \frac{v(\theta_j(2 - b\theta_j) + b\theta_{-j}^2)}{4 - 4b(\theta_H + \theta_L) - b^2(\theta_H - \theta_L)^2}, N_B^D = \frac{2v(2 - b(\theta_H + \theta_L))}{4 - 4b(\theta_H + \theta_L) - b^2(\theta_H - \theta_L)^2},$$

$$\Pi^D = \frac{v^2(\theta_H + \theta_L)^2}{4 - 4b(\theta_H + \theta_L) - b^2(\theta_H - \theta_L)^2}.$$

Total seller participation is then given as

$$N_S^D = N_L^D + N_H^D = \frac{v(2(2 + \theta_L) - 2b\theta_L - b\theta_H(2 - \theta_H - \theta_L))}{4 - 4b(\theta_H + \theta_L) - b^2(\theta_H - \theta_L)^2}.$$

Buyer surplus and type $j \in \{L, H\}$ sellers' surplus is respectively given by

$$CS^D = \frac{2v^2(2 - b(\theta_H + \theta_L))^2}{(4 - b^2(\theta_H - \theta_L)^2 - 4b(\theta_H + \theta_L))^2}, DS_j^D = \frac{v^2(\theta_j(2 - b\theta_{-j}) + b\theta_{-j}^2)}{2(4 - b^2(\theta_H - \theta_L)^2 - 4b(\theta_H + \theta_L))^2}.$$

Total welfare amounts to:

$$SW^D = \frac{v^2(16 + 12(\theta_H^2 + \theta_L^2) - 8b(\theta_H + \theta_L)(2 + \theta_H^2 + \theta_L^2) - b^2\Delta)}{2(4 - b^2(\theta_H - \theta_L)^2 - 4b(\theta_H + \theta_L))^2},$$

where $\Delta = (\theta_H^4 - 2\theta_H^3\theta_L - 4\theta_L^2 + \theta_L^4 - 2\theta_H^2(2 - \theta_L^2) - 2\theta_H\theta_L(4 + \theta_L^2))$.

Price discrimination vs. uniform pricing. Firstly, it is straightforward that the platform earns higher profit under price discrimination than under uniform prices.

Before we proceed further, it is informative to keep in mind how seller prices change under price discrimination. Comparing prices, we observe that

$$f^U - f_L^D = \frac{v(2 + b(\theta_H - \theta_L))(\theta_H - \theta_L)(2 - b(\theta_H + \theta_L))}{4(4 - b^2(\theta_H - \theta_L)^2 - 4b(\theta_H + \theta_L))} > 0$$

and

$$f^U - f_H^D = -\frac{v(\theta_H - \theta_L)((2 - b\theta_H)^2 - b^2\theta_L^2)}{4(4 - b^2(\theta_H - \theta_L)^2 - 4b(\theta_H + \theta_L))} < 0.$$

A corollary from the above price relations is that the low-type sellers are always better off.

Secondly, we find that total seller participation rises.

$$N_S^D - N_S^U = \frac{bv(\theta_H - \theta_L)^2(2 - b(\theta_H + \theta_L)}{2(1 - b(\theta_H + \theta_L))(4 - b^2(\theta_H - \theta_L)^2 - 4b(\theta_H + \theta_L))} > 0.$$

The above is always positive because $\theta_H > \theta_L$ and that the expressions in the denominator are positive to ensure an interior solution. A direct consequence of the above is that consumers' surplus rises. This is because consumer price is set at zero and seller participation increases under price discrimination, thus benefiting consumers.

Finally, in order to show Pareto improvement is a possibility, it is sufficient to find conditions under which the high-type sellers can be better off under price discrimination. A sufficient statistic for this result to hold is to show that the participation of high-type sellers is higher under price discrimination than under uniform pricing despite the fact that participation fee to the high-margin type rises. This can be formally demonstrated as follows. Taking the difference of participation of the high type under price discrimination with its participation under uniform prices yields

$$N_H^D - N_H^U = \frac{v(\theta_H - \theta_L)(2 - b(\theta_H + \theta_L))\Omega}{4(1 - b(\theta_H + \theta_L))(4 - b^2(\theta_H - \theta_L)^2 - 4b(\theta_H + \theta_L))},$$

where $\Omega = (2 - b^2(\theta_H + \theta_L) + b(3\theta_H + \theta_L))$. Note that the sign of $N_H^D - N_H^U$ follows that of Ω as all other terms are positive under the assumption that the problem is concave.

Differentiating Ω with respect to b, we observe that

$$\frac{\partial \Omega}{\partial b} = 3\theta_H + \theta_L + 2b(\theta_H - \theta_L)^2 > 0.$$

Further, computing Ω at the two bounds, we find that

$$\Omega|_{b=0} = -2, \ \Omega|_{b=\bar{b}'} = \frac{4(\theta_H^2 + \theta_H\theta_L + 2\theta_L^2) - 2(\theta_H + 3\theta_L)\sqrt{2(\theta_H^2 + \theta_L^2)}}{(\theta_H - \theta_L)^2} > 0.$$

Thus, by the intermediate value theorem, we can state there exists are cut-off denoted by $\hat{b}'$ above which $\Omega > 0$ and negative otherwise.

Equating Ω to zero and solving for b yields the following threshold

$$\hat{b}' = \frac{\sqrt{17\theta_H^2 - 10\theta_H\theta_L + 9\theta_L^2} - 3\theta_H - \theta_L}{(\theta_H - \theta_L)^2},$$

which is within the admissible parameter bounds, as it can be easily demonstrated.

Finally, comparing social welfare in the two cases, we find that $SW^D > SW^U$ if and only if $b > b_w$ with $b_w < \hat{b}'$; the analytical expression of b_w is very complex but can be provided upon request.