Interoperability in Digital Markets: Boon or Bane for Market Contestability?

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Policymakers worldwide discuss whether interoperability obligations are an appropriate regulatory tool to promote contestability and competition in digital markets where network effects are strong. In the EU, the Digital Markets Act imposes horizontal interoperability obligations on dominant messaging services, requiring them to allow rival services to connect to their network. The standard economic wisdom is that interoperability is pro-competitive in the presence of an incumbent with a large user base, because interoperability lowers the entry barriers constituted by network effects. However, we show that this logic is incomplete in the dynamic context of digital services, where only partial interoperability can be achieved and multihoming is economically feasible. Absent full interoperability some proprietary network effects remain and consumers still gravitate to the larger network in order to take advantage of the full richness of features. At the same time, horizontal interoperability lowers the incentives of consumers to multihome services, which is a powerful driver for contestability. We develop a dynamic multi-period model, which formalizes the trade-off between the (imperfect) sharing of network effects through interoperability and reduced incentives to multihome. We highlight that mandated interoperability can impede the ability of a more efficient entrant platform to contest the less efficient dominant platform. Hence, our results have immediate implications for the ongoing policy debate by demonstrating that horizontal interoperability obligations do not only have pro-competitive, but also anti-competitive effects and may thus not be an appropriate remedy for regulating dominant online platforms.

Key words: contestability of digital markets; interoperability; multihoming; online platform; regulation; Digital Markets Act

1. Introduction

Digital markets have achieved high levels of concentration due to significant economies of scale and network effects, leading to the emergence of large digital firms, the "Big Tech." This has raised growing concerns about the lack of competition in digital markets, their limited contestability, and the possible adverse effects it may have in terms of prices, innovation, privacy, consumer protection, or data governance.

Already over a decade ago, the European Commission (EC) identified the lack of interoperability as one of the most significant obstacles to digitalization, planning to examine measures to encourage significant market players to pursue interoperability-friendly business policies. The Digital Agenda called for standard-setting by the industry, supported by public policy, to promote greater interoperability. Since then, scholars and policymakers around the globe have called for mandated interoperability to strengthen competition in digital markets (see, e.g., Gans 2018, Borgogno and Colangelo 2019, Chao and Schulman 2020, Nadler and Cicilline 2020, Cyphers and Doctorow 2021, Graves 2021, Riley 2020, Santesteban and Longpre 2021, Scott Morton et al. 2021, Stella 2021).

Recently, these efforts culminated in the Digital Markets Act (DMA), a regulatory framework with a number of *ex-ante* rules and obligations for digital gatekeepers intended to improve fairness and contestability in digital markets. Specifically, the DMA includes an interoperability obligation for 'number-independent interpersonal messenger services' of designated gatekeepers, i.e., messaging services like WhatsApp if Meta (formerly Facebook) were to be designated as a gatekeeper under the DMA.[1] This means that providers of dominant messaging services must make their services interoperable, so that users of other messaging services are able to interact with the users on the dominant network (European Parliament 2022). A similar law, the ACCESS Act (Scanlon et al. 2021) is under debate in the US Congress.

In this paper, we study the effects of such *horizontal interoperability* obligations for market contestability, i.e., the ability of a rival network (with a significantly lower number of users and

[1] Article 7 of the Regulation of the European Parliament and of the Council on contestable and fair markets in the digital sector and amending Directives (EU) 2019/1937 and (EU) 2020/1828 (Digital Markets Act).

network effect) to gain a foothold in the market. Horizontal interoperability is believed to facilitate market contestability, because it requires competing platforms to share their direct network effects, which benefits especially the smaller network (Crémer et al. 2000, Jullien and Sand-Zantman 2021). For instance, by virtue of interoperability, the users of a small email provider, with only a few customers, can send and receive emails from as many users as the customers of a large email provider. Thus, with interoperability the number of fellow users of the same service does not matter anymore, leveling the playing field between the two services. This is true if interoperability were indeed perfect. However, interoperability may come in different degrees, and we can think of a continuum between no interoperability and full interoperability (Kerber and Schweitzer 2017). For instance, Scott Morton et al. (2021) suggest that an interoperability requirement for a social media platform like Facebook would apply to a set of 'standard' functionalities (e.g., exchange of text, images, video, or calendar), leaving aside other, 'non-standard' functionalities. The DMA also foresees interoperability obligations only for 'basic functionalities' of messaging services. The regulated degree of interoperability can then be defined as the relative amount of interoperable ('standard') functionalities.

There are various reasons that provide economic rationale to opt for partial rather than full interoperability. First, achieving interoperability may come at a cost for the different players (Kerber and Schweitzer 2017), pushing for an intermediate level of interoperability, for instance, if some functionalities would be too complicated or too costly to be made interoperable. Second, a higher degree of interoperability may reduce the possibilities of differentiation between market players. Partial interoperability would allow them to differentiate with respect to the 'non-standard', non-interoperable functionalities (Scott Morton et al. 2021), increasing variety to the benefit of users. It would also stimulate innovation for new, 'non-standard' functionalities.

In addition, there are also several reasons why full interoperability is rarely attainable in practice, especially in the context of digital services. First, digital services, even if they seemingly fulfill similar consumer needs, differ in their functionalities and feature-richness. Interoperability can only be achieved on a common set of features, and as we have argued above, firms will seek differentiation through implementing new (not yet interoperable) features. Second, innovation in digital services is fast-paced. New features can be rolled out in software and occur at high frequency. For example, in 2016, WhatsApp rolled out end-to-end encryption of its messaging service virtually overnight (Budington 2016). This would not have been possible - at least not so quickly - if WhatsApp had to maintain interoperability with rival networks. By contrast, in most other industries in which interoperability obligations have been in place so far, like telecommunications, innovation on core functionalities (e.g. the transition from circuit-switched to packet-switched transmission) occurs at a much slower pace. And yet, empirical research has found that full interoperability was not attainable in practice in the telecommunications industry to the extent that firm-specific network effects remained despite a full interoperability obligation (Grajek 2010).

The competitive effects of partial interoperability obligations have to date not been investigated in detail. If the degree of interoperability is imperfect, competition is still shaped by the level of proprietary network effects specific to each firm. With a larger customer base, a dominant firm would thus keep a competitive advantage due to its larger network. In this case, consumers may be inclined to opt for the 'focal' dominant network, i.e., the network where most other users are, to communicate with the full set of functionalities, using interoperability to communicate with users of small networks.

However, in this paper we highlight an even more powerful and novel economic argument why partial interoperability may in fact be anti-competitive and run counter to its objective of achieving more market contestability: *Interoperability can reduce multihoming.* Multihoming occurs when consumers use more than one platform for the same or a similar service (Belleflamme and Peitz 2019, Bakos and Halaburda 2020). Multihoming is an alternative to interoperability, because both allow consumers to interact with the consumers on several platforms. Multihoming may involve additional costs (e.g., for maintaining two accounts) compared to interoperability; however, the benefit of multihoming relative to interoperability (if imperfect) is that users can use each platform with the full set of functionalities.

For hardware products, multihoming can be very costly for users (e.g., by requiring multiple smartphones) and thus remains of marginal impact in practice. However, for software products (e.g., apps), especially 'free' apps, the costs of multihoming are typically relatively low, yielding high levels of multihoming. For instance, a representative survey of the German Federal Network Agency (2021) shows that 73% of users of messaging services multihome. Therefore, from a policy perspective, interoperability and multihoming may represent substitute means to achieve market contestability. Reducing the extent of multihoming may have an ambiguous effect on the likelihood that this goal is attained. Especially, interoperability and multihoming relate to two different types of competition in digital markets: Competition *in* the market and competition *for* the market.

Interoperability only allows for competition *in* the market (Jullien and Sand-Zantman 2021). First, this is because users can stick to the dominant platform and still benefit from an entrant's new network via interoperability. In other words, the dominant players are likely to remain 'focal,' limiting the possibility of entrants to expand and develop. Second, interoperability reduces the possibilities of differentiation between suppliers, as it induces a degree of commonality between their products or services. As dominant players remain focal, it also means that implementing horizontal interoperability requires ongoing regulation, because if it were lifted, the market would risk tipping again towards the focal firm. Besides, a dominant firm may constantly seek to undermine technical interoperability, which requires regulatory oversight.

Moreover, from a long-term perspective, competition *for* the market is often more desirable. If an innovative entrant with higher quality or superior technology enters the market, it would be efficient that it takes over the market and replaces the incumbent. Therefore, it is desirable from a social point of view that competition for the market remains possible. Multihoming, in contrast to (full) interoperability, keeps the possibility of competition *for* the market intact. Therefore, introducing horizontal interoperability may involve a trade-off. On the one hand, interoperability allows competition in the market to emerge swiftly and be sustainable, increasing static (short-term) efficiency. On the other hand, interoperability may undermine the incentives of consumers to multihome, reducing the possibility of competition for the market, which would lower dynamic (long-term) efficiency.

In this paper, we formalize this trade-off by means of a multi-period game-theoretical model. We show that partial interoperability can indeed undermine market contestability, and characterize the conditions under which this occurs. Our model highlights that partial interoperability is harmful to contestability especially when an entrant network has an intermediate quality advantage over the incumbent network and is on the brink of achieving a critical mass. In this case, interoperability prevents those entrants to achieve a critical mass (because it reduces the incentives to multihome), which would have let to a tipping of the market towards the new entrant otherwise. In many other settings, especially for entrants with a relatively low or relatively high quality advantage over the incumbent, partial interoperability does not lead to more or less contestability than multihoming, rendering such a remedy ineffective. In reverse, interoperability is conducive to market contestability especially if the costs of multihoming are high or interoperability is near perfect.

The remainder of this article is structured as follows. In Section 2 we briefly locate our work in the related literature, and in Section 3 we outline our model. In Section 4 we fully characterize consumers' adoption decisions in a single period of the multi-period game. Our main results are derived in Section 5, where, based on the single-period interaction, we highlight how interoperability can undermine digital market contestability in the long-run. We conclude with managerial and policy implications in Section 6.

2. Related Literature

Our paper relates to several streams of the literature on compatibility and competition in the presence of network effects. First, our paper relates to the literature considering firms' incentives to make their network products or services compatible.[2] This literature highlights two main forces

[2] We consider two-way compatibility, as we are interested in scenarios where consumers can interact between two platforms. The literature on compatibility has also considered situations of one-way compatibility (Manenti and Somma 2008, Liu et al. 2011, Adner et al. 2020), e.g., through the use of converters (Farrell and Saloner 1992a), which, however, is not meaningful in the context of communications platforms.

driving the incentive to choose compatibility. First, by making their products compatible, firms increase the value for users due to larger network effects, which may lead to demand expansion, and hence, increased profits. Second, compatibility neutralizes the competitive advantage of firms with larger installed bases vis-à-vis smaller rivals. Thus, compatibility may involve a trade-off between demand expansion and reduced competitive advantage for large firms. By contrast, small firms always benefit from compatibility. Therefore, compatibility is likely to emerge endogenously when firms do not compete directly (and thus only the demand expansion effect prevails), or if firms are relatively symmetric (see, e.g., Katz and Shapiro 1985, Xie and Sirbu 1995), even in the presence of random shocks that temporarily lead to a difference in the installed base advantage (Chen et al. 2009). However, when firms are asymmetric, large firms may resist compatibility, making it less likely to arise (Xie and Sirbu 1995, Crémer et al. 2000, Malueg and Schwartz 2006, Viecens 2011). Furthermore, the possibility of multihoming may also reduce firms' incentives for compatibility (Crémer et al. 2000, Doganoglu and Wright 2006).[3]

Our contribution to this literature is twofold. First, we build a dynamic model with partial compatibility (interoperability), endogenous multihoming, and the possibility of tipping, whereas most of the literature builds on static models,[4] with either no- or full-compatibility,[5] singlehoming,[6] and without tipping.[7] Second, we address a different question. Whereas the literature investigates firms' incentives to make their products or services compatible, we take the level of 'compatibility'

[3] In Doganoglu and Wright (2006), the possibility to multihome has a negative impact on consumer welfare, because multihoming raises firms' prices and profits. This result hinges crucially on the fact that horizontal differentiation between firms is assumed to be strong relative to network effects, while there is no vertical differentiation. By contrast, we assume that network effects are strong relative to horizontal differentiation and that there exists vertical differentiation.

[4] Chen et al. (2009) study compatibility decisions in a dynamic model in discrete time with an infinite horizon.

[5] Two exceptions are de Palma et al. (1999) and Crémer et al. (2000), who also consider partial compatibility.

[6] Crémer et al. (2000), Doganoglu and Wright (2006), and de Palma et al. (1999) consider the possibility of multihoming.

[7] Malueg and Schwartz (2006) and Chen et al. (2009) also consider the possibility of tipping.

as given and set by a regulator, and study how the degree of compatibility affects endogenous multihoming and market contestability.

We also contribute to the literature on incumbency advantage and contestability in markets with network effects. In the presence of network effects, an installed base of locked-in users may prevent potential entrants to penetrate the market even if they offer a higher quality, and thus represent a source of incumbency advantage (Farrell and Saloner 1986, 1992b, Fudenberg and Tirole 2000). Without locked-in users and switching costs, an incumbent can also maintain its dominant position if it benefits from favorable beliefs from potential users (Halaburda and Yehezkel 2019, Halaburda et al. 2020), or because users might wish to wait before migrating to an entrant platform that other users have already adopted (Biglaiser et al. 2021). We contribute to this literature by considering the degree of interoperability and its interaction with endogenous multihoming in mitigating incumbency advantage and restoring market contestability.

3. The model

3.1. Preliminaries and main assumptions

We build a model of competition between two platforms (or 'networks'), an incumbent platform, I, and an entrant platform, E, with and without (partial) interoperability. We seek to build this model in the most parsimonious way, leaving out unnecessary details so that we can highlight and focus on the main strategic trade-offs. Our main assumptions are driven by some general observations about digital markets and digital services that we highlight in the following using the example of messaging apps, albeit our model is not limited to this context.

First, due to strong network effects, digital markets are prone to 'market tipping.' In a tipped market all consumers singlehome on one platform. The need to overcome the strong network effects of the incumbent's services is considered the main impediment to digital markets' contestability. Therefore, we require that network effects are 'strong,' i.e., dominate consumers' innate (horizontal) preferences for a platform. For instance, consumers may have personal preferences for one messaging app over another, but ultimately the app is useless if they cannot communicate with others. Thus, consumers choose the messaging app with a larger network. If network effects were weak and consumers' horizontal preferences would dominate, the market would not tip, and several platforms could coexist in equilibrium even if all consumers were to singlehome. Thus, the assumption that network effects are strong implies that the market can tip. The two platforms can coexist in equilibrium only when some consumers multihome, but not when every consumer singlehomes.

Second, we consider that consumers differ in their valuation of network benefits, i.e., the value of interactions derived when joining a platform. Some consumers value more the ability to communicate with a large number of other users, or some may value more to be able to interact with the best possible quality and richness of features. In contrast, for other users, the network size and the feature richness are less important. For instance, in the context of messaging apps, some consumers extensively use advanced features such as video telephony, embedded gifs, or animated emoticons, while others merely send plain text messages.

Third, consumers may choose to multihome, i.e., use more than one digital service at a time. For example, as highlighted in the introduction, many users of messaging apps use more than one such app, e.g., WhatsApp and Signal. However, multihoming entails some additional (transaction) costs for users. Such costs may, for instance, arise from additional learning costs or the costs for maintaining and managing contacts across several platforms, or because installing a second app may negatively impact the hardware performance. In digital markets, the additional costs for multihoming can be relatively low, especially with regard to software platforms that do not require consumers to purchase additional (costly) hardware. For example, while multihoming is prevalent in the context of messaging apps, this is not the case in the context of operating systems. We are interested in markets in which multihoming costs are not prohibitively high and thus multihoming remains an economically feasible option for at least some consumers.

Fourth, we consider situations where the current market outcome is inefficient, and thus contestability is desirable from a social point of view. That is, we consider situations where an entrant platform, which is currently smaller (in terms of network size), provides a service of higher 'quality' (e.g., feature-richer, or more secure) service. This means that – in the absence of horizontal preferences and differences in network size – all consumers agree that the entrant platform would be the better network. Hence, the efficient market outcome involves all consumers eventually singlehoming on the entrant platform. However, we assume that the incumbent platform initially enjoys a larger customer base than the entrant platform, which may counter-balance the entrant's quality advantage.

Finally, we assume that the degree of interoperability is set by a regulator, e.g., by defining a common set of features. Further, as motivated in the introduction, we assume that interoperability is imperfect, and thus some proprietary features remain. This implies that, even though services are 'interoperable', the quality of interactions is higher within a platform than across platforms.

In summary, our main assumptions are as follows:

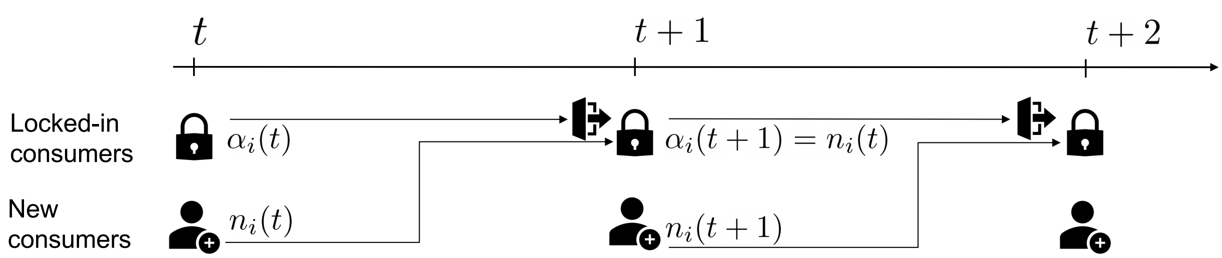
1. Network effects are strong: Network effects dominate consumers' horizontal preferences for platforms, favoring market tipping.
2. Consumers differ in their marginal valuation for interactions ($v \in [0,1]$).
3. Consumers can choose whether to singlehome or to multihome the platforms' services, but multihoming yields an additional cost over singlehoming ($m > 0$).
4. The entrant platform provides a better service (higher interaction quality) than the incumbent platform ($f_E > f_I$)
5. The incumbent platform has a larger installed base of users than the entrant platform ($\alpha_I > \alpha_E$).
6. Regulated interoperability, if it exists, is imperfect ($f_R < f_I$).

3.2. Detailed model setup

Consider a market with two competing platforms (or 'networks'), an incumbent platform, I, and an entrant platform, E, with the same marginal cost of delivering services, which we normalize to zero. Firms compete for a potentially infinite number of periods ($t = 0, 1, 2, ...$), but in each period, competition takes place only over a cohort of new users that have not yet affiliated themselves with one or both platforms. However, platforms do not act strategically in our model. The value they offer to consumers depends on the network size and the quality of interactions. We assume the interaction value to be exogenous and fixed over time, albeit different for the two firms, as we are interested in a situation where a higher quality entrant competes against a lower quality incumbent. However, the network size forms endogenously through consumers' (strategic) adoption decisions in each period.

Further, we assume that consumers' adoption decisions are sticky: At the end of a period, the new users become locked-in with the platform(s) they have chosen. Each user cohort has a mass of one and 'lives' for two periods. Thus, in each period t we only need to consider the current new cohort, denoted by $n_i(t)$ and the previous one, denoted by $\alpha_i(t)$ (see Figure 1). In order to ease notation, we omit the time index whenever we consider only a single period.

Figure 1 Timing and stickiness of consumers' adoption decisions



Note. In each period, t, new consumers enter and consider which platform to adopt, observing an installed base of locked-in consumers. In the next period, $t+1$, the new consumers of the previous period form the installed base of locked-in consumers and the next cohort of new consumers enters. Consumers 'live' for two periods and then exit.

In the previous period, a mass $\alpha_I(t) \geq 0$ of (new) users singlehomed on platform I, a mass $\alpha_E(t) \geq 0$ singlehomed on platform E, and finally, a mass $\alpha_M(t) \geq 0$ multihomed on both platforms, with $\alpha_I(t) + \alpha_E(t) + \alpha_M(t) = 1$. We assume that the number of locked-in users observed by new consumers in the initial period is such that $\alpha_I(0) \geq \alpha_E(0)$; so, I represents the 'large' platform enjoying the larger network effect initially.

The two platforms compete for a unit mass of new, unattached consumers that make their adoption decision in the current period. New consumers decide which platform to join, observing the adoption decisions of the previous cohort and making expectations about the decision of other unattached users. However, we consider that consumers are 'myopic' in the sense that they do not take into account their next period's payoff (when they will be locked-in) for their adoption decision in the current period.

New consumers can choose to singlehome on either platform or to multihome. We normalize the cost of joining a single platform to zero, and therefore, all consumers join at least one platform in equilibrium. However, multihoming yields an additional cost of $m>0$ and thus, depending on m, multihoming may or may not occur in equilibrium. In any period t, we denote by $n_I(t)$ and $n_E(t)$ the number of new consumers who choose to singlehome on platform I and E, respectively; and we denote by $n_M(t)$ the mass of new consumers who multihome, with $n_I(t)+n_E(t)+n_M(t)=1$.

Consumers are heterogeneous in how much they value interactions with other users.[8] We assume that each user receives a positive utility from interacting with each other user.[9] More precisely, we assume that a consumer is characterized by its marginal value for interactions, v, uniformly distributed on $[0,1]$,[10] and let

$$u(v)=vN_if_i$$

[8] Consumers derive utility only from interactions; there is no stand-alone value of joining a platform.

[9] In the literature on 'interconnection' (i.e., interoperability) of telecommunications networks, this is a standard assumption referred to as a 'uniform calling pattern' (Armstrong 1998). It is not without criticism, but has proven to establish a useful and robust benchmark (Armstrong 2004). In our case, it is a conservative assumption that favors the incumbent platform even more, as the incumbent platform enjoys the largest network effect and thus, there are more 'on-net' interactions compared to the smaller entrant network.

[10] Our model can be readily extended to a more general distribution. Further, note that we are allowing for two types of heterogeneity in the valuation of the network effect. First, consumers may differ in their marginal value for network size, but are homogeneous in their marginal valuation of the quality of an interaction. Second, consumers may differ in their marginal valuation for the quality of each interaction, but are heterogeneous in their marginal valuation of network size. Our model considers both types of heterogeneity jointly. In other words, let $v_N \in [0,1]$ denote a consumer's marginal valuation for network size and $v_f \in [0,1]$ denote the marginal valuation for quality, then $v=v_N\, v_f$. Hence, we can consider the distribution of v as the convolution of the distributions of v_N and v_f.

be the utility that the consumer derives from interacting with N_i other consumers using platform i, with the platform providing the interaction quality (or features) f_i. Since we are interested in the possibility of entry of an innovative entrant, we focus on the case where $f_E > f_I$ and, without loss of generality, we normalize $f_I = 1$. We denote platform E's quality advantage as $\delta = f_E - 1 > 0$. Since $\alpha_I(0) \geq \alpha_E(0)$ in the first period, platform I has an installed base advantage, but this is counter-balanced to some extent by the richer features offered by platform E.

No interoperability: In the absence of interoperability, singlehoming consumers can only interact with the other users on the same network, i.e., all other singlehoming users on the same network (from the current and previous periods), but also all multihoming users (from the current and previous periods). Hence, a consumer of type v singlehoming on platform $i = I, E$ derives the utility

$$u_i = v(\alpha_i + \alpha_M + n_i + n_M) f_i. \tag{1}$$

Alternatively, the consumer can multihome, i.e., join both platforms, in which case she derives the utility

$$u_M = v\left((\alpha_I + n_I) f_I + (\alpha_E + n_E + \alpha_M + n_M) f_E\right) - m. \tag{2}$$

We thereby assume that a multihomer reaches any other user through the platform offering the richest features (i.e., platform E when possible). Furthermore, we assume that multihomers derive utility from only one interaction with other multihomers (i.e., as in Bakos and Halaburda (2020) there is no 'double-counting' of the network benefit).

With interoperability: With regulated interoperability, a singlehomer on one platform can interact with singlehomers on the other platform. We study the impact of a regulation mandating a level f_R of features to be interoperable between competing platforms. Since $f_I < f_E$, full interoperability would mean that $f_R = f_I = 1$. Clearly, with full interoperability, no unattached user would have an incentive to join platform I or to multihome, because all users of I could be reached with the maximum possible interaction quality from platform E. However, as motivated above, we are interested in partial interoperability and thus assume that $f_R < 1$. Therefore, a user singlehoming on platform $i = I, E$ derives utility

$$u_i = v((\alpha_i + \alpha_M + n_i + n_M)f_i + (\alpha_j + n_j)f_R), \tag{3}$$

where $j \neq i$ denotes the other platform. Since a multihoming user always uses the best network to reach another user, interoperability does not affect the utility u_M from multihoming given by (2).

4. Platform adoption in a single period

We now consider the adoption decision of new users in a single period, $n_i(t)$, given the adoption decision of (new) consumers in the previous period, $\alpha_i(t) = n_i(t-1)$. Understanding the interaction in one period is crucial for understanding the dynamics of the multi-period game and the long-run outcomes highlighted in Section 5.

In each period, a consumer's decision depends on her beliefs regarding the action of other users. We look for the fulfilled expectations equilibria of this single-period game. This means that in equilibrium, consumers' expectations, which they have formed when adopting a certain platform, have to be fulfilled. Otherwise, a consumer may want to change her decision, violating the notion of an equilibrium. As is typical for games with network effects, there may exist multiple equilibria (for some parameter ranges). In this case, we choose among the multiple equilibria by applying the following selection criteria. First, we select stable equilibria over instable equilibria. Second, we consider optimistic beliefs and choose equilibria with high adoption levels over equilibria with low (or zero) adoption (a Pareto dominance criterion). Third, we consider the incumbent platform as 'focal' (cp. Caillaud and Jullien 2001, 2003, Halaburda and Yehezkel 2019), and select equilibria that favor the adoption of the incumbent over equilibria that favor the adoption of the entrant.[11]

The first important insight of the analysis is to note that in any given period there is no stable equilibrium where some consumers singlehome on I and some on E. In such an equilibrium,

[11] Note that all our results still hold qualitatively if we assume 'partial focality' (Halaburda and Yehezkel 2019), i.e., a lesser degree of focality for the incumbent platform, as long as the degree of partial focality is high enough. However, our results would not extend to the (arguably unreasonable) case where the entrant platform has full focality.

singlehoming consumers would necessarily be indifferent between joining I or E. But then, any unilateral deviation of a singlehoming user from one platform to the other would make all other singlehoming consumers switch to the same platform. Therefore, in each period only one of two cases can occur: either (1) all new consumers that choose to singlehome join platform I, or (2) all singlehoming new consumers join platform E. In both cases, some consumers may choose to multihome, but no consumer chooses to singlehome on the other platform. We denote Case (1) as a (candidate) equilibrium in which I *wins* the market, and Case (2) as a (candidate) equilibrium in which E *wins*. Next, we consider each candidate equilibrium in turn. Also, note that we do not need to consider the case without interoperability separately in the analysis, since this case can be deduced by setting $f_R = 0$.

4.1. Singlehoming on platform I

First, consider that all singlehoming users join platform I. It is an equilibrium if, for all singlehoming users of type v, we have $u_I \geq u_E$, i.e.,

$$v((\alpha_I + \alpha_M + n_I + n_M)f_I + \alpha_E f_R) \geq v\left((\alpha_I + n_I)f_R + (\alpha_M + \alpha_E + n_M)f_E\right), \tag{4}$$

where $n_E = 0$ as all singlehoming users join I and thus $n_M + n_I = 1$, and n_M represents an expected number of multihomers. Condition (4) states that it should not be profitable for a singlehoming user to deviate unilaterally from the 'winning' platform, I, to the other platform, E.

We now proceed by determining the number of multihomers in this candidate equilibrium. When all singlehoming users join I, a consumer of type v prefers multihoming over singlehoming on I if $u_M \geq u_I$, i.e.,

$$v((\alpha_I + n_I)f_I + (\alpha_E + \alpha_M + n_M)f_E) - m \geq v((\alpha_I + \alpha_M + n_I + n_M)f_I + \alpha_E f_R). \tag{5}$$

A consumer multihomes if the incremental benefit of multihoming, stemming from richer interactions when using platform E for communicating with other multihomers or with E's installed base, exceeds the cost of multihoming.

Recall that we have defined $\delta = f_E - f_I = f_E - 1$ as the quality advantage of platform E over platform I. To simplify the exposition, we also define $\rho = f_I - f_R = 1 - f_R$ as the quality loss from interoperability, with a lower ρ meaning a higher quality of interoperability, and $\rho = 1$ no interoperability. The multihoming condition (5) can then be rewritten as:

$$v \geq \hat{v}(n_M) = \frac{m}{\delta n_M + \gamma}, \tag{6}$$

where the term δn_M represents the incremental benefit from improved interactions with unattached multihomers, and

$$\gamma \equiv (f_E - f_R)\alpha_E + (f_E - f_I)\alpha_M = \rho\alpha_E + \delta(\alpha_E + \alpha_M)$$

the incremental benefit from improved interactions with E's singlehoming or multihoming locked-in users. In a fulfilled expectations equilibrium the realized and the expected numbers of multihomers must be equal. The (fulfilled-expectations) number of multihomers is thus the solution of

$$n_M = 1 - \frac{m}{\delta n_M + \gamma}, \tag{7}$$

which we denote by $\hat{n}_M$. To simplify the exposition further and focus on the relevant intuitions, we make the additional (convenient, but not crucial) assumption.

Assumption 1. *[E's quality advantage]*

$$\frac{\delta}{\rho} > \frac{\alpha_E}{\alpha_I}$$

This assumption states that E's advantage must be sufficiently large, relative to the degree of interoperability and I's advantage in the installed based of singlehomers. It is always satisfied, for example, if $\alpha_E = 0$, or if interoperability is near perfect ($\rho \to 0$). The assumption implies that $\delta > \gamma$, which allows us to reduce the number of cases that need to be considered when solving for the multihoming equilibria. However, no additional types of equilibria arise when this assumption is relaxed and thus our results are qualitatively unchanged by this assumption.[12] Using Assumption 1 and our equilibrium selection criteria, the following lemma characterizes the equilibrium number of multihomers. All proofs are relegated to the Appendix.

[12] See the detailed discussion in the proof of Lemma 1, where we lift the assumption to solve for the multihoming equilibrium in the general case.

Lemma 1. *[Endogenous multihoming when I wins]*

In case all unattached singlehomers choose platform I, there exists a critical quality advantage for platform E, denoted by $\underline{\delta}$, such that in equilibrium:

(i) If $\delta < \underline{\delta}$, there is no multihoming ($\hat{n}_M = 0$);

(ii) If $\delta \geq \underline{\delta}$, there is a strictly positive number of multihomers ($\hat{n}_M > 0$), with

$$\hat{n}_M = \frac{1}{2} - \frac{\gamma}{2\delta} + \frac{1}{2}\sqrt{\left(1 + \frac{\gamma}{\delta}\right)^2 - \frac{4m}{\delta}}. \tag{8}$$

Intuitively, multihoming occurs in equilibrium when platform E's quality advantage is sufficiently high. Figure 2 shows the number of multihomers with and without interoperability. The figure conveys two important insights. First, platform E must offer a sufficiently high level of quality, given by $\underline{\delta}$, for multihoming to take off. At $\underline{\delta}$, the number of multihomers ($\hat{n}_M$) is discontinuous and jumps from zero to a strictly positive number. Thus, multihoming does not occur gradually, but at the critical quality advantage, the entrant is suddenly able to achieve a 'critical mass' of multihoming users. Such discontinuous adoption patterns are typical in the presence of network effects, where small changes in the market environment can suddenly induce an unraveling and tip the market into a new equilibrium. Second, Figure 2 suggests that interoperability tends to reduce multihoming.[13] The next proposition formalizes this important insight.

Proposition 1. *[Interoperability and multihoming] Assume that platform E has locked-in users ($\alpha_E > 0$). If all unattached singlehomers choose platform I, an increase in interoperability (f_R)*

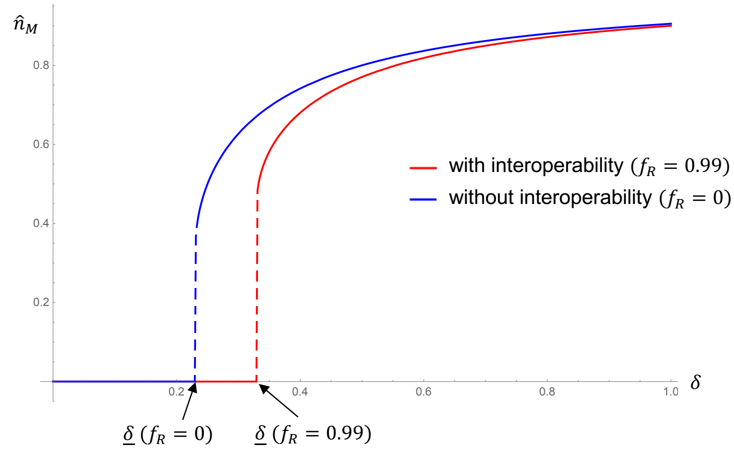
(i) increases the critical quality advantage ($\underline{\delta}$) at which multihoming occurs,

(ii) strictly reduces the number of multihomers in case multihoming occurs ($\hat{n}_M > 0$).

Proposition 1 highlights the main mechanism through which interoperability can undermine market contestability: a higher level of interoperability reduces the propensity of users to multihome, as interoperability and multihoming are two alternative means to reach platform E's locked-in singlehoming users. As interoperability becomes better (e.g., incorporates more features), the trade-off

[13] In addition, the number of multihomers increases with the entrant's quality advantage, δ, and decreases with the cost of multihoming, m. This is shown formally in Appendix A.

Figure 2 **Number of multihomers when I wins the singlehomers, with and without interoperability.**



Note. Numerical example derived for $\alpha_I = 0.9, \alpha_E = 0.05, \alpha_M = 0.05, m = 0.1$. Qualitatively similar results are obtained for other parameter constellations.

between singlehoming on I (i.e., relying on interoperability) and multihoming is shifted in favor of singlehoming, and multihoming is reduced. However, this mechanism is not at play if the market has completely tipped in favor of I in the previous period (hence, $\alpha_E = 0$), in which case interoperability does not affect multihoming.

Now that we have determined the (endogenous) number of multihomers, we can study the conditions under which the candidate equilibrium, in which I wins the market, exists. Replacing for $\hat{n}_M$ into the equilibrium condition (4), and using the fact that $n_I = 1 - \hat{n}_M$, there is an equilibrium where all singlehomers join I if

$$\hat{n}_M \leq \frac{(1 + \alpha_I - \alpha_E)\rho - (\alpha_E + \alpha_M)\delta}{\delta + \rho} \equiv \tilde{n}. \tag{9}$$

Condition (9) is central to understanding the trade-offs involved by interoperability. It highlights that whether or not the incumbent platform I wins the market depends crucially on the amount of (endogenous) multihoming. By Proposition 1, an increase in interoperability tends to decrease the number of multihomers, $\hat{n}_M$, i.e., the left hand side of this inequality. Treating the right hand side $\tilde{n}$ as constant, interoperability thus *increases* I's chances to win the market, undermining contestability. This is the anti-competitive effect of interoperability that we highlight here. By reducing multihoming, an increase in interoperability reduces the incentive to switch to the entrant platform to benefit from richer interactions with multihomers.

By contrast, the right hand side of condition (9) represents the well-known pro-competitive effect of interoperability. Taking the number of multihomers as given, an increase in interoperability (i.e., a lower ρ) provides consumers with a higher incentive to switch to the entrant platform. Indeed, taking the number of multihomers as fixed, the marginal effect of an increase in interoperability on the net gain of deviating to the entrant platform is equal to $(1+\alpha_I-\alpha_E)$, which is positive. With full interoperability ($\rho=0$), the costs of switching to E in terms of less rich interactions with I's singlehoming users vanish completely, and the right hand side of Condition (9) becomes negative. In this case, the condition can never be satisfied, and I can never win the market.

Taken together, an increase in interoperability can have an anti-competitive and a pro-competitive effect on market contestability, and *a priori*, it is ambiguous which effect dominates. However, the next Proposition shows that there is a simple rule allowing to determine the *net effect* of interoperability.

Proposition 2. *[Interoperability and contestability] There exists a unique threshold* $\hat{\delta}^{(I)}$ *such that condition* (9) *holds, hence,* I *wins the market, if and only if* $\delta \leq \hat{\delta}^{(I)}$. *An increase in interoperability makes it less likely that platform* I *wins the market in the parameter region where* $\hat{\delta}^{(I)} < \underline{\delta}$ *or* $\hat{\delta}^{(I)} > \underline{\delta}$. *Conversely, an increase in interoperability makes it more likely that* I *wins the market in the parameter region where* $\hat{\delta}^{(I)} = \underline{\delta}$.

Intuitively, Proposition 2 shows that there is a 'tipping point,' given by $\hat{\delta}^{(I)}$, such that platform I wins the market if the entrant's quality advantage is below the tipping point. Otherwise, the incumbent platform cannot win the market (and, as we will see below, the entrant then wins the market). This is because, if the entrant's quality advantage is high enough, consumers gain from switching to the entrant platform to enjoy richer interactions with E's singlehoming users and with multihomers.

The second important insight from the proposition is that the effect of an increase in interoperability on contestability (i.e., the likelihood that platform I loses the market in a single period) depends on whether the tipping point coincides with the critical quality advantage, at which multihoming occurs, or lies below or above it.

Consider first the case where the tipping point lies below the critical quality advantage (i.e., $\hat{\delta}^{(I)} < \underline{\delta}$). In this case, at the critical quality advantage, platform I always loses the market. Thus, multihoming is not necessary for contestability. Here only the pro-competitive effect of interoperability is at play, since there is no multihoming at the tipping point. Therefore, an increase in interoperability unambiguously favors contestability.

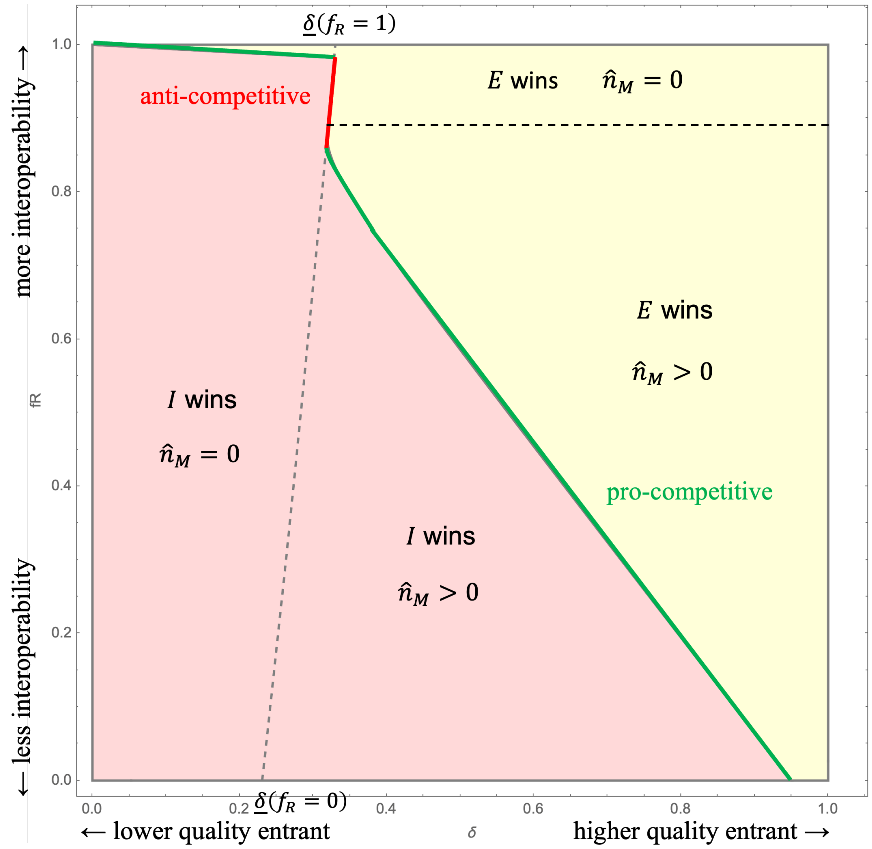
Next, consider the case where the tipping point lies above the critical quality advantage (i.e., $\hat{\delta}^{(I)} > \underline{\delta}$). In this case, at the critical quality advantage, multihoming is prevalent, but platform I still wins the market. Therefore, multihoming is necessary for contestability, but not sufficient. Here both the pro- and anti-competitive effect of interoperability operate. However, at the tipping point boundary, the magnitude of the anti-competitive effect of interoperability, which is due to the reduction of multihoming, is small, and the pro-competitive effect dominates.

Finally, consider the case where the tipping point coincides with the critical quality advantage (i.e., $\hat{\delta}^{(I)} = \underline{\delta}$). In this case, multihoming is a necessary and sufficient condition for contestability. Proposition 2 shows that in the third case ($\hat{\delta}^{(I)} = \underline{\delta}$), an increase in interoperability reduces contestability already in a single period. Here, a positive level of multihoming is a necessary and sufficient condition for I to lose the market. If platform E has locked-in singlehoming users ($\alpha_E > 0$), interoperability undermines contestability by increasing the critical quality advantage at which multihoming takes off. If platform E has no locked-in users ($\alpha_E = 0$), interoperability does not affect interoperability.

Figure 3 visualizes Proposition 2 and provides some further intuition.[14] It depicts the boundary $\hat{\delta}^{(I)}$ defined in Proposition 2. It can be seen that the boundary is comprised of three regions (identified in Proposition 2) that exhibit different properties. In the first region, it holds that

[14] In addition to the results from Proposition 1, Figure 3 assumes that E wins all singlehoming users in case $\delta > \hat{\delta}^{(I)}$. We derive this result formally in Proposition 3.

Figure 3 Effect of various degrees of interoperability on market outcomes in a single period.



Note. Four cases can emerge: (1) I wins the singlehoming users and there is no multihoming ($\hat{n}_M = 0$); (2) I wins the singlehoming users, but there is a positive level of multihoming ($\hat{n}_M > 0$); (3) E wins the singlehoming users and there is no multihoming ($\hat{n}_M = 0$); and (4) E wins the singlehoming users and there is a positive level of multihoming ($\hat{n}_M > 0$). When the entrant's quality advantage is intermediate ($\underline{\delta}(f_R = 0) < \delta < \underline{\delta}(f_R = 1)$), an increase in interoperability can lead to I winning the market instead of E.

Numerical example derived for $\alpha_I = 0.9, \alpha_E = 0.05, \alpha_M = 0.05, m = 0.1$. Qualitatively similar results are obtained for other parameter constellations.

$\hat{\delta}^{(I)} < \underline{\delta}$, which means that there is no prospect of multihoming in equilibrium. Here, only the pro-competitive effect of interoperability is present and thus, the boundary $\hat{\delta}^{(I)}$ is decreasing with more interoperability, meaning that E is more likely to win.

In the third region it holds that $\hat{\delta}^{(I)} > \underline{\delta}$, which means that the equilibrium condition (4) is satisfied at the critical quality advantage where multihoming occurs, and there exists a positive number of multihomers. Here, both the pro-competitive and the anti-competitive effect of interoperability are present, but the pro-competitive effect dominates. Thus, again $\hat{\delta}^{(I)}$ is decreasing with more interoperability. However, as we will show in Section 5, despite the fact that due to interoperability I wins more likely in *this* period, interoperability can yet be anti-competitive in the long-run, as it reduces the prevalence of multi-homing, which makes it more likely that E can win in the *next* period.

In the second (middle) region, the tipping point coincides with the critical quality advantage at which multihoming occurs, i.e., $\hat{\delta}^{(I)} = \underline{\delta}$. In this case, the anti-competitive effect of interoperability dominates, and hence the tipping point $\hat{\delta}^{(I)}$ is increasing with more interoperability. Consider what happens at this (middle region) boundary when δ is just below $\hat{\delta}^{(I)}$. The threshold at which multihoming occurs has not yet been reached and thus there is no multihoming. However, if one were to reduce the level of interoperability just a bit, then multihoming would suddenly jump to a positive level, which then unravels into an equilibrium where E wins the market. Consequently, a necessary (but not sufficient) condition for this to occur is that δ is intermediate and lies in a region where the degree of interoperability is decisive for whether or not multihoming occurs, i.e., $\underline{\delta}(f_R = 0) < \delta < \underline{\delta}(f_R = 1)$.

4.2. Singlehoming on platform E

A second possible equilibrium involves all singlehoming consumers joining the entrant platform, E. It is an equilibrium if a singlehoming user of type v has no incentive to switch unilaterally from platform E to the competing platform, I, that is, if

$$v\left((\alpha_E + \alpha_M + 1)f_E + \alpha_I f_R\right) \geq v\left((\alpha_I + \alpha_M + n_M)f_I + (\alpha_E + n_E)f_R\right), \tag{10}$$

where $n_I = 0$, so that $n_E + n_M = 1$. In this candidate equilibrium, a consumer of type v prefers multihoming over singlehoming on E if $u_M \geq u_E$, that is, if

$$v\left(\alpha_I f_I + (\alpha_E + \alpha_M + 1)f_E\right) - m \geq v\left(\alpha_I f_R + (\alpha_E + \alpha_M + 1)f_E\right). \tag{11}$$

For a user, the benefit of multihoming stems from the possibility to communicate with platform I's singlehoming locked-in users with richer features (i.e., with a quality $f_I = 1$ instead of

$f_R < 1$). In contrast to the candidate equilibrium where all singlehomers join I, the number of multihomers does not depend on the quality advantage of the entrant platform in the candidate equilibrium where all singlehomers join E. Otherwise, the number of multihomers exhibits similar properties as in Lemma 1 and Proposition 1, as detailed in the following lemma.

LEMMA 2. *[Interoperability and multihoming when E wins] In case all unattached singlehomers choose platform E, the endogenous number of multihomers is given by $\hat{n}_M = \max\{0, 1 - \frac{m}{\alpha_I(1-f_R)}\}$. Thus, an increase in interoperability (f_R) strictly reduces the number of multihomers in case multihoming occurs ($\hat{n}_M > 0$).*

Moreover, we obtain the following result.

LEMMA 3. *[Interoperability and contestability] There exists a unique threshold $\hat{\delta}^{(E)}$ such that condition* (10) *holds, hence, there exists an equilibrium where E wins the market, if and only if $\delta \leq \hat{\delta}^{(E)}$. In this equilibrium, a higher level of interoperability f_R makes it more likely that platform E wins the market ($\partial\hat{\delta}^{(E)}/\partial f_R \leq 0$).*

Hence, in this candidate equilibrium, interoperability has an unambiguously positive effect on market contestability.

4.3. Equilibrium platform adoption

We can now characterize the equilibrium of the game in a single period. First, we note that $\hat{\delta}^{(E)} \leq \hat{\delta}^{(I)}$ (see Appendix F). Thus, for $\delta < \hat{\delta}^{(E)}$, there is only one equilibrium, where all singlehoming users join I (I wins), and for $\delta > \hat{\delta}^{(I)}$ there is only one equilibrium where all singlehoming users join E (E wins). However, for $\hat{\delta}^{(E)} \leq \delta \leq \hat{\delta}^{(I)}$, there are two possible equilibria where singlehomers either all join I or E. In this case, we apply the equilibrium selection criterion of 'platform focality' (Caillaud and Jullien 2001, 2003, Halaburda and Yehezkel 2019) and consider the incumbent platform I as 'focal'. Focality implies that when there are multiple equilibria, all singlehoming users join the focal platform, i.e., in our case, platform I.[15] This yields the following proposition.

[15] In case of partial focality (Halaburda and Yehezkel 2019), the relevant threshold would be a weigthed sum of $\hat{\delta}^{(I)}$ and $\hat{\delta}^{(E)}$, the weight representing the degree of focality. It would still exhibit the qualitatively same properties as $\hat{\delta}^{(I)}$ to the extent that the degree of focality is high enough.

Proposition 3. *[Equilibrium platform adoption] Assume that there are α_I locked-in singlehoming users on platform I, α_E locked-in singlehoming users on platform E, and α_M locked-in multihoming users in the previous period. Platform I is focal and a level $f_R < 1$ of interoperability is mandated. Then, in equilibrium all unattached singlehoming users in the current period join platform I if $\delta \leq \hat{\delta}^{(I)}$; otherwise, if $\delta > \hat{\delta}^{(I)}$, all singlehoming users join platform E.*

The proposition shows that the incumbent platform wins the market in a single period if the entrant's quality advantage is below the tipping point $\hat{\delta}^{(I)}$; otherwise, the entrant platform wins the market. However, note that tipping is not always complete. The incumbent platform completely wins the market (i.e., there is no multihoming) if the entrant's quality advantage is very low (i.e., lower than the critical quality advantage $\underline{\delta}$). Conversely, for $\delta \in (\underline{\delta}, \hat{\delta}^{(I)})$, platform I wins the singlehomers but there is multihoming in equilibrium ($\hat{n}_M > 0$). Therefore, tipping is only partial, and as we show in Section 5, E can eventually still win the market. Similarly, the entrant platform completely wins the market if $\delta > \hat{\delta}^{(I)}$ and $f_R > 1 - m/\alpha_I$. If the latter condition does not hold, tipping on E is only partial (i.e., $\hat{n}_M > 0$). However, as we show in Section 5 in this case the market will eventually always tip towards E.

The impact of interoperability on contestability in any given period is therefore fully characterized by the analysis in Section 4.1, especially Proposition 2. From this proposition, we know that we can have either $\partial\hat{\delta}^{(I)}/\partial f_R \leq 0$ or $\partial\hat{\delta}^{(I)}/\partial f_R \geq 0$. Thus, a higher level of interoperability f_R may either decrease or increase the likelihood that platform I wins the market in any given period. On top of that, from Proposition 1, interoperability reduces multihoming. So, if platform I wins the market, platform E is left with less users in the next period, which increases platform I's chances to win in the next period. However, as long as there is multihoming, it is not inevitable that I wins all subsequent periods. Albeit the interaction in a single period is crucial for understanding the effects of interoperability on the dynamics of contestability, the main mechanism through which interoperability negatively affects contestability only shows in the multi-period setting, which we consider next.

5. Long-run market dynamics and tipping

Based on the consumers' adoption decisions in a single period, characterized in Section 4, we can now characterize the resulting long-run market dynamics and outcomes. We consider that platforms compete for an infinite number of periods, and that the sequence of consumers' adoption decisions is given as in Figure 1. We then look for the long-run steady state of this infinitely repeated adoption game. In particular, we will show that there exist parameter constellations in which a market with (imperfect) interoperability tips permanently in favor of the incumbent platform I, whereas it would have tipped permanently in favor of the entrant platform E without interoperability, demonstrating that interoperability can indeed undermine market contestability.

Our first insight is that if multihoming does not arise in the initial period, the market tips permanently in favor of the firm that wins the market in that period. This can be shown more generally for any given period.

Lemma 4. *[Tipping without multihoming] Assume that platform $i = I, E$ wins the market in period t and there is no multihoming (i.e., $\hat{n}_M(t) = 0$). Then, the market tips completely in favor of platform i, i.e., it wins the market in all subsequent periods $t' > t$ and multihoming does not emerge ($\hat{n}_M(t') = 0$).*

Therefore, the long-run steady state is determined by the market outcome in the initial period, $t = 0$, if multihoming does not emerge. If the entrant's quality advantage is below the critical quality advantage and the interoperability level is not too high ($\delta < \min\{\underline{\delta}, \hat{\delta}^{(I)}\}$), the market tips permanently in favor of the incumbent. Conversely, if the entrant's quality advantage is above the tipping point ($\delta > \hat{\delta}^{(I)}$) and the interoperability level is high enough ($f_R > 1 - m/\alpha_I$), the market tips permanently in favor of the entrant. (See Figure 3 where these two areas are shown for which $\hat{n}_M = 0$ in the first period.)

If a firm wins the market in a given period but multihoming emerges, the market dynamics are more involved. Indeed, in this case the firm that has lost in the current period still acquires new customers due to multihoming, which may improve its chances to win the market in the next period. However, we can show that when a firm wins the market in a given period, a sufficient condition for the market to permanently tip in its favor is that by winning the firm (weakly) increases its number of singlehomers.

LEMMA 5. *[Sufficient condition for tipping with multihoming] Assume that platform $i = I, E$ wins the market in period t and multihoming emerges (i.e., $\hat{n}_M(t) > 0$). Then, platform i wins the market in all subsequent periods if $\alpha_i(t+1) \equiv n_i(t) \geq \alpha_i(t)$.*

Using Lemma 5 in particular, we can then show that the market always converges to a long-run steady state and that it can be of three types.

PROPOSITION 4. *[Existence and uniqueness of long-run steady state] For any given starting values $\alpha_I(0), \alpha_E(0), \alpha_M(0)$, with $\alpha_I(0) > \alpha_E(0)$, the market converges to exactly one of the following steady states:*

(i) Tipping towards I: $\alpha_I(\infty) = 1$, $\alpha_E(\infty) = \alpha_M(\infty) = 0$;

(ii) Tipping towards E: $\alpha_E(\infty) = 1$, $\alpha_I(\infty) = \alpha_M(\infty) = 0$;

(iii) Multihoming: $\alpha_M(\infty) = \frac{1}{2}\left(1 + \sqrt{1 - \frac{2m}{\delta}}\right)$, $\alpha_I(\infty) = 1 - \alpha_M(\infty)$, $\alpha_E(\infty) = 0$.

Proposition 4 establishes that the market always converges to a long-run steady state in which either I or E wins the market. Interestingly, multihoming can persist in the long run, even when I attracts all the singlehomers.

While we provide important insights on the market dynamics in Lemma 4 and Lemma 5, characterizing the long-run outcome based on the starting values is complex in the general case, because the starting values can be chosen arbitrarily and thus can be "out of equilibrium" in the first period. Contrary to the sufficient condition required by Lemma 5, the share of singlehomers of a winning firm may therefore fall in the next period. However, we can fully characterize the market dynamics if we assume that the share of multihomers in the first period is not too high.

ASSUMPTION 2. *[Initial share of multihomers]*

$$\alpha_M(0) < \hat{\alpha}_M \equiv \frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{2m}{\delta}}.$$

This assumption is without loss of generality in terms of effects at play, but ensures that the number of multihomers converges to its long-term value from below. This limits the possible dynamics that have to be considered and makes the characterization of the long-run steady state with multihoming tractable. Besides, it also embodies that the focus of our analysis is on situations in which the entrant does not yet have a strong customer base (through singlehoming or multihoming) in the market.

PROPOSITION 5. *[Characterization of long-run steady state] For any given starting values* $\alpha_I(0), \alpha_E(0), \alpha_M(0)$*, with* $\alpha_E(0) < \alpha_I(0)$ *and* $\alpha_M(0) < \hat{\alpha}_M$*, the long-run steady state is as follows:*

(i) If platform I *wins period* 0 *and there is no multihoming, then the market tips towards* I*;*

(ii) If platform I *wins period* 0 *and there is multihoming, then the market converges to multihoming if* $\delta < \frac{\rho}{\sqrt{2\rho/m-1}}$*, and tips towards* E *otherwise;*

(iii) If platform E *wins period* 0*, then the market tips towards* E*.*

Based on the characterization of the long-run steady state, we can now discuss the impact of interoperability on market contestability.

COROLLARY 1. *[Interoperability and long-run market contestability] An increase in the degree of (partial) interoperability can increase or decrease market contestability (i.e., the share of users in the long-run steady state) for the more efficient entrant platform* E*.*

At first, we need to define market contestability in this context precisely. Let $\alpha_I^\infty(f_R)$ denote the number of singlehomers of the incumbent platform I that emerge in the long-run when a level $f_R < 1$ of interoperability is mandated. Clearly, by Proposition 4, $\alpha_I^\infty(f_R) \in [0,1]$, whereby a lower value of $\alpha_I^\infty(f_R)$ indicates that E was able to achieve more adoption in the market (either through singlehomers or through multihomers). Consequently, we can define $1 - \alpha_I^\infty(f_R)$ as the *degree of market contestability* at interoperability level f_R. The higher α_I^∞, the less the share of users that have adopted entrant E, and the lower market contestability.

It is evident from our analysis in Section 4 and the insights provided by Proposition 5 that a higher interoperability level can increase market contestability. For instance, starting from a situation where I wins the first period, increasing interoperability close to $f_R = 1$ leads to E winning in the first period instead (see Figure 3), and thus leads to market tipping towards E .

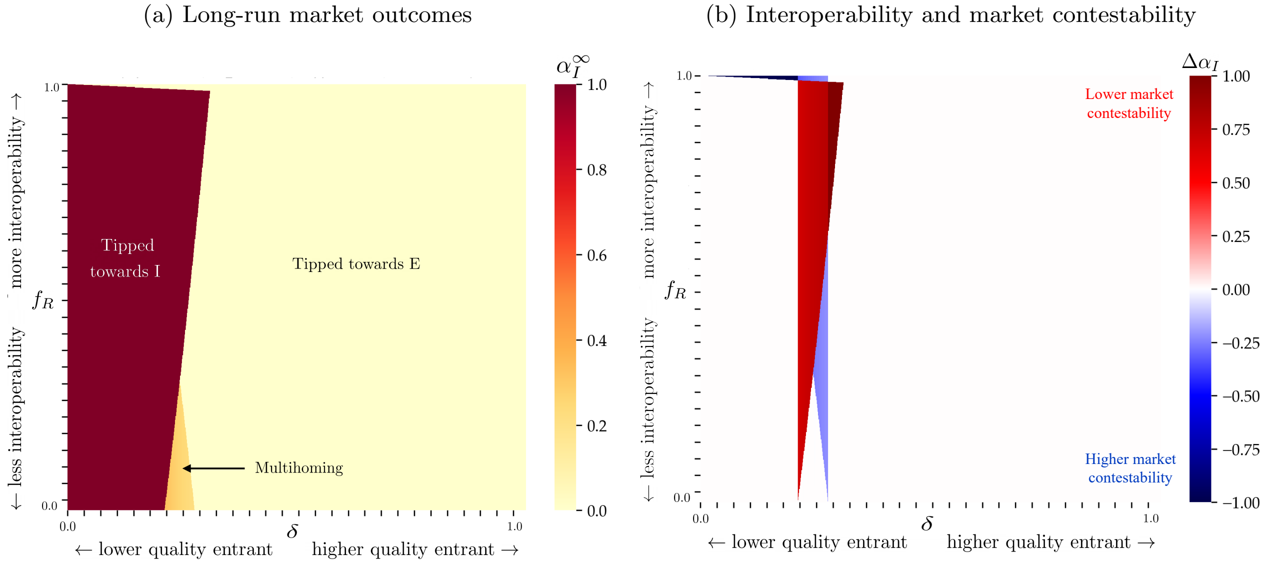
However, increasing interoperability may also reduce market contestability, through two different mechanisms. First, by Proposition 2, when the tipping point coincides with the critical quality advantage at which multihoming occurs, i.e., $\hat{\delta}^{(I)} = \underline{\delta}$, multihoming is a necessary and sufficient condition for E winning the first period. Since increasing interoperability reduces multihoming (Proposition 1), an increase in interoperability may lead from a situation where there is tipping on E to a situation where there is tipping on I.

Second, and possibly even more powerful, interoperability can negatively impact market contestability through its effect on market dynamics. Indeed, Proposition 5 shows that a customer base of multihomers may serve as a stepping stone for the entrant to grow over time and eventually win the market. Since a higher interoperability level decreases multihoming, it also reduces the possibility for the entrant to grow through multihoming, which harms market contestability.

For an illustration of the market dynamics and their effect on market contestability, consider Figure 4, which has been derived through numerical computation of the long-run steady state. Figure 4a visualizes the singlehoming market share of platform I, α_I^∞, in the long-run steady state. In particular, Figure 4a highlights the three market outcomes described in Propositions 4 and 5. The figure has been derived for the same starting values as Figure 3. Thus, Figure 3 offers the corresponding visualization of the first period outcomes, whereas Figure 4a shows the ensuing long-run steady states. It can clearly be seen that differences in who wins the market exist only in the area where I has won in the first period and there was multihoming. In particular, the multihoming steady state arises for intermediate values of $\underline{\delta}(f_R = 0) \leq \delta < \rho/(\sqrt{2\rho/m} - 1)$, but only if interoperability is relatively low. This is intuitive since more interoperability reduces the incentives to multihome (by Proposition 1), and hence multihoming is most likely in the absence of interoperability.

Finally, Figure 4b demonstrates that interoperability – as compared to no interoperability ($f_R = 0$) – can lead to a reduced market contestability for the more efficient entrant. Based on our

Figure 4 **Long-run market outcomes and the impact of interoperability on market contestability**



Note. Panel (a) shows the share of singlehomers of the incumbent firm, α_I, in the long-run steady state and demonstrates that three possible steady states can occur. Panel (b) shows the difference in the share of the incumbent's singlehomers, $\Delta\alpha_I$, with (partial) interoperability ($f_R > 0$) and without interoperability ($f_R = 0$). When $\Delta\alpha_I > 0$ (red area) the incumbent has a larger share of singlehomers with interoperability, signifying a reduction in market contestability for the entrant E. In the white area, interoperability has no impact on market contestability, and in the blue area, interoperability increases market contestability.

Numerical example derived for $\alpha_I = 0.9, \alpha_E = 0.05, \alpha_M = 0.05, m = 0.1$. Qualitatively similar results are obtained for other parameter constellations.

definition of contestability we can define $\Delta\alpha_I = \alpha_I^\infty(f_R) - \alpha_I^\infty(0)$ as the difference in market contestability with and without interoperability. Hence, $\Delta\alpha_I \in [-1, 1]$, where $\Delta\alpha_I < 0$ indicates that interoperability has increased market contestability, and $\Delta\alpha_I > 0$ indicates that interoperability has reduced market contestability.

Figure 4b shows the difference in market contestability, $\Delta\alpha_I$, using the same set of parameters as in Figure 4a. In the red shaded areas, interoperability reduces market contestability, whereas in the blue shaded areas, interoperability increases market contestability. This highlights again that interoperability can have pro-competitive and anti-competitive effects, and that either effect can dominate. In line with our analysis in Section 4, it can be seen that near perfect interoperability indeed increases market contestability in case the entrant's quality advantage, δ, is rather low.

However, in line with Proposition 1 and Proposition 5, it can also be seen that partial interoperability has an ambiguous effect on market contestability in the region where δ is intermediate ($\underline{\delta}(0) < \delta < \underline{\delta}(1)$). This is the region where the entrant is on the brink of achieving a positive number of multihomers. Interoperability can be especially harmful to contestability here, because it can undermine the emergence of multihoming, which could then unravel into a tipping of the market towards E. For example, notice that there exists a region in Figure 4b where $f_R > 0$ and $\Delta\alpha_I = 1$, meaning that with interoperability the market has tipped in favor of I, whereas without interoperability, the market would have tipped in favor of E. Similarly, in the region where $0 < \Delta\alpha_I < 1$ the market has tipped completely towards I with interoperability, but a multihoming steady state – in which E has a positive number of users – would have emerged without interoperability. Here, partial interoperability destabilizes this multihoming steady state, and leads instead to a tipping towards I, where no user adopts E. However, for intermediate values of δ, interoperability may also increase market contestability, i.e., $-1 < \Delta\alpha_I < 0$. In this region the level of multihoming is already quite high without interoperability, and the destabilization of the multihoming steady state through interoperability facilitates a tipping towards E. In the remaining regions, there is no difference in market outcomes with (partial) interoperability, as the quality difference of the entrant is either too low, so that tipping towards I is inevitable, or the quality difference is so high, that tipping towards E occurs regardless.

6. Conclusions and implications

Theoretical contribution: This paper makes a novel theoretical contribution by demonstrating that partial interoperability can not only have pro-competitive, but also anti-competitive effects. The standard economic wisdom is that interoperability is pro-competitive in the presence of an incumbent with a large user base, because interoperability lowers the entry barriers constituted by network effects (Crémer et al. 2000). With full interoperability, network effects are not important anymore for the consumers' platform choices, and they will choose the platform that offers better service. This is the direct (pro-competitive) effect of interoperability.

However, when interoperability is imperfect, some network effects remain, and consumers are inclined to stay with the incumbent platform. The anti-competitive effect arises because interoperability undermines consumers' incentives to multihome. Without interoperability, consumers who value interaction quality highly are inclined to join also the entrant platform, so that they can communicate with the full set of features on both platforms. Partial interoperability provides an imperfect substitute for multihoming; hence, some users that would have multihomed without interoperability, now stay as singlehomers with the incumbent platform. This is the indirect (anti-competitive) effect of interoperability.

We show that either the pro-competitive or the anti-competitive effect of interoperability can dominate. In particular, we show that the pro-competitive (direct) effect of interoperability dominates when interoperability is (near) perfect, or when interoperability does not significantly change the number of multihomers, e.g., because multihoming is too costly and never arises, or because the level of multihomers is already quite large. However, the anti-competitive (indirect) effect dominates when the entrant is on the brink of achieving a critical mass of multihomers. The extant literature has so far not considered partial interoperability in combination with endogenous multihoming and therefore overlooked the anti-competitive effect of interoperability.

Managerial implications: Our results also bear important implications for entrant platforms, who find themselves up against an incumbent platform with strong existing network effects. For those entrants, it may seem tempting to call for regulators to mandate interoperability, or to choose interoperability if it is already provided for through regulation, in order to quickly gain a foothold in the market. However, our results caution that innovative entrants, who offer a significantly better service than the incumbent, should rather attempt to win multihomers through their own efforts than to rely on regulated interoperability. With interoperability, further quality enhancements and innovations are not immediately experienced by those consumers that rely on interoperability and singlehome at the incumbent platform. Instead, multihoming allows the entrant to gain own customers who experience the full benefits of the better service and future innovation. In many cases, attracting a positive number of multihomers is a necessary precursor for 'winning the market' eventually. Relying on interoperability reduces the number of multihomers and can impede this competitive process. On the contrary, if multihoming is very costly or infeasible for consumers (e.g., as with multihoming operating systems, where consumers must purchase additional hardware), or if the quality advantage of the entrant is low, opting for interoperability can yet be the best strategic option for entrants.

Policy implications: Policymakers around the world are struggling to find the best regulatory framework in order to promote competition in digital markets. Interoperability obligations for dominant online platforms seem to be an obvious choice to increase market contestability. However, in digital markets, where both the speed of innovation and the potential for service differentiation are high, interoperability can only be imperfect and is in practice restricted to simple core features that are common to different platforms.

Our analysis reveals that in such an environment, interoperability can have the opposite of its intended effect and can thus harm market contestability rather than facilitate it. This is especially the case if entrants already exist in the market that have been adopted by some users, but have not yet attracted a significant user base. For example, this seems to be the case for messaging services, where next to an incumbent platform like WhatsApp, a number of smaller rivals, like Signal, exist. Small changes in the relative quality advantage of the entrant may push such entrants over the hump, leading to a tipping of the market towards the more efficient entrant. In the case of messaging apps this could, for example, occur when WhatsApp introduces less favorable privacy terms, increasing the relative quality advantage of more privacy-preserving messaging apps like Signal.[16] Interoperability could endanger this competition *for* the market, and may instead enshrine

[16] Indeed, in early 2021 WhatsApp announced that it would update its privacy policy. This highlighted that the company was sharing meta data with Facebook. Albeit this part of the privacy policy was not updated, the announcement already led to a public backlash and a significant increase in download numbers of alternative messaging apps, such as Signal (Hutchinson 2021).

incumbency. Yet, such a partial interoperability obligation, covering only basic functionalities of messaging services, has been included in the Digital Markets Act (DMA).

Our research suggests that only relatively low-quality entrants will opt for such regulated interoperability, whereas entrants that have already attracted some customer base will refrain from it. In order to facilitate competition for the market, policymakers should instead focus on policies that reduce consumers' cost of multihoming. Often such costs arise from a contractual nature (e.g., when consumers are not allowed to side-load apps) or because incumbents artificially inflate transactions costs of multihoming (e.g., by sending security warning messages when alternative apps are installed), rather than a technical necessity. Thus, these costs could potentially be reduced by regulation. Another proposal is to oblige incumbents to make their users aware that one of their contacts can also be reached through another platform, which may encourage consumers to multihome. Furthermore, interoperability regulation would require constant regulatory oversight and enforcement, as incumbents will generally object to it and may engage in non-price discrimination. Moreover, interoperability may also bear additional security and privacy risks, e.g., because it makes end-to-end encryption more challenging or even infeasible (Burgess 2022). Facilitating and encouraging multihoming does not have these caveats. However, if the costs of multihoming are inherently high or it is infeasible in practice, then interoperability may indeed be an appropriate policy option to increase contestability.

Limitations and future research: As with any analytical model, we have made a number of assumptions and simplifications in order to derive a parsimonious model. Although we believe that our insights are quite robust, the model could be challenged on any of these assumptions. Thus, we see several avenues that could be explored by future research. For example, we have assumed that the platforms are only vertically but not horizontally differentiated. Our results continue to hold qualitatively as long as network effects and vertical characteristics dominate horizontal characteristics. However, if consumers have a strong innate preference for one platform, then multihoming and market tipping becomes less likely; but this should also facilitate entry for a new platform, as network effects are less important for consumers' adoption decisions. Further, we have assumed that the platforms' quality difference is exogenously given and thus, platforms do not act strategically (e.g., by investing in quality first). Future research could investigate the strategic interaction of platforms in this context in more detail, for example, by allowing platforms to adjust prices or to innovate at a cost. Although this would not eliminate the pro-competitive and anti-competitive effect of interoperability, the platforms' strategic interaction could either increase or decrease these effects. Moreover, we have assumed a (fixed) quality difference between the incumbent and the entrant with respect to the interaction quality (e.g., more secure, more reliably or more private interactions). Technological progress could affect not only the entrant's quality advantage, (δ), but also the level of interoperability (ρ). This would alter the consumers' decision in a given period, and it would thus affect market contestability according to our analysis in Section 4, but it would not change our insights qualitatively. In reverse, our analysis is unaffected even when technological progress affects the interaction features (f_i), as long as the quality difference ($\delta = f_E - f_I > 0$) and the level of interoperability ($\rho = f_I - f_R > 0$) remain constant. Further, quality differences could also exist with respect to features that do not interact with the network size, such as the app interface. However, note that in this case, interoperability – whether present or not – would not affect consumers' adoption decisions, as it does not interact with network effects.

References

Adner R, Chen J, Zhu F (2020) Frenemies in platform markets: Heterogeneous profit foci as drivers of compatibility decisions. *Management Science* 66(6):2432–2451.

Armstrong M (1998) Network interconnection in telecommunications. *The Economic Journal* 108(448):545–564.

Armstrong M (2004) Network interconnection with asymmetric networks and heterogeneous calling patterns. *Information Economics and Policy* 16(3):375–390.

Bakos Y, Halaburda H (2020) Platform competition with multihoming on both sides: Subsidize or not? *Management Science* 66(12):5599–5607.

Belleflamme P, Peitz M (2019) Platform competition: Who benefits from multihoming? *International Journal of Industrial Organization* 64:1–26.

Biglaiser G, Crémer J, Veiga A (2021) Should I stay or should I go? Migrating away from an incumbent platform. *RAND Journal of Economics* forthcoming.

Borgogno O, Colangelo G (2019) Data sharing and interoperability: Fostering innovation and competition through apis. *Computer Law & Security Review* 35(5):105314.

Budington B (2016) Whatsapp rolls out end-to-end encryption to its over one billion users. Electronic Frontier Foundation, `https://www.eff.org/deeplinks/2016/04/whatsapp-rolls-out-end-end-encryption-its-1bn-users`.

Burgess M (2022) Forcing whatsapp and imessage to work together is doomed to fail. Wired, `https://www.wired.co.uk/article/dma-interoperability-messaging-imessage-whatsapp`.

Caillaud B, Jullien B (2001) Competing cybermediaries. *European Economic Review* 45(4-6):797–808.

Caillaud B, Jullien B (2003) Chicken & egg: Competition among intermediation service providers. *RAND Journal of Economics* 309–328.

Chao B, Schulman R (2020) Promoting platform interoperability. New America, `https://www.newamerica.org/oti/reports/promoting-platform-interoperability/`.

Chen J, Doraszelski U, Harrington JE (2009) Avoiding market dominance: product compatibility in markets with network effects. *RAND Journal of Economics* 40(3):455–485.

Crémer J, Rey P, Tirole J (2000) Connectivity in the commercial internet. *Journal of Industrial Economics* 48(4):433–472.

Cyphers B, Doctorow C (2021) Privacy without monopoly: Data protection and interoperability. Electronic Frontier Foundation, `https://www.eff.org/files/2021/06/14/privacy_without_monopoly.pdf`.

de Palma A, Leruth L, Regibeau P (1999) Partial compatibility with network externalities and double purchase. *Information Economics and Policy* 11:209–227.

Doganoglu T, Wright J (2006) Multihoming and compatibility. *International Journal of Industrial Organization* 24:45–67.

European Parliament (2022) Deal on digital markets act: Eu rules to ensure fair competition and more choice for users. Press release, https://www.europarl.europa.eu/news/en/press-room/20220315IPR25504/deal-on-digital-markets-act-ensuring-fair-competition-and-more-choice-for-users.

Farrell J, Saloner G (1986) Installed base and compatibility: Innovation, product preannouncements, and predation. *American Economic Review* 76(5):940–955.

Farrell J, Saloner G (1992a) Converters, compatibility, and the control of interfaces. *The Journal of Industrial Economics* 9–35.

Farrell J, Saloner G (1992b) Product introduction with network externalities. *Journal of Industrial Economics* 40(1):55–83.

Federal Network Agency (2021) Use of online communications services in Germany 2021 consumer survey results. https://www.bundesnetzagentur.de/SharedDocs/Downloads/DE/Sachgebiete/Digitales/OnlineKom/befragung_kurz21-ENG.pdf?__blob=publicationFile&v=2.

Fudenberg D, Tirole J (2000) Pricing a network good to deter entry. *Journal of Industrial Economics* 48(4):373–390.

Gans J (2018) Enhancing competition with data and identity portability. The Hamilton Project, Policy Proposal 2018-10, Brookings.

Grajek M (2010) Estimating network effects and compatibility: Evidence from the polish mobile market. *Information Economics and Policy* 22(2):130–143.

Graves Z (2021) The promise and perils of interoperability. Lincoln Policy, https://lincolnpolicy.org/2021/the-promise-and-perils-of-interoperability/.

Halaburda H, Jullien B, Yehezkel Y (2020) Dynamic competition with network externalities: how history matters. *Journal of Industrial Economics* 51(1):3–31.

Halaburda H, Yehezkel Y (2019) Focality advantage in platform competition. *Journal of Economics & Management Strategy* 28(1):49–59.

Hutchinson A (2021) Whatsapp is re-launching its controversial privacy policy update, will penalize users that don't accept. https://www.socialmediatoday.com/news/whatsapp-is-re-launching-its-controversial-privacy-policy-update-will-pena/599991/.

Jullien B, Sand-Zantman W (2021) The economics of platforms: A theory guide for competition policy. *Information Economics and Policy* 54:100880.

Katz ML, Shapiro C (1985) Network externalities, competition, and compatibility. *American Economic Review* 75(3):424–440.

Kerber W, Schweitzer H (2017) Interoperability in the digital economy. *Journal of Intellectual Property, Information Technology and Electronic Commerce Law* 8(1):39.

Liu CZ, Gal-Or E, Kemerer CF, Smith MD (2011) Compatibility and proprietary standards: The impact of conversion technologies in it markets with network effects. *Information Systems Research* 22(1):188–207.

Malueg DA, Schwartz M (2006) Compatibility incentives of a large network facing multiple rivals. *Journal of Industrial Economics* 54(4):527–567.

Manenti FM, Somma E (2008) One-way compatibility, two-way compatibility and entry in network industries. *International Journal of the Economics of Business* 15(3):301–322.

Nadler J, Cicilline DN (2020) Investigation of competition in digital markets. U.S. House of Representatives, Majority Staff Report and Recommendations: Subcommittee on Antitrust, Commercial and Administrative Law of the Committee on the Judiciary, https://judiciary.house.gov/uploadedfiles/competition_in_digital_markets.pdf.

Riley C (2020) Unpacking interoperability in competition. *Journal of Cyber Policy* 5(1):94–106.

Santesteban C, Longpre S (2021) Invigorating competition in social networking: An interoperability remedy that addresses data network effects and privacy concerns, https://dx.doi.org/10.2139/ssrn.3733851.

Scanlon MG, Owens B, Cicilline DN, Buck K, Nadler J (2021) Augmenting compatibility and competition by enabling service switching act of 2021. H. R. 3849, https://www.congress.gov/117/bills/hr3849/BILLS-117hr3849ih.xml.

Scott Morton FM, Crawford GS, Crémer J, Dinielli D, Fletcher A, Heidhues P, Schnitzer M, Seim K (2021) Equitable interoperability: the super tool of digital platform governance. Tobin Center for Economic Policy, Discussion Paper No. 4, `http://dx.doi.org/10.2139/ssrn.3923602`.

Stella S (2021) Public knowledge competition policy director Charlotte Slaiman to testify before house judiciary on how interoperability can rein in big tech. Public Knowledge, `https://bit.ly/3uxTeH8`.

Viecens MF (2011) Compatibility with firm dominance. *Review of Network Economics* 10(4).

Xie J, Sirbu M (1995) Price competition and compatibility in the presence of positive demand externalities. *Management Science* 41(5):909–926.

Appendix A: Proof of Lemma 1

Lemma 1 presents the number of endogenous multihomers after our equilibrium selection criteria have been applied. An extended version of the same Lemma is as follows:

LEMMA 6. *[Endogeneous Multihoming with Multiple Equilibria] There exist two thresholds, $\underline{\delta}$ and $\overline{\delta}$, with $\overline{\delta} > \underline{\delta}$, such that in equilibrium:*

(i) If $\delta < \underline{\delta}$, there is no multihoming ($\hat{n}_M = 0$);

(ii) If $\delta \in [\underline{\delta}, \overline{\delta}]$, there are 3 equilibria: an equilibrium with no multihoming ($\hat{n}_M = 0$); an unstable equilibrium with a low level of multihoming, corresponding to the lowest solution to (7); and a stable equilibrium with a high level of multihoming, corresponding to the highest solution to (7);

(iii) If $\delta > \overline{\delta}$, there is a strictly positive number of multihomers ($\hat{n}_M > 0$), given by the unique solution to (7).

Proof. Let $f(n) = n$ and $g(n) = 1 - m/(\delta n + \gamma)$. We look for the solutions to $f(n) = g(n)$. First, consider the case where $m < \gamma$, which holds if

$$\delta > \frac{m - \rho\alpha_E}{\alpha_E + \alpha_M} \equiv \overline{\delta}.$$

We have $g(0) = 1 - m/\gamma > 0 = f(0)$ and $g(1) < 1 = f(1)$. Since $g' > 0$ and $g'' < 0$, there exists a unique $\hat{n}_M \in (0, 1]$ such that $f(\hat{n}_M) = g(\hat{n}_M)$. Therefore, if $\delta > \overline{\delta}$, there is a strictly positive number of multihomers ($\hat{n}_M > 0$), given by the unique solution to (7).

Second, consider the case where $m > \gamma$ (i.e., $\delta < \overline{\delta}$) and $m < (\gamma + \delta)^2/(4\delta)$, which holds if

$$\delta > \frac{2m - \alpha_E(1 + \alpha_E + \alpha_M)\rho + 2\sqrt{m(m - \rho\alpha_E(1 + \alpha_E + \alpha_M)}}{(1 + \alpha_E + \alpha_M)^2} \equiv \underline{\delta}. \tag{12}$$

The equation $f(n) = g(n)$ can be expressed as $\delta n^2 + (\gamma - \delta)n + m - \gamma = 0$, which has two roots:

$$\hat{n}_{\text{inf}} = \frac{1}{2} - \frac{\gamma}{2\delta} - \frac{1}{2\delta}\sqrt{(\gamma + \delta)^2 - 4m\delta},$$
$$\hat{n}_{\text{sup}} = \frac{1}{2} - \frac{\gamma}{2\delta} + \frac{1}{2\delta}\sqrt{(\gamma + \delta)^2 - 4m\delta}.$$

The product of the two roots is positive as $m > \gamma$, and their sum is positive as $\gamma < \delta$ (per our assumption $\delta/\rho > \alpha_E/\alpha_I$). Therefore, the roots are both positive. Furthermore, since $m > 0$, we have $\hat{n}_{\text{sup}} \leq 1$. Therefore, there are two multihoming equilibria: one with a low level of multihoming, $\hat{n}_{\text{inf}}$, and one with a high level of multihoming, $\hat{n}_{\text{sup}}$. The former is unstable, while the latter is stable. There is also a third equilibrium with no multihoming. Indeed, if consumers anticipate no multihoming, i.e., $n_M = 0$, the marginal consumer indifferent between multihoming and singlehoming has the type $m/\gamma > 1$. Therefore, the realized number of multihomers is equal to 0 as anticipated.

Third, if $m > (\gamma+\delta)^2/(4\delta)$ (i.e., $\delta < \underline{\delta}$), the equation $\delta n^2 + (\gamma-\delta)n + m - \gamma = 0$ has no root. However, since $m > \gamma$, there is an equilibrium with no multihoming. □

In Case (ii) in Lemma 6, there are multiple equilibria. According to our selection criteria, we prefer stable over unstable equilibria, and equilibria with high rather than low adoption levels. Accordingly, we select the stable equilibrium with a high level of multihoming. Under this assumption, if $\delta \leq \underline{\delta}$, there is no multihoming ($\hat{n}_M = 0$). Otherwise, if $\delta > \underline{\delta}$, there is multihoming in equilibrium, and the number of multihomers is given by:

$$\hat{n}_M = \frac{1}{2} - \frac{\gamma}{2\delta} + \frac{1}{2}\sqrt{\left(1+\frac{\gamma}{\delta}\right)^2 - \frac{4m}{\delta}}. \tag{13}$$

Finally, if our assumption $\delta/\rho > \alpha_E/\alpha_I$ is not satisfied, we have $\gamma < \delta$, and the two roots $\hat{n}_{\inf}$ and $\hat{n}_{\sup}$ are negative. In this case, which is a subcase of Case (ii), there is no equilibrium with a positive number of multihomers, only the equilibrium with no multihoming.

Appendix B: Proof of Proposition 1

We have $\underline{\delta}$ (given in Appendix A) decreasing with ρ, and hence, increasing with f_R. Thus, a higher level of interoperability increases the chances that $\hat{n}_M = 0$. Besides, when multihoming occurs, from (8), we have $d\hat{n}_M/df_R \leq 0$, as $\partial\hat{n}_M/\partial\gamma \geq 0$ and $\partial\gamma/\partial f_R \leq 0$. Therefore, a higher level of interoperability reduces multihoming.

Besides, the threshold $\underline{\delta}$ is increasing with m. Therefore, when multihoming is more costly, there are more chances that $\hat{n}_M = 0$. Furthermore, from (8), we have $d\hat{n}_M/dm \leq 0$. Therefore, the number of multihomers decreases with m.

Finally, from (8), we have

$$\frac{d\hat{n}_M}{d\delta} = \frac{\partial\hat{n}_M}{\partial\delta} + \frac{\partial\hat{n}_M}{\partial\gamma}\frac{\partial\gamma}{\partial\delta}.$$

The first term on the right-hand side is positive, i.e., $\partial\hat{n}_M/\partial\delta \geq 0$. Indeed, if $m < \gamma$, we have $\partial\hat{n}_M/\partial\delta \geq \min\{\partial\hat{n}_M/\partial\delta|_{m=0}, \partial\hat{n}_M/\partial\delta|_{m=\gamma}\} = 0$. If $\gamma < m < (\gamma+\delta)^2/(4\delta)$, we have $\partial\hat{n}_M/\partial\delta \geq \partial\hat{n}_M/\partial\delta|_{m=\gamma} \geq 0$. The second term on the right-hand side is also positive, as $\partial\hat{n}_M/\partial\gamma \geq 0$ and $\partial\gamma/\partial\delta \geq 0$. Therefore, we have $d\hat{n}_M/d\delta \geq 0$. That is, the number of multihomers increases with E's quality advantage.

Appendix C: Proof of Proposition 2

To see the first part of the proposition, recall from the proof of Proposition 1 in Appendix B that the number of multihomers $\hat{n}_M$ is increasing with δ and going to 1 when δ becomes large. Conversely, $\tilde{n}(\delta)$ is decreasing with δ and becomes negative if δ is high enough. Therefore, there exists a unique threshold $\hat{\delta}^{(I)}$ such that (9) holds if and only if $\delta \le \hat{\delta}^{(I)}$.

For the second part of the proposition, we need to consider three possible cases: (i) $\hat{n}_M(\delta)$ and $\tilde{n}(\delta)$ cross at a point where $\hat{n}_M(\delta) > 0$, (ii) they cross at a point where $\hat{n}_M(\delta) = 0$, and finally, (iii) they do not cross and the threshold $\hat{\delta}^{(I)}$ lies at the point where $\hat{n}_M$ becomes strictly positive.

First, consider Case (i), where $\hat{n}_M(\hat{\delta}^{(I)}) > 0$. Note that Condition (4) can be rewritten as

$$\alpha_I + 1 \ge \frac{\gamma}{1 - f_R} + \frac{(1 + \delta - f_R)\hat{n}_M}{1 - f_R},$$

which holds if and only if $\delta \le \hat{\delta}^{(I)}$. Using the implicit function theorem, we find that $\partial\hat{\delta}^{(I)}/\partial f_R \le 0$. A higher level of interoperability f_R thus reduces the likelihood that I wins the market. This case occurs if $\hat{\delta}^{(I)} > \underline{\delta}$.

Second, consider Case (ii), where $\hat{n}_M(\hat{\delta}^{(I)}) = 0$. Using (9), the threshold is then equal to

$$\hat{\delta}^{(I)} = \frac{(1 + \alpha_I - \alpha_E)\rho}{\alpha_E + \alpha_M}.$$

Since $\rho = 1 - f_R$, the threshold $\hat{\delta}^{(I)}$ is decreasing in f_R. This case happens if $\hat{\delta}^{(I)} < \underline{\delta}$.

Third, and finally, there is also the possibility that $\hat{n}_M(\delta)$ and $\tilde{n}(\delta)$ do not cross, which corresponds to Case (iii). In this case, the threshold $\hat{\delta}^{(I)}$ lies precisely at the point where $\hat{n}_M$ is discontinuous and becomes strictly positive. Therefore, we have $\hat{\delta}^{(I)} = \underline{\delta}$. Since $\partial\underline{\delta}/\partial f_R > 0$, a higher level of interoperability makes I more likely to win the market.

Appendix D: Proof of Lemma 2

Using the fact that $\alpha_M + \alpha_E = 1 - \alpha_I$, inequality (11) holds if and only if $v \ge \frac{m}{\alpha_I(1-f_R)}$. Therefore, the (fulfilled-expectations) number of multihomers is $\hat{n}_M = 1 - \frac{m}{(\alpha_I(1-f_R))}$. Note that $\partial\hat{n}_M/\partial f_R \le 0$ and $\partial\hat{n}_M/\partial m \le 0$; therefore, a higher level of interoperability or a higher cost of multihoming reduces multihoming.

Appendix E: Proof of Lemma 3

Replacing for $\hat{n}_M$ into the equilibrium condition (10), and using the fact that $n_E = 1 - \hat{n}_M$, there is an equilibrium where all singlehomers join E if

$$\delta \ge \frac{(\alpha_I - \alpha_E)\rho\alpha_I - m}{\alpha_I(2 - \alpha_I)} \equiv \hat{\delta}^{(E)}. \tag{14}$$

We have $\partial\hat{\delta}^{(E)}/\partial m \le 0$ and $\partial\hat{\delta}^{(E)}/\partial f_R \le 0$ as $\alpha_I \ge \alpha_E$. Therefore, a higher cost of multihoming or a higher level of interoperability increases the chances that E wins the market constitutes an equilibrium.

Appendix F: Proof of Proposition 3

First, note that at $m = 0$, the two thresholds $\hat{\delta}^{(I)}$ and $\hat{\delta}^{(E)}$ determined in Proposition 1 and Lemma 3 are equal:

$$\hat{\delta}^{(E)}(0) = \frac{(\alpha_I - \alpha_E)\rho}{2-\alpha_I} = \hat{\delta}^{(I)}(0).$$

Since $\hat{\delta}^{(I)}(m)$ is increasing and $\hat{\delta}^{(E)}(m)$ is decreasing, the equilibrium outcome is as follows:

1. If $\delta < \hat{\delta}^{(E)}$, there is only one equilibrium where all singlehoming users join I;
2. If $\hat{\delta}^{(E)} \le \delta \le \hat{\delta}^{(I)}$, there are two equilibria where singlehomers all join either I or E;
3. If $\delta > \hat{\delta}^{(I)}$, there is only one equilibrium where all singlehoming users join E.

For intermediate values of platform E's quality advantage δ, there are multiple equilibria, where either platform I or platform E wins. Applying the equilibrium selection criterion of 'platform focality' (Caillaud and Jullien 2001, 2003, Halaburda and Yehezkel 2019) and assume that the incumbent platform I is 'focal' yields that the relevant threshold for determining the relevant candidate equilibrium is $\hat{\delta}^{(I)}$.

Appendix G: Proof of Lemma 4

I wins in period t. First, consider the case where platform I wins the market in period t and there is no multihoming, i.e., $\hat{n}_M(t) = 0$. Then, the firms' locked-in installed bases in the next period are such that $\alpha_I(t+1) = 1$ and $\alpha_E(t+1) = \alpha_M(t+1) = 0$. Besides, since I won the market, we know that Condition (9) holds in period t, that is, $\hat{n}_M(t) = 0 \le \tilde{n}(t)$. We are going to show that $\tilde{n}(t+1) \ge \tilde{n}(t)$ and $\hat{n}_M(t+1) = 0 = \hat{n}_M(t)$, which will imply that $\hat{n}_M(t+1) \le \tilde{n}(t+1)$, and thus, that I wins the market in period $t+1$ as well. We have

$$\tilde{n}(t+1) = \frac{(1+\alpha_I(t+1) - \alpha_E(t+1))\,\rho - (\alpha_E(t+1) + \alpha_M(t+1))\,\delta}{\delta+\rho}.$$

Since $1+\alpha_I(t+1) - \alpha_E(t+1) = 2 \ge 1 + \alpha_I(t) - \alpha_E(t)$ and $\alpha_E(t+1) + \alpha_M(t+1) = 0 \le \alpha_E(t) + \alpha_M(t)$, then $\tilde{n}(t+1) \ge \tilde{n}(t)$.

Besides, since there is no multihoming in period t, the entrant's quality advantage is below its critical level, i.e., $\delta < \underline{\delta}(t)$. The critical quality threshold $\underline{\delta}$ can be written as

$$\underline{\delta} = \frac{2m - \rho\alpha_E(2-\alpha_I) + 2\sqrt{m\,(m - \rho\alpha_E(2-\alpha_I))}}{(2-\alpha_I)^2}.$$

We have $\partial\underline{\delta}/\partial\alpha_E < 0$ and $\partial\underline{\delta}/\partial\alpha_I > 0$. Since $\alpha_E(t+1) = 0 \le \alpha_E(t)$ and $\alpha_I(t+1) = 1 \ge \alpha_I(t)$, then $\underline{\delta}(t+1) \ge \underline{\delta}(t) > \delta$, and thus, $\hat{n}_M(t+1) = 0$.

E wins in period t. Now, consider the case where platform E wins the market in period t and there is no multihoming, i.e., $\hat{n}_M(t)=0$. Then, the firms' locked-in installed bases in the next period are such that $\alpha_E(t+1)=1$ and $\alpha_I(t+1)=\alpha_M(t+1)=0$. Since E has won the market, Condition (9) does not hold, i.e., $\hat{n}_M^{(I)}(t)>\tilde{n}(t)$, where to avoid confusion, we denote by $\hat{n}_M^{(I)}(t)$ the number of multihomers in the candidate equilibrium where I wins, which is given by (8). We are going to show that $\tilde{n}(t+1)\leq\tilde{n}(t)$ and $\hat{n}_M^{(I)}(t+1)\geq\hat{n}_M^{(I)}(t)$, which will imply that $\hat{n}_M^{(I)}(t+1)\geq\hat{n}_M^{(I)}(t)>\tilde{n}(t)\geq\tilde{n}(t+1)$, and thus, that E wins the market in period $t+1$ as well.

Since $1+\alpha_I(t+1)-\alpha_E(t+1)=0\leq 1+\alpha_I(t)-\alpha_E(t)$ and $\alpha_E(t+1)+\alpha_M(t+1)=1\geq\alpha_E(t)+\alpha_M(t)$, then $\tilde{n}(t+1)\leq\tilde{n}(t)$.

Now, assume that $\hat{n}_M^{(I)}(t)>0$. We have $\partial\hat{n}_M^{(I)}/\partial\gamma\geq 0$ and $\gamma(t+1)=\rho\alpha_E(t+1)+\delta(\alpha_M(t+1)+\alpha_E(t+1))=\rho+\delta\geq\gamma(t)$. Therefore, $\hat{n}_M^{(I)}(t+1)\geq\hat{n}_M^{(I)}(t)>0$. Finally, if $\hat{n}_M^{(I)}(t)=0$, we have also necessarily $\hat{n}_M^{(I)}(t+1)\geq\hat{n}_M^{(I)}(t)=0$.

Finally, we have

$$\hat{n}_M(t+1)=\max\left\{0,1-\frac{m}{\alpha_I(t+1)(1-f_R)}\right\}=0,$$

since $\alpha_I(t+1)=0$.

Appendix H: Proof of Lemma 5

I wins in period t. First, consider that I wins in period t. We show that if $n_I(t)\geq\alpha_I(t)$, then I will also win period $t+1$.

Since I wins period t, Condition (9) holds, that is,

$$\hat{n}_M(t)\leq\tilde{n}(t)=\frac{(1+\alpha_I(t)-\alpha_E(t))\,\rho-(\alpha_E(t)+\alpha_M(t))\,\delta}{\delta+\rho}.$$

Since $\alpha_I(t+1)=n_I(t)\geq\alpha_I(t)$, we have $\alpha_E(t+1)+\alpha_M(t+1)=1-\alpha_I(t+1)\leq 1-\alpha_I(t)=\alpha_E(t)+\alpha_M(t)$. Besides, $1+\alpha_I(t+1)-\alpha_E(t+1)=1+\alpha_I(t+1)\geq 1+\alpha_I(t)\geq 1+\alpha_I(t)-\alpha_E(t)$. Therefore, $\tilde{n}(t+1)\geq\tilde{n}(t)$.

To prove that $\hat{n}_M(t+1)\leq\hat{n}_M(t)$, we use the fact that $\partial\hat{n}_M/\partial\gamma\geq 0$. We have $\gamma(t)=\rho\alpha_E(t)+\delta(1-\alpha_I(t))$ and $\gamma(t+1)=\rho\alpha_E(t+1)+\delta(1-\alpha_I(t+1))=\delta(1-\alpha_I(t+1))$. Since $\alpha_I(t+1)\geq\alpha_I(t)$, it follows that $\gamma(t+1)=\delta(1-\alpha_I(t+1))\leq\delta(1-\alpha_I(t))\leq\gamma(t)$. Therefore, $\hat{n}_M(t+1)\leq\hat{n}_M(t)$.

Therefore, we obtain that $\hat{n}_M(t+1)\leq\hat{n}_M(t)\leq\tilde{n}(t)\leq\tilde{n}(t+1)$, which implies that I wins period $t+1$.

Finally, we have $n_I(t+1)=1-\hat{n}_M(t+1)\geq 1-\hat{n}_M(t)=n_I(t)=\alpha_I(t+1)$. Since $n_I(t+1)\geq\alpha_I(t+1)$, then I wins also period $t+2$, etc.

E wins in period t. Now, consider that E wins in period t, with $n_E(t) \geq \alpha_E(t)$. Since E won the market in period t, Condition (9) does not hold, that is, $\hat{n}_M(t) > \tilde{n}(t)$.

We have $\alpha_E(t+1) + \alpha_M(t+1) = 1 \geq \alpha_E(t) + \alpha_M(t)$ and $1 + \alpha_I(t+1) - \alpha_E(t+1) \leq 1 + \alpha_I(t) - \alpha_E(t)$ as $\alpha_I(t+1) = 0 \leq \alpha_I(t)$ and $\alpha_E(t+1) = n_E(t) \geq \alpha_E(t)$. Therefore, $\tilde{n}(t+1) \leq \tilde{n}(t)$.

Besides, we have $\gamma(t+1) = \rho\alpha_E(t+1) + \delta(\alpha_E(t+1) + \alpha_M(t+1))$. Since $\alpha_E(t+1) \geq \alpha_E(t)$ and $\alpha_E(t+1) + \alpha_M(t+1) = 1 \geq \alpha_E(t) + \alpha_M(t)$, then $\gamma(t+1) \geq \gamma(t)$. Using the fact that $\partial\hat{n}_M/\partial\gamma \geq 0$, we conclude that $\hat{n}_M(t+1) \geq \hat{n}_M(t)$.

Since $\hat{n}_M(t+1) \geq \hat{n}_M(t) > \tilde{n}(t) \geq \tilde{n}(t+1)$, then E wins period $t+1$. Note that since $\alpha_I(t+1) = 0$, there is no multihoming in equilibrium in period $t+1$. Therefore, by Lemma 4, E wins the market permanently.

Furthermore, we have

$$\alpha_E(t+1) = 1 - \alpha_M(t+1) = \min\left\{1, \frac{m}{(1-f_R)\alpha_I(t)}\right\}.$$

Since we assumed multihoming at period t, we have necessarily $\alpha_E(t+1) = m/((1-f_R)\alpha_I(t))$. Therefore, the condition $\alpha_E(t+1) \geq \alpha_E(t)$ is equivalent to

$$\alpha_I(t)\alpha_E(t) \leq \frac{m}{1-f_R}.$$

Appendix I: Proof of Proposition 4

We proceed in two steps. First, we prove that the market always converges to a steady state. Second, we characterize the possible long run steady states.

Market convergence. To begin with, we prove that the market permanently tips after two periods if (i) E wins the first two periods, or (ii) one firm wins the first period and the other firm the second period.

First, consider the case where E wins the first two periods. Then, we have $\alpha_I(1) = 0$ since E wins period 0. This implies that in the second period, won by E as well, we have $\hat{n}_M(1) = 0$. Therefore, $n_E(1) = 1 \geq \alpha_E(1)$. Using Lemma 5, this proves that the market tips permanently in favor of E.

Second, consider the case where firm $i = I, E$ wins period 0 and firm $j \neq i$ wins period 1. Since firm j loses in period 0, we have $\alpha_j(1) = 0$. However, firm j wins in period 1, so $n_j(1) > 0 = \alpha_j(1)$. Applying Lemma 5, this proves that the market tips permanently in favor of firm j.

Now, consider that I wins repeatedly, starting from the initial period $t = 0$. Does the market converge to a steady state? (Note that if at some later period, E wins instead of I, then the previous reasoning applies and E wins permanently).

In such a case, the market dynamics are given by the share of multihomers $\alpha_M(t)$, since $\alpha_E(t)=0$ and $\alpha_I(t)=1-\alpha_M(t)$. The share of multihomers is a sequence defined by $\alpha_M(0)$ and the recurrence relation $\alpha_M(t+1)=f(\alpha_M(t))$, with

$$f(x)=\frac{1-x}{2}+\frac{1}{2}\sqrt{(1+x)^2-\frac{4m}{\delta}}.$$

The function f is defined on the interval $\Psi=[0,1]$, which is stable by f. Besides, f is strictly increasing; therefore, the sequence $\alpha_M(t)$ is monotone. We have $\alpha_M(1)>\alpha_M(0)$ if and only if

$$\alpha_M(0)<\hat{\alpha}_M=\frac{1}{2}+\frac{1}{2}\sqrt{1-\frac{2m}{\delta}}.$$

Therefore, $\alpha_M(t)$ is strictly increasing if $\alpha_M(0)<\hat{\alpha}_M$ and strictly decreasing if $\alpha_M(0)>\hat{\alpha}_M$. Finally, since Ψ is stable by f, $\alpha_M(t)$ is bounded from below and from above. Therefore, it is convergent. We find that it converges towards the fixed point

$$\alpha_M(\infty)=\hat{\alpha}_M=\frac{1}{2}+\frac{1}{2}\sqrt{1-\frac{2m}{\delta}},$$

which is defined if $\delta\geq 2m$. If $\delta<2m$, the number of multihomers becomes equal to 0 at some point and there is tipping towards I.

Finally, we check that the candidate long-run steady state with multihoming satisfies Condition (9). This is the case if

$$\delta\leq\frac{\rho}{\sqrt{2\rho/m}-1}.$$

If this condition does not hold, there is no long-run steady state with multihoming.

Characterization of long-run steady states. From the analysis above, it follows that the market always converges towards one of the three following steady states:

(i) Tipping towards I, with $\alpha_I(\infty)=1$, $\alpha_E(\infty)=\alpha_M(\infty)=0$;

(ii) Tipping towards E, with $\alpha_E(\infty)=1$, $\alpha_I(\infty)=\alpha_M(\infty)=0$;

(iii) Multihoming: $\alpha_M(\infty)=\frac{1}{2}(1+\sqrt{1-\frac{2m}{\delta}})$, $\alpha_I(\infty)=1-\alpha_M(\infty)$, $\alpha_E(\infty)=0$.

Since the repeated game is deterministic, there is always a single path for a set of starting values $\alpha_I(0)$, $\alpha_E(0)$, and $\alpha_M(0)$. Therefore, the long-run steady state is unique.

Appendix J: Proof of Proposition 5

Part (i) of the proposition follows from Lemma 4, which shows that if platform I wins the initial period and there is no multihoming, there is tipping towards I.

To prove part (ii) of the proposition, assume that platform I wins at period $t=0$ and there is multihoming. We first consider the candidate outcome where I wins repeatedly every period (as seen before, if E wins once, it wins permanently from Lemma 5). As detailed in Appendix I, the market dynamics are then given by the sequence $\alpha_M(t)$, which represents the share of locked-in multihomers. This sequence is increasing under Assumption 2, and converges towards $\hat{\alpha}_M$.

If platform I has won in t, I also wins the market in the next period $t+1$ if Condition (9) holds, that is, if

$$\hat{n}_M^{(I)}(t+1) = f(\alpha_M(t+1)) \leq \tilde{n}(t+1) = \frac{2\rho}{\delta+\rho} - \alpha_M(t+1). \tag{15}$$

This condition holds in particular for $t=0$, hence, gives the condition under which I wins period 1.

Note that $f(\alpha_M(t+1))$ is increasing in $\alpha_M(t+1)$, whereas the RHS of (15) is decreasing in $\alpha_M(t+1)$. Therefore, if condition (15) holds when $t \to \infty$, it holds for any period t. This is the case if

$$\delta \leq \frac{\rho}{\sqrt{2\rho/m}-1}. \tag{16}$$

Conversely, if $\delta > \rho/\left(\sqrt{2\rho/m}-1\right)$, there exists a period $T<\infty$ such that $f(\alpha_M(t)) \leq 2\rho/(\delta+\rho) - \alpha_M(t)$ for $t<T$ and $f(\alpha_M(t)) > 2\rho/(\delta+\rho) - \alpha_M(t)$ for $t \geq T$. Hence, E wins period T. From Lemma 5, it also wins the subsequent periods.

Summing up, if I wins the initial period and there is multihoming, there is a steady state with multihoming if (16) holds, and tipping towards E otherwise.

Finally, we prove part (iii) of the proposition. If E wins the initial period and there is no multihoming, there is tipping towards E from Lemma 4.

Now, consider that E wins the initial period and there is multihoming. However, assume that I wins the second period (if E wins the second period, E wins permanently; see Proposition 4). Then, starting from the third period, the market dynamics are given by the sequence $\alpha_M(t)$, which converges towards $\hat{\alpha}_M$. With a similar reasoning as above, there is tipping towards I with multihoming if (16) holds, and tipping towards E otherwise.

Finally, since (15) holds at $t=0$ for all $\delta \leq \hat{\delta}^I(0)$, then we have

$$\frac{\rho}{\sqrt{2\rho/m}-1} < \hat{\delta}^I(0).$$

Therefore, if E wins at $t=0$, there is tipping towards E.