Self-preferencing and Search Neutrality in Online Retail Platforms

Tianxin Zou

University of Florida

Bo Zhou

University of Maryland

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Abstract

Recent regulations on search neutrality prohibit retail platforms from self-preferentially prioritizing their first-party products over those of third-party sellers in consumers' search rankings. This paper shows that, despite its good intention, search neutrality may unintendedly harm consumers and third-party sellers due to the platform's and third-party sellers' strategic decisions. In the short term, search neutrality can weaken the price competition between the platform and third-party sellers, which will hurt consumers if most of them ex-ante prefer the third-party product, and can increase the platform's profit if most consumers ex-ante prefer the first-party product. In the long term, search neutrality can incentivize the platform to preempt the entry of third-party sellers if their entry cost is intermediate, further harming consumers and third-party sellers. Both unintended harms stem from two unique features of online retailing platforms: platforms personalize consumers' search rankings, and consumers observe product prices before searching a product. Alternative definitions of search neutrality, consumers' search costs, and their product match likelihoods are considered to demonstrate the robustness of the main results.

Keywords: platform self-preferencing, platform bias, search neutrality, platform regulation, antitrust, search ranking, market entry

1. Introduction

Major online retail platforms (e.g., Amazon, Apple's App Store) often sell first-party products and services that compete with third-party sellers' offerings on the platforms. For example, Amazon offered 22,617 private-label products across over 100 private-label brands in 2020 (DataWeave 2020). These practices raise antitrust concerns because platforms sometimes self-preferentially prioritize their first-party products, even if they are less relevant, over third-party products in consumers' personalized search rankings (Mattioli 2019). Amazon, for example, shows its private-label products in prominent positions in search rankings under the label "Featured From Our Brands," even if their organic rankings are much lower (Dudley 2020, also see Appendix Figure A1 for an example). Similarly, Apple's App Store consistently ranks Apple's apps ahead of competitors, even when consumers explicitly search for the latter (Hollister 2021; Mickle 2019; Nicas and Collins 2019). The platforms' self-preference in search rankings can significantly hurt third-party competitors' traffic (European Commission 2017), and most consumers are unaware of such practices (Konstantinovic 2021). To curb platform self-preferences, antitrust policymakers advocate for *search neutrality*—a platform's search results should be "comprehensive, impartial, and based solely on relevance." [1] The European Commission and the U.S. Federal Trade Commission are both investigating Amazon's self-preference in the search outcomes (Amaro 2020; Soper and Brody 2019), and legislations including the EU's *Digital Market Act* (DMA) and the U.S.'s *American Innovation and Choice Online Act* (AICOA) will ban platform self-preference in search results. U.S. Senator John Kennedy, who sponsored AICOA, claims that the legislation will "offer consumers more options at competitive prices."[2]

The direct impact of search neutrality is apparent: It benefits consumers and third-party sellers by restoring search relevance and fair competition. However, the indirect impact of search neutrality has drawn little attention. Search neutrality will affect price competition between the platform and

[1] http://www.searchneutrality.org/search-neutrality

[2] https://www.kennedy.senate.gov/public/2021/10/kennedy-klobuchar-grassley-introduce-american-innovation-and-choice-online-act-to-rein-in-big-tech

third-party sellers, as well as the platform's handling of third-party sellers' entry, which in turn influences consumers, platforms, and third-party sellers.

This paper builds an analytical model to examine the direct and indirect effects of search neutrality. An online platform (the first-party seller) offers a first-party product to compete with a third-party seller, who pays the platform a percentage commission. Consumers with unit demands have heterogeneous preferences. One group (type) of consumers has a higher match likelihood for the first-party product than for the third-party product, while the opposite is true for the other group of consumers. After entering the search query, a consumer sees a list of products along with their prices and basic information, from which she learns both products' prices and match likelihoods. Subsequently, the consumer needs to sequentially search the products in depth to learn a product's exact match. Consumers start by searching the top-ranked (prominent) product and then decide whether to inspect the lower-ranked (non-prominent) product by incurring a search cost. The platform can personalize its search rankings for each consumer based on her preference type.

Search neutrality affects how personalized search rankings are decided. Without search neutrality, the platform chooses each consumer's search ranking to maximize its total profit based on product prices and this consumer's type. Because the platform receives all the first-party product's revenue but only part of the third-party product's, the platform tends to self-preferentially rank the first-party product in front of the third-party one, even if the latter may be a better match to this consumer. Search neutrality forbids platform self-preference, so products are ranked based on their match likelihoods.[3] We examine both the short-term and the long-term impacts of search neutrality. In the short term, search neutrality influences the platform's and the third-party seller's pricing decisions, but the platform's commission and the third-party seller's entry decisions are yet to change. In the long term, the third-party seller decides whether to enter or exit the platform, and the platform can adjust its commission level to appropriate the third-party seller's profit and to accommodate or deter the third-party seller's entry.

[3] In Section 6, we analyze the setting with an alternative definition of search neutrality, where rankings are based on both product match likelihoods and their prices. Our main results remain robust.

This paper's main finding is that search neutrality can unintendedly harm consumers and the third-party seller in two ways. The first potential harm is *competition alleviation*: In the short term, search neutrality can raise the equilibrium prices for both products. Note that consumers observe a product's price before searching it, so a lower price of the non-prominent product can invite more consumers to search it after they have searched the prominent product. Without search neutrality, the platform will self-preferentially make the third-party product non-prominent for most consumers. Anticipating this, the third-party seller will dramatically lower its price to invite search because otherwise few consumers would visit the third-party seller due to its non-prominence, which triggers strong price competition. By contrast, search neutrality will guarantee each product's prominence for its high-match-likelihood consumers (because of personalized search rankings), and each product will be non-prominent for its low-match-likelihood consumers. Therefore, each seller is less willing to attract search by reducing its price, because the resulting profit-margin loss from high-match-likelihood consumers will outweigh the gain from more search from low-match-likelihood consumers. As a result, both sellers will set relatively high prices in equilibrium with search neutrality in place. Furthermore, we show that the competition-alleviation effect can be so strong that search neutrality will decrease consumer surplus if more consumers have higher match likelihoods for the third-party product than for the first-party product, and increase the platform's profit if the opposite is true. The latter result is counterintuitive because, without search neutrality, the platform can also make the third-party product prominent for its high-match-likelihood consumers. However, without search neutrality, the platform cannot credibly commit to doing so because after prices are set, the platform will self-preferentially deviate by making its own product prominent for these consumers. Anticipating this, the third-party seller will charge a low price that triggers strong competition. In essence, search neutrality solves the platform's commitment issue, so competition alleviation can be achieved.

The second potential harm from search neutrality is *entry deterrence:* In the long term, search neutrality may increase the platform's likelihood of increasing the commission to preempt the third-party seller's entry. Note that the third-party seller's entry has two opposing effects on the platform's profitability. On the positive side, the entry can benefit the platform by providing consumers with more options to search. On the negative side, to induce entry, the platform needs to lower its commission and leave enough revenue for the third-party seller to cover its entry cost. Search neutrality weakens the positive effect because it reduces both sellers' incentives to lower prices to attract search, so, in equilibrium, few consumers will search beyond their prominent product. We show that search neutrality increases the platform's likelihood of entry preemption when the third-party seller's entry cost is intermediate, in which case both consumers and the third-party seller become further worse off in the long term.

Our model captures two unique, important features of *online* (as opposed to *offline*) retail platforms, which are the target of major search-neutrality regulations. The first feature is *pre-search price observability*: online shoppers typically observe a product's price before clicking into its webpage to search for more detailed information. By contrast, offline shoppers often need to incur search costs to find a product before seeing its price. The second feature is *personalized search rankings*: online platforms can personalize different products' prominence for different consumers based on individual-level data. By contrast, although an offline retailer can adjust product prominence through in-store display advertising or shelf arrangement, a product's prominence uniformly applies to all visiting consumers. These two unique features are crucial for our mechanism and results. Without pre-search price observability, a seller cannot reduce its price to invite search from consumers who find its product in the non-prominent position. Without personalized search rankings, only one product will be prominent for all consumers, so search neutrality cannot alleviate competition by guaranteeing both products' prominence to their respective high-match-likelihood consumers. To our knowledge, no prior literature has jointly considered these two features, so our research provides novel insights regarding the impact of search neutrality on online retail platforms.

Our results have important policy implications for regulators and retail platforms. Regulators should be wary of the potential harms from search-neutrality regulations, such as DMA and AICOA, on alleviating competition and discouraging third-party sellers' entry. These outcomes are exactly against senator John Kennedy's wish that search neutrality will offer consumers more options at lower prices. Moreover, platforms should realize that they may be hurt by their abilities to self-preferentially manipulate search rankings. Search neutrality, by restricting such abilities, can benefit the platform.

The paper is organized as follows. Section 2 reviews the literature, and Section 3 introduces the model and discusses consumers' search process. Sections 4 and 5 examine the short-term and long-term impacts of search neutrality. Section 6 analyzes extensions and checks the robustness of the main results. Section 7 concludes the paper.

2. Literature Review

This paper contributes to the economics and marketing literature on intermediary biases. A stream of this literature examines how platforms may discriminate against different third-party sellers in search or recommendation results (Armstrong and Zhou 2011; Chen and He 2011; Hagiu and Jullien 2011; Inderst and Ottaviani 2012; Teh and Wright 2020; Zhou and Zou 2021). Closer to our context is research on a retail platform's self-preference for its products compared to those of third-party sellers (For a review, see Krämer and Schnurr 2018). For example, it can boost the search rankings of its own products (de Cornière and Taylor 2019; Song 2021; Zhu and Liu 2018), bundle them with other services or products on the platform (Bakos and Brynjolfsson 2000; Parker and Van Alstyne 2005), or completely block third-party sellers to eliminate any competition (Gu et al. 2023; Hagiu et al. 2022; Padilla et al. 2022). The platform can also create copycats of successful third-party sellers' products, which reduces these sellers' innovation and selling efforts (Etro 2021; Farrell and Katz 2000; Hagiu et al. 2022; Jiang et al. 2011; Wen and Zhu 2019). An event-ticket platform can limit consumers' capabilities of reselling their tickets on third-party platforms; interestingly, the restriction can lead to lower prices and thus benefit consumers (Zou and Jiang 2020). However, few papers have considered a platform's self-preference when both the first-party and the third-party products coexist on the platform and consumers' search rankings are personalized. de Cornière and Taylor (2019) show that a platform's recommendation bias towards its products can increase the platform's investment in quality improvement and thus benefit consumers, although the recommendation bias will hurt consumers if firms compete only on prices. Our paper differs from their framework by incorporating consumer search and shows that the platform's self-preference in search rankings can benefit consumers and harm the platform even if the platform and the sellers compete *only* on prices. Long and Amaldoss (2022) study a platform's tradeoff between making its product prominent and ceding the prominent position to third-party sponsored ads. By contrast, we consider personalized search rankings and how product prices affect consumers' willingness to search, which are not considered in their framework.

This paper also contributes to the literature on ordered consumer search, where consumers' search sequence is partially or fully predetermined (Arbatskaya 2007; Armstrong 2017; Armstrong et al. 2009; Armstrong and Zhou 2011; Wang et al. 2021; Zhou 2011; Zou and Jiang 2020). Most of these papers assume that consumers do not observe and thus need to search for product prices, which is a better depiction of offline retailing than of online retailing settings. By contrast, our paper considers the case where consumers observe product prices before searching and thus they search only for product-fit information, which better captures the online shopping environment (Armstrong 2017; Choi et al. 2018; Haan et al. 2018). This pre-search price observability feature is crucial for our key mechanism that a lower price of the non-prominent product can invite more consumer search. A closely related paper by Armstrong and Zhou (2011) also incorporates pre-search price observability so a seller can lower its price to attract search. However, this paper abstracts away personalized search rankings so consumers' search sequences depend only on product prices. By contrast, our paper focuses on how the platform's self-preference and search neutrality affect products' personalized search rankings (prominence), which in turn influence the sellers' search-invitation incentives and pricing competition.

3. Model

3.1. Firms and Consumers

A unit mass of consumers can choose from two competing products, whose marginal production costs are normalized to zero, on a retail platform. One product is sold by the platform itself (the first-party product, labeled as "F"), and the other by a third-party seller on the platform (the third-party product, labeled as "T"). We sometimes denote the platform (first-party seller) or the third-party seller simply as a seller. Consumers have a unit demand and may find a product to either match their preference or not. If product j matches consumer i's preference, this consumer's valuation is $v_{ij} = 1$. Otherwise, the product is a mismatch and her valuation is $v_{ij} = 0$. Two distinct groups of consumers differ in their preferences for the two products. A fraction $N_f = \alpha$ of consumers, labeled as type-f consumers, on average prefer product F to product T. Their match probability is ρ_H for product F and is ρ_L for product T, where $1 \geq \rho_H > \rho_L \geq 0$. The rest $N_t = 1 - \alpha$ of consumers, labeled as type-t consumers, prefer product T to product F on average. Their match probability is ρ_L for product F and is ρ_H for product T. Managerially, α captures the popularity of the first-party product relative to that of the third-party product. Let $k_i \in \{f, t\}$ denote consumer i's type, and ρ_{kj} the probability that product j matches a type-k consumer, so $\rho_{fF} = \rho_{tT} = \rho_H$ and $\rho_{fT} = \rho_{tF} = \rho_L$. We will call type-$k$ consumers the high-match-likelihood consumers to product j if $\rho_{kj} = \rho_H$, and low-match-likelihood consumers if $\rho_{kj} = \rho_L$. For example, type-f consumers are high-match-likelihood consumers for product F but low-match-likelihood consumers for product T.

Let $p_j \in [0,1]$ denote product j's price. The platform charges the third-party seller a percentage commission r based on the latter's price p_T. In Section 4, we consider the case with an exogenous r, which captures the short-term effect of search neutrality and illustrates the core mechanism of our paper. In Section 5, we examine the situation where the platform endogenously chooses its commission rate and the third-party seller endogenously decides whether to enter the platform, which illustrates the long-term effect of search neutrality.

In practice, after consumers enter their search queries, the platform returns a list of products with their prices and basic information (e.g., the product title and picture) that provides consumers with an initial impression of the product match. We define this as "pre-search" in our context. Consumers then need to click into a specific product's webpage to learn about the exact product match. We define this in-depth investigation as a "search" in our framework. We ignore any search cost in pre-search, so in pre-search, all consumers make their search inquiry and learn both products' prices and their expected valuations $E[v_{ij}]$ before searching any product. By contrast, a consumer needs to incur some search costs to search a product to learn the precise value of v_{ij}.

The platform can personalize the search rankings for each consumer based on her expected product valuations, i.e., this consumer's type $k \in \{f, t\}$. We call the product that appears in the first place in consumers' search outcomes the prominent product, and the second-place one the non-prominent product. Note that a product can be prominent for one type of consumers but non-prominent for the other type due to search ranking personalization. Consumers' search costs are lower for the prominent product than for the non-prominent product.[4] To simplify the exposition, the main analysis assumes that the search cost is zero for the prominent product and is c_i for the non-prominent product, where c_i follows a uniform distribution on $[0,1]$ across consumers.[5] Thus, consumers' optimal search sequence is to search the prominent product first, and then decide whether to search the non-prominent product. Section 6 shows that all the main insights qualitatively hold when the search cost is also positive for the prominent product. Section 3.4 will elaborate on consumers' search decisions.

3.2. Personalized Search Ranking

The platform personalizes the search ranking (i.e., which product is prominent) for each consumer based on her type and product prices. Without search neutrality, the platform chooses each consumer's search ranking to maximize the platform's total profit given the product prices. Since a consumer's purchase decision does not influence other consumers, for each consumer the platform will choose her ranking to maximize its expected profit from her. Because the platform receives only a proportion r of the third-party product's revenue as the commission, the platform tends to self-preferentially make product F prominent even for type-t consumers, for whom it is product T that tends to be a better match. Note that the platform can still choose to make product T prominent if doing so is more profitable. Hence, our model allows the third-party seller to influence the platform's search-ranking decision by strategically adjusting its price. By contrast, search neutrality prohibits platform self-preference, so the platform has to make product F prominent for

[4] This is consistent with the empirical findings that a product's search ranking affects consumers' search costs (with the top positions requiring lower search costs) but not their valuations for this product after searching it (Ursu 2018).
[5] We have also considered an alternative assumption where c_i uniformly distributes between $[0, C]$, and find the main results qualitatively the same as long as C is not too low.

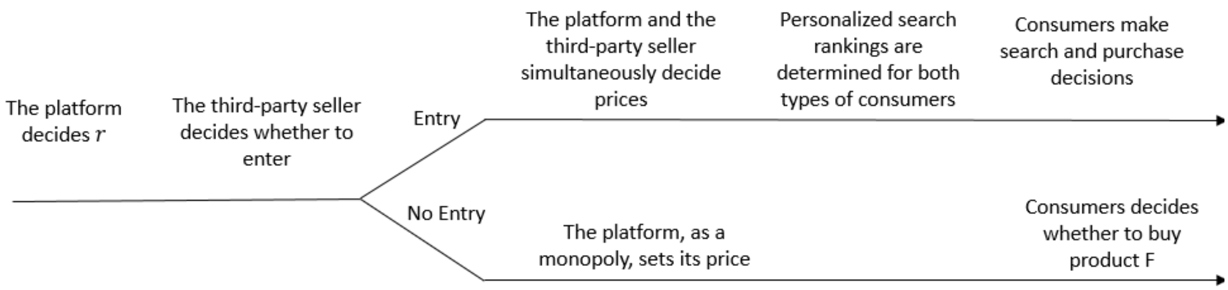
type-f consumers and product T prominent for type-t consumers. Section 6 shows that all our results are qualitatively robust under an alternative definition for search neutrality—a consumer's personalized search ranking should maximize her expected surplus given the product prices. We label the scenario with search neutrality as "S" (with search neutrality) and the scenario without as "NS" (no search neutrality).

We highlight that our model captures two important features unique to the setting of online (but not offline) retail platforms: pre-search price observability and personalized search rankings. As we will elaborate on later, these two unique features of online retailing are essential to our mechanism.

3.3.Game Sequence

The game sequence depends on whether we are focusing on the short-term or the long-term setting. In the short-term scenario (Section 4), both sellers first simultaneously set their prices, then the platform generates personalized search rankings for each consumer, and finally, consumers will make search and purchase decisions. In the long-term scenario (Section 5), first, the platform sets the commission r, and then the third-party seller decides whether to enter the platform by incurring a fixed entry cost E. If the third-party seller does not sell on the platform, then only the first-party product will be available and thus it will be prominent to all consumers. Instead, if the third-party seller joins the platform, the subsequent subgame will be the same as the short-term scenario. Figure 1 demonstrates the game sequence for the long-term scenario, and the sequence for the short-term scenario is represented by the top branch (the "Entry" subgame).

Figure 1 Game Sequence



3.4. Consumer Search Process

This section delineates consumers' search and purchase decisions. Note that these decisions are conditional on product prices and the search ranking outcome for each consumer. Consider consumer i of type k_i, and let j_1 and j_2 respectively denote her prominent and non-prominent products, where $j_1, j_2 \in \{F, T\}$ and $j_1 \neq j_2$. She knows her ex-ante valuations for the two products and their prices p_{j_1} and p_{j_2} before searching any product. She will first search the prominent product and learn her valuation v_{ij_1}, and then decide whether to search the non-prominent product j_2. If she does not search j_2, she can either buy j_1 or leave the market, so her total payoff will be $u_i^{\{j_1\}} = \max\{v_{ij_1} - p_{j_1}, 0\}$, where the superscript $\{j_i\}$ represents the product(s) she already searched. By contrast, if she searches j_2 by incurring the search cost c_i, her total payoff will be $u_i^{\{j_1,j_2\}} = \max\{v_{ij_1} - p_{j_1}, v_{ij_2} - p_{j_2}, 0\} - c_i$. As a result, she will search j_2 if and only if $E_{v_{ij_2}}\left[u_i^{\{j_1,j_2\}}\right] > u_i^{\{j_1\}}$. The consumer's search and purchase decisions depend on whether $p_{j_1} > p_{j_2}$. This is because conditional on j_1 being a match, the consumer will not search j_2 if it is more expensive ($p_{j_2} \geq p_{j_1}$), but she may do so if the latter is cheaper ($p_{j_2} < p_{j_1}$) and her search cost c_i is low. We discuss the two cases sequentially.

Case 1: $1 \geq p_{j_1} > p_{j_2}$. In this case, the prominent product is more expensive. The discussion depends on whether the prominent product j_1 is a match.

First, suppose j_1 is a mismatch ($v_{ij_1} = 0$), which happens with probability $1 - \rho_{k_i j_1}$. The consumer's expected payoff from searching the non-prominent product j_2 is $E_{v_{ij_2}}\left[u_i^{\{j_1,j_2\}}\right] =$

$E_{v_{ij_2}}[\max\{v_{ij_2} - p_{j_2}, 0\}] - c_i = \rho_{k_i j_2}(1 - p_{j_2}) - c_i$. She will search it if and only if her search cost is relatively low: $c_i < \rho_{k_i j_2}(1 - p_{j_2})$. If she chooses to search, with probability $\rho_{k_i j_2}$, she will find j_2 a match and then purchase it; with probability $1 - \rho_{k_i j_2}$, she will find it also a mismatch and will not buy either product. Alternatively, if $c_i \geq \rho_{k_i j_2}(1 - p_{j_2})$, she will not search j_2 or purchase either product. Therefore, conditional on j_1 being a mismatch, type-k consumers' average purchase probability is zero for j_1 and is $\Pr\left(c_i \leq \rho_{kj_2}(1 - p_{j_2})\right) \cdot \rho_{kj_2} = \rho_{kj_2}^2(1 - p_{j_2})$ for j_2.

Second, suppose j_1 is a match ($v_{ij_1} = 1$), which happens with probability $\rho_{k_i j_1}$. In this scenario, the consumer's utility of buying j_1 right away is $u_i^{\{j_1\}} = 1 - p_{j_1}$, and her expected payoff from searching j_2 is $E_{v_{ij_2}}\left[u_i^{\{j_1, j_2\}}\right] = E_{v_{ij_2}}[\max\{1 - p_{j_1}, v_{ij_2} - p_{j_2}\}] - c_i = \rho_{k_i j_2}(1 - p_{j_2}) + (1 - \rho_{k_i j_2})(1 - p_{j_1}) - c_i$. She will search j_2 if and only if $E_{v_{ij_2}}\left[u_i^{\{j_1, j_2\}}\right] > u_i^{\{j_1\}}$, which happens if $c_i < \rho_{k_i j_2}(p_{j_1} - p_{j_2})$. Conditional on this search, with probability $\rho_{k_i j_2}$, she will find j_2 a match and then buy it due to its lower price; with probability $1 - \rho_{k_i j_2}$, she will find it a mismatch and then return to buy j_1. By contrast, if $c_i \geq \rho_{k_i j_2}(p_{j_1} - p_{j_2})$, she will purchase j_1 without searching j_2. Therefore, conditional on j_1 being a match, type-k consumers' average purchase probability for j_1 is $\Pr\left(c_i < \rho_{kj_2}(p_{j_1} - p_{j_2})\right) \cdot (1 - \rho_{kj_2}) + \Pr\left(c_i \geq \rho_{kj_2}(p_{j_1} - p_{j_2})\right) \cdot 1 = 1 - \rho_{kj_2}^2(p_{j_1} - p_{j_2})$, and their purchase probability for j_2 is $\Pr\left(c_i < \rho_{kj_2}(p_{j_1} - p_{j_2})\right) \cdot \rho_{kj_2} = \rho_{kj_2}^2(p_{j_1} - p_{j_2})$.

Accounting for all the possibilities, when $p_{j_1} > p_{j_2}$, the prominent product's demand from type-k consumers is $N_k\left[(1 - \rho_{kj_1}) \cdot 0 + \rho_{kj_1}\left(1 - \rho_{kj_2}^2(p_{j_1} - p_{j_2})\right)\right] = N_k \rho_{kj_1}(1 - \rho_{kj_2}^2 p_{j_1} + \rho_{kj_2}^2 p_{j_2})$, and the non-prominent product's demand from type-k consumers is $N_k[(1 - \rho_{kj_1})\rho_{kj_2}^2(1 - p_{j_2}) + \rho_{kj_1}\rho_{kj_2}^2(p_{j_1} - p_{j_2})] = N_k \rho_{kj_2}^2(1 - \rho_{kj_1} + \rho_{kj_1} \cdot p_{j_1} - p_{j_2})$.

Case 2: $p_{j_1} \leq p_{j_2} \leq 1$. In this case, because j_1 is less expensive, consumer i will always buy j_1 as long as it is a match, which happens with probability $\rho_{k_i j_1}$. Next, suppose that j_1 is a mismatch, which happens with probability $1-\rho_{k_i j_1}$. In this situation, the consumer's search and purchase behaviors are the same as that when $p_{j_1} > p_{j_2}$ and $v_{ij_1} = 0$, which we have discussed in Case 1. In sum, the prominent product's demand from type-k consumers is $N_k \cdot \rho_{kj_1}$, and the non-prominent product's demand from type-k consumers is $N_k \cdot \left(1-\rho_{kj_1}\right)\rho_{kj_2}^2 \cdot \left(1-p_{j_2}\right)$. When $p_{j_1} \leq p_{j_2}$, the prominent product's demand from type-k consumers is independent of its price p_{j_1} because all consumers who find j_1 a match will buy it and those who find it a mismatch will not. Combining both cases, the prominent product's demand function for type-k consumers is

$$D_{kj_1} = \begin{cases} N_k \rho_{kj_1}\left(1-\rho_{kj_2}^2 p_{j_1} + \rho_{kj_2}^2 p_{j_2}\right), \text{ if } p_{j_1} > p_{j_2} \\ N_k \rho_{kj_1}, \text{ if } p_{j_1} \leq p_{j_2} \end{cases}, \tag{1}$$

and the non-prominent product's demand function for type-k consumers is

$$D_{kj_2} = \begin{cases} N_k \rho_{kj_2}^2\left(1-\rho_{kj_1} + \rho_{kj_1} \cdot p_{j_1} - p_{j_2}\right), \text{ if } p_{j_1} > p_{j_2} \\ N_k\left(1-\rho_{kj_1}\right)\rho_{kj_2}^2 \cdot \left(1-p_{j_2}\right), \text{ if } p_{j_1} \leq p_{j_2} \end{cases}. \tag{2}$$

Both demand functions are continuous and have a kink at $p_{j_1} = p_{j_2}$. A key observation from Equations (1) and (2) is that other things being equal, a product's demand from a type-k consumer is more elastic to its own price if the product is non-prominent than if it is prominent. Specifically, let l denote the focal product and $-l$ denote the competing product. If $p_l > p_{-l}$, then for any consumer type $k \in \{f,t\}$, product l's demand elasticity of its own price is $\frac{\partial D_{k,j_1=l}/D_{k,j_1=l}}{\partial p_l/p_l} = -p_l\left(\frac{1}{\rho_{k,-l}^2} + p_{-l} - p_l\right)^{-1}$ if it is in the prominent position, which is less negative than its counterpart when product l is in the non-prominent position, $\frac{\partial D_{k,j_2=l}/D_{k,j_2=l}}{\partial p_l/p_l} = -p_l(1-p_l)^{-1}$. Similarly, if instead $p_l < p_{-l}$, then for any consumer type $k \in \{f,t\}$, product l's demand elasticity of its own price is zero if it is prominent, which is less negative than its elasticity if it is non-prominent. Intuitively, this observation reflects that if a product is non-prominent to more consumers, its seller has a stronger incentive to set a low price to invite more consumer searches. This is because very few consumers will search the non-prominent product unless it is sufficiently cheap. By contrast, even if the prominent product is relatively more expensive, many consumers who find it a match will buy it without further searching the non-prominent product to save the search cost.

It is important to note that the non-prominent seller's ability to invite search with lower prices crucially relies on the *pre-search price observability* feature of online retailing. A low price of the non-prominent product can invite search only if consumers can directly see this low price before search. If prices were unobservable before search, consumers would not believe that the non-prominent seller would charge such a low price because of the hold-up problem as of Diamond (1971). Consequently, our result is in sharp contrast to Proposition 1 in Armstrong, Vickers, and Zhou (2009), which assumes that prices are unobservable before search and finds that a prominent seller will have a higher demand elasticity and set a lower price.

4. The Short-term Impact of Search Neutrality

In the short term, the third-party seller has already entered the platform and the commission rate r is exogenous. We analyze the market outcomes with and without search neutrality and then compare them to study the impact of search neutrality.

4.1. The Case of Low Commission Rate

To see the consequence of platform self-preference, Section 4.1 analyzes the case of a sufficiently low commission rate r. Here, we present the results of the special case with $r \to 0$. Although this setup appears to be a bit extreme, it accentuates the platform's ability and incentive to engage in self-preference because the commission revenue from selling the third-party product far trails the revenue from selling the first-party product. This greatly simplifies the expositions and helps to cleanly illustrate the impact of search neutrality that bans self-preference of the platform. Moreover, by continuity, the results in this section will qualitatively remain when r is sufficiently small. Later in Section 4.2, we analyze the case with a general commission rate r and show that the key insights are robust as long as r is not too large.

4.1.1. The Case without Search Neutrality

We start with the case without search neutrality (denoted by the superscript "NS"). With $r \to 0$, the platform's profit is $\pi_F = p_F D_F$, and the third-party seller's profit is $\pi_T = p_T D_T$, where D_j is product j's demand. Given the product prices, it is optimal for the platform to make its own product F prominent for both types of consumers in equilibrium. Based on the discussion in Section 3.4 and Equations (1) and (2), we obtain the following demand for the first-party seller and the third-party seller, respectively:

$$D_F^{NS} = \begin{cases} [\alpha\rho_H(1-\rho_L^2 p_F + \rho_L^2 p_T) + (1-\alpha)\rho_L(1-\rho_H^2 p_F + \rho_H^2 p_T)], \ if\ p_F \geq p_T \\ \alpha\rho_H + (1-\alpha)\rho_L, if\ p_F < p_T \end{cases}, \text{ and}$$

$$D_T^{NS} = \begin{cases} [\alpha\rho_L^2(1-\rho_H + \rho_H p_F - p_T) + (1-\alpha)\rho_H^2(1-\rho_L + \rho_L p_F - p_T)], \ if\ p_F \geq p_T \\ [\alpha(1-\rho_H)\rho_L^2 + (1-\alpha)(1-\rho_L)\rho_H^2](1-p_T), if\ p_F < p_T \end{cases}.$$

The demand functions are continuous but have kinks at $p_F = p_T$. In the Appendix, we prove that there exists a unique pure-strategy equilibrium despite the non-smooth profit functions and present the equilibrium outcome.[6] We highlight some equilibrium properties. Because the platform's profit π_F strictly increases with its own price p_F when $p_F < p_T$, in equilibrium it must be that $p_F^{NS*} \geq p_T^{NS*}$. It turns out that depending on product F's popularity (i.e., the size of type-f consumers α), the equilibrium either consists of an interior solution or a non-interior one. Specifically, when α is below a threshold α_1^{NS}, the equilibrium is interior: $1 > p_F^{NS*} > p_T^{NS*}$. By contrast, when $\alpha \geq \alpha_1^{NS}$, the equilibrium is non-interior: $1 = p_F^{NS*} > p_T^{NS*}$. Proposition 1 compares the two sellers' prices and discusses how they change with product F's popularity. All the proofs of Propositions are presented in the Online Appendix A.

Proposition 1. *Suppose search neutrality is absent and the commission rate is sufficiently low ($r \to 0$). (1) The first-party seller's price is higher than that of the third-party seller: $p_F^{NS*} > p_T^{NS*}$. (2) Both sellers' prices p_F^{NS*} and p_T^{NS*} strictly increase with popularity α when $\alpha \in [0, \alpha_1^{NS})$, and stay constant at $p_F^{NS*} = 1$ and $p_T^{NS*} = \frac{1}{2}$ when $\alpha \in [\alpha_1^{NS}, 1]$.*

Because product T occupies the non-prominent spot for all consumers, few consumers will search product T unless it is sufficiently cheap. Consequently, the third-party seller will set a low price p_T to invite consumer search, so its equilibrium price is lower than product F's: $p_T^{NS*} < p_F^{NS*}$. The

[6] All expressions of thresholds and equilibrium variable values, unless otherwise stated, are provided in the Appendix.

third-party seller's search-invitation incentive is stronger when more consumers are type-t (when α is low), because these consumers have a high purchase likelihood for product T after searching it. This in turn forces the first-party seller to lower its price as well. Thus, as product F's popularity α increases, both p_F^{NS*} and p_T^{NS*} increase—the price competition becomes weaker. When α reaches the threshold of α_1^{NS}, p_F^{NS*} has reached the maximal price of one and thus stays constant even if α continues to rise, and the third-party seller will charge $p_T^{NS*} = \frac{1}{2}$ as the best response. Interestingly, Corollary 1 highlights that the third-party seller can benefit from a higher popularity of the competing product F because of the weakened price competition. Meanwhile, consumers become worse off.

Corollary 1. *Without search neutrality, when product F's popularity α increases, the third-party seller's profit may increase, and consumer surplus will decrease.*

4.1.2. The Effect of Search Neutrality

We now analyze the case with search neutrality (denoted by the superscript "S"). With search neutrality, the platform still makes its product prominent for type-f consumers, but now it has to make product T prominent for type-t consumers. Again, based on Equations (1) and (2), we obtain the following demand for the first-party seller and the third-party seller, respectively:

$$D_F^S = \begin{cases} [\alpha\rho_H(1-\rho_L^2 p_F + \rho_L^2 p_T) + (1-\alpha)(1-\rho_H)\rho_L^2(1-p_F)], \ if\ p_F \geq p_T \\ \alpha\rho_H + (1-\alpha)\rho_L^2(1-\rho_H + \rho_H \cdot p_T - p_F), if\ p_F < p_T \end{cases}, \text{ and}$$

$$D_T^S = \begin{cases} [\alpha\rho_L^2(1-\rho_H + \rho_H p_F - p_T) + (1-\alpha)\rho_H], \ if\ p_F \geq p_T \\ [\alpha(1-\rho_H)\rho_L^2(1-p_T) + (1-\alpha)\rho_H(1-\rho_L^2 p_T + \rho_L^2 p_F)], if\ p_F < p_T \end{cases}.$$

Note that when the commission rate $r \to 0$, the two sellers are symmetric except for the sizes of their high-match-likelihood consumers, α and $1-\alpha$. In the Appendix, we prove that there exists a unique pure-strategy equilibrium despite the non-smooth profit functions. The equilibrium outcome depends on four thresholds of α, which satisfy $0 \leq \alpha_1^S \leq \alpha_2^S < \frac{1}{2} < \alpha_3^S \leq \alpha_4^S \leq 1$. Proposition 2 compares the two sellers' prices and discusses how they change with product F's popularity α.

Proposition 2. *Suppose search neutrality is present and the commission rate is sufficiently low* ($r \to 0$)*. (1) The first-party seller's price is higher than the third-party seller's price,* $p_F^{S*} > p_T^{S*}$*, if and only if* $\alpha \in (\alpha_3^S, 1]$*. The first-party seller's price is lower,* $p_F^{S*} < p_T^{S*}$*, if and only if* $\alpha \in [0, \alpha_2^S)$*. (2) As* α *increases,* p_F^{S*} *increases on* $\alpha \in [0, \alpha_2^S)$*, equals to one on* $\alpha \in [\alpha_2^S, \alpha_4^S)$*, and decreases on* $\alpha \in [\alpha_4^S, 1]$*;* p_T^{S*} *increases on* $\alpha \in [0, \alpha_1^S)$*, equals to one on* $\alpha \in [\alpha_1^S, \alpha_3^S)$*, and decreases on* $\alpha \in [\alpha_3^S, 1]$*.*

Figure 2 Effects of α and Search Neutrality on Equilibrium Prices[7]

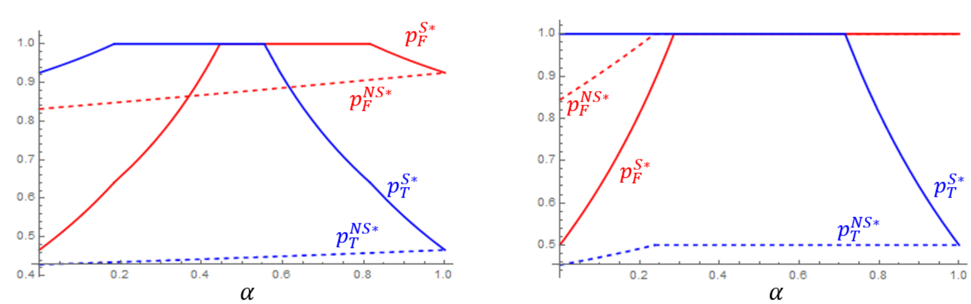


Figure 2 shows two examples of how search neutrality and product F's popularity α affect the equilibrium prices, with the solid lines representing the case with search neutrality and the dashed lines representing the case without. The first part of Proposition 2 shows that with search neutrality, product F's equilibrium price is higher ($p_F^{S*} > p_T^{S*}$) if it is sufficiently popular ($\alpha > \alpha_3^S$). Recall that search neutrality guarantees product F's prominence for type-f consumers and product T's prominence for type-t consumers. When α is large, product T is non-prominent for most consumers, so the third-party seller will set a low price p_T to invite search. Meanwhile, the platform does not need to lower its price p_F by much to invite search from type-t consumers, whose population $1 - \alpha$ is small. By contrast, and for a symmetric logic, product T's equilibrium price is higher ($p_F^{S*} < p_T^{S*}$) if it is sufficiently popular ($\alpha \leq \alpha_2^S$). This result is in stark contrast with the case without search neutrality, where product F is prominent for all consumers and thus

[7] In the left panel, $\rho_H = 0.9$ and $\rho_L = 0.85$. In the right panel, $\rho_H = 0.9$ and $\rho_L = 0.6$. Solid lines represent the case with search neutrality and dashed lines represent the case without.

its equilibrium price is always higher, $p_F^{NS*} > p_T^{NS*}$ (See Proposition 1). Essentially, this result shows that with search neutrality, sellers' pricing power only comes from their product popularities. The second part of Proposition 2 shows that the equilibrium prices of the two products are the highest when their popularities are relatively close (α is medium). With search neutrality, a seller has two different pricing strategies. On the one hand, it can set a high price to extract the surplus of its high-match-likelihood consumers, who find its product in the prominent position. On the other hand, it can set a low price to invite search from its low-match-likelihood consumers, who find its product in the non-prominent position. Because the low-match-likelihood consumers have relatively low purchase probabilities even after searching the product, the second strategy is often less appealing unless most consumers have low match likelihoods. Consequently, when α is medium, both sellers will focus on setting high prices to extract the surplus of their respective high-match-likelihood consumers, leading to weak price competition. By contrast, when α is high, most consumers are type-f, so the third-party seller needs to dramatically lower its price to invite their search, which in turn triggers strong price competition. The situation of a low α is symmetric. Importantly, this result relies on the *personalized search ranking* feature of online retailing. If instead all consumers would find the same product in the prominent position, then the non-prominent seller would have to lower its price to invite search. Finally, the result that equilibrium prices equal to one when α is intermediate should be interpreted as search neutrality significantly mitigates competition when the two products are comparable in popularity, rather than sellers must charge the maximal possible price.

Given the inverted-U shaped relationship between both sellers' prices and α, Corollary 2 shows that a seller's profit could decrease when its own product becomes more popular relative to the competitor's product.

Corollary 2. *With search neutrality, the third-party seller's profit can increase with α when α is sufficiently low, and the platform's profit can decrease with α when α is sufficiently high.*

After completely characterizing the equilibrium outcomes with and without search neutrality, we can now address the central question in this paper: What is the impact of search neutrality on seller competition and consumers? Proposition 3 summarizes the comparison between the two cases.

Proposition 3. *Suppose the commission rate is sufficiently low ($r \to 0$). (1) Search neutrality will strictly reduce consumer surplus when the two products' popularity difference is low, a sufficient condition of which is $\alpha \in [\frac{1}{4}, \frac{3}{4}]$. (2) Search neutrality is more likely to reduce consumer surplus when product T is predominantly popular ($\alpha < \frac{1}{4}$) than when product F is so ($\alpha > \frac{3}{4}$). Technically, if search neutrality reduces consumer surplus when $\alpha = \alpha_0 > \frac{3}{4}$, then it will also do so when $\alpha = 1 - \alpha_0 < \frac{1}{4}$. (3) Search neutrality will increase the third-party seller's profit, and may increase seller F's profit when α is sufficiently large.*

Figure 3 Effects of α and Search Neutrality on Equilibrium Payoffs[8]

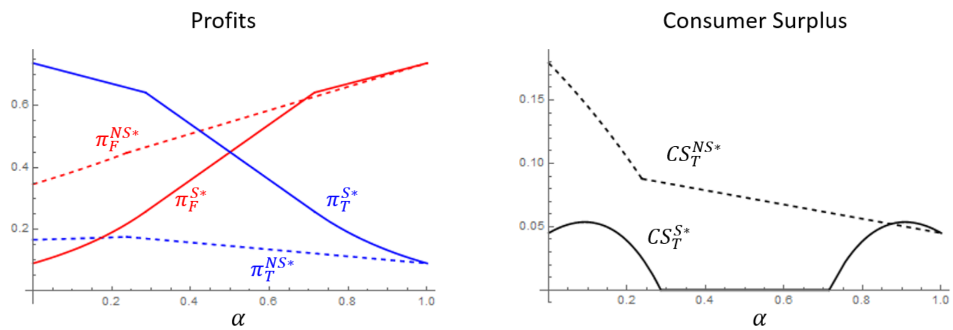


Figure 3 shows an example of how search neutrality and α affect the equilibrium profits (the left panel) and consumer surplus (the right panel). Search-neutrality regulations aim to benefit consumers by raising their search outcome relevance and facilitating competition between the platform and the third-party sellers. In addition, because search neutrality ensures that product T's prominence to type-t consumers, one may intuit that when there are more type-t consumers (i.e., when α is low), search neutrality is more likely to improve consumer surplus. Interestingly,

[8] $\rho_H = 0.9$ and $\rho_L = 0.6$. Solid lines represent the case with search neutrality and dashed lines represent the case without.

Proposition 3 shows that (1) search neutrality is most likely to reduce consumer surplus when the popularity of the two products does not differ by much, i.e., when α is moderate, and (2) search neutrality is more likely to benefit consumers when α is high rather than when it is low. The intuition hinges on how search neutrality and product popularities jointly affect the competition intensity between the two sellers. Without search neutrality, the price competition is stronger when α is smaller. By contrast, with search neutrality, the competition is weak when α is moderate (See Propositions 1, 2, and Figure 2.). Therefore, the likelihood for search neutrality to hurt consumers is the highest when α is medium, is moderate when α is low, and is lowest when α is high.

Additionally, Proposition 3 shows that although search neutrality curbs the platform's self-preference ability, it may increase the platform's profit when α is high (See the left panel of Figure 3 when $\alpha > 0.68$.). This is because search neutrality potentially hurts the platform by making product F non-prominent to type-t consumers, but this downside is limited when the size of these consumers is small (α is high). This finding perhaps sounds surprising because, without search neutrality, the platform can always replicate the search rankings from those under search neutrality by making product T prominent for type-t consumers. However, without search neutrality, the platform cannot credibly commit to doing so—after prices are set, the platform will self-preferentially deviate by making its own product prominent for these consumers. Anticipating this, the third-party seller will charge a low price, triggering strong competition. Search neutrality solves the platform's commitment issue, so competition alleviation can be achieved. Finally, in the short run, search neutrality will benefit the third-party seller because of the softened competition and the guaranteed prominence of product T to type-t consumers. In Section 5, we will show that search neutrality may also hurt the third-party seller in the long term, when the platform may block the third-party entry. In summary, search neutrality can make the platform, the third-party seller, and consumers all better off when product F is very popular (α is high), but otherwise may make the platform and consumers worse off.

Our results have important implications for policymakers and platforms. First, policymakers should be aware of the potential unintended consequences of search neutrality. In particular, given the commonality of pre-search price observability and personalized search rankings on online platforms, search neutrality could severely alleviate the competition between first-party and third-party sellers and harm consumers, especially when the popularities of the first-party and third-party products are similar. By contrast, search neutrality can benefit *all* market players in the short term if the first-party product is significantly more popular. Second, platforms should realize that they may be hurt by their abilities to self-preferentially manipulate search rankings. The platform could benefit from committing not to self-preference, for example, by hiring third-party audits or publicizing the ranking algorithms. On the other hand, such measures can be costly, hard to implement or communicate, or lack of credibility to sellers and consumers. Search neutrality can serve the platform as a less costly and more credible commitment to alleviate competition. Finally, instead of search neutrality, policymakers can design alternative policies to improve consumer welfare. One possibility is to make less relevant products prominent for all consumers, which will strengthen the sellers' incentives to invite search by lowering their prices. Our model indicates that such a policy may lower equilibrium prices and increase consumer surplus.[9]

4.2. The Effect of Commission Rate r

In this section, we relax the assumption that the commission r is sufficiently close to zero and analyze how r will affect the market outcome. To obtain pure-strategy equilibria and focus on the effect of r, we consider the situation where the two products have equal popularities, i.e., $\alpha = \frac{1}{2}$.[10]

4.2.1. The Case Without Search Neutrality

Without search neutrality, the platform chooses personalized search rankings to maximize its expected profit given the product prices, and sellers set prices anticipating the platform's search ranking decisions.

[9] We thank an anonymous reviewer for suggesting this possibility.
[10] When $r > 0$, pure strategy equilibria may not exist when α is close to 0 or 1.

Proposition 4 summarizes the equilibrium outcomes.

Proposition 4. *Suppose $\alpha = \frac{1}{2}$ and search neutrality is absent.*

(1) *When the commission $r \geq \frac{\rho_L}{\rho_H}$, the platform makes product F prominent for type-f consumers and product T prominent for type-t consumers. The equilibrium prices are $p_F^{NS*} = p_T^{NS*} = 1$.*

(2) When the commission $r < \frac{\rho_L}{\rho_H}$, the platform makes product F prominent for all consumers. The platform's equilibrium price is higher than the third-party seller's: $p_F^{NS*} > p_T^{NS*}$. Both p_F^{NS*} and p_T^{NS*} increase with r and decrease with ρ_H and ρ_L when $r \leq 3 - \frac{2}{\rho_H \rho_L}$. They stay constant at $p_F^{NS*} = 1$ and $p_T^{NS*} = \frac{1}{2}$ when $3 - \frac{2}{\rho_H \rho_L} < r < \frac{\rho_L}{\rho_H}$.

Without search neutrality, the equilibrium outcomes crucially depend on the commission level r. When r is high, the platform shares a large portion of product T's revenue, so its self-preference incentive is weak. Specifically, when $r \geq \rho_L/\rho_H$, the platform will make product T prominent for type-t consumers in equilibrium even without search neutrality, and search neutrality does not have any impact on the equilibrium outcome (Later, Proposition 5 formally proves this.). The more interesting, policy-relevant situation is $r < \rho_L/\rho_H$, when search neutrality is impactful. Without search neutrality, the platform has strong self-preference incentives such that in equilibrium it will make product F prominent for all consumers. In Section 5, we verify that when the platform endogenously decides r, it will indeed set $r < \rho_L/\rho_H$ as long as the third-party seller's entry cost E is neither too low nor prohibitively high. The equilibrium outcomes with $r < \rho_L/\rho_H$ are qualitatively similar to Proposition 1, where r is sufficiently low. Without search neutrality, the third-party seller has a strong incentive to invite search, so product T's equilibrium price is lower than product F's: $p_F^{NS*} > p_T^{NS*}$. The price competition is stronger when r is lower, or when ρ_H or ρ_L is higher. First, when r is low, the platform shares little revenue of product T. Second, when ρ_H or ρ_L is high, consumers have high purchase probabilities for product T if they search it, so the third-party seller tends to lower p_T to invite search, triggering strong competition. Specifically, and in the same spirit of Proposition 1, when $r \leq 3 - \frac{2}{\rho_H \rho_L}$, p_F^{NS*} and p_T^{NS*} increase with r and decrease with ρ_H and ρ_L. When r continues to increase such that $r > 3 - \frac{2}{\rho_H \rho_L}$, p_F^{NS*} reaches the maximal price of one, and the third-party seller will charge $p_T^{NS*} = \frac{1}{2}$ in response.

4.2.2. The Effect of Search Neutrality

With search neutrality, product F will be made prominent for type-f consumers and product T prominent for type-t consumers. Lemma 1 summarizes the equilibrium with search neutrality.

Lemma 1. *Suppose $\alpha = \frac{1}{2}$. When search neutrality is present, $p_F^{S*} = p_T^{S*} = 1$. The platform's profit is $\pi_F^{S*} = \frac{\rho_H(1+r)}{2}$ and the third-party seller's profit is $\pi_T^{S*} = \frac{\rho_H(1-r)}{2}$.*

The result highly resembles Proposition 2. When the two products have the same popularity ($\alpha = \frac{1}{2}$), search neutrality significantly alleviates the competition between the sellers, as they both set high prices to extract the surplus of their respective high-match-likelihood consumers. In this case, a higher commission r appropriates more product T's revenue to the platform, increasing the platform's profit and reducing the third-party seller's profit.

Finally, Proposition 5 examines the impact of search neutrality.

Proposition 5. *Suppose $\alpha = \frac{1}{2}$. When $r \geq \rho_L/\rho_H$, search neutrality does not affect the equilibrium outcome. When $r < \rho_L/\rho_H$, search neutrality will reduce consumer surplus and improve the third-party seller's profit. It strictly improves the platform's profit when ρ_H and ρ_L are sufficiently large.*

Following the discussion of Proposition 4, when $r \geq \rho_L/\rho_H$, search neutrality does not have any effect. By contrast, in the policy-relevant situation of $r < \rho_L/\rho_H$, when the two products are equally popular, search neutrality can significantly weaken price competition, hurting consumers and benefiting third-party sellers. The mechanism is similar to Proposition 3, where r is sufficiently low. It will also benefit the platform when ρ_H or ρ_L are large, because under these conditions the price competition is strong without search neutrality.

In summary, when the commission rate is high, search neutrality is moot. By contrast, when it is low, the platform will self-preferentially prioritize its own products in consumers' search rankings. In this case, search neutrality can alleviate competition, potentially harming consumers and benefitting the platform. The mechanisms are the same as those in Section 4.1, where the commission rate is sufficiently low. Hence, the level of commission rate, as long as it is not so high that the self-preference problem becomes irrelevant, does not affect our central insights.

5. The Long-term Impact of Search Neutrality

In the long term, the platform can change its commission r, and the third-party seller can decide whether to join the platform by incurring an entry cost E. The platform needs to consider several factors when deciding r. It can either (1) set a non-prohibitive r to induce the third-party seller's entry and squeeze its profit (down to just covering E), or (2) set a prohibitive r (e.g., $r \to 1$) to preempt the third-party seller's entry. Furthermore, in the former case, a lower r will strengthen the price competition because (1) it weakens the revenue sharing between the platform and the third-party seller, and (2) it strengthens the platform's self-preference incentive and thus the third-party seller tends to lower its price to invite search when search neutrality is absent. We will examine how search neutrality affects the platform's commission choice, the third-party seller's subsequent entry decision, as well as the welfare of all market players. For illustration, we present the case with $\alpha = \frac{1}{2}$. Pure-strategy subgame-perfect equilibria may not exist when $\alpha \neq \frac{1}{2}$. Proposition 6 summarizes the equilibrium outcomes with and without search neutrality. Note that if the third-party seller does not enter the platform, the platform will be a monopoly and optimally set $p_F = 1$, leading to zero consumer surplus.

Proposition 6. *There exist thresholds E_1 and E_2 such that*

(a) When $E < E_1$, regardless of the presence of search neutrality, the platform will set $r^* = 1 - \frac{2E}{\rho_H}$, and the third-party seller will enter the platform. The platform makes product F prominent

for type-f consumers and product T prominent for type-t consumers, and the equilibrium prices are $p_F^* = p_T^* = 1$.

(b) When $E_1 \leq E < E_2$, without search neutrality, the platform will set $r^* = 1 - \frac{8E}{\rho_H^2+\rho_L^2} < \frac{\rho_L}{\rho_H}$, and the third-party seller will enter the platform. The platform makes product F prominent for all consumers, and the equilibrium prices are the same as those in Proposition 4. Consumer surplus is $\frac{\rho_H^2+\rho_L^2}{16}$. By contrast, with search neutrality, the platform will set r sufficiently high such that the third-party seller does not enter the platform.

(c) When $E \geq E_2$, regardless of the presence of search neutrality, the platform will set r sufficiently high such that the third-party seller will not enter the platform.

The third-party seller's entry can benefit the platform in two aspects. The first benefit is increasing type-t consumers' purchase probabilities because of their higher match likelihood for product T. The second benefit is providing consumers "more search options"—consumers can still search the non-prominent product if they find the prominent one a mismatch. By contrast, inducing entry requires the platform to reduce r so the third-party seller's revenue can overcome the entry cost E—the higher E is, the lower r must be. Two relatively extreme cases are when E is either very low or very high. When E is very low ($E < E_1$), the platform can $r > \rho_L/\rho_H$ while still guaranteeing the third-party seller's entry. Proposition 5 has shown that search neutrality will have no effect in this case. By contrast, when E is very high ($E \geq E_2$), inducing the third-party seller's entry would be too costly for the platform, so it will deter entry regardless of search neutrality.

The more interesting case is when E is intermediate ($E_1 \leq E < E_2$), in which case the platform will induce the third-party seller's entry without search neutrality but will preempt its entry with search neutrality.[11] This is because search neutrality will weaken the entry's more-search-options benefit to the platform. As is discussed in Section 4, if the third-party seller enters the platform,

[11] In the region of $E_1 \leq E < E_2$, the optimal $r^* = 1 - \frac{8E}{\rho_H^2+\rho_L^2} < \frac{\rho_L}{\rho_H}$, which is consistent with the policy-relevant parameter range discussed in Propositions 4 and 5.

with search neutrality a seller has a weak incentive to lower the price to invite search from the low-match-likelihood consumers. These consumers find the seller's product in the non-prominent position, so few of them will search their non-prominent product. Hence, even though the third-party seller's entry provides consumers with more products to search, in equilibrium most consumers still search only one product. By contrast, without search neutrality, the third-party seller will reduce its price to attract search, so in equilibrium many consumers will indeed search the non-prominent product, and the more-search-options benefit is stronger.

Proposition 6 illustrates another potential harm of search neutrality—in addition to weakening the short-term price competition, in the long term, it may further hurt consumers by encouraging the platform's entry preemption, in which case the price competition becomes even weaker. In our model, search neutrality will not affect the third-party seller's profit because even if it enters the platform, the platform can set its commission to perfectly squeeze the third-party seller's revenue down to its entry cost. However, in a more general setting where the third-party seller's entry cost is non-deterministic and is its private information, the third-party seller may be able to earn a positive profit upon joining the platform, and search neutrality will strictly harm the third-party seller and consumers at the same time in the long term.[12]

Note that our model assumes a single third-party seller to concisely illustrate the key insights. This assumption leads to some seemingly extreme results: Third-party entry is either completely preempted or completely admitted, and when it is preempted, the short-term effect of search neutrality—competition alleviation—becomes obsolete. These extremities can be resolved in a more general model with multiple heterogeneous third-party sellers that potentially operate in different markets. In such a setting, the long-term effect of search neutrality will be that the platform may raise its commission to block more third-party sellers on the margin (rather than

[12] To see this, consider an example where $E \in (E_1, E_2)$ but with probability ϕ the third-party seller's entry cost equals E', which is slightly lower than E but still higher than E_1. If ϕ is sufficiently small, the platform will not change its commission decision for this low probability event. As a result, all the results in Proposition 6 will be the same except that without search neutrality, the third-party seller's expected profit will be $\phi(E - E') > 0$, which is higher than its zero profit with search neutrality.

blocking them all). At the same time, those who still enter the platform may compete less aggressively with the platform, so the short-term effect of search neutrality remains present.

6. Extensions

To obtain closed-form solutions and simplify expositions, the main model has made several assumptions: (1) With search neutrality, products are ranked based on their relevance (match likelihoods), independent of their prices; (2) The search cost for the prominent product is zero, so consumers will optimally search the prominent product first regardless of product prices; (3) There are two discrete consumer types in terms of product match likelihoods. In this section, we relax these assumptions and consider the following setup: (1) With search neutrality, products are ranked to maximize the expected surplus of each consumer, so lower-priced products tend to be favored. (2) The search cost for the prominent product is positive but lower than that for the non-prominent product, so consumers may optimally start from searching the non-prominent product or not search any product. (3) Consumers types are continuous in their product match likelihoods. Our analysis will show that our main insights are qualitatively unchanged in this alternative setting.

6.1. Model Setup

Consumer i's ex-post valuation (after a search) for product j ($j \in \{F, T\}$) is $v_{ij} = 1$ if it is a match, and is $v_{ij} = 0$ if it is a mismatch. Consumer types about product match likelihoods, x_i, are uniformly distributed on the interval $[0, 1]$. For the consumer with $x_i \in [0, 1]$, the match probability for product F is $1 - x_i$, and the match probability for product T is x_i. Valuations v_{iF} and v_{iT} can both turn out to be 1, which occurs with probability $x_i(1 - x_i)$. We can think of 0 to be product F 's location and 1 to be product T's location. The smaller (larger) x_i is, the more likely product F(T) will be a match for this consumer. The platform personalizes consumer i's search ranking based on her x_i. A consumer needs to incur a cost c_L to search the prominent product, and a cost c_H to search the non-prominent product, where $c_H > c_L > 0$. Consumers know their own x_i and the product prices, and they sequentially search the products. Search neutrality requires that given the product prices, personalized search rankings maximize each consumer's total expected utility on the platform, $EU(x_i)$, whose expressions will be given later. Other settings remain the same as the main model. In what follows, we start with the discussion on consumers' search process, and then discuss personalized search rankings with and without search neutrality. Finally, we complete the model setup with the tie-breaking rules and the game sequence.

Consumer search. Consumers' optimal search sequence and stopping rule follow the Weitzman (1979) reservation-value rule. Let $s_j(x_i, c)$ be consumer i's reservation value for product j if the search cost is c: $s_F(x_i, c) = 1 - p_F - \frac{c}{1-x_i}$ and $s_T(x_i, c) = 1 - p_T - \frac{c}{x_i}$. (See Weitzman (1979), p.648 for details.) Moreover, let $S_j(x_i)$ be consumer i's realized reservation value for product j given the search ranking: $S_j(x_i) = s_j(x_i, c_L)$ if product j is prominent, and $S_j(x_i) = s_j(x_i, c_H)$ if it is non-prominent. A consumer will optimally sequentially search products following the descending order of S_j, and will stop searching when all the unsearched S_j's are below her best searched option.

Search rankings with search neutrality. If product F is prominent, the reservation values of product F and product T are $s_F(x_i, c_L)$ and $s_T(x_i, c_H)$, respectively. Consumer i's expected surplus is

$$EU_F(x_i) = \max\{0, \underbrace{\mathbf{1}\big(s_F(x_i, c_L) \geq s_T(x_i, c_H)\big) \cdot [(1 - x_i)s_F(x_i, c_L) + x_i^2 \max\{s_T(x_i, c_H), 0\}]}_{\text{Product F has a higher reservation value}}$$

$$+ \underbrace{\mathbf{1}\big(s_F(x_i, c_L) < s_T(x_i, c_H)\big) \cdot [x_i s_T(x_i, c_H) + (1 - x_i)^2 \max\{s_F(x_i, c_L), 0\}]}_{\text{Product T has a higher reservation value}}\}. \quad (3)$$

In Equation (3), if both $s_F(x_i, c_L)$ and $s_T(x_i, c_H)$ are negative, the second term in the "max" function is negative, so consumers search nothing and $EU_F(x_i) = 0$. Next, suppose at least one of them is positive. The first indicator function, $\mathbf{1}\big(s_F(x_i, c_L) \geq s_T(x_i, c_H)\big)$, means product F has a higher reservation value, so the consumer starts searching from product F and will buy it right away if it is a match. Her expected surplus from searching product F is $(1 - x_i)(1 - p_F) - c_L = (1 - x_i)s_F(x_i, c_L)$. If product F turns out to be a mismatch, which happens with probability x_i, consumer i can continue to search product T or stop searching—in this case her expected surplus is $\max\{x_i(1-p_T)-c_H, 0\} = x_i \max\{s_T(x_i, c_H), 0\}$. In sum, if $s_F(x_i, c_L) > s_T(x_i, c_H)$, the consumer's expected utility is $[(1-x_i)s_F(x_i, c_L) + x_i^2 \max\{s_T(x_i, c_H), 0\}]$. Similarly, $\mathbf{1}\big(s_F(x_i, c_L) < s_T(x_i, c_H)\big) \cdot [x_i s_T(x_i, c_H) + (1-x_i)^2 \max\{s_F(x_i, c_L), 0\}]$ reflects consumer i's expected utility when product T has a higher reservation value, so she will start searching from the non-prominent product T.

Similarly, if product T is prominent, consumer i's expected surplus is

$$EU_T(x_i) = \max\{0, \mathbf{1}\big(s_T(x_i, c_L) \geq s_F(x_i, c_H)\big)[x_i s_T(x_i, c_L) + (1-x_i)^2 \max\{s_F(x_i, c_H), 0\}]$$
$$+ \mathbf{1}\big(s_T(x_i, c_L) < s_F(x_i, c_H)\big)[(1-x_i)s_F(x_i, c_H) + x_i^2 \max\{s_T(x_i, c_L), 0\}]\}. \quad (4)$$

With search neutrality, product F will be prominent if and only if $EU_F(x_i) \geq EU_T(x_i)$. The comparison between $EU_F(x_i)$ and $EU_T(x_i)$ depends on the relative orders of $s_F(x_i, c_L)$, $s_T(x_i, c_H)$, and 0, as well as the relative orders of $s_T(x_i, c_L), s_F(x_i, c_H)$ and 0, leading to 13 distinctive cases. Because of the complexity of the analysis, and to enhance exposition, we present all the detailed analysis of Section 6 in the Online Appendix B. Importantly, we prove that consumers' search rankings are determined by a threshold $\hat{x}$, which satisfies $EU_F(\hat{x}) = EU_T(\hat{x})$. Conditional on choosing to search, consumers with $x_i \leq \hat{x}$ will find product F prominent and search it first, and those with $x_i > \hat{x}$ will find product T prominent and search it first.

Search rankings without search neutrality. The platform will choose search rankings to maximize its expected profit. For analytical tractability and interpretability, we assume that the platform uses a threshold rule to decide consumers' search rankings: it will make product F prominent for consumers with $x_i < \hat{x}$, and product T prominent for consumers with $x_i \geq \hat{x}$.[13] We denote $\hat{x}$ as the platform's ranking threshold. The platform chooses $\hat{x}$ to maximize its profit given p_F and p_T.

Tie-breaking rules. As we will show, without search neutrality, multiple $\hat{x}$'s may exist which maximize the platform's profit. If this happens, $\hat{x}$ is chosen based on a lexicographical order: it

[13] Results are qualitatively robust without assuming the platform to use the threshold rule. See Online Appendix C for more details.

first maximizes consumer surplus, then maximizes the third-party seller's profit, and next the lowest $\hat{x}$ is chosen. Alternative tie-breaking rules lead to qualitatively similar results.

Game sequence. First, both sellers set the prices p_F and p_T. Second, depending on whether search neutrality is in effect or not, the ranking threshold $\hat{x}$ (and thus personalized search rankings) is determined. Third, consumers observe product prices and make search and purchase decisions.

6.2. Analysis and Results

In this section, we present the model analysis and the equilibrium outcomes. Table 1 summarizes consumer i's optimal search sequence and eventual purchase probabilities ϕ_{ij} for the two products, which depend on the relative orders of S_F, S_T, and 0, which in turn depend on the recommendation threshold $\hat{x}$, the product prices, and the consumer type x_i. Product F's demand is $D_F = \int_0^1 \phi_{iF} dx$, and product T's demand is $D_T = \int_0^1 \phi_{iT} dx$. The platform's profit is $\pi_F = p_F D_F + r p_T D_T$, and the third-party seller's profit is $\pi_T = (1-r) p_T D_T$.

Table 1 Consumer Search Sequence and Purchase Probability

	Search sequence	Probability of buying F (ϕ_{iF})	Probability of buying T (ϕ_{iT})
$S_F \geq S_T \geq 0$	Search F. If mismatch, search T.	$1-x$	x^2
$S_T > S_F \geq 0$	Search T. If mismatch, search F.	$(1-x)^2$	x
$S_F \geq 0 > S_T$	Search F. Quit if mismatch.	$1-x$	0
$S_T \geq 0 > S_F$	Search T. Quit if mismatch.	0	x
$0 \geq \min\{S_F, S_T\}$	No search.	0	0

First, following the main model in Section 4.1, we analyze the case with search neutrality in an important limiting case of $r \to 0$. This case has closed forms solutions that can cleanly demonstrate the strategic impact of search neutrality by accentuating the platform's self-preference incentive. Second, we numerically analyze the game when $r > 0$. Given the complexity of this model and the non-smoothness of the demand functions, pure-strategy equilibria may not exist in general. Therefore, the numerical analyses focus on showing the robustness of our main results on several sets of specific parameters within which pure-strategy subgame-perfect equilibria exist.

Proposition 7 presents the equilibrium outcome with search neutrality and zero commission.

Proposition 7. *Suppose search neutrality is in place and* $r \to 0$. *When* $c_H > 0.1301$ *and* $c_L < \frac{1}{8}$, *the equilibrium prices for both firms are* $p_T^* = p_F^* = \frac{3}{4}$ *and the ranking threshold is* $\hat{x} = \frac{1}{2}$. *Consumers with* $x_i < \frac{1}{2}$ *search only product F but not product T, and those with* $x_i \geq \frac{1}{2}$ *search only product T but not F. Both sellers' profits are* $\frac{9}{32}$. *Consumer surplus is* $\frac{3}{16} - c_L$.[14]

Proposition 7 qualitatively replicates the key findings of the main model. With search neutrality, each seller serves only consumers who find this seller's product in their prominent positions. Each seller charges a relatively high price to extract the surplus of its high-match-likelihood consumers, who find the seller's product in the prominent position; the seller does not serve their low-match-likelihood consumers. The only difference is that the equilibrium prices are now lower than one. This is because in this new model, search neutrality ranks products based on consumer surplus and thus favors lower-priced products, incentivizing sellers to lower their prices from one.

Numerical Analysis with $\boldsymbol{r > 0}$***:*** To facilitate illustration, we present the results with parameters $r = 0.2$, $c_H = 0.135$ and $c_L = 0.115$. We numerically verify that without search neutrality, a seller's equilibrium price globally maximizes its profit given the other seller's equilibrium price. With search neutrality, we numerically derive the platform's optimal choice of $\hat{x}$ as a function of p_F and p_T, and then verify a seller's equilibrium price globally maximizes its profit given the other seller charging its equilibrium price and the platform's subsequent choice of $\hat{x}$. [15]

Table 2 presents the equilibrium outcomes with parameters $r = 0.2, c_H = 0.135$, and $c_L = 0.115$. The core insights from the main model remain qualitatively robust. Without search neutrality, the platform makes product F prominent for all consumers with $s_F(x_i, c_L) \geq 0$, which corresponds to $x_i \leq 0.5607$. Specifically, for consumers with $x_i \in (0.4582, 0.5607]$, the platform self-

[14] The condition of $c_L < 1/8$ ensures a full market coverage. The condition $c_H > 0.1301$ means that a seller has to significantly lower its price to invite potential searches from consumers who find this seller's product non-prominent, so the seller will find doing so unprofitable.

[15] Please see Online Appendix C for more details and qualitatively similar results with other parameter values.

preferentially makes product F prominent, even though these consumers' expected surplus would have been higher if product T were made prominent (given the product prices). By contrast, search neutrality eliminates self-preference but softens competition, increasing equilibrium prices and profits for both sellers ($p_F^{S*} > p_F^{NS*}$, $p_T^{S*} > p_T^{NS*}$, $\pi_F^{S*} > \pi_F^{NS*}$, and $\pi_T^{S*} > \pi_T^{NS*}$). However, consumer surplus and social surplus become lower, and no consumer will search the non-prominent product.

Table 2 Equilibrium Outcomes

	Without search neutrality	With search neutrality
$\hat{x}^*$	0.5607 (from $s_F(\hat{x}, c_L) = 0$)	0.4830 (from $s_F(\hat{x}, c_L) = s_T(\hat{x}, c_L)$)
p_F^*	0.7382	0.7765
p_T^*	0.6995	0.7607
π_F^*	0.3409	0.3428
π_T^*	0.2161	0.2333
Consumer surplus	0.0916	0.0586
Social surplus	0.6486	0.6347
Consumers experiencing ranking self-preference	(0.4582,0.5607]	$\emptyset$

To summarize, the analysis of the more general model reiterates the key finding: With search neutrality, both sellers may have incentives to focus on extracting more surplus from their respective high-match-likelihood consumers. As a result, search neutrality can soften price competition and harm consumers despite its good intention. In fact, numerical studies further demonstrate that search neutrality can even decrease total social welfare, revealing its unintended consequences.

7. Conclusion

Antitrust regulators have proposed search-neutrality regulations (e.g., the U.S.'s AICOA and the EU's DMA) to prohibit platform self-preference in search results. This paper shows that search neutrality, despite its good intention, may unexpectedly harm consumers and third-party sellers in two ways after accounting for the strategic decisions of the platform and third-party sellers. The first potential harm from search neutrality is *competition alleviation*—in the short term, search neutrality can lead to higher equilibrium prices for both types of products. When a sufficient number of consumers ex ante prefer the third-party product, the competition-alleviation effect is so strong that search neutrality will reduce consumer surplus. By contrast, search neutrality can increase the platform's profit due to the alleviated pricing competition if many consumers ex ante prefer the platform's product. The second potential harm from search neutrality is *entry deterrence*—in the long term, search neutrality may increase the platform's likelihood of preempting the third-party seller's entry by raising the commission rate, which further harms consumers and third-party sellers. We find that this is more likely to happen when the third-party seller's entry cost is intermediate. These two unintended harms are caused by two unique features of online retail platforms—pre-search price observability and personalized search rankings.

Importantly, our results suggest that antitrust agencies should be wary of the potential harms from search neutrality on all market participants and should carefully carry out the search-neutrality regulations with the rule of reason. Broadly, our paper is in alignment with the recent criticisms on the potential unintended damages caused by the latest shift in antitrust paradigms from ex-post analyzing each case to ex-ante banning an entire category of conduct.

In our context, consumers need to fully inspect a product before making a purchase decision. In practice, partial inspection is possible (Doval 2018; Dukes and Liu 2016). One can decompose a consumer's search cost into two parts: the costs related to inspection, e.g., collecting and construing information, and the costs that are independent of inspection, e.g., the effort and time of clicking into and loading the product page. We conjecture that some consumers may partially inspect the product if several conditions are met: (a) the inspection-related search cost is much higher than the inspection-independent search cost, (b) consumers' valuation uncertainty is sufficiently low (e.g., ρ_j is very high), (c) if the product is cheap enough, and (d) consumers are sufficiently risk-tolerant. By contrast, consumers will either not search or search in full depth if some or all conditions are violated. For example, the inspection-independent search cost can be significant for many consumers—the worldwide webpage load time is 10.3 seconds on desktops and 27.3 on mobile and a 100 millisecond extra load time can reduce Amazon sales by 1%.[16]

One could extend our framework to more general settings. For example, we have assumed that consumers' match values have binary supports and that their match likelihoods for both products are perfectly correlated. We conjecture that our results can qualitatively remain when consumers' match values are continuously distributed and when their match likelihoods are independently distributed, as long as some consumers still have higher match likelihoods for one product and some for the other. Additionally, the platform may benefit from a lower entry cost from third-party sellers, so the platform may want to subsidize their entrance. This may be hard to implement because fake sellers, who do not need to incur the entry cost as the real sellers, may simply register on the platform to fraudulently earn the subsidy.

There are multiple directions for future research. First, our framework focuses on the case in which the platform competes with one third-party seller. Future studies can consider the situation with multiple third-party sellers. Second, one can study how search neutrality influences the sellers' other strategic decisions. Finally, more empirical research will be useful to test our theoretical predictions and quantify the overall impact of search neutrality.

[16] https://backlinko.com/page-speed-stats/, https://www.forbes.com/sites/steveolenski/2016/11/10/why-brands-are-fighting-over-milliseconds/

Appendix

Figure A1 An Example of Amazon's "Featured From Our Brand"

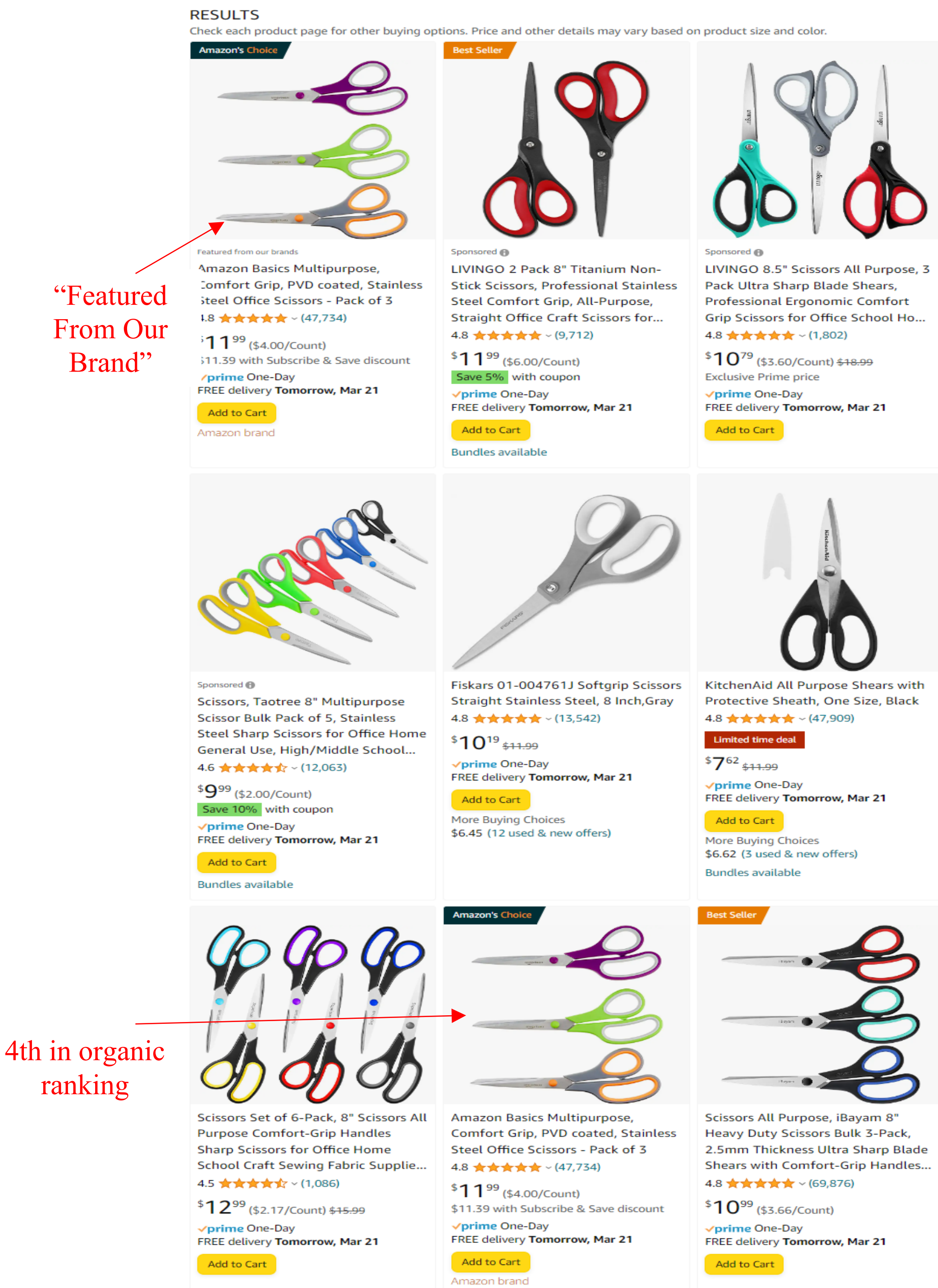


Figure A1 shows the search rankings for the keyword "scissors." The Amazon Basics product is the fourth in organic rankings but appears as the first outcome under the label "Featured From Our Brands."

Equilibrium derivation when search neutrality is absent and $r \to 0$. In this case of $r \to 0$, the platform makes product F prominent for all consumers, and hence the two sellers' demand are $D_F^{NS} = \mathbf{1}(p_F \geq p_T) \cdot [\alpha\rho_H(1-\rho_L^2 p_F + \rho_L^2 p_T) + (1-\alpha)\rho_L(1-\rho_H^2 p_F + \rho_H^2 p_T)] + \mathbf{1}(p_F < p_T)[\alpha\rho_H + (1-\alpha)\rho_L]$ and $D_T^{NS} = \mathbf{1}(p_F \geq p_T) \cdot [\alpha\rho_L^2(1-\rho_H+\rho_H p_F - p_T) + (1-\alpha)\rho_H^2(1-\rho_L+\rho_L p_F - p_T)] + \mathbf{1}(p_F < p_T)[\alpha(1-\rho_H)\rho_L^2 + (1-\alpha)(1-\rho_L)\rho_H^2](1-p_T)$. Their profits are $\pi_F^{NS} = p_F D_F^{NS}$ and $\pi_T^{NS} = p_T D_T^{NS}$. Both are continuous and have a kink at $p_T = p_F$, and π_F^{NS} strictly increases with p_F when $p_F < p_T$, so conditional on p_T, seller F will not choose any $p_F < p_T$, and the equilibrium prices must satisfy $p_F^{NS*} \geq p_T^{NS*}$. We discuss our analysis by whether $p_F^{NS*} < 1$ or $p_F^{NS*} = 1$.

(A) $p_T^{NS*} \leq p_F^{NS*} < 1$. In this case, $\left(p_F^{NS*}, p_T^{NS*}\right)$ is an equilibrium if and only if (1) p_j^{NS*} satisfies the first-order condition (FOC) $\frac{\partial \pi_j^{NS*}}{\partial p_j}|_{p_{-j}=p_{-j}^{NS*}, p_F \geq p_T} = 0$ for $j \in \{F, T\}$, (2) $p_T^{NS*} \leq p_F^{NS*} < 1$, and (3) seller T will not deviate to $p_F < p_T$.

From (1), we obtain $p_F^{NS*} = \frac{(1-\alpha)^2\rho_H^2\left(2+\rho_H^2(1-\rho_L)\right)\rho_L + \alpha^2\rho_H\rho_L^2(2+(1-\rho_H)\rho_L^2) + \alpha(1-\alpha)(\rho_H^2\rho_L^2(\rho_H+\rho_L-2\rho_H\rho_L)+2(\rho_H^3+\rho_L^3))}{\rho_H\rho_L[(1-\alpha)\rho_H+\alpha\rho_L]((1-\alpha)\rho_H^2(4-\rho_L)+\alpha(4-\rho_H)\rho_L^2)}$ and $p_T^{NS*} = \frac{(1-\alpha)(\rho_L+2(1-\rho_L)\rho_H^2)+\alpha(\rho_H+2(1-\rho_H)\rho_L^2)}{(1-\alpha)\rho_H^2(4-\rho_L)+\alpha\rho_L^2(4-\rho_H)}$. The SOCs are satisfied.

For (2), $p_F^{NS*} - p_T^{NS*} = \frac{1}{\rho_H\rho_L[\rho_H(1-\alpha)+\rho_L\alpha][\rho_H^2(4-\rho_L)(1-\alpha)+\rho_L^2(4-\rho_H)\alpha]} \cdot \{-[(\rho_H-\rho_L)^2(\rho_H^2(1-\rho_L)\rho_L + (2-(1-\rho_L)\rho_L)\rho_H + 2\rho_L)]\alpha^2 + (\rho_H-\rho_L)[2\rho_H^3(1-\rho_L)\rho_L + (2-\rho_L+\rho_L^2)\rho_H^2 - \rho_H(2-\rho_L)\rho_L - 2\rho_L^2]\alpha + \rho_H^2\rho_L[2-\rho_L-\rho_H^2(1-\rho_L)]\}$. The first term is positive. The second term (the expression in "{}") is also positive because (i) it is concave in α, (ii) it equals to $\rho_H^2\rho_L[2-\rho_L-\rho_H^2(1-\rho_L)] > 0$ when $\alpha = 0$, and (iii) it equals to $\rho_L^2\rho_H[2-\rho_H-\rho_L^2(1-\rho_H)] > 0$ when $\alpha = 1$. Hence $p_F^{NS*} - p_T^{NS*} > 0$ is always satisfied. Moreover, $p_F^{NS*} < 1$ if and only if $\alpha(\rho_H-\rho_L)(2+3\rho_H\rho_L) + \rho_L(2-3\rho_H^2) < 0$, or equivalently $\alpha < \alpha_{NS,1} \stackrel{\text{def}}{=} \mathbf{1}\left(\rho_H < \frac{\sqrt{6}}{3}\right) \cdot 0 + \mathbf{1}\left(\rho_L < \frac{\sqrt{6}}{3} \leq \rho_H\right) \cdot \frac{\rho_L(3\rho_H^2-2)}{(\rho_H-\rho_L)(2+3\rho_H\rho_L)} + \mathbf{1}\left(\frac{\sqrt{6}}{3} \leq \rho_L\right) \cdot 1$.

For (3), $\pi_T|_{p_F<p_T} \propto p_T(1-p_T)$, which strictly decreases with $p_T > \max\{p_F, \frac{1}{2}\}$. Note that $p_F^{NS*} \geq \frac{1}{2}$, because $p_F^{NS*} - \frac{1}{2} = \frac{1}{2\rho_H\rho_L[\rho_H(1-\alpha)+\rho_L\alpha][\rho_H^2(4-\rho_L)(1-\alpha)+\rho_L^2(4-\rho_H)\alpha]} \cdot \{\rho_H^2\rho_L\left(4-\rho_H^2(2+\rho_L)\right) + 2(\rho_H-\rho_L)(2\rho_H^2 - 2\rho_H\rho_L + 2\rho_H^3\rho_L - 2\rho_L^2 + \rho_H^2\rho_L^2 + \rho_H^3\rho_L^2)\alpha - (\rho_H-\rho_L)^2[(4+2\rho_H\rho_L)(\rho_H+\rho_L) + \rho_H^2\rho_L^2)]\alpha^2\}$. The term in "{}" is positive because (i) it is concave in α, (ii) it equals to $\rho_H^2\rho_L\left(4-\rho_H^2(2+\rho_L)\right) > 0$ when $\alpha = 0$, and (iii) it equals to $\rho_L^2\rho_H\left(4-\rho_L^2(2+\rho_H)\right)$ when $\alpha = 1$. So, seller T will not deviate to $p_T > p_F^{NS*}$.

Summarizing (1)-(3), there is an equilibrium with $p_T^{NS*} \leq p_F^{NS*} < 1$ if and only if $\alpha < \alpha_1^{NS}$.

(B) $p_T^{NS*} \leq p_F^{NS*} = 1$. If $p_F^{NS*} = 1$, $p_T \leq p_F$ is necessary, so $\pi_T^{NS} = \frac{1}{C}[\alpha(1-\rho_H)\rho_L^2 + (1-\alpha)(1-\rho_L)\rho_H^2](1-p_T)p_T$, $\forall p_T \in [0,1]$, which is maximized at $p_T = \frac{1}{2}$. So, the equilibrium must satisfy $p_F^{NS*} = 1$ and $p_T^{NS*} = \frac{1}{2}$. We also need to guarantee that seller F will not deviate to $p_F \in (\frac{1}{2}, 1)$. Because π_F is concave, quadratic in p_F on $p_F > p_T$, so the no-deviation condition is equivalent to $0 \leq \frac{d\pi_F}{dp_F}|_{p_F=1, p_T=\frac{1}{2}} = \frac{\alpha(\rho_H-\rho_L)(2+3\rho_H\rho_L)+\rho_L(2-3\rho_H^2)}{2}$, which is equivalent to $\alpha \geq \alpha_1^{NS}$.

The parameter regions for parts (A) and (B) are disjoint and collectively cover all parameters. Hence, there exists a unique pure-strategy equilibrium for all parameters. Table A1 summarizes the equilibrium outcome.

Table A1. Equilibrium Outcome Without Search Neutrality When $r \to 0$

α		Seller F	Seller T
$[0, \alpha_1^{NS})$	p_j^{NS*}	$\frac{(1-\alpha)^2\rho_H^2\left(2+\rho_H^2(1-\rho_L)\right)\rho_L+\alpha^2\rho_H\rho_L^2(2+(1-\rho_H)\rho_L^2)+\alpha(1-\alpha)(\rho_H^2\rho_L^2(\rho_H+\rho_L-2\rho_H\rho_L)+2(\rho_H^3+\rho_L^3))}{\rho_H\rho_L[(1-\alpha)\rho_H+\alpha\rho_L]((1-\alpha)\rho_H^2(4-\rho_L)+\alpha(4-\rho_H)\rho_L^2)}$	$\frac{(1-\alpha)(\rho_L+2(1-\rho_L)\rho_H^2)+\alpha(\rho_H+2(1-\rho_H)\rho_L^2)}{(1-\alpha)\rho_H^2(4-\rho_L)+\alpha\rho_L^2(4-\rho_H)}$
	π_j^{NS*}	$\frac{\left[2(\rho_H\alpha+\rho_L(1-\alpha))(\rho_H^2(1-\alpha)+\rho_L^2\alpha)+\rho_H\rho_L(\rho_L\alpha+\rho_H(1-\alpha))\left(\rho_L^2(1-\rho_H)\alpha+\rho_H^2(1-\rho_L)(1-\alpha)\right)\right]^2}{\rho_H\rho_L(\rho_L\alpha+\rho_H(1-\alpha))\left(\rho_L^2(4-\rho_H)\alpha+\rho_H^2(4-\rho_L)(1-\alpha)\right)^2}$	$\frac{\left((\rho_L(1-\alpha)+\rho_H\alpha)+2\rho_L^2(1-\rho_H)\alpha+2\rho_H^2(1-\rho_L)(1-\alpha)\right)^2(\rho_H^2(1-\alpha)+\rho_L^2\alpha)}{\left(\rho_L^2(4-\rho_H)\alpha+\rho_H^2(4-\rho_L)(1-\alpha)\right)^2}$
$[\alpha_1^{NS}, 1]$	p_j^{NS*}	1	$\frac{1}{2}$
	π_j^{NS*}	$\rho_H\alpha + \rho_L(1-\alpha) - \frac{\rho_H\rho_L}{2}[\rho_L\alpha + \rho_H(1-\alpha)]$	$\frac{\rho_L^2\alpha+\rho_H^2(1-\alpha)}{4}$

Equilibrium consumer surplus without search neutrality. Since $p_F^{NS*} \geq p_T^{NS*}$, a type-f consumer's surplus is

$CS_f^{NS} = \rho_H\left[1 - p_F^{NS*} + \left(\rho_L\left(p_F^{NS*} - p_T^{NS*}\right) - c_i\right)^+\right] + (1-\rho_H)\cdot\left(\rho_L\left(1 - p_T^{NS*}\right) - c_i\right)^+.$

A type-t consumer's surplus is

$CS_t^{NS} = \rho_L\left[1 - p_F^{NS*} + \left(\rho_H\left(p_F^{NS*} - p_T^{NS*}\right) - c_i\right)^+\right] + (1-\rho_L)\cdot\left(\rho_H\left(1 - p_T^{NS*}\right) - c_i\right)^+.$

The total consumer surplus is $CS^{NS} = \alpha\mathbf{E}_{c_i}[CS_f^{NS}] + (1-\alpha)\mathbf{E}_{c_i}[CS_t^{NS}] = [\alpha\rho_H + (1-\alpha)\rho_L]\left(1 - p_F^{NS*}\right) + \frac{\alpha\rho_L^2}{2}\left[\rho_H\left(p_F^{NS*} - p_T^{NS*}\right)^2 + (1-\rho_H)\left(1 - p_T^{NS*}\right)^2\right] + \frac{(1-\alpha)\rho_H^2}{2}\left[\rho_L\left(p_F^{NS*} - p_T^{NS*}\right)^2 + (1-\rho_L)\left(1 - p_T^{NS*}\right)^2\right]$.

Equilibrium derivation when search neutrality is absent and $r \to 0$. In this case, the two sellers are symmetric except for the sizes of their popularities α and $1-\alpha$. We will present the analysis with $\alpha \geq \frac{1}{2}$; The case with $\alpha < \frac{1}{2}$ is a mirror case by switching the roles of the two sellers. The two sellers' demand are $D_F^S = \mathbf{1}(p_F \geq p_T)\cdot[\alpha\rho_H(1 - \rho_L^2 p_F + \rho_L^2 p_T) + (1-\alpha)(1-\rho_H)\rho_L^2(1-p_F)] + \mathbf{1}(p_F < p_T)[\alpha\rho_H + (1-\alpha)\rho_L^2(1 - \rho_H + \rho_H \cdot p_T - p_F)]$ and $D_T^S = \mathbf{1}(p_F \geq p_T)\cdot[\alpha\rho_L^2(1 - \rho_H + \rho_H p_F - p_T) + (1-\alpha)\rho_H] + \mathbf{1}(p_F < p_T)[\alpha(1-\rho_H)\rho_L^2(1-p_T) + (1-\alpha)\rho_H(1 - \rho_L^2 p_T + \rho_L^2 p_F)]$, respectively. Their profits are $\pi_F^S = p_F D_F^S$ and $\pi_T^S = p_T D_T^S$. Both are continuous and have a kink at $p_T = p_F$.

Note that if $\alpha \geq \frac{1}{2}$ and $p_F < p_T$, $\frac{d\pi_F^S}{dp_F}|_{p_F<p_T} = (1-\alpha)\rho_L^2(1 - \rho_H - 2p_F + \rho_H p_T) + \rho_H\alpha > 0$. So, the equilibrium prices must satisfy $p_F^{S*} \geq p_T^{S*}$. Also, conditional on any p_T, the seller F will not choose any $p_F < p_T$. We divide the discussion into three exhaustive and exclusive cases.

(A) $p_T^{S*} \leq p_F^{S*} < 1$. In this case, $\left(p_F^{S*}, p_T^{S*}\right)$ is an equilibrium if and only if (1) p_j^{S*} satisfies the first-order condition (FOC) $\frac{\partial\pi_j^{S*}}{\partial p_j}|_{p_{-j}=p_{-j}^{S*}, p_F \geq p_T} = 0$ for $j \in \{F, T\}$, (2) $p_T^{S*} \leq p_F^{S*} < 1$, and (3) seller T will not deviate to $p_F < p_T$.

From (1), we obtain $p_F^{S*} = \frac{\rho_H\rho_L^2(3\alpha-2)+2\rho_L^2(1-\alpha)+2\rho_H\alpha+\rho_H^2(1-\alpha-\rho_L^2\alpha)}{\rho_L^2[4(1-\rho_H)+(8\rho_H-\rho_H^2-4)\alpha]}$ and $p_T^{S*} = \frac{(1-\rho_H)\rho_L^2\alpha(2-2\alpha-\rho_H(1-3\alpha))+\rho_H\left(2(1-\alpha)^2-\rho_H(2-6\alpha+3\alpha^2)\right)}{\alpha\rho_L^2[4(1-\rho_H)+(8\rho_H-\rho_H^2-4)\alpha]}$. The SOCs are satisfied.

For (2), $p_F^{S*} - p_T^{S*} = \frac{\rho_H(2\alpha-1)[2(1-\rho_H)+(\rho_H-(1-\rho_H)\rho_L^2)]}{\alpha\rho_L^2[4(1-\rho_H)+(8\rho_H-\rho_H^2-4)\alpha]} > 0$. Moreover, $p_F^{S*} < 1$ if and only if $[(5\rho_H - 2)\rho_L^2 - (2-\rho_H)\rho_H]\alpha > \rho_H^2 - 2(1-\rho_H)\rho_L^2$. When $\rho_L < \frac{\sqrt{6}}{3}$, this is false $\forall\alpha \in [0,1]$; when $\rho_L \geq \frac{\sqrt{6}}{3}$, the condition is true if and only if $\alpha \in \left(\frac{\rho_H^2-2(1-\rho_H)\rho_L^2}{(5\rho_H-2)\rho_L^2-(2-\rho_H)\rho_H}, 1\right)$. Define $\alpha_4^S \stackrel{\text{def}}{=} \mathbf{1}\left(\rho_L \geq \frac{\sqrt{6}}{3}\right) \cdot \frac{\rho_H^2-2(1-\rho_H)\rho_L^2}{(5\rho_H-2)\rho_L^2-(2-\rho_H)\rho_H} + \mathbf{1}\left(\rho_L < \frac{\sqrt{6}}{3}\right) \cdot 1$. ($\alpha_1^S$ to α_3^S will be defined later.) So, (2) is true if and only if $\alpha \in (\alpha_4^S, 1)$.

For (3), we will show seller T will not deviate to $p_T > p_F^{S*}$ when $\rho_L \geq \frac{\sqrt{6}}{3}$ and $\alpha \in (\alpha_4^S, 1)$. If seller T deviates to $p_T > p_F^{S*}$, its deviation profit is

$$\pi_{T,dev}^S = \left[\alpha(1-\rho_H)\rho_L^2(1-p_T) + (1-\alpha)\rho_H\left(1-\rho_L^2 p_T + \rho_L^2 p_F^{S*}\right)\right]p_T \leq_{\frac{\partial\pi_{T,dev}^S}{\partial p_T}=0} \pi_{T,dev}^{S*}$$

$$\stackrel{\text{def}}{=} \frac{\left(\rho_H\left((2-\rho_H)^2+2(1-\rho_H)\rho_L^2\right)+\left((4-\rho_H)\rho_H(3\rho_H-2)+(1-\rho_H)(4-8\rho_H+\rho_H^2)\rho_L^2\right)\alpha+2(2-5\rho_H+\rho_H^2)(\rho_H-(1-\rho_H)\rho_L^2)\alpha^2\right)^2}{4\rho_L^2[4(1-\rho_H)+(8\rho_H-\rho_H^2-4)\alpha]^2(\alpha+\rho_H(1-2\alpha))}.$$

(The equality may not be reached when $p_T \leq 1$.) We finish the proof by showing $\pi_T^{S*} - \pi_{T,dev}^{S*} \geq 0$. Note that $\pi_T^{S*} - \pi_{T,dev}^{S*} = \frac{\rho_H^2(1-2\alpha)^2}{4\rho_L^2[4(1-\rho_H)+(8\rho_H-\rho_H^2-4)\alpha]^2\alpha(\alpha+\rho_H(1-2\alpha))} \cdot \{-(8\rho_H + \rho_H^2 - 4)(\rho_H(1+\rho_L^2) - \rho_L^2)^2\alpha^3 + 2(\rho_H(1+\rho_L^2) - \rho_L^2)(8 - 28\rho_H + 22\rho_H^2 + \rho_H^3)\alpha^2 + (-32\rho_H + 88\rho_H^2 - 56\rho_H^3 - \rho_H^4 + 16\rho_L^2 - 48\rho_H\rho_L^2 + 44\rho_H^2\rho_L^2 - 12\rho_H^3\rho_L^2 - 4\rho_L^4 + 8\rho_H\rho_L^4 - 4\rho_H^2\rho_L^4)\alpha + 16\rho_H(1-\rho_H)^2\}$. The term in "{}" is positive on $\alpha \in [\frac{1}{2}, 1]$ because (i) it equals to $\frac{(2+\rho_H)(\rho_H(1-\rho_L^2)+\rho_L^2)[1+(1-\rho_H)(15+\rho_H-(6+\rho_H)\rho_L^2)]}{8} > 0$ when $\alpha = \frac{1}{2}$, (ii) it equals to $\rho_H(1-\rho_H)[4\rho_H + (1-\rho_H)\rho_L^2(16-(8+\rho_H)\rho_L^2)] > 0$ when $\alpha = 1$, and (ii) it is concave in α since its second derivative, $[4(\rho_H(1+\rho_L^2) - \rho_L^2)(8 - 28\rho_H + 22\rho_H^2 + \rho_H^3) - 6(8\rho_H + \rho_H^2 - 4)(\rho_H(1+\rho_L^2) - \rho_L^2)^2\alpha$, is negative on $\alpha \in [\frac{1}{2}, 1]$.

Summarizing, $p_T^{S*} \leq p_F^{S*} < 1$ if and only if $\alpha \in (\alpha_4^S, 1)$.

(B) $p_T^{S*} < p_F^{S*} = 1$. Given $p_F^{S*} = 1$, $p_T \leq p_F$ is necessary, so $\pi_T^S = [\alpha\rho_L^2(1-p_T) + (1-\alpha)\rho_H]p_T$. Because $p_T^{S*} < 1$, it must satisfy the FOC $\frac{d\pi_T^S}{dp_T} = 0$, so $p_T^{S*} = \frac{1}{2} + \frac{\rho_H(1-\alpha)}{2\rho_L^2\alpha}$. We need to verify that (1) $p_T^{S*} < 1$ and (2) seller F will not deviate to any $p_F \in [p_T^{S*}, 1)$. For (1), $p_T^{S*} < 1$ if and only if $\alpha > \alpha_3^S \stackrel{\text{def}}{=} \frac{\rho_H}{\rho_H+\rho_L^2}$. For (2), $\pi_F^{S*}|_{p_F \geq p_T^{S*}} = \left[\alpha\rho_H\left(1-\rho_L^2 p_F + \rho_L^2 p_T^{S*}\right) + (1-\alpha)(1-\rho_H)\rho_L^2(1-p_F)\right]p_F$, which is concave in p_F on $(p_T^{S*}, 1]$. So, $p_F^{S*} = 1$ if and only if $\frac{d\pi_F^S}{dp_F}|_{p_F=1} \geq 0$, which simplifies to $[(5\rho_H - 2)\rho_L^2 - (2-\rho_H)\rho_H]\alpha \leq \rho_H^2 - 2(1-\rho_H)\rho_L^2$, or equivalently $\alpha \leq \alpha_4^S$. It is easy to verify $\frac{1}{2} < \alpha_3^S \leq \alpha_4^S$. In summary, $p_T^{S*} \leq p_F^{S*} = 1$ if and only if $\alpha \in (\alpha_3^S, \alpha_4^S]$.

(C) $p_T^{S*} = p_F^{S*} = 1$. Given $p_T^{S*} = 1$, seller F will not deviate to $p_F < p_T^{S*}$. Given $p_F^{S*} = 1$, $\pi_T^S = [\alpha\rho_L^2(1-p_T) + (1-\alpha)\rho_H]p_T$, which is maximized at $p_T = 1$ if and only if $\frac{d\pi_T^S}{dp_T}|_{p_T=1} \geq 0$, or equivalently $\alpha \in [\frac{1}{2}, \alpha_3^S]$.

The parameter regions for parts (A) to (C) are disjoint and collectively cover all parameters. Hence, there exists a unique pure-strategy equilibrium for all parameters when $\alpha \geq \frac{1}{2}$. The analysis for $\alpha < \frac{1}{2}$ is analogous. Let $\alpha_1^S = 1 - \alpha_4^S$, and $\alpha_2^S = 1 - \alpha_3^S$. One can see that $0 \le \alpha_1^S \le \alpha_2^S < \frac{1}{2} < \alpha_3^S \le \alpha_4^S \le 1$. Table A2 summarizes the equilibrium outcome with search neutrality when $r \to 0$. All the equilibrium prices and profits are continuous in α, ρ_H, and ρ_L.

Table A2. Equilibrium Outcome with Search Neutrality When $r \to 0$

α		Seller F	Seller T
$(0, \alpha_1^S)$	p_j^{S*}	$\frac{(1-\rho_H)\rho_L^2(1-\alpha)(2\alpha-\rho_H(3\alpha-2))+\rho_H(2\alpha^2-\rho_H(3\alpha^2-1))}{(1-\alpha)\rho_L^2[4(1-\rho_H)-(4-8\rho_H+\rho_H^2)(1-\alpha)]}$	$\frac{\rho_H\rho_L^2(1-3\alpha)+2\rho_L^2\alpha+2\rho_H(1-\alpha)+\rho_H^2(\alpha-\rho_L^2(1-\alpha))}{\rho_L^2[4(1-\rho_H)-(4-8\rho_H+\rho_H^2)(1-\alpha)]}$
	π_j^{S*}	$\frac{\left[(3\rho_H-2)\left(\rho_H-\rho_L^2(1-\rho_H)\right)(1-\alpha)^2-[2(3\rho_H-2)\rho_H-(2-\rho_H)(1-\rho_H)\rho_L^2](1-\alpha)-2\rho_H(1-\rho_H)\right]^2}{(1-\alpha)\rho_L^2[4(1-\rho_H)-(4-8\rho_H+\rho_H^2)(1-\alpha)]^2}$	$\frac{[\alpha+\rho_H(1-2\alpha)]\cdot\left[(2-\rho_H)\left(\rho_H-\rho_L^2(1-\rho_H)\right)(1-\alpha)+\rho_H^2+2\rho_L^2(1-\rho_H)\right]^2}{\rho_L^2[4(1-\rho_H)-(4-8\rho_H+\rho_H^2)(1-\alpha)]^2}$
$[\alpha_1^S, \alpha_2^S)$	p_j^{S*}	$\frac{1}{2}+\frac{\rho_H\alpha}{2\rho_L^2(1-\alpha)}$	1
	π_j^{S*}	$\frac{[\rho_L^2(1-\alpha)+\rho_H\alpha]^2}{4\rho_L^2(1-\alpha)}$	$\frac{\rho_H[(2-\rho_L^2)(1-\alpha)+\rho_H\alpha]}{2}$
$[\alpha_2^S, \alpha_3^S)$	p_j^{S*}	1	1
	π_j^{S*}	$\rho_H\alpha$	$\rho_H(1-\alpha)$
$[\alpha_3^S, \alpha_4^S)$	p_j^{S*}	1	$\frac{1}{2}+\frac{\rho_H(1-\alpha)}{2\rho_L^2\alpha}$
	π_j^{S*}	$\frac{\rho_H[(2-\rho_L^2)\alpha+\rho_H(1-\alpha)]}{2}$	$\frac{[\rho_L^2\alpha+\rho_H(1-\alpha)]^2}{4\rho_L^2\alpha}$
$[\alpha_4^S, 1)$	p_j^{S*}	$\frac{\rho_H\rho_L^2(3\alpha-2)+2\rho_L^2(1-\alpha)+2\rho_H\alpha+\rho_H^2(1-\alpha-\rho_L^2\alpha)}{\rho_L^2[4(1-\rho_H)-(4-8\rho_H+\rho_H^2)\alpha]}$	$\frac{(1-\rho_H)\rho_L^2\alpha(2-2\alpha-\rho_H(1-3\alpha))+\rho_H(2(1-\alpha)^2-\rho_H(2-6\alpha+3\alpha^2))}{\alpha\rho_L^2[4(1-\rho_H)-(4-8\rho_H+\rho_H^2)\alpha]}$
	π_j^{S*}	$\frac{[1-\alpha+\rho_H(2\alpha-1)]\cdot\left[(2-\rho_H)\left(\rho_H-\rho_L^2(1-\rho_H)\right)\alpha+\rho_H^2+2\rho_L^2(1-\rho_H)\right]^2}{\rho_L^2[4(1-\rho_H)-(4-8\rho_H+\rho_H^2)\alpha]^2}$	$\frac{\left[(3\rho_H-2)\left(\rho_H-\rho_L^2(1-\rho_H)\right)\alpha^2-[2(3\rho_H-2)\rho_H-(2-\rho_H)(1-\rho_H)\rho_L^2]\alpha-2\rho_H(1-\rho_H)\right]^2}{\alpha\rho_L^2[4(1-\rho_H)-(4-8\rho_H+\rho_H^2)\alpha]^2}$

Equilibrium consumer surplus with search neutrality. (1) Suppose $\alpha \ge \frac{1}{2}$, then $p_F^{S*} \ge p_T^{S*}$. A type-f consumer's surplus is $CS_f^S = \rho_H\left[1 - p_F^{S*} + \left(\rho_L\left(p_F^{S*} - p_T^{S*}\right) - c_i\right)^+\right] + (1-\rho_H)\cdot\left(\rho_L\left(1 - p_T^{S*}\right) - c_i\right)^+$. A type-$t$ consumer's surplus is $CS_t^S = \rho_L(1 - p_T^{S*}) + (1-\rho_L)\cdot\left(\rho_H\left(1 - p_F^{S*}\right) - c_i\right)^+$. (2) Suppose $\alpha \le \frac{1}{2}$, then $p_F^{S*} \le p_T^{S*}$. A type-f consumer's surplus is $CS_f^S = \rho_H\left(1 - p_F^{S*}\right) + (1-\rho_H)\cdot\left(\rho_L\left(1 - p_T^{S*}\right) - c_i\right)^+$. A type-$t$ consumer's surplus is $CS_t^S = \rho_L(1 - p_T^{S*} + \left(\rho_L\left(p_T^{S*} - p_F^{S*}\right) - c_i\right)^+) + (1-\rho_L)\cdot\left(\rho_H\left(1 - p_F^{S*}\right) - c_i\right)^+$. Combining both cases, the total consumer surplus is $CS^{NS} = \alpha\mathbf{E}_{c_i}\left[CS_f^{NS}\right] + (1-\alpha)\mathbf{E}_{c_i}\left[CS_t^{NS}\right] = \alpha[\rho_H(1-p_F^{S*}) + (1-\rho_H)\frac{\rho_L^2\left(1-p_T^{S*}\right)^2}{2}] + (1-\alpha)[\rho_H\left(1-p_T^{S*}\right) + (1-\rho_H)\frac{\rho_L^2\left(1-p_F^{S*}\right)^2}{2}] + \frac{\rho_H\rho_L^2\left(p_F^{S*}-p_T^{S*}\right)^2}{2}\left[\mathbf{1}\left(p_F^{S*} \ge p_T^{S*}\right)\cdot\alpha + \mathbf{1}\left(p_F^{S*} < p_T^{S*}\right)\cdot(1-\alpha)\right]$.

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