Multi-homing Across Platforms: Friend or Foe

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Abstract

The gig economy offers flexible work opportunities where contractors enjoy the freedom to control when and how they do their work. In this paper, we look at one particular form of freedom: ability to work on multiple platforms, a practice termed multi-homing. We study whether it is in the best interest of the platforms to allow multi-homing and what users have to gain from it, if any. The answer depends on the characteristics of the market. With multi-homing, a platform has less incentive to invest in supply capacity. It also has less incentive to differentiate itself through scale, dampening the excessive competition between platforms. In markets that exhibit economies of scale (e.g. ride-sharing, rental, e-commerce), the latter effect is more prominent. In those markets, platforms have their cake and eat it too. By allowing multi-homing, platforms extract more revenue from the market and also give their workers more freedom, a key move that strengthens platforms' case against regulators that its workers are free from platform's control. Although workers have more freedom, they are not better off financially. In markets with multi-homing, the supply market is usually thin, and customers receive poor service quality.

1 Introduction

The rise of the gig economy has driven a fundamental change in how workers do their jobs and what channels they can access to offer their services. Many gig economy workers are free to choose when they work and for whom they work. They can work on multiple competing platforms, a practice commonly termed *multi-homing*. For example, drivers can search for rides simultaneously on Uber and Lyft, rental places can be posted on AirBnb and Booking.com, and sellers can open their online store on both eBay and Etsy.

While a firm can restrain its employees from working for other firms, platforms workers are independent contractors and, therefore, there are limits to how much control a platform can exert on them. A platform that wants to impose exclusivity needs to either change the nature of its relationship with the workers (e.g. classify them as employees), or implement indirect mechanisms like switching costs that make it costly to multi-home. In most platforms, the switching costs are not high. Workers can freely multi-home, and a considerable number of platform workers do. Some estimate that approximately 40% of gig economy drivers use multiple platforms over the course of a day (Davalos and Bennett 2022). Given its prevalence, we find it important to understand in what ways a market with multi-homing workers differs from one with single-homing workers, and what policy the users prefer.

Answering this question not only improves our understanding of how gig economy works, but also puts together another piece of the puzzle in a long-standing question: Should gig economy workers be classified as employees? This is a question that is being debated in many parts of the world, including California, Massachusetts, and the United Kingdom, and is one that likely will have the greatest impact on the future of the gig economy (Conger 2021, Browning 2022, Marshall 2021). Platforms such as Uber argue that if workers are to be reclassified as employees, they will no longer enjoy the flexibility they have right now (Stein 2020), including the ability to use multiple apps (Scheiber 2019). Surely, reclassification of workers has large direct economic implications for platforms (see Reich (2020) for a detailed analysis) and in this paper we abstract away from those. Instead, we aim to understand a question that is important for both the platforms and regulators: do platform get hurt by multi-homing, and would they restrict workers from multi-homing if given the option (e.g. after workers become employees)?

To study these questions, we build a mathematical model of a market with two platforms that can each choose its price and wage, and also whether it will operate with dedicated or multi-homing workers. In our model, we refer to the workers on the platform as *servers*. The servers choose whether to work and select which platform to work for on a short-term basis. Customers care about the prices they are charged and also the level of service quality in the platform from which they are taking service, which itself depends on whether servers multi-home or not.

Contrary to the common contention that multi-homing is bad from platforms, we find that there are various ways in which they benefit from it. More specifically, we find that multi-homing influences (i) overall efficiency of the market, (ii) the platform's incentive to invest in servers, and (iii) the nature of competition between the platforms. Depending on the characteristics of market the platforms operate in and relative strengths of these effects, platforms may prefer to operate with multi-homing workers. Instead, we find that in most of those instances, it is the customers and the servers who are worse off.

The value a platform provides to the market depends significantly on the composition of those two sides: An Uber rider can be matched with a closer ride if there is more driver supply; an Uber driver can find more rides if there is more demand. Due to these reinforcing effects, platforms generally operate more efficiently as they gain scale. Multi-homing allows servers to work for any platform, giving them the possibility to find ride requests that are possibly closer to them, reducing the service time associated with serving a customer. In markets where scale is beneficial, doing so increases the overall efficiency of the market.

As servers are limited resources with fixed capacity, only a single platform can utilize them at any given time. When servers multi-home, without an explicit control over servers, platforms are unable to take full advantage of the servers they hire. When a platform increases its wage to attract more servers to the market, some of that capacity is shared between the platforms. This disincentivizes investment in server capacity. With lower wages, the service quality typically deteriorates.

Multi-homing also changes the nature of competition between platforms. There are advantages to operating a large market (e.g., operating at a higher efficiency, which lends itself to lower costs). When platforms operate with their dedicated workers, each platform is responsible for hiring its own workers and building its own scale. Multi-homing combines the scale of two platforms and reduces a platform's ability to its own scale. Unable to get the full benefits of building scale, platforms have less incentive to compete for market share. In markets where the outside option is costly and platforms offer substitute services, the competition for scale is mostly destructive. In those cases, multi-homing dampens the excessive competition between platforms and improves profits.

2 Literature Review

Our paper is at the intersection of two streams of work that are closely related: (i) the design and management of service platform and (ii) competition under congestion or capacity limitations.

In recent years, there has been extensive study of two-sided platforms. Much of this work focuses on understanding how a two-sided monopolistic platform should be managed in the presence of on-demand workers. Some of the seminal papers on this are Gurvich et al. (2016), Cachon et al. (2017), Castilllo et al. (2017), Taylor (2018), Wu et al. (2020). In our model, however, we focus on two platforms competing in a market.

Similarly to us, papers like Ahmadinejad et al. (2019), Lian et al. (2022) look at competition between platforms in the presence of multi-homing, however, they do not compare it with a single-homing duopoly. Bakos and Halaburda (2020) look at how multi-homing affects the strategic interdependence between a platform's optimal pricing decision across two sides of the market. They consider a market where at least some agents multi-home, whereas we allow both sides of the market to single-home.

Papers like Bryan and Gans (2019), Liu et al. (2019), Belleflamme and Peitz (2019), Zhang et al. (2022) compare multi-homing with single-homing, however, they do not incorporate the effects on congestion on the competition, a key characteristic of many platforms markets.

Some focus on competition between platforms where the customers' utility from a service depends on the scale of a platform (e.g. number of users on one or both sides of the market), but not on utilization. Some seminal works in this area are (Rochet and Tirole 2003, Caillaud and Jullien 2003, Doganoglu and Wright 2006, Armstrong 2006). This is applicable to the analysis of social media platforms or payment systems. In our system, one side of the market offers a service and the other side receives a service, and it takes a nontrivial amount of time to complete the service. Hence, the utility users get from participating in the platform depend on how busy they expect servers to be.

Our paper is most closely related to those that explicitly study platform competition in the presence of multi-homing. Among these papers, Tadepalli and Gupta (2020), Benjaafar et al. (2020), Bernstein et al. (2021) also compare multi-homing with single-homing similar to us. Unlike us, though, they limit their analysis to those markets that exhibit constant returns to scale. Liu et al. (2017) compares multi-homing with single-homing in a market that exhibits economies of scale. Unlike us, they treat price and wage as exogenous decisions.

Bai and Tang (2018) look at a similar problem for two types of markets: markets that exhibit the "multi-homing effect" and markets that do not. The authors are interested in understanding the conditions under which there is a market equilibrium where two platforms co-exist. Unlike us, they do not compare the platforms' and users' utility under multi-homing with single-homing.

Nikzad (2017) also looks at a similar setting as ours, where two platforms compete over prices and wages in a market where customer waiting times depend on idle server capacity. While the paper compares monopoly and competition, our work compares multi-homing duopoly with a single-homing duopoly. By doing so, we identify a new competitive dynamic present in platform markets: as platforms gain scale, they tend to operate more efficiently, and therefore platforms price compete not only over market share, but also over scale. Multi-homing can dampen the excessive price competition by reducing a platform's ability to control its scale.

Li and Zhu (2021) use a dataset consisting of Groupon's deal offerings to empirically analyze how information sharing policies of a platform influence multi-homing behavior of its rival. They do not focus on pricing.

There are also papers in which the effects of economies of scale in a system are implicitly defined; however, this is not a primary area of focus. In (Levhari and Luski 1978, Allon and Federgruen 2007), firms compete in a market where customers form queues. Although the markets in these settings exhibit economies-of-scale, the behavior of markets with varying levels of economies-of-scale is not considered. Instead, we look at platform competition with varying levels of scale characteristics.

Another set of related papers examine how economies of scale influence competition between firms in supply chain or other one-sided service settings. In this line of work, Johari et al. (2010) examine the pricing and investment decision of firms when the congestion costs of customers decrease in the firms' investments. While their results are in line with our observations in our dedicated model, they do not provide any results that are analogous to our multi-homing setting. In a similar type of market, Cachon and Harker (2002) explores the trade-offs associated with the potential outsourcing decisions of firms. They find that firms have a strong incentive to outsource production in the presence of scale economies due to the effect of outsourcing on competitive incentives. There is a fundamental difference between outsourcing and multi-homing, which sets us apart from this line of work. When a firm outsources its production, its costs become insensitive to scale. With multi-homing, platform's costs remain sensitive to scale, but a platform's ability to control its own scale is limited. While a platform's selection of price and wage indirectly influences its own scale, so does the decision of other platforms. This means that a platform's strategy always depends on its competitors' decisions and it cannot be reduced to a single dimension, e.g. a game of prices, posing significant analytical complications.

3 Model

Consider a market with two profit-maximizing service platforms, a pool of customers and another pool of potential servers. The two platforms are denoted by subscripts i and j, and all characteristics defined on platform i apply analogously to platform j, unless otherwise noted. In this market, customers look for a service that requires a non-negligible amount of work (e.g. transportation), and the servers bring the capacity to the market to carry out the service in exchange for monetary compensation. Platforms are tasked with matching customers with servers, and a platform collects a commission from every transaction that occurs through its matching.

3.1 Agents

Customers take into account explicit and implicit costs when choosing a platform. The explicit cost associated with taking service from platform i is the amount paid for the service, p_i. The implicit cost is the disutility incurred by a customer in relation to the quality of service on the platform, g_i. In ride-sharing, this cost is analogous to the inconvenience of waiting for the service. We call the sum of these two costs the "full price", $f_i = p_i + g_i$ (see Cachon and Harker 2002).

Each platform can control the full price it charges customers, f_i, and the wage it pays to servers per unit of service time, w_i. If platform i serves λ_i customers and the average time it takes to serve a customer is τ_i, platform i earns a profit of

$$\begin{aligned}\Pi_i &= \lambda_i(p_i - \tau_i w_i) \\ &= \lambda_i\left(f_i - g_i - \tau_i w_i\right).\end{aligned}$$

Adopting a demand assumption frequently used in the platforms literature (Bai and Tang 2018, Bernstein et al. 2021), we let the customer demand for platform i be linear, decreasing in its full price and increasing in its competitor's full price:

$$\lambda_i = 1 - f_i + bf_j. \tag{1}$$

The parameter $b \in [0, 1]$ regulates the degree of differentiation of the services offered by the two platforms. If $b = 0$, the platforms offer independent services and each platform operates as a monopoly within its respective market. This is similar to a partially covered Hotelling model. When $b > 0$, services are substitutes and the degree of differentiation decreases in b. At the extreme $b = 1$, services are perfect substitutes. In this case, the total market size, $\lambda_i + \lambda_j$, is fixed and, similar to a fully covered Hotelling model, the full prices simply shift the market share allocated to each platform.

Following Lian et al. (2022), we assume that the supply market is sufficiently large and fully elastic. Servers have a homogeneous outside option with a pay-off of w_0 per unit time. They do not incur an explicit cost for working on or switching between platforms (other than their opportunity costs).

In the market, the service time associated with serving a customer is non-negligible. In ride-sharing, it involves moving to a customer's location, picking them up and dropping them off at a new location. The time it takes to complete this work is not static and can change depending on the size of the system. In a market with ample demand, it may be easier for servers to find nearby customers, reducing the total time it takes to complete a service. In other systems, a large market can increase congestion and increase workload. If servers of platform i fulfill a demand of λ_i, we let the average service time associated with that task be $\tau_i = \lambda_i^{\phi-1}$ where $\phi > 0$. If $\phi < 1$, the marginal workload to serve each customer decreases in size. In other words, it takes less work to serve an additional customer. We define this as a market with *economies of scale.* Alternatively, if $\phi > 1$, then serving each additional customer requires more work than before. This is a market with *diseconomies of scale.* Markets with $\phi = 1$ exhibit constant returns to scale. We let the service capacity of individual servers (e.g. amount of service time a server can provide) be equal to 1 without loss of generality. In our analytical results, we will focus on those markets that exhibit economies of scale, though our analysis can easily be extended to study diseconomies of scale. In our numerical study, we provide results on both types of markets.

Customers care about how busy the servers are. Typically, it is easier to find an available server if the servers are more idle. Consistent with this observation, we let customers' disutility for platform i's service, g_i, be linear, increasing in the average utilization of the servers that serve customers for platform i with slope $c \in \mathbb{R}$. That is, if platform i has μ_i many servers, these servers fulfill a demand of λ_i and the average service time is τ_i, the customers' implicit cost for taking service from platform i is $g_i = c\lambda_i\tau_i/\mu_i$.

Servers are paid for the duration they actively work and, therefore, they also care about their utilization when anticipating earnings. They expect to earn more in markets where supply is small and there is a lot of work to complete and adjust their entry accordingly. If μ_i many servers collectively serve λ_i customers with an average service duration of τ_i and earn w_i per unit time of service, each server expects to earn

$$\frac{\lambda_i\tau_i w_i}{\mu_i}.$$

Platforms send service offers to the servers upon the arrival of a customer on the system; hence, the servers also care about the timing of the offers. If two platforms pay different wages, a server may prefer an immediately available offer from a lower-wage platform to waiting for a higher-wage offer. A server accepts an offer if and only if the expected amount to be earned for the duration of the service is higher than what the server would expect to earn during that same duration if the server rejected the offer.

3.2 Operating policies

The platforms may individually adopt either of two available policies: a *dedicated policy* where the platforms make exclusive arrangements with servers to restrict them from working for the other platform, and a *multi-homing policy* where platforms allow multi-homing and the platforms operate with a combination of the two server pools. Given that there are only two platforms, if any of the platforms requires exclusivity, this forces the other platform to operate in a similar fashion (e.g. if platform i requires exclusivity and platform B does not, workers on platform j will not be able to work for platform i without abandoning platform j altogether). If neither platform makes exclusive arrangements, then the market operates with multi-homing servers. Throughout the text, we denote the dedicated operating policy with superscript $\mathcal{D}$ and the multi-homing policy with superscript $\mathcal{M}$.

Under the dedicated policy, each server works exclusively on a single platform. Any server working for a platform only performs the work associated with serving that platform's customers. Therefore, in these markets, the average time it takes to serve a customer on platform i is $\tau_i = \lambda_i^{\phi-1}$. If μ_i many servers work for platform i, the implicit cost customers observe for taking a service from platform i is $g_i = c\lambda_i^{\phi}/\mu_i$.

With multi-homing policy, a server can work for both platforms. Under the scenario where all servers accept all jobs in the market, all servers collectively serve the whole market In the analysis section, we will show that this is always the case when platforms adopt multi-homing. In these markets, the average time it takes to serve a customer on either platform is $\tau_i = (\lambda_i + \lambda_j)^{\phi-1}$. If there are μ many servers in the market, the implicit cost that a server observes for taking a service from either platform is $g_i = c(\lambda_i + \lambda_j)^{\phi}/\mu$.

3.3 Timing of decisions

Figure 1 displays the sequence of events. For both models, decisions in the system occur in three sequential stages. In the first stage, the platforms simultaneously choose whether to make exclusive arrangements with the workers. The platforms observe the outcome, and then in the second stage, both platforms simultaneously choose what price to charge to the customers and how much to pay servers. In the third stage, after observing these terms, the servers decide which platform(s) to join, if at all. After all decisions are completed, customers receive service and payments are made.

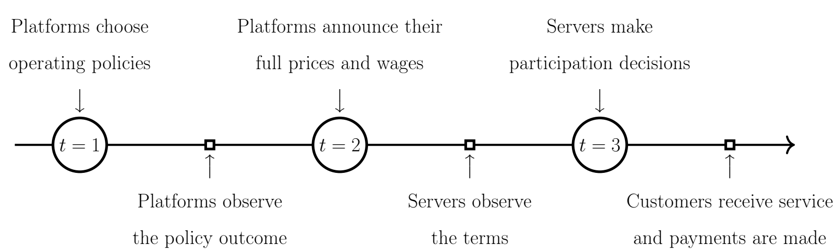

Figure 1: Sequence of events

4 Analysis

In this section, we analyze the equilibrium behavior of our model. First, in Section 4.1, we define the equilibrium characteristics. Then, we analytically study the equilibrium behavior of the market for the special case where platforms offer fully substitute services ($b = 1$) in Section 4.2 and the case without economies of scale ($\phi = 1$) in Section 4.3.

4.1 Analysis of Equilibrium

It is known that games with economies of scale may not behave well and an equilibrium is not guaranteed to exist (Cachon and Harker 2002). Our model is susceptible to similar challenges. We find that there are cases in which the equilibrium does not exist or there are multiple equilibria. In the analysis, we define these equilibrium conditions and characterize a particular type of market where uniqueness is guaranteed.

Since the game we analyze has three sequential decision steps, we use backward induction to solve for the equilibrium. First, in Section 4.1.1, we look at the servers' entry decisions. Then, in Sections 4.1.2 and 4.1.3, we first examine platform's pricing and wage decisions, and then their policy decisions.

4.1.1 Third Stage

In the third stage, prices, wages, and operating mode (dedicated or multi-homing) are fixed, and the servers decide whether to work.

With dedicated policy, servers individually prefer to work for the platform that yields the highest earnings, assuming that it pays higher than their outside option. Since the supply is fully elastic and expected earnings decrease in supply, equilibrium is achieved when all servers earn exactly w_0. Proposition 1 characterizes the equilibrium supply and service quality of a platform as a function of the decisions of the two platforms.

Proposition 1. *When the market operates with dedicated policy, there exists a unique equilibrium to the third stage game. In equilibrium, (i) the total number of servers working for platform i is*

$$\mu_i^{\mathcal{D}} = \frac{\lambda_i^{\phi} w_i}{w_0};$$

(ii) the customers' implicit cost for taking service from platform i is

$$g_i^{\mathcal{D}} = c\frac{w_0}{w_i}.$$

The equilibrium supply of a platform is proportional to the total revenue it distributes to the servers. In larger markets, there is more work to be completed and revenue to be earned. Therefore, the size of the supply market scales positively with demand. It is also linear, increasing in wage paid by the platform. With higher wages, servers earn more from each ride, which attracts more interest. Interestingly, customers' implicit cost is insensitive to the size of the demand. This is due to the free-entry characteristic of the supply market. As demand grows, more servers enter the market, thereby compensating for any direct effect of demand on utilization.

With multi-homing, a server may accept rides from both platforms, only from one platform or none of the platforms. When the two platforms pay different wages, servers naturally like to take service offers that pay more. However, they also prefer to take offers that are immediately available rather than waiting for a better offer. Proposition 2 shows that as long as both platforms pay wages above w_0, then all participating servers accept any ride offers from either platform. Furthermore, a platform that pays a wage below w_0 cannot attract any servers. This behavior is analogous to the supply market defined in Lian et al. (2022).

Proposition 2. *When multi-homing is allowed, the dominant strategy of a participating server is to accept all service offers that pay equal to or above* w_0. *If a platform pays strictly below* w_0, *no server accepts offers from that platform.*

Since participating servers accept all rides, servers of the two platforms that operate with a multi-homing policy operate as one large pool. Similar to dedicated policy, the equilibrium is achieved when all servers earn exactly w_0. Proposition 3 characterizes equilibrium supply and service quality of a platform as a function of the decisions of the two platforms.

Proposition 3. *When the market operates with multi-homing policy, there exists a unique equilibrium to the third stage game. In equilibrium, (i) the total number of servers participating in the market is*

$$\mu^{\mathcal{M}} = \frac{(\lambda_i + \lambda_j)^{\phi}}{w_0}\overline{w};$$

(ii) the customers' implicit cost for taking service from a platform is

$$g_i^{\mathcal{M}} = c\frac{w_0}{\overline{w}}$$

where $\overline{w}$ *is the average wage earned by the servers:*

$$\overline{w} = \frac{\lambda_i w_i + \lambda_j w_j}{\lambda_i + \lambda_j}. \tag{2}$$

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Since servers accept all incoming rides, there is no difference between the average utilization of a server working on platform i and platform j. As a result, both platforms operate with the same server workload. Furthermore, with multi-homing, service quality is a function of both platform's prices and wages, which means that a platform does not have full control over its service quality.

4.1.2 Second Stage

In the second stage, the platforms know the result of the first stage game, and each platform simultaneously chooses the full price and the wage that maximize its profits, taking into account the server decisions that are expected to occur in the third stage.

Analyzing the equilibrium in the second stage game is challenging because the platforms' profit-maximizing objective function is not always quasi-concave. As a result, a platform's best-response function may not necessarily be continuous, and an equilibrium to the second stage game may not exist.

Another complication is the substitutability characteristics of the platforms' wage investments. With multi-homing, number of servers accepting rides from a platform depends on the total wages paid in that market, but not on how much of that wage is paid by one particular platform. This means platforms can profitably free-ride (e.g. choose a very low wage and send ride offers to servers attracted to the market by higher-wage paying platform). Although this deviation is never an equilibrium, it can be a dominant choice, resulting in market with no equilibrium.

In order to provide a structure to the game, we focus only on those equilibria where both platforms serve non-zero customer and platforms pay wages that are strictly above the servers' outside option pay-off, w_0. We call these types of equilibria "full participation above minimum wage equilibria". This type of equilibria is expected to emerge when the supply is cheap (w_0 is small) and customers care significantly about the quality of the service (c is large). Propositions 4 and 5 show that there exists at most one such equilibrium to the game.

Proposition 4. *If the platforms operate with dedicated workers, there is a unique symmetric candidate full participation above minimum wage equilibrium to the second stage game. If an equilibrium exists, the equilibrium price of each firm, $f_i = f_j = f$, is equivalent to the smallest solution to the following implicitly defined function:*

$$\sqrt{c}\sqrt{w_0}(1+\phi)(1-(1-b)f)^{\frac{\phi-1}{2}} + 1 - 2f + bf = 0.$$

The equilibrium wages, $w_i = w_j = w$, are uniquely defined in closed form by the following equation:

$$w = \sqrt{c}\sqrt{w_0}(1-(1-b)f)^{\frac{1-\phi}{2}}.$$

The equilibrium wages of the platforms are increasing in the servers' outside option, w_0, and the service quality cost parameter, c. When it is expensive to hire servers or customers care a lot about service quality, platforms naturally need to invest more in supply. In markets with economies of scale, wages scale positively with the size of the market $\lambda_i(f, f) = 1 - f + bf$. This is due to two reinforcing effects. In a large market, platforms operate more efficiently and platforms can afford paying more to the servers. At the same time, higher wages increase service quality and drive more customers to the market. In markets with diseconomies of scale, the first effect works in reverse and wages scale inversely with market size.

Proposition 5. *If the platforms operate with multi-homing servers, any full participation above minimum wage equilibrium to the second stage game is symmetric. There exists a unique candidate full participation above minimum wage equilibrium. If the equilibrium exists, equilibrium price of each firm, $f_i = f_j = f$, is equivalent to the smallest solution to the following implicitly defined function:*

$$(5 + b - b\phi + \phi)2^{\frac{\phi}{2}-2}\sqrt{c}\sqrt{w_0}(1 - (1 - b)f)^{\frac{\phi-1}{2}} + 1 - 2f + bf = 0.$$

The equilibrium wages, $w_i = w_j = w$, are uniquely defined in closed form by the following equation:

$$w = \sqrt{c}\sqrt{w_0}2^{-\frac{\phi}{2}}(1 - (1 - b)f)^{\frac{1-\phi}{2}}.$$

The optimal wage under multi-homing satisfies the same properties as the dedicated model. However, for any set of full prices, $f_i = f_j = f$, multi-homing gives strictly lower wages. To see why, let us compare the partial derivative of a platform's objective function with respect to its wage at any symmetric set of decisions under the two policies.

$$\frac{\partial}{\partial w_i}\Pi_i^{\mathcal{P}}(f_i, f_j, w_i, w_j)\Bigg|_{\substack{f_i = f_j = f \\ w_i = w_j = w}} = \frac{cw_0(1 - (1 - b)f)}{w^2} - (1 - (1 - b)f)^{\phi},$$

$$\frac{\partial}{\partial w_i}\Pi_i^{\mathcal{M}}(f_i, f_j, w_i, w_j)\Bigg|_{\substack{f_i = f_j = f \\ w_i = w_j = w}} = \frac{cw_0(1 - (1 - b)f)}{2w^2} - 2^{\phi-1}(1 - (1 - b)f)^{\phi}.$$

These equations show us the effect of a small wage hike on the profits of platform i. In both equations, the first terms on the right-hand side capture the marginal profit gain resulting from improved service quality. The second term is profit loss due to rising direct server costs. When choosing an optimal wage, the platform strikes a balance between these two terms. Whether a policy generates higher or lower wages then depends on the relative sizes of these terms. With multi-homing, the ratio of the first term to the second term is always smaller in magnitude. This primarily occurs due to the presence of 2 in the denominator of the first term, which signifies that a platform operating with multi-homing servers gets less returns to its service quality from paying higher wages. This is due to the capacity-sharing nature of multi-homing. With multi-homing, the servers platforms attract to a market (with a wage hike) divide their capacity between the two platforms; whereas with dedicated, platforms utilize the full capacity of each server they attract to the market.

On the pricing side, a platform considers a balance between earning sufficiently high margins and having a sufficiently large market. For a set of symmetric full prices, f, and wages, w, we have

$$\frac{\partial}{\partial f_i}\Pi_i^{\mathcal{D}}(f_i, f_j, w_i, w_j)\Big|_{\substack{f_i = f_j = f \\ w_i = w_j = w}} = (1-(1-b)f) - \left(f - w(1-(1-b)f)^{\phi-1} - \frac{cw_0}{w}\right) \\ -w(1-\phi)(1-(1-b)f)^{\phi-1}. \tag{3}$$

The first two terms capture the effects of price increase that are not directly related to system efficiency. They correspond to the change in revenue associated with charging higher prices and operating a smaller market, respectively. The third term is the increase in server expenses due to the lower efficiency associated with operating a smaller market. The direction and magnitude of this term are directly related to the scale characteristics of the system. Due to economies of scale, a larger system operates more efficiently and charging higher prices decreases the size of the market and therefore the revenue.

With multi-homing, we have

$$\frac{\partial}{\partial f_i}\Pi_i^{\mathcal{M}}(f_i, f_j, w_i, w_j)\Big|_{\substack{f_i = f_j = f \\ w_i = w_j = w}} = (1-(1-b)f) - \left(f - 2^{\phi-1}w(1-(1-b)f)^{\phi-1} - \frac{cw_0}{w}\right) \\ -w(1-\phi)(1-b)2^{\phi-2}(1-(1-b)f)^{\phi-1}. \tag{4}$$

Multi-homing exhibits the same three effects. However, there are two differences. Comparing the second terms of Equations (3) and (4), we see that markets with and without multi-homing attain different profit margins, which influences pricing dynamics. More specifically, in markets with economies (diseconomies) of scale, multi-homing servers operate more (less) efficiently, and as a result, multi-homing generates larger (smaller) margins, which incentivizes platforms that allow multi-homing to set lower (higher) prices. Comparing third terms, we see that multi-homing also influences competitive dynamics through its effect on platform's ability to control the level of efficiency in the system. With a dedicated policy, the efficiency of a platform depends on its own customer market. With multi-homing, since all servers serve all customers, a platform's efficiency depends on the total size of the market. This distinction becomes important from a competitive perspective, because a platform's demand is more responsive to the platform's price than the total demand is:

$$\frac{\partial \lambda_i(f_i, f_j)}{\partial f_i} = -1,$$
$$\frac{\partial \left(\lambda_i(f_i, f_j) + \lambda_j(f_i, f_j)\right)}{\partial f_i} = -(1-b),$$

Especially in markets where platforms offer substitute products and the outside option is costly (b large), a platform with multi-homing servers cannot significantly influence its efficiency through prices. In markets with economies-of-scale, this means that platforms have less to gain from cutting their prices and gaining more market share. This can be advantageous for platforms. Not only can they charge higher prices, but they can also do so without any loss of efficiency. Through this dynamic, multi-homing dampens price competition between platforms and incentivizes platforms to operate at elevated prices.

Although Propositions 4 and 5 define the equilibrium characteristics of markets with economies of scale, the results naturally extend to markets with diseconomies of scale. Only minor difference is that, in markets with diseconomies of scale, the equilibrium full prices are equal to the largest root of the given implicitly defined price function.

4.1.3 First Stage

In the first stage, the platforms choose whether to operate with a dedicated or multi-homing policy, anticipating the implications of their decisions in the second and third stages. Let $\Pi_i^{\mathcal{D}} = \Pi_j^{\mathcal{D}} = \Pi^{\mathcal{D}}$ be the equilibrium payoffs of the platforms in the game if the outcome of the first decision stage is dedicated and $\Pi_i^{\mathcal{M}} = \Pi_j^{\mathcal{M}} = \Pi^{\mathcal{M}}$ be the payoff if the outcome of the first decision stage is multi-homing.

In the first decision stage, the platforms choose the policy that maximizes the profits they expect to earn. If they expect to earn more with a dedicated policy, then the platforms are better off with a $(\mathcal{D}, \mathcal{D})$ outcome (e.g. both platforms choose to operate with dedicated workers). Instead, if platforms expect to earn more with a multi-homing policy, they are better off with a $(\mathcal{M}, \mathcal{M})$ outcome. However, in both cases, the equilibrium is not necessarily unique. Since one platform choosing dedicated forces the competitor to operate with dedicated workers as well, platforms generate the same profits in scenarios $(\mathcal{D}, \mathcal{D})$, $(\mathcal{D}, \mathcal{M})$, $(\mathcal{M}, \mathcal{D})$. This leaves $(\mathcal{D}, \mathcal{D})$ as a potential equilibrium, even when platforms could perform better with multi-homing servers. Fortunately, as Proposition 6 shows, there exists a unique Pareto optimal outcome for the first stage game. Under the Pareto optimal equilibrium, both platforms make symmetric decisions and choose the policy that will give the highest pay-off under the assumption that their competitor will do the same.

Proposition 6. *Assuming that the second stage dedicated and multi-homing games have equilibrium solutions, a Pareto optimal outcome of the first stage game is* $(\mathcal{D}, \mathcal{D})$ *if* $\Pi^{\mathcal{D}} \geq \Pi^{\mathcal{M}}$. *Otherwise, if* $\Pi^{\mathcal{D}} < \Pi^{\mathcal{M}}$, *then the Pareto optimal outcome of the game is* $(\mathcal{M}, \mathcal{M})$.

A viable first stage equilibrium does not always exist. If platforms need to distribute a lot of revenue to attract servers to the market and customer demand for service is small, platforms may not achieve profitability. Alternatively, there exist cases where the market can be profitable, but the competitive forces in the market prevent such an equilibrium from arising. An interesting example of this is a scenario where a candidate equilibrium fails due to one platform deviating to a "free-riding" solution. In this scenario, one platform chooses to pay lower wages and operate a smaller market, and by doing so, capitalize on the scale built by a competing platform. This deviation cannot sustain a profitable equilibrium and instead leads to non-existence of a full participation above minimum wage multi-homing equilibrium.

4.2 Fully substitute services

In this section, we analyze the equilibrium characteristics of the game when platforms offer services that are perfect substitutes. This is the case where $b = 1$ in the platform's demand function in Equation (1). Following our structure in Section 4.1, we go after the unique symmetric full participation above minimum wage equilibrium of the game. Proposition 7 characterizes the equilibrium decisions and earnings of the platforms.

Proposition 7. *(i) Platforms prefer to operate with multi-homing servers if and only if there are economies of scale, that is,* $\phi < 1$. *(ii) If platforms operate with dedicated workers, the equilibrium full prices, wages and profits in the market are*

$$f_i^{\mathcal{D}} = f_j^{\mathcal{D}} = 1 + (1+\phi)\sqrt{cw_0 a^{\phi-1}},$$
$$w_i^{\mathcal{D}} = w_j^{\mathcal{D}} = \sqrt{cw_0},$$
$$\Pi_i^{\mathcal{D}*} = \Pi_j^{\mathcal{D}*} = 1 - (1-\phi)\sqrt{cw_0}.$$

(iii) If platforms operate with multi-homing servers, the equilibrium full prices, wages and profits in the market are

$$f_i^{\mathcal{M}} = f_j^{\mathcal{M}} = 1 + \frac{3}{2}\sqrt{2^{\phi} cw_0 a^{\phi-1}},$$
$$w_i^{\mathcal{M}} = w_j^{\mathcal{M}} = \sqrt{2^{-\phi} cw_0}.$$
$$\Pi_i^{\mathcal{M}*} = \Pi_j^{\mathcal{M}*} = 1.$$

Proposition 7 tells us that in markets where platforms offer fully substitute service, platforms prefer operating with multi-homing servers if and only if the market exhibits economies of scale ($\phi < 1$). In these markets, multi-homing dampens the competition among the platforms and allows platforms to operate with increased efficiency.

The stability of such an equilibrium requires that the equilibrium wages be above w_0. That is, a necessary condition for this equilibrium to be stable is

$$\sqrt{2^{-\phi}c} \geq \sqrt{w_0}.$$

This means that the equilibrium we define here is applicable to those markets where there are sufficiently strong economies of scale, the demand intercept is large, customers care about service quality and servers' outside options are not too high.

Proposition 8. *When platforms offer fully substitute services, in equilibrium, platforms operating with multi-homing policy (i) charge higher full price; (ii) pay less wage; (iii) attract less supply to the market; (iv) offer inferior service quality.*

Proposition 8 shows that multi-homing's benefits to the platforms come at the expense of other stakeholders. Multi-homing leads to higher full prices charged to the customer and a smaller supply attracted to the market. This is consistent with our earlier observation that multi-homing dampens excessive price competition between platforms and incentivizes underinvestment in supply capacity.

4.3 No economies of scale

In this section, we analyze a market that does not exhibit economies of scale. This is true if $\phi = 1$. Similarly, we solve for the unique symmetric full participation above minimum wage equilibrium of the game. Proposition 9 characterizes the equilibrium decisions and the earnings of the platforms.

Proposition 9. *(i) Platforms always prefer to operate with dedicated workers, except at $b = 1$, in which case the platforms are indifferent between the two policies. (ii) If platforms operate with dedicated workers, the equilibrium full prices, wages and profits in the market are*

$$f_i^* = f_j^* = \frac{1 + 2\sqrt{cw_0}}{(2-b)},$$

$$w_i^* = w_j^* = \sqrt{cw_0},$$

$$\Pi^{\mathcal{D}*} = \frac{(1 - 2(1-b)\sqrt{cw_0})^2}{(2-b)^2}.$$

(iii) If platforms operate with multi-homing servers, the equilibrium full prices, wages and profits in the market are

$$f_i^* = f_j^* = \frac{2 + 3\sqrt{2}\sqrt{cw_0}}{2(2-b)},$$
$$w_i^* = w_j^* = \sqrt{\frac{cw_0}{2}},$$
$$\Pi^{\mathcal{M}*} = \frac{(2 - 3\sqrt{2}(1-b)\sqrt{cw_0})^2}{4(2-b)^2}.$$

Proposition 9 tells us that in markets that do not exhibit economies of scale, platforms prefer to operate with dedicated workers. In these markets, multi-homing's efficiency and competition dampening effects disappear. The only remaining effect is the supply underinvestment, which works against platforms' interests. Hence, platforms have nothing to gain from multi-homing.

The stability of such an equilibrium requires that the equilibrium wages be above w_0:

$$\sqrt{\frac{c}{2}} \geq \sqrt{w_0}.$$

Consistent with our earlier observation, this means that the equilibrium we define is applicable to those markets where customers care about service quality and servers' outside options are not too high.

Proposition 10. *In markets that exhibit constant returns to scale, in equilibrium, platforms operating with multi-homing policy (i) charge higher full price; (ii) pay less wage; (iii) attract less supply to the market; (iv) offer inferior service quality.*

By Proposition 10, the demand and supply markets are larger with dedicated policy. Hence, stakeholders have nothing to gain from multi-homing in markets that do not exhibit economies of scale.

5 Numerical Study

In this section, we numerically evaluate the game and analyze the equilibrium behavior of the model over a wide range of parameters. Throughout the numerical study, unless we explicitly vary a parameter, we assign them the following values: $w_0 = 0.1$, $c = 0.5$, $b = 0.5$, $\phi = 0.9$.

In Figure 2, we plot how the optimal policy chosen by the platforms changes with respect to a change in the substitutability of services offered, b, and the economies of scale exhibited in the market, ϕ. Consistent with our observation in Section 4.2, platforms prefer multi-homing only if the market exhibits sufficiently strong economies of scale. In such markets, the efficiency of multi-homing and its dampening effects on competition dominate the downsides. The latter effect is stronger when platforms offer substitute services, making multi-homing more attractive when b is high.

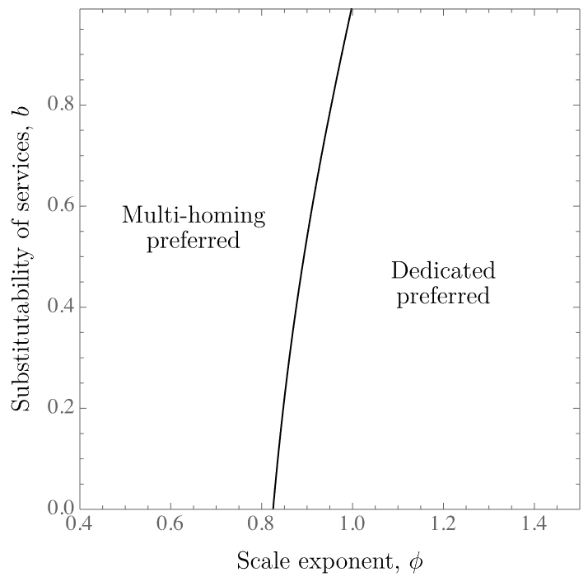

Figure 2: The platform's best policy across the (ϕ, b) space.

To verify the validity of this observation for a wider range of scenarios, we numerically check across the parameters $b \in \{0.1, 0.2, ..., 0.9\}$, $w_0 \in \{0.1, 0.2, ..., 0.9\}$, $c \in \{1, 2, ..., 5\}$ and $\phi \in \{0.5, 0.6, ..., 1.5\}$ when platforms prefer a multi-homing policy. In all instances, we identify a $\tilde{\phi} \in [0, 1]$ such that the platforms prefer multi-homing if and only if $\phi < \tilde{\phi}$.

In Figure 2, most of the variation in optimal policies appears to occur in the vicinity of $\phi = 0.9$. Around that region, we also want to understand how the cost of attracting supply influences the optimal policy. To this end, we plot the outcome of the game in the (w_0, c) space in Figure 3. Since multi-homing dampens competition and reduces the need to invest in a large supply, it is naturally more advantageous when supply is expensive. In fact, if supply is very costly, multi-homing can be the only option for platforms to generate a profitable equilibrium. If supply is cheap and customers care a lot about service quality, then platforms are better off operating dedicated workers. In such cases, multi-homing disincentivizes investment in supply capacity. As a result, platforms offer inferior service quality, which hurts their margins.

Now, we turn our attention to other stakeholders. We do not have a utility model for the customers and all servers in our model earn their outside options. Therefore, we cannot obtain meaningful results by evaluating those agents' utilities. Instead, we use the total demand and the total number of servers on the market as proxies for the collective utility of those agent groups. In a large set of scenarios, multi-homing leads to a smaller customer and server market. Figure

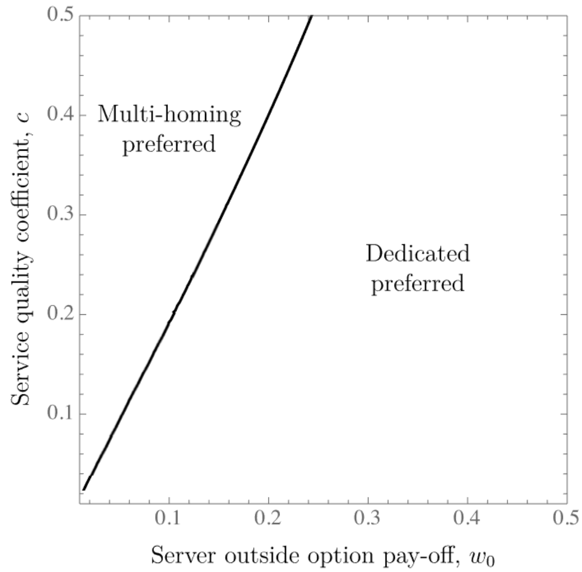

Figure 3: The platform's best policy across the (w_0, c) space.

4 illustrates an exception to this. When it is expensive to hire servers, service substitutability is low and customers care little about service quality, customers benefit from multi-homing. In these markets, high supply costs and lack of concern for quality incentivize platforms to operate with a smaller market. Multi-homing, through its efficiency benefits, can help platforms earn higher margins. When that efficiency is very strong, the benefits can spill over to customers. These are also the regions in which platforms operating with multi-homing policy set their prices more competitively, leading to lower equilibrium full prices.

6 Conclusion

In the freelance economy, it is becoming a norm rather than an exception for servers to work on multiple platforms. Despite its prevalence, though, multi-homing in the digital economy and its implications for the stakeholders are not well understood. Our aim with this paper was to shed light on the following question: How does a market with multi-homing workers differ from one where platforms have dedicated workers and who does multi-homing benefit?

There are upsides and downsides to working with multi-homing workers. We find that the upsides are present only in those markets that exhibit strong economies of scale. In these markets, multi-homing promotes efficient allocation of servers to tasks that are close to them, allowing them to operate more efficiently. At the same time, by reducing platforms' ability to fully control their scale, multi-homing dampens competition between the platforms. These dynamics can push platforms to

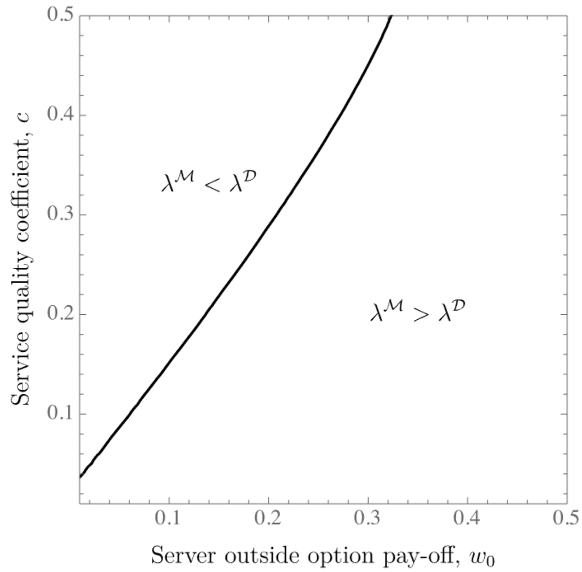

Figure 4: Equilibrium demands across (w_0, c) space at b=0.1

prefer multi-homing.

However, in other markets, platforms may be better off with dedicated workers. With multi-homing, the servers' capacity is shared between platforms. With less to gain from building a large supply, a platform can get away with investing less in servers and free-riding on the servers attracted to the market by the competing platform. This *tragedy of the commons* can result in markets with a small supply market and poor service quality.

The potential benefits of multi-homing generally do not always spill over to customers and servers. Customers and servers like to work on large platforms since they are more efficient. The dedicated policy incentivizes investment in supply and promotes price competition between platforms, generally working to the advantage of customers and workers. Though the dedicated policy does a good job of aligning platforms' incentives with customers and servers, it is not always very efficient. Especially in markets where supply is very expensive, platforms operating with a dedicated policy cannot easily gain scale. In these markets, multi-homing helps platforms operate more efficiently by combining the scales of the two platforms. In instances like these, where the efficiency of multi-homing allows the platform to operate in an otherwise unattractive market, the benefits of multi-homing spill over to the customers and the servers. In other situations, though, multi-homing leads to a weaker supply market and higher full prices charged to the customers.

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Appendix

Proof of Proposition 1. If there are μ_i many servers participating on platform i, the average earning of a server on platform i is

$$\frac{\lambda_i \tau_i w_i}{\mu_i} = \frac{\lambda_i^\phi w_i}{\mu_i}.$$

Since the supply market is fully elastic, an equilibrium is achieved when servers earn exactly w_0 on average:

$$\frac{\lambda_i^\phi w_i}{\mu_i} = w_0.$$

Hence, in equilibrium, the number of servers working on platform i is

$$\mu_i = \frac{\lambda_i^\phi w_i}{w_0}.$$

The customers' implicit cost for taking service from platform i is

$$g_i = c\frac{\lambda_i^\phi}{\mu_i} = c\frac{\lambda_i^\phi}{\frac{\lambda_i^\phi w_i}{w_0}} = \frac{w_0}{w_i}.$$

Proof of Proposition 2. First, let us note that since servers have an outside option that pays w_0 per unit of time, a necessary condition for a platform to be able to attract servers is to pay at least w_0. If a platform pays strictly less than w_0, then that platform cannot attract any servers.

Let us now show that, in equilibrium, all participating servers accept all rides from both platforms. Without loss of generality, assume that platform i pays lower wages than platform j, $w_0 \leq w_i < w_j$. Since the market is fully elastic, all servers in equilibrium earn exactly w_0 per unit time of service (if not, some servers would leave or exit the market, breaking that equilibrium).

Now consider a server's decision-making when the server receives a ride offer from platform i. Let us say that the terms of the offer is to pay w_i per unit time of service for a duration of τ_i. For the duration of τ_i, the best a server can expect to earn in equilibrium is $\tau_i w_0$ no matter what strategy it follows. If server accepts ride from platform i, the server expects to earn from that service is $\tau_i w_i$. Since $w_i \geq w_0$, server's decision to accept ride from platform i is a weakly dominating strategy. Since this is true for any marginal server, all servers are weakly better off accepting rides from both platforms.

Proof of Proposition 3. By Proposition 1, a server participating in the market accepts service from both platforms. If there are μ many servers participating in the market, the average earnings of a server is

$$\frac{\lambda_i \tau_i w_i + \lambda_j \tau_j w_j}{\mu} = \frac{\lambda_i (\lambda_i + \lambda_j)^{\phi-1} w_i + \lambda_j (\lambda_i + \lambda_j)^{\phi-1} w_j}{\mu} = (\lambda_i + \lambda_j)^{\phi-1} \frac{\lambda_i w_i + \lambda_j w_j}{\mu}.$$

Similar to Proposition 1, an equilibrium is achieved when servers earn exactly w_0 on average:

$$(\lambda_i^{\phi-1} + \lambda_j^{\phi-1}) \frac{\lambda_i w_i + \lambda_j w_j}{\mu} = w_0.$$

Therefore, in equilibrium, the number of servers working in the market is

$$\mu = (\lambda_i + \lambda_j)^{\phi-1} \frac{\lambda_i w_i + \lambda_j w_j}{w_0}.$$

The customers' implicit cost for taking service from platform i is

$$g_i = c \frac{(\lambda_i + \lambda^j)^\phi}{\mu} = c \frac{(\lambda_i + \lambda^j)^\phi}{(\lambda_i + \lambda_j)^{\phi-1} \frac{\lambda_i w_i + \lambda_j w_j}{w_0}} = c(\lambda_i + \lambda_j) \frac{w_0}{\lambda_i w_i + \lambda_j w_j}.$$

Proof of Proposition 4. Platform i has a profit function:

$$\Pi_i^{\mathcal{D}} = \lambda_i \left(f_i - w_i \lambda_i^{\phi-1} - c \frac{w_0}{w_i} \right).$$

Platform chooses the full price and the wage that maximizes its profits subject to the constraint that the utilization of servers does not exceed 100%. Formally, the constraint is

$$\frac{\lambda_i \tau_i}{\mu_i} = \frac{w_0}{w_i} \leq 1. \tag{5}$$

Total utilization, as defined in the inequality above, is monotone decreasing in w_i. Hence, for a given full price f_i, Equation (5) defines a lower bound on the wage selected by platform i.

Let us consider platform's problem as a sequence of optimization problems, where the platform chooses a wage first and then a full price. Within the feasible region (e.g. region where platforms have non-negative demand and supply), for any selection of f_i, platform's problem is concave in w_i:

$$\frac{\partial^2}{\partial w_i^2} \Pi_i^{\mathcal{D}} = -2 c w_0 \frac{\lambda_i}{w_i^3} < 0.$$

Ignoring the constraint on utilization, the wage that maximizes platform's objective function is uniquely defined by the equation

$$\frac{\partial}{\partial w_i} \Pi_i^{\mathcal{D}} = \lambda_i \left(c \frac{w_0}{w_i^2} - \lambda_i^{\phi-1} \right) = 0. \tag{6}$$

For a fixed full price, the optimal wage of the platform i will be the maximum of the solution to (6) and the lower bound imposed by Equation (5):

$$\begin{aligned} w_i^* &= \max\left\{\frac{\sqrt{c}\sqrt{w_0}}{\sqrt{\lambda_i^{\phi-1}}}, w_0\right\} \\ &= \begin{cases} \frac{\sqrt{c}\sqrt{w_0}}{\sqrt{\lambda_i^{\phi-1}}} & , w_0 < c\lambda_i^{1-\phi}, \\ w_0 & , w_0 \geq c\lambda_i^{1-\phi}. \end{cases} \end{aligned} \tag{7}$$

Plugging this into platform's objective:

$$\begin{aligned} \Pi_i^{\mathcal{D}} &= \begin{cases} \lambda_i \left(f_i - 2\sqrt{c}\sqrt{w_0}\sqrt{\lambda_i^{\phi-1}}\right) & , w_0 < c\lambda_i^{1-\phi}, \\ \lambda_i \left(f_i - w_0\lambda_i^{\phi-1} - c\right) & , w_0 \geq c\lambda_i^{1-\phi}. \end{cases} \\ &= \begin{cases} (1-f_i+bf_j)\left(f_i - 2\sqrt{c}\sqrt{w_0}\sqrt{(1-f_i+bf_j)^{\phi-1}}\right) & , w_0 < c(1-f_i+bf_j)^{1-\phi}, \\ (1-f_i+bf_j)\left(f_i - w_0(1-f_i+bf_j)^{\phi-1} - c\right) & , w_0 \geq c(1-f_i+bf_j)^{1-\phi}. \end{cases} \end{aligned} \tag{8}$$

The first case in (8) corresponds to the interior case, where the utilization of the servers on platform is strictly below 100%. The second case is a boundary case where the utilization of servers is 100%.

Since we are interested in full participation above minimum wage equilibria and a platform's optimal wage is equal to the minimum wage in the boundary case, we can restrict our search for a candidate equilibrium point to the region where the interior case holds. Such a region is defined by the inequality

$$w_0 < c(1-f_i+bf_j)^{1-\phi}.$$

The condition above defines an upper bound on the full prices chosen by platform i. There is also a natural restriction on how low a platform can set its price. In a full participation equilibrium where both platforms earn non-negative profits, a platform's choice of full price needs to be strictly larger than 0, since otherwise platform's margin from each customer will be strictly negative.

Furthermore, since the region defined by these upper and lower bounds are strict inequalities, conditional on the equilibrium being a full participation above minimum wage one, best response function of each platform will be characterized by the First Order Condition:

$$\frac{\partial \Pi_i^{\mathcal{D}}}{\partial f_i} = \sqrt{c}\sqrt{w_0}(\phi+1)\sqrt{(1-f_i+bf_j)^{\phi-1}} + 1 + bf_j - 2f_i = 0$$

The Second Order Condition shows that the objective function is concave when $\phi \geq 1$, but not necessarily when $\phi < 1$:

$$\frac{\partial^2 \Pi_i^{\mathcal{D}}}{\partial f_i^2} = -\frac{\sqrt{c}\sqrt{w_0}\left(\phi^2-1\right)\sqrt{(1-f_i+bf_j)^{\phi-1}}}{2(1-f_i+bf_j)} - 2 < 0\,, \forall \phi \geq 1$$

For $\phi < 1$, we can show that the function is concave-convex by taking the third derivative of the objective function with respect to f_i and showing that it is positive:

$$\frac{\partial^3 \Pi_i^{\mathcal{D}}}{\partial f_i^3} = \frac{\sqrt{c}\sqrt{w_0}(\phi-3)(\phi-1)(\phi+1)\sqrt{(1-f_i+bf_j)^{\phi-1}}}{4(1-f_i+bf_j)^2} > 0.$$

Both concave and concave-convex functions have a single interior maximum. This means that for all ϕ values, the first and second order conditions define the interior maximizer. In a symmetric equilibrium, the optimal prices are then defined by the following set of equations:

$$\begin{aligned} \sqrt{c}\sqrt{w_0}(\phi+1)\sqrt{(1-(1-b)f)^{\phi-1}}+1+b-f=0, \\ -\frac{\sqrt{c}\sqrt{w_0}\left(\phi^2-1\right)\sqrt{(1-(1-b)f)^{\phi-1}}}{2(1-(1-b)f)}-2<0. \end{aligned} \tag{9}$$

While any full participation above minimum wage equilibria is defined through the set of equations above, the existence and uniqueness of such equilibria are not guaranteed. To show uniqueness, first note that the left-hand side of the first equation above is convex when $\phi < 1$:

$$\frac{\partial^2}{\partial f^2}\left(\sqrt{c}\sqrt{w_0}(\phi+1)\sqrt{(1-(1-b)f)^{\phi-1}}+1+b-f\right) \\ = \frac{(b-1)^2\sqrt{c}\sqrt{w_0}(\phi-3)(\phi-1)(\phi+1)(1-(1-b)f)^{\phi-3}}{4\sqrt{(1-(1-b)f)^{\phi-1}}} > 0.$$

This means that there are at most two points where the first equation is satisfied: one point where the equation crosses 0 from above and another where the equation crosses 0 from below. We can eliminate one of these solutions by showing that the following set of equations cannot simultaneously hold in Mathematica:

$$\begin{aligned} \sqrt{c}\sqrt{w_0}(\phi+1)\sqrt{(1-(1-b)f)^{\phi-1}}+1+b-f=0, \\ -\frac{\sqrt{c}\sqrt{w_0}\left(\phi^2-1\right)\sqrt{(1-(1-b)f)^{\phi-1}}}{2(1-(1-b)f)}-2<0, \\ \frac{\partial}{\partial f}\left(\sqrt{c}\sqrt{w_0}(\phi+1)\sqrt{(1-(1-b)f)^{\phi-1}}+1+b-f\right)>0. \end{aligned}$$

This means the only possible equilibrium is one where the expression on the left-hand side of the equality constraint in Equation (9) crosses 0 from above. Since that expression is convex, this solution corresponds to the smallest solution of that equality.

Proof of Proposition 5. Platform i has a profit function:

$$\Pi_i^{\mathcal{D}} = \lambda_i\left(f_i - w_i(\lambda_i+\lambda_j)^{\phi-1} - c(\lambda_i+\lambda_j)\frac{w_0}{\lambda_i w_i + \lambda_j w_j}\right).$$

Platform chooses the full price and the wage that maximizes its profits subject to the constraint that the utilization of servers does not exceed 100%. Formally, the constraint is

$$(\lambda_i + \lambda_j)\frac{w_0}{\lambda_i w_i + \lambda_j w_j} \le 1. \tag{10}$$

Total utilization, as defined in the inequality above, is monotone decreasing in w_i. Hence, for a given full price f_i, Equation (10) defines a lower bound on the wage selected by platform i.

Let us consider platform's problem as a sequence of optimization problems, where the platform chooses a wage first and then a full price. Within the feasible region (e.g. region where platforms have non-negative demand and supply), for any selection of f_i, platform's problem is concave in w_i:

$$\frac{\partial^2}{\partial w_i^2}\Pi_i^{\mathcal{M}} = -\frac{2c(\lambda_i+\lambda_j)\lambda_i^3 w_0}{(\lambda_i w_i + \lambda_j w_j)^3} < 0.$$

Ignoring the constraint on utilization, the wage that maximizes platform's objective function is uniquely defined by the equation

$$\frac{\partial}{\partial w_i}\Pi_i^{\mathcal{M}} = \lambda_i\left(\frac{c(\lambda_i+\lambda_j)\lambda_i w_0}{(\lambda_i w_i + \lambda_j w_j)^2} - (\lambda_i+\lambda_j)^{\phi-1}\right) = 0. \tag{11}$$

For a fixed full price, the optimal wage of the platform i will be the maximum of the solution to (11) and the lower bound imposed by Equation (10):

$$\begin{aligned} w_i^* &= \max\left\{\frac{\sqrt{c}\sqrt{\lambda_i}\sqrt{w_0} - \lambda_j w_j\sqrt{(\lambda_i+\lambda_j)^{\phi-2}}}{\lambda_i\sqrt{(\lambda_i+\lambda_j)^{\phi-2}}}, \frac{(\lambda_i+\lambda_j)w_0 - \lambda_j w_j}{\lambda_i}\right\} \\ &= \begin{cases} \frac{\sqrt{c}\sqrt{\lambda_i}\sqrt{w_0}-\lambda_j w_j\sqrt{(\lambda_i+\lambda_j)^{\phi-2}}}{\lambda_i\sqrt{(\lambda_i+\lambda_j)^{\phi-2}}} & , w_0\lambda^\phi < c\lambda_i, \\ \frac{(\lambda_i+\lambda_j)w_0-\lambda_j w_j}{\lambda_i} & , w_0\lambda^\phi \ge c\lambda_i. \end{cases} \end{aligned} \tag{12}$$

Plugging this into platform's objective:

$$\begin{aligned} \Pi_i^{\mathcal{M}} &= \begin{cases} -2\sqrt{c}\sqrt{\lambda_i}\sqrt{w_0}(\lambda_i+\lambda_j)\sqrt{(\lambda_i+\lambda_j)^{\phi-2}} + f_i\lambda_i + \lambda_j w_j(\lambda_i+\lambda_j)^{\phi-1} & , w_0\lambda^\phi < c\lambda_i, \\ -c\lambda_i + f_i\lambda_i + (\lambda_i+\lambda_j)^\phi\left(\frac{\lambda_j w_j}{\lambda_i+\lambda_j} - w_0\right) & , w_0\lambda^\phi \ge c\lambda_i. \end{cases} \\ &= \begin{cases} -2\sqrt{c}\sqrt{w_0}\sqrt{1-f_i+bf_j}\sqrt{(2-(1-b)(f_i+f_j))^\phi} \\ \quad +w_j(1-f_j+bf_i)(2-(1-b)(f_i+f_j))^{\phi-1} \\ \quad +f_i(1-f_i+bf_j) & , w_0(2-(1-b)(f_i+f_j))^\phi < c(1-f_i+bf_j), \\ (c-f_i)(-1-bf_j+f_i) - w_0(2-(1-b)(f_i+f_j))^\phi \\ \quad +w_j(1-f_j+bf_i)(2-(1-b)(f_i+f_j))^{\phi-1} & , w_0(2-(1-b)(f_i+f_j))^\phi \ge c(1-f_i+bf_j). \end{cases} \end{aligned} \tag{13}$$

The first case in (8) corresponds to the interior case, where the utilization of the servers on platform is strictly below 100%. The second case is a boundary case where the utilization of servers is 100%.

Since we are interested in full participation above minimum wage equilibria and a platform's optimal wage is equal to the minimum wage in the boundary case, we can restrict our search for a candidate equilibrium point to the region where the interior case holds. Such a region is defined by the inequality

$$w_0(2+(b-1)(f_i+f_j)^{\phi} < c(1-f_i+bf_j).$$

When $\phi < 1$, the condition above defines an upper bound on the full prices chosen by platform i. There is also a natural restriction on how low a platform can set its price. In a full participation equilibrium where both platforms earn non-negative profits, a platform's choice of full price needs to be strictly larger than 0, since otherwise platform's margin from each customer will be strictly negative.

Furthermore, since the region defined by these upper and lower bounds are strict inequalities, conditional on the equilibrium being a full participation above minimum wage one, best response function of each platform will be characterized by the First Order Condition:

$$\begin{aligned}\frac{\partial \Pi_i^{\mathcal{M}}}{\partial f_i} &= w_j((b-1)\phi(1-f_j+bf_i)+(b+1)(1-(1-b)f_j))(2-(1-b)(f_i+f_j))^{\phi-2} \\ &\quad+\frac{\sqrt{c}\sqrt{w_0}(-b\phi+\phi+2)\sqrt{(2-(1-b)(f_i+f_j))^{\phi-2}}}{\sqrt{1-f_i+bf_j}}+1+bf_j-2f_i \\ &\quad+\frac{(b-1)\sqrt{c}\sqrt{w_0}(-bf_j\phi+f_i\phi+f_i+f_j)\sqrt{(2-(1-b)(f_i+f_j))^{\phi-2}}}{\sqrt{1-f_i+bf_j}}=0\end{aligned}$$

For $\phi < 1$, we can show that the function is concave-convex by taking the third derivative of the objective function with respect to f_i and showing that it is positive for all set of full prices and wages where both platforms participate (e.g. non-negative demands):

$$\begin{aligned}\frac{\partial^3 \Pi_i^{\mathcal{M}}}{\partial f_i^3} &= \frac{3\sqrt{c}\sqrt{w_0}(2-(1-b)(f_i+f_j))^{\phi/2}}{4(1-f_i+bf_j)^{5/2}}+\frac{3(b-1)^2\sqrt{c}\sqrt{w_0}(\phi-2)\phi(2-(1-b)(f_i+f_j))^{\frac{\phi}{2}-2}}{4\sqrt{1-f_i+bf_j}} \\ &\quad-\frac{1}{4}(b-1)^3\sqrt{c}\sqrt{w_0}(\phi-4)(\phi-2)\phi\sqrt{1-f_i+bf_j}(2-(1-b)(f_i+f_j))^{\frac{\phi}{2}-3} \\ &\quad+4(b-1)^2 w_j(\phi-2)(\phi-1)((b-1)\phi+3)(2-(1-b)(f_i+f_j)) \\ &\quad+4(b-1)^2 w_j(\phi-2)(\phi-1)((1-b)(\phi-3)(1-f_i+bf_j)) \\ &\quad+\frac{3(b-1)\sqrt{c}\sqrt{w_0}\phi(2-(1-b)(f_i+f_j))^{\frac{\phi}{2}-1}}{4(1-f_i+bf_j)^{3/2}}>0.\end{aligned}$$

Both concave and concave-convex functions have a single interior maximum. This means that for all ϕ values, the first and second order conditions define the interior maximizer. In a symmetric equilibrium, the optimal prices are then defined by the following set of equations:

$$\begin{aligned}(5+b-b\phi+\phi)2^{\frac{\phi}{2}-2}\sqrt{c}\sqrt{w_0}(1-(1-b)f)^{\frac{\phi-1}{2}}+1-2f+bf=0, \\ \sqrt{c}\sqrt{w_0}2^{\frac{\phi}{2}-3}\left(b^2(3\phi-2)+2b\phi-5\phi+6\right)(1-(1-b)f)^{\frac{\phi-3}{2}}-2<0.\end{aligned} \tag{14}$$

Note here that while any full participation above minimum wage equilibria is defined through the set of equations above, the existence and uniqueness of such equilibria is not guaranteed. To show uniqueness, first note that the left-hand side of the first equation above is convex when $\phi < 1$:

$$\frac{\partial^2}{\partial f^2}\left((5+b-b\phi+\phi)2^{\frac{\phi}{2}-2}\sqrt{c}\sqrt{w_0}(1-(1-b)f)^{\frac{\phi-1}{2}}+1-2f+bf\right)$$
$$=-(b-1)^2\sqrt{c}\sqrt{w_0}2^{\frac{\phi}{2}-4}(\phi-3)(\phi-1)(b(\phi-1)-\phi-5)(1-(1-b)f)^{\frac{\phi-5}{2}}>0.$$

This means that there are at most two points where the first equation is satisfied: one point where the equation crosses 0 from above and another where the equation crosses 0 from below. We can eliminate one of these solutions by showing that the following set of equations cannot simultaneously hold in Mathematica:

$$(5+b-b\phi+\phi)2^{\frac{\phi}{2}-2}\sqrt{c}\sqrt{w_0}(1-(1-b)f)^{\frac{\phi-1}{2}}+1-2f+bf=0,$$
$$\sqrt{c}\sqrt{w_0}2^{\frac{\phi}{2}-3}\left(b^2(3\phi-2)+2b\phi-5\phi+6\right)(1-(1-b)f)^{\frac{\phi-3}{2}}-2<0,$$
$$\frac{\partial}{\partial f}\left((5+b-b\phi+\phi)2^{\frac{\phi}{2}-2}\sqrt{c}\sqrt{w_0}(1-(1-b)f)^{\frac{\phi-1}{2}}+1-2f+bf\right)>0.$$

This means that the only possible equilibrium is one in which the expression on the left-hand side of the equality constraint in Equation (14) crosses 0 from above. Since that expression is convex, this solution corresponds to the smallest solution of that equality.

To show that the only possible equilibrium is a symmetric one, let us look at the optimal wage decision by the two platforms. We have

$$\begin{aligned}\frac{\partial}{\partial w_i}\Pi_i^{\mathcal{M}} &= \lambda_i\left(\frac{c(\lambda_i+\lambda_j)\lambda_i w_0}{(\lambda_i w_i+\lambda_j w_j)^2}-(\lambda_i+\lambda_j)^{\phi-1}\right)=0,\\ \frac{\partial}{\partial w_j}\Pi_j^{\mathcal{M}} &= \lambda_j\left(\frac{c(\lambda_i+\lambda_j)\lambda_j w_0}{(\lambda_i w_i+\lambda_j w_j)^2}-(\lambda_i+\lambda_j)^{\phi-1}\right)=0.\end{aligned} \tag{15}$$

In a full participation equilibrium, both conditions can only be satisfied if $\lambda_i = \lambda_j$, implying $f_i = f_j = f$. Now, let us look at the optimal price decision keeping wages fixed, and also plug in the symmetric prices and the optimal $w_i + w_j$ we get through the expression above:

$$\begin{aligned}\frac{\partial}{\partial f_i}\Pi_i^{\mathcal{M}} &= w_i 2^{\phi-2}(1-b\phi+b+\phi)(1-(1-b)f)^{\phi-1}+1+\frac{cw_0(-bw_i+(b+3)w_j+w_i)}{(w_i+w_j)^2}+(b-2)f=\\ &=2^{\phi-2}(1-(1-b)f)^{\phi-1}(-b(w_i(\phi-2)+w_j)+w_i\phi-3w_j)+1+(b-2)f=0,\\ \frac{\partial}{\partial f_j}\Pi_j^{\mathcal{M}} &= w_j 2^{\phi-2}(1-b\phi+b+\phi)(1-(1-b)f)^{\phi-1}+1+\frac{cw_0(-bw_j+(b+3)w_i+w_j)}{(w_i+w_j)^2}+(b-2)f=\\ &=2^{\phi-2}(1-(1-b)f)^{\phi-1}(-b(w_j(\phi-2)+w_i)+w_j\phi-3w_i)+1+(b-2)f=0.\end{aligned} \tag{16}$$

Equations together imply

$$-b(w_i(\phi-2)+w_j)+w_i\phi-3w_j=-b(w_j(\phi-2)+w_i)+w_j\phi-3w_i=2^{2-\phi}(a+(b-1)f)^{2-\phi},$$

which can hold if and only if $w_i = w_j = w$. Hence, in any full participation above minimum wage equilibria, the optimal full prices and wages chosen by the two platforms are symmetric.

Proof of Proposition 6. Since dedicated is a dominant solution, both platforms operate with the same policy in the second stage game, regardless of their decision in the first stage. This means that if both platforms choose to operate with multi-homing, they earns multi-homing profits. In all other scenarios, including in which exactly one platform operates with multi-homing, both platforms earn profits equivalent to the two platforms operating with dedicated policy.

Regardless of the pay-offs, the outcome $(\mathcal{D}, \mathcal{D})$ is a Nash equilibrium. That's because a platform has nothing to gain from deviation from that solution. That's due to the dominant nature of dedicated policy. As long as at least one platform chooses to operate with dedicated workers, the other platform is essentially forced. However, it can be a Pareto inferior outcome.

If $\Pi^{\mathcal{D}} \geq \Pi^{\mathcal{M}}$, then $(\mathcal{D}, \mathcal{D})$ is both a dominating and Pareto optimal outcome. However, if $\Pi^{\mathcal{M}} > \Pi^{\mathcal{D}}$, then $(\mathcal{D}, \mathcal{D})$ is Pareto inferior. Each platform is weakly better by choosing multi-homing, regardless of the opponent's decision. In that case $(\mathcal{M}, \mathcal{M})$ is the dominating and Pareto optimal outcome of the game.

Proof of Proposition 7. The FOC that define the equilibrium full prices and wages in a dedicated equilibrium are given in Proposition 4. We can solve them in explicit form when $b = 1$. In the interior case, the equalities transform to

$$1 + (\phi + 1)\sqrt{cw_0} - f = 0, \; w = \sqrt{cw_0},$$

giving the solution,

$$f = 1 + (1 + \phi)\sqrt{cw_0}, \; w = \sqrt{cw_0}.$$

Plugging the solution back into platform's objective, we get

$$\Pi^{\mathcal{D}} = 1 - (1 - \phi)\sqrt{cw_0}.$$

An interior equilibrium exists if and only if the interior equilibrium utilization is strictly below 100%. That holds true when

$$\frac{w_0}{w} = \frac{\sqrt{w_0}}{\sqrt{c}} < 1.$$

The FOC that define the equilibrium full prices and wages in a multi-homing equilibrium are given in Proposition 5. We can solve them in explicit form when $b = 1$. In the interior case, the equalities transform to

$$1 + \tfrac{3}{2}\sqrt{2^{\phi}cw_0} - f = 0, \; w = \sqrt{2^{-\phi}cw_0},$$

giving the solution,

$$f = 1 + \tfrac{3}{2}\sqrt{2^{\phi} c w_0},\ w = \sqrt{2^{-\phi} c w_0}.$$

Plugging the solution back into platform's objective, we get

$$\Pi^{\mathcal{M}} = 1.$$

An equilibrium is interior if and only if the interior equilibrium utilization is strictly below 100%. That holds true when

$$\frac{w_0}{w} = \frac{\sqrt{w_0}}{\sqrt{2^{-\phi} c}} < 1.$$

Notice that

$$\Pi^{\mathcal{D}} = 1 - (1-\phi)\sqrt{c w_0} < 1 = \Pi^{\mathcal{M}} \iff \phi < 1.$$

Proof of Proposition 8. Let $f^{\mathcal{D}}$ and $w^{\mathcal{D}}$ be the equilibrium full prices and wages under a dedicated policy when $b = 1$. Let $f^{\mathcal{M}}$ and $w^{\mathcal{M}}$ be same for multi-homing policy. Similarly, let $\mu^{\mathcal{D}}$, $\mu^{\mathcal{M}}$, $g^{\mathcal{D}}$, $g^{\mathcal{M}}$ define the equilibrium supplies and service quality cost in the market respectively. The expressions for the equilibrium full price and wage are defined in Proposition 7. We have

$$\begin{aligned}
f^{\mathcal{D}} &= 1 + (1+\phi)\sqrt{c w_0} < 1 + \frac{3}{2}\sqrt{2^{\phi} c w_0} = f^{\mathcal{M}},\\
w^{\mathcal{D}} &= \sqrt{c w_0} > \sqrt{2^{-\phi} c w_0} = w^{\mathcal{M}},\\
\mu^{\mathcal{D}} &= 2\frac{w^{\mathcal{D}}}{w_0} > \frac{2^{\phi} w^{\mathcal{M}}}{w_0} = \mu^{\mathcal{M}}\\
&\iff 2\sqrt{c}\sqrt{w_0} > 2^{\phi/2}\sqrt{c}\sqrt{w_0},\\
g^{\mathcal{D}} &= \frac{w_0}{w^{\mathcal{D}}} < \frac{w_0}{w^{\mathcal{M}}} = g^{\mathcal{M}}.
\end{aligned}$$

Proof of Proposition 9. The FOC that define the equilibrium full prices and wages in a dedicated equilibrium are given in Proposition 4. We can solve them in explicit form when $\phi = 1$. In the interior case, the equalities transform to

$$1 + (b-2)f + 2\sqrt{c}\sqrt{w_0} = 0,\ w = \sqrt{c w_0},$$

giving the solution,

$$f = \tfrac{1+2\sqrt{c}\sqrt{w_0}}{2-b},\ w = \sqrt{c w_0}.$$

Plugging the solution back into platform's objective, we get

$$\Pi^{\mathcal{D}} = \frac{\left(1 + 2(b-1)\sqrt{c}\sqrt{w_0}\right)^2}{(b-2)^2}.$$

An interior equilibrium exists if and only if the interior equilibrium utilization is strictly below 100%. That holds true when

$$\frac{w_0}{w} = \frac{\sqrt{w_0}}{\sqrt{c}} < 1.$$

The FOC that define the equilibrium full prices and wages in a multi-homing equilibrium are given in Proposition 5. We can solve them in explicit form when $\phi = 1$. In the interior case, the equalities transform to

$$1 + (b-2)f + \tfrac{3\sqrt{c}\sqrt{w_0}}{\sqrt{2}} = 0 \;, w = \sqrt{\tfrac{cw_0}{2}},$$

giving the solution,

$$f = \frac{1+\frac{3\sqrt{c}\sqrt{w_0}}{\sqrt{2}}}{2-b}, \; w = \sqrt{\tfrac{cw_0}{2}}.$$

Plugging the solution back into platform's objective, we get

$$\Pi^{\mathcal{M}} = \frac{\left(2+3\sqrt{2}(b-1)\sqrt{c}\sqrt{w_0}\right)^2}{4(b-2)^2}.$$

An equilibrium is interior if and only if the interior equilibrium utilization is strictly below 100%. That holds true when

$$\frac{w_0}{w} = \frac{\sqrt{2w_0}}{\sqrt{c}} < 1.$$

Notice that

$$\Pi^{\mathcal{D}} = \frac{\left(1+2(b-1)\sqrt{c}\sqrt{w_0}\right)^2}{(b-2)^2} \geq \frac{\left(2+3\sqrt{2}(b-1)\sqrt{c}\sqrt{w_0}\right)^2}{4(b-2)^2} = \Pi^{\mathcal{M}}$$

for all those equilibria with full participation.

Proof of Proposition 10. Let $f^{\mathcal{D}}$ and $w^{\mathcal{D}}$ be the equilibrium full prices and wages under a dedicated policy when $\phi = 1$. Let $f^{\mathcal{M}}$ and $w^{\mathcal{M}}$ be same for multi-homing policy. Similarly, let $\mu^{\mathcal{D}}$, $\mu^{\mathcal{M}}$, $g^{\mathcal{D}}$, $g^{\mathcal{M}}$ define the equilibrium supplies and service quality cost in the market respectively. The expressions for the equilibrium full price and wage are defined in Proposition 9. We have

$$f^{\mathcal{D}} = \frac{1+2\sqrt{c}\sqrt{w_0}}{2-b} < \frac{1+\frac{3\sqrt{c}\sqrt{w_0}}{\sqrt{2}}}{2-b} = f^{\mathcal{M}},$$

$$w^{\mathcal{D}} = \sqrt{cw_0} > \sqrt{\frac{cw_0}{2}} = w^{\mathcal{M}},$$

$$\mu^{\mathcal{D}} = 2\frac{(1-(1-b)f^{\mathcal{D}})w^{\mathcal{D}}}{w_0} > \frac{2(1-(1-b)f^{\mathcal{M}})w^{\mathcal{M}}}{w_0} = \mu^{\mathcal{M}},$$

$$g^{\mathcal{D}} = \frac{w_0}{w^{\mathcal{D}}} < \frac{w_0}{w^{\mathcal{M}}} = g^{\mathcal{M}}.$$