Platform competition with intertwined network effects: theory and evidence*

Bruno Carballa-Smichowski†

March 2025

Abstract

In many transaction two-sided markets, an aggregator platform (e.g., a lodging metasearch platform) serves as a gateway to other platforms while providing a competing matching service. Despite the prevalence of this phenomenon, including multiple prominent mergers, little is known about it. I model a firm acting as a matching platform and as an aggregator that gives buyers access to a competing matching platform's sellers. This creates "intertwined network effects" (INE) between the platforms. I show that while INE increase consumer surplus, they reduce seller surplus if the platforms are sufficiently differentiated for sellers, and increase it otherwise. In presence of INE, a non-consolidating merger harms consumers if the network effects they enjoy are sufficiently low, and vice versa. If the platforms are sufficiently homogeneous to sellers, the merger reduces their surplus. Using a Subgroup Difference-in-Differences design, I exploit the introduction of INE between the generalist classified platforms Adverts and DoneDeal to test the model's predictions. Consistent with these predictions, I show that INE caused an increase in the number of users in the aggregator (Adverts) and in the number of users across both platforms. I discuss the implications of the findings for merger control and asymmetric interoperability policies.

JEL Classification: D43, L13, L86, L41.

Keywords: intertwined network effects, two-sided markets, platform competition, platform merger, horizontal merger, competitive bottleneck.

*I thank Robert Somogyi for his helpful comments on a previous version of this article. The views and opinions expressed in this article are those of the author and do not necessarily reflect the official policy or position of the Joint Research Centre or the European Commission.

†European Commission's Joint Research Centre, Seville, Spain. E-mail: bruno.carballa-smichowski@ec.europa.eu.

1 Introduction

In many transaction two-sided markets, aggregator platforms serve as gateways to other matching platforms while providing a competing matching service. For example, Trivago allows travelers to be matched with hotels either directly or through a third-party lodging platform. In such settings, buyers (e.g., travelers) on an aggregator platform (e.g., Trivago) can access sellers (e.g, hotels) listed on a separate “source” platform (e.g., Booking). As illustrated in Figure 1 below, this allows platform users on one side of the market to benefit from the presence of users on the other side of the market participating in a *competing* platform. In such instances, I say the two platforms set up *intertwined network effects* (INE).

Examples of aggregator platforms with INE include retail marketplaces (Google Shopping, Bing Shopping), price comparison sites (Price Runner), lodging metasearch platforms (Google Hotels, Trivago), real estate platforms (real estate online platforms hosting listings from real estate agencies, Jinka) and digital wallets aggregating debit and credit cards (Apple Pay, We Chat, Pay Pal, Google Pay) or other digital wallets (Revolut/Bizum). Importantly, aggregators and source platforms might be horizontally differentiated on both sides of the market. For example, generalist marketplaces and price comparison sites aggregate specialized marketplaces that only partially compete with them (e.g., Coches.net/Milanuncios, Adverts/DoneDeal, Idealo/Otto). Moreover, across multiple industries, it is common for platforms having set up intertwined network effects to merge while maintaining the two firms active.[1] Examples include eBay/Motors, Adevinta/Gumtree, Se Loger/Logic Immo, Rightmove/PrimeLocation and eBay/StubHub.

Recent merger cases have featured intertwined network effects. Some of them involve aggregators merging with a source platform without consolidating. In others, platforms have (tried to) set up INE post-merger, typically arguing this practice constitutes a merger efficiency (e.g., Trade Me/Property NZ). Competition authorities' approaches to merger cases involving INE have been varied. Some have not factored INE into their assessment of the welfare effects of the merger (e.g., Schibsted/Nettbil, Trade Me/Property NZ, viagogo/Stubhub). Conversely, other competition authorities have deemed INE detrimental to specific user groups arguing that intensified network effects would increase the merged entity's market power (e.g., Booking/eTraveli, FDJ/Zeturf, Seek Asia Investments/JobStreet, Wedding Planner/Zank you).

The extant literature has investigated the effects of platform competition and mergers under various settings. However, little attention has been paid to how intertwined network effects could alter established results. This article studies the effect of intertwined network effects on platform participation, prices and the welfare of the two sides of the market. It also shows how a non-consolidating merger between the platforms affects these outcomes.

[1] In this case, the merger is said to be “non-consolidating”. For simplicity, I will hereafter say “merger” to refer to a *non-consolidating* merger.

I develop a model in which two horizontally-differentiated platforms compete to facilitate transactions between two types of users benefiting from positive indirect network effects: buyers and sellers. I consider a competitive bottleneck setting in which buyers singlehome and sellers can multihome. Sellers are charged a per-transaction price, while buyers interact for free. If the platforms decide to set up intertwined network effects, the buyers in the aggregator platform and the sellers in the source platform interact. For each of these cross-platform interactions, the aggregator platform charges an endogenously-determined per-transaction referral fee to the source platform. I first consider the scenario with intertwined network effects and compare it to one without intertwined network effects when the platforms are legally independent. I then compare a scenario in which the platforms set INE and merge without consolidating with one in which legally-independent platforms set INE. In both comparisons, I characterize, for each platform and in the aggregate, the effects on both user groups' participation, prices and surpluses.

I show that INE benefit consumers on the aggregator platform and harm consumers of the source platform. However, the overall effect on consumer surplus is always positive. The gain in consumer surplus brought about by the aggregator platform consumers' capacity to reach all sellers always more than compensates the harm to the source platform's consumers triggered by an INE-driven drop in seller participation. As for sellers, if the platforms are sufficiently differentiated for them, INE decrease their surplus, and vice versa. The reason is that INE increase participation and seller surplus in the aggregator platform, which is able to attract more consumers than without INE. The opposite happens in the source platform, leading some of its sellers to migrate to the aggregator. With sufficiently low seller transportation costs, enough sellers migrate from the source platform to the aggregator, leading to an increase in overall seller surplus, and vice versa.

If the network effects enjoyed by consumers are sufficiently low, the merger decreases their surplus, and vice versa. By eliminating the referral fee, the merger drives the aggregator platform to increase the per-transaction price it charges to sellers and the source platform to decrease it, while the average price drops. This leads some sellers and consumers to migrate from the aggregator to the source platform. If the stand-alone utility a consumer obtains from joining any of the two platforms is sufficiently low, network effects have a significant weight in her decision to join a platform over another. In this case, a sufficiently high amount of consumers switch from the aggregator (where the number of sellers decreases) to the source platform (where the number of sellers increases), which increases overall consumer surplus. The opposite happens if the stand-alone utility is sufficiently high.

A non-consolidating merger between two platforms having set up INE, in turn, reduces seller surplus if the platforms are sufficiently homogeneous to them. The decrease in the average price paid by sellers increases their surplus. However, the migration of sellers and buyers from the aggregator to the source platform reduces the overall number of interactions. Both single-homing sellers in the aggregator platform and multihoming sellers who, because of INE, meet the aggregator platform's buyers in both platforms, have less interactions post-merger. This decreases seller surplus. The lower seller transportation costs are, the more sellers and thus consumers change their participation decisions post-merger; hence, the lower the number of post-merger interactions.

The remainder of the article is organized as follows. Section 2 discusses the related literature. Section 3 presents the model. Section 4 studies the impact of intertwined network effects on platform participation, prices and the welfare of buyers and sellers when the platforms are legally independent. Section 5 analyzes how a non-consolidating merger affects these outcomes. Section 6 tests the predictions of the model by studying the effect of the introduction of INE between the classified platforms Adverts and DoneDeal on platform participation. Section 7 concludes and discusses the implications of the findings for merger control and asymmetric interoperability policies.

2 Related literature

This article relates to the vast literature on price competition between platforms, notably that focusing on a competitive-bottleneck setting (Belleflamme and Peitz, 2019; Armstrong, 2006). More precisely, given its focus on intertwined network effects, it contributes to the literature on interoperability with network effects (Rasch et al., 2023; Shekhar et al., 2022; Rasch and Wenzel, 2014; Doganoglu and Wright, 2006; Crémer et al., 2000; Katz and Shapiro, 1985; Farrell and Saloner, 1985). Within this literature, the article is particularly relevant to the work on interoperability between competing platforms with indirect network effects. This article also contributes to the growing literature on platform mergers (Ivaldi and Zhang, 2022; Farronato et al., 2020; Correia-da Silva et al., 2019; Tan and Zhou, 2019; Baranes et al., 2014; Chandra and Collard-Wexler, 2009), notably to the strand focusing on non-consolidating horizontal mergers.

Interoperability between competing platforms with indirect network effects. This article relates to the literature studying the effect of interoperability (also referred to as "compatibility") in markets subject to network effects. Indeed, I model INE as an *asymmetric* form of compatibility between two platforms in which, for each platform, only one of the user groups can interact with the other user group present in another platform.

Specifically, this article relates to the strand of this literature that, building on the seminal contributions focusing on the compatibility between network goods subject to direct network effects (e.g., Katz and Shapiro, 1985; Farrell and Saloner, 1985; Farrell and Saloner, 1986; and Katz and Shapiro, 1994), shifted to studying the choice and welfare effects of compatibility between competing platforms enabling interactions between user groups subject to *indirect* network effects. The closest contribution to ours is Maruyama and Zennyo (2015). Building on Rasch and Wenzel (2014)'s results, they consider a setting in which two symmetric platforms intermediating between consumers and content providers can independently decide whether to be compatible with each other or not at a fixed cost.

Consumers singlehome while content providers might multihome. When the fixed cost of interoperability is intermediate, the equilibrium is an "asymmetric case" in which one platform chooses compatibility while the other does not. This generates cross-platform network effects analogous to what I call "intertwined network effect" in this article. They find that the asymmetric case leads to a drop in consumer surplus. Moreover, as in this article, if the network effects enjoyed by content providers are sufficiently small, content providers' surplus increases, and vice versa.

To the best of my knowledge, other contributions to this literature have focused on cases in which interoperability is symmetric across platforms. An established result in this setting is that symmetric platforms serving single-homing users have an excessive incentive to be compatible with respect to a social planner. This is because compatibility renders demand less elastic, which benefits platforms but might hurt users in absence of sufficient market expansion, as in Crémer et al. (2000). Doganoglu and Wright (2006) make this argument in their study of the interplay between platforms' compatibility decisions and users' decisions to multihome. When users can multihome, compatibility leads to market contraction because it makes multihom unnecessary, as users can already benefit from all the network externalities without joining more than one platform (Doganoglu and Wright, 2006; Salim, 2010). It also makes demand more elastic (as consumers joining decisions are rival), which reduces prices and profits while increasing consumer surplus. Thus, platforms have insufficient incentives to be compatible with respect to the social optimum. Rasch and Wenzel (2014) encompass both cases in their study of a competitive-bottleneck model with single-homing users and multihoming content developers, both subject to a membership fee charged by horizontally-differentiated symmetric platforms. They find that the private incentives to choose compatibility can be insufficient or excessive. The key mechanism here is the change in content provision. From a social-welfare perspective, compatibility is desirable if it increases content, which happens if content providers' network effects are sufficiently strong, and vice versa.

This article contributes to this literature by examining the relatively under-researched, yet prevalent, case of asymmetric compatibility (i.e., INE) between platforms. This focus on asymmetric compatibility in platforms facilitating interactions between two user groups experiencing indirect network effects sets it apart from previous studies (with the exception of Maruyama and Zennyo (2015)). The main departure from this literature lies not only in the focus on INE, but also on some key modeling choices, namely per-transaction prices and referral fees, as well as differential transportation costs across user groups. Although the focus of this article is not determining when the choice of (asymmetric) interoperability is socially-optimal, it shows that platforms might or might not choose to be interoperable; however, it always benefits consumers. Moreover, I provide a new mechanism to explain the a priori ambiguous effect of compatibility on seller surplus that relies on the extent to which platform differentiation allows sellers to reallocate across platforms and increase multihoming. With sufficiently low seller transportation costs, enough sellers leave the source platform (where prices increase) for the aggregator platform (where prices decrease), leading to an increase in overall seller surplus, and vice versa

Non-consolidating horizontal platform mergers. The literature on non-consolidating horizontal platform mergers has explored the conditions under which the merged entity has incentives to lower prices and thus benefit at least one side of the market. Chandra and Collard-Wexler (2009) were the first to make this point in a model of mergers between newspapers intermediating between advertisers and readers. The latter are assumed to be heterogeneous and hence advertisers' value of being in the platform depends on the composition of consumers they can access. Because the platform cannot target consumers and price-discriminate, a marginal consumers' contribution might be negative. Therefore, price hikes might lead consumers to choose a newspaper to which their contribution to profit is negative. This can generate incentives to lower prices post-merger.

One of the main results that the literature subsequently established under various settings is that, if network effects are sufficiently strong, the merged entity has incentives to lower prices post-merger. Leonello (2010) makes this argument in a model of two platforms competing à la Hotelling on both sides of the market. If indirect network effects are sufficiently strong in side 2, the price decreases and demand increases in side 1. Important to this article, post-merger, the platforms also become interoperable in that side-1 users in platform A can access side-2 users of both platforms. This reinforces the merged entity's incentives to lower prices. Baranes et al. (2014) extend Leonello (2010)'s model to four platforms equidistantly located on a Salop circle with linear externalities and full market coverage. Similarly, they conclude that mergers between adjacent platforms may lead to lower prices if externalities are sufficiently strong. Tan and Zhou (2019) reverse this argument with a model that includes the possibility of non-linear externalities and in which consumers have a random utility function. Assuming full market coverage, they show that the merged entity always has incentives to increase prices unless there are strong cost-related efficiency gains.

I contribute to this literature by exploring a new setting that generates an unstudied reason why a non-consolidating horizontal platform merger might harm users. In a competitive bottleneck model with INE and per-transaction pricing on the multihoming side, the less differentiated the platforms are on the single-homing side, the more likely it is that the merger will harm both sides of the market. The reason is that, despite the overall increase in seller participation triggered by the merger, strong differentiation discourages single-homing users from sufficiently switching from the aggregator platform (where the number of sellers decreases) to the source platform (where the number of sellers increases).

3 The model

In this section I present the model in its two settings: with and without (i.e., the benchmark) intertwined network effects. In accordance with the observed features of matching platforms that set up INE, the model considers i) a competitive bottleneck setting in which buyers singlehome and sellers can multihome[2], ii) a per-transaction price on the seller side and zero-pricing on the buyer side and iii) when INE are set up, an endogenously-determined (per-transaction) referral fee charged by the aggregator to the source platform.

Model overview. There are two symmetric platforms in the market that compete to enable interactions between a unit mass of sellers and a unit mass of buyers. This interaction generates homogeneous, positive indirect network effects to both user groups. Platform $i \in \{1, 2\}$ competes for sellers through the per-transaction price p_i^s it charges them. The platforms have constant marginal costs, which, for simplicity, are normalized to zero. Buyers perceive the platforms as horizontally differentiated. I model horizontal differentiation à la Hotelling, where platform 1 is located at $x = 0$ and platform 2 at $x = 1$. Both consumers and sellers are uniformly distributed on a unit interval and face an opportunity cost of joining a platform that increases linearly over the distance at rates τ^b and τ^s, respectively.

Buyers singlehome while sellers can choose to multihome. Buyers and sellers interact with a seller every time they meet. Each interaction generates a benefit α^b for the buyer and α^s for the seller. Buyers obtain the same stand-alone utility v^b from joining a platform. I assume this benefit is sufficiently high for the market to be covered on the buyer side. Let n_i^b and n_i^s denote the mass of buyers and sellers active on platform i, respectively.

In the following subsections, two settings of the model are described: the benchmark (denoted with the superscript B) and the intertwined network effects setting (denoted with the superscript *INE*).

3.1 Benchmark setting

Utility and profit functions. In the benchmark, the utility of a buyer located at $x \in [0, 1]$ is:

$$U_i^{bB}(x, p_1^s, p_2^s) \equiv v^b + \alpha^b n_i^s(p_1^s, p_2^s) - \tau^b |x^i - x| \tag{1}$$

The utility of a seller located at $x \in [0, 1]$ is:

$$U_i^{sB}(x, p_1^s, p_2^s) \equiv n_i^b(p_1^s, p_2^s)(\alpha^s - p_i^s) - \tau^s |x^i - x| \tag{2}$$

The profit of platform $i \in \{1, 2\}$ is given by:

$$\Pi_i{}^B \equiv n_i^b(p_1^s, p_2^s) n_i^s(p_1^s, p_2^s) p_i^s \tag{3}$$

[2] Duch-Brown (2017) provides empirical evidence of the prevalence of seller multihoming in Europe for a wider scope of platforms that includes the following categories: “marketplaces”, “apps stores”, “social networks” and “online advertising”.

Timing. In the first stage, platforms simultaneously set the per-transaction prices charged to sellers (p_1^s, p_2^s). In the second stage, consumers and sellers simultaneously choose which platform(s) to join.

3.2 Intertwined network effects setting

Utility and profit functions. In a setting in which platforms choose to set up intertwined network effects (INE, hereafter) at no cost, the buyers served by platform 1 can access all the sellers that have joined platform 2 in addition to the sellers having joined platform 1. Conversely, the sellers that decide to join platform 2 can access not only the buyers served by platform 2, but also those served by platform 1. Hereafter, I refer to platform 1 as the *aggregator* and to platform 2 as the *source platform*.

To keep the model tractable, I assume "full double counting" of the network effects from overlapping agents. In other words, if a consumer and a multihoming seller meet twice, they both obtain two times the benefit of the interaction. In the motivating examples of platforms that set up INE, it is likely that there is "partial double counting", whereby meeting an agent on the other side of the market a the second time yields partial additional network benefits. For instance, in listing platforms, seeing the same listing in two platforms can provide consumers additional value by giving access to more photographs, richer product information and additional reviews. For sellers, it can increment the visibility of their offering, increasing so the probability of a sale (Bakos and Halaburda, 2020). Consequently, it is common in these platforms for sellers to be present both in an aggregator and a source platform. The results obtained through the *ad summum* assumption of full double counting provides an upper bound of the benefits of setting up INE for both user groups.

Transactions between a platform 1 buyer and a platform 2 seller take place in platform 2. Platform 1 charges a referral fee f to platform 2 for each of these transactions. Therefore, in the INE setting, the utility of a buyer located at $x \in [0,1]$ joining platform $i \in \{1,2\}$ is given by:

$$U_i^{b^{INE}}(x, p_1^s, p_2^s) \equiv \begin{cases} v^b + \alpha^b \left(n_1^s(p_1^s, p_2^s) + n_2^s(p_1^s, p_2^s)\right) - \tau^b |0 - x| & \text{if } i = 1 \\ v^b + \alpha^b n_2^s(p_1^s, p_2^s) - \tau^b |1 - x| & \text{if } i = 2 \end{cases} \tag{4}$$

The utility of a seller located at $x \in [0,1]$ joining platform $i \in \{1,2\}$ is in turn given by:

$$U_i^{s^{INE}}(x, p_1^s, p_2^s) \equiv \begin{cases} n_1^b(p_1^s, p_2^s)(\alpha^s - p_1^s) - \tau^s |0 - x| & \text{if } i = 1 \\ \left(n_1^b(p_1^s, p_2^s) + n_2^b(p_1^s, p_2^s)\right)(\alpha^s - p_2^s) - \tau^s |1 - x| & \text{if } i = 2 \end{cases} \tag{5}$$

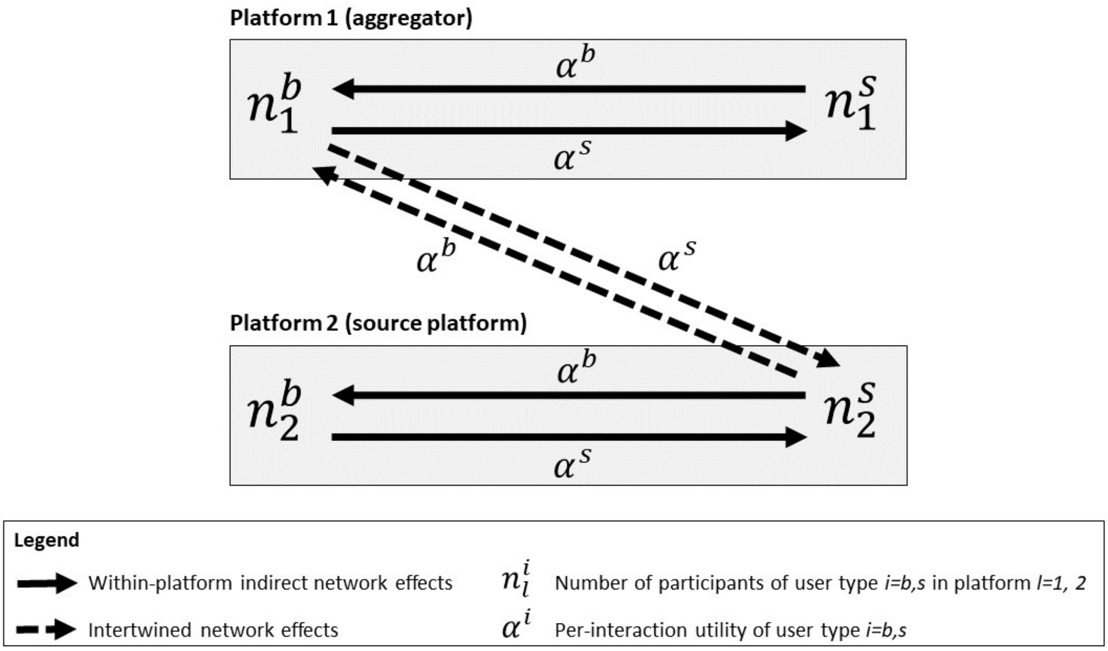
And the profit of platform $i \in \{1,2\}$ is given by:

$$\Pi_i^{INE} \equiv \begin{cases} n_1^b(p_1^s, p_2^s) n_1^s(p_1^s, p_2^s) p_1^s + n_1^b(p_1^s, p_2^s) n_2^s(p_1^s, p_2^s) f & \text{if } i = 1 \\ n_2^b(p_1^s, p_2^s) n_2^s(p_1^s, p_2^s) p_2^s + n_1^b(p_1^s, p_2^s) n_2^s(p_1^s, p_2^s)(p_2^s - f) & \text{if } i = 2 \end{cases} \tag{6}$$

Timing. In the first stage, platform 1 sets the per-transaction referral fee f. In the second stage, platforms simultaneously set the per-transaction prices charged to sellers (p_2^1, p_2^s). In the third stage, consumers and sellers simultaneously choose which platform(s) to join.

Figure 1 illustrates the network effects that exist in each setting.

Figure 1: Indirect network effects with and without intertwined network effects



4 Intertwined network effects between legally-independent firms

In this section I characterize the equilibria of the two settings described in Section 3. I assume full information for all participants in the model, i.e., each participant observes all the price decisions and knows all the parameters of the model. The equilibrium concept is the Subgame Perfect Nash equilibrium.

4.1 Equilibrium without intertwined network effects (benchmark)

In this subsection I consider the benchmark setting described in Section 3.1. The aim is to characterize the equilibrium in which both platforms are active.

Stage 2: participation decisions. In the second stage, the consumer indifferent between joining platform 1 and 2 is located at $\tilde{x}^B$ such that ${U_1^b}^B(\tilde{x}^B) = {U_2^b}^B(\tilde{x}^B)$. Thus, the number of consumers buying from platform 1 is equal to $\tilde{x}^B$ and the number of consumers buying from platform 2 is equal to $1 - \tilde{x}^B$. Sellers are divided into three sub-intervals on the unit interval. Sellers located "on the left" only join platform 1. Those located "on the right" join only platform 2. Those located "in the middle" join both platforms and can thus interact with both platforms' buyers. The seller indifferent between joining platform 1 and not joining it is located at x_{10}^B such that ${U_s^1}^B = 0$. The seller indifferent between joining platform 2 and not joining it is located at x_{20}^B such that ${U_s^2}^B = 0$. Then, the sellers that singlehome in platform 1 are located in the $[0, x_{20}^B]$ sub-interval, those who singlehome in platform 2 in the $[x_{10}^B, 1]$ sub-interval and those who multihome in the (x_{20}^B, x_{10}^B) sub-interval. To focus on the interesting case in which there is multihoming in the benchmark, I assume for the time being that $0 < x_{20}^B < x_{10}^B < 1$ (I provide the necessary and sufficient conditions below), so that $n_s^1 = x_{10}^B$ and $n_s^2 = 1 - x_{20}^B$.[3] Then, the number of buyers and sellers in each platform is found by solving the following system of four equations and four unknowns:

$$
\begin{aligned}
n_1^b &= \frac{\tau^b + n_1^s\alpha^b - n_2^s\alpha^b}{2\tau^b} \\
n_2^b &= 1 - \frac{\tau^b + n_1^s\alpha^b - n_2^s\alpha^b}{2\tau^b} \\
n_1^s &= \frac{n_1^b(\alpha^s - p_1^s)}{\tau^s} \\
n_2^s &= 1 - \frac{n_2^b(p_2^s + \tau^s - \alpha^s)}{\tau^s}
\end{aligned}
$$

Which yields buyers' and sellers' participation as a function of seller prices and the model's parameters.

$$
\begin{aligned}
n_1^b(p_1^s, p_2^s) &= \frac{\tau^b\tau^s + \alpha^b(p_2^s - \alpha^s)}{2\tau^b\tau^s + \alpha^b(p_1^s + p_2^s - 2\alpha^s)} \\
n_2^b(p_1^s, p_2^s) &= \frac{\tau^b\tau^s + \alpha^b(p_1^s - \alpha^s)}{2\tau^b\tau^s + \alpha^b(p_1^s + p_2^s - 2\alpha^s)} \\
n_1^s(p_1^s, p_2^s) &= \frac{(p_1^s - \alpha^s)(\alpha^b(\alpha^s - p_2^s) - \tau^b\tau^s)}{\tau^s(2\tau^b\tau^s + \alpha^b(p_1^s + p_2^s - 2\alpha^s))} \\
n_2^s(p_1^s, p_2^s) &= \frac{(p_2^s - \alpha^s)(\alpha^b(\alpha^s - p_1^s) - \tau^b\tau^s)}{\tau^s(2\tau^b\tau^s + \alpha^b(p_1^s + p_2^s - 2\alpha^s))}
\end{aligned}
\tag{7}
$$

Stage 1: platforms' choice of prices. In stage 1, each platform solves the maximization program $\max\limits_{p_i^s} \Pi^i(p_1^s, p_2^s)$. Solving the system of first-order conditions yields only one set of equilibrium symmetric prices that satisfy the second-order conditions and for which

[3] Note that the existence of multihoming on the seller side in the benchmark allows for an elastic demand on the money-making side of the market. This will be important to explain the effects of INE (cf. Section 4.3) and of a merger between platforms having set up INE (cf. Section 5.2).

$0 < x_{20}^B < x_{10}^B < 1$ (i.e., for which there is multihoming on the seller side):

$$p_i^s = \alpha^s - \frac{\tau^b \tau^s \sqrt{\tau^b \tau^s (\tau^b \tau^s - \alpha^b \alpha^s)}}{\alpha^b} \tag{8}$$

The equilibrium per-interaction price charged to sellers depends positively on the benefit they obtain from an interaction with buyers on the platform, net of transportation costs. This is evident from (8), where the price increases with α^s and decreases with τ^s.[4] This price also depends positively on the per-interaction net benefit buyers obtain from participating in the platform. A higher net benefit for buyers increases their participation on the platform, making the platform more valuable for sellers and thus leading to a higher price.

Replacing these equilibrium prices in (7) gives the equilibrium participation on the buyer and seller sides:

$$n_b^i = \frac{1}{2},$$
$$n_s^i = \frac{\tau^b}{2\alpha^b} - \frac{\sqrt{\tau^b \tau^s (\tau^b \tau^s - \alpha^b \alpha^s)}}{2\tau^s \alpha^b}.$$

Consumer and seller surplus are calculated respectively as:

$$CS^B \equiv \int_0^{\tilde{x}^B} U_1^{b\,B}(x)dx + \int_{\tilde{x}^B}^1 U_2^{b\,B}(x)dx,$$
$$SS^B \equiv \int_0^{x_{10}^B} U_1^{s\,B}(x)dx + \int_{1-x_{20}^B}^1 U_2^{s\,B}(x)dx.$$

Then, the equilibrium values of the model in the benchmark setting (i.e., without intertwined network effects) are the following.

[4] The term $-\frac{\tau^b \tau^s \sqrt{\tau^b \tau^s (\tau^b \tau^s - \alpha^b \alpha^s)}}{\alpha^b}$ is strictly negative, as per the second-order condition on the seller side reported in Assumption 1.

Lemma 1 **(Equilibrium without intertwined network effects).** *In the benchmark setting equilibrium:*

$$
\begin{aligned}
n_b^i &= \frac{1}{2},\\
n_b^B &= 1,\\
n_i^s &= \frac{\tau^b}{2\alpha^b} - \frac{\sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)}}{2\tau^s\alpha^b},\\
n_s^B &= \frac{\tau^b\tau^s - \sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)}}{\tau^s\alpha^b},\\
p_s^i &= \alpha^s - \frac{\tau^b\tau^s\sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)}}{\alpha^b},\\
\Pi_i^B &= \frac{\left(\tau^b\tau^s - \sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)}\right)\left(\alpha^b\alpha^s + \sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)} - \tau^b\tau^s\right)}{4\tau^s{\alpha^b}^2},\\
\Pi^B &= \frac{\left(\tau^b\tau^s - \sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)}\right)\left(\tau^b\tau^s - \alpha^b\alpha^s - \sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)}\right)}{2\tau^s{\alpha^b}^2},\\
CS^B &= v^b - \frac{\sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)}}{2\tau^s},\\
SS^B &= \frac{\tau^b\left(4\tau^s\alpha^b + \alpha^b\alpha^s + 2\sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)}\right) - 2{\tau^b}^2\tau^s - 2\alpha^b\left(\tau^s\alpha^b + 2\sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)}\right)}{4{\alpha^b}^2}.
\end{aligned}
$$

Assumptions. For the equilibrium reported in Lemma 1 to be valid, a series of conditions summarized in Assumption 1 have to hold. First, the second-order conditions of the platforms' maximization program are met if $\alpha^b\alpha^s < \tau^b\tau^s$. As it is usually the case in platform competition models, these conditions require that indirect network effects are sufficiently small compared to platforms' horizontal differentiation. They are also sufficient to ensure a stable and unique equilibrium in which both platforms are active. Second, to ensure that the market is covered on the buyer side, I need to verify that the consumer indifferent between the two platforms obtains a positive surplus, i.e., ${U_i^b}^B(\tilde{x}^B) > 0$. This condition is more stringent than the first. Third, I impose that some sellers multihome in equilibrium, which in turn implies full coverage on the seller side. Then, in equilibrium, $0 < x_{20}^B < x_{10}^B < 1$. Collecting these conditions, I obtain:

Assumption 1 (Benchmark conditions). *In the benchmark setting without intertwined network effects, the parameters satisfy the following conditions.*

$$\alpha^b\alpha^s < \tau^b\tau^s \tag{SOCs^B}$$

$$\alpha^b\alpha^s < \tau^b\tau^s and \tau_b^2\tau^s < 4v_b^2\tau^s + \tau^b\alpha^b\alpha^s \tag{FPB^B}$$

$$
\begin{aligned}
&-\tau^b\tau^s + \tau^s\alpha^b + \sqrt{-\tau^b\tau^s(\tau^b\tau^s - \alpha^b\tau^s)} < 0 \ \textit{ and}\\
&-\tau^b\tau^s + 2\tau^s\alpha^b + \sqrt{-\tau^b\tau^s(\tau^b\tau^s - \alpha^b\tau^s)} > 0
\end{aligned}
\tag{MHS^B}
$$

In equilibrium, the participation on both sides of the platform is strictly positive. This excludes the possibility of tipping in the benchmark.

4.2 Equilibrium with intertwined network effects

I now turn to a setting in which platforms decide to set up intertwined network effects and characterize its equilibrium. To present shorter mathematical expressions, let me introduce the following additional notation:

$$\Omega \equiv \sqrt{\tau^b\tau^s(16{\tau^b}^3\tau^{s3} - 8{\tau^b}^2\tau^{s2}\alpha^b\alpha^s + 5\tau^b\tau^s{\alpha^b}^2\alpha^{s2} - 2{\alpha^b}^3\alpha^{s3})} \tag{9}$$

In the third stage, the consumer that is indifferent between joining platform 1 and 2 is located at $\tilde{x}^{INE}$ such that ${U_1^b}^{INE}(\tilde{x}^{INE}) = {U_2^b}^{INE}(\tilde{x}^{INE})$. Thus, the number of consumers served by platform 1 is equal to $\tilde{x}^{INE}$ and the number of consumers served by platform 2 is equal to $1 - \tilde{x}^{INE}$. The seller indifferent between joining platform 1 and not joining it is located at x_{10}^{INE} such that ${U_1^s}^{INE} = 0$. The seller indifferent between joining platform 2 and not joining it is located at x_{20}^{INE} such that ${U_2^s}^{INE} = 0$. Then, the sellers that singlehome in platform 1 are located in the $[0, x_{20}^{INE}]$ sub-interval, those who singlehome in platform 2 in the $[x_{10}^{INE}, 1]$ sub-interval and those who multihome in the $(x_{20}^{INE}, x_{10}^{INE})$ sub-interval. As in the previous setting, I assume for the time being that $0 < x_{20}^{INE} < x_{10}^{INE} < 1$ (I provide the necessary and sufficient conditions below), so that ${n_1^s}^{INE} = x_{10}^{INE}$ and ${n_2^s}^{INE} = 1 - x_{20}^{INE}$. Then, the number of buyers and sellers in each platform is found by solving the following system of four equations and four unknowns:

$$\begin{aligned}
{n_1^b}^{INE} &= \frac{\tau^b + {n_1^s}^{INE}\alpha^b}{2\tau^b} \\
{n_2^b}^{INE} &= 1 - \frac{\tau^b + {n_1^s}^{INE}\alpha^b}{2\tau^b} \\
{n_1^s}^{INE} &= \frac{{n_1^b}^{INE}(\alpha^b - p_s^1)}{\tau^s} \\
{n_2^s}^{INE} &= 1 - \frac{{n_1^b}^{INE}p_s^2 + {n_2^b}^{INE}p_s^2 + \tau^s - {n_1^b}^{INE}\alpha^s - {n_2^b}^{INE}\alpha^s}{\tau^s}
\end{aligned}$$

Which yields buyers' and sellers' participation as a function of seller prices and the model's parameters.

$$
\begin{aligned}
n_1^{b\,INE}(p_s^1) &= \frac{\tau^b\tau^s}{2\tau^b\tau^s + p_s^1\alpha^b - \alpha^b\alpha^s} \\
n_2^{b\,INE}(p_s^1) &= \frac{\tau^b\tau^s}{-2\tau^b\tau^s + \alpha^b(\alpha^b - p_s^1)} - 1 \\
n_1^{s\,INE}(p_s^1) &= \frac{\tau^b(\alpha^s - p_s^1)}{2\tau^b\tau^s - \alpha^b(\alpha^s - p_s^1)} \\
n_2^{s\,INE}(p_s^2) &= \frac{\alpha^s - p_s^2}{\tau^s}
\end{aligned}
\tag{10}
$$

Note that, while in (7) sellers and buyers' participation depend on the price charged to sellers in both platforms, this is not the case in (10). With INE, the only price implicitly considered by buyers when deciding their participation level on both platforms is the per-transaction price charged to sellers in the aggregator (platform 1). Buyers know that, if they join platform 2, they can only interact with platform 2 sellers. If they join platform 1, they have access to both platforms' sellers and additional interactions with multihoming platform 1 sellers. Hence, the difference in the number of sellers they can expect to interact with depends only on platform 1 prices to sellers, which determines the number of sellers joining platform 1.

As for sellers, under INE, their participation in a platform does not depend on the rival platform's price charged to them. Sellers know that if they join platform 2 they will have access to all consumers at a p_2^s per-interaction price, and that if they (also) join platform 1 they will have (additional) interactions with consumers from platform 1 at p_1^s per interaction. This makes the decision to join each platform depend only of that platform's price. Additionally, note that, because platform 2 gives sellers access to all consumers, sellers' decision to join it do not depend on consumers' transportation costs in (10). This is not the case in absence of INE, as seen in (7).

In stage 2, each platform solves the maximization program $\max_{p_i^s} \Pi^i(p_1^s, p_2^s, f)$. Solving the system of first-order conditions yields a unique set of equilibrium prices:

$$
\begin{aligned}
p_s^{1*} &= \frac{f^2\tau^b\tau^s\alpha^b + 2\tau^b\tau^s\alpha^s(2\tau^b\tau^s - \alpha^b\alpha^s) + f\alpha^b(\alpha^b\alpha^s - 2\tau^b\tau^s)}{8\tau^{b^2}\tau^{s2} - 2\tau^b\tau^s\alpha^b\alpha^s + f\alpha^{b^2}\alpha^s} \\
p_s^{2*} &= \frac{f^2\alpha^{b^2}\alpha^s + 4\tau^b\tau^s(2\tau^b\tau^s - \alpha^b\alpha^s) + f\tau^b\tau^s(4\tau^b\tau^s - \alpha^b\alpha^s)}{f^2\alpha^{b^2} + 8\tau^b\tau^s(2\tau^b\tau^s - \alpha^b\alpha^s)}
\end{aligned}
\tag{11}
$$

In stage 1, platform 1 sets the optimal per-transaction referral fee that it charges platform 2 for every interaction between a platform 1 user and a platform 2 seller. To do so, it solves the maximization program $\max_f \Pi_1(p_1^{s*}, p_2^{s*}, f)$. The only value of f that satisfies the first and second-order condition of this maximization program is:

$$
f^* = \frac{2\big(\tau^b\tau^s(\alpha^b\alpha^s - 4\tau^b\tau^s) + \Omega\big)}{\alpha_b^2\alpha^s}
\tag{12}
$$

Replacing (11) and (12) in (10), I obtain the quantities on both sides of the market and the locations of the indifferent s in equilibrium. Consumer and seller surplus are calculated in the same way as in the benchmark setting and using the corresponding utility functions (cf. Section 3.2) and equilibrium threshold values.

Then, the equilibrium values of the model in the intertwined network effects setting are the following.

Lemma 2 (Equilibrium with intertwined network effects). *In the intertwined network effects setting equilibrium:*

$$
\begin{aligned}
{n_b^1}^{INE} &= \frac{4\tau^{b^2}\tau^{s2} - \tau^b\tau^s\alpha^b\alpha^s + \Omega}{16\tau^{b^2}\tau^{s2} - 8\tau^b\tau^s\alpha^b\alpha^s},\\
{n_b^2}^{INE} &= \frac{12\tau^{b^2}\tau^{s2} - 7\tau^b\tau^s\alpha^b\alpha^s - \Omega}{16\tau^{b^2}\tau^{s2} - 8\tau^b\tau^s\alpha^b\alpha^s},\\
{n_b}^{INE} &= 1,\\
{n_s^1}^{INE} &= \frac{\tau^b\tau^s(3\alpha^b\alpha^s - 4\tau^b\tau^s) + \Omega}{4\tau^s\alpha^b(2\tau^b\tau^s - \alpha^b\alpha^s)},\\
{n_s^2}^{INE} &= \frac{\alpha^s}{4\tau^s},\\
{n_s}^{INE} &= \frac{\alpha^s\left(2\tau^b\tau^s - \alpha^b\alpha^s\right) + \tau^b\tau^s\left(-4\tau^b\tau^s + 3\alpha^b\alpha^s\right)}{4\tau^s\alpha^b\left(2\tau^b\tau^s - \alpha^b\alpha^s\right)} + \frac{\Omega}{4\tau^s\alpha^b\left(2\tau^b\tau^s - \alpha^b\alpha^s\right)},\\
{p_s^1}^{INE} &= \frac{-16\tau^{b^3}\tau^{s3} + 4\tau^{b^2}\tau^{s2}\alpha^b\alpha^s - 2\tau^b\tau^s\alpha^{b^2}\alpha^{s2} + \alpha^{b^3}\alpha^{s3} + 4\tau^b\tau^s\Omega}{\alpha^{b^3}\alpha^{s2}},\\
{p_s^2}^{INE} &= \frac{3\alpha^s}{4},\\
f^{INE} &= \frac{2\big(\tau^b\tau^s(\alpha^b\alpha^s - 4\tau^b\tau^s) + \Omega\big)}{\alpha^{b^2}\alpha^s},\\
{\Pi^1}^{INE} &= \frac{(4\tau^b\tau^s - \alpha^b\alpha^s)(-4\tau^{b^2}\tau^{s2} + \tau^b\tau^s\alpha^b\alpha^s + \alpha^{b^2} + \alpha^{s2} + \Omega)}{16\tau^s\alpha^{b^2}(2\tau^b\tau^s - \alpha^b\alpha^s)},\\
{\Pi^2}^{INE} &= \frac{\alpha^{s2}}{16\tau^s},\\
\Pi^{INE} &= \frac{\alpha^{s2} + \left(-4\tau^b\tau^s + \alpha^b\alpha^s\right)\left(-4\tau^{b^2}\tau^{s2} + \tau^b\tau^s\alpha^b\alpha^s + \alpha^{b^2}\alpha^{s2}\right)}{16\tau^s\alpha^{b^2}\left(-2\tau^b\tau^s + \alpha^b\alpha^s\right)} + \frac{\Omega}{16\tau^s\alpha^{b^2}\left(-2\tau^b\tau^s + \alpha^b\alpha^s\right)},\\
CS^{INE} &= \frac{-16\tau^{b^4}\tau^{s4} + 32\tau^{b^3}\tau^{s3}\alpha^b\alpha^s - 2\alpha^{b^2}\alpha^{s2}\Omega + 8v^b\tau^s(2\tau^b\tau^s - \alpha^b\alpha^s)\big(\tau^b\tau^s(4\tau^b\tau^s - \alpha^b\alpha^s) + \Omega\big)}{32\tau^b\tau^{s2}(\alpha^b\alpha^s - 2\tau^b\tau^s)}\\
&\quad + \frac{\tau^{b^2}\tau^{s2}\big(13\alpha^{b^2}\alpha^{s2} + 4\Omega\big) + \tau^b\tau^s\alpha^b\alpha^s\big(\alpha^{b^2}\alpha^{s2} + 7\Omega\big)}{32\tau^b\tau^{s2}(\alpha^b\alpha^s - 2\tau^b\tau^s)},\\
SS^{INE} &= \frac{1}{32}\Bigg(8\tau^s\left(\frac{\tau^{b^2}}{\alpha^{b^2}} - 2\right) - \frac{3\alpha^{s2}}{\tau^s} + \frac{16\alpha^{b^4}\alpha^{s3} + 2\alpha^{b^2}\alpha^s\big(\alpha^{b^2}\alpha^s(32\tau^s + 3\alpha^s) - 4\Omega\big)}{\alpha^{b^2}(\alpha^b\alpha^s - 2\tau^b\tau^s)^2}\\
&\quad + 2\tau^b\alpha^b\alpha^s\big(-\alpha^{b^2}\alpha^s(32\tau^s + \alpha^s) + 3\Omega\big)\Bigg).
\end{aligned}
$$

For the equilibrium reported in Lemma 2 to be valid, a series of conditions summarized in Assumption 2 have to hold. These conditions are analogous to those presented in Assumption 1.

Assumption 2 (Intertwined network effects conditions). *In the intertwined network effects setting, the parameters satisfy the following conditions.*

$$\alpha^b\alpha^s < 2\tau^b\tau^s, \tag{SOCs^{INE}}$$

$$\frac{4(4v_B - 3\tau^b)\tau^b\tau^{s^2} + (-8v_B + 11\tau^b)\tau^s\alpha^b\alpha^s - 2\alpha^{b^2}\alpha^{s^2}}{8\tau^s\left(2\tau^b\tau^s - \alpha^b\alpha^s\right)} + \frac{\sqrt{\tau^b\tau^s\left(16\tau^{b^3}\tau^{s^3} - 8\tau^{b^2}\tau^{s^2}\alpha^b\alpha^s + 5\tau^b\tau^s\alpha^{b^2}\alpha^{s^2} - 2\alpha^{b^3}\alpha^{s^3}\right)}}{8\tau^s\left(2\tau^b\tau^s - \alpha^b\alpha^s\right)} > 0, \tag{FPB^{INE}}$$

$$0 < 1 - \frac{\alpha^s}{4\tau^s} < \frac{\tau^b\tau^s\left(-4\tau^b\tau^s + 3\alpha^b\alpha^s\right)}{4\tau^s\alpha^b\left(2\tau^b\tau^s - \alpha^b\alpha^s\right)},$$

$$\frac{\sqrt{\tau^b\tau^s\left(16\tau^{b^3}\tau^{s^3} - 8\tau^{b^2}\tau^{s^2}\alpha^b\alpha^s + 5\tau^b\tau^s\alpha^{b^2}\alpha^{s^2} - 2\alpha^{b^3}\alpha^{s^3}\right)}}{4\tau^s\alpha^b\left(2\tau^b\tau^s - \alpha^b\alpha^s\right)} < 1, \tag{MHS^{INE}}$$

In the equilibrium with intertwined network effects, there is tipping on the buyer side in favor of the aggregator platform if the platforms are sufficiently homogeneous on the buyer side (i.e., if $\tau^b \leq \frac{3\alpha^b\alpha^s(3+\sqrt{11})}{8\tau^s}$). However, this condition is incompatible with the second order condition of the no-INE benchmark set out in Assumption 1. Hence, in the parameter space in which platforms might choose to set up INE on which the remainder of this section focuses, there is no tipping in equilibrium.

4.3 Comparison between the equilibrium with and without intertwined network effects

In this subsection, I compare the equilibrium of the INE and the benchmark settings. Let me first introduce the following remark.

Remark 1. *Let $Cond^B$ and $Cond^{INE}$ be the parameter spaces defined by Assumptions 1 and 2, respectively.*

i) $\exists \quad S^B \quad \subseteq \quad Cond^B \quad | \quad S^B \cap Cond^{INE} = \varnothing$

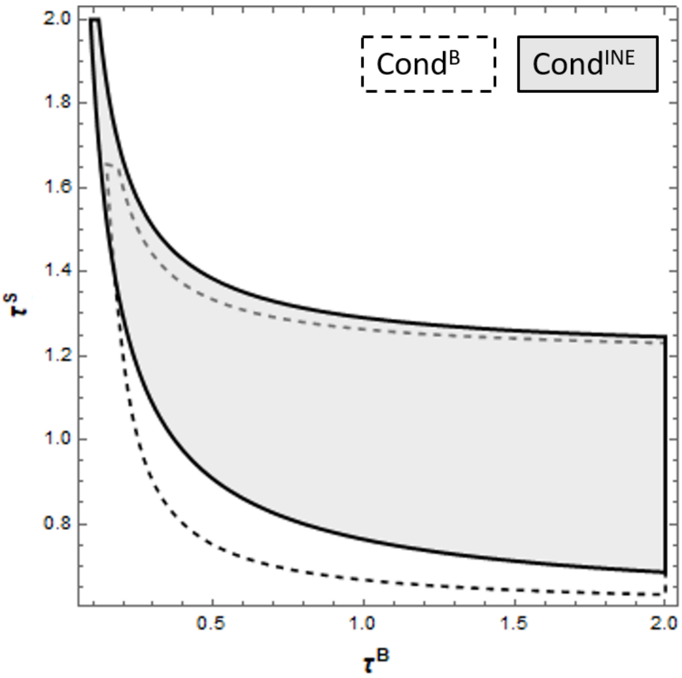
ii) $\exists \quad S^{INE} \quad \subseteq \quad Cond^{INE} \quad | \quad S^{INE} \cap Cond^B = \varnothing$

iii) $\exists \quad S^{B\cap INE} = Cond^B \cap Cond^{INE} \neq \varnothing$

Remark 1 shows that there are admissible parameter spaces in which i) only the benchmark case can take place, ii) only the INE case can take place and iii) both the benchmark and

INE case can take place. These are illustrated in Figure 2. Given that my focus is the effect of INE relative to the benchmark case, in the remainder of this section I will assume that Assumptions 1 and 2 hold, i.e., that the parameter space is the one defined as $S^{B\cap INE}$ in Remark 1 (overlapping areas in Figure 2). As illustrated in Figure 2, the parameter spaces of the benchmark and INE cases have a considerable overlap. When this is the case, as per Lemma 3, platforms always have an incentive to set INE.

Figure 2: Parameter spaces for the benchmark and the INE cases



Parameter setting: $\alpha^b = 0.1, \alpha^s = 2.4$ and $v^b = 1.3$.

Lemma 3 (**Platforms' incentives to set up intertwined network effects**). *If Assumptions 1 and 2 hold, setting-up intertwined network effects is a dominant strategy for the aggregator and the source platforms. Formally,* $\Pi_1{}^{INE} > \Pi_1{}^{B}$ *and* $\Pi_2{}^{INE} > \Pi_2{}^{B}$. ***Proof****: See Appendix A.1.*

Lemma 3 shows that, whenever both setting and not setting up intertwined network effects can be an equilibrium, setting INE is a dominant strategy for both platforms. The reason is that INE create new indirect network effects compared to the no-INE benchmark. With INE, sellers of the source platform (platform 2) can interact with consumers of the aggregator platform (platform 1). This generates additional surplus that the platforms can share among themselves. Note that, in this article's setting, part of this surplus stems from the assumption of full double counting of the network effects enjoyed by multihoming sellers. Therefore, if double counting is partial, Lemma 3 might be softened and be valid only if partial double counting is sufficiently strong. In the remaining of this section, I study the effects of setting up INE on platform participation, prices and participants' welfare.

4.3.1 Prices

Lemma 4 shows how INE affect prices.

Lemma 4 (Effect of intertwined network effects on prices charged to sellers). *When platforms set up intertwined network effects, the prices paid by sellers to the source platform increase ($p_2^{s\,INE} > p_2^{s\,B}$). In contrast, the prices paid by sellers to the aggregator platform can either increase or decrease depending on transportation costs. There exists a value $\tilde{\tau^b}(\alpha^b, \alpha^s, \tau^s)$ satisfying Assumptions 1 and 2 such that, if $\tau^b > \tilde{\tau^b}(\alpha^b, \alpha^s, \tau^s)$, they decrease ($p_1^{s\,INE} < p_1^{s\,B}$); otherwise (i.e., if $\tau^b < \tilde{\tau^b}(\alpha^b, \alpha^s, \tau^s)$), they increase ($p_1^{s\,INE} > p_1^{s\,B}$).*

***Proof*:** *See Appendix A.2.*

Platform 2 increases the price charged to sellers for two reasons. The first one is that the marginal cost of serving a seller that interacts with a consumer referred by platform 1 increases by f^{INE}. The second one is the increase in the network effects platform 2 offers to sellers given that post-INE, platform 2 sellers can interact with all buyers.

In platform 1, the price charged to sellers increases if consumers' transportation costs are sufficiently low, and decreases otherwise. The intuition is as follows. Post-INE, platform 1's quality to buyers increases, as they can now access both platforms' sellers through it. Hence, as shown in Lemma 5, post-INE, buyers' participation increases in platform 1. The lower transportation costs are, the stronger this increase is. With a sufficiently high increase in buyers' participation, platform 1 becomes so attractive to sellers that the platform maximizes profits by increasing the price it charges them. In this case, platform 1's price increases even if this reduces sellers' participation (and hence consumer's), leading to lower revenues through the referral fee. Conversely, if transportation costs are sufficiently low, the post-INE increase in buyers' participation in platform 1 is mild. Hence, the latter maximizes profit by lowering the price charged to sellers. This increases their participation, and hence consumers', leading to higher revenues through the referral fee.

4.3.2 Platform participation

Lemma 5 shows how INE affect platform participation in equilibrium.

Lemma 5 (Effect of intertwined network effects on platform participation). *When platforms set up intertwined network effects:*

i) The number of consumers increases in the aggregator platform ($n_1^{b\,INE} > n_1^{b\,B}$) and decreases in the source platform ($n_2^{b\,INE} < n_2^{b\,B}$)

ii) The number of sellers increases in the aggregator platform ($n_1^{s\,INE} > n_1^{s\,B}$) and decreases in the source platform ($n_2^{s\,INE} < n_2^{s\,B}$)

iii) The total number of sellers increases ($n_s^{INE} > n_s^{B}$) if and only if $\tau^b > \frac{(3+2\sqrt{3})\alpha^b\alpha^s}{6\tau^s}$ and decreases ($n_s^{INE} < n_s^{B}$) otherwise ($\tau^b < \frac{(3+2\sqrt{3})\alpha^b\alpha^s}{6\tau^s}$).

***Proof**: See Appendix A.3.*

The results regarding buyers' participation is intuitive. Post-INE, platform 1 becomes more attractive to buyers, as they can access all sellers through it. Given that buyers are assumed to singlehome, this results in an increase in platform 1 participation and a decrease in platform 2 participation on the buyer side.

Regarding sellers, in the aggregator platform (platform 1), the number of consumers increases, while the price can increase or decrease depending on transportation costs. However, the net effect on a seller's utility is always positive. Hence, seller participation increases in platform 1. In the source platform (platform 2), post-INE, the number of buyers that can be reached increases by $\frac{1}{2}$, while the price also increases. The net effect on a platform 2 seller's utility is a priori ambiguous. However, the increase in platform 1's seller utility drives some platform 2 sellers to migrate to platform 1, leading to a decrease in platform 2 seller participation.

Hence, overall seller participation can only increase if seller multihoming increases. This is the case if consumers' transportation costs are sufficiently high. As it can be analytically verified, the threshold of τ^b above which seller multihoming increases is above that for which prices in platform 1 decreases prices. In other words, if seller prices decrease sufficiently in the aggregator platform, the number of sellers that switch from singlehoming in platform 2 to multihoming will exceed that of sellers singlemoning that start to singlehome in platform 1, leading to an overall increase in seller multihoming.

4.3.3 Participants' surpluses

Propositions 1 and 2 show how INE affects consumer and seller surpluses, respectively.

Proposition 1 (Effect of intertwined network effects on consumer surplus). *When platforms set up intertwined network effects, consumer surplus increases ($CS^{INE} > CS^{B}$).*

***Proof**: See Appendix A.4.*

The introduction of intertwined network effects (INE) increases aggregate consumer surplus through two channels. For consumers on the **aggregator platform** (platform 1), surplus rises due to greater seller participation. These consumers gain access to all sellers on both platforms, amplifying the intensive margin (more interactions) while the platform attracts more buyers (extensive margin). In contrast, consumers on the **source platform** (platform 2) experience a decline in surplus. Platform 2 loses buyers to the aggregator, reducing its extensive margin, and its diminished seller base curtails the intensive margin.

Critically, the aggregator's gains always outweigh the source platform's losses. The cross-platform interactions enabled by INE allow aggregator consumers to capture surplus from both platforms' sellers, whereas source platform consumers lose access only to their own sellers. This asymmetry ensures that the net effect on consumer surplus is unambiguously positive, regardless of platform differentiation or pricing strategies.[5]

Proposition 2 (**Effect of intertwined network effects on seller surplus**). *When platforms set up intertwined network effects:*

i) If $\tau^s \leq \frac{5}{8}\alpha^s$, seller surplus increases ($SS^{INE} > SS^B$)

ii) If $\tau^s > \frac{5}{8}\alpha^s$, seller surplus decreases ($SS^{INE} < SS^B$)

Proof: *See Appendix A.5.*

Proposition 2 shows that if the platforms are sufficiently differentiated for sellers, INE decrease their surplus, and vice versa. As shown above, post-INE, the utility of a seller increases in platform 1 (the aggregator). Hence, some sellers leave platform 2 for platform 1, leading to an increase in the number of sellers in the former and a decrease in the latter. As a result, seller surplus increases in platform 1 and decreases in platform 2. The overall effect depends on which platforms' seller surplus increases the most, which in turn depends on how many sellers leave platform 2 for platform 1. With sufficiently low seller transportation costs, enough sellers leave the source platform for the aggregator platform, leading to an increase in overall seller surplus, and vice versa.

Finally, as shown in Corollary 1 below, setting-up INE is Pareto-improving if the platforms are sufficiently homogeneous to sellers.

Corollary 1 (Pareto improvement with intertwined network effects). *When platforms set up intertwined network effects, the surplus captured by consumers, sellers and the platforms increases if $\tau^s \leq \frac{5}{8}\alpha^s$.*

Proof: *The result follows directly from Lemma 3 and Propositions 1 and 2.*

The intuition behind this result is that, with INE, sellers from the source platform (platform 2) can interact with consumers from the aggregator platform (platform 1). This generates

[5] This result does not depend on the full double counting assumption.

additional surplus for sellers and consumers that the platforms can partially capture and share among themselves. If transportation costs are sufficiently low for sellers, post-INE, enough sellers leave the source platform (where seller surplus decreases) for the aggregator platform (where seller surplus increases). In this case, INE are Pareto-improving.

5 Platform merger with intertwined network effects

In this section I analyze the impact of a non-consolidating merger between two platforms having set up intertwined network effects on prices and the participation and surpluses of buyers and sellers. As in the previous section, I assume full information for all participants in the model, i.e., each participant observes all the price decisions and knows all the parameters of the model. The equilibrium concept is the Subgame Perfect Nash equilibrium.

5.1 Post-merger equilibrium with intertwined network effects

In this section I characterize the post-merger equilibrium in a setting with intertwined network effects. The superscript $INE-M$ is used to refer to this setting in which the platforms having set up intertwined network effects merged.

As in Section 3.2, the utility function of buyers and sellers are given by (4) and (5), respectively. However, given that the platforms are under common ownership, the referral fee f represents an internal transfer, and it is therefore not charged. Then, the profit of platform $i \in \{1, 2\}$ is given by:

$$\Pi^{i^{INE-M}} \equiv \begin{cases} n_1^b(p_1^s, p_2^s)n_1^s(p_1^s, p_2^s)p_1^s & \text{if } i = 1 \\ p_2^s\left(n_2^b(p_1^s, p_2^s)n_2^s(p_1^s, p_2^s) + n_1^b(p_1^s, p_2^s)n_2^s(p_1^s, p_2^s)\right) & \text{if } i = 2 \end{cases} \tag{13}$$

Thus, in this setting, the game has two stages. In the first stage, the platforms simultaneously set the prices charged to sellers (p_1^s, p_2^s). In the second stage, consumers and sellers simultaneously choose which platform(s) to join.

Given that their utility functions remain unchanged by the merger, in stage 2, consumers and sellers participation as a function of the prices charged to sellers and the model's parameters are given by (10). In stage 1, the platforms solve the joint maximization problem $\max_{p_1^s, p_2^s} \Pi(p_1^s, p_2^s)$, where $\Pi \equiv \Pi^1 + \Pi^2$. Solving the system of first-order conditions yields a unique set of equilibrium prices:

$$\begin{aligned} p_1^{s*} &= \frac{\alpha^s\left(\alpha^b\alpha^s - 2\tau^b\tau^s\right)}{\alpha^b\alpha^s - 4\alpha^b\alpha^s} \\ p_2^{s*} &= \frac{\alpha^s}{2} \end{aligned} \tag{14}$$

Replacing (14) in (10) I obtain the quantities on both sides of the market and the locations of the indifferent consumer and sellers in equilibrium. Consumer and seller surplus are calculated in the same way as in the benchmark setting and using the corresponding utility functions and equilibrium threshold values.

Then, the post-merger equilibrium values of the model in the setting with intertwined network effects are the following.

Lemma 6 (Post-merger equilibrium with intertwined network effects). *Post-merger, in the intertwined network effects setting equilibrium:*

$$
\begin{aligned}
n_1^{b\,INE-M} &= \frac{1}{4} + \frac{\tau^b\tau^s}{4\tau^b\tau^s - 2\alpha^b\alpha^s}\\
n_2^{b\,INE-M} &= \frac{3}{4} - \frac{\tau^b\tau^s}{4\tau^b\tau^s - 2\alpha^b\alpha^s}\\
n^{b\,INE-M} &= 1\\
n_1^{s\,INE-M} &= \frac{\tau^b\alpha^s}{4\tau^b\tau^s - 2\alpha^b\alpha^s}\\
n_2^{s\,INE-M} &= \frac{\alpha^s}{2\tau^s}\\
n^{s\,INE-M} &= \frac{\alpha^s}{2\tau^s} + \frac{\tau^b\alpha^s}{4\tau^b\tau^s - 2\alpha^b\alpha^s}\\
p_1^{s\,INE-M} &= \frac{\alpha^s\left(-2\tau^b\tau^s + \alpha^b\alpha^s\right)}{-4\tau^b\tau^s + \alpha^b\alpha^s}\\
p_2^{s\,INE-M} &= \frac{\alpha^s}{2}\\
\Pi^{1\,INE-M} &= \frac{\tau^b{\alpha^s}^2}{16\tau^b\tau^s - 8\alpha^b\alpha^s}\\
\Pi^{2\,INE-M} &= \frac{{\alpha^s}^2}{4\tau^s}\\
\Pi^{INE-M} &= \frac{1}{4}{\alpha^s}^2\left(\frac{1}{\tau^s} + \frac{\tau^b}{4\tau^b\tau^s - 2\alpha^b\alpha^s}\right)\\
CS^{INE-M} &= \frac{\left(4\tau^b\tau^s - \alpha^b\alpha^s\right)\left(4\left(4r_B - \tau^b\right)\tau^b{\tau^s}^2 + \left(-8r_B + 11\tau^b\right)\tau^s\alpha^b\alpha^s - 4{\alpha^b}^2{\alpha^s}^2\right)}{16\tau^s\left(-2\tau^b\tau^s + \alpha^b\alpha^s\right)^2}\\
SS^{INE-M} &= \frac{1}{8}\left(\tau^s\left(-4 + \frac{{\tau^b}^2}{{\alpha^b}^2}\right) + 8\alpha^s - \frac{3{\alpha^s}^2}{\tau^s} + \frac{4{\tau^b}^4{\tau^s}^3}{{\alpha^b}^2\left(-2\tau^b\tau^s + \alpha^b\alpha^s\right)^2} + \frac{4{\tau^b}^3{\tau^s}^2}{{\alpha^b}^2\left(-2\tau^b\tau^s + \alpha^b\alpha^s\right)}\right)
\end{aligned}
$$

For the equilibrium reported in Lemma 6 to be valid, a series of conditions summarized in Assumption 3 have to hold. These conditions are analogous to those presented in Assumption 1, with some caveats. First, the assumption that ensures the second order conditions hold ($\alpha^b\alpha^s < 2\tau^b\tau^s$) is replaced by the stricter NoTipping$^{\text{INE-M}}$ assumption ($\alpha^b\alpha^s < \frac{4}{3}\tau^b\tau^s$), which ensures there is no tipping on the buyer side in favor of the aggregator platform.[6] Second, the second order condition on the source platform's price ($-\frac{2}{\tau^s} < 0$) is not reported, as it is always met. In the same vein, the condition for the price vector that satisfies the first

[6]This is a mild additional assumption. As illustrated in Figure 3, the tipping case is a corner case.

order conditions to constitute a maximum is always met when the second-order condition on the aggregator's price is verified. Hence, for the sake of simplification, I do not include it in Assumption 3.

Assumption 3 (Platform merger with intertwined network effects conditions). *In the setting in which platforms having set up intertwined network effects merge, the parameters satisfy the following conditions.*

$$\alpha^b \alpha^s < \frac{4}{3}\tau^b \tau^s \qquad \textit{(NoTipping}^{\textit{INE-M}}\textit{)}$$

$$\frac{2\tau^b\tau^s}{\alpha^b} \neq \alpha^s \quad \textit{and} \quad 4v^b + \frac{2\alpha^b\alpha^s}{\tau^s} > \tau^b\left(3 + \frac{2\tau^b\tau^s}{-2\tau^b\tau^s + \alpha^b\alpha^s}\right) \qquad \textit{(FPB}^{\textit{INE-M}}\textit{)}$$

$$0 < 1 - \frac{\alpha^s}{2\tau^s} < \frac{\tau^b\alpha^s}{4\tau^b\tau^s - 2\alpha^b\alpha^s} < 1 \qquad \textit{(MHS}^{\textit{INE-M}}\textit{)}$$

5.2 Comparison between the pre- and post-merger equilibria

In this subsection, I compare the equilibrium with INE and legally-independent firms to a post-merger equilibrium with INE. Let me first introduce the following remark.

Remark 2. *Let $Cond^{INE}$ and $Cond^{INE-M}$ be the parameter spaces defined by Assumptions 2 and 3, respectively.*

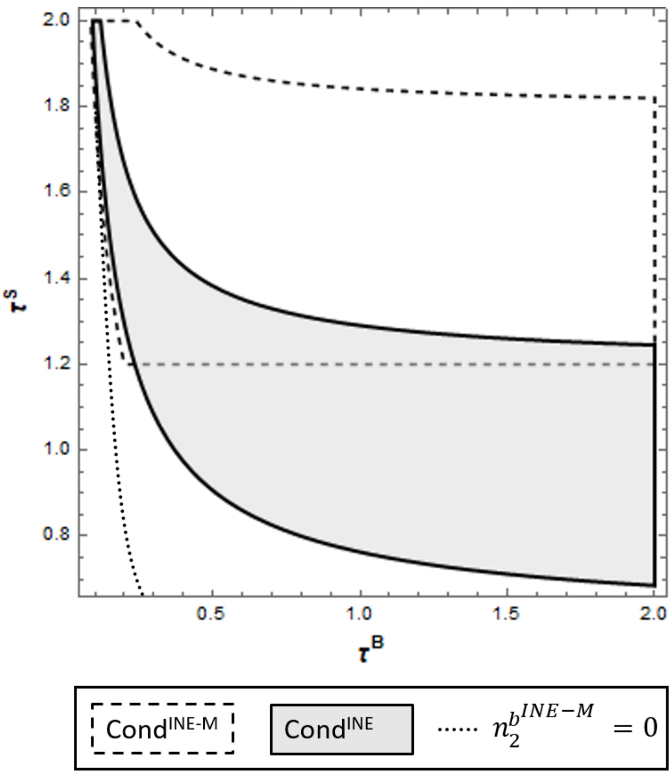
i) $\exists \quad S^{INE} \subseteq Cond^{INE} \quad | \quad S^{INE} \cap Cond^{INE-M} = \varnothing$

ii) $\exists \quad S^{INE-M} \subseteq Cond^{INE-M} \quad | \quad S^{INE-M} \cap Cond^{INE} = \varnothing$

iii) $\exists \quad S^{INE \cap INE-M} = Cond^{INE} \cap Cond^{INE-M} \neq \varnothing$

Remark 2 tells that there are admissible parameter spaces in which i) only the case with INE and legally-independent firms can take place, ii) only the case with INE and merged firms can take place and iii) both the cases with INE and either legally-independent or merged firms can take place. These are illustrated in Figure 3. Given that my focus is the effect of a non-consolidating merger relative to the case with INE and legally-independent firms, in the remainder of this section I will assume that Assumptions 2 and 3 hold, i.e., that the parameter space is the one defined as $S^{INE \cap INE-M}$ in Remark 2 (overlapping areas in Figure 3). As illustrated in Figure 3, the parameter spaces of the merger and INE cases overlap when transportation costs are sufficiently high for sellers. When this is the case, as per Lemma 7, platforms having set INE have an incentive to merge.

Figure 3: Parameter spaces for the merger and the INE cases



Parameter setting: $\alpha^b = 0.1, \alpha^s = 2.4$ and $v^b = 1.3$.

Lemma 7 (**Incentives to merge in presence of intertwined network effects**). *Consider two platforms having set up intertwined network effects. If Assumptions 2 and 3 hold, merging is a dominant strategy for both. Formally,* $\Pi^{1^{INE-M}} + \Pi^{2^{INE-M}} > \Pi^{1^{INE}} + \Pi^{2^{INE}}$.

Proof: *See Appendix A.6.*

Lemma 7 shows that, whenever setting INE without common ownership and merging can be an equilibrium, the platforms always have incentives to merge. The reason is that this allows them to better internalize the cross-platform network effects by jointly maximising profit. This provides a rationale of why it is common to see INE between transaction platforms belonging to the same group.

5.2.1 Prices

Lemma 8 shows how the merger affects equilibrium prices.

Lemma 8 (**Merger effect on prices**). *Consider two platforms having set up intertwined network effects. After they merge, the price paid by sellers to the aggregator platform increases* ($p_1^{s\,INE-M} > p_1^{s\,INE}$)*. In contrast, the prices paid by sellers to the source platform decrease* ($p_2^{s\,INE-M} < p_2^{s\,INE}$)*.*

Proof: *See Appendix A.7.*

Lemma 8 shows how the change in the inter-platform pricing structure brought about by the merger affects the prices charged to sellers. When the platforms are under common ownership, they maximize profit jointly. Hence, the referral fee f disappears. This lowers the marginal per-interaction cost of the source platform (platform 2) and hence the price it charges to sellers. Conversely, in absence of the referral fee, the aggregator (platform 1) loses a revenue stream that was allowing it to set lower prices, attract more sellers and increase its revenue through referral fees. Hence, post-merger, platform 1 increases the price charged to sellers.

5.2.2 Platform participation

Lemma 9 shows how the merger affects participation in equilibrium.

Lemma 9 (**Merger effect on platform participation**). *Consider two platforms having set up intertwined network effects. After they merge:*

i) The number of consumers decreases in the aggregator platform ($n_1^{b\,INE-M} < n_1^{b\,INE}$) and increases in the source platform ($n_2^{b\,INE-M} > n_2^{b\,INE}$)

ii) The number of sellers decreases in the aggregator platform ($n_1^{s\,INE-M} < n_1^{s\,INE}$) and increases in the source platform ($n_2^{s\,INE-M} > n_2^{s\,INE}$)

iii) The total number of sellers increases ($n_s^{INE-M} > n_s^{INE}$).

Proof: *See Appendix A.8.*

The changes in platform participation induced by the merger follow from the change in prices analysed in Lemma 8. Note that, contrary to what happens when comparing the effect of INE on platform participation (cf. Lemma 5), the merger does not alter neither firms' nor consumers' utility functions. Hence, their changes in participation are only explained by changes the changes prices shown in Lemma 8. In platform 1, the increase in the price charged to sellers drives some sellers to leave the platform, which in turn decreases consumers' participation. The opposite happens in platform 2.

Interestingly, the increase in seller participation in platform 2 is always stronger than the decrease in platform 1, leading to an overall increase in seller participation led by more sellers multihoming. This effect is analogous to the elimination of double marginalization effect in vertical mergers, which has been widely studied since Spengler (1950). Eliminating the referral fee results in a decrease in the price charged to sellers in the source platform

(platform 2) that is stronger than the increase in the price charged to sellers in the aggregator platform (platform 1).[7] This expands overall sellers' demand for interactions.

5.2.3 Participants' surpluses

To present shorter mathematical expressions, let me introduce the following notation.

$$\tilde{v^b} \equiv \frac{1}{16}\left(2\tau^b + \frac{32{\tau^b}^2\tau^s}{\alpha^b\alpha^s} - \frac{4\alpha^b\alpha^s}{\tau^s} + \frac{\tau^b\alpha^b\alpha^s}{2\tau^b\tau^s - \alpha^b\alpha^s} + \right.$$
$$\left.\sqrt{\frac{\tau^b\left(16\tau^b\tau^s - 5\alpha^b\alpha^s\right)^2\left(16{\tau^b}^3\tau^{s3} - 8{\tau^b}^2\tau^{s2}\alpha^b\alpha^s + 5\tau^b\tau^s{\alpha^b}^2\alpha^{s2} - 2{\alpha^b}^3\alpha^{s3}\right)}{\tau^s{\alpha^b}^2\alpha^{s2}\left(-2\tau^b\tau^s + \alpha^b\alpha^{s2}\right)}}\right)$$

Proposition 3 (Merger effect on consumer surplus). *Consider two platforms having set up intertwined network effects. After they merge, consumer surplus increases ($CS^{INE-M} > CS^{INE}$) if and only if $v^b < \tilde{v^b}(\tau^b, \tau^s, \alpha^b, \alpha^s)$, and decreases ($CS^{INE-M} < CS^{INE}$) otherwise (i.e., if $v^b > \tilde{v^b}(\tau^b, \tau^s, \alpha^b, \alpha^s)$).* ***Proof:*** *See Appendix A.9.*

The intuition is as follows. *Ceteris paribus*, the increase in seller participation in platform 2 (which benefits platform 2 buyers) and overall (which benefits the buyers served by both platforms) brought about by the merger benefits consumers in both platforms. This increases consumer welfare. However, due to the presence of INE, platform 1 consumers can interact with more sellers than platform 2 consumers irregardless of the ownership structure of the platforms. Hence, consumers switching from platform 1 to platform 2 decreases consumer surplus. If the stand-alone utility a consumer obtains from joining any of the two platforms (v^b) is sufficiently low, network effects have a significant weight in her decision to join a platform. In this case, a sufficiently high amount of consumers switch from platform 1 (where the number of sellers decreases) to platform 2 (where the number of sellers increases), which increases overall consumer surplus. The opposite happens if v^b is sufficiently high.

Proposition 4 (Merger effect on seller surplus). *Consider two platforms having set up intertwined network effects. After they merge, if $\tau^s \leq \frac{9\alpha^s}{16}$, seller surplus decreases.*

Proof: *See Appendix A.10.*

Seller surplus post-merger. To analyze the merger's impact on seller surplus, I distinguish between *singlehoming sellers* and *multihoming sellers*. Post-merger, platform 1 (the aggregator) raises its price to sellers, while platform 2 (the source platform) lowers its price.

[7] It can be shown that $({p_1^s}^{INE-M} - {p_1^s}^{INE}) + ({p_2^s}^{INE-M} - {p_2^s}^{INE}) < 0$ under Assumption 3.

For singlehoming sellers in platform 1, this price hike reduces their surplus. Conversely, singlehoming sellers in platform 2 benefit from the price reduction. Multihoming sellers, however, face conflicting forces: they pay a higher price in platform 1 but a lower price in platform 2. Because multihoming sellers interact with buyers from both platforms, the net effect depends on the balance between these price changes and the effect of the merger on the volume of interactions.

The merger also alters buyer participation: platform 1 loses buyers to platform 2. For multihoming sellers, this reallocation reduces the number of interactions with platform 1's buyers, while increasing interactions with platform 2's buyers. However, due to intertwined network effects (INE), multihoming sellers derive less value from platform 1's diminished buyer base. When seller transportation costs (τ^s) are sufficiently low, sellers reallocate their participation significantly, amplifying the decline in overall interactions. This interaction loss dominates the price-driven benefits for sellers, leading to a net decrease in seller surplus.

6 Empirical estimation

In this section, I test the model's predictions on the impact of intertwined network effects on platform participation. Using a Subgroup Difference-in-Differences design (Shahn, 2023), I exploit the introduction of INE between the generalist Irish classified platforms Adverts and DoneDeal that took place in December 2017. From that date onward, for some categories, whenever users would see less than 10 search results on Adverts, similar ads from DoneDeal started being displayed on Adverts.[8] In other words, in December 2017 INE were introduced, with DoneDeal being the source platform and Adverts being the aggregator.

Lemma 5 provides predictions on the effect of introducing INE on the number of buyers and sellers for the aggregator and the source platform. In the case of Adverts and DoneDeal, contrary to what the findings reported in Lemma 5 assume, the platforms were under common ownership before and after introducing INE. It can be shown that, if the two platforms are under common ownership, all the results in Lemma 5 but one hold.[9] The difference is that, when platforms are under common ownership, the total number of sellers unambiguously increases post-INE. Hence, I expect the introduction of INE to cause an increase in the number of sellers in both platforms and to an increase in the number of buyers in Adverts (the aggregator) coupled with a decrease in the number of sellers in DoneDeal (the source platform).

[8] See https://help.adverts.ie/hc/en-us/articles/360001288765-Ad-Sharing-from-to-DoneDeal-ie.

[9] To do so, one has to solve the baseline model presented in Section 3.1 with a caveat: in stage 1, the platforms solve the joint maximization program $\max_{p_1^s, p_2^s} \Pi(p_1^s, p_2^s)$. Then, one can compare the resulting equilibrium participation to the equilibrium participation of the post-merger with INE case reported in Lemma 6.

6.1 Data and methodology

To test these predictions, I use data from Sensor Tower, one of the main providers of app usage data. The unit of observation is an app in a given country. For each app, I observe the number of monthly active users (MAU).

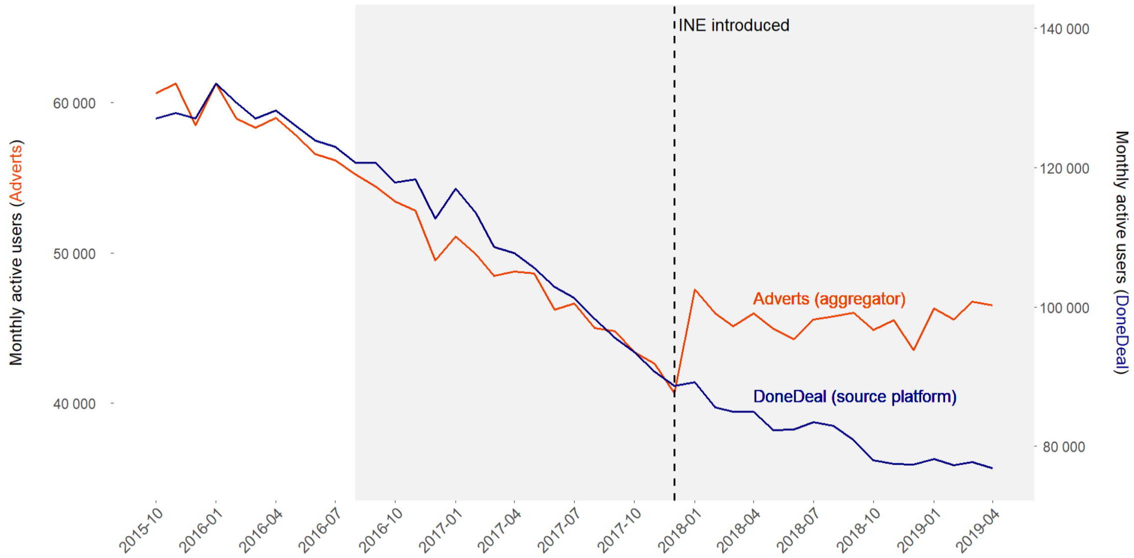
Since both apps are only active in Ireland, all the data refers to Irish users. I extract the number of monthly active users for Adverts and DoneDeal's apps between October 2015 (the first date at which the data is available) and April 2019. I limit the data collection to April 2019 because, since May 2019, sellers on Adverts can opt-in to share their content to DoneDeal. This introduces a new INE in the opposite direction, which complicates causal identification.

Sensor Tower does not distinguish between consumers and sellers in its counting of monthly active users.[10] Therefore, with this dataset, two falsifiable predictions can be tested. The first one is that the overall number of users (consumers and sellers) in the aggregator app (Adverts) should increase after the introduction of INE. The effect on the overall number of users of the source platform (DoneDeal), in turn, is a priori ambiguous. It depends on whether the expected increase in the number of sellers compensates the expected decline in the number of consumers. Second, the overall number of users on both apps should increase. Since the model assumes that consumers singlehome, the increase in the number of consumers in Adverts should be equal to the decrease in the number of consumers in DoneDeal. However, since the number of sellers should increase in both apps, I expect the total number of users (consumers and sellers in both apps aggregated) to increase after INE are introduced.

Figure 4 shows the evolution of the number of monthly active users for each app between October 2015 and April 2019.

[10] In generalist classified apps, the same user can simultaneously be a seller and a buyer.

Figure 4: Evolution of the number of monthly active users for Adverts' (left axis) and DoneDeal's (right axis) mobile apps (October 2015 to April 2019)



Note: the grey area corresponds to the time window used in the estimation equations (15)-(17). The vertical dashed line indicates the month of introduction of intertwined network effects.

The figure shows a declining parallel trend between the two apps' usage until the introduction of INE in December 2017 (the treatment period). After that, there is a discrete increase in the number of users of Adverts, followed by a flattening of the evolution of MAU over time. In the case of DoneDeal, the decline decelerates after the introduction of INE.

To ensure comparability between the pre- and post-treatment periods, I select an equal number of months (17) before and after the treatment event as the time window. Table 1 provides summary statistics for each app's MAU before and after the introduction of INE.

Table 1: Statistics on the number of monthly average users by app before and after the introduction of intertwined network effects.

	Adverts		**DoneDeal**	
	Before	After	Before	After
Monthly active users (mean)	49,238	45,339	10,873	81,669
Monthly active users (s.d.)	4,163	1,537	10,342	4,101
Monthly active users (min)	42,633	40,639	90,739	76,820
Monthly active users (max)	56,155	47,559	122,985	89,195
Number of months	17	17	17	17

To estimate the causal effect of intertwined network effects on each app's user base, I use a Subgroup Difference-in-Differences design (Shahn, 2023). The predictions of the theoretical model are agnostic as to whether INE should have a discrete and/or time-varying effect on platform participation. Therefore, I specify three models (referred to as Models 1, 2 and 3, hereafter) to capture all possible combinations. The estimation equations are, respectively:

$$\log(\text{MAU})_{i,t} = \alpha_0 + \alpha_1 \cdot \text{Donedeal} + \alpha_2 \cdot t + \alpha_3 \cdot \text{After} + \alpha_4 \cdot \text{After} \cdot \text{Donedeal} + \epsilon_{i,t} \quad (15)$$

$$\log(\text{MAU})_{i,t} = \beta_0 + \beta_1 \cdot \text{Donedeal} + \beta_2 \cdot t + \beta_3 \cdot \text{After} \cdot \text{Donedeal} \cdot t + \epsilon_{i,t} \quad (16)$$

$$\log(\text{MAU})_{i,t} = \gamma_0 + \gamma_1 \cdot \text{Donedeal} + \gamma_2 \cdot t + \gamma_3 \cdot \text{After} + \gamma_4 \cdot \text{After} \cdot \text{Donedeal} + \gamma_5 \cdot \text{After} \cdot \text{Donedeal} \cdot t + \epsilon_{i,t} \quad (17)$$

The models in Equations 15-17 estimate MAU_t, the logarithm of the number of Monthly Active Users (MAU) at month t, where t is centered so that $t = 0$ is the time of the introduction of INE. Variable $After$ is a dummy indicating the time periods in which the treatment (INE) has already taken place. Variable $Donedeal$ indicates the observation corresponds to app Donedeal, while ϵ_t is the error term.

The estimation equations (15)-(17) exploit variation in outcomes across time within apps to estimate the causal effect of the treatment (the introduction of INE) on the outcome variable (the number of monthly average users) for each app separately. This identification strategy is suited for cases like the one I analyze, in which there is no control group (other non-treated apps or countries)[11] and the effects of the treatment is expected to affect several subgroups (apps Adverts and DoneDeal) differently (De Chaisemartin and D'Haultfœuille, 2023; Shahn, 2023). The identification holds under two assumptions (Shahn, 2023). First, the subgroup parallel trends assumption. This assumption states that, in the absence of treatment, the untreated outcomes in the different subgroups would have followed the same trend over time. As seen in Figure 4 and confirmed by an auxiliary parallel trends test, this is the case in my dataset. The inclusion of a common pre-treatment time trend in the estimation equation follows. Second, there should not be time-varying confounders specific to subgroups that are not captured by the model. The lack of common trends both pre- and post- treatment for all the competing apps to Adverts and DoneDeal supports this assumption.

6.2 Results

Table 2 presents the estimation of the causal effect of INE on Adverts' and DoneDeal's monthly active users.

[11] The analysis of the variation of MAU during the same period for other competing apps showed there is no other suitable untreated control group. Moreover, since both Adverts and DoneDeal are only active in Ireland, other countries could not be used as control groups.

Table 2: Estimation of the effect of intertwined network effects on the number of monthly active users for Adverts and DoneDeal under different specifications.

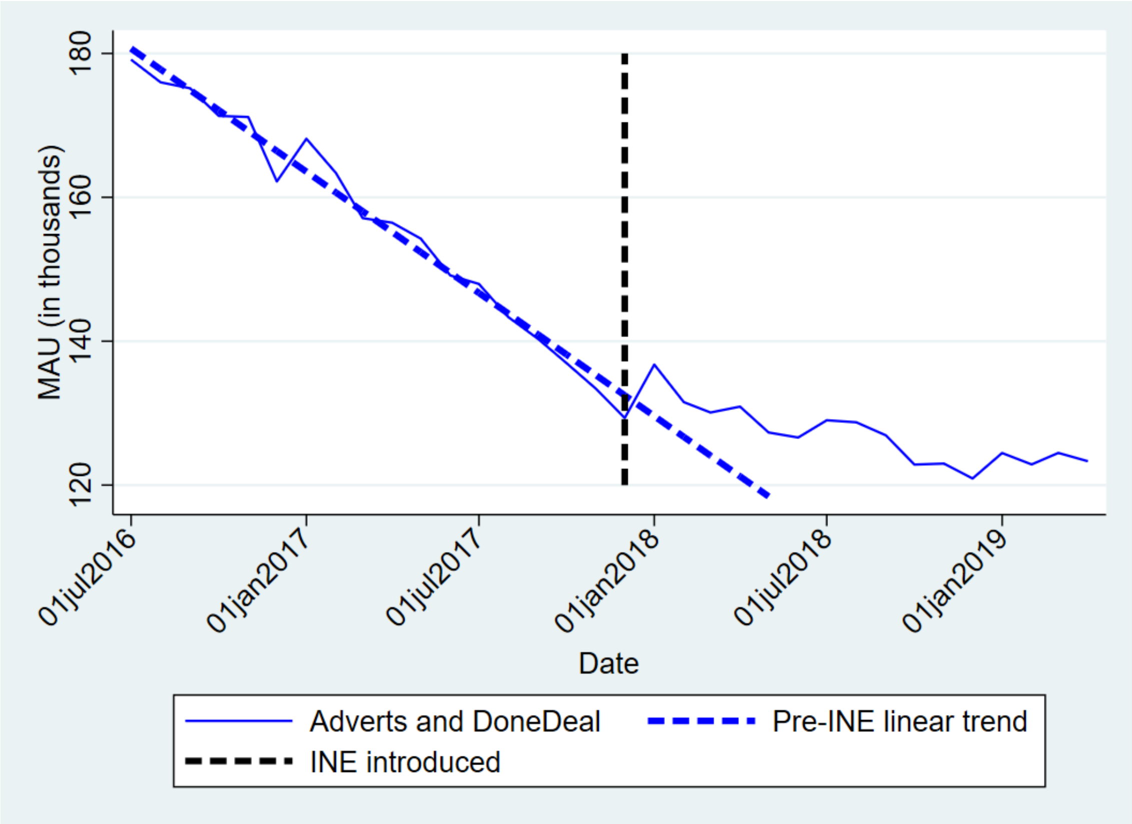
	Dependent variable: log(MAU)		
	(1)	(2)	(3)
After × Adverts	0.100***		0.052***
	(0.026)		(0.014)
After × Donedeal	-0.103***		-0.053***
	(0.034)		(0.023)
After × Adverts × t		0.024***	0.020***
		(0.002)	(0.001)
After × Donedeal × t		0.005***	0.008***
		(0.002)	(0.002)
App fixed effects	✓	✓	✓
Pre-treatment time trend	✓	✓	✓
R^2	0.9846	0.9944	0.9965
Observations	68	68	68

Note: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Standard errors in parentheses.

The three specifications corroborate the main prediction of the theoretical model: INE increase the number of users in the aggregator (Adverts). Model 3 is the preferred specification, as it aligns with the post-treatment trends observed in Figure 4. There we can observe a considerable discrete increase in the number of users of Adverts, followed by a stabilization of the time trend. This suggests that INE had an intercept and a slope effect on the evolution of the number of Adverts' users over time.

Finally, as shown in Figure 5, after the introduction of INE, we observe an increase in the number of total users when taking both apps together, compared to the pre-INE trend. This corroborates the prediction according to which INE increase the total number of users in both apps by encouraging seller multihoming.

Figure 5: Evolution of the number of monthly active users for Adverts' and DoneDeal's mobile apps aggregated (October 2015 to April 2019)



Note: the vertical dashed line indicates the month of introduction of intertwined network effects. The blue dashed line is the pre-INE linear trend.

7 Conclusion and discussion

This article studied how intertwined network effects influence platform participation, pricing, and user groups' welfare when platforms are independent and post-merger. It showed that while intertwined network effects increase consumer surplus, they reduce seller surplus if the platforms are sufficiently differentiated for sellers. In presence of INE, if the network effects they enjoy are sufficiently low, the merger harms consumers, and vice versa. If the platforms are sufficiently homogeneous to sellers, the merger reduces their surplus.

These results extend the literature on interoperability between platforms to an asymmetric case referred to as "intertwined network effects" in this article, a direction that had only been explored by Maruyama and Zennyo (2015). In the latter, when the network effects enjoyed by the multihoming side of the market are sufficiently small, their surplus increases, and vice versa. However, given this article's focus on platforms applying per-transaction pricing to the multihoming side of the market, I find that asymmetric interoperability (i.e., INE) benefits the single-homing side. The effect of INE on the multihoming user group, in turn, depends on the reallocation of the multihoming user group across platforms. With sufficiently low seller transportation costs, enough sellers leave the source platform (where prices increase) for the aggregator platform (where prices decrease), leading to an increase in overall seller surplus, and vice versa.

This article also extends the growing literature on platform mergers. It introduces per-transaction pricing to the multihoming side of the market to the canonical bottleneck setting to study a non-consolidating horizontal merger between platforms subject to INE. A novel mechanism through which a platform merger can harm users emerges from it: the reallocation of the single-homing user group across platforms. Despite the overall increase in the participation of the multihoming user group triggered by the merger, if platform differentiation is sufficiently strong, single-homing users fail to sufficiently switch from the aggregator platform (where the number of sellers decreases) to the source platform (where the number of sellers increases). As a result, the multihoming side is worse off post-merger. Moreover, if the network effects enjoyed by the users on the single-homing side are sufficiently low, post-merger, few consumers switch from the aggregator (where the number of sellers decreases) to the source platform (where the number of sellers increases). Hence, overall consumer surplus decreases.

The results of this article can help improving the analysis of mergers between transaction platforms subject to INE, or planning to set them up post-merger. Despite the existence of many platform mergers featuring INE, the literature on the topic remains scarce and competition authorities still lack clear guidance as to how they should be assessed. In that respect, Robertson provides a compelling case study in a note to the OECD (Robertson, 2023) and a subsequent article (Robertson, 2024). She shows how the ebay/Adevinta merger was differently analyzed in the eyes of the German (unconditional clearing), the Austrian (allowed with multiple structural and behavioral commitments) and the United Kingdom's (allowed with structural commitments) competition authorities. In these countries, the major concern was the overlap in the online classifieds market, in which both parties operate a transaction platform subject to per-transaction pricing on the seller side. Despite the different views expressed by these three competition authorities, an assessment of how post-merger intertwined network effects could affect consumers was absent in the three analyses. As in the ebay/Adevinta case, some competition authorities have not accounted for INE in their assessment of the merger's impact on welfare (e.g., Schibsted/Nettbil, Trade Me/Property NZ, viagogo/Stubhub). Other authorities, in turn, have judged them to be detrimental to at least one of the user groups (e.g., Booking/eTraveli, FDJ/Zeturf, Seek Asia Investments/JobStreet, Wedding Planner/Zank you). They typically argue that these effects would lessen competition by strengthening network effects.

This article shows that, in a competitive bottleneck setting with per-transaction pricing of multihoming sellers, not only should INE be considered; they should be encouraged. Even in an extreme case in which the platforms do not face competition from legally-independent firms, INE benefit consumers. Then, competition authorities guided by a consumer welfare standard could use INE as remedies in merger cases and ask platforms for a commitment not to consolidate whenever the merger might significantly limit competition. An INE remedy would not be entirely novel. Booking and eTraveli proposed such a remedy in their blocked merger. The flight metasearch platform Kayak, owned by eTraveli, proposed to set up a choice screen that would have displayed accommodation offers from Booking and other competing hotel booking portals on the check-out page of the flight booking process.[12] The European Commission dismissed it arguing this would pose a risk of self-preferencing.

However, sometimes consumers, not sellers, are on the multihoming side. This was the case in the merger between Française des Jeux (FDJ), France's national lottery, instant games and sports betting operator, and Zeturf, an online platform specialized in horse race betting. The analysis carried out by the Autorité de la Concurrence revealed that consumers (i.e., bettors) multihome to different extents depending on the game. Moreover, the French competition authority noted that the two firms had started establishing INE pre-merger and would have incentives to continuing doing so post-merger.[13] Judging this would give the merged entity too much market power, it asked firms to undo INE as a remedy to authorize the merger. As per the results in this article, harm to consumers from such a merger should only arise when network effects play a minor role in consumers' decision to join one platform over another. It follows that, as a general principle, to allow the merger, competition authorities could ask the parties to undo (or commit not to set up) INE when consumers multihome and network effects are weak, and ask them to set them up (or commit not to undo them) otherwise.

Moreover, as argued above, intertwined network effects are akin to asymmetric interoperability – as defined by Maruyama and Zennyo (2015) – between multi-sided platforms. In that respect, the results of this article can inform ongoing policy discussions about and the enforcement of asymmetric interoperability mandates imposed on some digital platforms. The European Commission's Digital Markets Act (DMA), currently under enforcement, implemented interoperability obligations for platforms considered to be "gatekeepers" in any of the defined eight "core platform services". Other major legislators and regulators are following suit within a narrower market scope. The United States' proposed Open App Markets Act includes provisions for interoperability between app stores and operating systems. India's Competition Commission has issued directives requiring Google to allow more interoperability for its services, including allowing third-party app stores and payment systems. If interoperability requirements to a specific platforms extend to transaction platforms charging per-transaction prices to the multihoming side (which is possible under the DMA), the findings of this articles could inform the design of such requirements. In this respect, it should be noted that interoperability mandates usually intend to help both business and end users participating in a platform. In the setting analysed in this article, INE might benefit one user group and harm the other. A possible solution could include regulating the per-transaction fee (if any is allowed) charged by the aggregator to make the user groups' interest converge.

[12] See para. 1196 et seq. in European Commission (2023).

[13] See paras 49-56 in Autorité de la Concurrence (2023).

Although this article's study of the effects of INE provides policy-relevant lessons for competition authorities and regulators, it leaves many questions open. Future work might complement this article's results by extending the analysis of INE to other settings. How are results affected when there is multihoming on both sides of the market? And how are results affected when the multihoming side is also charged a participation fee? One of the main limitations of this article's modelling is that it loses tractability when both sides of the market are charged a per-transaction price. It can therefore not provide insights in such cases, which are common in certain transaction platforms such as house rental platforms. Developing an alternative modelling of INE that allows to include per-transaction pricing to both sides of the market while preserving tractability might be a fruitful endeavour.

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A Proofs

A.1 Proof of Lemma 3

Subtracting the profits platforms 1 and 2 obtain in equilibrium with (cf. Lemma 2) and without (cf. Lemma 1) intertwined network effects yields the following simplified expressions, respectively.

$$\Pi^{1^{INE}} - \Pi^{1^{B}} = \frac{4\left(-\tau^b\tau^s + \sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)}\right)\left(-\tau^b\tau^s + \alpha^b\alpha^s + \sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)}\right)}{16\tau^s\alpha^{b^2}} +$$
$$\frac{(4\tau^b\tau^s - \alpha^b\alpha^s)\left(-4\tau^{b^2}\tau^{s2} + \tau^b\tau^s\alpha^b\alpha^s + \alpha^{b^2}\alpha^{s2} + \sqrt{\tau^b\tau^s(16\tau^{b^3}\tau^{s3} - 8\tau^{b^2}\tau^{s2}\alpha^b\alpha^s + 5\tau^b\tau^s\alpha^{b^2}\alpha^{s2} - 2\alpha^{b^3}\alpha^{s3})}\right)}{16\tau^s\alpha^{b^2}}$$

$$\Pi^{2^{INE}} - \Pi^{2^{B}} = \frac{(-2\tau^b\tau^s + \alpha^b\alpha^s + 2\sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)})^2}{16\tau^s\alpha^{b^2}}$$

Given that all the parameters are positive by definition, the two expressions are non-negative. *Q.E.D.*

A.2 Proof of Lemma 4

Subtracting the prices platforms 1 and 2 charge sellers in equilibrium with (cf. Lemma 2) and without (cf. Lemma 1) intertwined network effects yields the following simplified expressions, respectively.

$$p_1^{s^{INE}} - p_1^{s^{B}} = -\frac{16\tau^{b^3}\tau^{s3} - 4\tau^{b^2}\tau^{s2}\alpha^b\alpha^s + \alpha^{b^2}\alpha^{s2}\sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)}}{u^3\alpha^{s2}} +$$
$$\frac{\tau^b\tau^s(u^2\alpha^{s2} - 4\sqrt{\tau^b\tau^s(16\tau^{b^3}\tau^{s3} - 8\tau^{b^2}\tau^{s2}\alpha^b\alpha^s + 5\tau^b\tau^s\alpha^{b^2}\alpha^{s2} - 2\alpha^{b^3}\alpha^{s3})})}{u^3\alpha^{s2}}$$
$$p_2^{s^{INE}} - p_2^{s^{B}} = \frac{4\tau^b\tau^s - \alpha^b\alpha^s - 4\sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)}}{4\alpha^b}$$

$p_1^{s^{INE}} - p_1^{s^{B}}$ is positive if and only if $\frac{\alpha^b\alpha^s}{\tau^s} < \tau^b < \tilde{\tau^b}(\alpha^b, \alpha^s, \tau^s)$ and negative otherwise, where $\tilde{\tau^b}$ is the second root of the following polynomial:

$$P(\tau^b) = -\alpha^{b^4}\alpha^{s4} + 80\tau^{s2}\alpha^{b^2}\alpha^{s2}\tau^{b^2} - 384\tau^{s3}\alpha^b\alpha^s\tau^{b^3} + 256\tau^{s4}\tau^{b^4}$$

The expression for ${p_2^s}^{INE} - {p_2^s}^B$, in turn, is always positive given that all the parameters are strictly positive. *Q.E.D.*

A.3 Proof of Lemma 5

Platform 1 buyers. Subtracting the equilibrium number of buyers in platform 1 with (cf. Lemma 2) and without (cf. Lemma 1) intertwined network effects yields the following simplified expression.

$$ {n_1^b}^{INE} - {n_1^b}^B = \frac{\tau^b\tau^s\left(-4\tau^b\tau^s+3\alpha^b\alpha^s\right)+\sqrt{\tau^b\tau^s\left(16\tau^{b3}\tau^{s3}-8\tau^{b2}\tau^{s2}\alpha^b\alpha^s+5\tau^b\tau^s\alpha^{b2}\alpha^{s2}-2\alpha^{b3}\alpha^{s3}\right)}}{8\tau^b\tau^s\left(2\tau^b\tau^s-\alpha^b\alpha^s\right)} $$

This expression is non-negative when the second-order conditions with intertwined network effects (cf. Assumption 2), i.e., $\alpha^b\alpha^s < 2\tau^b\tau^s$.

Platform 2 buyers. Subtracting the equilibrium number of buyers in platform 2 with (cf. Lemma 2) and without (cf. Lemma 1) intertwined network effects yields the following simplified expression.

$$ {n_2^b}^{INE} - {n_2^b}^B = \frac{-2\tau^b\tau^s+\alpha^b\alpha^s+2\sqrt{\tau^b\tau^s(\tau^b\tau^s-\alpha^b\alpha^s)}}{4\tau^s\alpha^b} $$

Given that all the parameters are strictly positive by definition, this expression cannot be positive.

Platform 1 sellers. Subtracting the equilibrium number of sellers in platform 1 with (cf. Lemma 2) and without (cf. Lemma 1) intertwined network effects yields the following simplified expression.

$$ {n_1^s}^{INE} - {n_1^s}^B = \frac{2\sqrt{\tau^b\tau^s(\tau^b\tau^s-\alpha^b\alpha^s)}+\frac{\tau^b\tau^s(-8\tau^b\tau^s+5\alpha^b\alpha^s)+\sqrt{\tau^b\tau^s(16\tau^{b3}\tau^{s3}-8\tau^{b2}\tau^{s2}\alpha^b\alpha^s+5\tau^b\tau^s\alpha^{b2}\alpha^{s2}-2\alpha^{b3}\alpha^{s3})}}{2\tau^b\tau^s-\alpha^b\alpha^s}}{4\tau^s\alpha^b} $$

Given that all the parameters are strictly positive by definition, this expression is non-negative.

Platform 2 sellers. Subtracting the equilibrium number of sellers in platform 2 with (cf. Lemma 2) and without (cf. Lemma 1) intertwined network effects yields the following simplified expression.

Given that all the parameters are strictly positive by definition, this expression cannot be positive.

$$ {n_2^s}^{INE} - {n_2^s}^B = \frac{-2\tau^b\tau^s+\alpha^b\alpha^s+2\sqrt{\tau^b\tau^s(\tau^b\tau^s-\alpha^b\alpha^s)}}{4\tau^s\alpha^b} $$

Platforms 1 and 2 sellers. Subtracting the equilibrium number of sellers present in both platforms with (cf. Lemma 2) and without (cf. Lemma 1) intertwined network effects yields the following simplified expression.

$$ {n_s}^{INE} - {n_s}^B = \frac{\alpha^s+\frac{4(-\tau^b\tau^s+\sqrt{\tau^b\tau^s(\tau^b\tau^s-\alpha^b\alpha^s)})}{\alpha^b}+\frac{\tau^b\tau^s(-4\tau^b\tau^s+3\alpha^b\alpha^s)+\sqrt{\tau^b\tau^s(16\tau^{b3}\tau^{s3}-8\tau^{b2}\tau^{s2}\alpha^b\alpha^s+5\tau^b\tau^s\alpha^{b2}\alpha^{s2}-2\alpha^{b3}\alpha^{s3})}}{\alpha^b(2\tau^b\tau^s-\alpha^b\alpha^s)}}{4\tau^s} $$

Bearing in mind that all the parameters are strictly positive by definition, ${n_s}^{INE} - {n_s}^{B} < 0 \iff \frac{\alpha^b \alpha^s}{\tau^s} \leq \tau^b < \frac{(3+2\sqrt{3})\alpha^b \alpha^s}{6\tau^s}$ and ${n_s}^{INE} - {n_s}^{B} > 0 \iff \tau^b > \frac{(3+2\sqrt{3})\alpha^b \alpha^s}{6\tau^s}$.

A.4 Proof of Proposition 1

Denote Δn_i^l the absolute value of the variation in participation of user group $i = b, s$ in platform $l = 1, 2$ after the introduction of intertwined network effects (INE), compared to the non-INE benchmark. Given the uniform distribution of the mass 1 total number of consumers, the absolute change in consumer surplus generated by INE in platforms 1 (denoted ΔCS^1) and 2 (denoted ΔCS^2) can be expressed as:

$$\Delta CS^1 = (\frac{1}{2} + \Delta n_b^1)(n_1^{sB} + \Delta n_1^s + n_2^{sB} - \Delta n_2^s)$$
$$\Delta CS^2 = (\frac{1}{2} - \Delta n_b^2)\Delta n_2^s$$

Where the positive or negative signs are given by Lemma 5. The net effect of INE on consumer surplus on both platforms $\Delta CS \equiv \Delta CS^1 + \Delta CS^2$ is hence:

$$\Delta CS = \Delta\frac{1}{2}n_1^{sB} + \frac{1}{2}\Delta n_1^s + \frac{1}{2}n_2^{sB} + \Delta n_b^1 n_1^{sB} + \Delta n_b^1 \Delta n_1^s + \Delta n_b^1 n_2^{sB} - (\Delta n_b^1 + \Delta n_b^2)\Delta n_2^s$$

Denote $\Delta n_1^b \equiv \Delta n_b$. Given that the market is covered on the consumer side, $\Delta n_b = -\Delta n_2^b$, and hence $-\Delta n_2^b \equiv -\Delta n_b$. Then,

$$\Delta CS = \left(\frac{1}{2} + \Delta n_b\right) n_1^{sB} + \left(\frac{1}{2} + \Delta n\right) \Delta n_1^s + \left(\frac{1}{2} + \Delta n_b\right) n_2^{sB} > 0.$$

Q.E.D.

A.5 Proof of Proposition 2

Subtracting seller surplus equilibrium with (cf. Lemma 2) and without (cf. Lemma 1) intertwined network effects yields the following simplified expression.

$$SS^{INE} - SS^{B} = \frac{1}{32}\Bigg(8\tau^{s}\left(-2+\frac{\tau^{b^2}}{\alpha^{b^2}}\right) - \frac{3\pi_s^2}{\tau^s} - \frac{8}{\alpha^{b^2}}\Bigg(- 2\tau^{b^2}\tau^{s}$$
$$- 2\alpha^{b}\left(\tau^{s}\alpha^{b} + 2\sqrt{\tau^{b}\tau^{s}(\tau^{b}\tau^{s}-\alpha^{b}\pi_s)}\right) + \tau^{b}\left(4\tau^{s}\alpha^{b} + \alpha^{b}\pi_s + 2\sqrt{\tau^{b}\tau^{s}(\tau^{b}\tau^{s}-\alpha^{b}\pi_s)}\right)\Bigg)$$
$$+ \frac{1}{\alpha^{b^2}(-2\tau^{b}\tau^{s}+\alpha^{b}\pi_s)^2}\Bigg(16\alpha^{b^4}\pi_s^3 + 2\tau^{b^2}\tau^{s}\Big(\alpha^{b^2}\pi_s\left(32\tau^{s}+3\pi_s\right)$$
$$- 4\sqrt{\tau^{b}\tau^{s}\left(16\tau^{b^3}\tau^{s3} - 8\tau^{b^2}\tau^{s2}\alpha^{b}\pi_s + 5\tau^{b}\tau^{s}\alpha^{b^2}\pi_s^2 - 2\alpha^{b^3}\pi_s^3\right)}$$
$$+ 2\tau^{b}\alpha^{b}\pi_s\Big(- \alpha^{b^2}\pi_s\left(32\tau^{s}+\pi_s\right) + 3\sqrt{\tau^{b}\tau^{s}\left(16\tau^{b^3}\tau^{s3} - 8\tau^{b^2}\tau^{s2}\alpha^{b}\pi_s + 5\tau^{b}\tau^{s}\alpha^{b^2}\pi_s^2 - 2\alpha^{b^3}\pi_s^3\right)}\Big)\Big)\Bigg)\Bigg)$$

To simplify the mathematical expressions, and without loss of generality in the proof[14], denote $R^{n}_{(d,r)}(x)$ a function describing the r^{th} root a polynomial of degree $d \in \mathbb{N}$ in variable x. Superscript $n = [I, II, ...]$ identifies the equation describing the polynomial. Roots are ordered in increasing order.

Bearing in mind that all the parameters are strictly positive by definition, $SS^{INE} - SS^{B} < 0$ if and only if one of the following sets of conditions, labelled as $C^{SS-}_{i\in\mathbb{N}}$ are met:

$$C_1^{SS-} = \left\{\frac{5\alpha^{s}}{8} < \tau^{s} \leq \frac{17\alpha^{s}}{16} - \frac{1}{8}\sqrt{11}\alpha^{s}\right\} \text{ and } \left\{\tau^{b} > R^{I}_{(9,3)}\left(\tau^{b}\alpha^{b},\alpha^{s},\tau^{s}\right)\right)\right\}$$

$$C_2^{SS-} = \left\{\frac{17\alpha^{s}}{16} - \frac{1}{8}\sqrt{11}\alpha^{s} < \tau^{s} < R^{II}_{(4,2)}\left(\tau^{s}(\alpha^{s})\right)\right\} \text{ and } \left\{\tau^{b} \geq \frac{\alpha^{s}\alpha^{b}}{\tau^{s}})\right\}$$

$$C_3^{SS-} = \left\{R^{II}_{(4,2)}(\tau^{s}(\alpha^{s})) \leq \tau^{s} < R^{III}_{(16,3)}(\tau^{b}(\alpha^{b},\alpha^{s},\tau^{s}))\right\} \text{ and }$$
$$\left\{\left(\frac{\alpha^{s}\alpha^{s}}{\tau^{s}} \leq \tau^{b} < R^{I}_{(9,1)}\left(\tau^{b}\alpha^{b},\alpha^{s},\tau^{s}\right)\right)\right) \vee \left(\tau^{b} > 2\alpha^{s}\right)\right\}$$

$$C_4^{SS-} = \left\{\tau^{s} = R^{III}_{(16,3)}\left(\alpha^{s}\right)\right\} \text{ and } \left\{\left(\frac{\alpha^{s}\alpha^{s}}{\tau^{s}} \leq \tau^{b} < R^{I}_{(9,1)}\left(\tau^{b}(\alpha^{b},\alpha^{s},\tau^{s})\right)\right)\right\}$$

[14]The full expressions are available upon request.

$$C_5^{SS-} = \left\{\tau^s > R^{III}_{(16,3)}\left(\alpha^s\right)\right\} \text{ and } \left\{\tau^b \geq \frac{\alpha^s\alpha^s}{\tau^s}\right\}$$

In the same vein, $SS^{INE} - SS^B > 0$ if and only if one of the following sets of conditions, labelled as $C^{SS+}_{i\in\mathbb{N}}$ are met:

$$C_1^{SS+} = \left\{0 < \tau^s \leq \frac{5\alpha^s}{8}\right\} \text{ and } \left\{\tau^b \geq \frac{\alpha^s\alpha^s}{\tau^s}\right\}$$

$$C_2^{SS+} = \left\{\frac{5\alpha^s}{8} < \tau^s \leq \frac{17\alpha^s}{16} - \frac{1}{8}\sqrt{11}\sqrt{\pi_s^2}\right\} \text{ and } \left\{\frac{\alpha^b\alpha^s}{\tau^s} \leq \tau^b < R^I_{(9,1)}\left(\tau^b(\alpha^b, \alpha^s, \tau^s)\right)\right\}$$

$$C_3^{SS+} = \left\{\frac{17\alpha^s}{16} - \frac{1}{8}\sqrt{11}\sqrt{\pi_s^2} < \tau^s < R^{II}_{(4,2)}\right\} \text{ and } \left\{R^I_{(9,1)}(\tau^b\alpha^b, \alpha^s, \tau^s)) < \tau^b < R^I_{(9,3)}(\tau^b\alpha^b, \alpha^s, \tau^s))\right\}$$

$$C_4^{SS+} = \left\{\tau^s = R^{II}_{(4,2)}\right\} \text{ and } \left\{R^I_{(9,1)}(\tau^b\alpha^b, \alpha^s, \tau^s)) < \tau^b < 2\alpha^b\right\}$$

$$C_5^{SS+} = \left\{R^{II}_{(4,2)} < \tau^s < R^{III}_{(16,3)}(\tau^s, \alpha^b, \alpha^s, u)\right\} \text{ and } \left\{R^I_{(9,1)}(\tau^b\alpha^b, \alpha^s, \tau^s)) < \tau^b < R^I_{(9,2)}(\tau^b\alpha^b, \alpha^s, \tau^s))\right\}$$

Note that none of the C^{SS-} sets of conditions imply $\tau^s \leq \frac{5\alpha^s}{8}$. Moreover, condition C_1^{SS+} shows that $\tau^s \leq \frac{5\alpha^s}{8}$ is a sufficient condition for seller surplus to decrease, as the second part of the condition, $\tau^b \geq \frac{\alpha^s\alpha^b}{\tau^s}$, corresponds to the second order conditions in the pre-INE benchmark reported in Assumption 1. This proves point i) of Proposition 2.

By comparing the C^{SS-} sets of conditions, it can see that they all imply $\tau^s > \tilde{\tau^s}(\alpha^s) > \frac{5}{8}\alpha^s$. This proves point ii) of Proposition 2.

Q.E.D.

A.6 Proof of Lemma 7

Denote $\Pi^{INE-M} \equiv \Pi^{1^{INE-M}} + \Pi^{2^{INE-M}}$ and $\Pi^{INE} \equiv \Pi^{1^{INE}} + \Pi^{2^{INE}}$. Subtracting Π^{INE} from Π^{INE-M} and simplifying yields:

$$\Pi^{INE-M} - \Pi^{INE} = \frac{\left(-2\tau^b\tau^s + \alpha^b\alpha^s + 2\sqrt{\tau^b\tau^s\left(\tau^b\tau^s - \alpha^b\alpha^s\right)}\right)^2}{8\tau^s {\alpha^b}^2} > 0$$

Q.E.D.

A.7 Proof of Lemma 8

Subtracting the equilibrium price charged to sellers in platform 1 when the platforms are legally-independent (cf. Lemma 2) from the post-merger equilibrium price charged to sellers in platform 1 (cf. Lemma 6) and simplifying yields:

$$\begin{aligned} {p_1^s}^{INE-M} - {p_1^s}^{INE} = 2\tau^b\tau^s \Bigg(& \frac{1}{\alpha^b} - \frac{2\tau^b\tau^s}{{\alpha^b}^2\alpha^s} + \frac{\alpha^s}{-4\tau^b\tau^s + \alpha^b\alpha^s} \\ & - \frac{2\left(-4{\tau^b}^2\tau^{s2} + \sqrt{\tau^b\tau^s\left(16{\tau^b}^3\tau^{s3} - 8{\tau^b}^2\tau^{s2}\alpha^b\alpha^s + 5\tau^b\tau^s{\alpha^b}^2\alpha^{s2} - 2{\alpha^b}^3\alpha^{s3}\right)}\right)}{{\alpha^b}^3\alpha^{s2}} \Bigg) \end{aligned}$$

Which is strictly positive under the second order conditions of the merger case set out in Assumption 3, $\alpha^b\alpha^s < 2\tau^b\tau^s$

Subtracting the equilibrium price charged to sellers in platform 2 with INE (cf. Lemma 2) from the equilibrium price charged to sellers in platform 2 post-merger (cf. Lemma 6) yields:

$${p_2^s}^{INE-M} - {p_2^s}^{INE} = -\frac{\alpha^s}{4} < 0.$$

Q.E.D.

A.8 Proof of Lemma 9

Platform 1 consumers' participation. Subtracting the equilibrium participation on the consumer side in platform 1 with INE (cf. Lemma 2) from the post-merger one (cf. Lemma 6) and simplifying yields:

$${n_1^b}^{INE-M} - {n_1^b}^{INE} = \frac{4{\tau^b}^2\tau^{s2} + \tau^b\tau^s\alpha^b\alpha^s - {\alpha^b}^2\alpha^{s2} - \sqrt{\tau^b\tau^s(16{\tau^b}^3\tau^{s3} - 8{\tau^b}^2\tau^{s2}\alpha^b\alpha^s + 5\tau^b\tau^s{\alpha^b}^2\alpha^{s2} - 2{\alpha^b}^3\alpha^{s3})}}{8\tau^b\tau^{s2}\alpha^b - 4\tau^s{\alpha^b}^2\alpha^s}$$

Which is strictly negative given that all the parameters are strictly positive by definition.

Platform 2 consumers' participation. Subtracting the equilibrium participation on the consumer side in platform 2 with INE (cf. Lemma 2) from the post-merger one (cf. Lemma 6) and simplifying yields:

$${n_2^b}^{INE-M} - {n_2^b}^{INE} = \frac{\tau^b\tau^s(-4\tau^b\tau^s + \alpha^b\alpha^s) + \sqrt{\tau^b\tau^s(16{\tau^b}^3\tau^{s3} - 8{\tau^b}^2\tau^{s2}\alpha^b\alpha^s + 5\tau^b\tau^s{\alpha^b}^2\alpha^{s2} - 2{\alpha^b}^3\alpha^{s3})}}{8\tau^b\tau^s(2\tau^b\tau^s - \alpha^b\alpha^s)}$$

Which is strictly positive given that all the parameters are strictly positive by definition.

Platform 1 sellers' participation. Subtracting the equilibrium participation on the seller side in platform 1 when the platforms are legally independent (cf. Lemma 2) from the post-merger one (cf. Lemma 6) and simplifying yields:

$$n_1^{s\,INE-M} - n_1^{s\,INE} = \frac{4\tau^{b^2}\tau^{s2} - \tau^b\tau^s\alpha^b\alpha^s - \sqrt{\tau^b\tau^s(16\tau^{b^3}\tau^{s3} - 8\tau^{b^2}\tau^{s2}\alpha^b\alpha^s + 5\tau^b\tau^s\alpha^{b^2}\alpha^{s2} - 2\alpha^{b^3}\alpha^{s3})}}{8\tau^b\tau^{s2}\alpha^b - 4\tau^s\alpha^{b^2}\alpha^s}$$

Which is strictly negative given that all the parameters are strictly positive by definition.

Platform 2 sellers' participation. Subtracting the equilibrium participation on the seller side in platform 2 when the platforms are legally independent (cf. Lemma 2) from the post-merger one (cf. Lemma 6) and simplifying yields:

$$n_2^{s\,INE-M} - n_2^{s\,INE} = \frac{\alpha^s}{4\tau^s} > 0$$

Overall sellers' participation. Subtracting the equilibrium participation on the seller side in both platforms when the platforms are legally-independent (cf. Lemma 2) from the post-merger one (cf. Lemma 6) and simplifying yields:

$$n_s^{INE-M} - n_s^{INE} = \frac{-4\tau^{b^2}\tau^{s2} + \tau^b\tau^s\alpha^b\alpha^s + \sqrt{\tau^b\tau^s(16\tau^{b^3}\tau^{s3} - 8\tau^{b^2}\tau^{s2}\alpha^b\alpha^s + 5\tau^b\tau^s\alpha^{b^2}\alpha^{s2} - 2\alpha^{b^3}\alpha^{s3})}}{8\tau^b\tau^{s2}\alpha^b - 4\tau^s\alpha^{b^2}\alpha^s}$$

Which is strictly negative given that all the parameters are strictly positive by definition.

Q.E.D.

A.9 Proof of Proposition 3

Recalling that all the parameters are strictly positive, and assuming the second-order condition $SOCs^{INE}$ set out in Assumption 2 holds, $CS^{INE-M} > CS^{INE} \iff 2\tau^b\tau^{s2} > \tau^s\alpha^b\alpha^s$ and $v^b < \tilde{v^b}$, while $CS^{INE-M} < CS^{INE} \iff 2\tau^b\tau^{s2} > \tau^s\alpha^b\alpha^s$ and $v^b > \tilde{v^b}$.

Moreover:

$$CS^{INE-M} > CS^{INE} \cap v^b > \tilde{v^b} \cap Assumption\ 2 \cap Assumption\ 3 = \varnothing$$
$$CS^{INE-M} > CS^{INE} \cap v^b < \tilde{v^b} \cap Assumption\ 2 \cap Assumption\ 3 \neq \varnothing$$
$$CS^{INE-M} < CS^{INE} \cap v^b > \tilde{v^b} \cap Assumption\ 2 \cap Assumption\ 3 \neq \varnothing$$
$$CS^{INE-M} < CS^{INE} \cap v^b < \tilde{v^b} \cap Assumption\ 2 \cap Assumption\ 3 = \varnothing$$

Hence, $CS^{INE-M} > CS^{INE} \iff v^b < \tilde{v^b}$ and $CS^{INE-M} < CS^{INE} \iff v^b > \tilde{v^b}$.

Q.E.D.

A.10 Proof of Proposition 4

By comparing the region in which $SS^{INE-M} < SS^{INE} \cap Assumption\ 2 \cap Assumption\ 3$ holds with that in which $SS^{INE-M} > SS^{INE} \cap Assumption\ 2 \cap Assumption\ 3$ holds, it can be seen that

$$SS^{INE-M} > SS^{INE} \cap Assumption\ 2 \cap Assumption\ 3 \cap \tau^b > \hat{\tau^b} = \varnothing$$
$$SS^{INE-M} < SS^{INE} \cap Assumption\ 2 \cap Assumption\ 3 \cap \tau^b > \hat{\tau^b} \neq \varnothing$$

Therefore, $\tau^b > \hat{\tau^b} \implies SS^{INE-M} < SS^{INE}$.