A network approach to interoperability*

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Abstract

We study how interoperability, modeled as a weighted network of connections among competing platforms with user externalities, affects equilibrium prices and welfare. By characterizing equilibrium prices as functions of the eigenvalues of the interoperability network, we derive optimal network configurations and show that industry-wide full interoperability maximizes total welfare and is also platform-optimal. When user externalities are weak, the price effects of interoperability are bounded relative to the direct user-benefit effect, implying that full interoperability is Pareto-optimal. In contrast, coalitional interoperability does not maximize total welfare but can be consumer-optimal when user externalities are strong, as interoperability within coalitions endogenously generates demand complementarities among their members while intensifying competition across coalitions. For the case of four platforms, we fully characterize the set of feasible combinations of profit and consumer surplus and identify the associated Pareto frontier. Overall, the welfare effects of interoperability depend critically on both the strength of user externalities and the structure of interoperability within the industry.

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1 Introduction

Modern economies are increasingly shaped by competing platforms that exhibit positive demand-side user externalities. These effects are typically *within-platform*: a platform becomes more valuable to its users as its own user base grows, reflecting increased interactions that generate economic benefits. *Cross-platform* user externalities arise when users can interact across platforms, so that an increase in a platform's user base also generates interaction benefits for users on competing platforms. We use the term interoperability (or horizontal interoperability) to describe the channels or arrangements that enable such cross-platform user externalities.[1] To illustrate, we highlight three prominent examples.

- *Interoperability mandates.* Regulators may impose interoperability requirements to ensure that users on different platforms can interact and communicate seamlessly. Such mandates are common in industries characterized by strong user externalities, including telecommunications, social messaging applications, and internet service provision (ISPs).[2]
- *Data sharing arrangements.* Data-driven user externalities arise when firms learn from customer usage data to improve their products, attracting more users and generating additional data in turn.[3] Interoperability occurs when data collected by one firm partially enables another firm to improve its product, for example through bilateral or multilateral data-sharing agreements.[4]
- *Standardization and complementors.* A hardware device with a larger user base attracts more complementary products or services, increasing the device's value to users. For instance, a widely adopted video format (such as Betamax or VHS) attracts more content publishers to produce compatible media. When a format is standardized across multiple devices, these benefits extend to all users of the standard. Similar dynamics arise in the An-

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[1] The economics literature (e.g., Katz and Shapiro (1985); Crémer et al. (2000)) uses "compatibility" or "interconnectedness" to describe the same cross-platform demand-side externalities.

[2] A recent and important example arises under the EU Digital Markets Act (DMA), which imposes interoperability obligations on messenger services—such as WhatsApp—covering basic communication functionalities. For social media platforms such as Facebook and $\mathbb{X}$ (formerly Twitter), Scott Morton et al. (2023) provide detailed proposals for implementing interoperability requirements for standard functionalities, including the exchange of text, images, videos, and calendar features.

[3] Hagiu and Wright (2023) emphasize that data-driven user externalities operate through a process of "learning by using," whereby firms leverage accumulated user data to improve product quality, which in turn raises demand and generates additional data for further learning.

[4] For example, Everis et al. (2018) document a large and increasing prevalence of B2B data sharing. Consistent with this trend, a growing number of start-ups specialize in facilitating B2B data sharing, alongside policy initiatives such as GAIA-X in Europe.

droid ecosystem, where multiple original equipment manufacturers (OEMs) share a common operating system, thereby pooling the interaction benefits through shared complementary applications.

Taken together, these examples show that interoperability can take many forms, including industry-wide arrangements (e.g., regulatory mandates), coalition-based arrangements (e.g., standard-setting for file formats and digital ecosystems), and hub-and-spoke structures (e.g., centralized data-sharing agreements). This diversity of approaches raises several central economic questions: (i) How do different interoperability configurations affect platform competition and welfare? (ii) What limits, if any, constrain the welfare gains achievable through interoperability? (iii) Do more comprehensive forms of interoperability (such as industry-wide mandates) or more selective arrangements (such as coalitions) generate higher consumer welfare? (iv) What trade-offs do these approaches involve?

To address these questions within a unified framework, we develop a flexible, *network-theoretic* model of interoperability among platforms. The framework characterizes interoperability along two dimensions: strength, which captures the extent to which user externalities (or interaction benefits) are shared across platforms, and configuration, which specifies with whom such benefits are shared. More specifically, in Section 2 we consider an environment with $n \geq 2$ symmetric platforms competing for single-homing users by setting participation prices in a fully covered market. Users on each platform derive user externalities that increase with the effective user base that they can access and interact with. We represent this *effective user base* through an $n \times n$ interoperability matrix, where interoperability between any pair of platforms is encoded as a weighted link, as illustrated in Figure 1.[5] By appropriately specifying this matrix, the model accommodates industry-wide interoperability, coalition-based arrangements, combinations thereof, and a wide range of other configurations within a single oligopolistic framework.[6]

Our baseline model of symmetric platforms yields a tractable symmetric equilibrium characterization for any interoperability configuration. In Section 3, we show that equilibrium prices admit two equivalent representations. First, using a spectral decomposition of the interoperability matrix, we express the equilibrium price in closed form as a symmetric and concave function of the profile of

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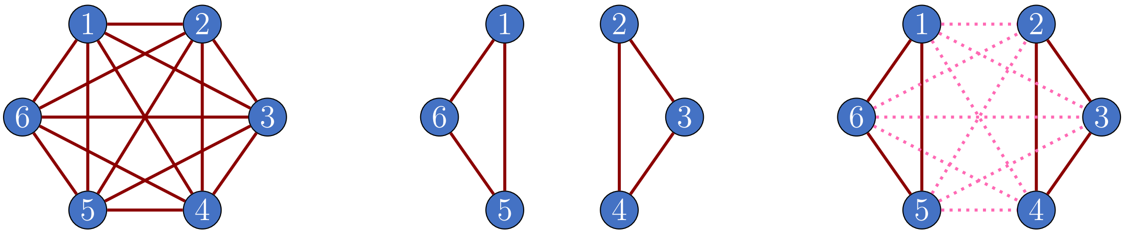
[5] Figure 1 illustrates the interoperability matrix for a set of $n = 6$ platforms, where each weighted link represents the fraction of user externalities shared between a pair of platforms.

[6] The interoperability matrix plays a role analogous to the adjacency matrix in social network analysis (see, e.g., Jackson (2008) for a textbook treatment): each entry encodes the intensity of bilateral links, which in our context captures the degree to which user externalities are shared across platforms.



Industry-wide

Coalitions

Cross-linked coalitions

Figure 1: Examples of interoperability configurations.

eigenvalues of the matrix, thereby facilitating a tractable analysis of optimal configurations.

Second, from the perspective of the platform-pricing literature, we show that the equilibrium price can be decomposed into three components: a standard markup that reflects market power arising from product differentiation, a user-externality subsidy, and a novel “interoperability internalization” term, which captures how each platform internalizes the effects of its pricing decisions on interoperable partners. This decomposition highlights that interoperability can either raise or lower prices relative to the zero-interoperability benchmark, depending on whether it induces demand substitution or complementarity among interoperable platforms.

Building on our equilibrium characterization and interpretations, we investigate the welfare consequences of interoperability in Section 4. We begin by identifying the configurations that are optimal under each welfare objective. Among all admissible configurations, full industry-wide interoperability maximizes total surplus and is also platform-optimal, as it yields the highest equilibrium price and achieves the maximum level of effective user externalities. The price-maximizing property of full industry-wide interoperability follows from a *convex analysis* argument that utilizes the symmetry and concavity of the equilibrium price with respect to the profile of eigenvalues of the interoperability matrix.

By contrast, the lowest prices and profits arise under zero interoperability when user externalities are weak. This result relies on a *bounding argument* that establishes a universal lower bound on the price for any admissible configuration, and that the bound is tight and attained under zero interoperability. Although this approach does not extend to environments with strong user externalities, we instead focus on the class of cross-linked coalition configurations, for which the eigenvalues of the interoperability matrix are linear in link weights. This property enables a stronger form of convex analysis: equilibrium prices are symmetric and concave in link weights directly, allowing us to show that the dual-coalition structure yields the lowest price.

The key mechanism underlying this novel result is the emergence of within-coalition demand complementarity, which we quantify using demand diversion ratios. Typically, platforms are substitutes in consumer demand: a price increase by one platform diverts users to its rivals. Interoperability among coalition partners, however, alters this logic. By sharing the positive externalities generated by their effective user bases, coalition members increase the coalition's overall attractiveness relative to competitors when any one member lowers its price. When user externalities are strong, a price cut by a single platform can even raise the total demand for its partners.

In the presence of such within-coalition demand complementarity, each platform internalizes the positive demand spillovers generated by its price reductions for interoperable partners. As a result, coalition-based interoperability induces platforms to adopt more aggressive pricing strategies.

Turning to consumer surplus, we identify a trade-off between expanding user externalities from the effective user base and inducing aggressive pricing by platforms. The analysis technique follows from those of the price analysis. Specifically, when user externalities are weak, the *bounding argument* shows that industry-wide interoperability is consumer-surplus maximizing. Likewise, when user externalities are strong, the *convex analysis* approach focusing on the class of cross-linked configurations implies the consumer surplus is symmetric and concave in the link weights. We then show that a dual-coalition configuration is the optimal configurations within that class.

Thus, even though coalitional interoperability does not maximize total welfare, it can generate higher consumer surplus than industry-wide interoperability, precisely because the former triggers more aggressive price competition.

In Section 5, we focus on $n = 4$ platforms, whereby the interoperability matrix of every admissible interoperability can be described by eigenvalues that are linear in the link weights. Hence, we again apply the convex analysis approach to identify the configurations that correspond to the extremum levels of price and consumer surplus. Then, an intermediate value theorem argument shows that every outcome between these extremes is achievable by some admissible configuration. We thus obtain a full characterization of achievable prices, profits, and consumer surplus—and hence of the entire Pareto frontier of welfare allocations in consumer surplus and industry profit—without restricting attention to specific configuration classes.

Finally, we discuss the policy relevance of our analysis for ongoing debates on interoperability. Our framework studies how interoperability among competing platforms affects prices and welfare. To keep the analysis general and consistent with the existing literature, we consider a competition setting that abstracts from industry-specific features. Despite this abstraction, our findings yield several insights that are relevant to regulatory discussions of interoperability mandates under platform legislation such as the EU Digital Markets Act (DMA).

First, prevailing debates often suggest that industry-wide interoperability intensifies competition in markets with asymmetric firms but may relax competition in relatively symmetric environments. Accordingly, proponents of interoperability emphasize its role in preserving market contestability and leveling the playing field between incumbents and entrants.[7] We contribute to this discussion by showing that this competition-relaxing effect is not inevitable when alternative interoperability configurations, such as coalition-based arrangements, are allowed. In such cases, interoperability induces demand complementarities between interoperable platforms and can mitigate, or even reverse, the perceived downside of interoperability.

Second, our results raise the question of whether regulatory interventions should primarily rely on mandatory industry-wide interoperability.[8] While this approach is non-discriminatory and maximizes total interaction benefits generated for users, it may also weaken platforms' incentives to compete for users within the interoperable group. Using a symmetric baseline model, we analyze general interoperability configurations, including the underexplored case of coalition-based interoperability, and we show that these arrangements can generate higher consumer surplus by balancing user base expansion with stronger price competition.

More broadly, our analysis suggests that regulatory policy may benefit from considering interoperability interventions that are more finely tailored than a uniform, industry-wide mandate.

This article contributes to the literature on canonical platform competition models, which has examined the importance of user externalities in driving competition outcomes.[9] Early works include Caillaud and Jullien (2003), Rochet and

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[7] For example, DMA requires gatekeepers offering number-independent interpersonal communication services (NI-ICS), classified as core platform services, to enable interoperability of personal information across competing entities upon users' request, without any associated fees. From a dynamic perspective, such requirements help maintain the contestability of digital markets primarily by reducing barriers to entry (see Line 105 of the DMA preface).

[8] For example, according to DMA's definition of interoperability, interoperable information must function across "all" elements of hardware and software in "all" possible ways (Article 1 of the DMA), which is most closely related to a full industry-wide interoperability in our framework.

[9] In this literature, user externalities are more commonly referred to as "network benefits." We avoid this terminology to prevent confusion between user externalities and the network representation of interoperability used in our analysis.

Tirole (2003, 2006), and Armstrong (2006), which have provided basic foundations for studying pricing by monopoly and duopoly platforms. Jullien et al. (2021) survey this literature. These works provide important insights into equilibrium pricing patterns and the impact of platform entry, but they do not allow for interoperability across platforms, which is our focus.[10]

The literature on interoperability or compatibility, pioneered by Katz and Shapiro (1985); Farrell and Saloner (1985); Crémer et al. (2000), initially focused on Cournot competition settings. Our study closely relates to the branch of this literature that focuses on price competition. Doganoglu and Wright (2006) develop a one-sided Hotelling model, showing that symmetric firms have excessive incentives for compatibility due to reduced price competition. Recent contributions include Ekmekci et al. (2025) and Peitz and Sato (2025), who introduce oligopolistic models with asymmetric platforms to study how the extent of asymmetry affects the implications of interoperability for the outcomes of price competition. A recurring theme in these studies is that industry-wide interoperability in a relatively symmetric setting makes it less attractive for each individual platform to subsidize and attract consumers, thus leading to higher prices.[11] Our paper adds to this literature by introducing a *network-based methodology* to model arbitrary pairwise interoperability configurations. This allows us to go beyond industry-wide interoperability and show that the aforementioned competition-relaxing downside is not universal. In particular, we identify a novel demand complementarity mechanism where interoperable platforms may actually price more aggressively than they would under zero interoperability —a feature that does not appear in these earlier works.[12] Beyond simple comparative statics, we further characterize the boundaries of welfare outcomes achievable through different interoperability configurations, providing a clearer look at the economic trade-offs associated with various interoperability configurations.

We introduce a flexible network-based methodology to model arbitrary pairwise interoperability configurations, following the analysis of social and economic networks (see e.g., Jackson, 2008; Goyal, 2023). This allows us to apply concepts and tools from that literature to obtain tractable results and sharper insights. Related work includes research on network games (Ballester et al., 2006), on the

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[10] Our price competition model is built upon an oligopolistic platform competition model with single-homing users, as developed by Tan and Zhou (2021). Other recent models featuring oligopolistic platform competition include Anderson and Peitz (2020); Tremblay et al. (2023); Teh et al. (2023), among others.

[11] See, e.g., recent surveys by Bianchi et al. (2023). A similar theme also occurs in the classic literature on mix-and-match and system competition, in which compatibility reduces each firm's incentive to cut prices (e.g., Matutes and Regibeau, 1988; Economides, 1989).

[12] A notable exception is Rey and Tirole (2019), which we discuss further in Section 4.2.

pricing of network information (Candogan et al., 2012; Bloch and Quérou, 2013; Fainmesser and Galeotti, 2016; Chen et al., 2018), network approaches to product differentiation and product variety (Ushchev and Zenou, 2018), and on patent licensing in multiproduct environments (Jeon et al., 2023).

We link the spectral properties of interoperability configurations to economic outcomes such as prices and welfare, and show that spectral methods and network eigenvalues provide analytical tractability and yield novel insights into interoperability across competing platforms. In this sense, we contribute to the literature that applies network methods to the analysis of industrial organization problems. Related work includes Galeotti et al. (2020) and Chen et al. (2022), as well as Galeotti et al. (2024), which apply spectral methods to other economic settings, including optimal targeted network interventions, price discrimination based on consumers' network positions, and robust market interventions.

Our characterization of optimal interoperability configurations relates to the literature on (centralized) network design. In this strand of the literature, the efficient network configuration is typically either complete (industry-wide interoperability) or empty (zero interoperability), or it belongs to specific classes such as hub-and-spoke networks (Hendricks et al., 1995, 1999) or nested split graphs (Belhaj et al., 2016). Our result on the consumer optimality of coalition-based interoperability is novel to our setting, as it captures a trade-off between enhancing users' interaction benefits and maintaining market contestability. To the best of our knowledge, this network structure has not been identified in either the network or industrial organization literature. Finally, we adopt a centralized point of view and abstract from strategic network formation, which is another central theme in the network literature (Jackson and Wolinsky, 1996; Bala and Goyal, 2000).

At a broader level, our model relates to recent applications in specific industry contexts and regulatory policies. These include issues in data-driven user externalities and data regulation (e.g., Hagiu and Wright, 2023; Rhodes et al., 2025), competition implications of data sharing (e.g., Bergemann et al., 2022; de Cornière and Taylor, 2025; Bhargava et al., 2025), and interoperability among messaging services (Bourreau and Krämer, 2025).

The rest of the paper proceeds as follows. Section 2 lays out the main model, the equilibrium of which is characterized in Section 3. Section 4 analyzes welfare outcomes across configurations. Section 5 provides a full characterization of welfare outcomes for the case of four platforms. All proofs and omitted derivations are relegated to the Appendix.

2 Model setup

There is a set $\mathcal{N} \equiv \{1, 2, \cdots, n\}$ of $n \geq 2$ one-sided platforms with a continuum of heterogeneous consumers (of measure 1).[13] Denote p_i as the membership price charged by platform $i \in \mathcal{N}$, and $x_i \in [0, 1]$ as the mass of consumers joining platform i. Each consumer knows her idiosyncratic match values (or membership benefits) $\boldsymbol{\epsilon} = (\epsilon_1, \cdots, \epsilon_n)$ with the n platforms. The vector $\boldsymbol{\epsilon}$ is drawn from a joint distribution $\mathbf{F}(\cdot)$ with support $[\underline{\epsilon}, \bar{\epsilon}]^n$, where $\underline{\epsilon} \geq -\infty$ and $\bar{\epsilon} \leq \infty$. We assume single-homing and full market coverage: each consumer joins one and only one platform.

☐ **Participation utility and interoperability.** The participation utility of a consumer from joining platform $i \in \mathcal{N}$ is:

$$\epsilon_i - p_i + \gamma z_i, \tag{1}$$

which consists of match values, membership prices, and the total user externality (or interaction benefits) received, i.e., total benefits from interacting with other consumers. Here, $\gamma > 0$ is the *user externality parameter* that indicates the per-interaction benefit, whereas z_i is the *effective user base* that a consumer on platform i can access and interact with.[14] Following the literature on platform interoperability (e.g., Katz and Shapiro, 1985; Crémer et al., 2000), we define

$$z_i = x_i + \sum_{j \in \mathcal{N} \setminus \{i\}} \lambda_{ij} x_j. \tag{2}$$

Here, $\lambda_{ij} = \lambda_{ji} \in [0, 1]$ indicates the strength of the two-way "interoperability link" between platforms i and j. That is, interoperability allows consumers on each platform i to interact with those on other platforms $j \neq i$.

The interoperability configuration between platforms is summarized by an $n \times n$ *interoperability matrix* (or weighted adjacency matrix) $\boldsymbol{\Lambda} = (\lambda_{ij})$, where $\lambda_{ij} = 0$ means that there is no interoperability link between i and j. We assume that the

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[13] Our focus on a one-sided, single-homing setting is meant to highlight the main economic implications of interoperability. See, e.g., recent works by Peitz and Sato (2025) and Ekmekci et al. (2025), which zoom into the same setting when analyzing the implications of interoperability. In an earlier version, we construct a two-sided microfoundation based on the models of app development by Jeon and Rey (2024); Teh and Wright (2025), whereby "platforms" correspond to smartphone OEMs that set device prices to single-homing consumers but earn nothing from app developers who can multihome. Interoperability is modeled as economies of scale when sellers multihome. The resulting pricing equilibrium is equivalent to our one-sided model.

[14] Our equilibrium characterization readily extends to $\gamma < 0$ (negative user externalities). We restrict attention to the empirically more relevant case of $\gamma > 0$ to streamline the exposition and simplify the result statements.

matrix $\mathbf{\Lambda}$ is *symmetric*, that is, $\lambda_{ij} = \lambda_{ji}$. By convention, the diagonal entries satisfy $\lambda_{ii} = 0$.

☐ **Platforms and symmetry.** Each platform $i \in \mathcal{N}$ makes an independent pricing decision to maximize its own profit, given by $\pi_i = (p_i - c_i)x_i$. We normalize the marginal cost to $c_i = 0$ for all i without loss of generality. We focus on price as a membership fee.

We introduce the following two symmetry assumptions. First, we assume that the joint distribution function $\mathbf{F}(\cdot)$ is continuously differentiable and symmetric across n platforms, in the sense that the joint distribution of $\boldsymbol{\epsilon} = (\epsilon_1, \cdots, \epsilon_n)$ is invariant under any permutation of these n random variables. These assumptions are general enough to permit several specifications commonly used in the literature, including the case of independent and identically distributed (IID) shocks across platforms (Perloff and Salop, 1985) and spatial settings such as the Hotelling model.

Second, the matrix $\mathbf{\Lambda}$ is *vertex-transitive*, meaning that the platforms are "identical" in terms of their positions in the interoperability configuration.[15] That is, switching the positions of any pair of platforms affects nothing in the model. Vertex-transitivity implies that every platform has the same total (weighted) number of interoperability links, i.e.,

$$\sum_{j \in \mathcal{N}} \lambda_{ij} = \hat{\lambda} \in [0, n-1] \quad \text{for all } i \in \mathcal{N}. \tag{3}$$

We will refer to $\hat{\lambda}$ as the *total interoperability strength.*

We say that an interoperability configuration is *admissible* if $\mathbf{\Lambda}$ is both symmetric and vertex-transitive. Below are some classes of admissible configurations $\mathbf{\Lambda}$ (shown in Figure 2, which is built upon Figure 1 by explicitly indicating the weights on the links):[16]

- *Industry-wide configuration* (denoted as IW(λ)). For all $i \neq j$, $\lambda_{ij} = \lambda$, where $\lambda \in [0,1]$, so that $\hat{\lambda} = (n-1)\lambda$. If $\lambda = 0$, this reduces to zero interoperability, which we will sometimes denote as IW(0).

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[15] An adjacency matrix $\mathbf{\Lambda}$ is vertex-transitive if, for any two vertices i and j, there exists an automorphism $\sigma : \mathcal{N} \to \mathcal{N}$ such that $\sigma(i) = j$. In this context, an automorphism is a permutation of the vertex set $\mathcal{N}$ that preserves the adjacency matrix, meaning $\lambda_{ij} = \lambda_{\sigma(i)\sigma(j)}$ for all $i, j \in \mathcal{N}$ (Godsil and Royle, 2001). This assumption is standard in network literature to ensure symmetric pricing equilibria (e.g., Ushchev and Zenou, 2018).

[16] These do not exhaustively characterize all admissible configurations (e.g., circular configurations). The class of admissible configurations can be fully parameterized for $n = 2, 3, 4$ (see Section 5), but as far as we know, such full characterizations are not available for larger n.

- *Coalition configuration* (denoted as $\mathrm{CA}(\lambda; m)$). All n platforms are partitioned into mutually exclusive coalitions, each consisting of m platforms. For all $i \neq j$, $\lambda_{ij} = \lambda \in [0, 1]$ if i and j belong to the same coalition, and $\lambda_{ij} = 0$ otherwise. That is, the interoperability strength is λ within each coalition and zero across coalitions. Each coalition has $m \geq 2$ members, so that $\hat{\lambda} = (m-1)\lambda$. We focus on integer numbers of coalitions with $\frac{n}{m} \in [2, \frac{n}{2}]$.[17]
- *Cross-link coalition configuration* (denoted as $\overline{\mathrm{CA}}(\lambda_{\mathrm{in}}, \lambda_{\mathrm{out}}; m)$). This is the same as the coalition configuration, except that $\lambda_{ij} = \lambda_{\mathrm{out}}$ for all $i \neq j$ that belong to different coalitions, and $\lambda_{ij} = \lambda_{\mathrm{in}}$ for all $i \neq j$ that belong to the same coalition, with $\lambda_{\mathrm{in}} \geq \lambda_{\mathrm{out}}$.[18] This nests $\mathrm{IW}(\lambda) = \overline{\mathrm{CA}}(\lambda, \lambda; n)$ and $\mathrm{CA}(\lambda; m) = \overline{\mathrm{CA}}(\lambda, 0; m)$ as special cases.

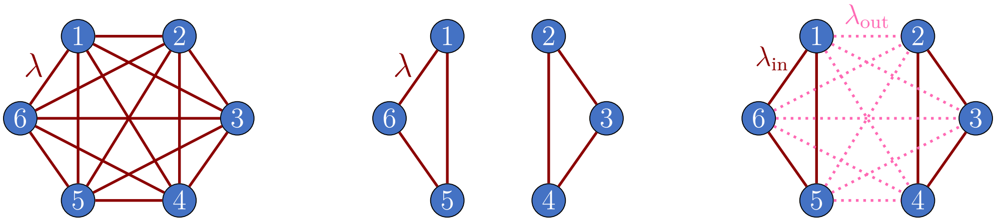

Figure 2: Illustrations of $\mathrm{IW}(\lambda)$, $\mathrm{CA}(\lambda;3)$, and $\overline{\mathrm{CA}}(\lambda_{\mathrm{in}}, \lambda_{\mathrm{out}}; 3)$ configurations, with $n = 6$.

☐ **Timing**. (1) Platforms simultaneously choose their prices; (2) observing all prices, users simultaneously decide which platform to join. The solution concept is the Subgame Perfect Nash Equilibrium (SPNE). Throughout, we denote by $\mathbf{I}$ the $n \times n$ identity matrix, $\mathbf{0}$ and $\mathbf{1}$ as the $n \times 1$ column vector of zeros and ones, and the superscript $\intercal$ as the transpose operator.

3 Equilibrium analysis

3.1 Participation equilibrium and demand properties

We first consider consumer decisions in the participation subgame for each given price profile $\mathbf{p} = (p_1, \cdots, p_n)^\intercal$ set by platforms. By definition, the demand profile $\mathbf{x} = (x_1, \cdots, x_n)^\intercal$ in the equilibrium of the participation subgame is such that each consumer joins the platform offering the highest utility, as specified in (1), while treating the participation decisions of other consumers as given.

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[17] Setting $\frac{n}{m} = 1$ amounts to an IW configuration (a single coalition), whereas $\frac{n}{m} = n$ amounts to no coalition. Any $m \in (1, 2)$ or $m \in (\frac{n}{2}, n)$ implies a non-integer number of coalitions and is thus omitted.

[18] We focus on $\lambda_{\mathrm{in}} \geq \lambda_{\mathrm{out}}$ to simplify the exposition by limiting the number of cases considered. Allowing for $\lambda_{\mathrm{in}} < \lambda_{\mathrm{out}}$ yields analogous results and does not alter our main conclusions.

Formally, define

$$Q_i(u_1, \cdots, u_n) \equiv \Pr\left(\epsilon_i + u_i \geq \max_{j \in \mathcal{N} \backslash \{i\}} \{\epsilon_j + u_j\}\right)$$

as the mass of consumers choosing platform i when the non-idiosyncratic component of consumer participation utility on each platform is given by $\mathbf{u} = (u_1, \cdots, u_n)$.

Let $\mathbf{Q}(\mathbf{u}) = (Q_1(\mathbf{u}), \cdots, Q_n(\mathbf{u}))^\intercal$, which lies in the unit simplex for any $\mathbf{u}$. From (1) and (2), we know that $\mathbf{u} = -\mathbf{p} + \gamma(\mathbf{I} + \mathbf{\Lambda})\mathbf{x}$. Then, in the participation equilibrium, the demand profile $\mathbf{x}$ is implicitly pinned down by the following system of equations:

$$\mathbf{x} = \mathbf{Q}(-\mathbf{p} + \gamma(\mathbf{I} + \mathbf{\Lambda})\mathbf{x}). \tag{4}$$

For any price profile, a solution to (4) is guaranteed by the continuity of the joint distribution $\mathbf{F}$ and Brouwer's fixed-point theorem. For instance, at any symmetric price $\mathbf{p} = p\mathbf{1}$, a symmetric participation equilibrium $\mathbf{x} = \frac{1}{n}\mathbf{1}$ is a solution to (4).

We now examine how demand responds to platform pricing by implicitly differentiating (4). Denote $H(\cdot)$ and $h(\cdot)$ as the CDF and PDF of the distribution $\epsilon_i - \max_{j \in \mathcal{N} \backslash \{i\}}\{\epsilon_j\}$, respectively.[19] Notice that full market coverage and a symmetric joint distribution $\mathbf{F}(\cdot)$ imply that, at any symmetric profile where $u_1 = u_2 = \cdots = u_n = u$, we have

$$\frac{\partial Q_i}{\partial u_i} = h(0), \qquad \frac{\partial Q_i}{\partial u_j} = -\frac{1}{n-1}h(0) \text{ for every } j \in \mathcal{N} \backslash \{i\}, \tag{5}$$

which reflects the standard demand substitution following a change in participation utility, with $h(0)$ inversely measuring the extent of product differentiation in standard discrete choice models (Perloff and Salop, 1985).[20]

Rewriting (5), we obtain the demand substitution matrix

$$-\mathbf{S} = \left.\frac{\partial \mathbf{Q}}{\partial \mathbf{u}}\right|_{\mathbf{u}=u\mathbf{1}} = \frac{nh(0)}{n-1}\left[\mathbf{I} - \frac{1}{n}\mathbf{1}\mathbf{1}^\intercal\right]. \tag{6}$$

Note that $\mathbf{S} = \mathbf{S}^\intercal$ (Slutsky symmetry) and $\mathbf{S}\mathbf{1} = \mathbf{S}^\intercal\mathbf{1} = \mathbf{0}$ (full market coverage).

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[19] Functions $H(\cdot)$ and $h(\cdot)$ are not indexed by $i \in \mathcal{N}$ because $\mathbf{F}(\cdot)$ is permutation-invariant.

[20] As an example, suppose that $(\epsilon_1, \cdots, \epsilon_n)$ follows IID Gumbel distributions with scale parameter $\beta > 0$. Then Q_i takes the *multinomial logit demand form* (Anderson et al., 1992):

$$Q_i(u_1, \cdots, u_n) = \frac{\exp\{u_i/\beta\}}{\sum_{j=1}^n \exp\{u_j/\beta\}},$$

where $h(0) = \frac{n-1}{\beta n^2}$. We will use the normalized logit form ($\beta = 1$) to show some of our results.

We define the *adjusted user externality coefficient* as

$$\delta \equiv \left(\frac{n}{n-1}\right) h(0)\gamma, \tag{7}$$

which indicates the strength of the user externality $\gamma > 0$ relative to the extent of product differentiation $1/h(0)$. [21] Clearly, $\delta = 0$ is equivalent to zero user externality. We focus on $\delta > 0$ and impose a restriction on the magnitude of the user externality coefficient.

Assumption I *The coefficient* $\delta < \frac{2}{n}$ *and* $\delta \neq \frac{1}{1+\hat{\lambda}}$.[22]

For $k = 1, \ldots, n$, let μ_k denote the k-th largest eigenvalue of the symmetric matrix $\mathbf{\Lambda}$, and let $\mathbf{e}_k$ denote the associated unit-length eigenvector, so that $\mathbf{\Lambda e}_k = \mu_k \mathbf{e}_k$. Vertex-transitivity implies that the largest eigenvalue of $\mathbf{\Lambda}$ is $\mu_1 = \hat{\lambda}$, with associated eigenvector $\mathbf{e}_1 = \frac{1}{\sqrt{n}}\mathbf{1}$. Define:[23]

$$\mathbf{A} \equiv \left[\mathbf{I} - \delta(\mathbf{I} + \mathbf{\Lambda})\right]^{-1}.$$

The following lemma provides expressions for the matrix of demand derivatives evaluated at the symmetric profile of prices and quantities:

Lemma 1 *Suppose Assumption I holds. Then,*

$$\left.\frac{\partial \mathbf{x}}{\partial \mathbf{p}}\right|_{(\mathbf{p},\mathbf{x})=(p\mathbf{1},\frac{1}{n}\mathbf{1})} = \mathbf{AS} = -\frac{nh(0)}{n-1}\sum_{k\neq 1}\frac{\mathbf{e}_k\mathbf{e}_k^{\intercal}}{1-\delta-\delta\mu_k}. \tag{8}$$

In particular, the own price effects are identical and negative.

The first equality in Lemma 1 states that the matrix of demand derivatives, that is, the effects of prices on demand, in our model equals the standard demand substitution matrix $\mathbf{S}$ multiplied by a matrix $\mathbf{A}$. Observe that when $\delta = 0$, $\mathbf{A} = \mathbf{I}$, so that $\mathbf{AS}$ reduces to the demand derivative matrix in the Perloff–Salop framework without user externalities or interoperability. We refer to $\mathbf{A}$ as an amplification matrix, as it captures how user externalities and interoperability

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[21] In our demand analysis, the effects of prices on demand are expressed in terms of the adjusted coefficient δ rather than γ. Intuitively, δ captures the change in demand following an initial price change after accounting for user externalities. This adjustment is necessary because γ measures the gain from user externalities (on the same scale as price), whereas $h(0)$ converts this gain into a marginal impact on demand. The factor $n/(n-1)$ reflects that any user attracted to a platform through user-externality gains must be drawn from the other $n-1$ rivals, i.e., the "loop effect" referred to in Tan and Zhou (2021).

[22] The latter condition is generic: it holds for all parameter values except on measure-zero sets.

[23] In the network literature, $\mathbf{A}$ is the Leontief inverse of matrix $\mathbf{I} + \mathbf{\Lambda}$ with decay factor δ.

amplify the standard demand substitution matrix embodied in $\mathbf{S}$. Moreover, $\mathbf{AS}$ is independent of p, as a consequence of the full coverage assumption.

The second equality in (8) shows that the product of the amplification matrix and the substitution matrix can be expressed in terms of the eigenvalues and eigenvectors of the interoperability configuration. To obtain this result, we rely on spectral decomposition techniques from the network literature. For example, in their analysis of optimal targeting interventions in networks, Galeotti, Golub and Goyal (2020) show that their amplification matrix can be diagonalized in the eigenspace of the network matrix. Similarly, we show that, in the eigenspace of $\mathbf{\Lambda}$, both $\mathbf{A}$ and $\mathbf{S}$ are diagonalizable, and hence so is their product $\mathbf{AS}$. In particular, vertex transitivity of $\mathbf{\Lambda}$ (and hence of $\mathbf{A}$) implies that all diagonal entries of $\mathbf{AS}$ are identical. As a result, the own-price effects can be computed from the trace of $\mathbf{AS}$, which equals the sum of its eigenvalues and therefore admits a simple closed-form expression via the simultaneous diagonalization of $\mathbf{A}$ and $\mathbf{S}$. Combined with Assumption I, this implies that the own-price effect is negative. This spectral decomposition approach yields the second representation of the effects of prices on demand in (8), which will be useful for our subsequent analysis.

3.2 Pricing equilibrium

In the pricing game, let the symmetric equilibrium be such that each platform sets a price p^* and has a participation mass of $x^* = 1/n$. To pin down p^*, suppose that platform i deviates to $p_i \neq p^*$ while all other platforms $j \neq i$ continue to set p^*, resulting in an off-path participation mass $x_i(p_i; p^*)$ on platform i, as determined by (4). We assume:

Assumption II *Any deviating platform i's profit function* $\pi_i(p_i; p^*) = p_i x_i(p_i; p^*)$ *is strictly quasiconcave in* $p_i > 0$.[24]

For all subsequent results, we maintain Assumptions I and II throughout. We now state our main equilibrium characterization.

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[24] In the logit demand form with $n = 4$, we have verified that profit quasi-concavity holds globally for any IW(λ) configuration if $\gamma < 2.5\beta$ (equivalently, $\delta < 0.625$) and in any CA($\lambda; m$) configuration if $\gamma < 1.8\beta$ (equivalently, $\delta < 0.45$). Moreover, under these conditions, the symmetric equilibrium is unique. More generally, under the log-concavity of $1 - H(\cdot)$, profit quasi-concavity holds when user externalities are absent (see, e.g., Caplin and Nalebuff, 1991) or not too strong relative to the extent of horizontal differentiation (e.g., Armstrong, 2006; Tan and Zhou, 2021; Ekmekci et al., 2025).

Proposition 1A (Pricing equilibrium) *There exists a symmetric SPNE in which all platforms have the same market share $x^* = 1/n$ and set the same price*

$$p^* = \frac{1}{nh(0)}\, d, \tag{9}$$

where d is defined by

$$\frac{1}{d} = \frac{1}{n-1} \sum_{k \neq 1} \frac{1}{1-\delta-\delta\mu_k}, \tag{10}$$

and μ_k is the k-th largest eigenvalue of the interoperability matrix $\mathbf{\Lambda}$.

First, Proposition 1A states that the equilibrium price is the product of two terms: $\frac{1}{nh(0)}$ and d. The first term, $\frac{1}{nh(0)}$, corresponds to the standard market power component for oligopolistic firms offering differentiated products (Perloff and Salop, 1985). The second term, d, captures the effects of user externalities and interoperability: if there are no user externalities ($\delta = 0$), we have $d = 1$ and p^* reduces to the Perloff–Salop price. More generally, observe that d is positive due to Lemma 1, since own-price effects are negative and platforms therefore earn a positive markup. As we show in the next section, d is at most one. Accordingly, we refer to d as a *price discount.*

Second, the equilibrium price is determined solely by the model primitives: the user externality (or interaction benefits) coefficient δ and the eigenvalues, which capture the structural properties of the interoperability matrix. For any given $\mathbf{\Lambda}$, one can readily compute the corresponding eigenvalues and, using Proposition 1A, obtain the discount d in (10), and hence the price p^* in (9). We illustrate this with the following class of examples:[25]

Example 1 (Cross-linked coalition configuration, $\overline{\mathbf{CA}}(\lambda_{\text{in}}, \lambda_{\text{out}}; m)$) *Let $\frac{n}{m} \geq 2$ be the number of coalitions. The eigenvalues, in descending order, are:*

$$\mu_1 = (m-1)\lambda_{\text{in}} + (n-m)\lambda_{\text{out}}$$

and

$$\mu_k = \begin{cases} (m-1)\lambda_{\text{in}} - m\lambda_{\text{out}}, & \textit{for } k = 2, \ldots, \frac{n}{m}, \\ -\lambda_{\text{in}}, & \textit{for } k = \frac{n}{m}+1, \ldots, n. \end{cases} \tag{11}$$

It follows that

$$d_{\overline{CA}} = (n-1)\left(\frac{\frac{n}{m}-1}{1-\delta-\delta\mu_2} + \frac{n-\frac{n}{m}}{1-\delta-\delta\mu_n}\right)^{-1}. \tag{12}$$

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[25] All derivation details related to the $\overline{\text{CA}}$ class are collected in Appendix A.2.

In the case of industry-wide interoperability IW(λ), $\lambda_{\text{in}} = \lambda_{\text{out}} = \lambda$, and we have:[26]

$$\mu_2 = \cdots = \mu_n = -\lambda, \qquad \textit{and} \qquad d_{IW} = 1 - \delta + \delta\lambda. \tag{13}$$

Third, note that the largest eigenvalue μ_1 does not appear in the pricing formula. Intuitively, under vertex transitivity, the eigenvector associated with μ_1, often interpreted as a measure of network centrality, is proportional to the all-ones vector. This property allows us to abstract from aggregate centrality effects and isolate the economically meaningful forces arising from the remaining eigenvalues, which have received far less attention in the literature.

Moreover, the discount factor d takes the form of a harmonic mean of the set $\{1-\delta-\delta\mu_k\}_{k\neq 1}$, all of which are positive under Assumption I. This representation yields several useful properties of the equilibrium price, stated in the following corollary.

Corollary 1 *The equilibrium price p^* is (i) symmetric, decreasing, and concave in the eigenvalue profile $(\mu_2, \ldots, \mu_n)$, and (ii) non-increasing and concave in the user externality parameter δ.*

Symmetry and monotonicity in $(\mu_2, \ldots, \mu_n)$ are immediate, whereas monotonicity in δ follows from a simple algebra verification. Meanwhile, since the harmonic mean is concave in its arguments (Hardy et al., 1952), it follows from (10) that d, and hence p^*, is concave in $\{1-\delta-\delta\mu_k\}_{k\neq 1}$, and therefore concave in $(\mu_2, \ldots, \mu_n)$ and δ respectively. These properties characterize how the equilibrium price (or, equivalently, the discount factor) depends on the eigenvalues of the interoperability matrix and the user externality coefficient.[27] They also allow us to establish lower and upper bounds on the equilibrium price and consumer surplus, which we exploit in Section 4.

3.3 A diversion-ratio perspective to equilibrium pricing

In this subsection, we restate Proposition 1A in a form that more explicitly relates our equilibrium pricing formula to the existing literature. In doing so, we also develop additional intuition that is instrumental for understanding the economic mechanisms underlying our subsequent welfare results.

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[26] This is consistent with the pricing equations in Doganoglu and Wright (2006) and Peitz and Sato (2025). Zero interoperability corresponds to IW(0), where $d = 1 - \delta$ (Tan and Zhou, 2021).

[27] Note that symmetry and concavity implies d is a Schur-concave function (Marshall et al., 2011), in $(\mu_2, \ldots, \mu_n)$.

We start by defining the demand diversion ratio (Katz and Shapiro, 2003) from platform i to another platform $j \neq i$, evaluated at any symmetric outcome, as:

$$\sigma_{ij} \equiv \left. \frac{\partial x_j / \partial p_i}{-\partial x_i / \partial p_i} \right|_{(\mathbf{p},\mathbf{x})=(p\mathbf{1},\frac{1}{n}\mathbf{1})}.$$

The ratio is an empirically measurable object commonly used in merger analysis.[28] In the case of substitute products, it measures the fraction of demand lost by platform i in response to an increase in p_i that is diverted to platform j. By Lemma 1, σ_{ij} is independent of the symmetric price p, and has the same sign as its numerator, $\partial x_j / \partial p_i$. Consequently, $\sigma_{ij} > 0$ ($\sigma_{ij} < 0$) corresponds to demand for i and j being substitutes (complements).

To state our result, we denote the weighted sum of diversion ratios as

$$\bar{\sigma} = \sum_{j \in \mathcal{N} \setminus \{i\}} \lambda_{ij} \sigma_{ij},$$

which is independent of the identity of platform i at any symmetric outcome and also independent of the symmetric price p.

Proposition 1B (Pricing equilibrium, continued) *The statements below hold:*

(i) *The equilibrium price in Proposition 1A can be written as*

$$p^* = \frac{1}{nh(0)} - \frac{\gamma}{n-1} + \frac{\gamma}{n-1}\bar{\sigma}. \tag{14}$$

(ii) *Demand complementarity (*$\sigma_{ij} < 0$*) for at least one pair of platforms i and j is a necessary condition for any interoperability configuration to yield a lower p^* than under zero interoperability.*

The pricing equation in Proposition 1B(i) is written to parallel those in the platform pricing literature that focuses on no-interoperability (e.g., Armstrong, 2006; Tan and Zhou, 2021). The first term in (14) corresponds to the Perloff–Salop price, where the second term reflects the subsidy arising from user externalities.

The third term in (14) captures the effect of interoperability.[29] Intuitively, interoperability means that the interaction benefits offered by a given platform i increase in the demand of its interoperable “partner” platform j where $\lambda_{ij} > 0$. Thus, platform i *internalizes* the spillover impact of its price p_i on the participation

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[28] Observe that $\sigma_{ij} = \sigma_{ji}$ by Slutsky symmetry and vertex transitivity in our setup.

[29] Ekmekci et al. (2025) considers industry-wide interoperability and obtains a similar decomposition of the pricing formula, but the exact expression differs due to their net-fee conduct assumption.

mass x_j, as measured by the diversion ratio σ_{ij}. Importantly, (14) shows that this incentive drives up p^* when $\sigma_{ij} > 0$ (internalizing positive spillovers of raising price), but decreases p^* when $\sigma_{ij} < 0$ (internalizing negative spillovers of raising price).

The equilibrium price formula in (14) further implies that platform interoperability reduces the equilibrium price if and only if the weighted aggregate diversion ratio is negative. Therefore, a necessary condition for any interoperability configuration to yield a lower price than that without interoperability is the presence of demand complementarity between at least one pair of platforms ($\sigma_{ij} < 0$). This result is highlighted in Proposition 1B(ii). In other words, for any configuration that does not generate complementarity, the equilibrium price is bounded below by the price under zero interoperability. At first glance, the possibility of demand complementarity between competing platforms may seem surprising, since in our model each individual consumer inherently views platforms as substitutes. In what follows, we identify the structural properties of interoperability configurations that give rise to such demand complementarity.

☐ **Diversion ratios and network centralities**. By Lemma 1, the first equality in (8) implies that the off-diagonal entries of the demand derivative matrix $\mathbf{AS}$ correspond to cross-price demand derivatives. Using the definition of $\mathbf{S}$, each of these entries can be written as

$$\frac{\partial x_j}{\partial p_i} = -\frac{nh(0)}{n-1}\left[a_{ji} - \frac{1}{n}\sum_{l=1}^{n} a_{li}\right], \tag{15}$$

where a_{ji} is the j-row–i-column entry of $\mathbf{A}$ (with $a_{ji} = a_{ij}$ by symmetry of $\mathbf{A}$).

For interpretation, we note that the matrix $\mathbf{A} = [\mathbf{I} - \delta(\mathbf{I} + \mathbf{\Lambda})]^{-1}$ admits an infinite Neumann series expansion when $\delta < \frac{1}{n}$:

$$\mathbf{A} = \mathbf{I} + \delta(\mathbf{I} + \mathbf{\Lambda}) + \delta^2(\mathbf{I} + \mathbf{\Lambda})^2 + \cdots . \tag{16}$$

This representation shows that each entry a_{ji} is the discounted sum of paths originating from platform i and ending at platform j in the graph $\mathbf{I} + \mathbf{\Lambda}$. Each path of length $r = 1, 2, \cdots$ (i.e., a path that passes through r interoperability links) is discounted by δ^r, which scales down the relative weight of longer paths.

Substituting (15) for $j \neq i$ and $j = i$ into the definition of the demand diversion ratio yields several properties that are crucial for understanding our subsequent results.

Lemma 2 (Properties of diversion ratios) *At any symmetric profile of prices and quantities, the following hold:*

(i) *There exists at least one pair of platforms* (i, j) *such that* $\sigma_{ij} > 0$.

(ii) *For any* $i, j, l \in \mathcal{N}$, $\sigma_{ij} < \sigma_{il}$ *if and only if* $a_{ji} > a_{li}$.

(iii) *For any* $i, j \in \mathcal{N}$, $\sigma_{ij} < 0$ *if and only if* $a_{ji} > \frac{1}{n}\sum_{l=1}^{n} a_{li}$.

Lemma 2 delivers three insights. First, positive diversion (demand substitutes) holds for at least one pair of platforms, which is intuitive given that individual consumers inherently view competing platforms as substitutes. Second, the lemma establishes a pecking order: the extent of demand substitution between i and j is weaker than that between i and l if and only if the discounted sum of paths between i and j is larger than that between i and l. Equivalently, any negative diversion from i to another platform—if it arises at all—must first occur with the platforms most strongly connected to i in terms of discounted network paths. Third, negative diversion (demand complementarity) between i and j arises if and only if $a_{ji} > \frac{1}{n}\sum_{l=1}^{n} a_{li}$; that is, when the discounted sum of paths linking i and j exceeds the average of those linking i to all platforms.

As an illustration of these properties of diversion ratios, we now specialize Lemma 2 to the class of $\overline{\text{CA}}$ (Example 1). Let $\sigma_{ij} = \sigma_{\text{in}}$ if the pair $i \neq j$ belongs to the same coalition and $\sigma_{ij} = \sigma_{\text{out}}$ otherwise. Within this class, σ_{in} and σ_{out}, and hence $\bar{\sigma}$, are completely determined by the model primitives m, n, λ_{in}, λ_{out}, and δ, and exhibit the following properties:

Lemma 2′ (Properties of diversion ratios in $\overline{\text{CA}}$) *At any symmetric profile of prices and quantities, the following hold:*

(i) $\sigma_{\text{out}} > 0$.

(ii) $\sigma_{\text{in}} \leq \sigma_{\text{out}}$ *(strict inequality holds if and only if* $\lambda_{\text{in}} > \lambda_{\text{out}}$*).*

(iii) $\sigma_{\text{in}} < 0$ *if and only if* $(n-1)\lambda_{\text{in}} - n\lambda_{\text{out}} > \frac{1}{\delta} - 1$.

Lemma 2′ says that a coalition characterized by strong within-coalition interoperability attenuates demand diversion among its members relative to diversion across coalitions, i.e., $\sigma_{\text{in}} \leq \sigma_{\text{out}}$. While diversion across coalitions is always positive, diversion within a coalition may become negative when λ_{in} is sufficiently large relative to λ_{out}.[30] We discuss the intuition and implications of negative within-coalition diversion ratios in Section 4.1.

☐ **Bottom line**. We close this section with a brief summary. Proposition 1A establishes a novel link between equilibrium prices and the spectral properties of

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[30] We have verified that the condition for $\sigma_{\text{in}} < 0$ does not necessarily contradict the requirement of profit quasi-concavity.

the interoperability configuration—a relationship that is central to our analysis of optimal structures. Proposition 1B provides an alternative representation of equilibrium prices in terms of demand diversion ratios, showing that interoperability interventions, relative to zero interoperability, can lead to either higher or lower equilibrium prices depending on their effects on these ratios. We examine these configurations in detail in the following section.

4 Welfare outcomes of interoperability

In the equilibrium characterized in Proposition 1A, industry profit equals the equilibrium price:

$$\Pi \equiv n \times \pi_i(p^*; p^*) = p^*.$$

Consumer surplus is given by

$$CS \equiv \mathbb{E}[\max_{i \in \mathcal{N}} \epsilon_i] - p^* + \gamma z^*, \tag{17}$$

which consists of the expected maximum match value from n platforms, the equilibrium price, and the total user externalities evaluated at the effective user base $z^* = \frac{1}{n}(1 + \hat{\lambda})$. Total welfare is therefore

$$TW \equiv \Pi + CS = \mathbb{E}[\max_{i \in \mathcal{N}} \epsilon_i] + \gamma z^*.$$

The maximum match value is constant and independent of interoperability, and it is omitted in the subsequent analysis.

In what follows, we analyze how interoperability influences the welfare measures introduced above. Specifically, Section 4.1 identifies the configurations that yield the lowest and highest levels of equilibrium price and industry profit; Section 4.2 focuses on consumer surplus. Section 5 then fully characterizes the set of all achievable welfare outcomes.[31]

4.1 Optimal configurations: price and profits

As indicated in Proposition 1A, the equilibrium price is fully determined by the eigenvalue structure of the interoperability matrix. Determining the optimal configuration that either maximizes industry profit or minimizes the con-

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[31] Total welfare increases in the effective user base z^* and is therefore maximized by the IW(1) configuration. An alternative approach is to define TW as a weighted sum of consumer surplus and platform profit, with a discount on the latter to capture the possibility that higher prices lower welfare. This leads to insights similar to those obtained from analyzing CS and p^*.

sumer price therefore requires comparing eigenvalue structures across configurations. Such comparisons, however, are generally challenging over the full set of admissible configurations. To address this, we first solve a relaxed version of the problem—imposing only the regularity condition on configurations as in (3)—to derive upper and lower bounds on prices, and then show that these bounds are attained by specific admissible configurations. The resulting characterizations are summarized in the next two propositions.

Proposition 2 (Maximization of industry profits) *The statements below hold:*

(i) *Among all admissible configurations of given* $\hat{\lambda}$, Π *is maximized at* $\mathrm{IW}(\frac{\hat{\lambda}}{n-1})$.

(ii) *Among all admissible configurations,* Π *is maximized at* $\mathrm{IW}(1)$*. The corresponding equilibrium price equals the Perloff–Salop price.*

Proposition 2 delivers two implications. Part (i) demonstrates the economic importance of exploring configurations beyond the IW class: it implies that, starting from an IW configuration of total strength $\hat{\lambda}$, reorganizing the links into any other configuration (of the same total strength) must lead to a lower equilibrium price and profit (recall $\Pi = p^*$). Part (ii) says that the highest price and profits are achieved under IW(1). Therefore, although IW(1) maximizes the effective user base (hence total interaction benefits), platforms respond by raising prices, which in turn increases their profits. It also substantiates our earlier claim that the factor d captures a discount relative to the Perloff–Salop price.

Formally, Proposition 2(i) follows from comparing the eigenvalues (excluding the largest one) of the original configuration $\mathbf{\Lambda}$ and those of $\mathrm{IW}(\frac{\hat{\lambda}}{n-1})$:

$$(\mu_2, \ldots, \mu_n), \quad \text{and} \quad \left(-\frac{\hat{\lambda}}{n-1}, \ldots, -\frac{\hat{\lambda}}{n-1}\right).$$

Observe that both vectors have the same sum: $\mu_2 + \cdots + \mu_n = \mathrm{tr}(\mathbf{\Lambda}) - \mu_1 = -\hat{\lambda}$ because $\mathrm{tr}(\mathbf{\Lambda}) = \sum_i \lambda_{ii} = 0$ and $\mu_1 = \hat{\lambda}$ (and both configurations have the same total strength $\hat{\lambda}$). However, under the IW configuration, the eigenvalues are evenly distributed. By the symmetry and concavity — hence Schur-concavity — of d (see Corollary 1), it follows that[32]

$$d \leq 1 - \delta + \delta \left(\frac{\hat{\lambda}}{n-1}\right).$$

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[32] Schur-concavity of a function $f(x_1, \ldots, x_k)$ implies that for any $x_1, \ldots, x_k$,

$$f(x_1, \ldots, x_k) \leq f(\bar{x}, \bar{x}, \ldots, \bar{x}), \quad \text{where } \bar{x} = \frac{x_1 + \cdots + x_k}{k}. \tag{18}$$

This establishes the claimed maximal equilibrium price. The upper bound increases with $\hat{\lambda}$ and equals one when $\hat{\lambda} = n - 1$. Since $d = 1$ corresponds to the price in IW(1) and also the Perloff–Salop price, this immediately implies part (ii) of the proposition.

Proposition 3 (Minimization of prices) *The statements below hold:*

(i) *Among all admissible configurations, p^* is minimized at* IW(0) *if $\delta < \frac{1}{n}$.*

(ii) *Among all $\overline{\mathrm{CA}}(\lambda_{\mathrm{in}}, \lambda_{\mathrm{out}}; m)$ configurations, p^* is minimized at* $\mathrm{CA}(1; \frac{n}{2})$ *if $\delta > \frac{1}{n}$ and $n \geq 4$ is even.*

Proposition 3(i) shows that in our framework, introducing interoperability inevitably increases consumer price when $\delta < \frac{1}{n}$ is small. Moreover, it increases the price to the maximum at the industry-wide configuration (Proposition 2). Hence, there is a trade-off between enhanced positive user externalities and higher equilibrium prices. However, when $\delta > \frac{1}{n}$, Proposition 3(ii) shows that the trade-off described above partially disappears. In particular, coalition configuration $\mathrm{CA}(1; \frac{n}{2})$ leads to both lower prices and larger user interaction benefits than the zero interoperability benchmark.

To see the logic, note that the equilibrium price p^* is inversely related to the own-price derivative of demand characterized in Lemma 1. It therefore suffices to establish an upper bound on the own-price effect. For part (i), we first show that $\mathrm{tr}([\mathbf{I} + \mathbf{\Lambda}]^r) \leq n(1 + \hat{\lambda})^{r-1}$ for each $r \geq 1$. Recall when $\delta < \frac{1}{n}$, matrix $\mathbf{A} = [\mathbf{I} - \delta(\mathbf{I} + \mathbf{\Lambda})]^{-1}$ admits the Neumann series expansion form (16). Combining the inequality and the expansion form yields an upper bound on $\mathrm{tr}(\mathbf{A})$ hence an upper bound on the own-price effect. Moreover, the bound is tight and is achievable under zero interoperability when $\delta < 1/n$, establishing Proposition 3(i).

For part (ii), we note that when $\delta > \frac{1}{n}$, the bounding approach above no longer applies in general. Accordingly, we focus on the class of $\overline{\mathrm{CA}}$ configurations and apply convex analysis to simplify the optimization problem. The logic is as follows. First, d is concave in $\{\mu_k\}_{k \neq 1}$ by Corollary 1. Second, by (11), the eigenvalues $\{\mu_k\}_{k \neq 1}$ in the $\overline{\mathrm{CA}}$ class are linear in the parameters $(\lambda_{\mathrm{in}}, \lambda_{\mathrm{out}})$. Taken together, these properties imply that d—and therefore the price—is concave in $(\lambda_{\mathrm{in}}, \lambda_{\mathrm{out}})$. As a result, the lowest price is attained at the extreme points

$$(\lambda_{\mathrm{in}}, \lambda_{\mathrm{out}}) \in \{(1, 0), (0, 0), (1, 1)\},$$

corresponding to $\mathrm{CA}(1; m)$, IW(0), and IW(1), respectively. The last candidate yields the highest price and is therefore ruled out by Proposition 2. Finally, within the class of $\mathrm{CA}(1; m)$ configurations, a direct computation shows that the equilibrium price is decreasing in m up to $m = n/2$ when $\delta > 1/n$. A direct comparison between CA$(1; n/2)$ and IW(0) then yields the result.

☐ **Within-coalition complementarity intensifies competition**. The comparative statics above are proven based on Proposition 1A. Nonetheless, to intuitively understand why prices under coalition configurations can be lower than other configurations, it is useful to return to the pricing formula in Proposition 1B, which shows that the equilibrium price is *increasing* in $\bar{\sigma}$ (weighted sum of diversion ratios). In the $\overline{\text{CA}}$ class,

$$\bar{\sigma} = (m-1)\lambda_{\text{in}}\sigma_{\text{in}} + (n-m)\lambda_{\text{out}}\sigma_{\text{out}}.$$

That is, $\bar{\sigma}$ is a linear combination of within-coalition diversion, σ_{in}, and cross-coalition diversion, σ_{out}, with weights proportional to the strengths of within- and cross-coalition interoperability.

Recall from Lemma 2′(iii) that within-coalition complementarity ($\sigma_{\text{in}} < 0$) arises whenever λ_{in} is sufficiently large relative to λ_{out}. The key economic force is that a price cut by one coalition member has two opposing effects on its partners' demand. First, there is a direct substitution effect, which draws consumers away from the partner platform. Second, there is an indirect user-base-expansion effect: by increasing its own demand, the deviating platform expands the effective user base enjoyed by all coalition members, thereby raising their attractiveness relative to rival coalitions. When within-coalition interoperability is sufficiently strong, the user-base-expansion effect dominates, endogenously generating demand complementarity and lowering equilibrium prices.[33]

To illustrate, consider $n = 4$ platforms forming two coalitions, consisting of platforms 1 and 2, and platforms 3 and 4, respectively, with $\lambda_{\text{in}} > 0$ and $\lambda_{\text{out}} = 0$. Suppose platform 1 lowers its price p_1. The immediate effect is an increase in x_1, as consumers switch from other platforms, including its coalition partner, platform 2 (see Figure 3). However, because platforms 1 and 2 are interoperable, the increase in x_1 expands the effective user base available to platform 2, raising its utility relative to platforms 3 and 4. This induces an inflow of consumers from platforms 3 and 4 to platform 2 (see Figure 4). When λ_{in} is sufficiently large, this inflow outweighs the initial substitution away from platform 2, so that x_2 increases following a decrease in p_1. In this case, platform 1's price reduction raises its part-

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[33] This mechanism of demand complementarity requires (i) a non-trivial link between platform 1's demand and platform 2's attractiveness via interoperability, and (ii) out-of-coalition entities with positive market shares. Under full coverage in $\overline{\text{CA}}$, such entities are rival coalitions. In an earlier version with partial market coverage, we show that the outside option plays this role, so complementarities can arise even under IW.

ner's demand, implying $\sigma_{\text{in}} < 0$ and within-coalition demand complementarity.[34]

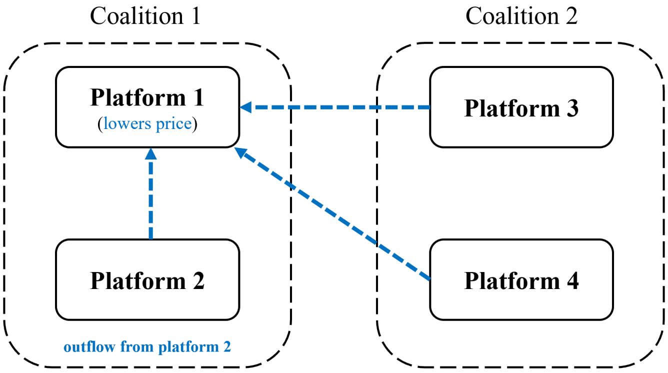

Figure 3: A decrease in p_1 causes an initial outflow of consumers from platforms 2, 3, and 4.

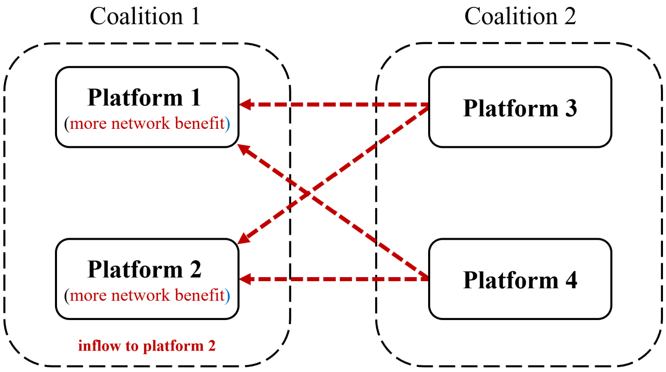

Figure 4: The initial expansion by platform 1 makes both platforms in coalition 1 more attractive, inducing an inflow of consumers to platform 2 from platforms 3 and 4.

Whenever there is within-coalition demand complementarity, a platform's price reduction would partially benefit its coalition partners, causing each platform to internalize a positive spillover from cutting its price. More generally, Proposition 1B says that stronger demand complementarities — measured in a weighted sense by $\bar{\sigma}$ — lead platforms to compete more aggressively on prices. This perspective provides an economic interpretation for Proposition 3: minimizing the equilibrium price is equivalent to minimizing $\bar{\sigma}$. Within the class of $\overline{\text{CA}}$ configurations, if $\delta < \frac{1}{n}$ then $\bar{\sigma} \geq 0$ for all configurations and it is minimized at $\bar{\sigma} = 0$, achieved at IW(0); if $\delta > \frac{1}{n}$, then CA$(1; \frac{n}{2})$ results in the most negative $\bar{\sigma} < 0$ — that is, it delivers the

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[34] A similar logic of interoperability-induced complementarity has appeared in Rey and Tirole (2019)'s analysis on price cap interventions. They demonstrate a demand system with a price-dependent degree of demand substitution by constructing a Hotelling example with user externalities that are shared between firms. In their setup, demand complementarity arises when firms set sufficiently high prices such that they effectively behave as local monopolies. We focus on a full market coverage setting and show how the demand complementarity can arise in more general demand systems and interoperability configurations.

maximal extent of demand complementarity, and hence the strongest incentives for platforms to cut price. This explains why coalition configurations can lead to lower equilibrium prices relative to other configurations (including zero-interoperability and industry-wide).

4.2 Optimal configurations: consumer surplus

In this subsection, we analyze the impact of interoperability on consumer surplus (CS). From the expression for CS in (17), there is a fundamental trade-off between expanded user externalities (or interaction benefits) and potentially higher equilibrium prices.

We ask whether interoperability in our setting enhances consumer surplus. The answer follows in two steps. First, Proposition 2 implies that for any configuration with total strength $\hat{\lambda}$, equilibrium prices are bounded above and, therefore, CS is bounded below by that under the IW(λ) configuration of the same total strength $\hat{\lambda} = (n-1)\lambda$. Second, (13) and (17) imply that within the class of IW(λ) configurations, CS is linear and increasing in λ when $n \geq 3$, as the gains from expanded user externalities dominate any adverse price effects.[35] Hence, higher levels of industry-wide interoperability yield higher consumer surplus. Taken together, these results imply that any admissible interoperability configuration generates higher consumer surplus than no interoperability. We summarize this result in the following corollary.

Corollary 2 *When $n \geq 3$, any admissible interoperability configuration increases CS compared to no interoperability.*

We next determine the optimal interoperability configuration that maximizes consumer surplus. To do so, we define, for any $n \geq 3$, two thresholds:

$$\underline{\delta} \equiv \frac{n^2 - 3n + 1}{n(n-1)^2} \quad \text{and} \quad \bar{\delta} \equiv \frac{2(n^2 - 3n + 1)}{n(n-1)(n-2)}.$$

Note that $\underline{\delta} < \frac{1}{n} < \bar{\delta} < \frac{2}{n}$. Our main results on CS are as follows:[36]

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[35] By contrast, when $n = 2$, the gain from expanded user externalities is dominated because consumers are concentrated on two platforms and therefore already benefit from substantial user externalities even without interoperability. Peitz and Sato (2025) similarly show that the result on consumer surplus from interoperability in duopoly settings does not generalize to markets with more than two platforms.

[36] We note that the threshold values of δ required for CS maximization differ from the $1/n$ threshold for price minimization, reflecting the additional user-base expansion effect in consumer surplus maximization.

Proposition 4 (Maximization of Consumer Surplus) *The statements below hold:*

(i) *Among all admissible configurations, CS is maximized at* IW(1) *if* $n \geq 3$ *and* $\delta < \underline{\delta}$.

(ii) *Suppose* $n \geq 4$ *is even. Among all* $\overline{\mathrm{CA}}(\lambda_{\mathrm{in}}, \lambda_{\mathrm{out}}; m)$ *configurations, CS is maximized at* IW(1) *if* $\delta < \bar{\delta}$, *and at* CA(1; $n/2$) *if* $\delta > \bar{\delta}$.

Proposition 4(i) shows that IW(1) is consumer-surplus optimal among all admissible configurations when the user externality coefficient δ is small. The argument parallels the bounding technique used in Proposition 3(i): when $\delta < 1/n$, there exists a common upper bound on the positive price difference between IW(1) and any admissible configuration. As δ decreases, this bound tightens, and for sufficiently small $\delta < \underline{\delta}$ (note that $\underline{\delta} < 1/n$), any price advantage of other configurations is outweighed by IW(1)'s advantage in expanding user externalities, making it consumer-surplus optimal.

Does IW(1) remain consumer-surplus optimal when δ exceeds the threshold $\underline{\delta}$? This is a challenging question, as a full characterization of admissible matrix configurations and the bounding approach developed in the previous subsection are limited. Nevertheless, we provide a partial answer by focusing on the class of configurations $\mathrm{CA}(\lambda_{\mathrm{in}}, \lambda_{\mathrm{out}}; m)$, which includes $\mathrm{IW}(\lambda)$ and $\mathrm{CA}(\lambda; m)$ as special cases.

Proposition 4(ii) shows that CA configurations yield higher consumer surplus than IW configurations when $\delta > \bar{\delta}$, reflecting that CA configurations induce lower equilibrium prices. The mechanism is the same as described in Section 4.1: under CA configurations, platforms compete more aggressively due to rivalry across coalitions. This rivalry is strongest when within-coalition demand complementarity is maximized, which occurs under the CA(1; $n/2$) configuration.

To build the intuition for why δ determines which configuration yields a higher consumer surplus, consider a direct comparison between IW(1) and CA(1; $n/2$):

$$CS_{\mathrm{IW}(1)} - CS_{\mathrm{CA}(1;n/2)} = \frac{1}{nh(0)}\left[\frac{\delta}{2} + d^*_{\mathrm{CA}(1;n/2)} - d^*_{\mathrm{IW}(1)}\right]. \tag{19}$$

It makes explicit the trade-off between expanding the user base and reducing prices. The term $\delta/2 > 0$ reflects the advantage of IW(1) over CA(1; $n/2$) in expanding total interaction benefits and increases linearly in δ. In contrast, the term $d^*_{\mathrm{CA}(1;n/2)} - d^*_{\mathrm{IW}(1)} < 0$ captures the advantage of CA(1; $n/2$) in inducing a lower equilibrium price. The relative strength of these two effects is governed by δ. Since $d^*_{\mathrm{IW}(1)} = 1$, whereas $d^*_{\mathrm{CA}(1;n/2)}$ is decreasing and concave in δ (Corollary 1), the magnitude of the price difference increases with δ. Consequently, for sufficiently large δ, the price effect dominates, and the CA configuration maximizes consumer surplus. Figure 5 illustrates consumer surplus under the two configurations for normalized logit demand with $n = 4$.[37]

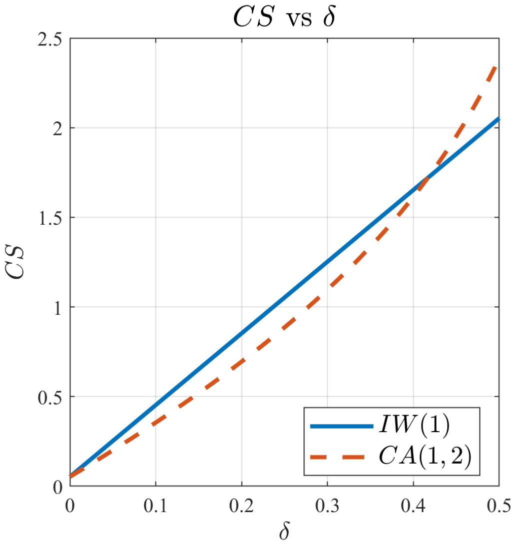

Figure 5: Consumer surplus in IW(1) and CA($1; n/2$) as functions of δ; the intersection occurs at $\bar{\delta} = \frac{5}{12}$.

☐ **Bottom line**. Propositions 2 - 4 characterize the configurations that maximize platform profits and consumer surplus, respectively. Two central insights emerge from our analysis. First, although industry-wide interoperability maximizes the effective user base and platform profits, it does not necessarily maximize consumer surplus when the user externality coefficient (δ) is sufficiently large. In such cases, platforms have stronger incentives to raise prices, thereby offsetting the benefits of full interoperability for consumers. Second, coalition-based configurations can improve consumer surplus by creating demand complementarity among platforms within a coalition and further intensifying price competition. These findings highlight a divergence between the configuration that is optimal for platforms and the one that maximizes consumer surplus, with important implications for the design of interoperability policies.

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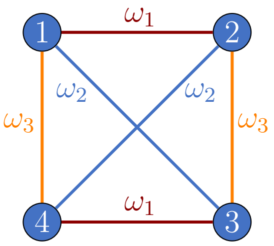
[37] For a fixed δ, consumer-surplus rankings across configurations are distribution-free: $h(0)$, which reflects the distribution of match values $\boldsymbol{\epsilon}$, enters only multiplicatively, as shown in (19). Thus, the normalized logit assumption affects only the absolute levels of consumer surplus reported in Figure 5, not the shape of the comparison or the threshold values of δ. The same observation applies to Figures 6 and 7 below.

5 Full characterization of welfare outcomes with $n = 4$ platforms

The welfare analysis in Section 4 raises two natural questions. First, when δ is large, what can be said about consumer-surplus–maximizing and price-minimizing configurations without restricting attention to the $\overline{\text{CA}}$ class? Second, rather than deriving bounds on consumer surplus CS and profit Π separately, what can be said about the set of jointly achievable (CS, Π) pairs and the Pareto frontier?

To answer these questions, we now focus on $n = 4$ and show that this leads to characterizations of the entire set of achievable pairs of (CS, Π). A notable observation is that for $n = 4$ *any* admissible interoperability matrix $\boldsymbol{\Lambda}$ admits a parametrization by the vector $\boldsymbol{\omega} = (\omega_1, \omega_2, \omega_3) \in [0, 1]^3$. Below we illustrate the associated matrix and its corresponding graph:

$$\boldsymbol{\Lambda} = \begin{pmatrix} 0 & \omega_1 & \omega_2 & \omega_3 \\ \omega_1 & 0 & \omega_3 & \omega_2 \\ \omega_2 & \omega_3 & 0 & \omega_1 \\ \omega_3 & \omega_2 & \omega_1 & 0 \end{pmatrix}$$



This formulation nests the $\overline{\text{CA}}(\lambda_{\text{in}}, \lambda_{\text{out}}; m = 2)$ class as $\boldsymbol{\omega} = (\lambda_{\text{in}}, \lambda_{\text{out}}, \lambda_{\text{out}})$, which further nests $\text{CA}(\lambda; m = 2)$ when $\lambda_{\text{out}} = 0$. However, a circular configuration, described by $\boldsymbol{\omega} = (\omega, \omega, 0)$, lies outside the $\overline{\text{CA}}$ class.

For a configuration parameterized by $\boldsymbol{\omega}$, its eigenvalues are linear functions of $\boldsymbol{\omega}$:[38]

$$\begin{aligned} \mu_1 &= \omega_1 + \omega_2 + \omega_3 \equiv \hat{\omega} \\ (\mu_2, \mu_3, \mu_4) &= (\omega_1 - \omega_2 - \omega_3, -\omega_1 + \omega_2 - \omega_3, -\omega_1 - \omega_2 + \omega_3) \end{aligned} \tag{20}$$

which, by Proposition 1A, yields the equilibrium price. Clearly, $p^*(\boldsymbol{\omega})$ is invariant under permutation of $\boldsymbol{\omega}$. Then, the concavity of the harmonic mean (Corollary 1) together with the linearity of eigenvalues in $\boldsymbol{\omega}$ implies that $p^*(\boldsymbol{\omega})$ is a symmetric and concave function of $\boldsymbol{\omega} \in [0, 1]^3$.

☐ **Achievable outcomes.** The next result characterizes the set of all achievable outcomes via interoperability configurations (rather than just the optimal con-

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[38] The corresponding eigenvectors are:

$$\mathbf{e}_1 = \frac{1}{2}(1, 1, 1, 1)^\intercal, \ \mathbf{e}_2 = \frac{1}{2}(1, 1, -1, -1)^\intercal, \ \mathbf{e}_3 = \frac{1}{2}(1, -1, 1, -1)^\intercal, \ \mathbf{e}_4 = \frac{1}{2}(1, -1, -1, 1)^\intercal.$$

figurations). Formally, we say that a price p is *achievable* if there exists $\boldsymbol{\omega} \in [0,1]^3$ such that $p^*(\boldsymbol{\omega}) = p$. An analogous definition applies for achievable welfare pairs (CS, Π). To state the result, we define the following two functions of $\hat{\omega} \in [0,3]$:

$$\underline{p}(\hat{\omega}) \equiv \begin{cases} p^*(\hat{\omega}, 0, 0) & \text{for } \hat{\omega} \in [0,1] \\ p^*(1, \hat{\omega}-1, 0) & \text{for } \hat{\omega} \in [1,2] \\ p^*(1, 1, \hat{\omega}-2) & \text{for } \hat{\omega} \in [2,3] \end{cases} \quad \text{and} \quad \bar{p}(\hat{\omega}) \equiv p^*(\hat{\omega}/3, \hat{\omega}/3, \hat{\omega}/3).$$

Both functions are easily computed from Proposition 1A together with the eigenvalues in (20), and they satisfy $\underline{p}(\hat{\omega}) \leq \bar{p}(\hat{\omega})$.[39]

Proposition 5 (Achievable outcomes) *Assume $n = 4$.*

(i) *For a fixed $\hat{\omega} \in [0,3]$, the set of prices achievable by $\boldsymbol{\omega}$ with total weight $\hat{\omega}$ is $[\underline{p}(\hat{\omega}), \bar{p}(\hat{\omega})]$;*

(ii) *The set of all achievable (CS, Π) is a disjoint union of line segments:*

$$\bigcup_{\hat{\omega} \in [0,3]} \left\{ \left(\frac{1}{4}\gamma(1+\hat{\omega}) - p, p \right) : \; p \in [\underline{p}(\hat{\omega}), \, \bar{p}(\hat{\omega})] \right\}. \tag{21}$$

For each $\hat{\omega} \in [0,3]$, define the compact, convex set $L(\hat{\omega}) \equiv \{\boldsymbol{\omega} \in [0,1]^3 : \omega_1 + \omega_2 + \omega_3 = \hat{\omega}\}$. Proposition 5(i) exactly pins down the set of prices achievable over $L(\hat{\omega})$. First, since $p^*(\boldsymbol{\omega})$ is continuous in $\boldsymbol{\omega}$ and the nonempty compact set $L(\hat{\omega})$ is connected, the intermediate value theorem implies that the set of achievable prices over $L(\hat{\omega})$ must be connected and therefore forms a compact interval with a well-defined minimum price and a maximum price. By the concavity of $p^*(\boldsymbol{\omega})$, the minimum price is attained at the extreme points of $L(\hat{\omega})$. For instance, when $\hat{\omega} \in [0,1]$, the extreme points of $L(\hat{\omega})$ are $(\hat{\omega}, 0, 0)$ and its permutations, all of which give the same price—namely the lower bound $\underline{p}(\hat{\omega})$. On the other hand, the symmetry and concavity of $p^*(\boldsymbol{\omega})$ imply that it is Schur-concave in $\boldsymbol{\omega}$. So, by inequality (18), for any $\boldsymbol{\omega} \in L(\hat{\omega})$, we have $p^*(\boldsymbol{\omega}) \leq p^*(\hat{\omega}/3, \hat{\omega}/3, \hat{\omega}/3) = \bar{p}(\hat{\omega})$, implying the maximum price is indeed $\bar{p}(\hat{\omega})$. Hence, the set of achievable prices is exactly the interval $[\underline{p}(\hat{\omega}), \bar{p}(\hat{\omega})]$. The cases for $\hat{\omega} \in [1,2]$ and $\hat{\omega} \in [2,3]$ follow from the same logic. The only difference is that a representative extreme point takes the form $(1, \hat{\omega}-1, 0)$, or $(1, 1, \hat{\omega}-2)$, respectively.

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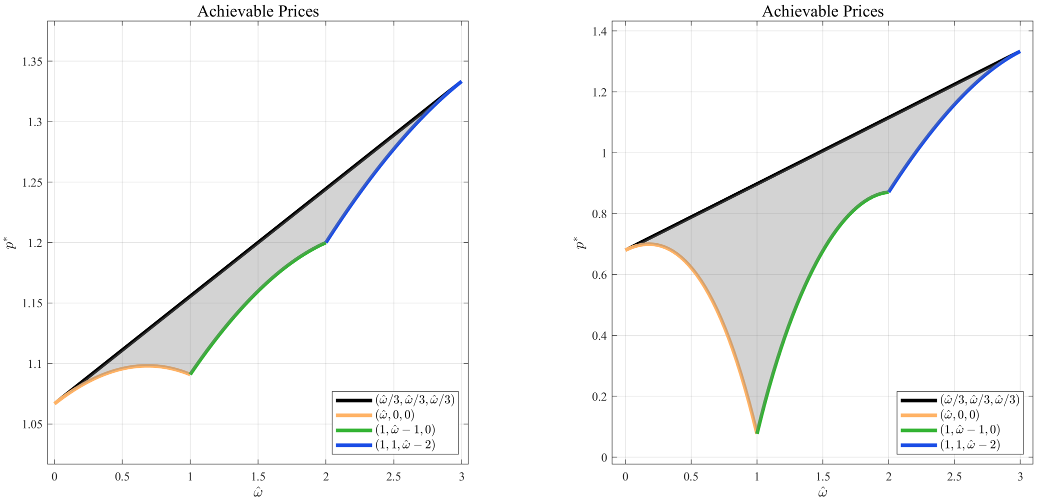
[39] Moreover, the concavity of $p^*(\boldsymbol{\omega})$ implies that $\underline{p}(\hat{\omega})$ is piecewise concave in $\hat{\omega}$, possibly exhibiting kinks at integer values of $\hat{\omega}$. Since $\bar{p}(\hat{\omega})$ coincides with the price under configuration IW$(\hat{\omega}/3)$, it is linear in $\hat{\omega}$ (see (13)).

Figure 6 illustrates Proposition 5(i), based on normalized logit demand, with $\delta = 0.2$ and $\delta = 0.49$. In each panel, the black curves trace the price upper bound $\bar{p}(\hat{\omega})$ for varying $\hat{\omega} \in [0, 3]$, and it is linear and increasing in $\hat{\omega}$. Likewise, the orange, green, and blue curves similarly trace the price lower bound $\underline{p}(\hat{\omega})$, and it is piecewise concave and generally non-monotone in $\hat{\omega}$. For each given $\hat{\omega}$, the vertical segment between $\bar{p}(\hat{\omega})$ and $\underline{p}(\hat{\omega})$ indicates the set of achievable prices when total interoperability strength is $\hat{\omega}$. Collecting these segments across $\hat{\omega} \in [0, 3]$ yields the set of all achievable prices by all admissible configurations (the shaded region).



(a) $\delta = 0.20$

(b) $\delta = 0.49$

Figure 6: The set of achievable prices by configurations with given $\hat{\omega} \in [0, 3]$.

Proposition 5(ii) is a direct consequence of part (i). For each $\hat{\omega} \in [0, 3]$, the set of achievable welfare pairs (CS, Π) over $L(\hat{\omega})$ forms a line segment. This is because the total welfare satisfies $\Pi + CS = \gamma(1 + \hat{\omega})/4$, which is constant over $L(\hat{\omega})$ (recall the total strength is just $\hat{\omega}$). Moreover, the set of achievable prices (or industry profits Π) is exactly the interval characterized in part (i). These line segments are disjoint since a larger $\hat{\omega}$ corresponds to a higher total welfare level. Taking the union of these disjoint line segments over $\hat{\omega} \in [0, 3]$ yields the entire set of achievable (CS, Π).

Figure 7 illustrates Proposition 5(ii) under the same parametric assumption as Figure 6. The figure shows the space of welfare allocations (CS, Π). The shaded region represents the set of achievable (CS, Π) across all admissible configurations, constructed in the same manner as in Figure 6. As an example of configurations that are not included in the $\overline{\text{CA}}$ class, note that the circular configuration $(\omega, \omega, 0)$ generates a locus (as ω rises from 0 to 1) that runs from the origin to the kink point at $(1, 1, 0)$, yet does not belong to $\overline{\text{CA}}$.

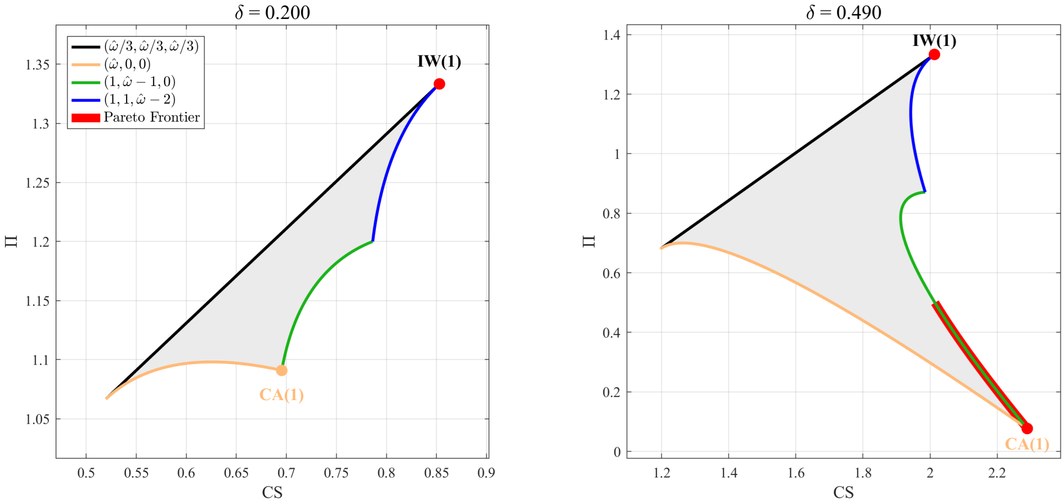


Figure 7: The set of welfare allocations (CS, Π) across all admissible configurations.

☐ **Pareto frontiers**. In Figure 7, the *Pareto frontiers* of achievable welfare allocations, corresponding to the top-right boundaries of the achievable allocations, are highlighted in red. When δ is low (left panel), the Pareto frontier is a single point, $\boldsymbol{\omega} = (1, 1, 1)$, corresponding to IW(1), which simultaneously maximizes consumer surplus and industry profits. When δ is large (right panel), the Pareto frontier consists of two disjoint components: the point $\boldsymbol{\omega} = (1, 1, 1)$ and a segment of the orange curve that delivers higher CS than IW(1). This latter segment is parameterized by configurations $\boldsymbol{\omega} = (1, \omega, 0)$ for $\omega \in [0, W_{\text{pareto}})$, where $W_{\text{pareto}} < 1$ is uniquely defined by the condition that consumer surplus under $(1, W_{\text{pareto}}, 0)$ equals that under IW(1). The disjointness of the Pareto frontier reflects the fact that, because IW(1) maximizes total user externalities, any consumer-surplus improvement must come from a qualitatively different configuration that sufficiently lowers equilibrium prices. As will be proven below, the latter is achieved by coalition-based configurations.

☐ **Revisiting optimal configurations.** Building upon Proposition 5, we derive the results on optimal configurations across *all* admissible configurations.

Corollary 3 *Suppose $n = 4$. Comparing across all $\boldsymbol{\omega} \in [0, 1]^3$, we have:*[40]

(i) *p^* is the lowest at $\boldsymbol{\omega} = (0, 0, 0)$ if $\delta < \frac{1}{4}$, and at $\boldsymbol{\omega} = (1, 0, 0)$ if $\delta > \frac{1}{4}$;*

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[40] For the sake of completeness, we note p^* and Π are highest at $\boldsymbol{\omega} = (1, 1, 1)$, whereas CS is lowest at $\boldsymbol{\omega} = (0, 0, 0)$. These follow immediately from Proposition 2.

(ii) *CS is the highest at $\boldsymbol{\omega} = (1, 1, 1)$ if $\delta < \frac{5}{12}$, and at $\boldsymbol{\omega} = (1, 0, 0)$ if $\delta > \frac{5}{12}$.*

Corollary 3(i) identifies the global minimum price (which equals the industry profit Π) over the entire parameter space $[0, 1]^3$. By the concavity of $p^*(\boldsymbol{\omega})$, the minimum is attained at one of the extreme points of $[0, 1]^3$, which, up to permutations, are:

$$\boldsymbol{\omega}: \quad (0, 0, 0),\ (1, 0, 0),\ (1, 1, 0),\ (1, 1, 1).$$

For each of these points, the corresponding eigenvalues (excluding the largest one μ_1) are:

$$(\mu_2, \mu_3, \mu_4): \quad (0, 0, 0),\ (1, -1, -1),\ (0, 0, -2),\ (-1, -1, -1).$$

The last two vectors are component-wise lower than the first one and so, by Corollary 1, imply that $(1, 1, 0)$ and $(1, 1, 1)$ yield higher price than $(0, 0, 0)$. Pointwise comparison between the remaining two cases, $(0, 0, 0)$ and $(1, 0, 0)$, yields the result. A similar technique applies to Corollary 3(ii). Given that the user externality component is linear in $\boldsymbol{\omega}$ and $p^*(\boldsymbol{\omega})$ is concave (by Corollary 1), we have that CS is convex in $\boldsymbol{\omega}$ and thus maximized at one of the extreme points of $[0, 1]^3$. Pointwise comparison across these points then delivers the result.[41]

Corollary 3(i) on the price-minimizing configuration is consistent with Proposition 3(i) when $\delta < \frac{1}{4}$ and further strengthens Proposition 3(ii) by extending the result to all admissible configurations when $\delta > \frac{1}{4}$. In particular, Corollary 3(i) implies that, among all admissible configurations, either the zero-interoperability configuration or the bilateral-coalition configuration minimizes the equilibrium price, depending on the magnitude of the user externality coefficient.

Turning to consumer surplus, note that when $n = 4$ the relevant thresholds are $\underline{\delta} = \frac{5}{36}$ and $\bar{\delta} = \frac{5}{12}$. Hence, Corollary 3(ii) strengthens Proposition 4(i), since $\frac{5}{12} > \underline{\delta}$. It is also consistent with—and stronger than—Proposition 4(ii), as the relevant threshold for $n = 4$ is $\bar{\delta} = \frac{5}{12}$. Importantly, Corollary 3 imposes no restriction on the class of configurations and therefore extends beyond the $\overline{\text{CA}}$ family considered in Proposition 4.

☐ **Bottom line**. For four platforms, the tractable parameterization of all admissible configurations delivers a complete picture: Proposition 5 characterizes the entire set of achievable welfare outcomes, whereas Corollary 3 identifies

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[41] The convex analysis approach here extends to weighted total welfare criteria $TW_\alpha = \alpha\Pi + CS$ for $\alpha \geq 0$, whereby TW_α is the highest at $\boldsymbol{\omega} = (1, 0, 0)$ if $\delta > \frac{5}{12} + \frac{\alpha}{3}$, and at $\boldsymbol{\omega} = (1, 1, 1)$ if $\delta < \frac{5}{12} + \frac{\alpha}{3}$.

the configurations that minimize price and maximize consumer surplus *among all* admissible configurations. The user externality coefficient δ alone determines whether full industry-wide interoperability or coalition-based arrangements are optimal; no restriction to specific configuration classes is needed.

6 Concluding remarks

In this paper, we propose a flexible network-based approach to study interoperability among competing platforms along two dimensions: strength and configuration, that is, the extent to which user externalities are shared across a set of platforms. This approach allows us to characterize equilibrium outcomes across a wide range of interoperability structures, including industry-wide and coalition-based arrangements, within a unified oligopolistic framework.

Our analysis yields several main insights. First, we identify a novel relationship between equilibrium prices and the spectral properties of the interoperability configuration matrix. Second, we characterize the boundaries of welfare outcomes that are implementable through interoperability configurations, showing that welfare extrema are attained under either industry-wide or coalition-based arrangements. Third, we explain the advantages of coalition-based interoperability by linking them to the degree of demand complementarity among interoperable platforms. This mechanism contrasts with existing analyses that focus on industry-wide interoperability, where demand complementarity does not arise.

These results lead to a novel empirical and policy implication: in markets with user externalities and interoperability, demand systems may exhibit negative diversion ratios (or negative cross-price elasticities), even though individual consumers initially perceive platforms as substitutes. This perspective offers a new lens through which to interpret the “co-opetition” observed among OEMs in the Android OS ecosystem (a coalition in our terminology) where member OEMs compete with one another while exhibiting complementarity when positioned against devices using alternative operating systems, such as Apple’s iOS.

Measuring interoperability and empirically distinguishing it from standard user externalities is challenging and has received relatively limited attention. User externalities are typically captured by the marginal value users derive as platform size expands. By contrast, interoperability refers to the extent to which users can interact with, or access services across, multiple platforms. A practical empirical proxy is the gap in interaction quality between within-platform and cross-platform interactions.

In social media platforms, randomized A/B tests can assign otherwise comparable users to treatments that differ in their degree of cross-platform access and then track outcomes such as daily usage, in-app purchases, and user ratings (e.g., Scott Morton et al., 2023). When randomization is infeasible, staggered adoption across markets can provide quasi-experimental variation. For example, in telecommunications, phased rollouts of cross-platform compatibility allow researchers to exploit variation in objective service-quality measures (e.g., latency, packet loss, and drop rates) complemented by user-reported quality. In two-sided markets such as ride-hailing platforms, where drivers and consumers are matched, observable matching outcomes (e.g., wait times and driver utilization) can similarly serve as informative proxies for the effectiveness of interoperability.[42]

We view our analytical framework as a proof-of-concept demonstrating that representing interoperability structures as matrices provides a useful modeling tool and opens several avenues for future research. First, our network-based approach to interoperability can be readily extended to other frameworks of platform competition, including the transaction-fee models of Rochet and Tirole (2003, 2006), the net-fee model of Ekmekci et al. (2025), and the aggregative game framework of Peitz and Sato (2025). Building on Armstrong (2006), our analysis illustrates the feasibility of integrating tools from network economics with core questions in industrial organization.

Second, some policy debates focus on whether interoperability can mitigate the market power of a leading platform facing several smaller rivals. In a companion paper, we examine this setting through a class of non-vertex-transitive configurations that nests a hub-and-spoke (star) structure, whereby the leading platform interoperates with each fringe rival, but fringe platforms do not interoperate among themselves. We show that this star configuration most effectively reduces the leading platform's market share when the platform is already dominant (holding more than half of the market), whereas industry-wide interoperability maximizes consumer surplus. Moreover, the leading platform has no incentive to sustain interoperability with fringe rivals, so neither the star nor the industry-wide configuration arises endogenously without intervention. More broadly, these findings suggest that applying a network-based interoperability framework to asymmetric environments offers a promising direction for future research.

Third, while our primary focus is on price competition between platforms, we analyze platforms' incentives to interoperate only indirectly, through comparisons

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[42] For example, in an empirical study of ride-hailing platforms, Rosaia (2025) model full interoperability as a counterfactual regime in which passengers can be matched with all drivers across competing platforms, while platforms continue to set prices independently. Using an estimated structural model, the paper quantifies the efficiency and welfare gains from this integration relative to the status quo of no interoperability.

of joint-industry profits across configurations. A more comprehensive approach would explicitly model the endogenous formation of interoperability coalitions, tracing industry dynamics from a no-interoperability benchmark to coalition-based and, ultimately, industry-wide interoperability. Such an extension could shed light on the strategic and structural forces that impede full adoption, even when broader interoperability may be socially desirable.

A Appendix

A.1 Proofs

Proof. (Lemma 1). Totally differentiating (4) ,

$$\left.\frac{\partial \mathbf{x}}{\partial \mathbf{p}}\right|_{(\mathbf{p},\mathbf{x})=(p\mathbf{1},\frac{1}{n}\mathbf{1})} = \mathbf{S} - \gamma \mathbf{S}(\mathbf{I}+\mathbf{\Lambda})\left.\frac{\partial \mathbf{x}}{\partial \mathbf{p}}\right|_{(\mathbf{p},\mathbf{x})=(p\mathbf{1},\frac{1}{n}\mathbf{1})}$$

We know $\mathbf{S} = -\frac{\delta}{\gamma}[\mathbf{I}-\frac{1}{n}\mathbf{1}\mathbf{1}^\intercal]$ (by definition) and $\mathbf{1}^\intercal[\mathbf{I}+\mathbf{\Lambda}] = (1+\hat{\lambda})\mathbf{1}^\intercal$ (vertex transitivity), so

$$\left.\frac{\partial \mathbf{x}}{\partial \mathbf{p}}\right|_{(\mathbf{p},\mathbf{x})=(p\mathbf{1},\frac{1}{n}\mathbf{1})} = \mathbf{S} + \delta(\mathbf{I}+\mathbf{\Lambda})\left.\frac{\partial \mathbf{x}}{\partial \mathbf{p}}\right|_{(\mathbf{p},\mathbf{x})=(p\mathbf{1},\frac{1}{n}\mathbf{1})} - \frac{\delta}{n}(1+\hat{\lambda})\mathbf{1}\underbrace{\mathbf{1}^\intercal\left.\frac{\partial \mathbf{x}}{\partial \mathbf{p}}\right|_{(\mathbf{p},\mathbf{x})=(p\mathbf{1},\frac{1}{n}\mathbf{1})}}_{=\mathbf{0}^\intercal \text{ by full coverage}}.$$

Rearranging,

$$[\mathbf{I}-\delta(\mathbf{I}+\mathbf{\Lambda})]\left.\frac{\partial \mathbf{x}}{\partial \mathbf{p}}\right|_{(\mathbf{p},\mathbf{x})=(p\mathbf{1},\frac{1}{n}\mathbf{1})} = \mathbf{S},$$

which yields the first equality in (8) after inversion, with $\mathbf{A} = [\mathbf{I}-\delta(\mathbf{I}+\mathbf{\Lambda})]^{-1}$. Denote

$$\theta_k = \frac{1}{1-\delta-\delta\mu_k} \tag{22}$$

as the k-th eigenvalue of $\mathbf{A}$. $\theta_1 = \frac{1}{1-\delta-\delta\hat{\lambda}}$ is well-defined by Assumption I. The remaining eigenvalues satisfy $\theta_k > 0$ for $k \neq 1$. To see this, note that $\mathbf{\Lambda}$ is $\hat{\lambda}$-regular, and hence

$$\mu_k \leq \min\{\hat{\lambda},\, n-2-\hat{\lambda}\} \leq \frac{n}{2}-1, \qquad \text{for } k \neq 1, \tag{23}$$

where the first bound follows from Lemma 8.5.1 of Godsil and Royle (2001), and the second bound is maximized at $\hat{\lambda} = n/2-1$. Therefore, for all $k \neq 1$,

$$1-\delta-\delta\mu_k > 1-\delta-\delta\Big(\frac{n}{2}-1\Big) > 0 \tag{24}$$

by Assumption I.

To prove the second equality in (8), we note from (6) that $\mathbf{S}\mathbf{e}_1 = \mathbf{0}$. For $k \neq 1$, the vector $\mathbf{e}_k$ is orthogonal to $\mathbf{e}_1 = \frac{1}{\sqrt{n}}\mathbf{1}$ (and to $\mathbf{1}$), so $\mathbf{S}\mathbf{e}_k = -\frac{nh(0)}{n-1}\mathbf{e}_k$. Thus, in the eigenspace of $\mathbf{\Lambda}$, the matrices $\mathbf{A}$ and $\mathbf{S}$ (and hence their product $\mathbf{AS}$) are simultaneously diagonalizable. Specifically,

$$\mathbf{E}^{-1}\mathbf{A}\mathbf{E} = \operatorname{diag}(\theta_1,\ldots,\theta_n) \qquad , \qquad \mathbf{E}^{-1}\mathbf{S}\mathbf{E} = -\frac{nh(0)}{n-1}\operatorname{diag}(0,1,\cdots,1)$$

$$\mathbf{E}^{-1}(\mathbf{AS})\mathbf{E} = \mathbf{E}^{-1}(\mathbf{SA})\mathbf{E} = -\frac{nh(0)}{n-1}\operatorname{diag}(0,\theta_2,\ldots,\theta_n).$$

Here $\mathbf{E} = [\mathbf{e}_1, \dots, \mathbf{e}_n]$ is an orthogonal matrix. Equivalently, we have

$$\mathbf{A} = \sum_{k=1}^{n} \theta_k \mathbf{e}_k \mathbf{e}_k^\intercal, \qquad \mathbf{S} = -\frac{nh(0)}{n-1} \sum_{k \neq 1} \mathbf{e}_k \mathbf{e}_k^\intercal, \quad \mathbf{AS} = \mathbf{SA} = -\frac{nh(0)}{n-1} \sum_{k \neq 1} \theta_k \mathbf{e}_k \mathbf{e}_k^\intercal.$$

■

Proof. (Proposition 1A). By Assumption II, the optimal price satisfies the first-order condition

$$\frac{1}{n} + p^* \left.\frac{\partial x_i}{\partial p_i}\right|_{(\mathbf{p},\mathbf{x})=(p\mathbf{1},\frac{1}{n}\mathbf{1})} = 0.$$

From Lemma 1, by vertex-transitivity of $\mathbf{\Lambda}$ and hence $\mathbf{A}$, we compute the demand derivative as

$$\left.\frac{\partial x_i}{\partial p_i}\right|_{(\mathbf{p},\mathbf{x})=(p\mathbf{1},\frac{1}{n}\mathbf{1})} = \frac{1}{n}\operatorname{tr}(\mathbf{AS}) = -\frac{h(0)}{n-1} \sum_{k \neq 1} \frac{1}{1-\delta-\delta\mu_k} < 0, \quad \text{for all } i \in \mathcal{N}. \tag{25}$$

Using (25) and rearranging yields the pricing equation with d defined in (10). ■

Proof. (Corollary 1). It remains to verify monotonicity in δ. Differentiating (10),

$$\frac{\partial}{\partial \delta}\left(\frac{1}{d}\right) = \frac{1}{n-1} \sum_{k \neq 1} \frac{1+\mu_k}{(1-\delta-\delta\mu_k)^2}$$

Using the elementary inequality that $\frac{y}{(1-\delta y)^2} \geq y$ for any scalar $y < \frac{1}{\delta}$ and that $1+\mu_k < \frac{1}{\delta}$ for $k \neq 1$ due to (24), we get

$$\frac{\partial}{\partial \delta}\left(\frac{1}{d}\right) \geq \frac{1}{n-1} \sum_{k \neq 1} (1+\mu_k) = \frac{1}{n-1}(n-1-\hat{\lambda}) \geq 0$$

where the last equality used the trace identity $\mu_2 + \cdots + \mu_n = -\mu_1 = -\hat{\lambda} \geq -(n-1)$. Hence, d is non-increasing in δ (and strictly decreasing for any non-IW(1) configurations). ■

Proof. (Proposition 1B). From (8), we can restate the matrix of demand derivatives using the definition of $\mathbf{A}$ as

$$\left[(1-\delta)\mathbf{I} - \delta\mathbf{\Lambda}\right] \left.\frac{\partial \mathbf{x}}{\partial \mathbf{p}}\right|_{(\mathbf{p},\mathbf{x})=(p\mathbf{1},\frac{1}{n}\mathbf{1})} = \mathbf{S}.$$

The i-th diagonal element of this matrix equation corresponds to

$$\left.\left((1-\delta)\frac{\partial x_i}{\partial p_i} - \delta \sum_{j \in \mathcal{N} \setminus \{i\}} \lambda_{ij} \frac{\partial x_j}{\partial p_i}\right)\right|_{(\mathbf{p},\mathbf{x})=(p\mathbf{1},\frac{1}{n}\mathbf{1})} = -h(0).$$

Using the definition of the diversion ratios, this equation can be simplified to

$$(1-\delta+\delta\bar{\sigma})\left.\frac{\partial x_i}{\partial p_i}\right|_{(\mathbf{p},\mathbf{x})=(p\mathbf{1},\frac{1}{n}\mathbf{1})}=-h(0).$$

Combining this identity with the pricing first-order condition yields the statement. ■

Proof. (Lemma 2). Lemma 2(i) is a consequence of full market coverage (which implies $\sum_{j\in\mathcal{N}\setminus\{i\}}\sigma_{ij}=1$). Next, following the in-text derivation, we have

$$\sigma_{ij}=-\frac{a_{ji}-\frac{1}{n}\sum_{l=1}^{n}a_{li}}{a_{ii}-\frac{1}{n}\sum_{l=1}^{n}a_{li}}, \tag{26}$$

Observe that $\sigma_{ij}=\sigma_{ji}$, since $\mathbf{A}$ is symmetric and has identical diagonal entries by vertex-transitivity. Lemma 2(ii)-(iii) are immediate from (26) because the denominator in (26) is always positive (it coincides with the inverse of the own-price derivative in Lemma 1). ■

Proof. (Lemma 2′). For $\overline{\text{CA}}$ class, σ_{in} and σ_{out} have closed-form expressions (29). Assumption I implies $\theta_2\geq\theta_n>0$, which immediately implies $\sigma_{\text{in}}\leq\sigma_{\text{out}}$ and $\sigma_{\text{out}}>0$ from (29). The sign of σ_{in} is the same sign as its numerator:

$$\begin{aligned}\left(\frac{1}{n}-\frac{1}{m}\right)\theta_2+\frac{1}{m}\theta_n &= \left(\frac{1}{n}-\frac{1}{m}\right)\left(\frac{1}{1-\delta-\delta\mu_2}\right)+\frac{1}{m}\left(\frac{1}{1-\delta-\delta\mu_n}\right)\\ &= \frac{(1-\delta)m-n\delta\mu_2+(n-m)\delta\mu_n}{mn(1-\delta-\delta\mu_2)(1-\delta-\delta\mu_n)}.\end{aligned}$$

Substituting $\mu_2=(m-1)\lambda_{\text{in}}-m\lambda_{\text{out}}$ and $\mu_n=-\lambda_{\text{in}}$ from (11) and simplifying, we obtain:

$$=\frac{1-\delta-\delta(n-1)\lambda_{\text{in}}+n\delta\lambda_{\text{out}}}{n(1-\delta-\delta\mu_2)(1-\delta-\delta\mu_n)}.$$

This expression is negative if and only if $1-\delta-\delta(n-1)\lambda_{\text{in}}+n\delta\lambda_{\text{out}}<0$, as required. ■

Proof. (Proposition 3). Consider an arbitrary admissible configuration, and denote $\mathbf{G}\equiv\mathbf{I}+\mathbf{\Lambda}$. Rewrite the matrix $\mathbf{A}$ as an infinite series (which is well defined because $\delta<\frac{1}{n}$ implies that the leading eigenvalue of $\mathbf{G}$ satisfies $1+\hat{\lambda}\leq 1/\delta$):

$$\mathbf{A}=\mathbf{I}+\delta\mathbf{G}+\delta^2\mathbf{G}^2+\delta^3\mathbf{G}^3+\cdots.$$

It follows that $\text{tr}(\mathbf{A})=n+\sum_{m=1}^{\infty}\delta^m\,\text{tr}(\mathbf{G}^m)$. Observe that $\text{tr}(\mathbf{G})=n$, whereas

$$\text{tr}(\mathbf{G}^m) = \sum_{i=1}^{n}[\mathbf{G}^m]_{ii}=\sum_{i=1}^{n}\sum_{j=1}^{n}[\mathbf{G}^{m-1}]_{ij}g_{ji}$$

$$
\begin{aligned}
&\le \sum_{i=1}^{n}\sum_{j=1}^{n}[\mathbf{G}^{m-1}]_{ij} = \mathbf{1}^{\intercal}\mathbf{G}^{m-1}\mathbf{1} \\
&= \mathbf{1}^{\intercal}(1+\hat{\lambda})^{m-1}\mathbf{1} = n(1+\hat{\lambda})^{m-1},
\end{aligned}
$$

where the second line uses $g_{ji} \in [0,1]$, and the last line uses the fact that $(1+\hat{\lambda})^{m-1}$ is an eigenvalue of $\mathbf{G}^{m-1}$ with eigenvector $\mathbf{1}$. This upper bound is tight and is attained by any CA$(1;m)$ configuration. Therefore,

$$
\mathrm{tr}(\mathbf{A}) \le n + n\sum_{m=1}^{\infty}\delta^{m}(1+\hat{\lambda})^{m-1} = n\left(\frac{1-\hat{\lambda}\delta}{1-\delta-\hat{\lambda}\delta}\right).
$$

Consequently, we obtain a universal bound that applies to any admissible configuration:

$$
\frac{1}{d} = \frac{1}{n-1}\sum_{k=2}^{n}\frac{1}{1-\delta-\delta\mu_k} = \frac{1}{n-1}\left(\mathrm{tr}(\mathbf{A}) - \frac{1}{1-\delta-\delta\hat{\lambda}}\right) \le \frac{1}{n-1}\left(\frac{n-1-n\hat{\lambda}\delta}{1-\delta-\delta\hat{\lambda}}\right) \equiv B(\hat{\lambda}). \tag{27}
$$

Here, $B(\hat{\lambda})$ is decreasing in $\hat{\lambda}$ when $\delta \le \frac{1}{n}$ and is maximized at $B(0) = \frac{1}{1-\delta} = \frac{1}{d_{\mathrm{IW}}}|_{\lambda=0}$.

For part (ii), we denote the discount factor under $\overline{\mathrm{CA}}(\lambda_{\mathrm{in}},\lambda_{\mathrm{out}};m)$ by $d(\lambda_{\mathrm{in}},\lambda_{\mathrm{out}};m)$. Following the discussion in the text, it suffices to compare $d(1,0;m)$ (CA configurations) and $d(0,0;m)$ (zero-interoperability). For CA configurations with any $m \in [2, n/2]$, we have

$$
\frac{1}{d(1,0;m)} = \frac{1}{n-1}\left[\frac{\frac{n}{m}-1}{1-\delta-\delta(m-1)} + n - \frac{n}{m}\right].
$$

The right-hand side is strictly increasing in m when $\delta > 1/n$, and hence $d(1,0;m)$ is minimized at the upper bound $m = n/2$. A direct comparison then shows that $d(1,0;n/2) < d(0,0;m)$ if and only if $\delta > 1/n$, as required. ■

Proof. (Proposition 4). To prove part (i), we assume $\delta \le 1/n$. For any admissible configuration, inequality (27) establishes a lower bound on the equilibrium price, which implies the following upper bound on consumer surplus:

$$
CS \le \frac{\gamma}{n}(1+\hat{\lambda}) - \frac{n-1}{nh(0)}\left(\frac{1-\delta-\hat{\lambda}\delta}{n-1-n\hat{\lambda}\delta}\right) = \frac{\gamma}{\delta}\left(\frac{\delta}{n}(1+\hat{\lambda}) - \frac{1-\delta-\hat{\lambda}\delta}{n-1-n\hat{\lambda}\delta}\right).
$$

Meanwhile, $CS_{\mathrm{IW}(1)} = \frac{\gamma}{\delta}\left(\delta - \frac{1}{n-1}\right)$. Thus, $CS < CS_{\mathrm{IW}(1)}$ if

$$
\frac{\delta}{n}(1+\hat{\lambda}) - \frac{1-\delta-\hat{\lambda}\delta}{n-1-n\hat{\lambda}\delta} \le \delta - \frac{1}{n-1} \;\Leftarrow\; \hat{\lambda}\delta \le \frac{n^2-3n+1}{n(n-1)},
$$

which holds for all $\hat{\lambda} \in [0, n-1]$ if $\delta \le \underline{\delta}$.

To prove part (ii), we assume $\delta > \frac{1}{n}$. Given $\overline{\mathrm{CA}}(\lambda_{\mathrm{in}},\lambda_{\mathrm{out}};m)$, we express CS as

$$
CS(\lambda_{\mathrm{in}},\lambda_{\mathrm{out}};m) = \frac{\gamma}{n}(1+(m-1)\lambda_{\mathrm{in}}+(n-m)\lambda_{\mathrm{out}}) - \frac{\gamma}{\delta(n-1)}d_{\overline{\mathrm{CA}}}, \tag{28}
$$

which is convex in $(\lambda_{\text{in}}, \lambda_{\text{out}})$ given (11) and (12). Therefore, CS must be maximized at one of the extreme points $(\lambda_{\text{in}}, \lambda_{\text{out}}) \in \{(1,0),(1,1),(0,0)\}$. The last point is dominated by $(\lambda_{\text{in}}, \lambda_{\text{out}}) = (1,1)$ by Proposition 2. Next, observe that $CS(1,0;m)$ is maximized at $m = n/2$ because

$$\begin{aligned} CS(1,0;m) &= \frac{\gamma m}{n} - \frac{\gamma}{\delta}\left(\frac{\frac{n}{m}-1}{1-\delta-\delta(m-1)} + n - \frac{n}{m}\right)^{-1}, \\ \frac{n}{\gamma}\frac{\partial CS(1,0;m)}{\partial m} &= 1 - \frac{n(1-n\delta)}{\left(n-1-(m-1)n\delta\right)^2}. \end{aligned}$$

If $\delta \geq 1/n$, then clearly $\frac{\partial CS(1,0;m)}{\partial m} > 0$. If $\delta < 1/n$, then using $m \leq n/2$ yields

$$\frac{n}{\gamma}\frac{\partial CS(1,0;m)}{\partial m} > 1 - \frac{n(1-n\delta)}{(n-m)^2} \geq 1 - \frac{4}{n} \geq 0.$$

Hence, it remains to compare $CS(1,1;m)$ and $CS(1,0;n/2)$. Direct algebraic comparisons show $CS(1,1;m) > CS(1,0;n/2) \iff \delta < \bar{\delta}$. ■

Proof. (Corollary 3). We express CS as

$$CS(\omega_1,\omega_2,\omega_3) = \frac{\gamma}{n}(1+\omega_1+\omega_2+\omega_3) - p^*(\omega_1,\omega_2,\omega_3),$$

which is convex (since $p^*(\boldsymbol{\omega})$ is concave). Therefore, CS must be maximized at one of the extreme points $\boldsymbol{\omega} \in \{(1,0,0),(1,1,0),(1,1,1),(0,0,0)\}$. The last point is dominated by $(1,1,1)$ by Proposition 2. A direct comparison then shows that

$$CS(1,1,1) - CS(1,1,0) = \frac{9\gamma}{12(3+\delta)}\left(\frac{5}{9} - \delta\right),$$

which is positive given $\delta < 1/2$ (Assumption I). Similarly, we have

$$CS(1,1,1) - CS(1,0,0) = \frac{\gamma}{6(3-4\delta)}\left(-12\delta + 5\right),$$

which is positive if and only if $\delta < 5/12$. ■

A.2 Details for the $\overline{\text{CA}}$ class

☐ **Spectral decomposition.** In $\overline{\text{CA}}$ configurations, let $l = \frac{n}{m}$ be the number of coalitions. Denote $\mathbf{J}_y$, $\mathbf{I}_y$ as all-ones square matrix and identity matrix of dimension y. Define matrices

$$\mathbf{B} = \frac{1}{n}\mathbf{J}_n \quad \text{and} \quad \mathbf{B}_0 = \frac{1}{m}\mathbf{J}_m \otimes \mathbf{I}_l,$$

which satisfy (i) $\mathbf{B}\mathbf{B} = \mathbf{B}$ and $\mathbf{B}_0\mathbf{B}_0 = \mathbf{B}_0$; and (ii) $\mathbf{B}_0\mathbf{B} = \mathbf{B}\mathbf{B}_0 = \mathbf{B}$ by mixed-product properties of Kronecker products. Let $\boldsymbol{\Lambda} = n\lambda_{\text{out}}\mathbf{B} + m(\lambda_{\text{in}} - \lambda_{\text{out}})\mathbf{B}_0 - \lambda_{\text{in}}\mathbf{I}_n$.

To facilitate the spectral decomposition, we define projection matrices:

$$\mathcal{E}_{all} = \mathbf{B}, \quad \mathcal{E}_{\text{out}} = \mathbf{B}_0 - \mathbf{B}, \text{ and } \mathcal{E}_{\text{in}} = \mathbf{I}_n - \mathbf{B}_0$$

which satisfy: (i) $\mathcal{E}^\intercal = \mathcal{E}$ and $\mathcal{E}^2 = \mathcal{E}$ (symmetry and idempotence); and (ii) $\mathcal{E}_{all}\mathcal{E}_{\text{out}} = \mathcal{E}_{all}\mathcal{E}_{\text{in}} = \mathcal{E}_{\text{out}}\mathcal{E}_{\text{in}} = \mathbf{0}_{n\times n}$ (mutual orthogonality).

Matrix $\mathcal{E}_{all}$ projects to the eigenspace associated with $\mu_1 = (m-1)\lambda_{\text{in}} + (n-m)\lambda_{\text{out}}$:

$$\boldsymbol{\Lambda}\mathcal{E}_{all} = m(\lambda_{\text{in}} - \lambda_{\text{out}})\underbrace{\mathbf{B}_0\mathbf{B}}_{=\mathbf{B}} + n\lambda_{\text{out}}\underbrace{\mathbf{B}\mathbf{B}}_{=\mathbf{B}} - \lambda_{\text{in}}\mathbf{B} = \mu_1\mathcal{E}_{all}.$$

Matrix $\mathcal{E}_{\text{out}}$ projects to the eigenspace associated with $\mu_2 = (\lambda_{\text{in}} - \lambda_{\text{out}})m - \lambda_{\text{in}}$:

$$\boldsymbol{\Lambda}\mathcal{E}_{\text{out}} = m(\lambda_{\text{in}} - \lambda_{\text{out}})\underbrace{\mathbf{B_0}(\mathbf{B_0} - \mathbf{B})}_{=\mathbf{B_0}-\mathbf{B}} + \underbrace{n\lambda_{\text{out}}\mathbf{B}(\mathbf{B_0} - \mathbf{B})}_{=0} - \lambda_{\text{in}}(\mathbf{B_0} - \mathbf{B}) = \mu_2\mathcal{E}_{\text{out}}.$$

Matrix $\mathcal{E}_{\text{in}}$ projects to the eigenspace associated with $\mu_n = -\lambda_{\text{in}}$:

$$\boldsymbol{\Lambda}\mathcal{E}_{\text{in}} = m(\lambda_{\text{in}} - \lambda_{\text{out}})\underbrace{\mathbf{B_0}(\mathbf{I_n} - \mathbf{B_0})}_{=0} + \underbrace{n\lambda_{\text{out}}\mathbf{B}(\mathbf{I_n} - \mathbf{B_0})}_{=0} - \lambda_{\text{in}}(\mathbf{I_n} - \mathbf{B_0}) = \mu_n\mathcal{E}_{\text{in}}.$$

There are no other eigenvalues because $\mathcal{E}_{all} + \mathcal{E}_{\text{out}} + \mathcal{E}_{\text{in}} = \mathbf{I}_n$. So, the only distinct eigenvalues are μ_1, μ_2 and μ_n. The spectral decomposition theorem then implies

$$\boldsymbol{\Lambda} = \mu_1\mathcal{E}_{all} + \mu_2\mathcal{E}_{\text{out}} + \mu_n\mathcal{E}_{\text{in}}.$$

The multiplicities of the eigenvalues equals the dimensions of the associated eigenspaces, which equals ranks of the projection matrices: $\text{rank}(\mathcal{E}_{all}) = 1$, $\text{rank}(\mathcal{E}_{\text{out}}) = l - 1$, $\text{rank}(\mathcal{E}_{\text{in}}) = n - l$.

Matrix $\mathbf{A}$ shares the same set of eigenvectors as $\boldsymbol{\Lambda}$ with eigenvalues defined in (22). So, the same decomposition applies: $\mathbf{A} = \theta_1\mathcal{E}_{all} + \theta_2\mathcal{E}_{\text{out}} + \theta_n\mathcal{E}_{\text{in}}$, with diagonal and off-diagonal entries

$$a_{ii} = \frac{\theta_1 - \theta_2}{n} + \frac{\theta_2 - \theta_n}{m} + \theta_n, \quad \text{and} \quad a_{ij} = \begin{cases} \frac{\theta_1-\theta_2}{n} + \frac{\theta_2-\theta_n}{m}, & \text{if } \lambda_{ij} = \lambda_{in}, \\ \frac{\theta_1-\theta_2}{n}, & \text{if } \lambda_{ij} = \lambda_{out}. \end{cases}$$

Substituting these into (26) then leads to

$$\sigma_{\text{in}} = \frac{\frac{\theta_2}{n} - \frac{\theta_2-\theta_n}{m}}{(\frac{1}{m} - \frac{1}{n})\theta_2 + (1 - \frac{1}{m})\theta_n} \quad \text{and} \quad \sigma_{\text{out}} = \frac{\frac{\theta_2}{n}}{(\frac{1}{m} - \frac{1}{n})\theta_2 + (1 - \frac{1}{m})\theta_n}. \tag{29}$$

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