When does quality beat data? Gaining momentum in dynamic platform competition*

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Abstract

We consider a dynamic model with two horizontally differentiated platforms that serve heterogeneous users. In each period, platforms collect data from users that improves their services in future periods. The incumbent platform starts with a larger initial data base, while the entrant platform has a higher base quality. We find that when the value that users derive from data is low, the entrant gains momentum–gradually increasing its market share over time and eventually dominating the market. This occurs even if the entrant has an initial lower market share than the incumbent. When the value of data is high, the opposite occurs and the incumbent strengthens its position. We further find that greater product differentiation benefits the entrant, making it more likely for it to gain momentum and dominate the market. Finally, a policy that mandates reciprocal data sharing between platforms leads to long-run coexistence, with both platforms being active in the long run.

JEL Classification: L1

Keywords: Dynamic platform evolution, network effects, product differentiation, platform competition

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1 Introduction

In today's digital economy, platforms increasingly rely on user data accumulation to enhance their offerings—through personalization, algorithmic refinement, and predictive services. Unlike traditional network effects, where value is created by cross-users interactions, data accumulation can be leveraged across users and over time, even after users leave. This distinction introduces a novel source of advantage to platforms with a large market share. The more users a platform serves in a certain period, the more data it accumulates, which enables the platform to enhance its services and attract a larger market share in the next period.

Recent trends, however, suggest that this advantage is no inevitable. For example, in 2025, the market share of large generative AI chatbots, ChatGPT and Gemini *fell* by about 5%, while small platforms like Perplexity and ClaudeAI gained market share.[1] This raises the question: When do large platforms enjoy momentum where data accumulation enables them to continuously grow their market share? Under what conditions do small platforms gain momentum? Moreover, how does the evolution of the platforms' market share shape long-run market outcomes? In particular, do platforms with an early lead in market share always end up dominating in the long run?

This paper explores the dynamics of platform market share momentum in data-enabled environments. We present a dynamic model with heterogeneous users in which the market shares of two horizontally differentiated platforms evolve over time. Each platform improves its service by accumulating user data, but this data depreciates over time. Thus, the value of a platform's service at any point of time reflects not only its base quality but also the stock of user data it has acquired—even if those users are no longer with the platform. In that way, data acts as a persistent asset that can increase the platform's future appeal, thereby altering competitive dynamics. We focus on a non-trivial scenario in which one of the platforms, the entrant, enters after the competing platform, the incumbent, already accumulated data, but the entrant offers a higher base quality.

Our model explicitly assumes that platforms do not charge users a price—a realistic feature of many major platform data markets, such as search engines (e.g., Google), generative AI chatbots (e.g., ChatGPT, Gemini), and social media (e.g., Facebook, Instagram). In these markets, end-users typically enjoy services without direct mone-

[1]See: https://firstpagesage.com/reports/top-generative-ai-chatbots/

tary costs, while platforms monetize indirectly, primarily through advertising or data commercialization. By abstracting from user pricing, our model isolates the role data accumulation and product-quality differences play in determining platform success, particularly in environments where user expectations constrain platforms to provide services free of charge. This approach allows us to identify the conditions under which an entrant's superior base quality can overcome an incumbent's initial data advantage.

We show that even when a platform has a larger market share than its competitor and can exploit it to collect more data, it does not necessarily gain momentum over time. In particular, we show that momentum in our setting depends critically on the interaction between data-accumulation, base quality, and horizontal differentiation. When the entrant enjoys a larger market share, it will gain momentum–steadily increasing its share over time. When the incumbent is the one that enjoys a larger market share, it may either gain or lose momentum. Specifically, we identify a threshold in users' benefit from data, such that if this benefit is high, the incumbent gains momentum, while if it is low, the entrant gains momentum. At intermediate values of this benefit, market shares remain constant over time, with the incumbent capturing a per-period larger market share. Greater product differentiation benefits the entrant, making it more likely that the entrant gains momentum.

We further find that in the long-run steady-state, the market may converge to one of three outcomes: the incumbent dominates the market, the entrant dominates, or both platforms share the market. The short-run dynamics serves as an equilibrium selection mechanism. When users' benefit from data is high, the market converges to the long-run equilibrium in which the incumbent dominates the market. The opposite case occurs when users' benefit from data is low.

Our model offers important policy insights. In particular, while mandating a one-time data transfer to entrants, at the time of entry, eliminates the incumbent's informational head start and ensures that the highest-quality platform dominates, this outcome is not efficient because users with stronger preferences for one of the platforms are served by the other one. In contrast, continuous reciprocal data-sharing requirement implements the welfare maximizing outcome where both platforms maintain constant market shares, and the entrant–being the superior quality platform–holds a larger market share.

2 Related Literature

Our model integrates persistent yet depreciating data, heterogeneous user preferences, and horizontal product differentiation into a unified dynamic framework. We contribute to a growing body of work that examines how accumulated user data improves platform quality over time and drives learning. Hagiu and Wright (2023) develop a model of data-enabled learning with homogeneous users and all-or-nothing competition. They find that an increase in the strength of network effects reduces the incumbent advantage because entrants can catch up more quickly. Our model departs from theirs by incorporating heterogeneous users and horizontal differentiation, allowing for interior equilibria in which both platforms survive or in which either the entrant or the incumbent can gain momentum. Moreover, we find that in contrast to the case where competition is all-or-nothing, when users are heterogeneous and data persists beyond user participation, an increase in the strength of network effects strengthens the incumbent's grip.

Our analysis connects to the broader literature on platform competition with network effects (Katz and Shapiro, 1985; Farrell and Saloner, 1985; see Jullien et al., 2021, for a review of the literature). Hałaburda and Yehezkel (2013; 2016; 2019), and Markovich and Yehezkel (2022; 2024) consider platform competition and coordination in the context of a static game. Hałaburda et al. (2020), and Biglaiser and Crémer (2020) consider dynamic competition. This literature shows that because of network effects, there are multiple equilibria, depending on users beliefs regarding the decisions of other users to which platform to join. A key difference between this literature and our paper is that in our setting with data accumulation, users' value from the platform depends not only on its current user base but also on data from past users even if they are no longer on the platform. We contribute to this literature by showing how dynamics and the platforms' momentum over time serve as an equilibrium selection. In particular, while we find that in the long-run there are multiple steady-state equilibria, our analysis of the short-run dynamics and platforms' momentum shows that the market converges to a unique long-run steady state.

Our momentum results contribute to the literature on dynamic competition with switching and learning (Cabral, 2011) and connect to prior work on market tipping and path dependence. Markovich (2008) examines dynamic competition in differentiated markets and shows how market structure can evolve toward monopoly or coexistence depending on switching costs and network effects. Similar to that work, we focus on dynamic trajectories and thresholds that determine whether an entrant can overcome an incumbent's advantage; however, our model replaces switching costs with persistent data-driven advantages.

3 The Model

Consider a dynamic game between two horizontally differentiated platforms, platforms A and B , and heterogeneous users. Given that we want our analysis to take the entrant's point of view, we denote the entrant with its advanced quality as platform A, and the incumbent with its data advantage as platform B. The two platforms operate for an infinite number of discrete periods, $t = 0, 1, ..., \infty$. In each period, users decide which platform to join in the current period. Suppose that in each period users can join only one of the platforms and can switch platforms at no costs.

Platforms A and B offer users initial base qualities $Q_A = Q + v$ and $Q_B = Q + 1$, where $2 > v \geq 1$ measures the quality advantage of platform A. We assume that Q is sufficiently large such that the market is fully covered, i.e., all users join a platform. In addition to their initial base qualities, the platforms can collect data from users that enables them to enhance the quality of their services in the current period and in future periods. [2] Platforms accumulate data over time, but old data depreciates in future periods. When platform $i = A, B$ served $n_{i,0}, n_{i,1}, ..., n_{i,\widetilde{t}}$ users in periods $t = 0, 1, ..., \widetilde{t}$, respectively, its total amount of data accumulation in period $\widetilde{t}$ is $D_{i,\widetilde{t}} = n_{i,\widetilde{t}} + \gamma n_{i,\tilde{t}-1} + \gamma^2 n_{i,\tilde{t}-2}, ..., \gamma^{\widetilde{t}} n_{i,0}$, where γ $(0 < \gamma < 1)$ is the data-depreciation factor. By accumulating data $D_{i,t}$ in period t, the platform can provide users services of value $\beta D_{i,t}$. The parameter β measure the users' benefit from data collected on other users. Notice that while β seems equivalent to network effects, it incorporates the feature that a user's data benefits other users even after the user left the platform, because the user's data remains the platform's property. For this reason, $D_{i,t}$ is different from installed base of users, because users do not necessarily stay with the same platform.

Users are heterogeneous in their preferences for the two platforms. Suppose that users are uniformly distributed along the Hotelling line, where $x \in [0, 1]$ denotes user x's location. Platform A (B) is located at point 0 (1), and user x incurs the costs θx ($\theta(1 - x)$) from joining platform A (B). The parameter θ measures the degree of differentiation between the two platforms. To ensure the stability of the equilibrium in

[2] In Section 7, we assume that platforms can commercialize data; i.e., sell data to advertisers.

the short-run, we restrict the parameter space to $\beta < \theta$.[3]

We therefore have that a user's utilities from joining platforms A or B in period t are:

$$U_{A,t} = Q + v + \beta D_{A,t} - \theta x, \quad U_{B,t} = Q + 1 + \beta D_{B,t} - \theta(1-x).$$

In order to identify the incumbency advantage of data collection, suppose that platform B entered the market before the advanced-quality platform A. With a slight abuse of notations, we can define the period in which platform A entered as time $t = 1$. By the time platform A entered, platform B already accumulated data D_0. The size of D_0 depends on the number of periods in which platform B was a monopoly and on platform B's market coverage in these periods. Instead of solving for D_0 given arbitrary number of periods and market share, we solve the model given D_0, that measures the data-driven incumbency advantage. We restrict attention to the interesting case where $D_0 < \frac{1}{(1-\gamma)}$, such that the entering platform A can gain a positive market share.[4] We assume that platforms provide their services for free and compete to attract market share. In the background of the model, platforms collect and commercialize *anonymous* user data, which does not inflict a disutility on them. Hence, platforms' profits increase with their market share and their goal is to achieve as large a share of the market as possible. In Section 6, we allow platforms to make a strategic decision on whether, and to what extent, to commercialize personal data, where such commercializing imposes privacy costs on users.

The model introduces a tradeoff between platform B's data-driven incumbency advantage and platform A's quality advantage. This tradeoff raises the question of how the market evolves over time.

4 Short-run equilibrium

In this section we solve for the equilibrium allocation of users between the two platforms after a finite number of periods (we consider the long-run steady-state in the next section). The main conclusion of this section is that the market can be characterized by increased momentum of platform $i = A, B$, where platform i increases its market share over time, or no momentum, where market shares remain constant over time.

[3] If $\beta > \theta$, in the short-run all users join the same platform.

[4] In Proposition 2 below we show that at the equilibrium in which no platform gains momentum, platform A gains a positive market share if and only if $D_0 < \frac{1}{(1-\gamma)}$.

Consider a certain period after entry, $t > 1$. If both platforms have positive market share, there is an indifferent users, x_t, such that users with $x \in [0, x_t]$ join the entrant platform A while users with $x \in [x_t, 1]$ join platform B, where

$$v - \theta x_t + \beta(x_t + \gamma D_{A,t-1}) = 1 - \theta(1 - x_t) + \beta(1 - x_t + \gamma D_{B,t-1}), \tag{1}$$

hence:

$$x_t = \frac{1}{2} + \frac{v-1}{2(\theta - \beta)} - \frac{\beta\gamma(D_{B,t-1} - D_{A,t-1})}{2(\theta - \beta)}. \tag{2}$$

Intuitively, platform A gains a higher market share than platform B ($x_t > 1/2$), if its relative quality advantage, $v - 1$, is large and platform B's relative data-advantage, $D_{B,t-1} - D_{A,t-1}$, is low. Given x_t, in period $t+1$, the indifferent user, x_{t+1} solves:

$$v - \theta x_{t+1} + \beta\left(x_{t+1} + \gamma x_t + \gamma^2 D_{A,t-1}\right) \tag{3}$$

$$= 1 - \theta(1 - x_{t+1}) + \beta\left(1 - x_{t+1} + \gamma(1 - x_t) + \gamma^2 D_{B,t-1}\right),$$

hence,

$$x_{t+1} = \frac{1}{2} + \frac{(v-1)(\theta - \beta(1-\gamma))}{2(\theta - \beta)^2} - \frac{\beta\gamma^2\theta(D_{B,t-1} - D_{A,t-1})}{2(\theta - \beta)^2}. \tag{4}$$

We say that the entrant platform A gains momentum between periods t and $t+1$ if $x_{t+1} > x_t$, while platform B gains momentum otherwise. If $x_{t+1} = x_t$, the market is stable between periods. The following proposition shows that either platform A has momentum in all periods, platform B has momentum in all periods, or the market is stable between periods. The proposition further shows how the market's parameters affect the momentum of platforms A or B:

Proposition 1. *Suppose that:*

$$0 < \frac{v-1}{\gamma\theta} < D_0 < \frac{1}{(1-\gamma)}. \tag{5}$$

Then, in the short-run, regardless of t, there is a momentum threshold, $\overline{\beta} \in (0,1)$, where

$$\overline{\beta} = \frac{v-1}{D_0} + \theta(1-\gamma), \tag{6}$$

such that platform A gains momentum if $0 < \beta < \overline{\beta}$, platform B gains momentum if $\beta > \overline{\beta}$, and the market is stable if $\beta = \overline{\beta}$.

Importantly, Proposition 1 shows that the question of who gains momentum does not depend on t. If a platform gained momentum between periods t and $t+1$, it will continue to gain momentum in all future periods.

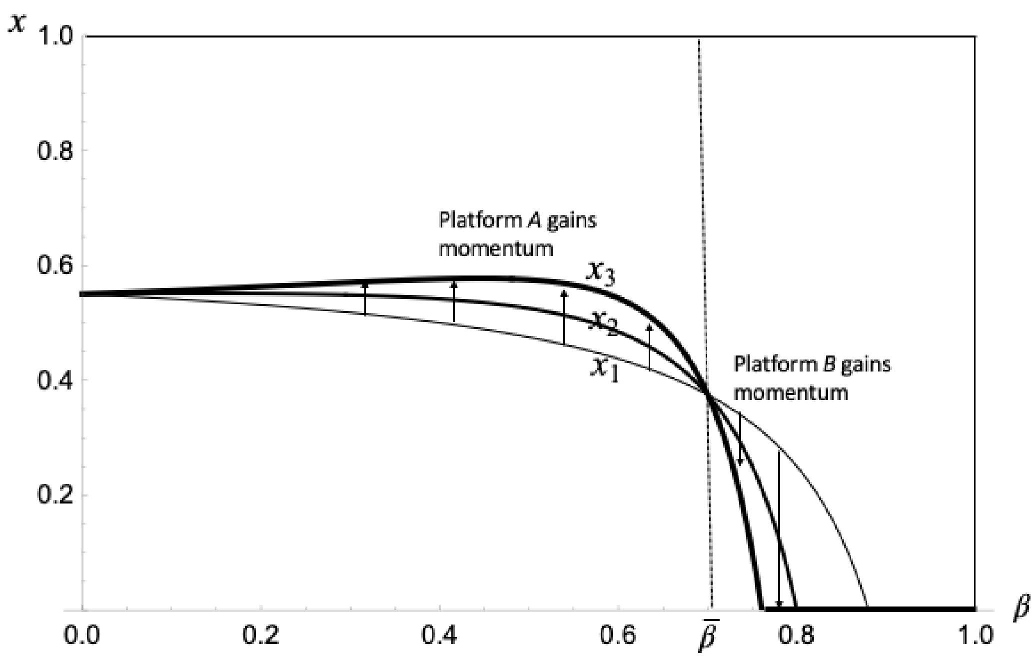

Figure 1: Market share of the two platforms ($v = 1.1$, $\gamma = 0.5$, $D_0 = 0.5$ and $\theta = 1$)

Figure 1 illustrates the market shares of the two platforms as a function of β, for $t = 1, 2, 3$. While we only show the market shares in the first three periods after entry, by Proposition 1, the results follow to all t (until the market reaches corner solutions, on which we elaborate in the next section). Starting with the case of $\beta = 0$, the figure shows that the market remains constant over time, because accumulating data does not increase the users' utility. When β is lower than the momentum threshold $\overline{\beta}$, platform B increases its market share over time. Even if platform B starts period $t = 1$ at a market share below 1/2, whenever $\beta < \overline{\beta}$, platform B gains momentum by increasing its market share as t increases. The opposite result holds when $\beta > \overline{\beta}$, in which case platform A gains momentum as t increases (until eventually covering the entire market). When $\beta = \overline{\beta}$, the market share of the two platforms remains constant over time. Notice that in this case, platform A gains higher market share than platform B in all periods.

The intuition for this result is the following. Recall that in the first period after entry ($t = 1$) platform B holds the data-driven incumbency advantage due to its initial data base, while platform A has a quality advantage, that does not depend on previous data collection. When β is higher than the momentum threshold (i.e., $\beta > \overline{\beta}$), platform B's data-driven incumbency advantage is stronger than platform A's quality advantage, resulting in an increase in platform B's market share, i.e., $x_2 > x_1$. This increases platform $B's$ data driven incumbency advantage even further and enables platform B to increase its market share even further in the next period: $x_3 > x_2$, and so on. The opposite case occurs if $\beta < \overline{\beta}$, in which case platform A's quality advantage is stronger than platform B's data-driven advantage, which enables platform A to increase its market share over time. In the case where $\beta = \overline{\beta}$, the two effects of initial data collection and initial quality advantage balance each other. Therefore, $x_2 = x_1$. As market share in the second period did not change, the two effects continue to balance each other such that $x_3 = x_2$ and so on.

Notice that momentum does not necessarily depend on the initial market share. As shown in Figure 1, if β is slightly lower than $\overline{\beta}$, platform B enjoys in the first period after entry a higher market share than platform A ($x_1 < 1/2$). Yet, because $\beta < \overline{\beta}$, the incumbency advantage due to data collection is small, and platform A can increase its market share in the next period ($x_2 > x_1$). Platform A accumulates the data for period $t = 3$, and as the gap in data accumulation decreases (following the increase in platform A's market share), platform A increases its market share in $t = 3$. Hence, platform A gains momentum despite starting with an inferior market share.

These results differ from Hagiu and Wright (2023) that show that an increase in the value of the new data (which is equivalent, in our model, to an increase in β) decreases the incumbent's competitive advantage.[5] In Hagiu and Wright (2023), competition is all - or nothing, as users are identical. Here, because users are heterogeneous in their preferences for the two platforms, it is possible to find a $\overline{\beta}$ and a market share that balances the data-advantage and the quality advantage. Then, if β increases above $\overline{\beta}$, platform B has the momentum because its data-driven advantage becomes stronger. As β decreases below $\overline{\beta}$, platform A has the momentum because platform B's data-driven advantage becomes weaker.

Next we move to how market conditions affect $\overline{\beta}$. Corollary 1 follows directly from Proposition 1:

Corollary 1. *Platform A is more likely to gain momentum (the momentum cutoff $\overline{\beta}$ increases) as:*

[5] In their Proposition 2, they show that when both platforms are in the increasing part of their learning curves increasing the value of new data by a multiplier factor grater than one decreases the incumbent's competitive advantage.

(i) *platform A's quality advantage, v, increases;*

(ii) *product differentiation, θ, increases;*

(iii) *platform B's initial data accumulation, D_0, decreases;*

(iv) *data depreciation factor, γ, decreases.*

Intuitively, as platform A's quality advantage, v, increases, it is easier for platform A to gain momentum, which enables it to gain momentum for higher values of β. Likewise, as platform B's data-advantage, D_0 (or the ability to accumulate data, γ), increases, it is easier for platform B to gain momentum, which enables it to gain momentum for lower values of β. A less straightforward result is that as differentiation (measured by θ) increases, it is easier for the entrant to gain momentum. To see the intuition for this result, define x^* as the market share evaluated at $\beta = \overline{\beta}$. We have:

Proposition 2. *At the momentum threshold $\overline{\beta}$, where market shares remain stable over time, platform B holds a larger market share than platform A. That is, $x^* = \frac{1}{2} - \frac{1}{2}D_0(1-\gamma)$. Hence, $0 < x^* < \frac{1}{2}$ and is independent of θ.*

Proposition 2 shows that in order to balance the data-driven advantage and the quality advantage (which occurs when $\beta = \overline{\beta}$), platform B needs to have a superior market share. The intuition is that B's data-driven advantage can be sustained over time only if platform B continues to collect more data than platform A in all periods.

This result explains why differentiation benefits the entrant. At $\overline{\beta}$, the indifferent user, $x^* < \frac{1}{2}$, has stronger preferences for platform A because this user is closer in preferences to platform A. As θ increases, and users' preferences become stronger, it becomes more costly for this user to join the incumbent that offers a more distant service than platform A. Hence, this user will no longer be indifferent and now strictly *prefers* to join platform A. As a result, a higher β is needed to make this user indifferent between the two platforms, resulting in an increase in $\overline{\beta}$.

Finally, we note that it is not always the case that $\overline{\beta} \leq \theta$. Figure 2, shows the case where $D_0 < \min\left\{\frac{v-1}{\gamma\theta}, \frac{1}{(1-\gamma)}\right\}$.[6] In this case, because D_0 is low, platform B's data-driven advantage is weak, such that the entrant gains momentum for all values of $\beta \leq \theta$. Platform A starts the first period with a larger market share than platform

[6]This condition holds if either $D_0 < \frac{v-1}{\gamma\theta} < \frac{1}{(1-\gamma)}$, where the last inequality holds whenever $\frac{v-1}{v-1+\theta} < \gamma < 1$, or $D_0 < \frac{1}{(1-\gamma)} < \frac{v-1}{\gamma\theta}$, where the last inequality holds whenever $0 < \gamma < \frac{v-1}{v-1+\theta}$.

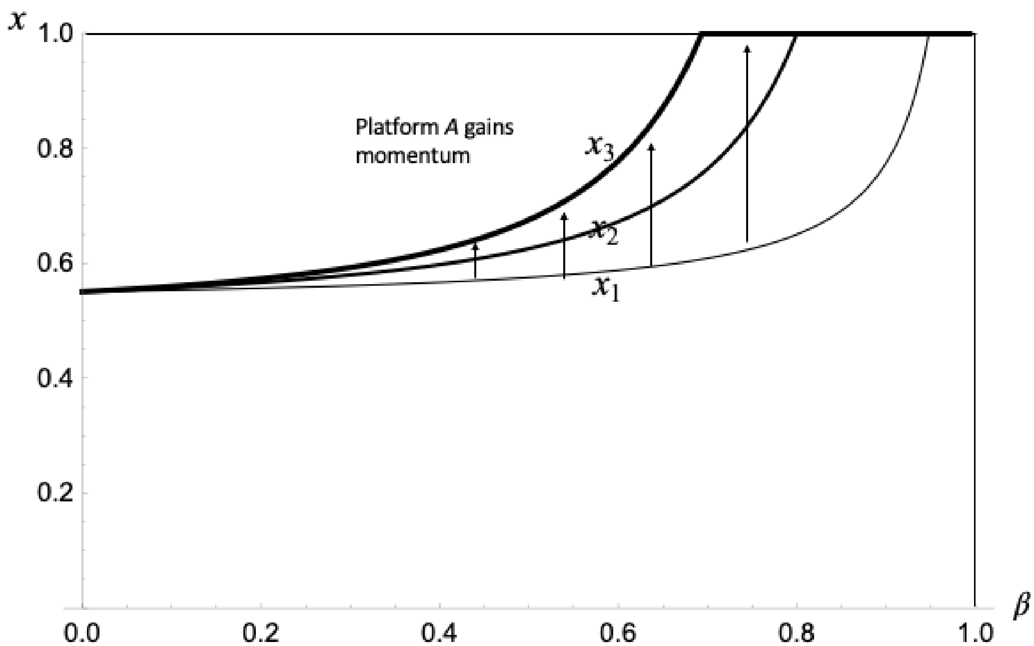

Figure 2: Market share of the two platforms ($v = 1.1$, $\gamma = 0.5$, $D_0 = 0.1$ and $\theta = 1$)

B ($x_1 > 1/2$ for all β) due to its superior quality advantage. This enables platform A to gain more data in the second period which further increases its advantage. For this reason, platform A's momentum increases, i.e., the gap $x_{t+1} - x_t > 0$ increases as β increases.

5 Long run steady state equilibrium

In this section we consider the equilibria in the steady-state, when market shares remain constant over time: $x_{t+1} = x_t$ for all t. These equilibria will hold in the long-run, after the market converges to one of these equilibria following the short-run dynamics that we identified in the previous section. We first solve for the potential steady-state equilibria, and then investigate which equilibrium the market converges to.

There are three potential steady-state equilibria: (i) both platforms have positive market shares; (ii) platform Awins; and (iii) platform B wins.

Starting with the equilibrium in which both platforms have positive market shares, define x^{**} as the indifferent user in this equilibrium. In each period, platform A collects x^{**} data and therefore accumulates $D_{t,A} = \frac{x^{**}}{1-\gamma}$. Likewise, platform B accumulates

$D_{t,B} = \frac{1-x^{**}}{1-\gamma}$. The indifferent user solves:

$$v - \theta x^{**} + \beta \frac{x^{**}}{1-\gamma} = 1 - \theta(1 - x^{**}) + \beta \frac{1 - x^{**}}{1-\gamma}, \tag{7}$$

or

$$x^{**} = \frac{1}{2} - \frac{(1-\gamma)(v-1)}{2(\beta - \theta(1-\gamma))}. \tag{8}$$

There is an equilibrium with positive market shares if $0 < x^{**} < 1$.

In a steady-state equilibrium in which platform A wins the market in all periods, user $x = 1$ prefers joining platform A over platform B, given that in all periods, all users join platform A:

$$v - \theta + \frac{\beta}{1-\gamma} > 1. \tag{9}$$

Likewise, in a steady-state equilibrium in which platform B wins, user $x = 0$ prefers joining platform B over platform A, given that in all periods, all users join platform B:

$$1 - \theta + \frac{\beta}{1-\gamma} > v. \tag{10}$$

To analyze when each steady-state equilibrium holds in the long run, it is useful to distinguish between the case of high-benefit and low-benefit from data, β.

High data benefits

Consider first the case of a high benefit from data: $\beta \in [(1-\gamma)(v+\theta-1), \theta]$. The following proposition shows that if β is sufficiently high, all three equilibria are possible:

Proposition 3. *Suppose that $(1-\gamma)(v+\theta-1) < \beta < \theta$. Then, in the long-run steady-state, the market converges to one of three equilibria:*

(i) *all users join platform A,*

(ii) *all users join platform B,*

(iii) *$x^{**} < \frac{1}{2}$ users join platform A and $1 - x^{**}$ users join platform B. $x^{**} = 0$ at $\beta = (1-\gamma)(v+\theta-1)$ and increases with β.*

Figure 3 shows the three potential steady-state equilibria. Notice first that all three equilibria are independent of D_0. This is because in the steady-state, the initial data-advantage of platform B already depreciated. As the figure and Proposition 3 show,

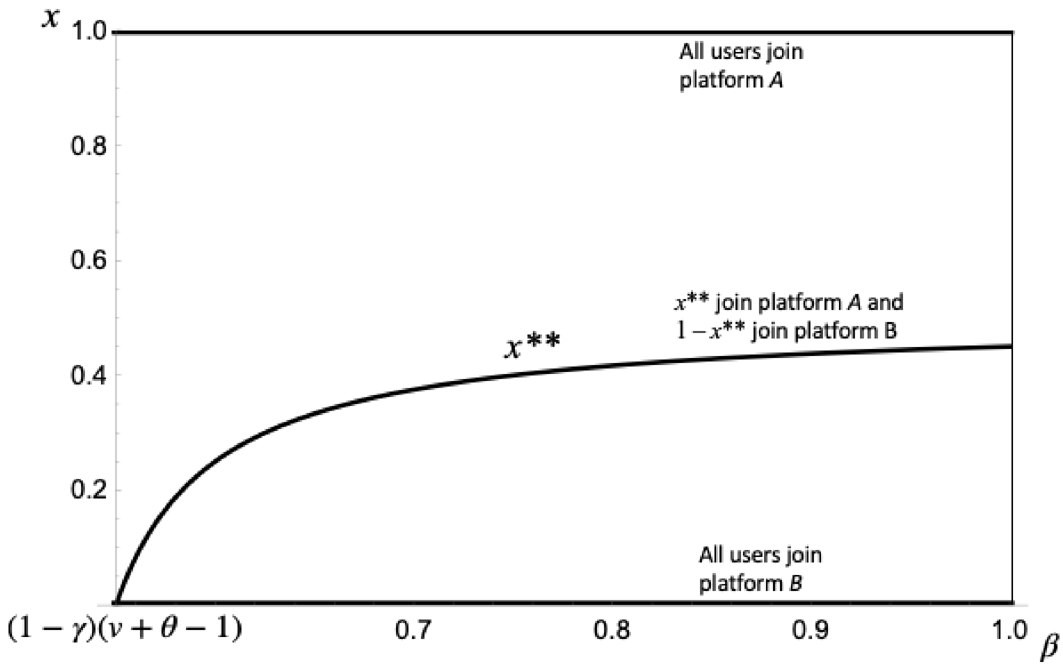

Figure 3: Potential steady-state equilibria ($v = 1.1$, $\gamma = 0.5$, and $\theta = 1$)

when β is high, either platform can dominate the market. Intuitively, as the value of data is high, if a platform accumulated sufficient data, it can dominate the market, regardless of the base quality advantage. Yet, there is also a potential steady-state equilibrium in which the platforms share the market. Perhaps counter intuitive, in this equilibrium the higher-quality platform A has lower market share than the lower-quality platform B. The intuition for this result is that in order to obtain steady-state equilibrium with positive market shares, platform B needs to balance the quality advantage of platform A by consistently accumulating more data, which requires it to capture a larger market share. In this equilibrium, platform A's market share increases with β, because as β increases a smaller amount of data accumulation is needed for platform B to balance the quality advantage of platform A.

We continue our focus on the case in which all three steady-state equilibria are possible, i.e., $(1-\gamma)(v+\theta-1) < \beta < \theta$, and analyze which steady-state equilibrium the market converges to. We note that because $D_0 < \frac{1}{(1-\gamma)}$, $\overline{\beta} > (1-\gamma)(\theta+v-1)$. Moreover, if $\frac{v-1}{\gamma\theta} < D_0$ then $\overline{\beta} < \theta$. This implies that when $\frac{v-1}{\gamma\theta} < D_0 < \frac{1}{(1-\gamma)}$, $\overline{\beta} \in [(1-\gamma)(v+\theta-1), \theta]$.[7] The following proposition connects between the short-run

[7] To see why, we have that $(1-\gamma)(v+\theta-1) < \theta$ if and only if $\frac{v-1}{\gamma\theta} < \frac{1}{(1-\gamma)}$. Furthermore, we have that $\overline{\beta} > (1-\gamma)(v+\theta-1)$ if and only if $D_0 < \frac{1}{1-\gamma}$. Finally, recall that $\overline{\beta} < \theta$ if and only if $D_0 > \frac{v-1}{\gamma\theta}$.

equilibrium of Section 4 and the long-run equilibria derived in this section:

Proposition 4. *Suppose that* $\frac{v-1}{\gamma\theta} < D_0 < \frac{1}{(1-\gamma)}$*, such that* $\overline{\beta} \in [(1-\gamma)(v+\theta-1), \theta]$ *and suppose that* $\beta \in [(1-\gamma)(v+\theta-1), \theta]$*. Evaluated at* $x^{**} = x^*$*:*

(i) *if* $\beta < \overline{\beta}$*, the market converges to the steady state equilibrium in which platform* A *dominates the market,*

(ii) *if* $\beta > \overline{\beta}$*, the market converges to the steady state equilibrium in which platform* B *dominates the market,*

(iii) *if* $\beta = \overline{\beta}$*, the market stays at steady state equilibrium in which* x^{**} *users join platform* A *and* $1 - x^{**}$ *users join platform* B*.*

Platform competition typically has multiple equilibria because of users' coordination: users join the platform they expect others to join. Proposition 4 shows that, in our model, the comparison between β and $\overline{\beta}$ serves as an equilibrium selection mechanism, and shows how the short-run analysis selects the long-run equilibrium.

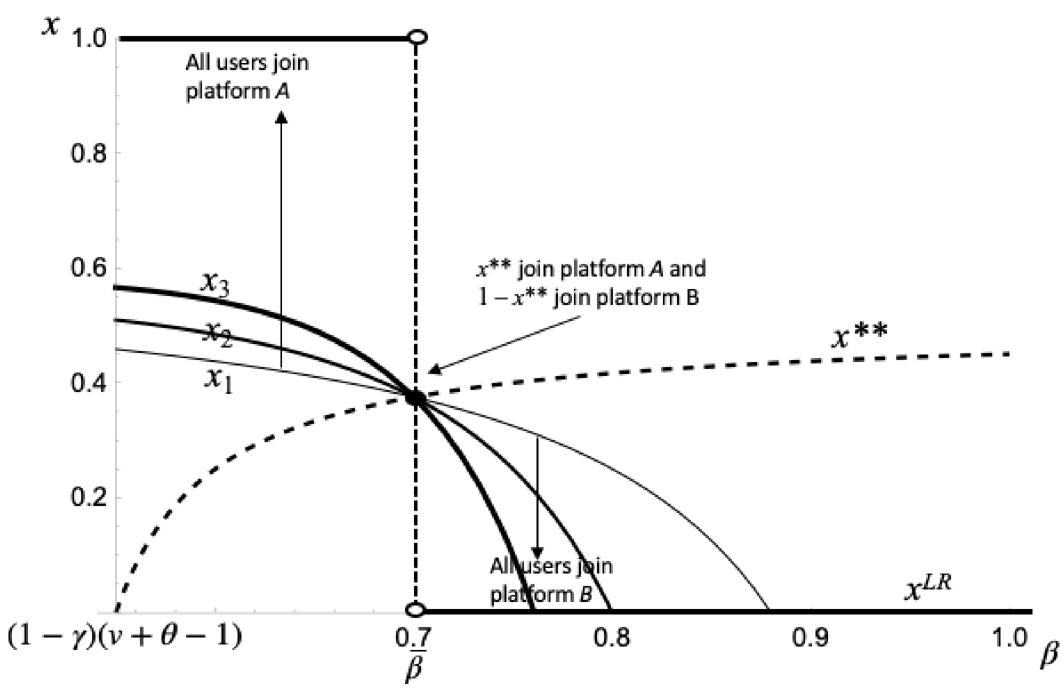

Figure 4: Market share of the two platforms ($v = 1.1$, $\gamma = 0.5$, and $\theta = 1$)

To see the intuition for Proposition 4, let x^{LR} denote the long run steady-state market share of platform A. Figure 4 illustrates the results of Proposition 4 and the equilibrium x^{LR} as a function of β. As the figure shows, all short-run x_t intersect at the same point, in which $\beta = \overline{\beta}$ and $x = x^{**}$. As a result, if $\beta < \overline{\beta}$, the short-run x_t increases over time and therefore in the steady-state platform A dominates the market. Likewise, if $\beta > \overline{\beta}$, the short-run x_t decreases over time and therefore in the steady-state platform B dominates the market. If $\beta = \overline{\beta}$, the short-run x_t is constant over time and therefore both platforms are active in the steady-state, with platform A attracting a smaller market share than platform B.

Next, we analyze the convergence rate to the steady-state equilibrium. As can be seen in Figure 4, when $(1-\gamma)(v+\theta-1) < \beta < \overline{\beta}$, $x_3 - x_2 > x_2 - x_1$. That is, platform A's momentum intensifies over time. Likewise, when $\overline{\beta} < \beta < \theta$, platform B's momentum intensifies over time, such that $x_2 - x_3 > x_1 - x_2$. The following proposition confirms this observation:

Proposition 5. *Suppose that* $\frac{v-1}{\gamma\theta} < D_0 < \frac{1}{(1-\gamma)}$*, such that* $\overline{\beta} \in [(1-\gamma)(v+\theta-1), \theta]$ *and suppose that* $(1-\gamma)(v+\theta-1) < \beta < \theta$*. Then the market converges in an increasing rate to the steady state. That is, for all* t*, when* $\beta < \overline{\beta}$*,* $x_{t+2} - x_{t+1} > x_{t+1} - x_t$ *and when* $\beta > \overline{\beta}$*,* $x_{t+1} - x_{t+2} > x_t - x_{t+1}$*.*

Our results in this subsection focused on the case where $\overline{\beta} \in [(1-\gamma)(v+\theta-1), \theta]$. We conclude this subsection by commenting that if $\overline{\beta} > \theta$, then for all $\beta \in [(1-\gamma)(v+\theta-1), \theta]$, the analysis above indicates that the market converges to the long-run steady state equilibrium in which all users join platform A. Moreover, they do so in an increasing rate.

Low data benefits

We now move to the the case of low benefit from data: $\beta \in [0, (1-\gamma)(v+\theta-1)]$. From our assumption that $D_0 < \frac{1}{1-\gamma}$, $\overline{\beta} > (1-\gamma)(v+\theta-1)$, so in the short-run platform A increases its market share over time. Notice that this holds regardless of whether $\overline{\beta}$ is higher or lower than θ. The following proposition derives the long-run, steady state that the market converges to:

Proposition 6. *Suppose that* $\beta \in [0, (1-\gamma)(v+\theta-1)]$*. Then,*

(i) *if* $\theta < v-1$*, the market converges to a long-run steady state in which platform* A *dominates:* $x^{LR} = 1$*.*

(ii) *if $\theta > v - 1$, the market converges to a long-run steady state in which the two platforms share the market if β is sufficiently low:*

$$x^{LR} = \begin{cases} x^{**}, & \text{if } \beta \in [0, (1-\gamma)(\theta - (v-1))], \\ 1, & \text{if } \beta \in [(1-\gamma)(\theta - (v-1)), (1-\gamma)(v + \theta - 1)], \end{cases}$$

where $\frac{1}{2} < x^{**} < 1$ and increasing in β and decreasing with θ.

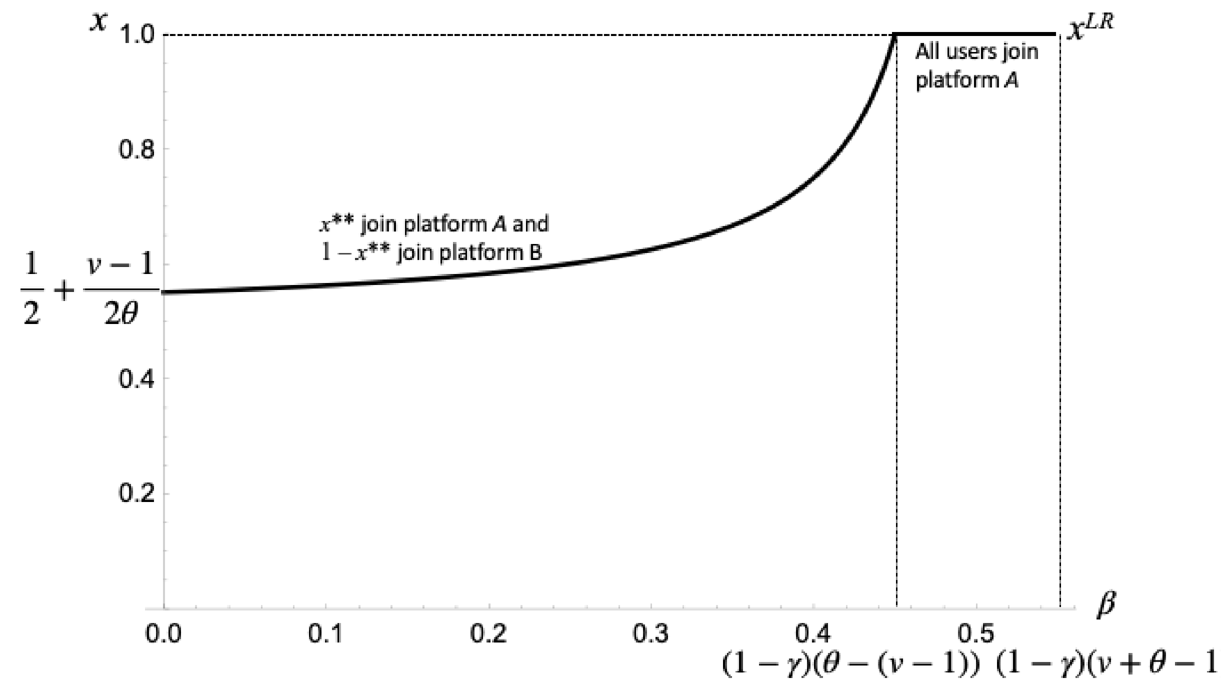

Figure 5: Market share of the two platforms ($v = 1.1$, $\gamma = 0.5$, and $\theta = 1$)

Figure 5 illustrates the second part of Proposition 6. Intuitively, when data accumulation is not very important to users such that β is small, and when product differentiation, θ is high, there is a long-run steady state with two active platforms. In this equilibrium, $x^{LR} > \frac{1}{2}$ because platform A offers a superior base quality. As β increases, platform A's market share increases, because data accumulation becomes more important to users and platform A, who has a higher market share, accumulates more data. Moreover, unlike the case of high β, where differentiation benefits platform A, here, θ decreases platform A's market share. The intuition for this result is that now platform A gains a higher market share than platform B, hence, attracts more distant users. As θ increases, it is more costly for these users to join platform A, and some users switch to platform B.

Finally, we analyze the convergence rate to this long-run steady state equilibrium.

Proposition 7. *Suppose that $\beta \in [0,(1-\gamma)(\theta+(v-1))]$. Then the market converges in a decreasing (increasing) rate to the long-run steady state if $\beta < (1-\gamma)\theta$ ($\beta > (1-\gamma)\theta$).*

Intuitively, when the long-run steady state involves two active platforms (when $0 < \beta < (1-\gamma)(\theta-(v-1))$), the market always converges to this equilibrium in a decreasing rate. In a way, the market approaches this equilibrium, without actually ever reaching it. Yet, when the long-run steady state involves platform A dominating the market, (when $0 < (1-\gamma)(\theta-(v-1)) < \beta <(1-\gamma)(\theta+(v-1))$), there is a threshold, $(1-\gamma)\theta \in [(1-\gamma)(\theta-(v-1)),(1-\gamma)(\theta+(v-1))]$, such that for $\beta < (1-\gamma)\theta$, the market approaches the long-run steady state in which platform A dominates in a decreasing rate, while for $\beta > (1-\gamma)\theta$, the market converges to this equilibrium in an increasing rate.

6 Data commercialization of personal data

The analysis so far assumed that platforms profit from commercializing *anonymous* data and therefore compete to attract market share. In this section, we assume that in additional to commercializing anonymous data, platforms can commercialize personal data, which inflicts a disutility on users.

Suppose that platforms A and B can strategically set the amount of data that they commercialize from each user, d_A and d_B, respectively. Platforms care about the long run so are very patient and want maximize their payoff in the steady-state. We focus on the parameter space in which $\beta \in [(1-\gamma)(v+\theta-1),\theta]$ and $\frac{v-1}{\gamma\theta} < D_0 < \frac{1}{(1-\gamma)}$, such that without data commercialization, in the long-run steady-state platform A wins if $\beta < \overline{\beta}$ and platform B wins otherwise. Platform A and B's profits in the long run from commercializing personal data are therefore $d_A x^{LR}$ and $d_B(1-x^{LR})$, respectively. Because in the relevant parameter space the steady state equilibrium that the market converges to is not affected by t, and because platforms only concern with their long-run profits, it does not matter in which period the two platforms make this decision. For simplicity, we assume that the two platforms make this decision simultaneously.

Given data commercialization d_A and d_B, users' utilities from joining each platform become:

$$U_{A,t} = Q + v - d_A + \beta D_{A,t} - \theta x, \quad U_{B,t} = Q + 1 - d_B + \beta D_{B,t} - \theta(1-x).$$

Therefore, applying our analysis from sections 4 and 5, it is straightforward to show that, there is a threshold,

$$\overline{\beta}(d_A, d_B) = \overline{\beta} + \frac{d_B - d_A}{D_0}, \tag{11}$$

such that platform A dominates in the steady-state if and only if $\beta < \overline{\beta}(d_A, d_B)$, where $\overline{\beta}(d_A, d_B)$ is decreasing in d_A and increasing in d_B.

Platforms compete by setting d_A and d_B, while taking into account that $\overline{\beta}(d_A, d_B)$ determines which platform dominates the market. Given d_B and the value of β, platform A sets the highest d_A possible subject to that it winning the steady state, i.e., $\beta < \overline{\beta}(d_A, d_B)$. At the same time, platform B sets the highest d_B possible subject to $\beta > \overline{\beta}(d_A, d_B)$. Therefore, if $\beta < \overline{\beta}$, the equilibrium strategies are $d_B^* = 0$ and the d_A^* that solves $\beta \leq \overline{\beta}(d_A, 0)$, resulting in platform A dominating the market in the steady state. Likewise, if $\beta > \overline{\beta}$, the equilibrium strategies are $d_A^* = 0$ and the d_B^* that solves $\beta \geq \overline{\beta}(0, d_B)$, resulting in platform B dominating the market. Solving $\beta = \overline{\beta}(d_A, 0)$ and $\beta = \overline{\beta}(0, d_B)$, yields the following equilibrium strategies:

$$d_A^* = \begin{cases} v - 1 - D_0(\beta - (1-\gamma)\theta), & \text{if } \beta < \overline{\beta}, \\ 0, & \text{if } \beta \geq \overline{\beta} \end{cases}$$

$$d_B^* = \begin{cases} 0, & \text{if } \beta \leq \overline{\beta}, \\ D_0(\beta - (1-\gamma)\theta) - (v-1), & \text{if } \beta > \overline{\beta}. \end{cases}$$

The following corollary summarizes this result:

Corollary 2. *Suppose that platforms commercialize personal data to maximize their steady-state profits. Then, the amount of data commercialization is non - monotonic in β: it decreases in β if $\beta < \overline{\beta}$ and increases in it otherwise.*

Interestingly, an increase in the strength of the network effects does not necessarily pushes the platforms to commercialize more data. Intuitively, a platfomr may want to take advantage of an increase in β to increase the data it commercializes, because β increases the users' benefit from data accumulation. Commercializing too much data, however, may negatively affects the platform's ability to win the market. That is, if β is substantially lower than $\overline{\beta}$, platform A would commercialize a high level of personal data as it can do so while still dominating in steady state, so d_A^* is high. As β increases, the gap $\overline{\beta} - \beta$ decreases, implying that platform A must reduce the amount of data it commercializes to ensure it does not affect its intrinsic value to the point that it will not continue to dominating the market. This continues until reaching $d_A^* = 0$ at $\beta = \overline{\beta}$. The opposite case occurs when $\beta > \overline{\beta}$, in which case platform B wins the steady state and increases the amount of data it commercializes as β increases. [8]

Notice that the non-monotonicity follows to other market parameters. In particular, when platform A wins, the amount of data commercialization is increasing in the degree of substitution, θ. This is because recall that substitution works in favor of platform A by making it easier for platform A to dominate the market. For the same reason, when platform B wins, the amount of data commercialization is decreasing in the degree of substitution.

7 Implications

Managerial implications

Short-run market share and long-run dominance

It might be expected that if a platform has a short-run market share above 50%, and data from current users accumulates over time, then this platform is bound to gain momentum, increase its market share further and end up dominating the market. Yet, the results of this paper show that this is not always the case. Consider platform B, whose competitive advantage is in its initial data accumulation. Suppose that β is slightly below $\overline{\beta}$. Because $x^{**} < \frac{1}{2}$, platform B has a market share of above 50%. However, because $\beta < \overline{\beta}$, platform B does not have momentum: it will gradually lose market share until eventually platform A will dominate the market. Therefore, platform B cannot rely on its current superior market share: it will need to take measures that reduce $\overline{\beta}$ (such as increasing its base quality) to prevent platform A from dominating the market.

Notice that the same argument does not apply to platform A, whose competitive advantage is in its superior quality. If platform A benefits from a short-run market share above 50%, it will always (taking other market conditions as given) have momentum and dominate the market. To see why, notice that x_t can be higher than $\frac{1}{2}$ only if $\beta < \overline{\beta}$, in which case platform A has momentum.

[8] At $\beta = \overline{\beta}$, $d_A^* = d_B^* = 0$ and the two platforms share the market. This is an equilibrium because given $d_j^* = 0$, if platform i collects even a slight amount of data, it loses the entire market.

Commercializing data and the threat of losing market dominance

Related to the previous policy implications, suppose that a platform (say, platform A) decides to start commercializing data in order to generate positive revenues. Suppose further that commercializing data inflicts a disutility of users and therefore decreases platform A's quality advantage. This reduces v, which, in turn, reduces $\overline{\beta}$. In the short-run, commercializing data result in losing a certain market share, due to the decrease in v. Yet, in the long-run, if, after data commercialization, it is still the case that $\beta < \overline{\beta}$, platform A still has momentum. Whatever market share lost due to data commercialization, platform A will gain it back in future periods and eventually dominate the market. If, however, the decrease in $\overline{\beta}$ results in $\overline{\beta} < \beta$, then platform A will start losing momentum and eventually loose the entire market.

It is possible to apply this argument for platform B, though recall from the previous policy implication the difference between the two platforms. Suppose, for example, that platform A has, say, 70% of the market, and due to data commercialization, in the short-run its market share drops to 60%. Then, it is always the case that platform A still has momentum and will gain its high market share back. This is not necessarily the case for platform B. Suppose, for example, that platform B has 70% and has momentum, with an increase in market share in each period. Suppose further that once platform B starts commercializing data, in the short-run its market share drops to 60%. Although its market share is still above 50%, it is not necessarily the case that platform B still has momentum. It could be that the decrease in its base quality (equivalent to an increase in v in our model) increases $\overline{\beta}$ from values under which platform B has momentum ($\overline{\beta} < \beta$) to values under which it does not ($\beta < \overline{\beta}$). In other words, for platform B, a market share above 50% does not necessarily place it in a position in which it can start commercializing data.

Increased product differentiation result in entrant's market dominance

It might be expected that as differentiation increases, it is more likely that competing platforms share the market. As a result, incumbents might be less fearful from entry threats as products are more substitutes. Yet, the results of this paper show that as differentiation increases (an increase in θ), $\overline{\beta}$ increases and it is more likely that platform A has momentum and will eventually dominate the market. Consequently, differentiation creates a threat of entry by providing a stronger competitive advantage to entrants. For example, when asking both ChatGPT and Gemini what are the differences between them, the two AI tools highlight differences, stating that each has unique advantages in serving specific needs. According to our model, such differentiation is more likely to benefit the entrant platform.

Policy implications

Before considering the policy implications of our results, notice that in this model, due to market dominance and product differentiation, there are two sources for loss of welfare. First, the equilibrium in which platform B dominates the market provides lower total welfare than the equilibrium in which platform A dominates the market. This is because platform B offers a lower base quality than platform A. Yet, because of product differentiation, the equilibrium in which platform A dominates the market does not maximize welfare. Users with strong preferences towards platform B (i.e., with x close to 1) may prefer to join platform B over platform A. Hence, a second loss of welfare occurs when a platform dominates the market, depriving users of the choice of a competing platform that is closer to their preferences.

Requiring the incumbent to share its initial data accumulation

Consider a policy that requires platform B to share with platform A the data it collected prior to platform A's entry. Only the initial data is shared: no platform is obligated to share data with its competitor in future periods. In our model, this implies that in period $t = 1$, platform B has to share D_0 with platform A, and then the two platforms continue to accumulate their own data, given their market shares. The following proposition shows that this policy solves the first source of loss of welfare:

Proposition 8. *Suppose that $D_{A,t} = D_{B,t}$. Then, platform A has momentum (i.e., $x_{t+1} > x_t$) if and only if $v > 1$.*

Proposition 5, along with our previous results, show that if platform B shares its initial data with platform A, then, the highest quality platform has momentum and will therefore dominate the market in the long-run. This result has three implications for welfare. First, it has been argued that if a high-quality incumbent platform share its data with a new platform, the incumbent will lose a welfare enhancing competitive advantage to a lower-quality platform. Yet, Proposition 8 shows that if platform B offers a superior quality than platform A (which corresponds to $v < 1$), sharing the initial D_0 with platform A will not prevent platform B from gaining momentum and dominating the market. Second, notice that in the long-run, this policy is relevant only if $\beta > \overline{\beta}$. Recall that if $\beta < \overline{\beta}$, platform A gains momentum and dominates the market without this policy, though in the short-run, this policy may expedite the convergence to the steady-state in which platform A wins. Finally, notice that while under this policy, the best platform wins the market, the market is not necessarily efficient. As explained above, welfare can be improved if both platforms are active in the market.

Requiring both platforms to continuously share their data

To show how welfare can be maximized, consider a rather extreme (and perhaps unpractical) policy in which both platforms need to share their data with each other in all periods. In this case, in each period, each platform collects data equals to 1 regardless of its market share. The indifferent user between the two platforms in period t, x_t, is:

$$v-\theta x_t+\beta\left(1+\sum_{\widehat{t}=1}^{t-1}\gamma^{\widehat{t}}\right)=1-\theta(1-x_t)+\beta\left(1+\sum_{\widehat{t}=1}^{t-1}\gamma^{\widehat{t}}\right) \iff x_t=\frac{1}{2}+\frac{v-1}{2\theta}. \quad (12)$$

Because both platforms have access to the same pool of data, β does not affect x_t and therefore x_t is constant over time. Moreover, $\frac{1}{2} < x_t < 1$, where the second inequality follows when $1 < v < 2$ and $\theta > 1$. We therefore have the following result:

Proposition 9. *Consider a policy that requires both platforms to share their data in all periods. Then, in all periods, users with $x < \frac{1}{2}+\frac{v-1}{2\theta}$ join platform A and users with $x > \frac{1}{2}+\frac{v-1}{2\theta}$ join platform B. Compared with the steady state equilibrium in which platform A (B) dominates, in the steady state users with $x > \frac{1}{2}+\frac{v-1}{2\theta}$ ($x < \frac{1}{2}+\frac{v-1}{2\theta}$) are better off following this policy and other users are indifferent.*

Proposition 6 shows that this policy can help the market to solve both types of inefficiencies. Both platforms are active, both have the same set of data and the higher-quality platform A gains a higher market share. Yet, we acknowledge that such a regulation can raise two problems. First, from a legal perspective, this regulation requires platforms to give up on their intellectual property – the data that they collect – in all periods. This is different from a one-time sharing of data from the incumbent to the entrant as in the previous subsection. Second, sharing data in a continuous manner may raise problems of data security and privacy that our paper does not model. Consequently, this policy should be interpreted as a benchmark case of what policy can enhance welfare in an ideal, unconstrained, market.

8 Conclusion

In the competitive landscape of digital platforms, does a data-rich incumbent hold an indisputable advantage against a higher-quality entrant? This paper demonstrates that this is not the case. The evolution of market dominance is not predetermined by initial conditions but hinges on a crucial, and often overlooked, trade-off: the economic value users derive from accumulated data versus the intrinsic quality of the service.

Our central finding is the existence of a sharp threshold in the user benefit from data (β). This parameter determines the market's trajectory. When this benefit is low, the entrant's superior quality provides a powerful engine for growth, allowing it to gain momentum and ultimately capture the market—even from an initial position of market share inferiority. Conversely, when the data benefit is high, the compounding force of the incumbent's data accumulation creates a gravitational pull that is too strong for a higher-quality entrant to escape.

This framework yields counter-intuitive results with strategic implications. First, current market share can be a poor predictor of long-run viability. An incumbent enjoying a majority share may already be on a path to obsolescence. Second, and perhaps more surprisingly, greater product differentiation does not insulate the incumbent. Instead, it amplifies the entrant's quality advantage among its core user base, making an eventual market takeover more, not less, likely.

These dynamics offer a clear warning for dominant platforms: a decision to monetize that degrades service quality can initiate a fatal loss of momentum, even if market share remains high in the short run. For policymakers, our analysis highlights the starkly different outcomes of regulatory interventions. A one-time mandated data transfer can level the playing field to ensure the best platform wins, but a more radical policy of continuous data pooling could sustain a duopoly, preserving user choice and potentially enhancing total welfare.

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Appendix A

Below are the proofs for all propositions in the text.

Proof of Proposition 1:

We first show that regardless of t, $x_{t+2} > x_{t+1}$ if and only if $x_{t+1} > x_t$. Hence, if a platform has momentum in a certain period, it has momentum in all periods. To this end, x_{t+2} is the solution to:

$$v - \theta x_{t+2} + \beta\left(x_{t+2} + \gamma x_{t+1} + \gamma^2 x_t + \gamma^3 D_{A,t-1}\right) \tag{13}$$

$$= 1 - \theta(1 - x_{t+2}) + \beta\left(1 - x_{t+2} + \gamma(1 - x_{t+1}) + \gamma^2(1 - x_t) + \gamma^3 D_{B,t-1}\right).$$

Substituting x_{t+1} and x_t from (4) and (2), yields:

$$x_{t+2} = \frac{1}{2} + \frac{(v-1)\left(\theta^2 + \beta^2(1-\gamma) - \beta\theta(2-\gamma-\gamma^2)\right)}{2(\theta-\beta)^3} - \frac{\beta\gamma^3\theta^2(D_{B,t-1} - D_{A,t-1})}{2(\theta-\beta)^3}. \tag{14}$$

We have that:

$$x_{t+1} - x_t = \frac{\beta\gamma}{2(\theta-\beta)^2}\left[v - 1 - (D_{B,t-1} - D_{A,t-1})(\beta - \theta(1-\gamma))\right], \tag{15}$$

and:

$$x_{t+2} - x_{t+1} = \frac{\beta\gamma^2\theta}{2(\theta-\beta)^3}\left[v - 1 - (D_{B,t-1} - D_{A,t-1})(\beta - \theta(1-\gamma))\right]. \tag{16}$$

The terms in the squared brackets in (16) and (15) are identical. Recalling that $\theta > \beta$, we have that given any $D_{B,t-1}$ and $D_{A,t-1}$, $x_{t+2} > x_{t+1}$ if and only if $x_{t+1} > x_t$.

Next, we turn to derive $\overline{\beta}$. As the result above holds for any history (i.e., any $D_{B,t-1}$ and $D_{A,t-1}$), there is no loss of generality in solving for $\overline{\beta}$ that ensures that $x_2 - x_1 > 0$. In $t = 1$,$D_{B,0} = D_0$ and $D_{A,0} = 0$. Substituting $D_{B,t-1} = D_0$ and $D_{A,t-1} = 0$ into the term squared brackets in (15) yields that $v - 1 - D_0(\beta - \theta(1-\gamma)) > 0$ if and only if $\beta < \overline{\beta}$.

Finally, notice that it is always the case that $\overline{\beta} > 0$. Moreover, $\overline{\beta} < \theta$ if and only if: $\frac{v-1}{\gamma\theta} < D_0$. As we also require that $D_0 < \frac{1}{(1-\gamma)}$, we have that $\overline{\beta} < \theta$ if and only if $\frac{v-1}{\theta\gamma} < D_0 < \frac{1}{(1-\gamma)}$. It is possible to show that $\frac{v-1}{\theta\gamma} < \frac{1}{(1-\gamma)}$ if and only if: $\frac{v-1}{v-1+\theta} < \gamma < 1$.

Proof of Proposition 2:

Because evaluated at $\beta = \overline{\beta}$, $x_t = x_{t+1}$ for all t, there is no loss of generality in evaluating x^* at $t = 1$. Substituting $D_{B,t-1} = D_0$ and $D_{A,t-1} = 0$ into x_t in (2), we have that x_1 is:

$$x_1 = \frac{1}{2} + \frac{v-1}{2(\theta-\beta)} - \frac{\beta\gamma D_0}{2(\theta-\beta)}. \tag{17}$$

Evaluating x_1 at $\beta = \overline{\beta}$ yields $x^* = \frac{1}{2} - \frac{1}{2}D_0(1-\gamma)$. Because $D_0 < \frac{1}{(1-\gamma)}$, $x^* > 0$, and because $\gamma < 1$ and $D_0 \geq 0$, $x^* < \frac{1}{2}$.

Proof of Proposition 3:

There is a steady state equilibrium in which all users join platform A if (9) holds, or: $\beta > (1-\gamma)(\theta - (v-1))$. Likewise, there is a steady state equilibrium in which all users join platform B if (10) holds, or: $\beta > (1-\gamma)(\theta + (v-1))$. Because $v - 1 > 0$, we have that $(1-\gamma)(\theta + (v-1)) >(1-\gamma)(\theta - (v-1))$, so if $(1-\gamma)(\theta + (v-1)) < \beta < \theta$, both equilibria holds. Notice that $(1-\gamma)(\theta + (v-1)) < \theta$ if and only if $\frac{v-1}{v-1+\theta} < \gamma < 1$ and recall from the proof of Proposition 1 that in this case $\frac{v-1}{\theta\gamma} < \frac{1}{(1-\gamma)}$.

Next, consider the steady-state equilibrium with two active platforms. Straightforward calculation shows that $x^{**} = 0$ at $\beta = (1-\gamma)(\theta + (v-1))$. Moreover, x^{**} is increasing in β. Evaluating x^{**} at $\beta = \theta$ yields $x^{**} \mid_{\beta=\theta} = \frac{1}{2} - \frac{(1-\gamma)(v-1)}{2\gamma\theta} < \frac{1}{2}$.

Proof of Proposition 4:

Substituting $\beta = \frac{v-1}{D_0} + \theta(1-\gamma)$ into x^{**} in (8) yields $x^{**}|_{\beta=\overline{\beta}} = \frac{1}{2} - \frac{1}{2}D_0(1-\gamma) = x^*$. As $x_{t+1} > x_t$ if and only if $\beta < \overline{\beta}$ and $x_{t+1} = x_t$ if $\beta = \overline{\beta}$, we have that for $\beta < \overline{\beta}$, platform's A's market share expands over time until reaching the steady-state where it dominates the market. The opposite case occurs when $\beta > \overline{\beta}$. When $\beta = \overline{\beta}$, $x^{**} = x^*$ and the market remains stable over time at $x_t = x^{**}$.

Proof of Proposition 5:

Given any time t, we have from the proof of Proposition 1 (and using $x_{t+2} - x_{t+1}$ in (16) and $x_{t+1} - x_t$ in (15)) that:

$$x_{t+2} - x_{t+1} - (x_{t+1} - x_t) =$$

$$\frac{\beta\gamma(\beta - (1-\gamma)\theta)}{2(\theta-\beta)^3}\left[v - 1 - (D_{B,t-1} - D_{A,t-1})(\beta - \theta(1-\gamma))\right] \tag{18}$$

Suppose that $\beta < \overline{\beta}$, such that the term in the squared brackets of (18) is positive. The gap $x_{t+2} - x_{t+1} - (x_{t+1} - x_t)$ is positive if $\beta > (1-\gamma)\theta$, which holds because $\beta > (1-\gamma)(\theta + (v-1))$. Suppose now that $\beta > \overline{\beta}$, such that the term in the squared brackets of (18) is negative. Then, because $\beta > (1-\gamma)\theta$, $x_{t+2} - x_{t+1} - (x_{t+1} - x_t) < 0$, implying that $x_{t+1} - x_{t+2} > x_t - x_{t+1}$.

Proof of Proposition 6:
From the proof of Proposition 3, there is no steady state in which platform B dominates, and there is a steady state in which platform A dominates if $\beta > (1-\gamma)(\theta - (v-1))$. If $(1-\gamma)(\theta-(v-1)) < 0$ (such that $\theta < v-1$), this equilibrium holds for all $\beta > 0$. If $(1-\gamma)(\theta-(v-1)) > 0$ (such that $\theta > v-1$), this equilibrium holds only for $\beta > (1-\gamma)(\theta-(v-1)) > 0$. For $\beta \in [0, (1-\gamma)(\theta-(v-1))]$, there is a unique equilibrium with $x^{LR} = x^{**}$, where x^{**} is given by (8). It is straightforward to see that $x^{**}|_{\beta=0} = \frac{1}{2} + \frac{v-1}{2\theta} < 1$ where the inequality follows if and only if $\theta > v-1$. Moreover, $x^{**}|_{\beta=0} > \frac{1}{2}$ because $v > 1$. As β increases, x^{**} increases and $x^{**}|_{\beta=(1-\gamma)(\theta-(v-1))} = 1$. Moreover, x^{**} is decreasing with θ because:

$$\frac{\partial x^{**}}{\partial \theta} = -\frac{(1-\gamma)^2(v-1)}{2(\theta(1-\gamma)-\beta)^2} < 0.$$

Finally, to show that the market converges to this x^{LR}, we need to show that $x_1 < x^{**}$ (as recall that for $\beta < \overline{\beta}$, x_t increases over time). To this end, the gap:

$$x^{**} - x_1 = \frac{\beta\gamma(v-1)}{2(\theta-\beta)(\theta(1-\gamma)-\beta)} + \frac{\beta\gamma D_0}{2(\theta-\beta)} > 0, \tag{19}$$

where the the first term is positive because $v > 1$ and because $\beta < (1-\gamma)(\theta-(v-1))$ implies that $\beta < \theta(1-\gamma)$ and the second term is positive because $\theta > \beta$.

Proof of Proposition 7:
The proof follows directly from the first part of the proof of Proposition 5.

Proof of Proposition 8:
Substituting $D_{A,t-1} = D_{B,t-1}$ into the gap $x_{t+1} - x_t$ in (15) yields $x_{t+1} - x_t = \frac{\beta\gamma}{2(\theta-\beta)^2}[v-1]$, which is positive if and only if $v > 1$.

Proof of Proposition 9:
The result that under this policy, users with $x < \frac{1}{2} + \frac{v-1}{2\theta}$ join platform A and users with $x > \frac{1}{2} + \frac{v-1}{2\theta}$ join platform B follows directly from the text. Moving to part (i), we have that under this policy, in the steady state a user of type x that joins platform A gains a utility of $Q + v - \theta x + \beta$, which is the same utility that the user gains in the

steady state in which all users join platform A. A user that joins platform B under this policy (a user with $x > \frac{1}{2} + \frac{v-1}{2\theta}$) gains a utility of $Q + 1 - \theta(1 - x) + \beta > Q + v - \theta x + \beta$, where the inequality follows because $x > \frac{1}{2} + \frac{v-1}{2\theta}$. Hence, this use benefits from the policy. Part (ii) is equivalent.